Strategic Equilibria in Centralized Electricity Markets

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Introduction

Competitive electricity markets in which the participants individually submit their preferred schedules, and subsequently the IGO centrally sets the trading solution based on those schedules, have been put in practice. This market model is usually known as PoolCo and the PJM Interconnection is a specific example of its implementation [1]. The trading solution set by the IGO aims usually at short-term efficiency, which may be measured by the social welfare or surplus [2], and incorporates rules to enforce feasibility. We define as feasible a trading solution that respects both the operational and physical constraints of the electric system. An OPF (Optimal Power Flow) tool, which maximizes the participants' welfare - based on their bids and offers - subject to system constraints, seeks both short-term efficiency and feasibility.

Participants in a market are expected to behave rationally, where rationality may be translated into the attempt to maximize individual profit. In the PoolCo model, players may do that by untruthfully revealing their cost/benefit in their offer/bid curves [3].

Given specific market rules, it is of great interest to be able to model the strategic behavior (or gaming) of the market participants and to identify solutions of those games, which we modeled as static non-cooperative continuous-kernel games under complete information [4],[5]. Non-cooperative means that each individual player is pursuing his or her own interests, and continuous-kernel stands for the fact that each player has at his or her disposal a continuum of choices and also that the utility functions are continuous. The game is static due to the fact that the process of decision-making is simultaneous for all players involved. It is implicit that the game is not repeated. Complete information means that every player knows his or her own costs/benefits – and those of all the other players in the market – and that constitutes common knowledge. This is believed to be true for producers operating in electricity markets, for which a good estimate of costs may be obtained from the daily spot price of fuels (natural gas or other) and from the technological capacity and corresponding heat rates of the power plants [6]. Consumers, on the other hand, have a weak responsiveness to price changes due, in great extent, to technological restrictions. This fact may be translated into limited ability to game the market. We will, therefore, focus primarily on the supply-side gaming.

The solutions prescribed by the proposed game are the so-called Nash equilibria, either in pure or in mixed strategies. They constitute the strategically stable or self-enforcing points in the bidding space from which the players have no incentive to deviate. Stability may be alternatively defined with respect to the readjustment scheme employed by the players when searching for the equilibrium points.

The paper addresses the problem of how to find multiple equilibria in PoolCo model where the line constrains have been considered. It makes use of a modified version of a previously proposed readjustment scheme (or algorithm) for a single equilibrium problem and shows a systematical approach for solving the multiple equilibria problem[7][8].

Modeling strategic behavior in electricity markets is of special importance if opportunities for market power abuse based on gaming by participants are to be identified. The present situation in the Californian market enforces the feeling that carefully designed market rules cannot be overrated. Furthermore, the goals of a competitive market may be improved by acknowledging the players' opportunities to game, by correctly identifying them, and consequently either preventing those opportunities or controlling their consequences. Therefore, this work is not specially aimed at providing players with tools to pursue their objectives, but, in contrary, to clarify in which context gaming opportunities may take place.

The paper has four more sections and one appendix. In section two we revisit the algorithm that identifies a Nash equilibrium in a PoolCo model. Section three illustrates the possibility of existence of multiple of those equilibria and describes the proposed procedure to systematically find them. The method is exemplified in section four through a simple test system. In section five we draw some conclusions about the proposed method. In the appendix we give the data and the results for the example presented in section four.

The Individual Welfare Maximization

The original Individual Welfare Maximization (IWM) algorithm assumes that the IGO runs a centralized economic dispatch subject to the system constraints (OPF) [7][8]. This OPF, which uses bids and offers freely submitted by the participants, sets the nodal prices (Lagrangian multipliers) that are used to charge/pay consumption/generation on every node of the grid [9]. The participants game by untruthfully revealing their costs/benefits on their offer/bid curves or schedules and they may do so by continuously changing one or more parameters of the marginal cost/benefit curves that they submit to the IGO. Which parameter or parameters they change is not relevant. What is important is the state or operational equilibrium that is attained by changing those parameters. We assume that the players have reasonable estimates about all participant's true costs/benefits and we also assume that these estimates constitute common knowledge among players; moreover, that the players are rational also constitutes common knowledge. The schedules submitted by the players are valid for one time period of the market, typically one hour. Therefore, they need to optimize their bid/offer for a specific time period, which is viewed as a snapshot in time.

Each player in the market may find the equilibrium points through his or her own choice of the parameters of the reported schedule and by mimicking the other players' choices. This is possible from the rationality and the complete information assumptions. Hence, the equilibria found by each player alone will match those found by the other players. This constitutes the main strength of the Nash equilibrium concept.

The individual welfare maximization problem is cast as a nested optimization problem. The inner problem is the OPF, and the outer problem is the optimization of the individuals' utility functions subject to the OPF solutions. We denote by $f_p()$ the utility function of player p. The vectors of generation and load controlled by player p are denoted by $\mathbf{P}_p$ and $\mathbf{D}_p$, respectively. Each player p controls a vector of reported variables that is represented by $\boldsymbol{\alpha}_p$. The nodal prices applied to the generation and load controlled by player p are a byproduct of the OPF and appear as $\boldsymbol{\lambda}_p$. The cost and benefit functions of each generator and load are denoted by C_i and B_i, respectively. $\mathcal{G}$ represents the set of generators and $\mathcal{D}$ represents the set of loads. The cost and benefit function are assumed to be well described by quadratic functions.

$$C_i(P_i) = a_{P,i}.P_i^2 + b_{P,i}.P_i + c_{P,i}\ , \quad i \in \mathcal{G} \tag{1}$$

and

$$B_i(D_i) = a_{D,i}.D_i^2 + b_{D,i}.D_i + c_{D,i}\ , \quad i \in \mathcal{D} \tag{2}$$

In this context, α may, for example, replace the true cost/benefit variable a in the reported marginal curve. The equality and inequality constraints are represented by $\mathbf{g}()$ and $\mathbf{h}()$, respectively, where $\mathbf{P}$ is the vector of all generated power, $\mathbf{D}$ is the vector of all loads, and $\mathbf{x}$ represents the vector of state variables. The individual welfare maximization problem may be thus described as follows.

$$\begin{aligned} \max_{\boldsymbol{\alpha}_p} \quad & f_p(\mathbf{P}_p, \mathbf{D}_p, \boldsymbol{\lambda}_p) \\ \text{s.t.} \quad & (\mathbf{P}_p, \mathbf{D}_p, \boldsymbol{\lambda}_p) \text{ are determined by} \\ & \left(\begin{array}{ll} \max\limits_{\mathbf{x},\mathbf{P},\mathbf{D}} & \sum\limits_{i\in\mathcal{D}} B_i(D_i, \alpha_{D,i}) - \sum\limits_{i\in\mathcal{G}} C_i(P_i, \alpha_{P,i}) \\ \text{s.t.} & \mathbf{h}(\mathbf{x},\mathbf{P},\mathbf{D}) = \mathbf{0} \\ & \mathbf{g}(\mathbf{x},\mathbf{P},\mathbf{D}) \leq \mathbf{0} \end{array} \right) \end{aligned} \tag{3}$$

The utility function of each player p is given by the difference between the sum of benefits minus charges and the sum of payments minus costs that results from the set of his or her controlled generators and loads. We write it below, where the fixed components have been dropped.

$$\begin{aligned} f_p(\mathbf{P}_p, \mathbf{D}_p, \boldsymbol{\lambda}_p) = & \sum_{i\in\mathcal{D}_p} [B_i(D_i, a_{D,i}, b_{D,i}) - \lambda_i.D_i] \quad + \\ & \sum_{i\in\mathcal{G}_p} [\lambda_i.P_i - C_i(P_i, a_{P,i}, b_{P,i})] \\ = & \mathbf{D}_p{}^T.\mathbf{A}_{D,p}.\mathbf{D}_p + \mathbf{B}_{D,p}{}^T.\mathbf{D}_p \quad - \\ & \boldsymbol{\lambda}_{D,p}^T.\mathbf{D}_p + \boldsymbol{\lambda}_{P,p}^T.\mathbf{P}_p \quad - \\ & \mathbf{P}_p{}^T.\mathbf{A}_{P,p}.\mathbf{P}_p + \mathbf{B}_{P,p}{}^T.\mathbf{P}_p \end{aligned} \tag{4}$$

The quadratic parameters are represented in the diagonal elements of matrices $\mathbf{A}_{D,p}$ and $\mathbf{A}_{P,p}$. The linear parameters appear in $\mathbf{B}_{D,p}$ and $\mathbf{B}_{P,p}$. $\mathcal{G}_p$ and $\mathcal{D}_p$ are the set of generators and the set of loads, respectively, controlled by player p.

In order to determine the best set of bid/offer parameters, the players – whose set is denoted by $\mathcal{P}$ – readjust $\boldsymbol{\alpha}_p$ on each iteration k by using Newton's method, until a stationary point is reached.

$$\boldsymbol{\alpha}_p^{(k+1)} = \boldsymbol{\alpha}_p^{(k)} - (\nabla^2_{\boldsymbol{\alpha}_p} f_p)^{-1}\big|_k . \nabla_{\boldsymbol{\alpha}_p} f_p)\big|_k\ , \quad \forall p \in \mathcal{P} \tag{5}$$

If a stationary point is found, then it is stable with respect to the selected readjustment scheme and it constitutes, by definition, a Nash equilibrium.

The readjustment schemes presented so far in the literature propose that it be done for each unit (generator or load) of each player at a time [7],[3]. Although this made sense in a discontinuous kernel – for it avoids cycling when near the solution – it is not necessary for the IWM readjustment scheme. So, the change proposed to the original IWM is to let all the players change all their bids'/offers' reported parameters on a single iteration. So, instead of optimizing the parameters for each unit of each player at a time before proceeding to the next one, a complete Newton-step is performed on each individual utility function – in order to optimize all his or her reported parameters – for each readjustment step. This change accelerates the algorithm because the readjustment evolves using all the dimensions of the bidding space, instead of using only one variable controlled by a single player. The Gradient and the Hessian of the individual utility functions may be obtained from the Newton-based OPF solutions, and the proposed change implies using the cross derivatives of each individual utility function with respect to the control variables of the corresponding player.

Another part of the original algorithm that is neglected in this work is the step-size selection – originally used to prevent crossing regions of the bidding space where the utility functions are not differentiable due to the system constraints [7]. As we will see in the next section, this problem is circumvented by enforcing some of the system constraints and relaxing others, ensuring, in this way, differentiable utility functions in every specified region of the bidding space.

Searching for Multiple Equilibria

In the search for multiple equilibria we consider the problem in which only the generators game. We do this because this is the problem faced by present-day electricity markets. One should, however, consider the following characteristic of equilibria: if both generators and loads game, and two of them happen to be in the same bus, this may give origin to a continuum of equilibria. This is so because the load and the generator would work as substitutes with respect to each other.

The proposed search method

If the cost functions of the generators are convex and the constraints are linear, the problem is well behaved inside each region defined by the linear constraints. This implies the use of D.C. power flow equations, which, of course, implies some simplification. For example, the California ISO uses a D.C. model to allocate transmission during market re-dispatch [10]. The differentiability of the utility functions breaks down only when linear constraints – for example, line constraints – are hit. Therefore, if we are able to start the IWM algorithm with an initial solution inside each region defined by the constraints, then the convex functions and the linear constraints guarantee convergence if a solution exists in the region [5]. In case the algorithm crosses the boundaries of the region, it means that there is no equilibrium in the specified region. However, initializing the algorithm inside a specific region is not easy. Instead, we run the IWM considering the linear constraints defining each region as equality constraints and we relax the remaining inequalities so they are never hit.

In Figs. 6-9 we show the contour of the utility function – resulting from the OPF solution – of player 1 in the 4-bus test system used in example of section 4, as he or she changes the parameters of the two generators under his or her control. In Figs. 6 and 7 the limit on line 1-3 is modeled as an inequality constraint, whereas in Figs. 8 and 9 this limit is changed to an equality constraint. Fig. 6 of the appendix shows two equilibrium points: one when the line constraint is binding (or active) and another one when this constraint is not binding (or inactive). The observation of the figures indicates that, if one equilibrium exists for the original problem with the constraint active, then it will also exist for the modified problem wherein the line is forced to operate exactly at that limit (Figs. 6 and 7) and the remaining line limits are relaxed. Yet, that equilibrium will only appear in the modified problem if it does not exist in the original problem (Figs. 8 and 9). This gives rise to the necessity of checking the solutions of the modified problem back in the original problem.

In summary, we proceed in the following way: the IWM is run for each region of the bidding space by forcing the correspondent lines to operate at their limits and relaxing the remaining ones; if an equilibrium is found, then the IWM is run for the original problem, using the possible equilibrium as starting point, for the for the purpose of checking its existence. The biggest drawback resulting from this procedure is, of course, the necessity of checking all the possible feasible regions given by the feasible combinations of constraints. As a result of that, the problem is of combinatorial nature and the number of possibilities may render a non-manageable problem. In the next subsections we give some insights on how to overcome this difficulty.

Feasible cases

A feasible case is one that is supported by a power flow solution. Since we assume D.C. flows on lines, the feasible solutions may be easily checked by running a LP (linear programming) problem.

The algorithm to find multiple equilibria must, therefore, be run only for the feasible cases (or regions) in the bidding space. The unfeasible regions are never reached and we need not to consider them. So, prior to initiating the search, we build a database of feasible cases corresponding to the feasible combinations of binding constraints. In case we take into consideration only the line constraints, we have a database comprising the cases associated with feasible regions of the bidding space, and this database may be represented as $\mathcal{C}_i = \{c_1, c_2, ..., c_m\}$, where $c_i = \{0, 1, -1\}$, and m is the number of lines in the system. The triplet c_i allows us to represent each line to be either uncongested, or congested in each of the two possible directions.

Problem size reduction

Due to the non-polynomial nature of the problem – $\mathcal{O}(3^m)$, if only the line constraints are taken into account – it is impossible to manage any real life examples in the absence of any form of reducing its size. Fortunately, the LP program, together with careful propagation and pruning, allows the elimination of a substantially number of cases. In the tree of Fig. 1 it is shown the sequence in which the cases must be generated in order to allow pruning. Every new child case is generated from the parent case by including more congested lines that are represented further to the right, until the last represented line is reached. This simple procedure avoids repetition of cases. Then, for each new level of the tree, all cases are tested, but only the feasible ones are propagated to the next level. The unfeasible ones are pruned (eliminated and not propagated) and the cases that are feasible for a configuration further down in the tree are eliminated but still propagated. For real system where the players have the ability to congest a reduced number of lines, this method may prove very efficient.
Secondly, in real systems, the operators know from their experience which ones are the lines that, under certain operating conditions, are likely to be congested. In the California

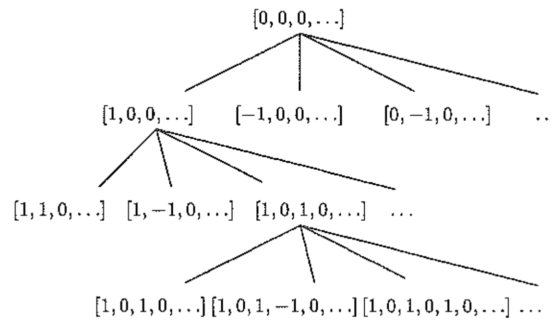

Fig. 1. Generation of cases

ISO, for example, this knowledge is used by the system operators to define the inter-zonal interfaces [10]. The market participants may now use this same knowledge in order to avoid running an incommensurable number of cases. In addition, once the history of congested lines has been established no other cases beyond the ones inferred from that history need to be investigated.

Mixed-strategy equilibria

The method, as it was proposed, is suitable for finding equilibria in pure strategies. For mixed strategies, the algorithm needs to be modified and that is object of current research. First, the algorithm must be allowed the crossing of regions in the strategy space. It is the discontinuities caused by the system constraints that might cause the reaction curves of players not to cross and, therefore, give origin to the equilibria in mixed strategies [7]. Secondly, this would require the algorithm to be initialized precisely in the specified regions, without relaxing any of the remaining constraints.

The equilibria in mixed strategies may be identified because they make the proposed algorithm cycle back and forth across a given constraint (or constraints). The solution for a mixed-strategy equilibrium is the one that maximizes the expected profit of all players, for strategies with assigned probabilities, in the spanned regions. The expected profit, e_p, for player p may be written as in (6), where $f_p^r()$ denotes the profit of player p in region r and $\mathcal{S}$ denotes the set of regions spanned by the equilibrium. The probability with which a given strategy α_p^r is played is denoted by y_p^r. The maximum number of players is denoted by P.

$$\begin{cases} e_p = \sum_{r \in \mathcal{S}} \left[f_p^r(\alpha_1^r, \ldots, \alpha_p^r, \ldots, \alpha_P^r) \cdot \prod_{p \in \mathcal{P}} y_p^r \right] \\ \sum_{r \in \mathcal{S}} y_p^r = 1 \,, \quad \forall p \in \mathcal{P} \end{cases} \tag{6}$$

Example

We ran the proposed method on the 4-bus test system whose configuration can be seen in Figs. 2-5 of the appendix. Table I contains the line reactances. The producers and consumers participating in this small market are assumed to be well characterized by their cost and benefit curves as in (1) and (2), whose parameters' values are showed in Table II and Table III. Table IV describes the ownership of the generators.

Because the test system has 4 lines, we could expect a maximum of 81 feasible regions. In fact, there exists a power flow solution for only 27 of those regions and, hence, a database of only those cases is built prior to initiating the search.

In this experiment only the generators game and, if both of the players run the search algorithm, then they will find the equilibrium solutions of Table V. Upon confirmation, only equilibriums 1-4 survive, and those are illustrated in Figs. 2-5 of the appendix. There we can observe which lines are congested for each equilibrium operating point. Equilibrium 5 does not survive confirmation and, if we allowed the confirmation algorithm to proceed, it would end up being attracted to a cycle that is nothing else than a mixed-strategy equilibrium that spans regions $\{0, 1, 0, 0\}$ and $\{0, 0, 1, 0\}$.

In Table VI we show the profits attained by the players in the different pure-strategy equilibria that were found using the search algorithm. It is easy to verify that the first three equilibria provide lower revenues for both the players than the revenues obtained in the fourth equilibrium. In other words, the first three equilibria are strictly dominated by the fourth equilibrium and could be thus eliminated. This is not always the case – and might not be the case were we in possession of the solutions in mixed strategies – and the players may be left in the end with more than one equilibrium point that is not eliminated by any dominating strategy to choose from.

In Figs. 6-9 of the appendix we illustrate how the utility function of player 1 may change when the limit on line 1-3 is varied. It is possible to observe that one of the equilibrium points vanishes when the limit on the line is sufficiently high (Fig. 8). This fact is not apparent in the modified problem wherein the limit is enforced as an equality (Fig. 9). In the making of Figs. 6-9, the price schedule parameters of generators 2 and 4 controlled by player 2 were fixed to $\hat{a}_2 = 0.089$ and $\hat{a}_4 = 0.181$, respectively.

Conclusions

The work presented in this paper addresses the problem of multiple strategic equilibria that constitute the natural operating solutions in the bidding space for players participating in centralized electricity markets. Moreover, the paper shows that under specified assumptions it is possible to employ a systematic procedure to find those multiple equilibria.

When the players are unaware of the set of lines that can be congested by gaming, they must be able to tell the feasible cases from the non-feasible ones. We showed a simple method, based on careful propagation and pruning, that helps building a meaningful database.

In the absence of any straightforward rule to eliminate equilibrium solutions, as elimination of dominated strategies, the players are left with a multitude of answers for the gaming problem. Future research on the paper's topic will have necessarily to address this problem.

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Biographies

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Acknowledgments

This work was supported by the Science and Technology Foundation of the Portuguese Ministry of Science and Technology, under the scholarship PRAXIS XX1/BD/13801/97, and by the National Science Foundation through grant ECS 00-80279.

Appendix

TABLE I
Test System Line Parameters.

Line	Reactance ($p.u.$)	Maximum flow ($p.u.$)
1-2	0.790	1.87
1-3	0.116	1.81
2-4	0.122	2.11
3-4	1.030	1.93

Base MVA = 100, Base kV = 345

TABLE II
Price Schedules' Parameters

Generator	Parameters	
	a_P ($\$/MW^2h$)	b_P ($\$/MWh$)
1	0.05	15.0
2	0.05	6.0
3	0.10	1.0
4	0.10	1.0

TABLE III
Value Schedules' Parameters

Load	Parameters	
	a_D ($\$/MW^2h$)	b_D ($\$/MWh$)
1	-0.10	80.0
2	-0.10	80.0
3	-0.50	440.0
4	-0.50	440.0

TABLE IV
Generator's Ownership

Player	Generator			
	1	2	3	4
1	√		√	
2		√		√

TABLE V

REPORTED PARAMETERS FOR EACH EQUILIBRIUM POINT

Eq.	Parameter			
	$\hat{a}_1$ ($\$/MW^2h$)	$\hat{a}_2$ ($\$/MW^2h$)	$\hat{a}_3$ ($\$/MW^2h$)	$\hat{a}_4$ ($\$/MW^2h$)
1	0.0931	0.0883	0.1552	0.1662
2	0.0787	0.0922	0.1975	0.1783
3	0.0965	0.0868	0.1638	0.1748
4	0.0826	0.0906	0.2063	0.1879
5	*0.0000*	*0.0000*	*0.2696*	*0.2697*

TABLE VI

PLAYERS' PROFITS FOR EACH EQUILIBRIUM POINT ($\$/h$)

Equilibrium	Player	
	1	2
1	16,514	20,226
2	17,303	21,040
3	16,628	20,694
4	17,523	21,579

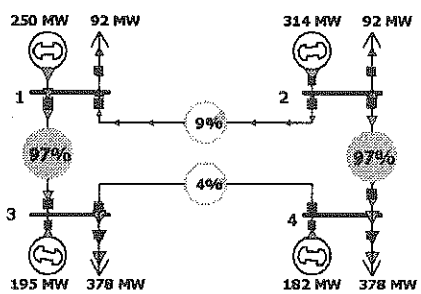

Fig. 2. Equilibrium 1

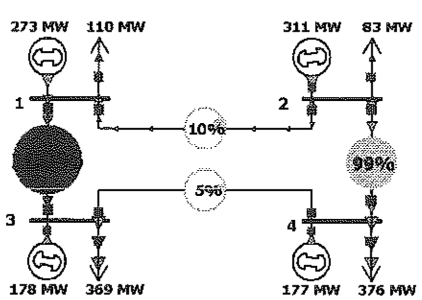

Fig. 3. Equilibrium 2

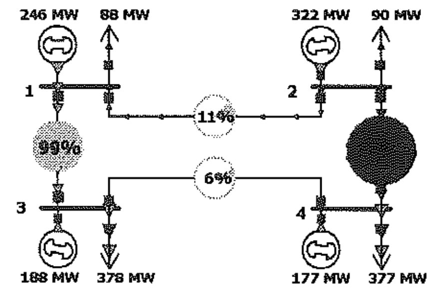

Fig. 4. Equilibrium 3

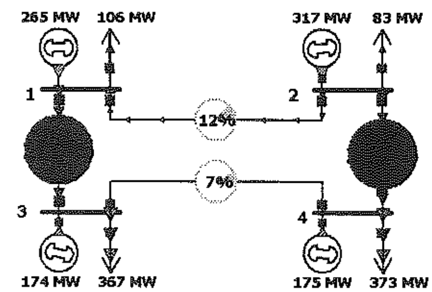

Fig. 5. Equilibrium 4

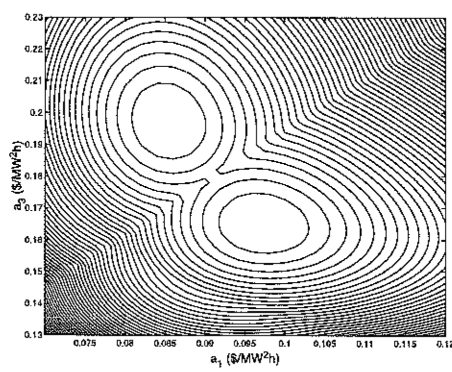

Fig. 6. Player 1 utility function; $P_{13} \leq 195MW$

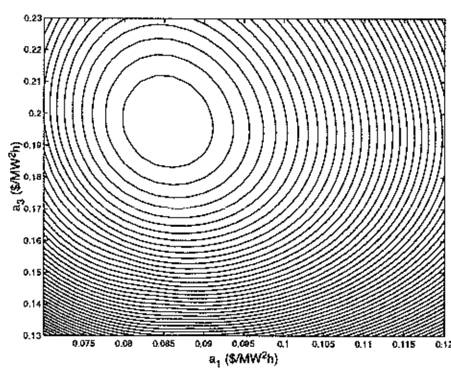

Fig. 7. Player 1 utility function; $P_{13} = 195MW$

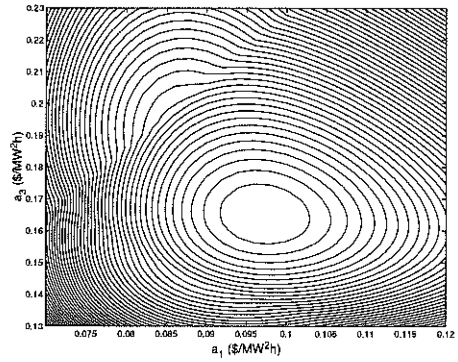

Fig. 8. Player 1 utility function; $P_{13} \leq 210MW$

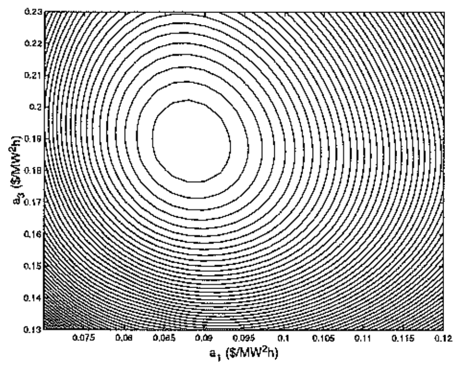

Fig. 9. Player 1 utility function; $P_{13} = 210MW$