

Bulk Power System Voltage Phenomena



Voltage Stability Security and Control

22-26 August 1994

Davos, Switzerland

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BULK POWER SYSTEM VOLTAGE PHENOMENA III
VOLTAGE STABILITY, SECURITY, AND CONTROL

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PREFACE

A first International Seminar on Bulk Power System Phenomena was held under the auspices of the Engineering Foundation, the National Science Foundation, and the Electric Power Research Institute in Potosi, Missouri during 19-24 September 1988. The Proceedings of that Seminar were published by the Electric Power Research Institute in January 1989, as EL-6183. A second was held under the auspices of the National Science Foundation and of ECC, Inc. in Deep Creek Lake, Maryland during 4-7 August 1991, and the Proceedings were published by ECC, Inc.

As was stated in the Preface to the second Seminar, concern with voltage related operational problems on power systems, as potential causes of system disruption, has continued unabated. Since then, increased attention is being paid to control of voltage dynamics, as suggested by the expanded title of the Seminar, and initial applications of research findings are now appearing in the field. As before, this Seminar was planned to present and examine continuing advances in understanding of large scale power system voltage phenomena that have been achieved since 1991, and to stimulate further research and development on algorithmic tools adequate to the burgeoning needs of the industry.

The Seminar was organized into sessions addressing recent results in the phenomenology of voltage collapse, utility response to the threat of voltage collapse, methodologies of security analysis, analytical methods, load modeling, voltage control, and simulation of voltage phenomena.

As in the case of the first Seminar, organization and participation has been a delight. Responses to the announcement of the Seminar and to the call for papers was enthusiastic and gratifying. The support of Dr. Vijay Vittal during his tenure at the National Science Foundation, was critical to the success of the Seminar. Special thanks are due to the authors who agreed to serve as session chairmen, and as editors of the discussion periods. Most important of all, of course, is the contribution by the authors of the results of their investigations.

IN MEMORIAM - YASUO TAMURA

This Proceedings is dedicated to the memory of Professor Yasuo Tamura, who died on 22 November 1993 of a heart attack, in Tokyo, Japan, at the age of 62. The community of power system engineers owes him much gratitude for his contributions.

Professor Tamura was born on 21 September 1931. He studied electrical engineering at Waseda University, Tokyo, Japan, and received the BE, ME, and PhD degrees in 1953, 1955, and 1961, respectively, from Waseda University. He joined the faculty of electrical engineering of Waseda University in 1955, and became a full professor in 1969. He recognized the importance of computers for power system studies at the beginning of his career, and studied computers at the University of Michigan in 1957-58. His research interests were in the field of power system engineering, in particular, applications of computer and control technologies to power systems.

The contributions of Professor Tamura are well known to power system engineers. He authored more than 150 technical papers.



Professor Yasuo Tamura

He was world renowned, especially in the field of voltage stability. In this field, he was the first person who raised the idea of the relationship between voltage instability and multiple load flow solutions. Based on this idea, he developed the voltage stability index called VIPI (Voltage Instability Proximity Index), for which he defined an angle between the present and the critical operating conditions. The idea and the index are widely cited in publications. He was deeply involved in activities of the IEEE PES Working Group on Voltage Stability and made significant contributions in this field. He was also active in the field of AI applications to power systems, chairing Task Force 38.01.07 on Expert Systems for Power System Analysis and Techniques in CIGRE, the technical committee on Expert Systems Applications to Power Systems in Japan, and the 2nd International Forum on ANNPS (Artificial Neural Network Applications to Power Systems) held in Yokohama, Japan in 1993.

Professor Tamura was elected a Fellow of the IEEE in 1982. He contributed to power system engineering education in China, and became Honorary Professor of Wuhan Institute of Hydraulic and Electrical Engineering in 1985. In Belgium, he received the George Montefiore award in 1985. He received the prestigious Electric Power Award from the IEE of Japan in 1986, for his distinguished contributions in power system engineering, and was Vice President of the IEE of Japan from 1987 to 1989.

He was a very hard working and responsible man. He devoted much time to his students, many of whom have become outstanding in the field of power system engineering research. To his students, he once said that his hobby was wandering between dreams and reality. We can no longer see him wandering between dreams and reality, but his spirit remains in our hearts. Professor Tamura was not only a distinguished scholar, but also a very fine gentleman, and was well respected by his colleagues and students.

- *Shinichi Iwamoto*

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Bulk Power System Voltage Phenomena - III
Voltage Stability, Security & Control
22-26 August 1944, Davos, Switzerland

Introduction

"Concerning Voltage Control"
Lester H. Fink
ECC, Inc.

Concerning Voltage Control

Lester H. Fink
ECC, Inc.

Background

This is the sixth in an informal, somewhat sporadic series of conferences, the first of which was held at South Berwick, Maine, in July of 1972. It is also the second to be held in Davos, the first having been in September/October of 1979. Finally, it is the third on the topic of Bulk Power System Voltage Phenomena, the first of these having been held at Potosi, Missouri, in 1988, and the second at Deep Creek Lake, Maryland, in 1991.

The Berwick conference resulted from a fortuitous misunderstanding. I was participating in an Engineering Foundation Conference on underground transmission research. In a casual conversation with the late Prof. Simpson Linke of Cornell University, I suggested it would be good to have a similar conference on power system control. Sam, who at that time was temporarily at the National Science Foundation, thought I was requesting an NSF grant to support such a conference, and responded enthusiastically that he would be glad to provide such support (in my innocence, I had not known such support was possible). Among the attendees who attended the resulting conference at Berwick were Jacques Carpentier of Electricité de France, and the late Giorgio Quazza of ENEL. A few of the others are in attendance here.

The second in the series, at Henniker, New Hampshire, in August 1975, initiated the program in Systems Engineering for Power, for which I was responsible during my time with the U.S. Government, and the third, here at Davos in 1979, presented the final reports on three-year research projects in two (out of the sixteen) areas funded by that program: Emergency Power System Control, and Organizational Forms for Large Scale Systems. From the first of these came the modern work on energy methods applied to stability analysis, which was later supported by EPRI, and has now been reduced to practice. From the second came the work on asymptotic methods, notable singular perturbation techniques for power system model reduction, which also has seen significant applications. The content of the proceedings for the more recent Voltage Phenomena seminars provide reason to expect that similar substantive results will follow from the work reported. It is noteworthy that one person, Professor Zaborsky of Washington University, has made very significant contributions to every one of these six conferences.

Trends Affecting Voltage Control

At the time of the Potosi Seminar, six years ago, it was suggested that voltage phenomena were not yet well understood, and that much work and reflection would be necessary before they were. By the time of the second Seminar, a much clearer picture was emerging. Now, ongoing work by many investigators, while not a close-harmony chorus, suggests that a commonly accepted basic understanding is taking shape. A significant movement of interest from theory to practice is reflected in the mix of papers that will be presented at this meeting. By rough count, 40% of the 1991 papers dealt with theory, 60% with applications, while for this meeting, 20% deal with theory, 65% with applications, and 15% with utility practices. This shift of emphasis is welcome.

However, it may be worth reviewing the present situation in the industry to make sure that all bases are indeed covered.

To that end, I will make a few rather commonplace observations, followed by some implications that do not seem to be commonly accepted. I am raising questions, not suggesting solutions; the questions might not be well-posed, hence not permitting of clean cut solutions.

The day of massive building of bulk power transmission is over. For the foreseeable future - pending, for instance, development and deployment of superconducting transmission technology - additional transmission capabilities often will be eked out by upgrading, reconductoring, etc. At the same time, barring catastrophic developments, load will continue to increase, so that systems will be subject to increasingly heavy loading. This means that the importance of voltage phenomena in determining system security and performance will continue to increase, and that system operation will be increasingly vulnerable to voltage disturbances. An obvious conclusion is that the control of voltage will become an increasingly important factor in system security control.

Other trends, some old and some newer, are affecting the voltage control problem. There has been a long term trend toward higher power factors in generators. Although it dates back to the period when the control of system voltage levels was not a significant problem, it has been driven by economics - high power factor machines are less expensive, other things being equal - and is not likely to be reversed. More recently, the development of static VAR compensators, and more recently the promise of static condensers, is providing prospective, and more effective, alternative means for controlling voltage profiles. Another practice that is beginning to emerge is that of compensating inductive loads at the distribution level, so that the bulk power system is operated at nearly unity power factor [1].

Other, institutional, trends are significant. At the end of its first century, the electric energy utility industry, worldwide, is in the midst of an unprecedented restructuring, even a reconceptualization. This radical process has by no means run its course, and no one can say with certainty what will be the corporate characteristics of the industry even twenty years hence. To those who have been active in the industry since mid-century, this is an astounding fact. Twenty years ago, it was commonplace to reflect that, because of the glacial rate at which changes propagated throughout the industry, the capital intensive plant which characterized it, and the long lead times required to plan, acquire, and install new plant - because of all this, then ten years hence the industry would be pretty much as it already was!

All this has changed, and perhaps the one thing that is certain, despite the radical changes that have already occurred, is that twenty years hence the industry will be very different from what it is now. The best that we can do to anticipate the changes to come, is to reflect on current trends, and extrapolate them in light of influences likely to be exerted by the interests and potential incentives of various parties that will be involved.

We are familiar with the traditional utility structure: government regulated, vertically integrated utilities, obligated to serve a native service area. At present, after a process of change that has gathered momentum over the past decade or so, this concept is an endangered species. Around the world, the various elements of the traditional utility are differentiating, and are being disaggregated. Services are being

"unbundled." The newly independent, or potentially independent, elements, where they are not achieving independent status, are being shuffled (and reshuffled) into a variety of new patterns, almost as in a kaleidoscope.

Some have suggested that six prominent elements may be distilled from the traditional vertically integrated utility: (A) generation, (B) transmission, (C) distribution, (D) system planning and operations (others would distinguish these two), (E) bulk power markets, and (F) retail sales. The variety of ways in which these elements may be shuffled may be illustrated by the current (fluid) situation in four countries: Chile, North Ireland, the Netherlands, and Norway. In Chile, five entities are currently identified, essentially each of the above except that (D) and (E) are combined. In North Ireland, this is reduced to four entities by the additional combination of (B) and (C). In the Netherlands, while (A) is separated, (B), (D), and (E) are combined into a second element, and (C) and (F) into a third. Finally, in Norway, the six elements are further subdivided before being reassembled into three entities: (i) (A), (C), (D), (F), and *regional* (B); (ii) local (B) and pool (E); (iii) bilateral (E).

In order to assess what kind of a more common pattern might emerge after the present experiments in restructuring have run their course, it is prudent to consider what parochial interests have been unleashed, and what responsibilities may be identified with those interests. Elsewhere [2], I have suggested that long term implications of current trends in the industry could result in a very different control structure and control objectives. Briefly, it seems likely to me that, should the emerging industry structure run its course, AGC as it has been known will no longer be much used. My inference is based on consideration of the implicit incentives - and disincentives - that will be built into the new structure.

Historically, even within the familiar, vertically integrated utility, plant operators (and plant superintendents) have resisted and even sabotaged regulation (e.g. by placing units on "load limit," or countenancing widening governor deadbands). This has been a major factor in poor system performance for many years. To this writer's personal knowledge, one major utility (not the one with which he was affiliated) avoided contributing regulation to its pool for many years by maintaining wide governor deadbands. Again, it was noted in a recent paper [3] that "it is not uncommon to operate a 75 psig H_2 machine at 60 psig H_2 . Such operation reduces the windage losses in the synchronous generator... There is also less hydrogen consumption at the lower H_2 pressure... Lower hydrogen consumption results in the purchase of less hydrogen which reduces the operating cost of the synchronous generator." The significance of this is that a machine's reactive capability, which is extremely important to voltage security, is a direct function of hydrogen pressure.

With these factors weighing so heavily in a conventional vertically integrated utility structure, it is hard to imagine that they will not become overwhelming, even as generating plants become profit centers, not to mention when they are subsumed within an independent production company! Plant owners and operators will not want their units "jerked around." Even if they were willing to let their plants be regulated by a third party, it would not be easy to assign clear responsibility for components of the regulating requirement (dealing with sums of random processes), and few distributors would be willing to pay for some one else's problem.

Hence, with independent production (generation), transmission, and distribution entities, a likely scenario will be for production (by generating companies) and sale (to distribution companies, or even power brokers) of blocks of energy under contracts similar to fuel contracts, with transmission companies responsible for control of the network, and distribution companies responsible for whatever remains of regulation as it is now practiced. Regulation would thus be-

come a distribution responsibility, because of the impossibility of assigning responsibility and credit for necessary elements of regulation at the bulk system level. It might be handled by the installation of regulating generation at the distribution level, or by some type of direct or indirect load control (e.g., on-line price signals). Network control (i.e. what remains of the traditional AGC) will focus on control of the distribution of power (flows), of voltage, and of network security; provision of necessary reactive sources will be embedded in network capital costs.

System Voltage Profile Control

Voltage Control vis-a-vis AGC

Over the past half-century, conventional AGC has become a mature body of practice. EMS specifications are built around vendors' standard offerings, and the trend toward open systems with "advanced applications" selected from "boutique" software offerings presupposes a basic maturity.

The same cannot be said, however, about system voltage control. A number of reasons for this disparity may be cited. First, until relatively recently, voltage phenomena were not a major issue: systems were overbuilt to a degree that resulted in lightly loaded transmission, and generally were either transient-stability limited or thermal-load limited. Second, reactive support is limited in range, thus permitting of local control, and for many years voltage control meant local control. Control of voltage levels on the bulk power network was for many years treated almost casually. When I was involved in developing functional specifications for the first digital-computer-based AGC, operators spent hours "chasing VAr's around the system" to correct perceived imbalances, but a suggestion to provide automatic voltage control was rejected on the grounds that operators needed that exercise to develop and maintain a feel for the system.

Such practice is no longer adequate. Optimal power flows are being invoked to provide operator guidance, and power electronics are being utilized to provide more responsive control means. Nevertheless, one facet of the overall voltage control problem seems to be overlooked. Given the increasing vulnerability of systems to voltage disturbances, system voltage profiles are an increasingly important factor in determining system security levels, yet little if any attention seems to be given to consideration of just what determines the proper voltage profile for a system.

Automatic Control

Automatic control of bulk power network voltage was not universally ignored. By 1968, Kyushu Electric Power Company had augmented their AGC with automatic control of system voltage [4].

Hierarchical Regimes

Given the restricted range of effectiveness of reactive injections for affecting voltage levels, voltage control is even more suited to hierarchical implementation than is power control. The hierarchical nature of the voltage control problem has long been recognized by theorists [4,5,6], and been implemented in practice. The need for system-wide coordination of the control of voltage was discussed by Bertigny et al. [7] in the 1972 Session of CIGRE. This paper described a hierarchical structure being developed by Electricité de France, utilizing "pilot points" to control voltage levels within "zones." This scheme has since been used effectively in France, Italy [8], and elsewhere, but is now being replaced within EdF by a more advanced regime [9] necessitated by decreased sparsity of the French network. Thus far, however, this praxis has received only theoretical attention in the United States.

Objective Functions

One of the most important elements of an effective control system is the objective function which it is designed to satisfy. If we agree that the voltage profile is a determining factor in the performance of a power system, then it would seem that proper definition of the "optimal" voltage profile should be prerequisite to design of tertiary level system voltage control. Initially, however, control of voltage was discussed within the context of overall economic system control, as a generalization of economic dispatch [10,11], predating concerns with security control. This objective has generally dominated discussion of the topic to date.

The broader question has not been entirely neglected, of course, having been discussed in the literature some years ago [12,13,14]. Capasso et al. [13] stated the problem as "determination of the reactive control variables in such way that the system is in the best state to cope with perturbances (load increase, generator or line tripping) while still complying with specific constraints (minimum and maximum voltages, minimum and maximum reactive power injections). In other words, the system must have available the maximum reactive power margins, and these margins must be distributed in an appropriate way among the generators." Despite the fundamental significance of the problem discussed by the paper, it appeared only as an unpublished Conference Paper, and I am unaware of any subsequent citations other than the subsequent paper [14] having a common author, C.Sabelli. (At the same meeting, Aldrich et al. [15] mentioned in passing that, by optimal voltage scheduling, "Power system security is improved through better utilization of reactive resources by allowing greater reactive reserves to be made available for emergency conditions.")

Capasso et al. [13] seek to ensure the availability of reactive power margins on all generators, and utilize a forecast of a schedule of reactive control variables. To achieve these objectives, they evaluate the performance of four objective functions on two types of networks. The objective functions considered are: (i) minimization of the sum of the absolute values of the reactive injections from the generators; (ii) Minimization of the weighted reactive injection; (iii) minimization of active transmission losses; (iv) minimization of the deviation of voltages from preselected values. They conclude, inter alia, that the second of these objectives is most effective in ensuring a desirable distribution of reserves.

Obviously, a wider variety of objective functions could be considered. Possible objective functions in addition to those evaluated by Capasso et al. might include, e.g. minimum VAR flow, and concentration of VAR reserves in most vulnerable areas of system. Equally obviously, the objectives of maximum reserve margins and minimum active power losses are in conflict - the latter requires a maximum voltage level, and the former a minimum voltage level. Two questions, whose answers are not obvious, are thus posed: (i) how should the tradeoff between economy and security be resolved? and (ii) what distribution of reactive reserves provides the best overall security, and under what conditions?

Conclusions

This "reflection" obviously has not reached a conclusion. A summary of its somewhat rambling content might be that, while the industry is undergoing unprecedented reconfiguration, and voltage control is emerging as an increasingly important problem, that function, overall, has not yet received serious attention. This is not to disparage the significant, even brilliant work that has been done in unfolding the fundamental nature and characteristics of voltage phenomena, nor the significant achievements of the industry in providing effective on-line control and stable system operation. It is meant, however, to pose a challenge to determine what more effective and efficient operation might be achieved by serious

reflection on the entire problem, now that we are achieving some grasp of its important elements.

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Bulk Power System Voltage Phenomena - III
Voltage Stability, Security & Control
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Session 1

Survey I

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Voltage Stability and Collapse - A Review of Basic Phenomena and Methods of Assessment

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Abstract

Voltage instability and collapse have to be ascribed to the inability of a power system to satisfy the load, in principle any load. Early investigations have made it clear that an excess of reactive load is essentially responsible for voltage stability problems and thus the emphasis on the strong correlation between voltage and reactive load has been maintained. A static model is able to explain the basic phenomena and indicators can be derived whenever a load flow solution is available. For better understanding a few basic definitions are introduced which classify the problems according to time frames. Among the methods of analysis eigenvalues and singular values are explained which seem to be the most effective tools for characterizing the state of a system. However, the developments show that a more detailed dynamic analysis with improved models, limitations, generator dynamic, etc. is of interest. Additional effects are mentioned which are due to the angular movement of generators. As these tendencies persist there will be no difference between a dynamic analysis for voltage stability and the conventional transient stability analysis. Hence, a separate approach to voltage stability analysis will become superfluous.

Introduction

As future transmission of electric power over medium and long distances has to rely on existing right-of-way and circuits are utilized to a high degree of loading the maintenance of an acceptable voltage profile in the overall system during normal conditions and after being subjected to a disturbance is becoming of prime importance. Serious incidents in large power systems, which had to be ascribed to a failure of mastering the voltage problem, have given an indication of how disastrous it can be if a system is prone to conditions and disturbances which lead to a loss of voltage. This experience and subsequent analytical studies have created and termed the problem area of voltage stability and voltage collapse.

Early investigations and findings about a strong correlation between reactive power and voltage as well as about effects which could be explained by the static load flow model have justified the approach of limiting all considerations to a steady state model and primarily to reactive power and voltage. Working groups and task forces of engineering societies and institutions have devoted their work strictly to this problem and numerous reports, papers and monographs [1-4] have been published. This conference has also followed this path and has chosen "Voltage Stability and Security" in 1991 [5] as the key topic.

This emphasis on voltage and reactive power has certainly aided the understanding of the phenomena and has produced a series of means and methods for the analysis and for the assessment of the stability problem. However, when the recent publications in this domain are examined more thoroughly it becomes quite clear and obvious that the models and methods have been extended and enriched such that the original voltage stability problem can still be seen as the central theme but that the whole methodology must be termed as "general stability analysis" and is strongly related to the well-known transient stability analysis or long-term stability analysis. There are additions as far as modelling is concerned and extensions of methods but what is being done today is essentially the same as done in conventional stability analysis.

Although this opinion is given as a preview of one of the conclusions of this paper the material presented will deal with voltage stability and collapse. It will give a brief outline of the basic mechanisms and phenomena of and associated with the voltage problem and will concentrate on up-to-date methods of stability assessment. Thereby it will be necessary to repeat a few definitions and concepts which have been lacking in a clear and concise form in the past. This, in addition, should help towards a thorough understanding of the problem.

The interested reader is referred to the wealth of published material which is partly referenced at the end of the paper, in particular the special publications by IEEE, CIGRE and EPES [1-5, 11].

Basic mechanism and phenomena

The problem as treated over the years [6-10] has been strongly oriented towards reactive power and its repercussions on voltage. In essence, however, it is a problem associated with the inability of the system to satisfy the load - any load. Due to the dominance of the reactive component of the system impedance over the resistive part reactive load and its variation are considered responsible for any difficulties.

Hence, drawing power - active and reactive - from a multi-node power system is limited by a simple non-linear relation which is essentially quadratic when the active and reactive power components, P and Q , are considered constant, i.e. independent of the voltage magnitude at the node from where the power is drawn.

If, for the moment, the rest of the nodal loads are neglected and a Thévenin equivalent consisting of $Z = R + jX$ (equivalent impedance) and E (equivalent source voltage) is formed then the nodal voltage V at the particular bus is governed by

$$V^2 - E \cdot V = (P + jQ)(R + jX)$$

The right-hand side being a complex number can be replaced by

$$(P + jQ)(R + jX) = a + jb$$

and thus the voltage magnitude is given by

$$V = \sqrt{\frac{E^2}{4} + a} \pm \sqrt{\frac{E^4}{4} + aE^2 - b^2}$$

A real solution for V exists only if the expression

$$\frac{E^4}{4} + aE^2 - b^2 > 0$$

and thus it can be taken as a stability criterion in a static sense for the voltage.

In [8] this criterion has been extended such that the reaction of the other PQ-loads to changes in voltage are included. This is achieved by correcting the complex load of a particular node by adding a complex power derived from the nodal impedance matrix and the

other $n-1$ nodes. This complex power S_j at node j has to be formed by

$$S_j = P_j + jQ_j + S_j^{corr}$$

where

$$S_j^{corr} = V_j \left(\sum_{\substack{i \in \alpha \\ i \neq j}} \frac{Z_{ji}^*}{Z_{ij}^*} \cdot \frac{S_i}{V_i} \right)$$

$S_j, S_j^{corr}, V_j, V_i, Z_{ji}, Z_{ij}$ are complex quantities.

Using this correction an indicator L_j for each node can be formed

$$L_j = \left| 1 + \frac{E_j}{V_j} \right|$$

which must be smaller than one for the voltage magnitude to have a real solution. The maximum of L_j becomes the stability criterion for the system.

Such a criterion obviously provides the information whether a solution to a particular load demand exists or where the limits are.

Approaching the limit is conceptually an easy matter as visualized by the so-called nose curve as used in many explanations of the voltage stability problem. However, the numerical evaluation of the limits in terms of loads and voltages is a difficult matter. In any case the value of the max. (L_j) will give an indication of how far away the particular solution from the limit is which is also called the bifurcation point.

Another way of assessing and explaining the voltage stability problem is given by examining the Jacobian of the load flow problem. Since it is recognized that voltage stability or instability is related to the existence or non-existence of a load flow solution this property must inherently be given by the Jacobian, i.e. by the eigenvalue or singular values of it. This has been shown by early investigations [9-10] and has been further worked out in more recent results.

Again, however, a load flow solution has to be available for the formation of the Jacobian. Then the minimum positive eigenvalue or the minimum singular value contains the relevant information as to the distance to the bifurcation point. However, its size will give a poor indication as to the actual distance.

Other tools have been developed for the assessment of the voltage state [6, 7, 12] which are all based on the ordinary load flow and as tests have shown they are able to come up with similar results [11].

The central point of having a solved load flow is not only a numerical burden but also a principal drawback when it comes to the determination of the point of collapse. The ordinary load flow suffers from the fact that the basic gradient technique (Newton - Raphson) fails near the bifurcation point and sophisticated measures have to be applied to come near it. An elegant method is given in [13] where the nonlinear problem is replaced by successive linear problems and whereby difficulties of convergence are avoided.

Up to this point in the historical development the basic mechanism as being the inability of the power system to satisfy the load requirement has become more clear and understandable.

For the understanding of the evolution of real incidents in actual systems additional features, models and actions had to be included. One is the initiating event which kicks off the generation of a depressed voltage and finally a collapse. According to the records this is always a switching operation, the loss of a transmission circuit or the loss of generation. Thereby the active power balance is not necessarily upset or may be restored fairly quickly. However, the change in load flow causes a change in the reactive balance and results in lower voltage levels. This is probably the main reason for the correlation with the reactive demand and supply.

Once the voltage is depressed other mechanisms become active the most important being the load dependence on voltage and the restoration of subtransmission voltage levels by OLTC (on-load-tap-changers). Another factor is the reaction of induction motors to a reduction of voltage which causes a sudden increase in reactive load, i.e. within seconds. OLTC's act much slower and result in noticeable changes after 30 seconds or more. Thermostatically controlled heat loads have a similar effect. When the distribution voltage drops the controlled heaters will draw less power at first, however, the temperature control system will restore the load after minutes, more precisely, the average of the heater load.

These various types of load reactions are quite important for the assessment of the voltage behavior and for its stability over longer time periods. Thereby it is obvious that one component tends to restore the active and reactive load irrespective of the voltage magnitude whereas another component shows a certain dependence on the voltage.

Definitions

From these introductory remarks it will be clear that the voltage behavior has to be considered dependent on the time frame and the models in use which are determined by the effects the analyst is looking for.

Hence, for better understanding and communication definitions have been introduced [4] which are being accepted as time goes on. A few are given below.

Transient voltage stability: Transient voltage stability or collapse is characterized by a large disturbance and a rapid response of the power system and its components, e.g. induction motors. The time frame is one to several seconds which is also a period in which automatic control devices at generators react.

Longer-term voltage stability: Longer-term voltage stability or collapse is characterized by a large disturbance and subsequent process of load restoration or load change of long duration. The time frame is within 0.5 - 30 minutes.

In both of these regimes it is conceivable to proceed by using a dynamic model or by a steady state (static) model. However, the literature is not quite clear as to the implications of dynamic and static. These terms and definitions are used in a somewhat pragmatic way and are related to the method of analysis. Anyway, a characterization is given here which will allow a rough classification of the models and methods.

Static model and analysis: Static refers to the basic load flow model, static loads and static representations of generators or compensation. Static does not mean constant, i.e. a static analysis can very well consider a time variation of a parameter. However, a static model does not involve differential equations and their solution in the time domain.

Dynamic model and analysis: A dynamic model comprises all the features of a static model but involves differential equations, e.g. representing regulators, governors and angular movements of generators.

Analytical methods for stability assessment

Static analysis

General

Since the early period when the voltage stability problem was recognized as such load flow analysis has been considered as the main tool. No particular effort was made to model the loads or generation in the beginning. PQ- and PV-nodes were standard. Using this model indicators, multiple solutions, eigenvalues and singular values, etc. were derived. Computing exercises employing various methods on a small test network have shown that the results obtained agree quite well. From today's viewpoint it seems that the basic load flow model and the methods mentioned have been treated extensively.

Indicators and modal analysis

The idea of an indicator comes quite naturally when the problem is basically understood and, of course, it is also initiated by the classical stability criteria. One example of an indicator has already been given in the section on basic mechanisms, others can be found in the literature [7, 9, 10, 12, 19], e.g. multiple solutions of the load flow. In contrast, it has to be emphasized that the magnitude of the nodal voltage does not give an indication of the proximity of a collapse.

More recently, modal analysis and singular values have evolved [14, 15, 20] as tools for generating an indication of the risk of a collapse. The advantage of these methods is their ability to show a more transparent insight into the system and various mutual relationships. Thus, it will be worthwhile exploring the main features of these methods.

The starting point is the iterative step of a load flow solution where the incremental update of the nodal voltage is carried out by a functional matrix, namely the Jacobian matrix J which can be separated into several submatrices related to the subvectors of active and reactive nodal powers, as well as to voltage magnitudes of PV-nodes. If PQ-nodes are considered only in order to keep the analysis simple the following matrix relation is obtained

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{PV} & J_{P\Theta} \\ J_{QV} & J_{Q\Theta} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \Theta \end{bmatrix}$$

Based on the reasoning that the iterative steps will converge to a stable solution if the matrix is non-singular one can take the eigenvalues of J , in particular the smallest positive eigenvalue as an indication of the proximity of a collapse. Numerical routines are readily available to work out eigenvalues and eigenvectors allowing the transformation

$$\Delta P_T = X^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = X^{-1} \begin{bmatrix} J_{PV} & J_{P\Theta} \\ J_{QV} & J_{Q\Theta} \end{bmatrix} X \cdot X^{-1} \begin{bmatrix} \Delta V \\ \Delta \Theta \end{bmatrix} = \Lambda \Delta W$$

where Λ = diagonal matrix of eigenvalues
 X = matrix of eigenvectors
 ΔW = transformed vector of voltages/angles
 ΔP_T = transformed nodal powers

Since a small eigenvalue has the effect that a small variation in ΔP_T will result in a large deviation in a voltage or an angle the eigenvectors take on the function of sensitivity parameters. Actually, the row in X^{-1} related to the smallest eigenvalue contains these parameters, i.e. weights, saying which nodal power has the most important effect on a voltage variation. Hence, eigenvalues and eigenvectors represent comprehensive information about critical nodes and most sensitive changes.

The important prerequisite for the application of modal analysis is the possibility of approaching the critical load point in terms of load flow solutions which is a problem of its own. However, assuming that this is possible, then the eigenvectors will give quite a clear picture as to which node is in danger and for which nodes countermeasures are most effective. Thereby the absolute value of the smallest eigenvalue does not have the same value as the indicator given in a previous chapter. However, the entry in the eigenvector gives the information as to how many mvars will cause a drop in voltage of e.g. 5 % which is quite practical.

In [2] the use of a reduced Jacobian is suggested under the assumption that active nodal powers do not change anyway.

The reduced Jacobian J_R is obtained by setting the subvector

$\Delta P = 0$ and relating ΔQ to ΔV .

$$\Delta Q = [J_{QV} - J_{Q\Theta} J_{P\Theta}^{-1} J_{PV}] \Delta V = J_R \Delta V$$

In a pure static analysis this approach may be justified. However, in a contingency analysis when the loss of generators is included it seems necessary to maintain the original matrix and to be able to exhibit all nodal power variations.

Preserving the original matrix allows the inclusion of reactive power limits or the changeover from PV nodes to PQ nodes by replacing the corresponding matrix coefficients.

Hence, modal analysis offers systematic approaches without sacrificing details of the model or properties of nodal powers.

Judging the proximity of a voltage collapse in terms of the minimum singular value of the Jacobian [14] is related to eigenvalue analysis. However, the computational procedure to determine the minimum singular is different from eigenvalue analysis.

Singular values of the Jacobian are the square roots of the r positive eigenvalues of $J^T J$ where r is the rank of J .

Similar to the eigenvalue transformation there is a transformation to the diagonal matrix of singular values

$$J = U \Sigma V = \sum_{i=1}^n \sigma_i U_i V_i^T$$

where U and V are $n \times n$ orthonormal matrices, U_i, V_i are the columns of U and V and Σ is the diagonal matrix of σ_i .

Hence the effect of nodal power variations on the nodal voltage components can be represented by

$$\begin{bmatrix} \Delta V \\ \Delta \Theta \end{bmatrix} = V \Sigma^{-1} U^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

or for the minimum singular value σ_n

$$V_n^T \begin{bmatrix} \Delta V \\ \Delta \Theta \end{bmatrix} = \sigma_n^{-1} U_n^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$$

Since σ_n^{-1} is very large any finite $\Delta P, \Delta Q$ will produce a large change in ΔV and $\Delta \Theta$ where U_n gives the most sensitive direction

of the change in nodal powers and V_n gives the most sensitive voltages and angles.

This result, however, is not basically different from the information contained in eigenvalues and eigenvectors.

The advantage lies in the computation of σ_n which uses techniques of the load flow iteration. Exploiting the sparsity of J and forward-backward substitution the following algorithm is as simple as an ordinary load flow solution [14]

1. Solve $J^T u_n^{(k)} = v_n^{(k)}$
2. Estimate $\sigma_n^\wedge = \frac{\|v_n^{(k)}\|_2}{\|u_n^{(k)}\|_2}$
3. Solve $J^T v_n^{(k+1)} = u_n^{(k)}$
4. Estimate $\sigma_n^\wedge = \frac{\|u_n^{(k)}\|_2}{\|v_n^{(k+1)}\|_2}$

Hence the minimum singular value can be taken as an indicator for the proximity to voltage collapse. Tests have shown that the singular vector v_n is strongly related to the voltage profile and the minimum singular value is related to the minimum voltage magnitude of a system [14].

All of these methods are able to produce some indication of the proximity to collapse, e.g. by a certain numerical value, by the distance of multiple solutions, by the size of the singular value. However, a certain assumption about the increase of the load has to be made and a change of topology or of the impedances is not permitted. Hence, such a distance is not a unique quantity but depends on many factors and it is therefore of limited value, as the subsequent treatment of the problem will show.

At the stage of the static model contingency analysis has already been applied, which was quite obvious since it is known that voltage problems have been initiated by disturbances. The procedures are similar to contingency analysis as applied for a classical security analysis the only difference given by a different measure, namely a proximity indicator instead of a percentage overload.

The contingency analysis certainly has a higher practical value than an indicator for the undisturbed load case. This is true because the network would be at risk if following a disturbance the indicator approaches the critical value or if the reserve margin were exhausted.

Dynamic analysis

Dynamic analysis within the realm of voltage stability includes the well-known procedures as used for transient stability as well as consecutive steps of static analysis. The latter method is a consequence of the static concepts as initially introduced. Thereby static relations are exploited only and static changes introduced from step to step. This way load changes and changes of transformer taps can be easily included. Dynamics in the true sense, however, is not developed, the more so since there is no

electromechanical imbalance which would accelerate a generator rotor.

The tendency to add further modelling details [17] suggests that a full dynamic analysis is performed and there is very little which would speak against such a move. This type allows the inclusion of regulators and exciters as well as the action of static var compensators. As a result the interaction between voltage and angular movement is exhibited.

Usefulness of indicators

Indicators as mentioned above have been worked out and developed based on the idea that their numerical value is a measure of the risk of voltage instability or a distance to voltage collapse. Examples are the indicator in [8], the distance between load flow solutions, the magnitude of the smallest eigenvalue or of the minimum singular value.

In the early development this concept seemed quite reasonable. However, with more detailed insight and experience it had to be concluded that the change from the present state where the indicator has been evaluated to the point of collapse is almost unpredictable and hence the risk of collapse is difficult to define or otherwise just worst case assumptions are possible.

A brief discussion of possible transitions to a point of collapse may clarify this point.

Quite often a transition in terms of a load increase is considered. Such a load increase may occur:

- by increasing the reactive load in a single node
- by increasing the active and reactive load in a single node
- by a uniform increase of loads in all nodes
- by a loss of a generator
- by a loss of a transmission line

In each of these cases the distance to the point of collapse is different even if a purely static model is presumed and the load change is abrupt.

The situation varies even further when a dynamic development is included. As the efforts in modelling and dynamic analysis show it is considered mandatory to take the load behavior, the action of OLTC's and of var compensators into account. Other items which affect the evolution of a voltage problem are the action of protective relays and the var limits of generators.

Hence there is a wide variety of arguments for considering indicators with a certain caution, although it cannot be denied that the accumulated experience with a certain system and with an indicator will be an important aid in preventing a voltage collapse.

Load behavior and OLTC

An important effect in the initiation of a voltage collapse is the reaction of the loads to a decrease of the voltage magnitude in the load area, i.e. at the distribution level. Although there are load models in use which are based on a power law where the exponent is larger than one, like

$$P = P_0 (V_i / V_{i0})^\alpha$$

$$Q = Q_0 (V_i / V_{i0})^\beta$$

it is to be realized that there are load components which lead to an increase in either active and/or reactive load when the voltage decreases. A typical example is the induction motor which draws

more reactive power when the voltage goes down. At the lower voltage the reactive current increases which makes things worse.

Thermostatically controlled heat loads draw less load following a voltage decrease. However, after a delay the load is fully restored which again causes a current increase.

The action of OLTC's has a negative effect on the above factors [18]. They restore the programmed distribution voltage level which results in a restoration of the load, maybe even with a certain increase. The original load is supplied by a lower transmission voltage and therefore a higher current is drawn.

This leads to the conclusion that in a first approach the basic PQ-model is quite reliable for a voltage stability study.

Countermeasures

All the aids for supporting voltage stability [16] are a direct consequence of the insight into the mechanism of voltage collapse. Since the maintenance of the nominal voltage is of prime importance any means which supports the voltage is a valid countermeasure to voltage instability.

Since voltage stability is essentially the ability to satisfy the load, a voltage problem due to an active load increase a compensation of the load would be the natural remedial measure. This is obviously not possible, hence, the next best approach is given by a reactive compensation. It works if the system impedance has a dominant reactive component. A critical situation due an excessive reactive load must be obviously eliminated by an additional reactive supply. Therefore var compensators as countermeasures to voltage collapse are foremost. These devices are all based on controllable capacitors which can reach their limit of supply. At the limit they act as fixed condensers and this may be a disadvantage. A simple analysis will show that adding a fixed capacitor at a critical node will extend the nose curve. However, near the nose the drop in voltage is quite abrupt.

Therefore, it is mandatory that a var compensator has an ample control range.

Dynamic effects

Although voltage stability has already been enriched by a series of dynamic effects [17] there are a few items which have not been related to the voltage problem. These are caused by the angular movement of synchronous generators.

As the elementary analysis above shows voltage stability depends on the equivalent source voltage (Thèvenin). In the static model this voltage is assumed to be constant although static var compensators when activated will change this voltage. During an angular swing of a generator there can be a significant decrease of the source voltage as seen from a critical node. The schematic in Fig. 1 will make this clear:

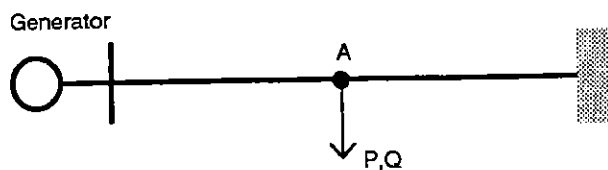


Fig. 1 Schematic of a power system with an intermediate load

At node A somewhere between a power station and a network (infinite bus) a load is assumed.

If the generators are accelerated due to a fault the voltage at A decreases and the source voltage seen there as well. The load when acting as constant P and constant Q will further lower the voltage and will adversely affect the restoring torques in the generators.

This is a case for transient voltage stability and represents an example where the analysis has to take the complete power system model into consideration. A static load flow will not be sufficient.

Another case closely related to the above example is given by the action of the damper windings of the generators. Damper windings act like the rotor windings of an induction motor when the rotor moves relative to the field. Thereby the generator absorbs reactive power which is taken from the generator itself and from the external network. The experience with transient stability studies shows that the terminal voltage drops during the movement of the rotor. This reactive power can upset the reactive power balance and can thus be detrimental to voltage stability.

Again it has to be remembered that the restoring torques in the generators are proportional to the square of the voltage magnitude and thus there is a repercussion on the angular stability of the system.

Consequences

At this point it should have become quite clear that the phenomenon "voltage instability" can be explained in a simple way and as long as a static model is adhered to an assessment of the state of the power system is fairly easy. Assuming that a load flow solution is available a variety of indicators becomes accessible.

The attempt to determine the distance to a possible collapse, however, is somewhat questionable since the path to the limiting point is unknown.

Hence, it seems mandatory to be able to perform a dynamic simulation which includes the initiating disturbance, the action of regulators and OLTC's, the models of the loads, the limits of components supplying reactive power and the settings of protective relays.

The recent developments in the area of voltage stability indicate as some publications show that a detailed simulation becomes more and more important.

A good model which includes the angular movement of generators, regulators and exciters, etc. will have the benefit that a variety of effects can be studied and countermeasures be tested. Apparatus taken to act against a collapse has to be modelled with the same detail as the other equipment.

It must be realized that finally it will be a question of how critical the risk of an instability will be and what kind of countermeasures will be necessary. From an engineering standpoint it is obvious that such an assessment is to be done on the basis of a detailed analysis, e.g. by a simulation.

What remains is a final discussion of the worth of an indicator. It seems that the role of an indicator lies in the online use in an EMS environment. Although the reliability of the indicator as regards the distance to a collapse cannot be very high the qualitative information will be of value. Of course, the application of an indicator will have to be oriented towards a security assessment based on outages.

There again it is seen that the assessment of voltage stability originated as a separate discipline and now tends towards a merger with the routines of security assessment already applied. Thereby it appears that there is no justification for the use of separate models and procedures.

Conclusions

The basic phenomena of voltage instability both from a static as well as a dynamic point of view are fairly well understood. This is a result of a step-by-step improvement of the models and associated methods of analysis and assessment. The emphasis when considering the vast volume of publications has been the assessment of the state by indicators and by a longer term simulation.

Interestingly enough a closed form criterion has not evolved which is due to the complexity of the overall phenomenon.

Because of the desire to obtain a better insight over the years models have been continuously improved. These include nonlinearities, switching operations and control actions. The result is that the model used for assessing voltage stability exhibits diminishing differences against that used for angular stability, which is quite justified since there are important interactions between voltage variations and angular movement.

This leads to the recommendation that the voltage stability problem should be treated within the framework of transient stability.

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*) There is a misprint on the volume indicating No. 2 but it is actually No. 1.

CURRENT STATUS OF THE TAXONOMY THEORY OF LARGE POWER SYSTEM DYNAMICS – DAE SYSTEMS WITH HARD LIMITS –

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ABSTRACT

The objective of this ongoing research is to develop a mathematically precise and physically meaningful theory for the large nonlinear dynamic system with a clear structure of the details and their interactions throughout the complete state and parameter spaces—a taxonomy. This effort led to a large number of new results developed in an accidental synchronism with this sequence of International symposia. At Potosi, Mo in 1988 the first beginnings were reported and by 1991 in Deep Creek Lake, Md the full theory was presented for the algebraically constrained ordinary differential equation (DAE) model and in 1994 in this paper the theory is extended to the DAE System which contains a variety of hard limits as is common in large nonlinear systems. In addition the question of using phasors for dynamic systems which became a bit murky over the decades with the use of the “quasi stationary assumption” is clarified by analyzing the time varying phasor concept as a mathematical transformation. This lays the groundwork for precise and meaningful power system models beyond DAE which can be used for more sensitive analysis such as in the vicinity of singularities. Accordingly the paper divides into three Chapters.

Chapter I summarizes the taxonomy theory as it existed at the time of the 1991 Symposium and some additional work.

This analysis of the taxonomy reveals a two level cellular structure in both the parameter and state spaces. The *parameter space* (which contains structural parameters like system composition, reactances, etc. and operating parameters like loads, generation, etc.), is composed of *typal regions*, bounded by bifurcation surfaces and characterized by uniform topology of equilibria and trajectories.

Typal regions are grouped into *feasibility regions* within which a specific operating point (a stable equilibrium or load flow) can be shifted around smoothly by operator action (such as in remedying an alert state) without danger of collapse. The boundary of this region can be calculated as zero sets of explicit analytic functions and is thus relatively accessible to computation. The boundary of the feasibility region is composed of three different kinds of bifurcation surfaces corresponding to 1) the common (P-V curve type) saddle node bifurcation, 2) the Hopf bifurcation, and 3) the singularity induced bifurcation, a relatively

new type, which was first rigorously studied by us in at the Deep Creek Lake Symposium.

Extensive additional analysis is given for phenomena in the vicinity of the singularity which has basic consequences to the structure of the constrained vector field.

The *state space* (which defines the dynamic stability of voltages, rotor angles, frequency etc.) is separated into connected components by singular surfaces. No trajectories pass through the singularity. In fact, trajectories entering a small vicinity of the singularity, physically generate a very fast random process, so that events after the entrance into the singular region are unpredictable. Within each component stable equilibria have their regions of attractions bounded by stability boundaries and other regions where no convergence or convergence to a singularity may occur. The stability boundary consists of segments associated with different kinds of anchor points including stable manifolds of unstable equilibria and periodic orbits, and of portions of trajectories anchored to the singular surface at what were called pseudo equilibria or semi-singular points. Also certain sections of the singular surface where trajectories originate and the system is transversal to the singularity may lie in the stability boundary.

Chapter II analyzes systematically the phasor problem which has become vexing and somewhat nebulous since the 1991 Symposium where we raised questions of the validity of the traditional quasi stationary assumption of the “transient stability” era which became questionable for the faster transients in today's voltage stability problems. The use of phasor calculus is closely connected with the proper and precise model to be used for the power system, a system model which retains its validity in the fast events near the singularity of the DAE model and in general off the constraint surface. A time varying phasor is defined precisely and its properties analyzed. When combined with the true physical model of the transmission lines of the network which are boundary value problems on distributed constant four terminal networks a precise model for the system can be defined. *Surprisingly the true mathematically and physically precise dynamic model for the power system turns out to be ordinary differential equations constrained by algebraic equations which contain line specific time delays.* The latter represent the travel times through the transmission lines. These are relatively

small and when neglected the DAE model emerges as an approximation. A good approximation away from the singularity but in the fast world around the singularity the fast internal dynamics of the transmission lines plays the decisive role in determining the details of the events so neglecting the time delays makes it impossible to get the physical picture. Injecting the nonphysical dynamics of the common nominal π or completely arbitrary singular perturbations of the form of $\epsilon \dot{y}$ term in the algebraic constraints will lead to unrealistic results.

So in Chapter II the precise models are established for studying events near the singularity and in other locations off the constraint surface. Analytic studies of the time-delay models are now proceeding but time delays are a mathematically difficult subject, since this represents a system with "memory". In the meantime analyzing the singular events of the DAE model has established a sort of backbone structure which will still guide the precise overall phenomena when and if they are established.

Chapter III contains the first systematic study of the large DAE systems which contains arbitrary numbers of hard limits of the three dominant types 1.) the IEEE windup (here called actuation) limits where no state is blocked outright but an internal path which actuates some action or information transfer has a hard limit 2.) the IEEE non windup type where the hard limit directly blocks a state variable and 3.) the relay or switching type where the reaching of a hard limit of either an actuation variable or a state variable, while not blocking either, is the starting point of structural type changes. All three of these are analyzed for both the state space and the parameter space.

The state space analysis of the system with hard limits includes definition of entities like, trajectories, definition of equilibria and of stability and other phenomena encountered in smooth nonlinear systems. The segments (and their anchors) which constitute the stability boundary are analyzed and new segments are discovered and identified. An important and interesting aspect of the hard constrained state space picture is the way in which some parts of the state space are excluded or become empty. The limit boundaries here play novel roles in that they carry special trajectories and generate equilibrium like phenomena.

The parameter space analysis also parallels that of systems without hard limits. Particular attention is paid to the feasibility boundary which turn out to be a very much richer than without hard limits. In fact a number of new bifurcation segments directly connected with hard-limits are identified along the boundary which were not hitherto recognized. The new phenomena range from limit induced static bifurcations (which annihilate the operating point) to limit induced dynamic bifurcations (such as connected with hard-limit induced oscillations). Numerical simulations of the rudimentary system now with hard-limits illustrate the results.

The material in Chapter III represents ongoing research reported here for the first time. While this material adds up to an essentially complete taxonomy like the one reported in 1991 and summarized in Chapter I for the DAE's without hard limits still there are statements which are reported as conjectures (which we feel rather strong about) still to be completely proven.

The bifurcations discussed in this paper are mostly of the local type. Some of the global bifurcations were covered in the 1991 paper [86] in this series. Systematic studies of global bifurcations on the large system are now in progress and so far we published one paper on this subject connected with the persistence of homoclinic orbits [95]. Other studies including excellent episodic exploration studies on very small system are appearing in the literature.

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INTRODUCTION

This paper represents the newest stages in the development of the Taxonomy theory a general and structured theoretical analysis of the dynamic behavior of very large nonlinear systems. The authors motivation for this study derives mostly from the large electric power system (which indeed may be the most complex and varied among large nonlinear systems, except the biological ones) which also includes a network component. The theory however applies for all systems of its class far beyond power systems. The work on this theory started around about 1985 and the present step, on systems with hard limits, brings it near to completion. This was strategic timing. In mathematics several fundamental results appeared in the last couple decades on nonlinear dynamics particularly in stability [74, 75, 57, 10] and bifurcations [76, 37, 22, 11], while at the same time the increased load demand, spreading interconnections and the computer age which brought a wide redevelopment of practices in the power industry bringing with it more complex dynamic phenomena (in normal as well as in distributed constant circumstances). These fast outstripped the meager toolkit of the quasi stationary approximation of dynamics which has served the industry well during the preceding decades of the century, mostly in a electromechanical "transient stability" techniques and load flow solutions more recently aided by computers.

The research community responded vigorously but in piecemeal fashion. Interesting and challenging problems were posed and solved giving important and useful results (see References) but overall in a haphazard assortment of detail with only partial coverage and no clear understanding of the systemwide interactions of the detail problems.

In this project a general theory of broad perspective is sought with a clear structure of the details and their interactions throughout the complete state and parameter spaces. This effort led to a very large number of new results which all fit in the general theory in a clear structure.

More specifically at this point the Taxonomy Theory emerged and was comprehensively reported in 1991 [86, 90, 91]. This is a general theory of the dynamics of large nonlinear systems with algebraic constraints which is novel in many of its aspects and is new in the sense that

1. no other comprehensive general theory of this type now exists;
2. it identifies a two level cellular (hierarchical) structure for a class of algebraically constrained large systems (Tables 1 and 2) which is directly relevant for large power system operation;
3. it is unraveling for the first time the complex taxonomy which sustains the nonlinear interactions connecting the state space and the parameter space. Indeed the new theory opens alternate approaches to practical problems through the parameter space and the state space;
4. it develops a rich collection of new results (for any system size and type of smooth model) such as the full characterization of the stability and feasibility boundaries in precise and detailed mathematical terms; it introduces new concepts of direct relevance to the stability boundary such as the pseudo-equilibria and semi-singular points; it develops necessary and sufficient conditions for the principal an-

chors of the stability boundary; it gives precise results for the contributions of the singularity to the stability boundary; it provides a rigorous analysis of the bifurcation points connected with equilibria at the singularity thus establishing the new bifurcation as a generic instability mechanism for the large power system and a component in the feasibility boundary; it gives a rigorous analysis of the instabilities caused by discrete automatic control devices (for instance, the bifurcations connected with cascading voltage descent of ULTC's); it identifies the composition of the feasibility boundary as zero sets which implies the computability of this powerful security monitoring concept even for the large system; and much more.

5. The major part of this paper (Chapter III) develops the Taxonomy of Large DAE systems which contain hard limits. This new work is very extensive. It covers in detail the state space and parameter spaces and their interactions in the manner of the preceding work on smooth systems.

In no sense is this new theory a mere compilation of existing results, although a modicum of earlier results by us and others is included in the theory where appropriate.

The main objective of our ongoing research is to develop a mathematically precise and physically meaningful theory base for system stability, with emphasis on the general large nonlinear power system (including both voltage and angle dynamics) and to design efficient practical techniques for monitoring and enhancing the security of the large power system operation.

This work goes back some ten years with the initial participation of Garng Huang, Baohua Zheng and Tin-Chi Leung [112, 114, 113, 125, 123] and became especially productive and rewarding in the last couple of years with Vaithianathan Venkatasubramanian and Heinz Schättler [85]–[103] and currently Xin Jiang. John Zaborszky has led the team. The earlier phase of this work produced novel general results for the stability of large smooth nonlinear systems such as the electric power system in a field which for decades restricted the scope of its thinking to rather small systems.

In the way of preliminaries it may be mentioned that in the 1970's extensive work was carried out on online stabilization of the power system using a concept of observation decoupled equilibria introduced by J. Zaborszky and A. K. Subramanian [118] in the early 1970's. This work used simple classical dynamic models [120]–[122] for the power system. The inadequacies of these models especially in voltage stability led to a shift of interest to the dynamics of the system itself in precise mathematical terms. The first efforts here were studying the dynamics of large smooth nonlinear systems. This work resulted in the theory of the dynamics, the details of the stability boundary and the introduction of a new and essential concept of a quasi stability boundary which eliminates internal segments [113, 114, 125]. The importance of the algebraic constraints in voltage stability stimulated our interest in smooth systems with algebraic constraints (DAE). Initial results which recognized the role of singularity and pseudo equilibria (among others) were presented in 1988 [110] at the first of these symposia and in [123]. By the time of the second conference [127] a fully developed Taxonomy theory was presented [86] followed by numerous other publications on this subject [85]–[103]. It also laid the foundations for the development in the last couple of years [85]–[103] of the theory of large dynamic nonlinear systems with algebraic constraints

Table 1. Structure of the parameter space

	Region	Sections of the boundary
<i>Feasibility regions contain</i>	<u>FEASIBILITY REGION</u> Operating point movable continuously as a stable equilibrium	<u>BIFURCATIONS</u> 1) Saddle node 2) Hopf 3) Singularity Induced
	<u>TYPAL REGION</u> State space behavior maintains its type, i.e. the topology of the trajectories	Various kinds of bifurcation surfaces consisting of local and global bifurcations

Table 2. Structure of the state space

	Region	Sections of the boundary
<i>State space components contain</i>	<u>COMPONENT</u> State moves on predictable trajectories over regular points	<u>SINGULAR SURFACE</u> Source and termination of predictable trajectories
	1) REGION OF ATTRACTION 2) Region of no convergence 3) Region of voltage collapse near singularity 4) Additional types	<u>QUASI-STABILITY BOUNDARY</u> 1) Stable manifolds of four types of anchors: i) Unstable equilibria ii) Unstable limit cycles iii) Pseudo-equilibrium surface iv) Semi-singular surface 2) Singular points

(differential algebraic systems or DAEs in short). This theory was presented in 1991 with an engineering emphasis for power systems but with detailed content of the mathematical results [86]. The mathematical theory as separate from the specifics of the power system was published in 1992 in a copyrighted Washington University Report # SSM 9201 [91]. The paper [92] from the November 1992 issue of the IEEE Transactions on Automatic Control can be viewed as a detailed illustrative example serving as a stepping stone study for the theory. This example is particularly interesting because most of the phenomena observed and analyzed in this rudimentary power system are also meaningful for the large system as proved by our theory.

This was the status of this research three years ago at the previous meeting of this series of symposia at Deep Creek Lake Maryland in 1991. This phase of the work is summarized here in Chapter I. Major progress was made in the last three years. Mathematically precise and thorough studies of complex events around the singularity in the DAE models was carried out (also included in Chapter I). Interesting and unexpected phenomena (e.g. a smooth manifold passing through the singularity [98, 100]) emerged. When the system state moves around the singularity the movements become fast. This invalidates the traditional quasi stationary phasor dynamics (a fact sometimes overlooked in the literature, producing questionable results). To clarify this rather confusing situation extensive studies were carried out (Closure in [86], [116], [101], [102], [99]). The results

introduce a time varying phasor dynamics which can deal with fast transient situations in a precise manner. These results are presented concisely in Chapter II. An interesting conclusion is, that for power systems which are constrained by algebraic conditions (such as the power flow on the network) the precise mathematical model consists of ordinary differential equations and algebraic equations where the latter may contain line specific time delays.

In fact the bulk of this paper is devoted in Chapter III to results obtained for the algebraically constrained system when it contains hard limits of various types. This is a really challenging problem. We succeeded however to formulate it in a mathematically tractable (precise-not approximate) manner which then allowed us to develop a full state space-parameter space theory for such systems. Interesting new features arise. In the state space the constraints combine or interact with the segments of the stability boundary to generate previously unknown segments and anchor points. In the parameter space previously unrecognized types of bifurcations (both static and dynamic) arise and are analyzed.

The work in Chapter III is the initial publication of this precise and comprehensive mathematical results. No prior publication has been made on the research in this context.

The three Chapters of this paper then report important new developments toward the complete theory of the dynamics of very large nonlinear systems which are subject to algebraic constraints, such as the network power flow, and also to hard limits such as limits on certain states (e.g. excitation current) or on certain "actuation" quantities (functions of states) which are internally transmitted for actuating certain actions (e.g. control errors). Relays can also conceived and construed as hard limits.

We are not aware of any similar comprehensive theory for large nonlinear algebraically and hard limit constrained systems. We are however enclosing at the end of this introduction a rather rich Bibliography of more than one hundred items (which we consider pertinent) some of which have some relationship to our work reported in our own publications also included in this list.

As the details unfold in the following text connections to the literature or individual publications will be carefully examined and reported with proper credit to other authors.

This is a rather long paper written mostly in historical order of the developing Taxonomy theory. Accordingly it is divided into three Chapters.

Chapter I summarizes the status of Taxonomy theory at the time of the second of these Symposia in 1991 [86] and subsequent substantial developments in the analysis of behavior around the singularity in the DAE models which were obtained since 1991. This state of the theory is well presented in the conference proceedings and in complete detail in a departmental report [91]. Only a short summary of the theory is offered here in Chapter I.

Chapter II is devoted to the problems of the inadequacy of the quasi stationary phasor theory and their remedy. Fast movements around the singularity necessitated a complete rethinking of the vital phasor concept and the introduction of a time varying phasor theory. Once accomplished this new phasor eliminates some of the fundamental modeling ambiguities that exist in voltage stability studies.

Chapter III the dominant piece in this paper introduces for the first time the broad new and nearly complete theory of large systems with algebraic constraints and hard limits.

The two level cellular structure of the dynamics with hardlimits is summarized in Tables 3 and 4 for the case of windup limits and in Tables 5 and 6 for the case of nonwindup limits. In each case, it is shown that both the parameter space (Tables 3 and 5) and the state space (Tables 4 and 6) admit a two-level cellular structure much like the algebraically constrained DAE systems in [86,91]. In Section 1 below, the state space structure is summarized and in Section 2, the parameter space features are discussed.

For easy comparison of the differences between the windup and nonwindup, and for separation of the phenomena related to each of the two types, in Chapter III, we study two subproblems: 1) a smooth system with only windup limits present say Σ_{al} (where al stands for actuation limits, the functional name introduced in Chapter III for windup limits), and 2) a smooth system with only nonwindup limits present say Σ_{sl} , (where sl stands for state limits the functional name given to nonwindup limits in Chapter III). Moreover the presence of singularity is deliberately excluded from the model to isolate the phenomena connected with hardlimits. (see Section III.1 for details). As emphasized throughout Chapter III, in the real system, all types of hardlimits along with the possible interaction from the singularity of the DAE model would affect the overall behavior of the system. The results for the separate subsystems Σ_{al} (windup), Σ_{sl} (nonwindup) and Σ_{rl} (relay type) in Chapter III together with the DAE results from [86,91] (in Chapter I) should naturally lead to such an overall treatment.

In the next two sections, we provide brief overviews of the hardlimit related state space and parameter space phenomena for windup and nonwindup limits.

1. Overview of the state space phenomena

Tables 4 and 6 summarize the two-level cellular structure of the state space for the system Σ_{al} with windup limits and for the system Σ_{sl} with the nonwindup limits. Note that the structure is similar to the case of the DAE system (shown in Table 2) though important differences exist as we see below. The structure shown in these two tables 4 and 6 1) provides an insight into how the hardlimits influence the transient behavior in the state space, and 2) lays the foundation for developing Lyapunov function type (transient) stability monitoring algorithms (for non-hard-constrained models, a small sample- [36,41,56,62,112]) which now directly incorporate the effects of the two hardlimit types in the model. It is hoped that practical Lyapunov theory techniques emerge for large power system models with hardlimits which are aided by the development of the dynamic theory for the hardlimits in Chapter III.2.

1.1. Windup limits or actuation limits

The windup limits are hardlimits which appear in the internal transfer (the actuation) of certain functional elements such as control errors. These hardlimits do not directly affect state variables (unlike the nonwindup limits as we see below in Section 1.2), rather the effect is through the overall dynamics. Specifically the input to such a device could "wind-up" while the output is stuck at a limit, hence the name the windup limits.

For the case of windup limits (actuation limits), the state space consisting of the set of all system trajectories (or transients in a loose sense) divides into a number of open components such that the system is smooth within each of the components (see Table 4). The boundaries of these components are the limiting surfaces where one of the windup variables reaching its limiting values. Note that reaching the limit changes the functional behavior of the "output" i.e. the actuation signal internally in the system, hence causing a nonsmooth change in the system equations when the windup limit is encountered. In other words, the trajectories are smooth as long as they stay within open components (the number of limits that have been reached, does not change so no nonsmoothness). Upon reaching the boundary of a component, one or more windup limits change their behavior (say going from actuation to saturation or vice versa), and hence the trajectories lose smoothness while moving from one component to another. Moreover when windup limits are modeled in the analysis, equilibria and their stability properties change (in the form of equations) since the smooth functional description within each component is different. We can define stability regions in this case as well, where the region of attraction of the operating point (essentially all the transients which eventually settle down to the given operating condition) consists of trajectories originating from various components. In other words, the possibility of some windup limits being encountered (and disengaged conversely) in the process of the transient converging to the stable equilibrium corresponds to the trajectory moving across various components before reaching the stable equilibrium. Since such trajectories would have "kinks" (are nonsmooth), at first sight, it appears doubtful whether Lyapunov function type results can be developed say based on the knowledge of boundary characterization such as the controlling equilibrium point methods [56,36,17,62].

Interestingly the main result in Chapter III.2 is the proof that the qualitative features of the stability regions remains topologically unchanged from those of the smooth system when the windup limits are included. Particularly it is proved that under Morse-Smale like assumptions, the boundary of the region of attraction again consists of stable manifolds of unstable equilibria and stable manifolds of unstable periodic orbits. This directly implies that extensions of Lyapunov function type algorithms can be developed for the model with windup limits by algorithms similar to existing ones for smooth systems. Moreover using the notion of quasi-stability boundary introduced in [114], it is proved that it is sufficient to consider only order one equilibria (and order two periodic orbits if present) for stability boundary analysis.

The differences compared to the smooth system which are caused by the presence of the windup hardlimits are as follows: 1) the unstable equilibria which anchor the stability boundary may correspond to conditions where some hardlimits may have been encountered, (hence results for these equilibria would differ from those of the smooth system); 2) the stable manifolds described above in general are not smooth manifolds (may have corners and kinks) but are topological manifolds (still well-behaved). The nonsmoothness means that the construction of Lyapunov functions or energy functions would be more difficult in this case. However with increasing interactions between hardlimits and the transients reported in utilities [105,131], research in this direction is well-justified based on the boundary characterization in Section III.2.1.

1.2 Nonwindup limits or state limits

Nonwindup limits are functionally quite different from the windup limits since a nonwindup limit directly affects the status of a state variable. That is, when a state with a nonwindup limit on it reaches its limit, the state variable gets “stuck” at this limit value as long as the dynamics is driving the state to move past the limit value. In other words, when a nonwindup limit is encountered, and is active (the “free” dynamics is trying to push the state past the limit), then the state variable becomes a constant in the model, and the system loses a degree of freedom (system dimension in the sense of dynamics drops by one). Note that this state variable stuck at the nonwindup limit will indeed return to be an unconstrained variable the moment the “free” dynamics allows it to do so. Hence there is no windup of the state involved here. The state variable is strictly limited between its limiting values at all times (hence the name state limit introduced in Chapter III for nonwindup limits in a general sense).

What this implies from a dynamic view-point is that the dimension of the system dynamics (the degrees of freedom in a loose sense) can vary from the full dimension say n (when no nonwindups limits are encountered) to a smaller dimension say $n - k$ when k limits are encountered and are active (then k states are constant at their limits: only $n - k$ states vary dynamically). Therefore the state space consists of several “active level k sets”, where k ranges from zero (no limits active) to n (all limits active - highly unrealistic, but included for completeness sake). During a voltage crisis, for a typical transient, k the number of nonwindup limits encountered may range from zero to something like ten (typically) depending on the severity. Therefore the state space analysis should provide an overview of how these different sets fit together and how the transition from one set to another set is effected as is done here in Section III.2.2. The salient features are summarized in the upper row of Table 6.

The state space description (in the form of functions) is smooth within each of the active level k sets. As noted above, however the complexity arises from the fact that the dimensions ($n - k$) of these different smooth subsystems are different. The fundamental question then is whether (if at all) these different subsystems fit together nicely in a global sense. That is, given an initial condition (say a post-fault clearing state), is it guaranteed that there exists a unique trajectory which is well-defined through all the transitions of encountering (and possibly disengaging) several nonwindup limits? If the answer is yes, then this trajectory can be used to assess stability properties such as whether it settles down to a stable operating point (see the second row of Table 6 on the properties of the stability regions). The answers to both questions are yes (fortunately) though this is nontrivial since we prove that this argument works only in positive time.

In other words, given any initial condition during a transient, we can compute exactly how the trajectory will be headed in forward time (mathematically we say that positive flow exists), but we cannot determine how the trajectory arrived at this given condition (nonuniqueness in backward time). Moreover during transitions to and out of hardlimits, the smooth properties of the trajectories are destroyed though continuity persists as can be seen intuitively.

In summary, the state space for the nonwindup case consists of several constrained smooth systems of varying dimensions which fit together nicely (imagine a match-box in higher dimen-

sions) so that the dynamics is such that the trajectories exist and are unique in forward time, but may suffer loss of existence and loss of uniqueness in negative time (Table 6). Therefore in the analysis of stability regions, the boundary characterization is far more challenging compared to the windup case (Table 4). This is because 1) some of the anchor points or the unstable equilibria on the stability boundary may occur on active level k sets where $k > 0$ (may be such that some limits are already encountered), and 2) the trajectories composing the boundary are nonsmooth and they possibly suffer loss of uniqueness along the way (stable “manifold”s may consist of several pieces of varying dimensions). However prior results in this direction (for the DAE case [86,91]) strongly indicate that such an analysis is still possible though this may be much more technical. The results in this direction are conjectured in the second row of Table 4.

With a “careful” extension of Morse-Smale like assumptions, the stability boundary can again be shown to be composed of the stable “manifolds” of unstable equilibria and unstable periodic orbits, possibly along with points on the limit boundaries where trajectories originate (loss of existence in negative time). Therefore Lyapunov theoretic extensions of the stability boundary analysis should be possible in this case as well, though the arguments may be somewhat more complicated in this case because of the varied system dimensions in different active level k sets of the state space.

2. Parameter space structure

As in the DAE case for non-hard-constrained models, the parameter space can again be divided into several typical regions bounded by local and global bifurcations. The bifurcations boundaries signify parameters where the topology (that is the qualitative structure and not to be confused with the connection topography of the network) of the state space undergoes changes. The bifurcations can be broadly divided into local bifurcations (affecting the stability properties of equilibria) and global bifurcations (all the non-local ones). While local bifurcations related to changes in the eigenproperties of the system Jacobian can be computed and analyzed (under certain transversality assumptions) using center manifold theory and normal form theory, global bifurcations are much more difficult to detect and analyze for the large system. Again certain global bifurcations which arise in the neighborhood of (degenerate cases of) local bifurcations can be analyzed using the normal form theory and examples can be seen in [95]. Specifically global bifurcations connected with complex behavior such as the presence of chaos have generated interest [34] in recent extensive studies of small-scale power system models (angle stability - [50,66], voltage stability - [5,19,2,106]). For the large system, a method for detecting the presence of homoclinic orbits (which upon perturbations could lead to complex behavior) in power system models was proposed recently in [95] using results from bifurcation theory.

When the hard-limits are introduced in the analysis the dynamic properties of the state space are affected (see Tables 4 and 6), and this naturally leads to new bifurcation phenomena both the local ones and some global bifurcations. An example of global phenomena is the birth of stable nonsmooth homoclinic orbits in our simulation studies (see Figure III.15 in Chapter III) from the presence of nonwindup limits. However in this paper, we only analyze the phenomena connected with the local bifurcations specifically those which lead to loss of stability of the operating point.

For analyzing such local bifurcations connected with loss of stability, we have proposed a notion of feasibility region in the parameter space [85,86,91,92] which consists of the set of all operating points which can be reached by quasi-static parametric variations from a given operating point. The feasibility region therefore delineates a safe operating region within which the operation can be shifted around by operator initiated actions (or by load changes etc.) without an imminent danger of losing small-signal stability at the operating condition. The boundary of this region consists of local bifurcations.

For a smooth unconstrained system, it is well-known that loss of stability corresponds to one of two eigenvalue conditions at the equilibrium point [37]: 1) zero eigenvalues (the system Jacobian is singular) lead to saddle node bifurcations generically [76,52], and 2) purely imaginary eigenvalues ("zero damping case") lead to Hopf bifurcations [37,1] generically. Among the two cases, the saddle node bifurcation is a static bifurcation in that the equilibrium structure of the system undergoes a change at this bifurcation (two equilibria meet and disappear) whereas the Hopf bifurcation is a dynamic bifurcation [22]. When algebraic constraints are introduced as in typical power system models [8,13] (the DAE case), there arises the new possibility of eigenvalues being unbounded and a theory for this bifurcation was developed in [86,91] under the name the singularity induced bifurcation. More details on these three cases follow in Section I.3.

When the model with hardlimits are considered, there arise new bifurcation phenomena which are of the static type as well as the dynamic type. When the operating point encounters a hardlimit, the local stability at the equilibrium is affected at these bifurcations. The possibility of such cases has been noted in the literature [32,72] connected with studies of specific power system models. Here we prove the occurrence of limit induced static and dynamic bifurcations in the model with hardlimits in the general large system, and provide a theory base for computing (and analyzing) them. These are briefly summarized below in Sections 2.1 and 2.2 for the windup limits and nonwindup limits respectively.

2.1 Windup limits or actuation limits

The boundary of the feasibility region consists of four types of bifurcations (in a generic sense) (see Table 3). The first two types, namely the saddle node and Hopf correspond to the respective boundaries for the smooth system with no hardlimits. The third type the actuation limit (windup) induced static bifurcations is connected with the annihilation of the operating point when the operating conditions reach the actuation hardlimits resulting in no equilibrium locally after this bifurcation is encountered (hence a collapse). Therefore the phenomenon is similar to the static saddle node bifurcation of the smooth systems even though the system Jacobian (for the smooth system) would be nonsingular at this bifurcation point. To be precise, these points correspond to a (nonsmooth) change in the description of the equations, hence technically there does not exist a well-defined Jacobian at the bifurcation point. Moreover since the limit is encountered abruptly, the disappearance of the equilibrium is sudden (imagine the nose of the PV curve becoming a sharp edge).

A simple direct algebraic formula is proved in Chapter III.3.1 for characterizing the occurrence of the static bifurcations (see Actuation limit induced static bifurcation Theorem). There also

exists the possibility of dynamic bifurcations (where the operating point "survives" but loses stability like in the case of Hopf), and these are analyzed in Section III.3.1 under the name of the actuation limit induced dynamic bifurcations. Under some simple conditions, it can be shown (Section III.3.1) that in two dimensional systems, these correspond to the birth of hardlimit induced nonsmooth oscillations. Extensions of these results for the large system are currently underway.

2.2 Nonwindup limits or state limits

As observed in Section 1.2, the state space for the nonwindup case is richer consisting of several subsystems of different dimensions. Therefore the construction of the feasibility region for the nonwindup case is more technical. However it can be still be proved that the bifurcation points which lead to loss of local stability (the feasibility boundary points in Table 6) consist of the well-known saddle-node and Hopf, as well as the new types, the limit induced static bifurcations and the limit induced dynamic bifurcations. As in the case of the limits, simple algebraic conditions are derived in Section III.3.2 for computing the state limit induced static bifurcations and the dynamic bifurcations.

Therefore these results would allow us to directly incorporate the effects of both the windup and nonwindup limits in the small-signal stability analysis and in local bifurcation analysis for precisely identifying when the operation faces the danger of losing local stability (leading to collapse). This analysis specifically brings out when reaching the hardlimits is "harmless" (local stability is sustained) and when it is disrupted (the static and dynamic bifurcations). It is hoped that the Theorems presented in Section III.3 provide the basis for developing practical computational results for detecting collapse mechanisms which now directly incorporate the effects of hardlimits.

With these latest additions the Taxonomy theory is nearly complete in covering the large nonlinear system theory under most practically important system types. Of course challenging problems still loom and the work will continue by us and we would like to hope by others.

Table 3. Structure of the parameter space with the windup (actuation) limits.

	Region	Sections of the boundary
Feasibility regions contain Typical regions	FEASIBILITY REGION Operating point movable continuously as a stable equilibrium	BIFURCATIONS 1) Saddle node bif. 2) Hopf bif. 3) Actuation limit induced static bif. 4) Actuation limit induced dynamic bif.
	TYPICAL REGION State space behavior maintains its type, i.e. the topology of the trajectories	BIFURCATIONS Various kinds of bifurcation surfaces consisting of local and global bifurcations

Table 4. Structure of the state space
with the windup (actuation) limits.

	Region	Sections of the boundary
<i>Stability regions</i>	<u>COMPONENT</u> State moves on smooth trajectories over regular points	<u>TRANSITION SURFACE</u> Trajectories cross-over with continuity but lose smoothness in transition.
<i>span over</i>	1) REGION OF ATTRACTION	<u>QUASI-STABILITY BOUNDARY</u>
<i>various components</i>	2) Region of convergence to stable limit cycle 3) Region of convergence to complex limit set 4) Region of no convergence.	Stable (nonsmooth topological) manifolds of two types of anchors: i) Order one equilibria ii) (Nonsmooth) order two periodic orbits.

Table 5. Structure of the parameter space
for the nonwindup (state) limits.

	Region	Sections of the boundary
<i>Feasibility regions</i>	<u>FEASIBILITY REGION</u> Operating point movable continuously as a stable equilibrium	<u>BIFURCATIONS</u> 1) Saddle node bif. 2) Hopf bif. 3) State limit induced static bif. 4) State limit induced dynamic bif.
<i>contain</i>		
<i>Typal regions</i>	<u>TYPAL REGION</u> State space behavior maintains its type, i.e. the topology of the trajectories	<u>BIFURCATIONS</u> Various kinds of bifurcation surfaces consisting of local and global bifurcations

Table 6. Structure of the state space
for the nonwindup (state) limits.

	Region	Sections of the boundary
<i>Stability regions</i>	<u>ACTIVE LEVEL k SETS</u> State moves on smooth trajectories with only $n - k$ degrees of state freedom; k states are constant at their limit values.	<u>LIMIT SURFACES</u> Trajectories cross-over with continuity but lose smoothness in transition; trajectories unique only in forward time; can originate trajectories.
<i>span over</i>	1) REGION OF ATTRACTION	<u>QUASI-STABILITY BOUNDARY</u>
<i>various level sets</i>	2) Region of convergence to stable limit cycle 3) Region of convergence to complex limit set 4) Region of no convergence.	1) Stable (nonsmooth topological) "manifolds" of two types of anchors: i) Order one equilibria ii) (Nonsmooth) order two periodic orbits. 2) limit transition points.

I. A SUMMARY OF THE ORIGINAL TAXONOMY THEORY FOR DAE SYSTEMS

Many problems in the theory of power systems like voltage stability can be modeled (within the quasi-stationary range) by a parameter dependent differential-algebraic system of the form

$$\dot{x} = f(x, y, p), \quad f: \mathbb{R}^{n+m+p} \rightarrow \mathbb{R}^n \quad (1)$$

$$0 = g(x, y, p), \quad g: \mathbb{R}^{n+m+p} \rightarrow \mathbb{R}^m \quad (2)$$

$$x \in X \subset \mathbb{R}^n, \quad y \in Y \subset \mathbb{R}^m, \quad p \in P \subset \mathbb{R}^p$$

In the state space $X \times Y$ dynamic state variables x and instantaneous state variables y are distinguished. Typical dynamic state variables are the time dependent values of generator voltages and rotor phases, instantaneous variables are bus voltages and other load flow variables. The parameter space P is composed of system parameters (the system topography, i.e. what is energized, and equipment constants e.g. inductances), and operating parameters (such as loads, generation, voltage set-points etc.). Both of these spaces have distinct structural features, sometimes intertwined (See Tables 1 and 2).

For a fixed parameter, p_0 , the state variables describe the time behavior of the system, the stationary solution or the transients. The equality constraints $g = 0$, which are mostly the power balance equations, limit the state to a constraint set, L ,

$$L = \{(x, y) \in X \times Y : g(x, y, p_0) = 0\} \quad (3)$$

at all times. Through every point $q \in L$ with the exception of points in a singular subset S ,

$$S = \{(x, y) \in L : \Delta(x, y) := \det D_y g(x, y, p_0) = 0\} \quad (4)$$

there exists a unique trajectory (i.e. transient) which passes through q and lies in L . Here $D_y g$ denotes the matrix of partial derivatives of the components of g with respect to y . The singular set S is defined as the set where the conditions of the implicit function Theorem for eliminating y from the algebraic constraint are violated. Typically S is a stratified set of maximal dimension $n - 1$ embedded in L and L is separated by S into open regions, the components C of the state space. The state space is confined to the complement of S in L . Typically no solution exists within the singular set S . In this work, we do not allow trajectories to cross the singular set. Our analysis simply stops at S . If one interprets the singular surface as the onset of unpredictable events like in the case of power system models [86], this is indeed a realistic interpretation for the actual behavior. The definition of stable and unstable manifolds given here reflects this interpretation, i.e. stable and unstable manifolds are defined to be subsets of a single component C . Since all dynamics are constrained to individual open components, C , the stability boundaries thus, typically also contain segments of S .

Note that in the next Chapter, we report on recent developments on power system dynamic models which are not restricted by the quasi-stationary phasor assumption. These results are based on a notion of fast time-varying phasor introduced recently by us for analyzing faster dynamic transients such as near the singularity and outside the constraint set L . However as we will see, the true dynamic model for analyzing the global behavior turns out to be of a time-delay differential-algebraic form, hence the analysis of such problems appears to be quite challenging at the moment. While the functions f and g are assumed to be smooth in the model Σ , in Chapter III, we extend the analysis to include nonsmooth elements always present in the system such as windup limiters, nonwindup limiters and relay type limiters, and the analysis in Chapter III is again based on the quasi-stationary phasor assumption. Naturally the next step would be to derive global results for a dynamic model which also includes in it the fast dynamics of the transmission line etc. in the form of the time-delay differential algebraic equations, for gaining insight into what goes on near the singularity and for the transient analysis outside the quasi-stationary domain. However in this Chapter, we summarize the results for the quasi-stationary phasor domain where indeed the models are of the differential-algebraic form above Σ assuming that the analysis simply stops at the singularity.

An analysis of the main structural features of the constrained dynamics with emphasis on those aspects which are caused by the presence of the algebraic constraints is given in Section I.1. The key tool is a singular transformation in Section I.1.3 which was proposed for local analysis in [78] and is extended in our theory for global analysis using the matrix adjoint property. This global transformation gives rise to a smooth transformed system which has the same phase portrait as the original system except for pointwise time rescaling along the trajectories and reversal of flow on some sections. Roughly speaking, *singular systems of the form (1) and (2) become smooth systems with a singular time scale*. The singular time rescaling proposed here completely eliminates all singular aspects of the system. All singular points either become regular or what will be called pseudo equilibria (which are equilibria only in the transformed state). We establish the structure (both local and global properties) of the singular system Σ by analyzing the transformed smooth system, and this formulation is a novel contribution of our work.

State space dynamics is analyzed in section I.1 and I.2. The main objective is to characterize the stability boundary ∂A of the region of attraction A of a stable equilibrium point. For studying the structure of the stability boundary, certain regularity assumptions are imposed which essentially identify the most regular structure of the dynamics. For instance it is assumed that the system (1) and (2) admits a dissipating energy function. With these assumptions, it is shown that the stability boundary consists of segments associated with different kinds of anchor points including stable manifolds of unstable equilibria and periodic orbits, and of portions of trajectories anchored to the singular surface at what will be called here the pseudo equilibria or semi-singular points. Also certain sections of the singular surface where trajectories originate and the system is transversal to the singularity may lie in the stability boundary.

The structure of the parameter space is summarized in Chapter I.3. The concept of a feasibility region is introduced, which is inspired by the notion of reachable set. It will be shown that three bifurcation surfaces constitute the boundary of the feasibility region, namely the saddle node, Hopf and the singularity induced bifurcation surfaces. The singularity induced bifurcation, which arises when the equilibrium is at the singularity, is rigorously analyzed in Sections I.3.3 and I.3.4. A brief introduction to the concept of typical regions is presented in Section I.3.5.

Note that the proofs for all the Lemmas and Theorems stated in this Chapter were published in the form of a technical report [91] and in several of our publications. In this Chapter, we only summarize the salient results.

I.1 State space structure of the constrained dynamics Σ

A fixed parameter p_0 is considered in equations (1) and (2) and for notational simplicity the parameter p is dropped. The equations are written in the form

$$\Sigma: \quad \dot{x} = f(x, y), \quad f: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^n \quad (5)$$

$$0 = g(x, y), \quad g: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^m \quad (6)$$

I.1.1 An overview

The following sets largely define the structure of the state space:

$$L = \{(x, y) \in \mathbb{R}^{n+m} : g(x, y) = 0\} \quad (7)$$

$$M = \{(x, y) \in L : \text{rank}(D_x g(x, y), D_y g(x, y)) = m\} \quad (8)$$

$$S = \{(x, y) \in L : \Delta(x, y) := \det D_y g(x, y) = 0\} \quad (9)$$

$$\Psi = \{(x, y) \in S : \kappa(x, y) := \text{adj}(D_y g(x, y)) D_x g(x, y) f(x, y) = 0\} \quad (10)$$

$$\Xi = \{(x, y) \in S \setminus \Psi : D_y \Delta(x, y) \kappa(x, y) = 0\} \quad (11)$$

$$R = S \setminus (\Psi \cup \Xi) \quad (12)$$

L is the constraint surface determined by the algebraic constraint. The system must stay on L and henceforth all topological definitions are relative to L . M is a regular subset of L which, by the implicit function Theorem, is an n -dimensional manifold. S is called the singular surface or the singularity. Typically no solution exists for points in S . Ψ is called the *pseudo equilibrium surface*. This high dimensional subset displays the behavior of an equilibrium surface for the vector field Z (equation (17)) induced by the system Σ and indeed becomes an equilibrium surface for an “equivalent” smooth dynamical system defined by a transformed vector field Z^T (equation (20)). Z and Z^T are related by a singular transformation. The details of this transformation follow in Section I.1.3. The pseudo equilibrium surface is a true equilibrium surface for the transformed vector field Z^T . It is useful to distinguish the following subsets of Ψ :

$$N_\psi = \{(x, y) \in \Psi \cap M : \text{rank}(D_y g) = m - 1 \text{ and the Jacobian of } Z^T \text{ has exactly two eigenvalues with nonzero real parts}\} \quad (13)$$

$$B_\psi = \Psi \setminus N_\psi \quad (14)$$

Typically, the Jacobian of Z^T (restricted to M) has at least $(n - 2)$ zero eigenvalues at a point in Ψ (see Section I.1.5.2). Hence N_ψ corresponds to the subset of Ψ where the most regular type behavior takes place. It will be shown that near every point in N_ψ the integral curves of the system locally behave like for a linear system, and the points in the set N_ψ will be

called the *nice* pseudo equilibrium points. The local dynamics near the points in the set B_ψ is more complex, and we call these points the *bad* pseudo equilibrium points. Finally, Ξ corresponds to points where Z^T is tangent to the singularity (see Section I.1.5.1). Henceforth points in Ξ will be called *semi-singular*. Let

$$N_\xi = \{(x, y) \in \Xi : \text{rank} \begin{pmatrix} D_x g & D_y g \\ D_x \Delta & D_y \Delta \end{pmatrix} = m + 1, \text{ and } D_y \{(D_y \Delta) \kappa\} \neq 0\} \quad (15)$$

$$B_\xi = \Xi \setminus N_\xi \quad (16)$$

We call the points in the set N_ξ the *nice* semi-singular points, and the points in the set B_ξ the *bad* semi-singular points. The condition on the second derivative in the definition of N_ξ ensures that trajectories do not cross over as they touch the singularity. R is the complement of Ψ and Ξ in S and it consists of ordinary singular points i.e. at each point z in R either two trajectories originate from z or converge to z at infinite speed.

I.1.1.1 Singular dynamics hierarchy Theorem

The properties of these various subdivisions of the state space are summarized in the next Theorem which will be proved and illustrated in Sections I.1.2-I.1.5.

Theorem 1 (Singular dynamics hierarchy Theorem) [86]
If the subsets are not empty, then

1. $L \setminus S$ is an n -dimensional manifold. The solutions of the system (5) and (6) are the integral curves of the vector field Z on $L \setminus S$.
2. R is an $(n - 1)$ -dimensional manifold. At each point in R two trajectories either approach or leave the singularity transversally.
3. N_ψ is an $(n - 2)$ -dimensional manifold. Ignoring orientations, the solution curves are locally equivalent to those of a linear system near the origin.
4. N_ξ is an $(n - 2)$ -dimensional manifold. Trajectories are tangent to the singularity at N_ξ , but do not cross over. $\square$

In Section I.1.3, the singular transformation is introduced. Using the transformation, then we will analyze the constrained dynamics Σ by starting with the constraint surface L . At each step, we leave out a lower dimensional subset with a more degenerate behavior, thus identifying the most regular behavior for that level in the analysis.

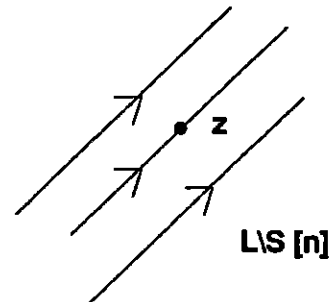


Figure I.1. Dynamics near Regular points in $L \setminus S$.

I.1.2 Codimension 0 dynamics

Lemma 1 *Restricted to $L \setminus S$, the dynamics defined by equations (5) and (6) is locally induced by the vector field Z (i.e. described by an ordinary differential equation),*

$$Z = \begin{pmatrix} f(x, y) \\ -(D_y g(x, y))^{-1} D_x g(x, y) f(x, y) \end{pmatrix} \quad (17)$$

For any point in the set $L \setminus S$, the Jacobian $D_y g$ is non-singular and hence $L \setminus S \subset M$. Therefore the set $L \setminus S$ is an n -dimensional manifold, and the dynamics Σ restricted to this set $L \setminus S$ is smooth. For any point $z \in L \setminus S$, there exists a unique trajectory which passes through z . When $z \in L \setminus S$ is not an equilibrium point for the vector field Z , it is well-known that the flow defined by Z can be nicely characterized locally near z (Flow Box Theorem [3]) as shown in Figure I.1. In the illustrations, the corresponding dimension for each set is shown in square brackets. The connected components of the set $L \setminus S$ will be called in this paper the *components* of the state space.

I.1.3 Singular transformation

In [78], Takens proposed a singular transformation for analyzing the local dynamics near the singular points, by multiplying the vector fields with the determinant $\det(D_y g)(x, y)$. For our system (1)-(2), the transformation can be extended globally using the standard properties of the matrix adjoint,

$$\text{adj}(D_y g) D_y g = D_y g \text{adj}(D_y g) = \det(D_y g) I_n \quad (18)$$

and

$$(D_y g)^{-1} = \frac{\text{adj}(D_y g)}{\det(D_y g)} \text{ when } \det(D_y g) \neq 0. \quad (19)$$

Using these two properties, let us define the transformed vector field Z^T to be

$$\begin{aligned} Z^T &:= Z \det(D_y g) = Z \Delta(x, y) \\ &= \begin{pmatrix} f(x, y) \Delta(x, y) \\ -\text{adj}(D_y g) D_x g(x, y) f(x, y) \end{pmatrix} \end{aligned} \quad (20)$$

The flow of the vector field Z describes the dynamics of the system. The transformed vector field Z^T is globally smooth and leaves the constraint surface invariant. Z^T will be used in the proofs thus facilitating standard nonlinear analysis. The results for the singular systems can be readily reinterpreted for the DAE system Σ as implied by the next Lemma. Therefore this singular transformation serves as a powerful tool for analyzing the singular dynamics of Σ using *standard* (not nonstandard which is the practice in [126]) mathematical analysis.

The vector field Z^T is obtained when the vector field Z is pointwise scaled by the determinant $\Delta(x, y)$. Define

$$(L \setminus S)_+ = \{(x, y) \in L \setminus S : \Delta(x, y) > 0\} \quad (21)$$

$$(L \setminus S)_- = \{(x, y) \in L \setminus S : \Delta(x, y) < 0\} \quad (22)$$

Clearly, $(L \setminus S) = (L \setminus S)_+ \cup (L \setminus S)_-$ as $\Delta(x, y) \neq 0$ in $L \setminus S$.

Lemma 2 *The vector fields Z and Z^T , restricted to $(L \setminus S)_+$, are equivalent. When restricted to $(L \setminus S)_-$, Z and $-Z^T$ are equivalent.*

In particular, if (x_0, y_0) is an equilibrium point for the vector field Z , then it is also an equilibrium for the new vector field Z^T , and the eigenvalues in the transformed system are simply

scaled by $\Delta(x_0, y_0)$. As curves, the trajectories of Z and Z^T are identical in $L \setminus S$. Whereas Z is not defined on S , Z^T defines a smooth vector field on all of the constraint surface L . The transformed vector field Z^T has an additional set of equilibrium points in the pseudo equilibrium surface Ψ .

The subset M of L is an n -dimensional manifold by implicit function Theorem. By definition $L \setminus M \subset S$. Moreover, it is interesting that all such "non-manifold" points on the constraint surface are pseudo equilibrium points.

Lemma 3 $L \setminus M \subset \Psi$

Furthermore, by the definition of the classical adjoint for matrices,

$$\text{adj}(D_y g) D_y g = D_y g \text{adj}(D_y g) = 0 \quad (23)$$

i.e.

$$\text{adj}(D_y g) \in \text{kernel}(D_y g) \quad (24)$$

Equations (23) and (24) imply that, at (x_0, y_0) , $\text{adj}(D_y g) D_x g = 0$, i.e. $(x_0, y_0) \in \Psi$. Q.E.D.

Lemma 4 *Any integral curve of the vector field Z which approaches the singular surface has a unique limit on the singularity and this limit agrees with the limit of the solution curve in the transformed system.*

Lemma 4 says that Z^T provides a canonical extension of the vector field Z to the singularity. It should be noted that the orientation of the trajectories is reversed for the points in $(L \setminus S)_-$, as $\Delta(x, y) < 0$.

I.1.4 Codimension 1 dynamics

In Section I.1.1, we could easily show that the Σ dynamics is smooth when restricted to $L \setminus S$. Now let us consider the next degenerate subset $S \subset L$.

Lemma 5 $R = S \setminus (\Psi \cup \Xi)$ is an $(n-1)$ -dimensional embedded submanifold of M and Z^T is transversal to R .

Since $R \cap \Psi = \emptyset$, by Lemma 3, we conclude that $R \subset M$. Thus by Lemma 5, it follows that $R \subset S$ is a codimension one submanifold of M . Let us consider any point $z \in R$. Then since $z \notin \Psi$, z is a regular point for the flow of the transformed vector field Z^T . Therefore by the Flow Box Theorem [3], there exists a unique trajectory of Z^T which passes through z . Again by Lemma 5, the vector field Z^T is transversal to R at z and so, the trajectory which passes through z crosses the singular surface R transversally at z . From the proof of Lemma 5, the determinant $\Delta(x, y)$ must change sign at $z \in R$ when the trajectory crosses $z \in R$. This is because along the trajectory $(x(t), y(t))$ which passes through $z \in R$, by transversal intersection, we have that

$$\frac{d}{dt} \Delta(x(t), y(t))|_z = (D_x \Delta, D_y \Delta) Z^T|_z = -((D_y \Delta) \kappa)|_z \neq 0. \quad (25)$$

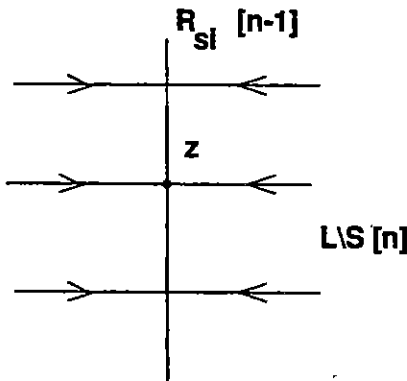
Transforming back to the original system Σ , we conclude that for each $z \in R$, either two trajectories of Z approach or leave z transversal to R . Hence depending on the sign of the term $(D_y g) \kappa$, we define the first case as singular sinks R_{ss} or singular sources R_{so} respectively, that is,

$$R_{si} := \{(x, y) \in R : (D_y \Delta) \kappa(x, y) > 0\} \quad (26)$$

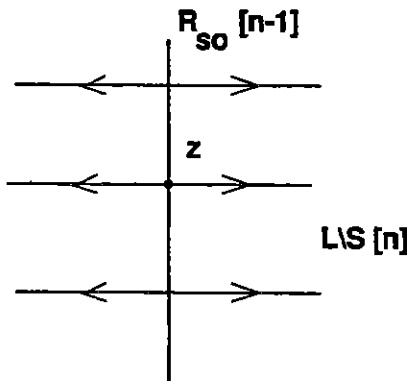
$$R_{so} := \{(x, y) \in R : (D_y \Delta) \kappa(x, y) < 0\} \quad (27)$$

By equation (25), we conclude that for any point $z \in R_{si}$, the determinant $\Delta(x(t), y(t))$ decreases along the transformed system trajectory $(x(t), y(t))$. Hence for Σ , two trajectories approach $z \in R_{si}$ transversally as shown in Figure I.2a. Similarly we conclude that at any $z \in R_{so}$, two trajectories of Σ leave z transversally away from R . Since the vector field Z^T is nonzero at $z \in R$ and $\Delta = 0$ at $z \in R$, it is clear that the two trajectories for the vector field Z (which is obtained by dividing Z^T by Δ) will have unbounded velocities as they approach the point $z \in R$.

Special singular points such as the transverse sources and transverse sinks above where the trajectories of the constrained dynamics Σ have been a subject of much interest in the theory of nonlinear networks. In [23], points where positive (negative) time-continuation of solutions is not possible are define *forward (backward) impasse points*. Using Lyapunov-Schmidt



(a) Transverse sinks



(b) Transverse sources

Figure I.2. Dynamics near Singular sinks R_{si} and Singular sources R_{so}

reduction arguments, it is proved in [23] that impasse points (forward and backward as defined above) must be generic within the set of singular points S . The analysis in this section proves that *transverse sources are backward impasse points and transverse sinks are forward impasse points for the DAE system Σ* . In our geometric analysis of codimension one dynamics, the transverse sources and transverse sinks have been derived analytically with explicit easily verifiable algebraic formulas (by construction), and these naturally emerge as the least degenerate objects within the singularity S . In other words, transverse

sources and transverse sinks tell us *how* to identify the forward impasse points and the backward impasse points for the model Σ (generically) by simple formulas. Now we proceed with the analysis of the next level of the hierarchy, namely with the codimension two dynamics.

I.1.5 Codimension 2 dynamics

We left out the subsets, the set of pseudo equilibria Ψ and the set of semi-singular points Ξ for establishing the principal structure R of the singular set S . In this section, we will look at the “nice” subsets of Ψ and Ξ and show that locally near these points, the singular dynamics Σ is well-understood.

I.1.5.1 Nice Semi-singular points:

Lemma 6 *If nonempty, then N_ξ is an $(n-2)$ -dimensional embedded submanifold of M . The vector field Z^T has a nonzero normal component at every point in N_ξ .*

Since Z^T has a normal component for any point $\xi \in N_\xi$, the trajectories of Z^T which reach any $\xi \in N_\xi$ strictly leave the set N_ξ . Let us proceed with a detailed analysis of the local picture near these points. First we construct a special set of chart coordinates, which reduce the dynamics of Z^T to a canonical form near the singular points.

Define the coordinates η_1 and η_2 in (x, y) coordinates as

$$\eta_1 = \Delta(x, y), \quad \eta_2 = D_y \Delta(x, y) \kappa(x, y) \quad (28)$$

and complete the coordinate system η_i in a suitable way. We claim that this is possible because the Jacobian say J_{ss} involved in the coordinate change is of full rank [91]. In η_i -coordinates, let the system be described as

$$\dot{\eta}_1 = \beta_1(\eta_1, \eta_2, \eta_r), \quad (29)$$

$$\dot{\eta}_2 = \beta_2(\eta_1, \eta_2, \eta_r), \quad (30)$$

$$\dot{\eta}_r = \beta_r(\eta_1, \eta_2, \eta_r). \quad (31)$$

Then the singular surface S is locally given as $\{\eta_1 = 0\}$ and N_ξ is the coordinate plane $\{\eta_1 = 0, \eta_2 = 0\}$. Furthermore, since $(D_y \Delta) \kappa = 0$ in Ξ ,

$$\beta_1(0, 0, \eta_r) = (D_y \Delta) \kappa|_{N_\xi} = (D_{y_1} \Delta_1) g_{1|N_\xi} = 0, \quad (32)$$

and since $D_y \{(D_y \Delta) \kappa\} \neq 0$ in N_ξ ,

$$\beta_2(0, 0, \eta_r) = D_y \{(D_y \Delta) \kappa\}|_{N_\xi} = (D_{y_1} \varphi_1) g_{1|N_\xi} \neq 0. \quad (33)$$

Since $\Xi \cap \Psi = \emptyset$, there are no equilibrium points of Z near the singularity. Hence the local picture is as shown in Figure I.3.

As observed earlier, it is clear that the flow of Z^T is tangent to the singularity at any point $\xi \in N_\xi$. But due to our assumption on the second derivative,

$$\dot{\eta}_1 = \frac{d}{dt}|_{(0,0,\eta_r)} \beta_1(\eta_1, \eta_2, \eta_r) = D_y \{(D_y \Delta) \kappa\}|_{N_\xi} \neq 0 \quad (34)$$

Suppose we look at one side of the singular surface first, say at a component C . Without loss of generality assume that $\Delta(x, y) > 0$ in C (otherwise redefine Δ as $-\Delta$). If $D_y \{(D_y \Delta) \kappa\} > 0$ at ξ , then

$$\frac{d}{dt}|_\xi \Delta(x, y) = 0 \quad \text{and} \quad \frac{d^2}{dt^2}|_\xi \Delta(x, y) > 0. \quad (35)$$

In C there exists a unique trajectory which reaches ξ in finite time and immediately leaves the singularity again to the same side. Combining this fact with the Flow Box Theorem [3] for the transformed vector field Z^T at ξ , we have established the local phase portrait near ξ is as shown in Figure I.3.

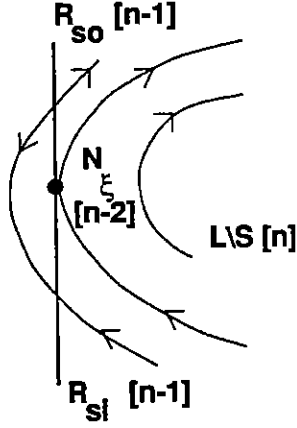


Figure I.3. Dynamics near nice semi-singular points N_ξ .

I.1.5.2 Nice pseudo equilibrium points:

The pseudo equilibrium surface Ψ which is an equilibrium surface for the transformed vector field Z^T can be split into a nice set N_ψ and a bad set B_ψ as defined in equations (13) and (14). The properties of the set N_ψ will be explored in this section.

Lemma 7 *If nonempty, then N_ψ is an $(n-2)$ -dimensional embedded submanifold of M .*

Lemma 8 [86] *For every point $\psi \in N_\psi$, there exists a neighborhood U of ψ in M , and a coordinate chart $(\eta_1, \dots, \eta_n)$ defined on U , such that*

- (o) *the chart is centered at ψ , i.e. $\eta_i(\psi) = 0$ for all i*
- (i) *the dynamics take the form*

$$\dot{\eta}_i = f_i^1 \eta_1 + f_i^2 \eta_2 \quad (36)$$

for some functions $f_i^k : \eta(U) \subset \mathbb{R}^n \rightarrow \mathbb{R}$, $k = 1, 2$ and $i = 1, \dots, n$

- (ii) $S \cap U = \{\eta_1 = 0\}$
- (iii) $N_\psi \cap U = \{\eta_1 = 0 \text{ and } \eta_2 = 0\}$. $\square$

In these coordinates, the origin is the pseudo equilibrium and the Jacobian of the vector field Z^T has $n-2$ zero eigenvalues and 2 nonzero eigenvalues. Hence, the equilibrium points in the set N_ψ are nonhyperbolic. But they are special in the following sense:

Lemma 9 [86] *The local center manifold for any point $\psi \in N_\psi$ in the nice set is N_ψ itself.*

The set N_ψ can be split into three different subsets, depending on the stability status. Define

$$N_{so} := \{\psi \in N_\psi : \text{Jacobian of } Z^T \text{ has two eigenvalues in } \mathbb{C}^+\} \quad (37)$$

$$N_{sa} := \{\psi \in N_\psi : \text{Jacobian of } Z^T \text{ has one eigenvalue in } \mathbb{C}^- \text{ and one in } \mathbb{C}^+\} \quad (38)$$

$$N_{si} := \{\psi \in N_\psi : \text{Jacobian of } Z^T \text{ has two eigenvalues in } \mathbb{C}^-\} \quad (39)$$

So $N_\psi = N_{so} \cup N_{sa} \cup N_{si}$. Note that any component of N_ψ entirely lies in one of the three sets. Let us call the points in N_{so} , N_{sa} and N_{si} , *pseudo sources*, *pseudo saddles* and *pseudo sinks* respectively. This definition is motivated by the next Lemma which shows that the local dynamics for Z^T near the nice pseudo equilibrium manifold N_ψ looks like the linear dynamics of a $(n-2)$ -dimensional continuum of second order systems.

Lemma 10 *For $\psi \in N_\psi$, the flow of Z^T is locally equivalent to a linear system defined as:*

$$\dot{x}_u = x_u, \quad \dot{x}_s = -x_s, \quad \dot{x}_c = 0, \quad x_c \in \mathbb{R}^{n-2}. \quad (40)$$

The dimensions of x_u and x_s depend on the particular set in N_ψ , i.e.

$$\begin{aligned} \dim(x_u) &= 2 \text{ and } \dim(x_s) = 0, \text{ when } \psi \in N_{so} \\ \dim(x_u) &= 1 \text{ and } \dim(x_s) = 1, \text{ when } \psi \in N_{sa} \\ \dim(x_u) &= 0 \text{ and } \dim(x_s) = 2, \text{ when } \psi \in N_{si} \end{aligned}$$

I.1.5.2.1 Transverse Pseudo-saddles:

The dynamics for the transformed vector field Z^T locally is equivalent to the dynamics of the linear system (40) near the pseudo saddle manifold with $\dim(x_u) = \dim(x_s) = 1$ and $\dim(x_c) = n-2$, by Lemma 10. Consider any $\psi \in N_{sa}$. Since the equivalent linear system has a one dimensional stable and unstable manifolds, it directly follows that ξ has one dimensional stable and unstable manifolds for Z^T . But we are interested in analyzing the dynamics of Z . Locally near ψ , the singular set is a manifold by Lemma 8 and the determinant Δ changes sign at the singular set in a local neighborhood. Therefore the trajectories of Z^T need to be reversed on one side of the singular set for finding the integral curves of Z by Lemma 2. Hence we need to carefully look at the relative position of the singular set, compared to the stable and unstable manifolds.

A pseudo saddle is called *transverse* if its stable and unstable manifolds are not tangent to the singularity. Define the set of transverse pseudo saddles as $N_{tr,sa}$. Let us analyze the local dynamics near pseudo saddles using the chart in Lemma 8. Locally near any pseudo saddle $\psi \in N_{sa}$, the singular surface is described by setting $\eta_1 = 0$ and the pseudo equilibria (which locally are pseudo saddles) are characterized by setting $\eta_1 = 0$ and $\eta_2 = 0$. Clearly the Jacobian of the dynamics evaluated at ψ has $n-2$ zero eigenvalues. Define e_k as the unit vector along the η_k axis. Then it is easy to see that $\{e_3, e_4, \dots, e_n\}$ form the set of eigenvectors for the zero eigenvalues. If the stable (unstable) manifold is tangential to the singularity at ψ , then the eigenvector for the stable (unstable) eigenvalue lies on the singular surface $\eta_1 = 0$. Hence in particular at ψ , f_1^2 must be zero for non-transverse pseudo saddles. Hence, generically tangentiality of the stable (unstable) manifold to the singularity, occurs only at a lower dimensional subset of N_{sa} . In other words, generically, pseudo saddles are non-transverse only at lower dimensional subsets. In the general spirit of our approach, let us first look at the transverse pseudo-saddles as the most regular structure in N_{sa} .

The local dynamics for the vector field Z near transverse pseudo saddles is shown in Figure I.4. Near any transverse pseudo saddle, as the stable and unstable manifolds are transversal to the singularity, the singularity divides a neighborhood of ψ into two parts, each containing one stable and one unstable trajectory as shown in Figure I.4. Since the trajectories get reversed on one side of the singular set from Z^T to Z , the unstable

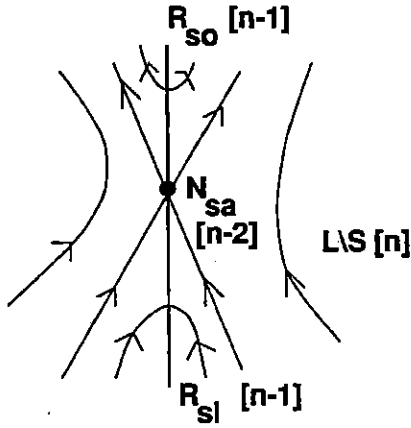


Figure I.4. Dynamics near transverse pseudo saddles $N_{tr,sa}$.

and stable manifolds of the pseudo saddles for Z^T become continuous trajectories for Z as shown in Figure 5.

I.1.5.2.2 Pseudo sinks and pseudo sources

Let us consider pseudo sinks first. By definition, the Jacobian of Z^T has two stable eigenvalues at a pseudo sink. By Lemma 10, it follows that the local dynamics of Z^T near a pseudo sink is topologically equivalent to the dynamics of the linear system $\dot{x}_1 = -x_1$ and $\dot{x}_2 = 0$ where x_1 is two dimensional and x_2 is $(n-2)$ -dimensional. For the transformed system defined by Z^T , then the local picture near N_{si} will look like that of a $(n-2)$ -dimensional continuum of stable 2-dimensional nodes or 2-dimensional stable foci, and this will depend on the nature of the two nonzero eigenvalues. Lemma 10 is not sufficient to distinguish between the node and focus, but using standard arguments for second order systems such as in [9], it can be proved that locally the dynamics for these cases (for nodes and foci) are both well-understood and are quite different. This is not shown here due to space limitations. The local picture near pseudo sources follows from the analysis of the pseudo sinks by reversing the time scale.

I.1.6 Codimension 3 and higher

I.1.6.1 Bad semi-singular points

When $\xi \in B_\xi$, since semi-singular points can not be equilibria by their definition, there still exists a unique nontrivial trajectory for Z^T which passes through any $\xi \in \Xi$ and is tangential to the singular surface.

I.1.6.2 Non-transverse pseudo saddles

Consider any non-transverse pseudo saddle ψ . By the local equivalence Lemma 10, for Z^T , there exist two stable trajectories and two unstable trajectories for the non-transverse pseudo saddle ψ . The actual dynamics of Σ depends on the relative position of the singular set at ψ . When the stable manifold is tangential to the singularity different possibilities arise, depending on one, two or three of the four stable and unstable trajectories on one side of the singular set. Also a stable or unstable trajectory may entirely lie in the singular set when bad semi-singular points are nearby.

I.1.6.3 Bad pseudo equilibrium points

Certainly the most interesting elements of the codimension three subsets are the bad pseudo equilibrium points. Here we have defined the nice pseudo equilibria as the set, where the Jacobian has exactly two nonzero eigenvalues ($n-2$ are zero since generally Ψ is $(n-2)$ -dimensional). Generally at a bad pseudo equilibrium, one or two of the remaining two eigenvalues lie on the imaginary axis. When the Jacobian has exactly one nonzero eigenvalue, and certain inequality conditions are satisfied, we define the points to be *pseudo saddle nodes*. The dynamics near these points has a rich structure, but will not be discussed here due to space limitations. The bad pseudo equilibria where the two eigenvalues are purely imaginary may also be interesting. More degenerate bad pseudo equilibria such as when all the eigenvalues are zero may be quite difficult to analyze and it is an open question whether the dynamics near those points will have any nice structure.

I.2. Analysis of the stability regions

The objective of this section is to study the stability boundary ∂A of the region of attraction A of a stable equilibrium point. Instead of restricting the analysis to the region of attraction A defined above, it is practically more meaningful to consider the bigger region, the interior of the closure, $\text{int}(\bar{A})$. With small perturbations always present in the system, any trajectory in this region $\text{int}(\bar{A})$, will converge to the stable equilibrium point [114]. Hence, we will analyze the quasi-stability boundary $\partial \bar{A}$, the boundary of the region $\text{int}(\bar{A})$.

The tools used in the analysis are 1) standard geometric tools such as the λ -Lemma [57] and its extensions for pseudo equilibria and semi-singular points [86, 91], 2) the invariance properties [91] and a technical result called here 3) the Basic Anchor Theorem (see Section I.2.1). After making some assumptions in Section I.2.2, the necessary and sufficient conditions for testing the presence of anchor points on the stability boundary ∂A will be developed in Sections I.2.3 and I.2.3. The characterization of the quasi stability boundary $\partial \bar{A}$ follows in Section I.2.4. Energy function based stability tests are developed in Section I.2.5.

I.2.1 Basic properties

Equilibrium points and periodic orbits are defined as usual for the singular dynamics Σ , with the proviso that they do not intersect the singular set S . Stable equilibrium points define the system operating points for general physical systems of the form Σ such as the large power system. Given a stable equilibrium point z_s , the set of all trajectories which converge to the point z_s generally define the region of attraction for z_s . However for the singular system of the form Σ , we restrict our definition of the region of attraction and other stable and unstable manifolds to the state space component C (connected component of $L \setminus S$) which contains z_s [86]. (see also the related notion of *causal regions* proposed in [42, 40] where a state space decomposition is proposed based on the eigenproperties of the Jacobian $D_\psi g$). This definition is strongly motivated by the dynamics of the large power system, where the singular set signals the onset of unpredictable system behavior [86].

The *region of attraction* of a stable equilibrium z_s is defined as

$$A = \{z \in C : \Phi_t(z) \in C \forall t \geq 0, \Phi_t(z) \rightarrow z_s \text{ as } t \rightarrow \infty\}. \quad (41)$$

Similarly define the stable and unstable manifolds for equilibria and periodic orbits in the usual way, but with the restriction to the component. The stable and unstable manifolds for a pseudo equilibrium Ψ and a semi-singular point ξ is similar to those for the equilibrium, but the convergence may occur in finite time. More details can be found in [86, 91].

Note that ignoring the speed, the trajectories of the vector fields Z and Z^T are identical (topologically equivalent) in C by Lemma 2. Since the trajectories (integral curves to be more precise) of Z coincide with the trajectories of Z^T within the component C , the stability regions defined above for the flow Φ_t of Z can be equivalently redefined in terms of the flow Φ_t^T of the transformed vector field Z^T , by restricting the trajectories of Z^T to the component C as shown for the region of attraction A .

$$A = \{z \in C : \Phi_t^T(z) \in C \forall t \geq 0, \Phi_t^T(z) \rightarrow z_s \text{ as } t \rightarrow \infty\} \quad (42)$$

Hence the boundary of the region of attraction ∂A (with respect to L) can be analyzed equivalently using the transformed vector field. Note that the trajectories of Z^T demand a more careful analysis near the singular surface to be consistent with the DAE Σ , as explained in Section I.1.

In this section, we present results on the internal structure of the stability boundaries. This analysis then paves the way for characterization of the stability boundary ∂A in the next section. The results presented in this section are extensions of the results for unconstrained systems in [57], [114] and [18]. Our results cover the boundary characterization results for smooth systems in [114] and [18] as the special case when the singular set S is empty. Though most of the results use the transformed vector field Z^T and its flow Φ_t^T , we want to emphasize that the results stated below are directly valid for stable manifolds and stability boundaries for the vector field Z .

For some of the results, we will assume that the intersections of stable and unstable manifolds are transversal. Let N_c denote a connected component of either N_ψ or N_ξ . Let B_c denote a connected component of the set of bad pseudo equilibria B_ψ . The assumption can be precisely stated as follows:

(A2) Restricted to $\bar{A}$, stable manifolds from the set $\{W^s(z_0), W^s(\gamma_0), W^s(N_c), W^s(B_c)\}$ intersect transversally with unstable manifolds from the set $\{W^u(z_0), W^u(\gamma_0), W^u(N_c), W^u(B_c)\}$. $\square$

I.2.1.1 Basic Anchor Theorem

Let us start with a definition.

Definition 1 *Equilibrium points, periodic orbits, pseudo equilibria and semi-singular points in the boundary of the region of attraction will be called anchor points.* $\square$

Our objective in this section is to study the properties of the stable and unstable manifolds of anchor points, when they are present on the stability boundaries. This preliminary analysis sets up the basis for stronger conditions to be developed in the Chapter 8. The main results in this section are summarized below. The proofs follow by breaking up the Theorem into several Lemmas after the statement of the Theorem.

Theorem 2 (Basic Anchor Theorem) *Suppose all equilibria z, z_0 and periodic orbits γ, γ_0 are hyperbolic and all pseudo equilibria ψ, ψ_0 and semi-singular points ξ, ξ_0 lie in the nice sets in the statement below. For the equivalences marked with an (*), also assume that assumption (A2) on the transversal intersections of stable and unstable manifolds holds. Furthermore, ν is used to label transverse pseudo saddles or semi-saddles. Then*

1. For an equilibrium point z_0

$$\begin{aligned} z \in \partial W^s(z_0) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(z_0)} \neq \emptyset \\ (*) z \in \partial W^s(z_0) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(z_0) \neq \emptyset \\ \gamma \in \partial W^s(z_0) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(z_0)} \neq \emptyset \\ (*) \gamma \in \partial W^s(z_0) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(z_0) \neq \emptyset \\ \nu \in \partial W^s(z_0) &\iff W^u(\nu) \cap \overline{W^s(z_0)} \neq \emptyset. \end{aligned} \quad (43)$$

[If z_0 is stable, no transversality assumptions are required.]

2. For a periodic orbit γ_0

$$\begin{aligned} z \in \partial W^s(\gamma_0) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(\gamma_0)} \neq \emptyset \\ (*) z \in \partial W^s(\gamma_0) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(\gamma_0) \neq \emptyset \\ \gamma \in \partial W^s(\gamma_0) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(\gamma_0)} \neq \emptyset \\ (*) \gamma \in \partial W^s(\gamma_0) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(\gamma_0) \neq \emptyset \\ \nu \in \partial W^s(\gamma_0) &\iff W^u(\nu) \cap \overline{W^s(\gamma_0)} \neq \emptyset. \end{aligned} \quad (44)$$

3. For a connected component N_C of the nice set, with $\nu \notin \partial N_C$,

$$\begin{aligned} z \in \partial W^s(N_C) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(N_C)} \neq \emptyset \\ (*) z \in \partial W^s(N_C) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(N_C) \neq \emptyset \\ \gamma \in \partial W^s(N_C) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(N_C)} \neq \emptyset \\ (*) \gamma \in \partial W^s(N_C) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(N_C) \neq \emptyset \\ \nu \in \partial W^s(N_C) &\iff W^u(\nu) \cap \overline{W^s(N_C)} \neq \emptyset. \end{aligned} \quad (45)$$

4. For a connected component B_C of the bad set, with $\nu \notin \partial B_C$,

$$\begin{aligned} z \in \partial W^s(B_C) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(B_C)} \neq \emptyset \\ (*) z \in \partial W^s(B_C) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(B_C) \neq \emptyset \\ \gamma \in \partial W^s(B_C) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(B_C)} \neq \emptyset \\ (*) \gamma \in \partial W^s(B_C) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(B_C) \neq \emptyset \\ \nu \in \partial W^s(B_C) &\iff W^u(\nu) \cap \overline{W^s(B_C)} \neq \emptyset. \end{aligned} \quad (46)$$

I.2.2 Assumptions

In this section, we state certain assumptions which are extensions of the assumptions for the Morse-Smale systems of smooth dynamical systems.

Assumptions (A): It will be assumed that the following assumptions are satisfied in the closure of the region of attraction $\bar{A}$:

(A0) The nice sets N_ψ and N_ξ are dense in Ψ and Ξ , respectively. Furthermore, at every point $\psi \in \Psi$ or $\xi \in \Xi$, the sets B_ψ and B_ξ have dimension at most $(n-3)$.

(A1) (a) Equilibria z_0 and periodic orbits γ_0 in the boundary of the region of attraction, ∂A , are hyperbolic.
(b) Except for sets of dimension at most $(n-3)$, all pseudo saddles in ∂A are transverse.

(A2) Stable manifolds from the set $\{W^s(z_0), W^s(\gamma_0), W^s(N_C), W^s(B_C)\}$ intersect transversally with unstable manifolds

from the set $\{W^u(z_0), W^u(\gamma_0), W^u(N_C), W^u(B_C)\}$. Here N_C stands for a connected component of either N_ψ or N_ξ and B_C stands for a connected component of B_ψ or B_ξ .

(A3) All trajectories converge to an equilibrium point, a periodic orbit, a pseudo equilibrium point or a semi-singular point. $\square$

It can be shown that generally assumptions (A0)–(A2) will be satisfied for the system Σ [90, 91]. However Assumption (A3) is not generic, but will be satisfied if there exists a dissipating energy function [86].

Lemma 11 *If there exists a Lyapunov-function $V = V(x, y)$ such that*

1. $D_x V f - D_y V (D_y g)^{-1} D_x g f \leq 0$ in A
2. V is proper in the closure of A
3. the ω -limit set in $\bar{A}$ consists only of equilibria and periodic orbits

then assumption (A3) is satisfied. All trajectories in ∂A converge to equilibria, periodic orbits, pseudo equilibria or semi-singular points.

I.2.3 Analysis of the stability boundary ∂A

From here onwards Assumptions (A) will be in effect in this chapter, unless stated otherwise. The composition of the stability boundary is established first.

Theorem 3 (Stability Boundary Theorem) [91] *Under assumptions (A) the stability boundary is composed of*

1. *Stable manifolds of unstable equilibria $z_0 \in \partial A$,*

$$\bigcup_{z_0 \in \partial A} \{z \in C : \Phi_t^T(z) \rightarrow z_0 \text{ as } t \rightarrow \infty \text{ and } \Phi_t^T(z) \in C \forall t > 0\} \quad (47)$$

2. *Stable manifolds of unstable periodic orbits $\gamma_0 \subset \partial A$,*

$$\bigcup_{\gamma_0 \subset \partial A} \{z \in C : \Phi_t^T(z) \rightarrow \gamma_0 \text{ as } t \rightarrow \infty \text{ and } \Phi_t^T(z) \in C \forall t > 0\} \quad (48)$$

3. *Stable manifolds of transverse pseudo saddles $N_{tr,sa} \cap \partial A$,*

$$\bigcup_{\psi \in (N_{tr,sa} \cap \partial A)} \{z \in C : \Phi_t^T(z) \rightarrow \psi \text{ as } t \rightarrow \infty \text{ and } \Phi_t^T(z) \in C \forall t > 0\} \quad (49)$$

4. *Stable manifolds of semi-saddles $N_{se,sa} \cap \partial A$,*

$$\bigcup_{\xi \in (N_{se,sa} \cap \partial A)} \{z \in C : \exists t_0 > 0 \text{ such that } \Phi_t^T(z) \rightarrow \xi \text{ as } t \rightarrow t_0 \text{ and } \Phi_t^T(z) \in C \forall 0 < t < t_0\} \quad (50)$$

5. *Stable manifolds of other pseudo equilibria $(\Psi \setminus N_{se,sa}) \cap \partial A$,*

$$\bigcup_{\psi \in ((\Psi \setminus N_{se,sa}) \cap \partial A)} \{z \in \partial A : \Phi_t^T(z) \rightarrow \psi \text{ as } t \rightarrow \infty \text{ and } \Phi_t^T(z) \in C \forall t > 0\} \quad (51)$$

6. *Stable manifolds of other semi-singular points $(\Xi \setminus N_{se,sa}) \cap \partial A$,*

$$\bigcup_{\xi \in ((\Xi \setminus N_{se,sa}) \cap \partial A)} \{z \in \partial A : \exists t_0 > 0 \text{ such that } \Phi_t^T(z) \rightarrow \xi \text{ as } t \rightarrow t_0 \text{ and } \Phi_t^T(z) \in C \forall 0 < t < t_0\} \quad (52)$$

7. *Singular boundary pieces $S \cap \partial \bar{A}$.*

In other words,

$$\begin{aligned} \partial A = & \bigcup_{z_0 \in \partial A} W^s(z_0) \cup \bigcup_{\gamma_0 \in \partial A} W^s(\gamma_0) \cup W^s(N_{tr,sa} \cap \partial A) \\ & \cup W^s(N_{se,sa} \cap \partial A) \cup (W^s(\Psi \setminus N_{tr,sa}) \cap \partial A) \\ & \cup (W^s(\Xi \setminus N_{se,sa}) \cap \partial A) \cup (S \cap \partial \bar{A}) \quad (53) \end{aligned}$$

We next state a set of necessary and sufficient conditions for distinguish the anchor points (which anchor the stable manifolds within ∂A) from among the various candidates such as unstable equilibria, unstable periodic orbits, pseudo saddles and semi-saddles.

First we consider order one equilibria on the stability boundary ∂A .

Lemma 12 *Suppose z_0 is an order 1 equilibrium point. Then*

$$(i) \quad z_0 \in \partial A \iff (W^u(z_0) \setminus \{z_0\}) \cap A \neq \emptyset \quad (54)$$

$$(ii) \quad z_0 \in \partial A \iff W^s(z_0) \subset \partial A \quad (55)$$

Note that for a general unstable equilibrium, a necessary condition is already available in the Basic Anchor Theorem, however this is weaker than for the order one equilibria in Lemma 12. The sufficient condition is of the same nature in general.

Lemma 13 *Suppose z_0 is a hyperbolic unstable equilibrium point. Then*

$$z_0 \in \partial A \iff W^s(z_0) \subset \partial A \quad (56)$$

A similar result follows for unstable periodic orbits.

Lemma 14 *Suppose γ_0 is a hyperbolic unstable periodic orbit. Then*

$$\gamma_0 \in \partial A \iff W^s(\gamma_0) \subset \partial A. \quad (57)$$

The conditions for pseudo saddles and semi-saddles are presented as part of the quasi-stability boundary analysis for convenience.

I.2.4 Analysis of the quasi-stability boundary $\partial \bar{A}$

To simplify the proofs in this section, the role of periodic orbits as anchor points on the quasi-stability boundary will not be investigated. We only consider the set of anchor points consisting of equilibria, pseudo equilibria and semi-singular points. The basic structure of the periodic orbits on the stability boundary ∂A has been illustrated by the Stability Boundary Theorem (Theorem 3). The contribution of the stable manifolds of the periodic orbits to the quasi-stability boundary $\partial \bar{A}$ can also be established in an analogous way. Such an extension will be presented elsewhere. Note that in the state space analysis of windup limits in Chapter III (section III.2.1), we prove the structure of the quasi-stability boundary for (actuation) hardlimit constrained models of the form Σ when the singularity is excluded from the analysis. When both the singularity and periodic orbits

are included in the quasi stability boundary analysis, standard Morse-Smale like induction arguments are not applicable, and special topological arguments such as developed in [91] become necessary. Essentially these arguments are much more technical when periodic orbits are present on the boundary, hence these are excluded from this presentation.

More precisely, in addition to Assumptions (A0)-(A3), we make the following assumption:

(A4) The ω -limit set for any trajectory in ∂A consists only of equilibrium points. $\square$

Assumptions (A3) and (A4) together imply that all the trajectories on the stability boundary converge to equilibrium points, pseudo equilibria or semi-singular points. (no periodic orbits on ∂A). The energy function Lemma (Lemma 11) can be restated to guarantee both assumptions (A3) and (A4) as shown next and the proof is exactly same as for Lemma 11. Note that all the existing energy functions for constrained power system models of the form Σ do not permit the presence of periodic orbits anyway [43, 41], hence this assumption (A4) is not restrictive.

Lemma 15 *If there exists a Lyapunov-function $V = V(x, y)$ such that*

1. $D_x V f - D_y V (D_y g)^{-1} D_x g f \leq 0$ in A

2. V is proper in the closure of A

3. the ω -limit set in $\bar{A}$ consists only of equilibria

then assumptions (A3) and (A4) are satisfied. All trajectories in ∂A converge to equilibria, pseudo equilibria or semi-singular points.

1.2.4.1 Anchor characterization Theorem

The basic structure of the anchors on the boundary $\partial \bar{A}$ is considered first. A set of necessary and sufficient conditions for some of the anchor points on the quasi-stability boundary are summarized as a Theorem below.

Theorem 4 (Anchor Characterization Theorem)

$$(a) \quad \begin{aligned} z_0^1 \in \partial \bar{A} &\iff W^u(z_0^1) \cap A \neq \emptyset \text{ and } W^u(z_0^1) \cap \bar{A}^c \neq \emptyset \\ z_0^1 \in \partial \bar{A} &\iff W^s(z_0^1) \subset \partial \bar{A} \end{aligned} \quad (58)$$

$$(b) \quad \begin{aligned} \nu \in N_{n-1}^\nu &\iff \text{there exists a } \delta_0 > 0 \text{ such that for} \\ &\text{every } \delta < \delta_0, W^u(\delta(\nu)) \cap A \text{ is dense in} \\ &W^u(\delta(\nu)), \text{ for any } \delta\text{-sized neighborhood,} \\ &\delta(\nu) \text{ in } N_\nu. \end{aligned} \quad (59)$$

$$\nu \in N_{n-1}^\nu \iff W^s(\nu) \subset \partial \bar{A} \quad (60)$$

where

$$N_{n-1}^\psi = \{\psi \in N_{tr,sa} \cap \partial \bar{A} : \text{there exists a } \delta > 0 \text{ such that} \\ \text{the } \delta\text{-sized neighborhood of } \psi \text{ in } N_\psi, \delta(\psi) \subset \partial \bar{A}\} \quad (61)$$

and

$$N_{n-1}^\xi = \{\xi \in N_{se,sa} \cap \partial \bar{A} : \text{there exists a } \delta > 0 \text{ such that} \\ \text{the } \delta\text{-sized neighborhood of } \xi \text{ in } N_\xi, \delta(\xi) \subset \partial \bar{A}\} \quad (62)$$

Here ν labels either ψ or ξ . $\square$

1.2.4.2 Quasi-stability boundary Theorem

Recall that the order of an equilibrium point z_0 is the dimension of its unstable subspace and it is denoted by a superscript.

Theorem 5 (Quasi-Stability Boundary Theorem)

Under Assumptions (A0)-(A4), the boundary of the closure of the region of attraction $\partial \bar{A}$, the quasi-stability boundary, is composed of the following principal elements,

1. Stable manifolds of order 1 equilibria $z_0^1 \in \partial \bar{A}$,

$$\bigcup_{z_0^1 \in \partial \bar{A}} \{z \in C : \Phi_t^T(z) \rightarrow z_0^1 \text{ as } t \rightarrow \infty \text{ and} \\ \Phi_t^T(z) \in C \forall t > 0\} \quad (63)$$

2. Stable manifolds of transverse pseudo saddles $N_{n-1}^\psi \cap \partial \bar{A}$,

$$\bigcup_{\psi \in (N_{n-1}^\psi \cap \partial \bar{A})} \{z \in C : \Phi_t^T(z) \rightarrow \psi \text{ as } t \rightarrow \infty \text{ and} \\ \Phi_t^T(z) \in C \forall t > 0\} \quad (64)$$

3. Stable manifolds of semi-saddles $N_{n-1}^\xi \cap \partial \bar{A}$,

$$\bigcup_{\xi \in (N_{n-1}^\xi \cap \partial \bar{A})} \{z \in C : \exists t_0 > 0 \text{ such that } \Phi_t^T(z) \rightarrow \xi \\ \text{as } t \rightarrow t_0 \text{ and } \Phi_t^T(z) \in C \forall 0 < t < t_0\} \quad (65)$$

4. Stable manifolds of bad anchors B_{n-1} ,

$$\bigcup_{\psi \in B_{n-1}} \{z \in \partial \bar{A} : \Phi_t^T(z) \rightarrow \psi \text{ as } t \rightarrow \infty \text{ and} \\ \Phi_t^T(z) \in C \forall t > 0\} \quad (66)$$

5. singular boundary pieces $S \cap \partial \bar{A}$.

More precisely

$$\partial \bar{A} = \overline{\bigcup_{z_0^1 \in \partial \bar{A}} W^s(z_0^1) \cup W^s(N_{n-1}) \cup (W^s(B_{n-1}) \cap \partial \bar{A}) \cup (S \cap \partial \bar{A})} \quad (67)$$

where $N_{n-1} = N_{n-1}^\psi \cup N_{n-1}^\xi$.

A qualitative representation of the boundary $\partial \bar{A}$ is shown in Figure 1.5 (periodic orbits are also included for conceptual clarity).

For the power system model, only polynomials and polynomial functions of sinusoidal functions are present. If the angles are restricted to a variation of 2π , it follows from algebraic stratification theory (essentially, from the Tarski-Seidenberg Theorem [45]), that the numbers of equilibria and the numbers of connected components of N , B and S are finite. Therefore

Corollary 1 *Suppose assumptions (A0)-(A4) hold. Then for a power system model there are only finitely many terms in the union*

$$\partial \bar{A} = \bigcup_{z_0^1 \in \partial \bar{A}} \overline{W^s(z_0^1)} \cup \overline{W^s(N_{n-1})} \cup \overline{(W^s(B_{n-1}) \cap \partial \bar{A})} \cup (S \cap \partial \bar{A}) \quad (68)$$

$\square$

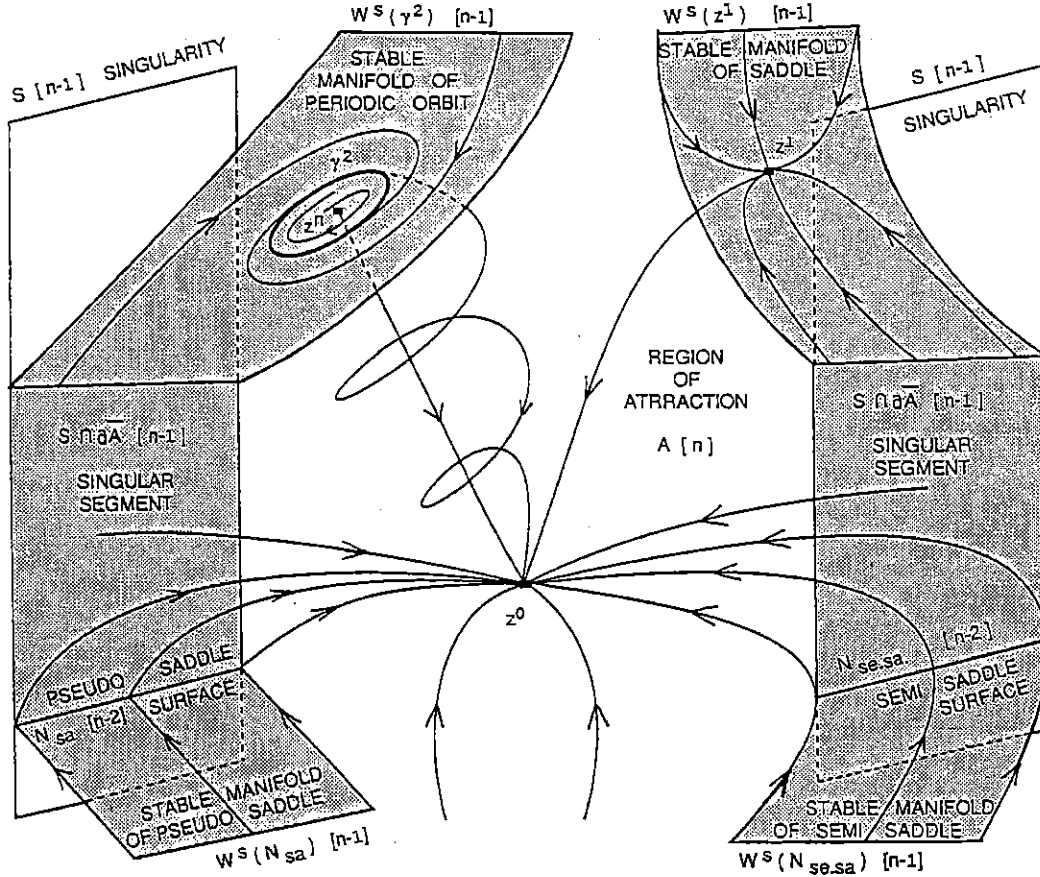


Figure I.5. Qualitative representation of the boundary of the region of attraction $\partial \bar{A}$.

I.2.5 Energy function results

Suppose an energy function V with the properties as defined in Lemma 15 is available. Then, define the set $\mathcal{R}(e)$ as the connected component of the set

$$\{z \in C : V(z) < e\} \quad (69)$$

that contains the stable equilibrium point z_s .

Lemma 16 Suppose $e_{\max}$ is the largest number such that $\overline{\mathcal{R}(e_{\max})}$ does not intersect with unstable equilibrium points, pseudo equilibrium points and semi-singular points. Then,

$$\mathcal{R}(e_{\max}) \subset A. \quad (70)$$

It follows by Theorem 5 and the definition of $\mathcal{P}$ that the quasi-stability boundary is composed of the closure of the stable manifolds of the principal anchors and the singular set $S \cap \partial \bar{A}$. The energy V is decreasing along the trajectories. Suppose α and β are anchor points. Then, by continuity, if $\alpha \in \partial W^s(\beta)$, then $V(\alpha) \geq V(\beta)$. By Theorem 5, every anchor point that is not principal, belongs to the boundary of the principal anchor points or the boundary of their stable manifolds. Hence we have proved the next result:

Lemma 17 The energy function attains its lowest values at the closure of the set of principal anchor points in the quasi-stability boundary. $\square$

The next result provides an energy function based estimate of the stability region $\bar{A}$.

Lemma 18 Suppose α is an anchor point. Then $\mathcal{R}(V(\alpha)) \subset \bar{A}$ if and only if α is the point in the set, $\bar{\mathcal{P}}$ with the lowest energy.

In the previous Lemma, we have developed a method for computing a conservative estimate of the region $\text{int}(\bar{A})$ (which better serves as the region of attraction A due to practical considerations [114]). However, given an initial condition z_0 , for testing whether z_0 belongs to the region of attraction or more generally to $\text{int}(\bar{A})$, we need to compute the stable manifolds of the principal anchors along with the singular segment (see Theorem 5) for a rigorous stability test. To provide a practical tool for judging the stability of the given point, next an energy function type method is developed for finding the "critical" sections of the stability boundary (which has multitudes of sections on a large system) where the boundary may be breached. Among all the principal anchor points, we will prove that those with energy higher than the energy of the given initial state, can be ignored for the stability test. The result thus considerably reduces the complexity of a rigorous stability test. This concept was originally introduced and analyzed in [114] and stability monitoring schemes based on this concept have been developed in [112]. Here we extend those results to include new segments in the quasi-stability boundary for our problem, such as the singularity and the pseudo saddles.

Definition 2 Given an initial condition z_0 , the critical anchor points are the principal anchor points with energy lower than $V(z_0)$. $\square$

So, $\mathcal{P}$ can be divided into two sets $\mathcal{P}_c$ and $\mathcal{P}_{nc}$ defined as

$$\mathcal{P}_c(z_0) := \{\alpha \in \mathcal{P} : V(\alpha) \leq V(z_0)\} \quad (71)$$

$$\mathcal{P}_{nc}(z_0) := \{\alpha \in \mathcal{P} : V(\alpha) > V(z_0)\} \quad (72)$$

The next Lemma enables us to consider only the critical anchor points $\mathcal{P}_c$ for stability tests.

Lemma 19 (Critical Anchor Lemma)

Suppose $z_0 \in \mathcal{C}$ is an initial condition. Then,

$$z_0 \text{ is inside (outside) } \bar{A}$$

$$\iff$$

z_0 is inside (outside) the portion of the stability boundary defined by the closure of stable manifolds of the critical anchor points.

I.3 Structure of the parameter space

The parameter space can be broadly divided (functionally) into the set of all system parameters (which do not change during system operation) and the set of operating parameters (that can change during operation). When these parameters change, the state space (consisting of system trajectories or transients) associated with each parameter value undergoes changes, however the structure of the state space remains topologically equivalent unless we encounter certain boundaries in the parameter space, namely the bifurcation boundaries. In other words, the bifurcation surfaces in the parameter space divide the parameter space into open connected regions called the typical regions [85, 86] within which the structure of the state space remains identical. This includes the static properties such as the equilibrium structure, dynamic local properties such as the stability status of equilibria, and dynamic global properties such as the composition of the stability boundary. Therefore this knowledge on the properties of typical regions would provide us with a sort of "complete" overview of the current system operation for security studies. However unfortunately at the present time, such an overview seems to be impossible even theoretically for large nonlinear systems. This is because the boundary of the typical regions consists of local and global bifurcations. Specifically the global bifurcations in general can themselves be dense in open sets of the parameter space (see [37, 10], complex behavior), which then renders the problem extremely difficult to analyze even for system dimension three, let alone the large system. For these reasons, we restrict the analysis in this section to the study of a simpler yet more practical concept in the parameter space called the notion of feasibility regions.

The concept of feasibility regions in the solution space of nonlinear problems has existed for a long time (for instance, refer to Optimization theory). In the power system context, Galiana et al. [46] introduced a notion of load-flow feasibility for quantifying the solvability of the load-flow equations in the parameter space. Later work of Fischl et al. [35] extended this concept to include the effects of small-signal stability by restricting the analysis to those solutions where the system is small-signal stable. The boundary of this region then consists of points where an operating point undergoes change in its eigenstructure, and these points therefore correspond to local bifurcations. In unconstrained system models, it is well-known that the generic local bifurcations are the saddle-node bifurcation (connected with zero eigenvalue of the system Jacobian) and the Hopf bifurcation (related to purely imaginary eigenvalues of the system Ja-

cobian). The first case of zero eigenvalues and the saddle-node bifurcation has been studied extensively in the recent past connected with the voltage collapse phenomena [52, 31]. The latter bifurcation can be generally associated with oscillatory phenomena [98, 100] leading to either stable oscillations (supercritical Hopf, [37]) or oscillatory divergence (subcritical Hopf, [37]). Recently, occurrences of oscillatory voltage disturbances have been reported by the industry, even though this phenomenon is not well-understood in general. There has been extensive research progress recently (e.g. [7, 4, 33, 30]) on the problem of computing the nearest saddle-node and Hopf bifurcation points in the parameter space.

In [85, 86], we have proposed a notion of feasibility region in the parameter space as the set of all operating points in the parameter space which can be reached by quasi-static parametric variations during the system operation. This region then naturally identifies a safe operating region in the parameter space within which the operation can be shifted around freely (by slow continuous parameter changes) without an imminent danger of losing small-signal stability. Of course, it has to be remembered that the size of the region of attraction (roughly the size of the region of transient stability) decreases down to zero as one approaches the feasibility boundary (see Loss of Stability Theorem, [91], see Section I.3.2.4), therefore the proximity to the feasibility boundary, that is the boundary of the feasibility region, could indicate potential emergence of transient instability problems. So in summary, while the system operation has to stay inside the feasibility region for sustained small-signal stability, it is also desirable to stay well-inside the region and away from the boundary to lessen the danger of transient instability phenomena. All this demands a careful analysis of the feasibility boundary for gaining insight into its structure and for developing techniques to compute them.

We will show that the boundary of the feasibility region as defined here typically consists of three types of bifurcation surfaces, namely the saddle node, Hopf and the singularity induced bifurcations (Section I.3.2). It can be shown that the saddle node bifurcation boundary segments can be computed by solving for a zero set of explicit functions [76, 37], which forms the basis for computational methods which exploit the rank deficiency [73] (point of collapse methods, [7]). Computational methods have been developed for computing the distance to the saddle node (a small sample-[7, 4, 29]) and Hopf bifurcation boundaries [30]. Here we will show that all the three bifurcation surfaces can be characterized as zero sets of explicit functions and hence the whole feasibility boundary should amenable to computational methods for system security of the large system. Note that this characterization may need to be combined with model reduction techniques such as dynamic clustering approaches [119] and the participation factor approaches [83] for developing practical computer algorithms.

I.3.1 Feasibility regions and feasibility boundary

Let

$$\begin{aligned} EQ &= \{(x, y, p) \in X \times Y \times P : f(x, y, p) = 0, g(x, y, p) = 0\} \\ OP &= \{(x, y, p) \in EQ : D_y g \text{ is nonsingular and} \\ &\quad J = D_x f - D_y f (D_y g)^{-1} D_x g \\ &\quad \text{has eigenvalues with negative real part}\}; \end{aligned} \quad (73)$$

Here $D_x f$ etc. denote the matrix of partial derivatives of the components of f with respect to x . EQ is the set of equilibrium points and OP is the subset of stable equilibria. Note that the full Jacobian in x and y is nonsingular for $(x, y, p) \in OP$ and therefore, by the implicit function Theorem, the equations $f(x, y, p) = 0$ and $g(x, y, p) = 0$ can theoretically be solved uniquely for x and y as functions of the parameter p . Hence OP is a p -dimensional submanifold embedded in $EQ \subset X \times Y \times P$.

Definition 3 Given a stable equilibrium $x_s^0 = (x_0, y_0)$ (a load flow solution) for parameter value p_0 , the connected component F of OP which contains (x_0, y_0, p_0) is called the feasibility region of x_s^0 . Its boundary (relative to EQ) is the feasibility boundary.

Note that the feasibility region is defined as a subset of $X \times Y \times P$, not as a subset of the parameter space. This way, all possible stable equilibrium points which can be reached from (x_0, y_0, p_0) by continuous variations of the parameter while maintaining stability form the feasibility region.

1.3.1.1 Feasibility Boundary Theorem

Theorem 6 (Feasibility Boundary Theorem) For a system defined in equations (1) and (2), the feasibility boundary of a feasibility region F consists of three zero sets

$$\partial F = (\partial F \cap C_S) \cup (\partial F \cap C_Z) \cup (\partial F \cap C_H), \quad (74)$$

where

$$C_S = \{(x, y, p) \in EQ : \det D_y g = 0\}, \quad (75)$$

$$C_Z = \{(x, y, p) \in EQ : \det D_y g \neq 0, \det \begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix} = 0\} \quad (76)$$

and

$$C_H = \{(x, y, p) \in EQ : \det D_y g \neq 0, \det \begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix} \neq 0, \det(H_{n-1}(J)) = 0\} \quad (77)$$

where

$$J = D_x f - D_y f (D_y g)^{-1} D_x g \quad (78)$$

and H_{n-1} is the Hurwitz matrix,

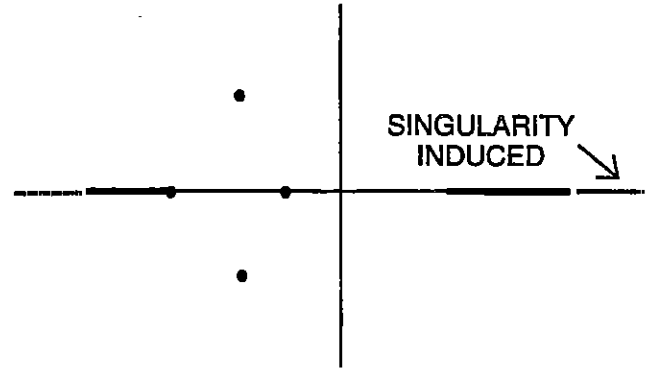
$$H_{n-1} = \begin{pmatrix} a_1 & a_3 & a_5 & * & * & a_{2n-3} \\ a_0 & a_2 & a_4 & * & * & a_{2n-4} \\ 0 & a_1 & a_3 & * & * & a_{2n-5} \\ 0 & a_2 & a_4 & * & * & a_{2n-6} \\ . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & * & a_{n-1} \end{pmatrix} \quad (79)$$

corresponding to the coefficients a_i of the characteristic polynomial of J

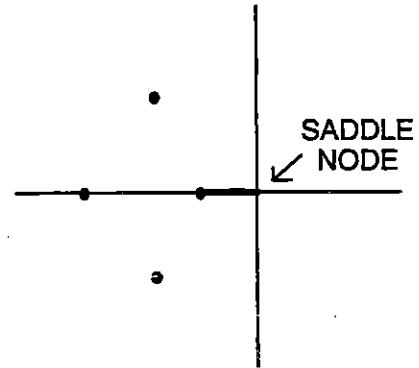
$$\det(sI - J) = a_0 s^n + a_1 s^{n-1} + \dots + a_n. \quad (80)$$

and $a_k = 0$ for $k > n$. $\square$

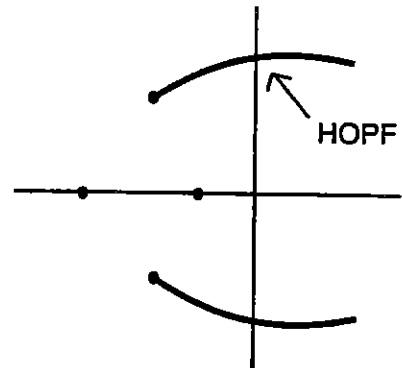
The three zero sets, which constitute the feasibility boundary, correspond to specific conditions on the eigenvalues of the system. As a point on EQ approaches C_S some eigenvalues diverge to infinity. The equilibrium has zero eigenvalues in the set C_Z and purely imaginary eigenvalues for points in the set C_H . (see Figure I.6). Under generic conditions local bifurcations occur at all these points, namely a singularity induced bifurcation



Unbounded eigenvalues in the set C_S



Zero eigenvalues in the set C_Z



Purely imaginary eigenvalues in the set C_H

Figure I.6. Locus of the eigenvalues near the three feasibility boundary segments.

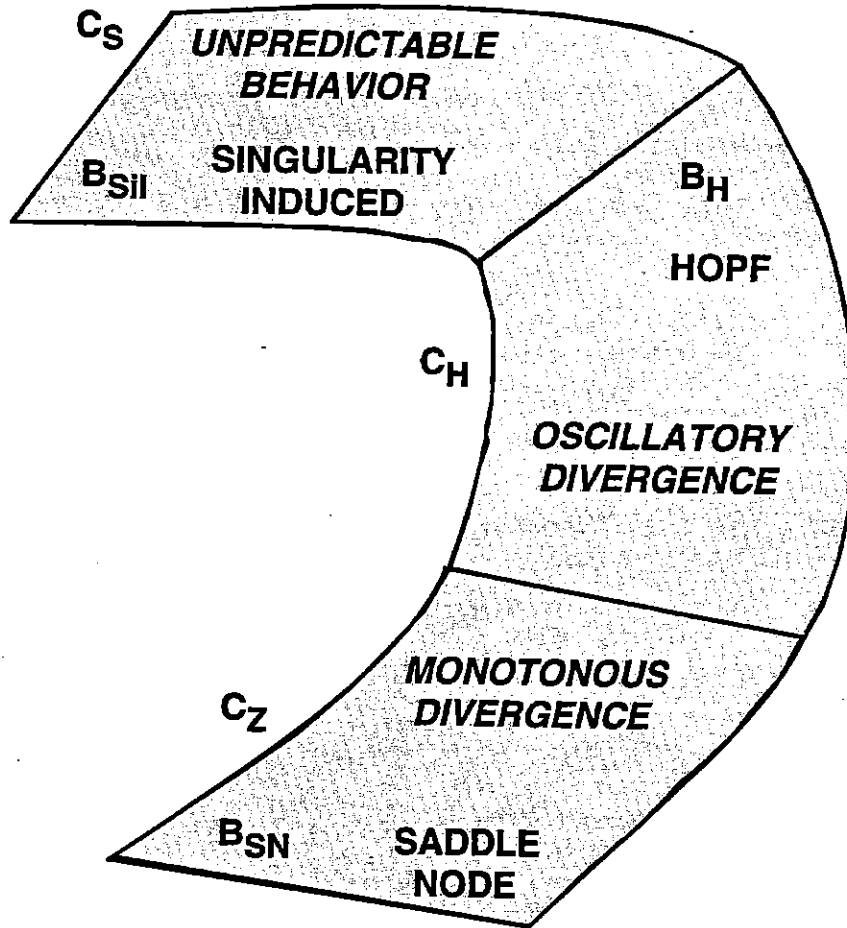


Figure I.7. Conceptual sketch of the feasibility boundary.

in C_S , a saddle-node bifurcation in C_Z and a Hopf bifurcation in C_H . Moreover, generically these bifurcations constitute a dense subset of the feasibility boundary for equations (1) and (2). Saddle-node and Hopf bifurcations are well-known in the literature and their definitions will be summarized here for the reader's convenience. The singularity induced bifurcation will be analyzed.

A qualitative representation of the feasibility boundary is shown in Figure I.7. In Section I.3.4, we will take a closer look at the local system behavior near the three feasibility boundary segments. First we need to establish the principal bifurcation structure in Sections I.3.2 and I.3.3.

I.3.2 The three types of bifurcations on the principal strata of the feasibility boundary

I.3.2.1 Saddle node bifurcation

A *saddle-node bifurcation* occurs when a system has a non-hyperbolic equilibrium with a simple zero eigenvalue at the bifurcation point and additional transversality conditions are satisfied. Points in C_Z can locally be described as $\dot{x} = f_R(x, p)$ for some reduced function f_R . Then points in C_Z are saddle-node bifurcations if the following conditions are satisfied:

(SN1) The matrix $D_x f_R = D_x f - D_y f (D_y g)^{-1} D_x g$ has a simple zero eigenvalue with right eigenvector v and left eigenvector w and there is no other eigenvalue on the imaginary axis.

(SN2) $w(D_p f_R) = w(D_p f - D_y f (D_y g)^{-1} D_p g) \neq 0$

(SN3) $w(D_x^2 f_R(v, v)) \neq 0$ □

I.3.2.2 Hopf bifurcation

A *Hopf bifurcation* occurs at points where the system has a nonhyperbolic equilibrium connected with a pair of purely imaginary eigenvalues, but no zero eigenvalue, and additional transversality conditions are met. Specifically, for the system (1) and (2) it is shown in the proof of Theorem 6 that the Hurwitz function H_{n-1} detects the presence of purely imaginary eigenvalues. The assumption $\det(J) \neq 0$ implies that locally there exists a smooth curve of equilibria, $(x(p), y(p), p)$, with eigenvalues $\lambda(p), \bar{\lambda}(p)$ of $D_x f_R$ which are imaginary and vary smoothly with p . A Hopf bifurcation occurs at any point in C_H where the following transversality conditions are satisfied:

(H1) $J = D_x f_R$ has a simple pair of purely imaginary eigenvalues and there is no other eigenvalue on the imaginary axis.

(H2) $D_p \text{Real}(\lambda(p)) \neq 0$. □

I.3.2.3 Singularity induced bifurcation

A *singularity induced bifurcation* occurs when an equilibrium crosses the singular surface

$$S = \{(x, y, p) \in EQ : \Delta(x, y, p) := \det(D_y g(x, y, p)) = 0\} \quad (81)$$

and, again, certain additional transversality conditions are satisfied. The analysis of the singularity induced bifurcation is presented in Section 5.3. The number of stable eigenvalues of the system changes by one when the eigenvalue blows up. In the full system, a singularity induced bifurcation occurs if the gradient of Δ along the local equilibrium locus $(x(p), y(p), p)$ does not vanish,

$$(SI3) \quad \frac{d}{dp} \Delta(x(p), y(p), p) \neq 0$$

It is clear that the three bifurcation types discussed above arise when additional independent inequality constraints are imposed on points in the sets C_S , C_Z and C_H . Generically these conditions will be satisfied on dense subsets. Henceforth it will be assumed that

(D1) The set of points where $\{(SN1), (SN2), (SN3)\}$ are satisfied, is dense in C_Z .

(D2) The set of points where $\{(H1), (H2)\}$ are satisfied, is dense in C_H .

(D3) The set of points where $\{(SI1), (SI2), (SI3)\}$ are satisfied, is dense in C_S . $\square$

I.3.2.4 Loss of Stability Theorem

The feasibility boundary ∂F may contain some internal segments of the boundary, i.e. in $\text{int}(\bar{F})$ (closure is with respect to EQ , the set of all equilibrium points). Since such boundary segments $\partial F \cap \text{int}(\bar{F})$ lie in the interior, they do not correspond to loss of stability. If $(z, p) \in \partial F \cap \text{int}(\bar{F})$, then we can find a neighborhood U of (z, p) in EQ such that most of the operating states in U are stable. Hence, with the model (1) and (2), the feasibility boundary segments in the interior $\partial F \cap \text{int}(\bar{F})$ do not imply loss of stable operation at the present equilibrium state. Next Theorem proves that generically there are no internal segments of the feasibility boundary, i.e.

$$\partial F \cap \text{int}(\bar{F}) = \emptyset. \quad (82)$$

In other words, generically indeed the local stability at the equilibrium is lost at most of the feasibility boundary points.

Theorem 7 (Loss of Stability Theorem)

Suppose system (1), (2) satisfies assumptions (D1), (D2) and (D3). Then the saddle-node bifurcation, the Hopf bifurcation and the singularity induced bifurcation occur on a dense subset of the boundary ∂F . The feasibility boundary ∂F coincides with the quasi-feasibility boundary $\partial \bar{F}$, i.e. the stability of the equilibrium is lost at most of the boundary points.

$$\partial F = \partial \bar{F} = (\partial F \cap \bar{C}_H) \cup (\partial F \cap \bar{C}_S) \cup (\partial F \cap \bar{C}_Z). \quad (83)$$

$\square$

Moreover, for a power system model, as observed earlier, only polynomials and polynomial functions of sinusoidal functions are present. If the angles are restricted to a variation of 2π , it follows from algebraic stratification theory (essentially, from the Tarski-Seidenberg Theorem [45]) that there are finitely many connected components of codimension one bifurcations in the quasi feasibility boundary.

Lemma 20 For the power system model, there are finitely many connected components in the union,

$$\partial F = \partial \bar{F} = (\partial F \cap \bar{B}_H) \cup (\partial F \cap \bar{B}_{SI}) \cup (\partial F \cap \bar{B}_{SN}). \quad (84)$$

$\square$

In short, the feasibility boundary can be computed as zero sets. Generically, for the system (1), (2) most of the boundary points lead to a saddle-node, Hopf or a singularity induced bifurcation.

I.3.3 Analysis of the Singularity Induced Bifurcation

When the steady state solution or the equilibrium defined by EQ approaches the singularity S , it has been observed that some eigenvalues of the system Jacobian at the equilibrium point become unbounded [52, 72, 85, 21, 92]. In [85, 92], it was shown that this case generally is 'equivalent' to a transcritical bifurcation ('exchange of stability' bifurcation [37]) for a two-dimensional system. In this section, we analyze this scenario for the general system (1)–(2), when the equilibrium approaches the singularity. As the change in the structure is due to the presence of the singular surface, we call this bifurcation the *singularity induced bifurcation*. For analyzing the bifurcation, we use the singular transformation, which was introduced in Section I.1.3. By analyzing the transformed system Z^T under certain transversality conditions, the local eigen-structure of the equilibrium point is established near the bifurcation point.

I.3.3.1 Singularity Induced Bifurcation Theorem

Theorem 8 (Singularity Induced Bifurcation Theorem)

Consider the system Σ_s with a 1-dimensional parameter (say μ) space. Suppose the following conditions are satisfied at $(0, 0, \mu_0)$:

(SI1) $f(0, 0, \mu_0) = 0$, $g(0, 0, \mu_0) = 0$ and $D_y g$ has a simple zero eigenvalue

(SI2) $\begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix}$ is nonsingular

(SI3') $\begin{pmatrix} D_x f & D_y f & D_\mu f \\ D_x g & D_y g & D_\mu g \\ D_x \Delta & D_y \Delta & D_\mu \Delta \end{pmatrix}$ is nonsingular.

Then there exists a smooth curve of equilibria, say $EQ(\mu) := (x(\mu), y(\mu), \mu)$ in $\mathbb{R}^{n+m+1}$ which passes through $(0, 0, \mu_0)$ and is transversal to the singular surface S at $(0, 0, \mu_0)$. When μ increases through μ_0 , one eigenvalue of the system, (i.e. an eigenvalue of

$$J = D_x f - D_y f (D_y g)^{-1} D_x g \quad (85)$$

evaluated along the equilibrium locus), moves from the open left half complex plane $\mathbb{C}^-$ to the open right half complex plane $\mathbb{C}^+$ if $\frac{b}{c} > 0$ (respectively from $\mathbb{C}^+$ to $\mathbb{C}^-$ if $\frac{b}{c} < 0$) along the real axis by diverging through ∞ . The other eigenvalues remain bounded and stay away from the origin. Here c is defined as the derivative of Δ along $EQ(\mu)$

$$c := \frac{d}{d\mu} \Delta(x(\mu), y(\mu), \mu), \quad (86)$$

and b is defined as the only nonzero eigenvalue of the Jacobian $-D_y f \text{adj}(D_y g) D_x g$ at $(0, 0, \mu_0)$ where $\text{adj}(A)$ stands for the classical matrix adjoint of the matrix A . $\square$

1.3.4 Local dynamics near the feasibility boundary

A detailed study of the qualitative behavior of the system Σ near the three principal boundaries, namely the saddle-node, Hopf and the singularity induced bifurcations would provide us with the insight on how the instability develops as the system operation approaches one of the feasibility boundary segments.

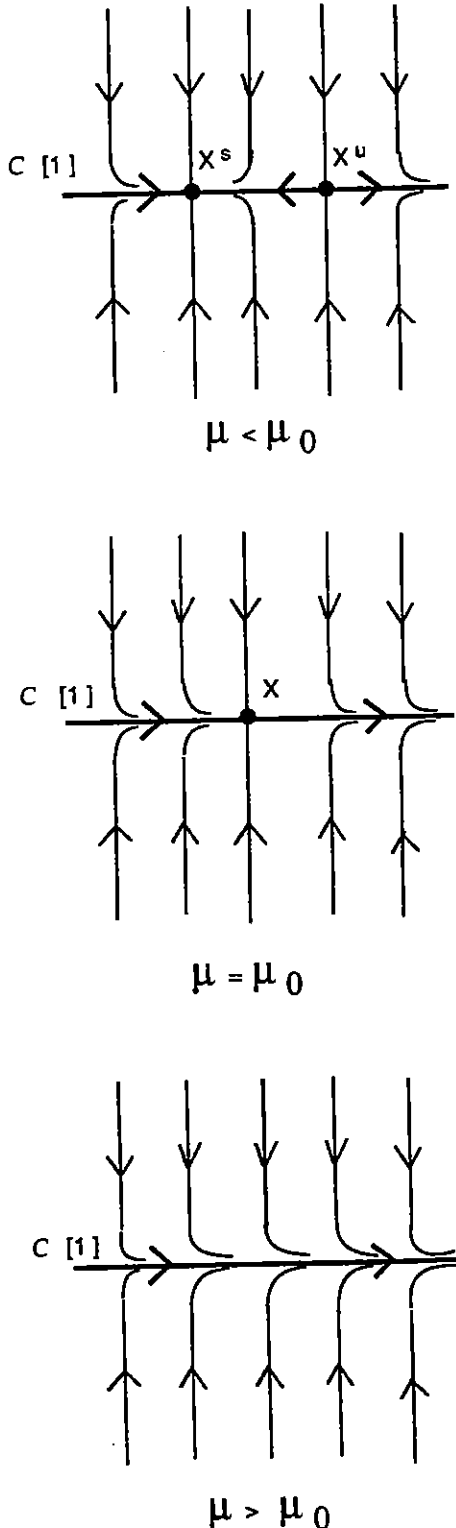


Figure I.8. Instability mechanism of the saddle node bifurcation boundary: annihilation of the operating point results in monotonous divergence.

That is, the eigenstructure analysis of the previous sections needs to be extended to explore the geometry of the system trajectories near all three bifurcation boundaries. For the saddle node bifurcation boundary, such an analysis was presented in [31] using center manifold theory for certain unconstrained power system models. It was shown that the occurrence of the saddle node bifurcation is related to the voltage collapse phenomenon under some conditions. In [98, 100], we have considered all the three principal local bifurcations for the large constrained system Σ , and established the qualitative local behavior of Σ for the three cases. These results are briefly stated below. Details can be found in [98, 100].

① The saddle node bifurcation segments of the feasibility boundary correspond to monotonous divergence. (see Figure I.8).

② The supercritical Hopf implies the transition from stable equilibrium to sustained oscillations. (see Figure I.9). Near the subcritical Hopf segments, the transients will be diverging in an oscillatory fashion. (see Figure I.10).

③ For the singularity induced bifurcation, it can be shown that generically even when the equilibrium is at the singularity, there exists an $(n-1)$ -dimensional invariant manifold W^{res} that passes through the singularity such that the singular dynamics Σ restricted to the invariant manifold is smooth. (see Figure I.11). Moreover, this invariant manifold is locally “attracting” on one side of the singular surface, and “repelling” on the other side, thereby providing a “conduit” for the trajectories from one side of the singular surface to cross over to the other side at finite speed, possibly either diverging towards a new steady state condition or ending at the singular surface, where a different model takes over. The constrained model Σ does not provide sufficient information to distinguish between the two different types of behaviors, so the singularity induced bifurcation denotes a true barrier to *predictable* behavior with the model Σ from a mathematical as well as an engineering viewpoint. Moreover the loss of reliable system prediction using Σ at this bifurcation follows for any constrained model of the form Σ under certain generic transversality assumptions and is not related to the limitations of the quasi-stationary assumption in phasor power system models. A detailed description of the local dynamics near the singularity induced bifurcation can be seen in [98, 100] and is summarized in the form of a Theorem in the next subsection.

1.3.4.1. Local dynamics decomposition Theorem

The Singularity Induced Bifurcation Theorem proves that the other eigenvalue which is unbounded moves from $\mathbb{C}^-$ into $\mathbb{C}^+$ by diverging through infinity. Hence the system is unstable on one side of the feasibility boundary segment B_{SI} at this equilibrium because the system Jacobian has at least one unstable eigenvalue. However when some of the bounded eigenvalues are purely imaginary at the bifurcation point, a more degenerate situation arises which will be ruled out for further analysis by the next transversality condition.

(SI4) All the $n-1$ bounded eigenvalues $\lambda_i(\mu_0)$ for $i = 1, 2, \dots$ have nonzero real parts.

The main structural features of the system Σ near the singularity induced bifurcation are presented in the next theorem.

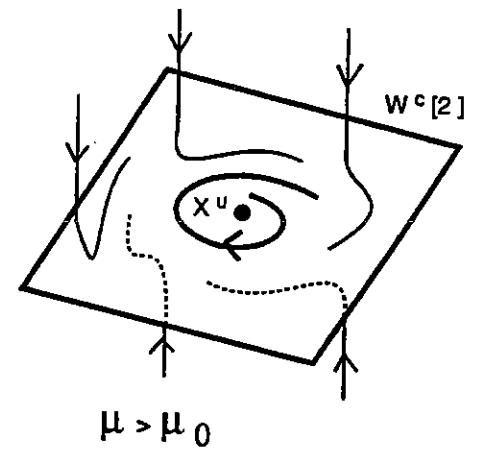
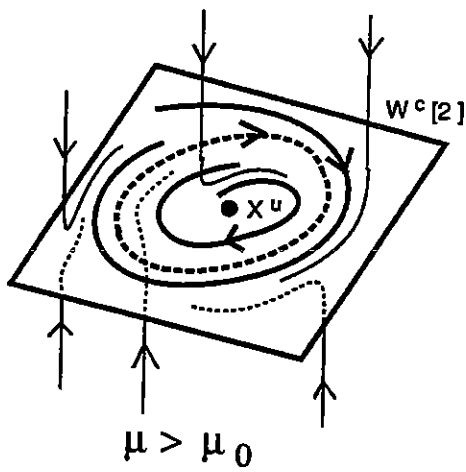
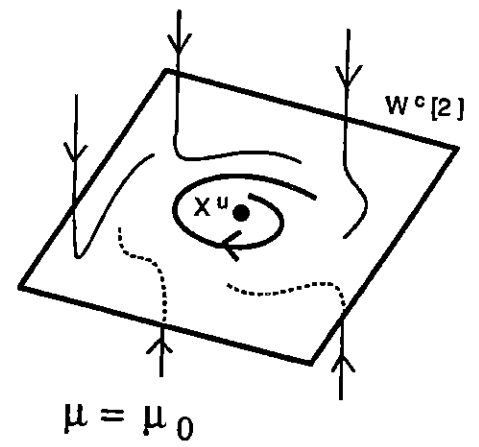
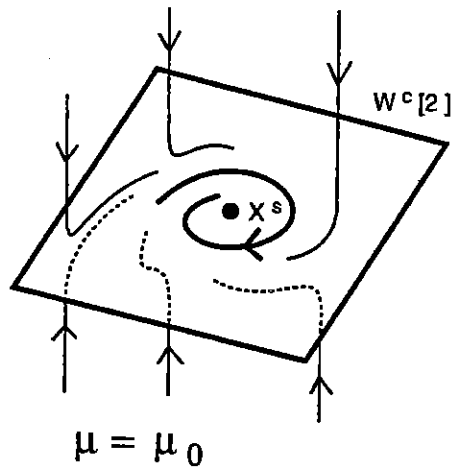
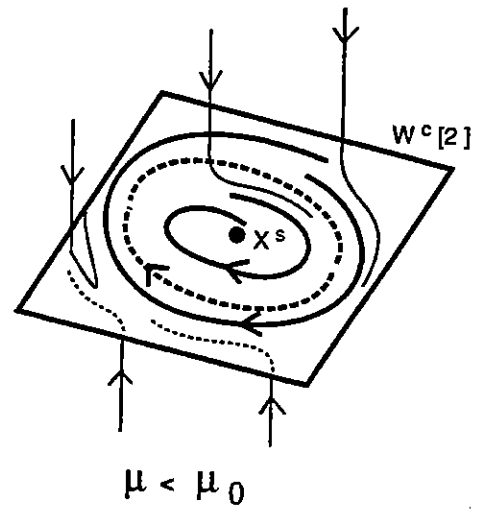
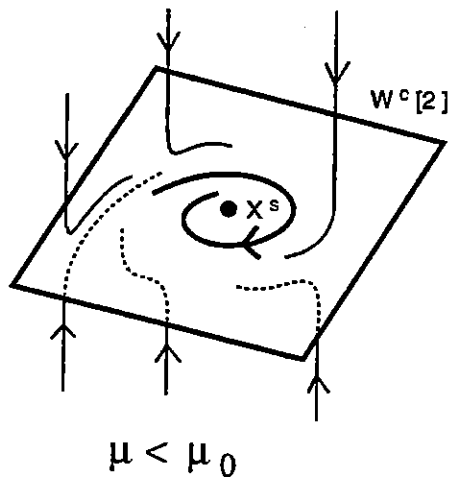


Figure I.9. Instability mechanism of the supercritical Hopf bifurcation boundary: operation changes from an operating point to stable oscillations.

Figure I.10. Instability mechanism of the subcritical Hopf bifurcation boundary: annihilation of the operating point leads to oscillatory diverging transient.

Figure I.11 summarizes the statements as an illustration. The number of unstable eigenvalues n_u in the theorem is assumed to be zero in the illustration. For instance, on the feasibility boundary $B_{SiI} \cap \partial F$, the constant n_u will indeed be zero so that the local dynamical properties of the feasibility boundary $B_{SiI} \cap \partial F$ are as shown in Figure I.11.

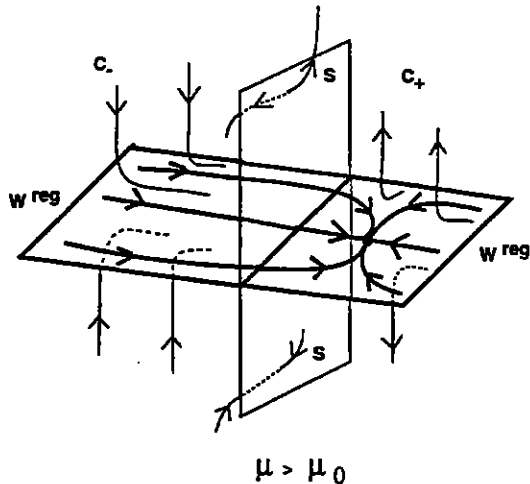
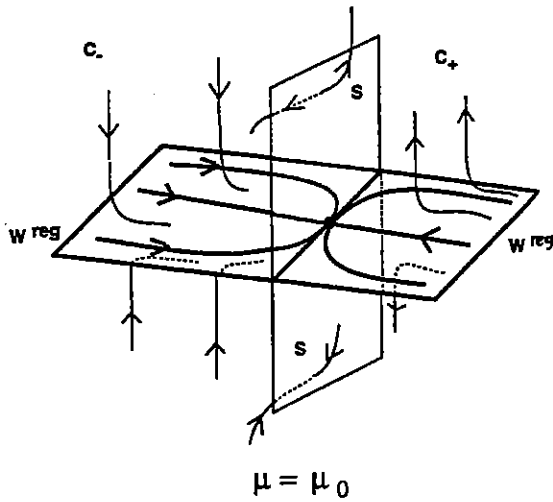
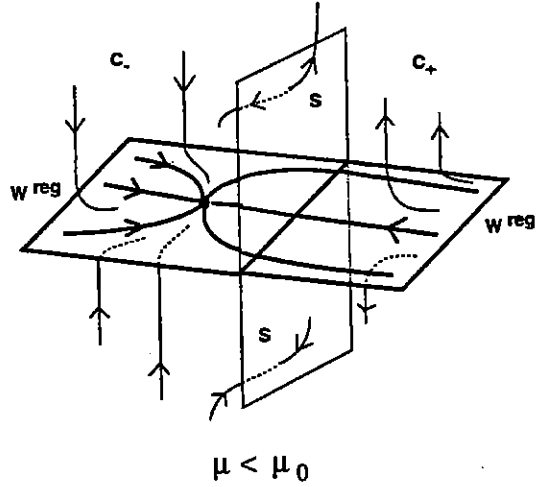


Figure I.11. Instability mechanism of the singularity induced bifurcation boundary: unpredictable system behavior.

Theorem 9 (Local Dynamics Decomposition Theorem)
Consider equations (1), (2) with a one dimensional parameter space. Suppose the conditions (SI1), (SI2), (SI3') and (SI4) are satisfied at $(0,0,\mu_0)$. Let the $(n-1)$ bounded eigenvalues of J at $(0,0,\mu_0)$ be divided into n_s stable eigenvalues and n_u unstable eigenvalues. Then we have

(LD1) There exists an $\epsilon > 0$ such that for $\mu \in (\mu_0 - \epsilon, \mu_0 + \epsilon)$, the following statements are satisfied:

(LD1.1) There exists an n -dimensional manifold $W \subset L$,

$$W = \bigcup_{|\mu - \mu_0| < \epsilon} W^{reg}(\mu) \quad (87)$$

which is foliated into $(n-1)$ -dimensional embedded submanifolds $W^{reg}(\mu)$ by the slices $\mu = \text{constant}$. The submanifold $W^{reg}(\mu)$ contains the equilibrium point $(x(\mu), y(\mu), \mu)$, is locally invariant, and the restriction of the system Σ to $W^{reg}(\mu)$ is smooth.

(LD1.2) There exist locally invariant submanifolds $W^s(\mu)$ and $W^u(\mu)$ of dimensions n_s and n_u respectively, with the property that the restriction of the flow of the dynamics Σ to $W^s(\mu)$ is a contraction, and the restriction of Σ to $W^u(\mu)$ is an expansion.

(LD1.3) There also exists locally invariant one-dimensional manifolds $W^{sing}(\mu)$ containing $(x(\mu), y(\mu), \mu)$ with the property that the flow of the system Σ restricted to $W^{sing}(\mu)$ is a contraction for $\mu < \mu_0$ ($\mu > \mu_0$) and is an expansion for $\mu > \mu_0$ ($\mu < \mu_0$), when $\frac{b}{c} > 0$ ($\frac{b}{c} < 0$).

(LD2) For $\frac{b}{c} > 0$ and $\mu \neq \mu_0$, locally near $(x(\mu), y(\mu), \mu)$, the flow Φ_t of the system Σ is topologically equivalent to the flow of the following partitioned system

$$\dot{x}_s = -x_s, \quad x_s \in W^s(\mu) \quad (88)$$

$$\dot{x}_u = x_u, \quad x_u \in W^u(\mu) \quad (89)$$

$$\dot{x}_{sing} = \frac{x_{sing}}{\mu - \mu_0}, \quad x_{sing} \in W^{sing}(\mu). \quad (90)$$

When $\frac{b}{c} < 0$, the equivalence is valid with $\mu - \mu_0$ replaced by $-(\mu - \mu_0)$ in equation (90) above.

(LD3) When $\mu = \mu_0$, the flow Φ_t restricted to each of the locally invariant manifolds $W^s(\mu_0)$, $W^u(\mu_0)$ and $W^{sing}(\mu_0)$, is equivalent to the respective smooth flows as shown below.

$$\dot{x}_s = -x_s, \quad x_s \in W^s(\mu_0). \quad (91)$$

$$\dot{x}_u = x_u, \quad x_u \in W^u(\mu_0). \quad (92)$$

$$\dot{x}_{sing} = -x^2, \quad x_{sing} \in W^{sing}(\mu_0). \quad (93)$$

(LD4) Locally near the bifurcation point $(0,0,\mu_0)$, for $\mu \in (\mu_0 - \epsilon, \mu_0 + \epsilon)$, the following statements are satisfied for the system Σ :

(LD4.1) There exists an $(n-1)$ -dimensional manifold of singular points S which separates the constraint surface $L \setminus S$ into two connected components, $C_-(\mu)$ and $C_+(\mu)$ such that the determinant $\Delta(x, y, \mu)$ is positive in $C_+(\mu)$ and negative in $C_-(\mu)$.

(LD4.2) When μ increases through μ_0 , the equilibrium point $(x(\mu), y(\mu), \mu)$ moves from $C_-(\mu)$ to $C_+(\mu)$ when $c > 0$ and vice versa when $c < 0$.

(LD4.9) When $\frac{b}{c} > 0$, the segments of the invariant manifold $W^{res}(\mu) \cap C_-(\mu)$ and $W^{res}(\mu) \cap C_+$ are locally attracting (in the sense that locally trajectories not on W^{res} move towards W^{res}) and repelling (in the sense that locally trajectories not on W^{res} move away from W^{res}) respectively.

1.3.5 Typal Regions

A feasibility region consists of possibly several open subregions, called *typal regions* in this paper, and some of the respective boundaries. Typal regions are defined by the property that the phase-portraits of the system are topologically equivalent (i.e. structurally stable) within one subregion. The boundaries between these regions are determined by various local and global bifurcation surfaces. The possibility of complex behavior connected with global bifurcations has been noted in angle [50, 66] and voltage [85, 19, 2] power system models. For a rudimentary power system model (with $n = 2$ and $m = 1$ in equations (1) and (2)), a complete classification of the typal regions has been given in [85, 92] for a realistic range of two parameters (power and control). However for a general n -dimensional system defined by equations (1) and (2), for reasons of dimensionality and because of the difficulty connected with complex system behavior, a similar complete description is unrealistic.

II. A TAXONOMY OF THE PHASOR CALCULUS - ANALYSIS OF FAST TRANSIENT DYNAMICS USING TIME-VARYING PHASORS -

Phasor dynamics for many years was considered a prominent *nonproblem* in power engineering. The conventional quasi-stationary phasor (also called the quasi-static phasor) was established as a good approximation and a convenient method for computing static and dynamic events on the power system provided they were not varying faster than those studied in conventional transient stability and in load flow.

During the last couple of decades the nature of the power system has gradually changed by growing loads on the transmission and generation, increasing interconnections, increasing use of power electronic devices like HVDC, SVC, thyristor controlled motors, etc. The result, among others, is an emergence of voltage stability (voltage collapse) and voltage control as critical problems connected with some much faster moving phenomena than those in the traditional rotor angle transient stability. Thus the theoretical and practical questions connected with the validity of phasor formulation are now playing a fundamental role in the dynamic theory of the modern AC system. Most of us were unaware of this change which in turn resulted in numerous unexplained disturbances and questionable results in the literature. Awareness has now been raised by practical occurrences and also in the literature [86], [116] but a clear-cut comprehensive understanding of phasor dynamics, still needs to be offered. This is the objective of this section.

Recent results in this area have made considerable progress on this topic [86, 116, 68, 69, 101, 28, 102, 99], even though excepting [99], all these efforts are based on a lumped parameter π circuit representation of the transmission system.

Earlier approaches (e.g. [68]) use physical reasoning and intuitive deduction to try to reach a general time dependent phasor concept. Under the assumption that the system (including the transmission network and the loads) only consists of resistors and inductors, detailed power system models purely in terms of flux linkages (without the quasi-stationary approximation) have been proposed for the large system in [69]. A concept of transient algebraic circuits is introduced in [69] which model the impedance variations as functions of frequency deviations, and the resulting singularly perturbed dynamics is simplified using the integral manifold theory.

Some of the fundamental limitations connected with the general time-varying phasor concept have been recognized recently in [28]. Based on a filtering approach, it is shown in [28] that time-varying power flow equations can be rewritten as autonomous differential equations in the phasor domain under certain bandwidth restrictions. An elegant method for formulating these phasor domain equations using a notion of "phasor domain circuits" is proposed in [28] which is related to the concept of "transient algebraic circuits" in [69] for the R-L networks.

The main difference in our approach from the earlier work is the recognition that phasors essentially represent a mathematical transformation. This transformation can lead to tremendous simplification in analysis (for all cases: stationary/quasi-stationary/fast transients) provided necessary caution is exercised in establishing it rigorously. The mathematical properties of the phasor transformation as used in this paper have been

proved recently in [101, 102]. The results in [101] are based on the Blondel transformation where the time-varying phasor transformation is established precisely for the balanced system. These results are summarized here in Section II.2.1. In [102], the methodology of time-varying phasors is extended to the general large system (valid for both balanced and unbalanced conditions) using the long-existing notion of time-varying phasors from the communication theory under certain low pass bandwidth assumptions (see Section II.2).

Here we provide an overview (essentially a taxonomy) of the various phasor concepts and their implications for power system dynamic analysis. Two basic questions need to be answered:

- 1.) Removal of the restrictive "quasi stationary" assumption requiring the relatively slow movement (not exceeding the typical speeds of traditional "transient stability"). This restriction is sometimes ignored in the literature resulting in questionable results. Here a new time varying phasor concept is offered which allows a wide range of speed in the dynamics. This question is analyzed in Subsection II.1. The mathematical formulation presented in Section II.1 is summarized from [102].
- 2.) The second question then is the precision of the model for the transmission system dynamics. First it is necessary to establish the fully precise model for use with the new tool of time varying phasor for obtaining correct results. The RLC or π circuit models are quite inadequate because they grossly distant the actual circuit which defines the fast oscillations of the transmission system. This calls among other things for the consideration of the distributed nature of R , L and C . A time delay model will be developed in Section II.5 below.

Finally in subsection II.7 these results are applied to a number of special cases and the question of approximation is expressed.

II.1 The generalized time-varying phasor transformation

Current engineering practice involving modulated signals such as AC voltages and currents, $e(t)$, $i(t)$, is largely based on transforming modulated signals into a phasor form, $\vec{E}(t)$, $\vec{I}(t)$ through the relationship

$$\begin{aligned} e(t) &= E(t) \cos(\omega_c t + \delta(t)) \\ &= \operatorname{Re}(\vec{E}(t) e^{j\omega_c t}) = \operatorname{Re}(E(t) e^{j(\omega_c t + \delta(t))}) \forall t \in \mathbb{R} \quad (94) \end{aligned}$$

In this formulation, the phasor definition is tacitly conceived as a mathematical operator say $\mathcal{P}$ which maps the modulated signals to those time varying phasor signals which satisfy the modulation equation (94).

Notations: Lower case characters such as $e(\cdot)$ and $i(\cdot)$ denote modulated signals. Upper case characters with arrows such as $\vec{E}(\cdot)$ and $\vec{I}(\cdot)$ stand for the phasor signals. The capital letters such as $E(t)$ and $I(t)$ denote the magnitudes of the phasors or equivalently the amplitudes of the modulated signals for notational convenience $E(t)$, $I(t)$ will be interpreted as RMS values ($E(t) = (1/\sqrt{2})\mathcal{E}(t)$, $I(t) = (1/\sqrt{2})\mathcal{I}(t)$ where $\mathcal{E}(t)$ and $\mathcal{I}(t)$ are the actual peak values of the modulated sinusoids for voltage and current. This is customary in power engineering and lower case Greek letters such as $\delta(t)$ and $\phi(t)$ denote the phases of the phasors.

Note that the frequency of the carrier signal ω_c is itself time varying in general. In our analysis, the carrier frequency is taken to be the reference frequency (e.g. $\omega_c = 60\text{Hz}$) and ω_c is assumed fixed in this analysis. Note that in slow dynamics such as in Automatic Generation Control (AGC) the frequency of the center of angle, $C(t)$, itself may be time varying, defining $\omega_c = \omega_c(t)$ where the dynamics governing $\omega_c(t)$ may be part of the dynamics modeled in the problem. Another approach may be to treat ω_c as an input from the outside. Either of these formulations could be used (depending on the problem studied) using the basic methodology presented here. Essentially since our primary objective in this section is for the *faster* transient analysis (such as the dynamics of the transmission system) using phasors, we assume that ω_c is fixed at 60 Hz. Note however that this does not imply that $\omega = 60\text{ Hz}$ for the signal $e(t)$, since the actual frequency of the signal $e(t)$ (assuming that the amplitude $E(t)$ is constant) is $\omega = \omega_c + \delta$. The concept of “a signal frequency” becomes more difficult when the amplitude is also time-varying such as in emerging voltage problems. Therefore the notion of time-varying phasor needs to be carefully thought out starting from the fundamentals and this will be done in the next section.

To develop a precise dynamic time-varying phasor for the general system, the following definitions are now made:

Definition 4 (Phasor signal) *The set of all functions*

$$\tilde{E}(t) = E(t)e^{j\delta(t)} = E(t)\angle\delta(t) \quad (95)$$

which are complex valued functions of time (i.e. $\tilde{E} : \mathbb{R} \rightarrow \mathbb{C}$ and $\tilde{E}$ is C^∞) is defined as the set $\mathcal{T}$ of phasors. $\square$

Definition 5 (Modulated signal) *The set of all functions*

$$e(t) = E(t)\cos(\omega_c t + \delta(t)) \quad (96)$$

which are real valued smooth functions of time (i.e. $e : \mathbb{R} \rightarrow \mathbb{R}$ and e is C^∞) as the set $\mathcal{M}$ of modulated functions. $\square$

Since power system engineering is mostly about moving power around, the respectively modulated and phasor form of the instantaneous power $p(t) = e(t)i(t)$ through a cross section or into an impedance is equally important to voltage and current (Definitions 4, 5). In this respect, phasor use in power systems differs from most other applications. The following readily results by simple manipulation.

Definition 6 (Complex power) *Given a voltage signal $e(\cdot) \in \mathcal{M}$ and a current signal $i(\cdot) \in \mathcal{M}$ the phasor power operator $\tilde{S}(e, i)$ is given by*

$$\tilde{S}(e, i) := \mathcal{P}(e)\mathcal{P}(i)^* = \tilde{E}\tilde{I}^* = P + jQ \quad (97)$$

where $\tilde{I}^*$ denotes the complex conjugate of $\tilde{I}$. The real power operator $\mathcal{P}$ is the real part of $\tilde{S}(e, i)$:

$$P(e, i) := \text{Real}(\tilde{S}(e, i)) = \text{Real}(\tilde{E}\tilde{I}^*) \quad (98)$$

Similarly the reactive power operator $\mathcal{Q}$ is the imaginary part:

$$Q(e, i) := \text{Imag}(\tilde{S}(e, i)) = \text{Imag}(\tilde{E}\tilde{I}^*) \quad (99)$$

Suppose $\tilde{E}(t) = E(t)\angle\delta(t)$ and $\tilde{I}(t) = I(t)\angle(\delta(t) + \varphi(t))$. Then it is easily seen that the real power $P(e, i) = EI\cos(\varphi)$ and the reactive power $Q(e, i) = -EI\sin(\varphi)$. Definitions locally

follow from (94), (95), and (96) which leads to the equation,

$$\begin{aligned} p(t) &= e(t)i(t) \\ &= E(t)I(t)\cos(\phi(t))\left[1 + \cos 2(\omega_c t + \delta(t))\right] \\ &\quad - E(t)I(t)\sin(\phi(t))\left[\sin 2(\omega_c t + \delta(t))\right] \\ &= P(e(t), i(t))\left[1 + \cos 2(\omega_c t + \delta(t))\right] \\ &\quad + Q(e(t), i(t))\left[\sin 2(\omega_c t + \delta(t))\right] \end{aligned} \quad (100)$$

which could be interpreted as a phasor relationship at double the carrier frequency ω_c .

Remarks: Note that the sign of $P(e(t), i(t))\left[1 + \cos 2(\omega_c t + \delta(t))\right]$ always matches the sign of P and, in the stationary case where E, I, δ, φ and P are all constants, P represents a net cumulative power flow which is thus a “real” or “active” transfer of energy. On the other hand $Q(e(t), i(t))\left[\sin 2(\omega_c t + \delta(t))\right]$ does not generally match the sign of Q and in the stationary case it represent a fixed amount of energy swinging across the cross section $A - A$, at double frequency with no net exchange over time periods of integer number of half cycles of ω_c . This latter property is denoted by the name of “reactive” (which probably derives from the word reactance or vice versa).

Definitions 4, 5 and 6 define the phasor form for voltage current and power in a manner applicable whether the modulating signals E, δ are time varying or constant. As originally defined only for the stationary case, where time derivatives of $E, I, \delta, \varphi, P, Q$ are identically zero, this concept is precise, such as in problem areas like power flow computation. In traditional engineering practice the stationary relations are also used, under the name of quasi stationary (or quasi-static) cases, as approximations for slowly varying E and δ i.e. $\dot{E} = \eta_E \neq 0$, $\dot{\delta} = \eta_\delta \neq 0$ where η_E and η_δ are small. The limits of the necessary slowness for good approximation were established both by experience and simulation and recently also analytically [116] to cover conventional “transient stability” analysis primarily for angle (electromechanical) dynamics and marginally also voltage (electromagnetic) dynamics. New problems resulting from new equipment and practice results in voltage dynamic conditions where the quasi stationary approximation is no longer valid [86]. Often this fact is obscured by the deeply ingrained status of phasor representation in our engineering minds resulting in some dubious results reported in the literature.

This situation clearly calls for a precise analysis of the properties of phasors introduced through (94), (95), (96) for the entire time and frequency domain. Such an analysis is presented here.

First of all it needs to be observed that the transformation (e to $\tilde{E}$ or $\tilde{E}$ to e) implied by (94)–(96) is not unique or one to one unless certain assumptions are made on e and δ .

To illustrate the mathematical difficulty in the definition, let us consider a very simple sinusoidal signal of the form $e_0(t) = E_0 \cos \omega_c t$. By inspection, it is clear that the phasor $\tilde{E}_0(t)$ should be uniquely defined as $E_0 \angle 0$. However the time-varying phasor

$$\tilde{E}_1(t) = E_0 \cos \omega_c t \angle (-j\omega_c t) \quad (101)$$

also satisfies the basic relationship (94), hence qualifies as a can-

didate for being a time-varying phasor associated with $e_0(t)$. In fact, it is easy to construct an infinite number of different time-varying phasors which all satisfy (94) but they are not of practical interest. The point is that in general, an explicit representation of the form $\delta_0(t)$ is not available for modulated signals, and the problem of finding the phasor is not mathematically meaningful unless we impose some assumptions to tighten the field of possible candidates to the practically interesting ones. For instance, it can be observed that the degenerate phasors such as $\tilde{E}_1(t)$ in (101) associated with the signal $e_0(t)$ all have their bandwidths (as defined by their Fourier transforms) greater than or equal to ω_c whereas the bandwidth of the phasor $\tilde{E}_0(t)$ is strictly less than ω_c . In other words, if we restrict the choice of phasors to those with bandwidths less than the carrier frequency ω_c , then $\tilde{E}_0(t)$ is the unique phasor associated with $e_0(t)$ and the problem is well-defined. Next we will show that this property holds for any modulated signal by imposing this low pass assumption on the signal bandwidths.

II.1.1 Low pass phasor signals

The sets $\mathcal{M}$ and $\mathcal{P}$ are easily seen (Definitions 4 and 5) to be vector spaces. By the engineering definition of phasors, an element say $\tilde{I}(\cdot) \in \mathcal{T}$ can be considered to be the corresponding time-varying phasor for a modulated signal $\tilde{I}(\cdot)$ provided $\tilde{I}$ and i satisfy the modulation equation (94). Therefore in this formulation, the phasor definition becomes a mathematical operator, say $\mathcal{P}$, which maps the modulated signals in the space $\mathcal{M}$ to those time-varying phasor signals $\mathcal{T}$ which satisfy the modulation equation (94). Precisely, $\tilde{I} \in \mathcal{P}i$ provided (94) holds. However the example in (101) above shows that even a simple signal such as e_0 may be mapped to an infinite number of time-varying phasor signals such as $\tilde{E}_0$ and $\tilde{E}_1$. In other words, the phasor operator $\mathcal{P}$ based on the modulation property (95) is not one-to-one (may be set-valued, not single valued) in general. But as mentioned before, from a practical view-point, it is quite reasonable to restrict the phasor analysis to those with bandwidths smaller than the carrier frequency.

Definition 7 (Low pass phasor) A signal $\tilde{E}(\cdot) \in \mathcal{T}$ is a low-pass phasor if its Fourier transform $\tilde{E}(\omega)$ satisfies

$$(LP) \quad \tilde{E}(\omega) = 0, \quad \omega \geq \omega_c \text{ and } \omega \leq -\omega_c \quad (102)$$

The set of all low-pass phasors is defined as $\mathcal{L}$ and clearly $\mathcal{L} \subset \mathcal{T}$ is a subspace. An assumption that is inherent in the definition of low-pass phasors is the existence of its Fourier transform. $\square$

Notation: The Fourier transforms of time-varying phasors are denoted by corresponding bold-face capital letters such as $\tilde{E}$ denoting the Fourier transform of $\tilde{E}$. Similarly plain bold capital letters such as $\mathbf{E}$ denote the Fourier transform of modulated signals such as $e \in \mathcal{M}$.

The corresponding subset of $\mathcal{M}$ which is related to $\mathcal{L}$ through the property (96) can be defined as the set of band-pass signals.

Definition 8 (Band-pass signal) A signal $e(\cdot) \in \mathcal{M}$ is a band-pass signal if its Fourier transform $\mathbf{E}(\omega)$ satisfies

$$(BP) \quad \mathbf{E}(\omega) = 0, \quad \omega \leq 0 \text{ and } \omega \geq 2\omega_c \quad (103)$$

$\square$

The subset of band-pass signals within the space $\mathcal{M}$ is defined as the space of band-pass signals, denoted by $\mathcal{B}$.

Restriction to these subspaces will establish the phasor transformation precisely as a linear operator as follows.

II.2 The time-varying phasor and its properties

The preceding analysis makes it clear that while the quasi stationary phasor (an approximation) proved useful and reliable throughout the transient stability age when the only dynamics encountered was the one associated with analyzing the stability of transient angular oscillations of generator rotors faster events, especially in dynamic voltage conditions, require precise time varying phasor foundations to give reliable dynamic results. Such a precise time varying phasor transformation will now be introduced and defined in

Theorem 10 (Low pass phasor transformation Theorem) [102] Assume that definitions 4-8 apply and that $e(t)$ is restricted to $\mathcal{L}$, i.e. $e(t) \in \mathcal{B}$ (and so also $\tilde{E}(t) \in \mathcal{L}$) then the map $\mathcal{P} : \mathcal{B} \rightarrow \mathcal{L}$ defined by (94) is a linear operator which possesses the following properties 1-4. Q.E.D.

Note: In the statement of the Theorem, the symbols such as $e(t)$, $\tilde{E}(t)$ correspond to any voltage e , current i , power p , etc. Each Property to be stated will be proved individually.

Property 1 $\mathcal{P} : \mathcal{B} \rightarrow \mathcal{L}$ is a one-to-one and onto map. Moreover $\mathcal{P} : \mathcal{B} \rightarrow \mathcal{L}$ (or simply $\mathcal{P}$) is linear.

Proof: (One-to-one property): Let $e(\cdot) \in \mathcal{B}$. First it will be proved that there exists a unique $\tilde{E} \in \mathcal{L}$ which satisfies $\mathcal{P}(e) = \tilde{E}$. The proof directly follows from the concept of analytic signal from communication theory and is summarized here for continuity. Let $\tilde{e}(t)$ denote the Hilbert transform of the signal $e(t)$, and define the pre-envelope or the analytic signal $e_+(t)$ by $e_+(t) := e(t) + j\tilde{e}(t)$. By the assumption (BP) on the band-pass signals in $\mathcal{B}$, it follows that the Fourier transform of $e_+(t)$ is nonzero only within the interval $0 < \omega < 2\omega_c$. Therefore the signal $e_+(t)$ can be defined as $e_+(t) = \tilde{E}(t) \exp(j\omega_c t)$ for a unique function $\tilde{E}(t)$ whose Fourier transform can be seen to be

$$\tilde{E}(\omega) = \begin{cases} 2\mathbf{E}(\omega + \omega_c), & -\omega_c < \omega < \omega_c \\ 0, & \text{otherwise} \end{cases} \quad (104)$$

Since the pre-envelope $e_+(t)$ by construction satisfies $e(t) = \text{Re}(e_+(t))$, the function $\tilde{E}(t)$ satisfies the equation (94), hence $\tilde{E} \in \mathcal{P}(e)$. From the Fourier transform property (94) and the band-pass property (BP), it follows that $\tilde{E} \in \mathcal{L}$. The fact that $\tilde{E}$ is the only such function in $\mathcal{L}$ with this property can be easily seen by the following equivalent construction of $\tilde{E}$ based on the classical demodulation technique (Figure 2.35a in [38], [28]): 1) The real part $e_d(t)$ of the signal $\tilde{E}(t)$ denoted the *in-phase component* [38] or the direct component is recovered from $e(t)$ by passing the product $e(t) \cos \omega_c t$ through an ideal low-pass filter with cut-off frequency at $\omega = \omega_c$. 2) Similarly the imaginary part $e_q(t)$ of $\tilde{E}(t)$ denoted the *quadrature component* [38] can be reconstructed by passing the product $-e(t) \sin \omega_c t$ again through an ideal low-pass filter with the cut-off frequency at $\omega = \omega_c$. This demodulation technique is significant for real-time practical implementations because low pass filters can be effectively emulated in real-time using standard digital signal processing techniques. The converse that a phasor signal $\tilde{E} \in \mathcal{L}$ maps into a unique e under the inverse of the phasor operator is straightforward from (94). It is easily seen that $\mathcal{P}$ is also onto.

(Linearity): The linearity follows simply by the linearity of the construction of real and imaginary parts of $\vec{E}$ using the low-pass filter shown above. Q.E.D.

Property 2. Assume that $e(\cdot) \in \mathcal{B}$ and $i(\cdot) \in \mathcal{B}$ then instantaneous power balance equations can be equivalently rewritten in terms of the real power and reactive power balance equations.

Proof: Since all the signals are in $\mathcal{B}$ it follows that the phasor operator $\mathcal{P}$ is linear by Property 1 and so is the complex conjugate operator: therefore the composition $\mathcal{P}(\cdot)^*$. By the generalized form of Tellegen's Theorem [60] applying the linear operator $\mathcal{P}$ on the voltages and the linear operator $\mathcal{P}^*$ on the currents it follows that

$$\begin{aligned} \sum_{k=1}^n e_k(t) i_k(t) = 0 &\Leftrightarrow \sum_{k=1}^n \vec{E}_k(t) \vec{I}_k(t) = 0 \\ &\Leftrightarrow \sum_{k=1}^n P(e_k(t), i_k(t)) = 0 \\ &\quad \sum_{k=1}^n Q(e_k(t), i_k(t)) = 0 \end{aligned} \quad (105)$$

Q.E.D.

Note: It will be shown in Section 3 that stronger results can be obtained for various specific problem areas defined by additional assumptions.

Property 3 Suppose $e(\cdot) \in \mathcal{B}$ and $\frac{d}{dt}e(\cdot) \in \mathcal{B}$. Then

$$\mathcal{P}\left(\frac{d}{dt}e(t)\right) = \frac{d}{dt}\mathcal{P}(e(t)) + j\omega_c \mathcal{P}(e(t)) \quad (106)$$

Proof: Suppose $e(\cdot) \in \mathcal{B}$ and let $\vec{E} = \mathcal{P}(e)$. Then these two are related in a unique way by ([5], see the proof of Property 1)

$$e(t) = \text{Re}(\vec{E}(t)e^{j\omega_c t}) \quad (107)$$

By direct differentiation, we find

$$\frac{d}{dt}e(t) = \text{Re}\left[\left(\frac{d}{dt}\vec{E}(t) + j\omega_c \vec{E}(t)\right)e^{j\omega_c t}\right] \quad (108)$$

On the other hand, since the derivative $\dot{e} \in \mathcal{B}$, there exists a unique derivative in the phasor domain $\mathcal{P}(\dot{e})$ which satisfies

$$\dot{e}(t) = \text{Re}[\mathcal{P}(\dot{e}(t))e^{j\omega_c t}] \quad (109)$$

Since the phasor operator is one-to-one (Property 1), comparing equations (108) and (109), the result follows. Q. E. D.

Remarks: 1. Note that for the conventional stationary/quasi stationary phasor calculus the derivative consists of only the second term.

2. Using Property 3, the instantaneous equations for a linear capacitor C , a linear inductor L (and a linear resistor) elements in the network are easily rewritten using the derivative rule above.

Property 4

1. (Capacitor) The current $i_C(t)$ flowing through a capacitor C with the terminal voltage $e_C(t)$ can be represented by

$$\vec{I}_C(t) = C \frac{d}{dt}\vec{E}_C(t) + j\omega_c C \vec{E}_C(t) \quad (110)$$

in the phasor domain when i_C and e_C belong to $\mathcal{B}$.

2. (Inductor) The voltage $e_L(t)$ across an inductor L when current $i_L(t)$ is flowing through, can be represented by

$$\vec{E}_L(t) = L \frac{d}{dt}\vec{I}_L(t) + j\omega_c L \vec{I}_L(t) \quad (111)$$

in the phasor domain when i_L and e_L belong to $\mathcal{B}$.

3. (Resistor) The voltage $e_R(t)$ across a resistor R when the current $i_R(t)$ is flowing through can be represented by

$$\vec{E}_R(t) = R \vec{I}_R(t) \quad (112)$$

□

Remarks. 1. These three equations (110), (111), (112) along with (105) provide the relationships needed for writing the dynamic models in the time-varying representation. These properties are summarized in the form of a table (Table II.1) for convenience.

2. An equivalent definition for the time-varying phasor can be given for balanced three phase signals without any speed restrictions by using the Blondel transformation approach [101]. We briefly summarize the definition in the next section, and note that the main properties of the phasor transformation, namely the linearity is preserved by the Blondel transformation.

II.2.1 Fast time-varying phasors for three phase balanced signals

Consider a three phase signal $\underline{e}(t)$ of the form

$$\underline{e}(t) = \begin{pmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{pmatrix} = \begin{pmatrix} \sqrt{2}E_a(t)\cos(\omega_c t + \delta_a(t)) \\ \sqrt{2}E_b(t)\cos(\omega_c t + \delta_b(t)) \\ \sqrt{2}E_c(t)\cos(\omega_c t + \delta_c(t)) \end{pmatrix} \quad (113)$$

Clearly $\underline{e}(\cdot) \in \mathcal{M}^3 := \mathcal{M} \times \mathcal{M} \times \mathcal{M}$. Among these three phase signals, the set of all balanced three phase signals can be distinguished. Precisely we say that $\underline{e}(t) \in \mathcal{M}^3$ is balanced if there exist smooth scalar functions $E : \mathbb{R} \rightarrow \mathbb{R}$ and $\delta : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$\begin{aligned} \underline{e}(t) &= \begin{pmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{pmatrix} = \begin{pmatrix} \sqrt{2}E(t)\cos(\omega t + \delta(t)) \\ \sqrt{2}E(t)\cos(\omega t + \delta(t) - \frac{2\pi}{3}) \\ \sqrt{2}E(t)\cos(\omega t + \delta(t) + \frac{2\pi}{3}) \end{pmatrix} \\ &= \sqrt{3}B(t) \begin{pmatrix} E(t)\cos\delta(t) \\ E(t)\sin\delta(t) \\ 0 \end{pmatrix} \end{aligned} \quad (114)$$

where $B(t)$ is the orthonormal matrix,

$$B(t) := \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) & \frac{1}{\sqrt{2}} \\ \cos\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t - \frac{2\pi}{3}\right) & \frac{1}{\sqrt{2}} \\ \cos\left(\omega t + \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (115)$$

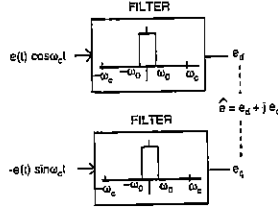
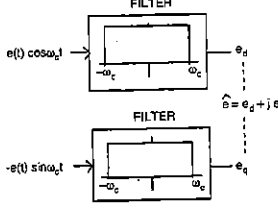
The set of all three phase balanced signals satisfying (114) as the set $\mathcal{M}_B^3$. Since the operation (114) is linear, it follows that $\mathcal{M}_B^3 \subset \mathcal{M}^3$ itself is a vector space.

Now for the balanced signals $\underline{e}(\cdot) \in \mathcal{M}_B^3$, we can define the associated time-varying phasor $\vec{E}(\cdot)$ by inverting the linear transformation $B(t)$ in (114). To distinguish the definition for the balanced three phase signals from the single phase band-limited signals of the previous section, we represent the phasor operator by $\mathcal{P}_B$ her B stands for the balanced case.

Definition 9 The phasor representation $\mathcal{P}_B(\underline{e}(t)) := \vec{E}(t)$ of a balanced sinusoidal three phase signal $\underline{e}(t)$ is

$$\mathcal{P}_B(\underline{e}(t)) = (1 \quad j \quad 0) \frac{B^{-1}(t)}{\sqrt{3}} \underline{e}(t) \quad (116)$$

Table II.1. Properties of the low pass phasors.

Property	Instantaneous Signal Domain	Quasi-stationary Phasor Domain	Low pass Phasor Domain
Assumptions	None	Phasor speeds very small compared to ω_c (Narrow band assumption)	Phasor bandwidths less than ω_c (Assumptions (BP) and (LP))
Estimation	$e(t)$		
Power Concepts	Instantaneous power $p(t) = e(t)i(t)$	Complex Power	$S = \hat{e}i^*$
		Real Power	$P = \text{Re}(\hat{e}i^*)$
		Reactive Power	$Q = \text{Im}(\hat{e}i^*)$
Time Differentiation	$\frac{d}{dt}e(t)$	$j\omega_c \hat{e}$	$\frac{d}{dt}\hat{e}(t) + j\omega_c \hat{e}(t)$
Capacitor Equation	$i_C(t) = C \frac{d}{dt}e_C(t)$	$\hat{i}_C = j\omega_c C \hat{e}_C$	$\hat{i}_C(t) = j\omega_c C \hat{e}_C(t) + \frac{d}{dt}\hat{e}_C(t)$
Inductor Equation	$e_L(t) = L \frac{d}{dt}i_L(t)$	$\hat{e}_L = j\omega_c L \hat{i}_L$	$\hat{e}_L(t) = j\omega_c L \hat{i}_L(t) + \frac{d}{dt}\hat{i}_L(t)$
Network Equations	$\sum_{k=1}^n e_k(t) = 0$ (KVL)	$\sum_{k=1}^n \hat{e}_k = 0$ (KVL)	$\sum_{k=1}^n \hat{e}_k(t) = 0$ (KVL)
	$\sum_{k=1}^n i_k(t) = 0$ (KCL)	$\sum_{k=1}^n \hat{i}_k = 0$ (KCL)	$\sum_{k=1}^n \hat{i}_k(t) = 0$ (KCL)
Power Balance Equations	$\sum_{k=1}^n e_k(t)i_k(t) = 0$	$\sum_{k=1}^n \hat{e}_k \hat{i}_k^* = 0$	$\sum_{k=1}^n \hat{e}_k(t)\hat{i}_k(t)^* = 0$

Or in alternate (and conventional) notations

$$\begin{aligned}
 \mathcal{P}_B(\underline{e}(t)) &:= \vec{E}(t) := E_d(t) + jE_q(t) \\
 &:= E(t) \cos \delta(t) + jE(t) \sin \delta(t) \quad (117) \\
 &= E(t)e^{j\delta(t)} = E(t)\angle\delta(t) \quad (118)
 \end{aligned}$$

□

Remarks. 1. Given a balanced three phase signal $\underline{e}(t)$, the phasor operation above essentially identifies the associated phasor $\vec{E}(t)$ by exploiting the invertibility of the Blondel transformation matrix (114).

2. Blondel transformation above has been routinely used in machine theory for decades for simplifying the rotating flux equations within the machine by defining a rotating reference frame B where the entries of the matrix are usually functions of the machine angle say θ . Mainly this transformation identifies the direct (the d -axis in short) and quadrature (the q axis in short) axis components which are stationary with respect to the rotor thus considerably simplifying the electromagnetic equations. However when multi-machine models are considered, the d - q axes of the various machines are each rotating at different reference frames resulting from different machine speeds. To unify the analysis in the computation of stator transients, under

the quasi-stationary assumption, it is customary to transform all the different machine references to a common reference which is usually taken to be the 60 Hz synchronous speed. For instance, see the treatment in Chapter 9 of [8] where this transformation to a common reference frame is explicitly derived in the matrix form under the constant impedance representation of the network and the connected loads.

Recent work of [68, 69] extends this approach to a more general formulation by transforming the transmission network and the loads into equivalent R - L networks in d and q axes in the form of flux equations. Under the assumption that the network with the loads only consist of resistors and inductors, it is shown in [69] that all the machine equations along with the network-load flux equations can be transformed to a common d - q axis reference frame where the computations of stator transients are carried out. However such an approach by requiring equivalent physical networks in the d - q axes becomes restrictive. Among other factors, they have been forced to exclude capacitors and general load types.

On the other hand, we approach the problem of phasor analysis as one of a convenient mathematical transformation. The motivation is to facilitate the problem solution in the transformed domain. It is important to realize that the states of

Table II.2. Properties of the fast time-varying balanced phasors.

Property	Instantaneous (actual) time Domain	Quasi-stationary Phasor Domain	Fast time-varying Phasor Domain		
Assumptions	None	Phasor speeds are very small compared to ω (Quasi-stationary assumption)	The three phases are balanced.		
Power Concepts	Instantaneous power $p(t) = e(t) \cdot i(t)$ $= \begin{pmatrix} e_a(t)i_a(t) \\ e_b(t)i_b(t) \\ e_c(t)i_c(t) \end{pmatrix}$	Complex Power	$\tilde{S} = \tilde{E}\tilde{I}^*$	Complex Power	$\tilde{S} = \tilde{E}\tilde{I}^*$
		Real Power	$P = \text{Re}(\tilde{E}\tilde{I}^*)$	Real Power	$P = \text{Re}(\tilde{E}\tilde{I}^*)$
		Reactive Power	$Q = -\text{Im}(\tilde{E}\tilde{I}^*)$	Reactive Power	$Q = -\text{Im}(\tilde{E}\tilde{I}^*)$
Time Differentiation	$\frac{d}{dt}e(t)$	$j\omega\tilde{E}$	$\frac{d}{dt}\tilde{E} + j\omega\tilde{E}$		
Capacitor Equation	$i_C(t) = C\frac{d}{dt}e_C(t)$	$\tilde{I}_C = j\omega C\tilde{E}_C$	$\tilde{I}_C = j\omega C\tilde{E}_C + C\frac{d}{dt}\tilde{E}_C$		
Inductor Equation	$e_L = L\frac{d}{dt}i_L$	$\tilde{E}_L = j\omega L\tilde{I}_L$	$\tilde{E}_L = j\omega L\tilde{I}_L + L\frac{d}{dt}\tilde{I}_L$		
Network Equations (Kirchoff)	$\sum_{k=1}^n e_k(t) = 0 \quad (\text{KVL})$	$\sum_{k=1}^n \tilde{E}_k = 0 \quad (\text{KVL})$	$\sum_{k=1}^n \tilde{E}_k = 0 \quad (\text{KVL})$		
	$\sum_{k=1}^n i_k(t) = 0 \quad (\text{KCL})$	$\sum_{k=1}^n \tilde{I}_k = 0 \quad (\text{KCL})$	$\sum_{k=1}^n \tilde{I}_k = 0 \quad (\text{KCL})$		
Power Balance Equations	$\sum_{k=1}^n e_k(t) \cdot i_k(t) = 0$	$\sum_{k=1}^n \tilde{E}_k \tilde{I}_k^* = 0$	$\sum_{k=1}^n \tilde{E}_k \tilde{I}_k^* = 0$		

the transformed system need not have direct physical meaning. If properly solved mathematically, however, they will give the correct physical result after inverse transformation. Think of Laplace transformation. The transforms do not have direct physical meaning (although after working with them for some time, one may have the feeling that they do – as it also happened in the phasor case), yet the inverse Laplace transform of the solution gives the right physical time domain results. Understanding this fact opens up a wide horizon as it does in this paper yielding a set of dynamic load-flow equations for analyzing fast transients.

The properties of the time-varying phasor transformation $\mathcal{P}_B$ for the case of balanced three phase signals are similar to the operation defined previously for band limited signals (low pass phasors).

Theorem 11 (The balanced three phase phasor transformation operator) The map $\mathcal{P}_B : \mathcal{M}_B^3 \rightarrow \mathcal{T}$ defined by (116) is a linear operator, and is one-to-one and onto. Moreover it satisfies the four properties (Properties 1 through 4) restated for this case. Q. E. D.

The properties of the transformations are summarized in a tabular form presented as Table II.2.

Remarks. Note that the fundamental properties, Properties 1 through 4, on linearity (Property 1), on the complex power-balance equations (Property 2), on the differentiation rule (Property 3), and on the linear network equations (Property 4) are exactly the same for the two notions of time-varying phasors introduced namely for low-pass phasors $\mathcal{P}$ and the three-phase balanced time-varying phasor. This is to be expected since when the three phase balanced signals are also band-limited signals and the network is symmetric, the results between the two definitions must coincide so that they are consistent. The first case of low-pass phasors $\mathcal{P}$ by not requiring the balanced symmetric op-

eration is more useful when unbalanced conditions such as under asymmetric fault conditions are considered (see an extension of the symmetric components method in [102]). The Blondel transformation approach $\mathcal{P}_B$ on the other hand is not restricted by any bandwidth assumption, hence is suited for analyzing faster transients such as the fast electromagnetic transients arising in transmission networks. For these reasons, the first concept is denoted the *low-pass phasors* (connected with band-limited signals) and the latter one, the *fast balanced phasors* (connected with balanced three phase signals). From Properties 1–4, both concepts are powerful tools for simplifying the analysis as shown in the next section. Specifically the fast balanced phasors will be used in the next few sections for deriving a set of time-delay algebraic equations that describe the network dynamics of the interconnected transmission system.

II.3 The set of tools provided by the time-varying phasors

The set of tools offered by the three basic formulations for writing models for network type systems

- 1.) Modulated time domain signals
- 2.) Traditional quasi stationary phasor signals
- 3.) Time varying low pass phasor signals
- 4.) Fast time-varying balanced phasor signals

are summarized in Tables II.1 and II.2.

Important questions arise when one looks into such details as three phase balance or the lack of it, bandwidths of modulating signals etc., and the presence of distributed parameter components in the power network. A quick look at the familiar structure of the power system in Figure II.1 will help to identify domains of phasor calculus on this system.

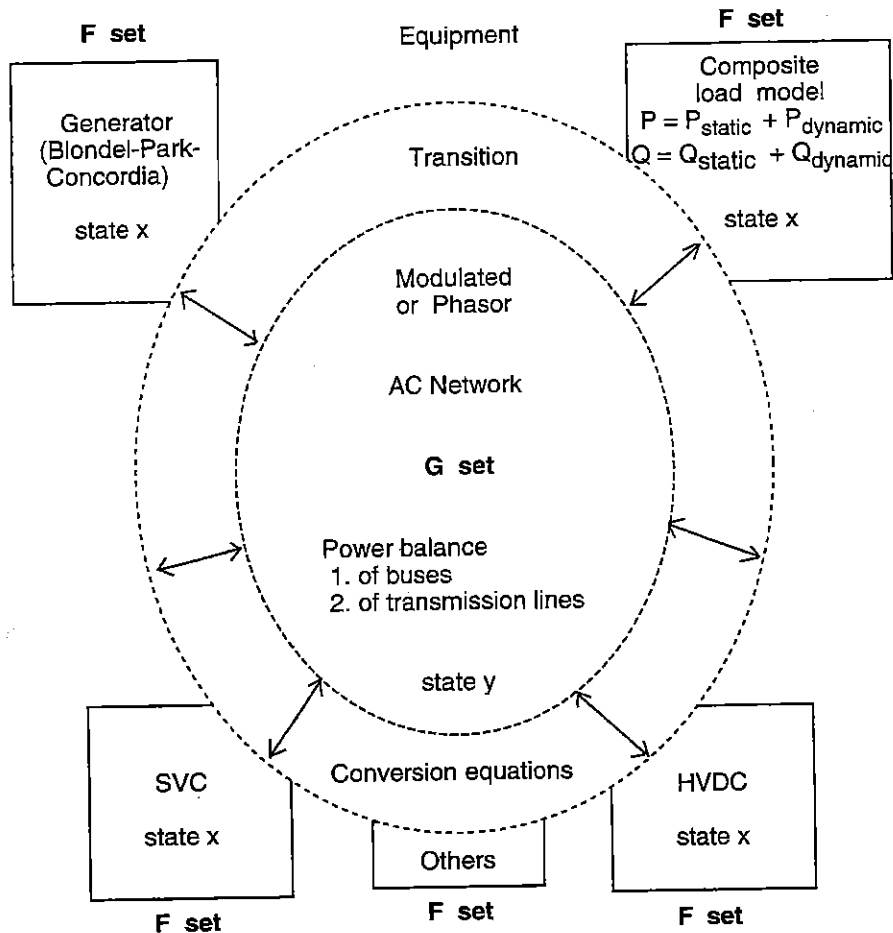


Figure II.1. Overall structure of the large power system.

II.4 Parts of the power system where phasors apply

The large electric power system consists of those fundamental elements:

- 1.) *Transmission*, i.e. a network of high voltage transmission lines connecting buses (nodes), is at the core of the system (see sketch in Figure II.1). All states or signals within are modulated AC (or appear to be such at the endpoints for high voltage DC lines and SVC). It has a sparse connection matrix (from 2 to 10 lines per bus) even for large systems. This part is represented by a set G of equations.
- 2.) *Equipment* such as generators, loads and various power electronic (thyristor, SVC, HVDC) devices are connected to individual buses. Each item has its own dynamics in various forms, usually differential equations (e.g. Park equations for the machines). None of the equipment is physically in modulated AC form and there is no direct coupling between individual items outside their connection to the buses (Figure II.1). Accordingly, the equipment dynamics is defined mostly by a set F (Figure II.1) of ordinary differential equations and need not be discussed in this paper.
- 3.) Since the mathematical form of describing the dynamics is thus quite different for respectively transmission and the attached equipment, conversion equations (e.g. Blondel) are typically needed for the interaction between the G and F sets (Figure II.1).

Our interest is here directed to the dynamics of the transmission system i.e. the G set where phasor representation nat-

urally applies. It will be proved in subsections 4-7 that the G set under certain assumptions can be represented by a set of differential-algebraic equations in the phasor domain which are time-delayed equations in nature where the time-delays are contributed by various transmission lines.

In a stationary or steady-state condition (a normal operating condition), a large power system is defined by two sets of algebraic equations

$$F \text{ set: } f(x, y) = 0, \quad x \in \mathbb{R}^n, \quad f: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^n \quad (119)$$

$$G \text{ set: } g(x, y) = 0, \quad y \in \mathbb{R}^m, \quad g: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^m \quad (120)$$

When the system enters a dynamic condition, equations of the F set naturally change into ordinary differential equations in the x states ($\dot{x} = f(x, y)$). A special feature of the power system is that interconnections of x variables occurs exclusively at individual buses. Bus to bus interconnections are by transmission lines through the y states. Each transmission line represented by a single phase circuit for a balanced system is a four terminal (three if the circuit is represented by one phase and a neutral for balanced system). The differential equations of the F set are not in phasors generally (e.g. the generators) since their state variables are not modulated signals. (although a transformation to phasors via the Blondel transformation may be convenient as explained above). The area to use phasors is typically in the G set that is the transmission network. For balanced networks, this set consists of buses interconnected by the four terminal networks of the transmission line dynamically. Each such four terminal network is a boundary value problem. This dichotomy of representations between the F and G set calls for a (mathematical) transition layer consisting of the Blondel transforma-

tion and others. In this section our interest is the G set and its representation.

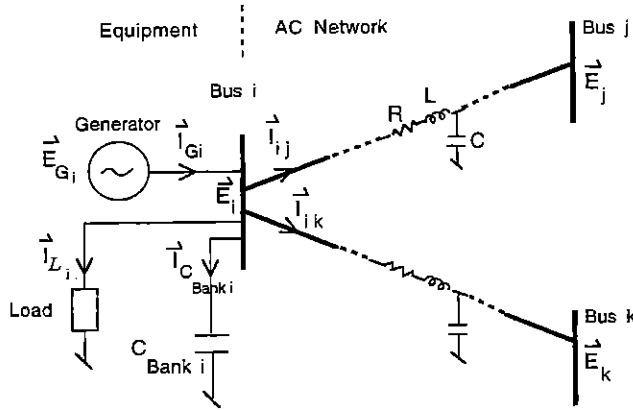


Figure II.2. Sketch for the power balance equations.

II.4.1 Phasors in the G set i.e. the transmission system

The electric power transmission system is an electric network which connects the buses. A simplified approximate example of one bus i and two neighbors j and k is shown in Figure II.2. In a general and precise sense the G set has two types of components

- 1.) The buses or simply nodes which must balance flows of all connected branches and
- 2.) The transmission lines which are basically four terminal networks that may involve mutual inductances and may be in distributed parameter form which produce time delay effects.

In linear system theory, Kirchoff's well-known laws

- 1.) The sum of all currents at every node is zero at every instant of time.
- 2.) The sum of all voltages around every loop is zero at every instant of time.

are invoked to define the dynamics of such a system. However since power system is about moving electric power around, the power balance equations which follow are preferable

- 1.) At the buses the sum of instantaneous power flowing into a bus from all connected equipment and all connected transmission lines is zero. Buses have no appreciable storage ability.
- 2.) The lines: the difference of instantaneous power at the two ends of a line is stored in the line or dissipated as heat for loss.

These are easily shown to be equivalent or homeomorphic to Kirchoff by Tellegen's Theorem [12, 60].

There is little that can be added to 1.). However the general three terminal definition of the line (which assumes balanced three phase operation and is used here to save space and improve clarity. The four terminal case follows directly) in phasor form the solution of the four terminal network takes the form

$$\vec{I}_i = \vec{Y}_{ij}(\vec{E}_i, \vec{E}_j, \tau_{ij}, \dots) \quad (121)$$

$$\vec{I}_j = \vec{Y}_{ji}(\vec{E}_i, \vec{E}_j, \tau_{ij}, \dots) \quad (122)$$

can be quite a bit more complex than the familiar linear admittance constant case

$$\vec{I}_i = \vec{Y}_{ii}\vec{E}_i + \vec{Y}_{ij}\vec{E}_j \quad (123)$$

$$\vec{I}_j = \vec{Y}_{ji}\vec{E}_i + \vec{Y}_{jj}\vec{E}_j \quad (124)$$

In more complex situations, it may involve the solution of boundary value problems to get a form like (121)–(122) and the presence of nonlinearities. Furthermore the power balance form

$$\vec{S}_{ij} = \vec{E}_i \vec{I}_{ij}^*, \quad \vec{S}_{ji} = \vec{E}_j \vec{I}_{ji}^*$$

is nonlinear even if the line circuit is linear as it is on the power system.

Overall then the broad general form of the equations to define the G set is in phasor form

1.) Bus i power balance in phasor form

$$\sum_i \vec{S}_{Gi} + \sum_i \vec{S}_{Li} + \sum_i \vec{S}_{SVC, HVDCi} + \sum_i \vec{S}_{miz} + (j\omega C_i E_i^2 + C_i \vec{E}_i \vec{E}_i^*) - \sum_j \vec{S}_{ij} = 0 \quad (125)$$

2.) Line (ij, ji) power balance equations if the modulation is averaged out by the balanced nature of the three phase system or by some other means (see the complex power balance equations, Property 2)

$$\vec{S}_{ij} = \vec{E}_i Y_{ij}(\vec{E}_i, \vec{E}_j, \tau_{ij}, \dots) = \vec{E}_i \vec{I}_{ij}^* \quad (126)$$

$$\vec{S}_{ji} = \vec{E}_j Y_{ji}(\vec{E}_i, \vec{E}_j, \tau_{ij}, \dots) = \vec{E}_j \vec{I}_{ji}^* \quad (127)$$

Note that $\vec{E}_i$ and $\vec{E}_j$ up front can usually be divided out of these latter two equations.

Here subscript G refers to the generator, L to the load, subscript $SVC, HVDC$ to thyristor controls, subscript miz means unspecified equipment, C_i is a capacitor bank (if any) at bus i , Y_{ij}, Y_{ji} are generally nonlinear functions of the voltages (but for G sets consisting of simple ac transmission lines they are elements of the familiar admittance matrix or Y bus matrix used in power flow analysis) at the two line ends, time delay and potentially other parameters.

II.5 Precise and detailed analysis of the transmission line dynamics including distributed constants and line losses

Important components of both power and communications systems are transmission lines which are distributed parameter systems modeled precisely by a special type of partial differential equations known as the wave equations because they define traveling waves along the line. Typically in power system networks, the line length is an (often small) fraction of the wave length of the carrier on the line, while in communications the reverse situation prevails with thousands or millions of carrier wave lengths along the line (a wave guide). The basic problem is the same but the situation is more complex on the power system. Wave equations and their solutions are well known in the literature including the engineering use of phasors for the stationary system (e.g. Chapter 6 in [115] 1955). A time-delay solution of the wave equations in the instantaneous time domain was proposed in [15] under the assumption of lossless transmis-

sion lines, commonly known (in electromagnetics/transmission line theory) under the name of the *Branin algorithm*. In fact, it appears that such time-delay solutions were known earlier in the power system context, and [6] appears to be one of the first references. In this paper, our aim is to derive explicit solutions to the wave equations using time-varying phasors so that we can formulate the dynamics of the distributed parameter type interconnected transmission network all from within the familiar phasor analytical domain.

A mathematically precise formulation of the time-varying phasors was established in the previous sections (II.1 to II.3). In this section, using the properties derived there, the construction of time-delay type exact solutions to the wave equation is briefly summarized to demonstrate our claim that the general phasor operator will apply directly to solving the partial differential equation problem as well.

Looking at a short segment of the transmission line (Figure II.2), it is readily recognized as an ordinary linear circuit for which relations between Δe , Δi and $\Delta \ell$ and Δt are readily written. Taking limits for $\Delta \ell \rightarrow 0$ the wave equations, twin partial differential equations result, which are of general validity not depending on conditions like stationarity. They also account for line losses

$$\frac{\partial e(\ell, t)}{\partial \ell} = R_{ij} i(\ell, t) + L_{ij} \frac{\partial i(\ell, t)}{\partial t} \quad (128)$$

$$\frac{\partial i(\ell, t)}{\partial \ell} = G_{ij} e(\ell, t) + C_{ij} \frac{\partial e(\ell, t)}{\partial t} \quad (129)$$

When these are combined with the conditions at the line ends i and j

$$e(0, t) = e_i(t), \quad e(\ell_{ij}, t) = e_j(t) \quad (130)$$

$$i(0, t) = i_{ij}(t), \quad i(\ell_{ij}, t) = -i_{ij}(t) \quad (131)$$

The dynamics of the line is then defined in terms of the boundary value problems of (128)–(131) or more precisely one pair of the four conditions in (128) and (131). The other pair represents the free variables of (130), (131). Assuming zero initial conditions, a solution for the lossless case ($R_{ij} = G_{ij} = 0$) can be obtained in the form [6, 15]

$$e_i(t) = \tilde{R}_{ij} i_{ij}(t) + \tilde{R}_{ij} i_{ji}(t - \tau_{ij}) + e_j(t - \tau_{ij}) \quad (132)$$

$$e_j(t) = \tilde{R}_{ij} i_{ji}(t) + \tilde{R}_{ij} i_{ij}(t - \tau_{ij}) + e_i(t - \tau_{ij}) \quad (133)$$

$$\tilde{R}_{ij} = \sqrt{\frac{L_{ij}}{C_{ij}}}, \quad \tau_{ij} = \frac{\ell_{ij}}{c} \quad (134)$$

The actual solution also included in it some terms corresponding to nonzero initial values for line voltages and currents but these technical issues will be postponed for the moment. Note that these equations depend on the characteristic line impedance $\tilde{R}_{ij}$ and include a time delay τ_{ij} which is line specific, it is the time the wave takes to pass through the line at the speed of light c . So when precisely calculated, each line is described by an (rather simple—in the lossless case) algebraic equation with a time delay which depends on the line length but not on any state variable. For three phase or parallel lines mutual impedances need to be considered, but again these can be eliminated precisely from the symmetry in balanced three phase lines [115].

These are classical results for the time domain although the form becomes more complicated when modulation is considered.

Using fast balanced time varying phasors as defined above, it develops that in general the phasor transformation is exchange-

able with the $\Delta \ell \rightarrow 0$ limit process. Accordingly a set of wave equations incorporated in a boundary value problem directly corresponding to (128)–(2) results

$$\frac{\partial \tilde{E}(\ell, t)}{\partial \ell} = R_{ij} \tilde{I}(\ell, t) + L_{ij} \frac{\partial \tilde{I}(\ell, t)}{\partial t} + j\omega_c L_{ij} \tilde{I}(\ell, t) \quad (135)$$

$$\frac{\partial \tilde{I}(\ell, t)}{\partial \ell} = G_{ij} \tilde{E}(\ell, t) + C_{ij} \frac{\partial \tilde{E}(\ell, t)}{\partial t} + j\omega_c C_{ij} \tilde{E}(\ell, t) \quad (136)$$

$$\tilde{E}(0, t) = \tilde{E}_i(t), \quad \tilde{E}(\ell_{ij}, t) = \tilde{E}_j(t) \quad (137)$$

$$\tilde{I}(0, t) = \tilde{I}_{ij}(t), \quad \tilde{I}(\ell_{ij}, t) = -\tilde{I}_{ij}(t) \quad (138)$$

Again for the lossless case this boundary value problem can be solved

$$\begin{aligned} \tilde{I}_{ij}(t) = & \tilde{I}_{ij}(t - 2\tau_{ij})e^{-j2\tilde{\theta}_{ij}} - \tilde{G}_{ij}\tilde{E}_i(t) + 2\tilde{G}_{ij}\tilde{E}_j(t - \tau_{ij})e^{-j\tilde{\theta}_{ij}} \\ & - \tilde{G}_{ij}\tilde{E}_i(t - 2\tau_{ij})e^{-j2\tilde{\theta}_{ij}} \end{aligned} \quad (139)$$

$$\begin{aligned} \tilde{I}_{ji}(t) = & \tilde{I}_{ji}(t - 2\tau_{ij})e^{-j2\tilde{\theta}_{ij}} - \tilde{G}_{ij}\tilde{E}_j(t) + 2\tilde{G}_{ij}\tilde{E}_i(t - \tau_{ij})e^{-j\tilde{\theta}_{ij}} \\ & + \tilde{G}_{ij}\tilde{E}_j(t - 2\tau_{ij})e^{-j2\tilde{\theta}_{ij}} \end{aligned} \quad (140)$$

where $\tilde{G}_{ij} = 1/\tilde{R}_{ij}$ is the characteristic admittance of the line ij and $\tilde{\theta}_{ij} = \omega_c \tau_{ij}$ is the phase shift introduced by the line ij . Note that (139)–(140) are more complex than (132)–(133) however remember that (132)–(133) are less specific. They are not restricted to modulated signals whereas the phasor form in (139)–(140) implies modulation.

When the line losses are included, then attenuation terms enter along with the time delays. Thus finding exact solutions to the phasor wave equations is much more difficult however quite precise solutions can be found under certain assumptions such as when the line resistance R_{ij} is much smaller than the line reactance $X_{ij} = \omega_c L_{ij}$. Actual practice typically satisfies this assumption very well. Laplace transform can then be used to find the solution.

In the Laplace domain, denoting the complex Laplace transforms of the phasors $\tilde{E}(z, t)$ and $\tilde{I}(z, t)$ by $\tilde{\mathbf{E}}(z, s)$ and $\tilde{\mathbf{I}}(z, s)$, the complex phasor wave equations (135)–(136) can be Laplace transformed to

$$\frac{d^2 \tilde{\mathbf{E}}(z, s)}{dz^2} = \gamma_{ij}^2 \tilde{\mathbf{E}}(z, s) \quad (141)$$

where

$$\gamma_{ij} = \sqrt{(R_{ij} + (s + j\omega_c)L_{ij})(s + j\omega_c)C_{ij}} \quad (142)$$

Also let

$$Y_{0ij} = \sqrt{\frac{(s + j\omega_c)C_{ij}}{R_{ij} + (s + j\omega_c)L_{ij}}} \quad (143)$$

Then solutions to the equation (141) can be found in the Laplace domain rather easily, but the challenge is to find valid inverse Laplace transforms for time domain solutions. Specifically the variables γ_{ij} and Y_{0ij} need to be simplified. Under the assumption that $R_{ij} \ll X_{ij}$, these quantities can be approximated as

$$\gamma_{ij} \approx (s + j\omega_c) \sqrt{L_{ij}C_{ij}} + \frac{R_{ij}}{2} \sqrt{\frac{C_{ij}}{L_{ij}}} \quad (144)$$

$$Y_{0ij} \approx \sqrt{\frac{C_{ij}}{L_{ij}}} \left(\frac{s + j\omega_c}{2L_{ij}} + (s + j\omega_c) \right) \quad (145)$$

Note that when the line losses are set to zero, (with $R_{ij}=0$), the quantities $Y_{0ij} = \tilde{G}_{ij}$ and $\gamma_{ij} = \sqrt{(s + j\omega_c)L_{ij}C_{ij}}$ are the lossless counterparts, hence equations (144) and (145) are indeed con-

sistent. With these approximations, an explicit solution to the wave equations (112)–(135), which now includes transmission line attenuation effects can be shown to be

$$\begin{aligned}\bar{I}_{ij}(t) &= \bar{G}_{ij}\bar{\phi}_i(t) + \bar{I}_{ij}(t - 2\tau_{ij})e^{-2(\bar{\alpha}_{ij} + j\bar{\theta}_{ij})} \\ &\quad - 2\bar{G}_{ij}\bar{\phi}_j(t - \tau)e^{-(\bar{\alpha}_{ij} + j\bar{\theta}_{ij})} \\ &\quad + \bar{G}_{ij}\bar{\phi}_i(t - 2\tau_{ij})e^{-2(\bar{\alpha}_{ij} + j\bar{\theta}_{ij})}\end{aligned}\quad (146)$$

$$\begin{aligned}\bar{I}_{ji}(t) &= \bar{G}_{ij}\bar{\phi}_j(t) + \bar{I}_{ji}(t - 2\tau)e^{-2(\bar{\alpha}_{ij} + j\bar{\theta}_{ij})} \\ &\quad - 2\bar{G}_{ij}\bar{\phi}_i(t - \tau_{ij})e^{-(\bar{\alpha}_{ij} + j\bar{\theta}_{ij})} \\ &\quad + \bar{G}_{ij}\bar{\phi}_j(t - 2\tau_{ij})e^{-2(\bar{\alpha}_{ij} + j\bar{\theta}_{ij})}\end{aligned}\quad (147)$$

where $\bar{\alpha}_{ij} = \frac{R_{ij}\ell_{ij}}{2} \sqrt{\frac{C_{ij}}{L_{ij}}}$ is the attenuation factor of the line.

The phasor functions $\bar{\phi}_i(t)$ and $\bar{\phi}_j(t)$ are given by the equations

$$\dot{\bar{\phi}}_i = -j\omega_c \bar{\phi}_i - \frac{R_{ij}}{2L_{ij}} \bar{\phi}_i + \bar{E}_i + j\omega_c \bar{E}_i \quad (148)$$

$$\dot{\bar{\phi}}_j = -j\omega_c \bar{\phi}_j - \frac{R_{ij}}{2L_{ij}} \bar{\phi}_j + \bar{E}_j + j\omega_c \bar{E}_j \quad (149)$$

Note that for the lossless case (by setting $R_{ij} = 0$), it is easily seen from (148) and (149) that the phasor variables $\bar{\phi}_i$ and $\bar{\phi}_j$ coincide with the bus voltages $\bar{E}_i$ and $\bar{E}_j$ respectively. Therefore when the line losses are neglected, i.e. if we set $R_{ij} = 0$ (hence $\bar{\alpha}_{ij} = 0$) in equations (146)–(147), these equations reduce to the lossless counterparts (139)–(140). Also better approximations to these solutions for the lossy case can be derived by considering finite series expansions for the quantities γ and Y_0 in the derivation, whereas only the first terms were used in the derivation thus far, given by the relations (144) and (145).

Equations (125), (126), (127) define the nature of the $g(x, y)$ equations shown in Figure II.1. The actual form here would vary widely depending on the circumstances ranging from the simple linear circuit relations to complex nonlinear algebraic equations with time delays and in fact the G set representation may have to remain a boundary value problem if suitable analytic solution can not be found.

It is the objective of this chapter to develop the basic models of the (125), (126), (127) type for the G set.

First such basic models will be developed in the next section for the most complete representation of the transmission line and then in consecutive sections for various simplifying approximations such as the commonly used RLC or π circuit representation.

II.6 Transmission system equations (power balance) - models for the G set using time-delays to represent PDE's

The form (125), (126) and (127) of the power balance equations that constitutes the model of the set G composed of distributed constant lines can now be given explicitly with reference to (125), (126) and (127).

1.) Bus power balance equation (see (125) for detail)

$$\begin{aligned}\sum_i \bar{S}_{Gi} + \sum_i \bar{S}_{Li} + \sum_i \bar{S}_{SVC, HVDC_i} + \sum \bar{S}_{mix_i} \\ + (j\omega C_i \bar{E}_i + C_i \bar{E}_i \bar{E}_i^*) - \sum_j \bar{S}_{ij} = 0\end{aligned}\quad (150)$$

2.) Line power balance equation.

Let $\bar{G}_{ij} = \frac{1}{\bar{R}_{ij}}$ represent the wave admittance of the transmission line between i and j ; let τ_{ij} be its time delay, and let $\bar{\theta}_{ij}$ ("s are dropped in the equation below for notational simplicity) be its phase shift, as defined. Then after simple algebraic manipulations, equations (110)–(111) can be rewritten as

$$\begin{pmatrix} \bar{I}_{ij}(t) \\ \bar{I}_{ji}(t) \end{pmatrix} = \begin{pmatrix} \bar{G}_{ij} \frac{\cosh \bar{\theta}_{ij}}{\sinh \bar{\theta}_{ij}} & -\bar{G}_{ij} \frac{1}{\sinh \bar{\theta}_{ij}} \\ -\bar{G}_{ij} \frac{1}{\sinh \bar{\theta}_{ij}} & \bar{G}_{ij} \frac{\cosh \bar{\theta}_{ij}}{\sinh \bar{\theta}_{ij}} \end{pmatrix} \begin{pmatrix} \bar{E}_i(t) \\ \bar{E}_j(t) \end{pmatrix} + \begin{pmatrix} \Delta \bar{I}_{ij}(t) \\ \Delta \bar{I}_{ji}(t) \end{pmatrix} \quad (151)$$

where

$$\begin{pmatrix} \Delta \bar{I}_{ij}(t) \\ \Delta \bar{I}_{ji}(t) \end{pmatrix} = \begin{pmatrix} -\bar{G}_{ij} \frac{\cosh \bar{\theta}_{ij}}{\sinh \bar{\theta}_{ij}} (\bar{E}_i(t) - \bar{E}_i(t - 2\tau_{ij})) \\ -\bar{G}_{ij} \frac{1}{\sinh \bar{\theta}_{ij}} (\bar{E}_i(t) - \bar{E}_i(t - \tau_{ij})) \\ -\bar{G}_{ij} \frac{1}{\sinh \bar{\theta}_{ij}} (\bar{E}_j(t) - \bar{E}_j(t - \tau_{ij})) \\ -\frac{e^{-j\bar{\theta}_{ij}}}{\sinh \bar{\theta}_{ij}} (\bar{I}_{ij}(t) - \bar{I}_{ij}(t - 2\tau_{ij})) \\ -\frac{e^{-j\bar{\theta}_{ij}}}{\sinh \bar{\theta}_{ij}} (\bar{I}_{ji}(t) - \bar{I}_{ji}(t - 2\tau_{ij})) \end{pmatrix} \quad (152)$$

Together the equations (150), (151) and (152) completely describe the voltage-current relationships of the transmission system, which is now the exact set of solutions for the lossless transmission system. By setting the current correction terms $\Delta \bar{I}_{ij}$ and $\Delta \bar{I}_{ji}$ terms to zero in the equations (151), (which will be justified where $\tau_{ij} \rightarrow 0$ as it would on short lines) the equations (151) and (152) simplify to the well-known bus admittance matrix or Y matrix formulation used in software for load flow or power flow computations. Also the correction terms drop out naturally for steady state computations: under steady state conditions, the quantities $\bar{E}(t) = \bar{E}(t - \tau) = \text{const.}$ so $\Delta \bar{I}_{ij}$ and $\Delta \bar{I}_{ji}$ reduce to zero. However these terms become significant under transient conditions, depending on the line lengths (which determines the time delays τ and the speeds of the trajectories (how much they change within the τ time delay period). Note that the simplification of the equations for the lossy lines would be similar starting from (146)–(147).

While in this context the basic model (125), (151), (152) takes the role as the defining equations of static events in the G set, i.e. on the transmission system, these equations are also partners to the dynamic F set $\dot{x} = f(x, y, p)$ in Figure II.1. Together they define the precise dynamic behavior of the overall interconnected power system.

II.6.1 Transients in the G set.

By tacit assumption the equations (125), (151), (152) of the section II.5, the variation of the modulation signal is smooth and the system does not undergo sudden changes. As a result there are no transients, the trajectory is not dislocated from the current state. This type of operation is correct much of the time on the power system. Note that the model (125), (151), (152) will automatically incorporate the dynamic contributions of the distributed parameter line itself to the path taken by the system trajectory. However this contribution will be minor (negligible) because of the fast internal response of the line.

When the trajectory is abruptly dislocated from the current state because of a disturbance or switching action this information can be incorporated into the Laplace transform and can be evaluated at least in principle. This is not included here and because of its complexity and its small effect associated with the speed of these transients it is conventionally neglected. This implies instant internal response within the network.

II.7 Power system dynamic models

Two major issues arise in analyzing and establishing dynamic models for a power system and especially its transmission network, the G set.

- 1.) What are the important details that need to be incorporated to assure proper and precise results. These will now be discussed.
- 2.) What if any limitations exist on the nature of the dynamic quantities especially their velocities given the fact that on the transmission system all transfers of "signals" such as voltage, current, power are via the modulated 60 Hz carrier.

II.7.1 Details and precision of the model, available approximations

In section II.5, a complete and precise model of the G set, the transmission system, was established in the form of power balance equations for the individual buses (125) and the individual transmission lines (151), (152). Note here that the terms "bus" and "transmission line" should be interpreted broadly. Bus simply means a node where data is available to establish a bus power balance type condition. This may be the terminal of a generator or load as examples where such formulation is desirable. Likewise transmission lines may incorporate transformers, tap changers, generator internal impedance or may be of the HV-DC type. Furthermore as defined above this complete model for the G set accounts for the time varying nature of the modulating signals, the presence of distributed parameter components, transmission line losses and other fine points (the effect of hard limits will be discussed in Chapter III). So equation (125), (151), (152) present as a detailed as broad and as precise model for the transmission lines as possible. When combined with the dynamic equations $\dot{x} = f(x, y, p)$ (F set) for the equipment (generators loads etc.) which are typically ordinary differential equations, the precise model of the power system is of the general type

$$\Sigma_{td}: \dot{x} = f(x, y, p) \quad x \in R \quad y \in \quad (153)$$

$$0 = \tilde{g}(x(t), y(t), x(t - \tau), y(t - \tau), p) \quad (154)$$

consisting of differential equations $\dot{x} = f$ for the equipment (F set) and algebraic equations $0 = g$ for the transmission system (G set) which also incorporate line specific time delays in the form of time-delayed variables $x_i(t - \tau_j)$ and $y_i(t - \tau_k)$. The latter account for the wave propagation on the transmission lines.

It is interesting, and will come as a surprise to many, that the precise model of the power system is still a DAE type albeit with time delays in the algebraic constraints. In fact our studies show that singularities still occur in the constraint manifold even with the time delays (if the speed of the internal transients is considered instant), however these appear to result in jump phenomena parasitic effects are modeled using the time-varying phasors (Tables II.1 and II.2).

Note that time delayed constraints are quite comfortable for numerical integration but a difficult type problem for analytic solution.

The time-delay DAE model Σ_{td} provides with the means for checking the *component stability* question which was posed at the

1991 Maryland Workshop (by DeMarco [127]). That is, suppose we are given a state (x, y) and a parameter p which satisfy the algebraic constraint $g = 0$ of the quasi-stationary DAE model Σ , the local stability of the state (x, y, p) in the time-delay model Σ_{td} defines the component stability. Our simulations confirm the conjecture made by DeMarco in [127] that some of the small-signal stable equilibria of the DAE model Σ can in fact, turn out to be unstable when the effect of transmission system dynamics is modeled in the analysis such as in Σ_{td} . In other words, our simulations show that there do exist small-signal equilibria in the sense of the DAE power system models Σ which are small-signal unstable for the time-delay model Σ_{td} . These results will be presented elsewhere.

At this point, we also need to point out certain potential problems associated with a prevalent practice in the literature of introducing arbitrary singular perturbations of the type $\epsilon \dot{y} = g(x, y)$ dynamics into the DAE equations, under the justification that the singular perturbations in the form of $\epsilon \dot{y}$ model the dynamics of the transmission network and other parasitic effects. Typically the load-flow equations in the form of $0 = g(x, y)$ are arbitrarily modified to be $\epsilon \dot{y} = g(x, y)$ by introducing a voltage derivative term in the reactive power flow equations and a phase derivative term in the real power equations. Such singular perturbation representations which convert the DAE model Σ into a "nice" analytical entity are then used for continuing the trajectory near singular points S and for tracking jump phenomena outside the constraint $g = 0$. This approach is of questionable validity for a number of different reasons:

- 1) Recall that the load-flow equations or the constraint equations $g = 0$ are based on the quasi-stationary phasor approximation. However outside the constraint $g = 0$ and near the singularity S of the constraint set $g = 0$, the quasi-stationary assumption loses validity, hence quasi-stationary phasor models of the form $\epsilon \dot{y} = g(x, y)$ are not applicable.

- 2) Note that the load-flow equations are based on the bus admittance matrix which assumes steady state conditions for analytical validity (see the classical derivation of the equivalent π representation of the distributed parameter wave equations). In other words, for steady state conditions, the distributed parameter effect can be incorporated accurately by using the notion of equivalent π representation (see Section II.6.2 below). However when fast transients are considered, there is no lumped parameter model. The line is physically and mathematically of the distributed parameter type, resulting in a boundary value problem of interconnected partial differential equations. Therefore singular perturbation models based on modeling the network to be of the lumped parameter type, would seem to lack physical validity. What aggravates the situation is the fact that the $\dot{y} = g(x, y, p)$ formulation introduces the fast transients connected with the π circuit model which have no physical validity.

"In the π circuit model, charge or energy dumps at arbitrary rates into the capacitive bucket, the leg of the π circuit at the bus, whereas on the real system it is forced to flow through a narrow pipe of a transmission line in the form of a wave which flows at only one speed regardless of the imposed voltage or energy. In fact, using π representation, the transmission system becomes a very ordinary looking RLC network, which it is not. Great care must be exercised then not to read results from such a representation which are not physically meaningful" [86].

Table II.3. General taxonomy of the time-varying phasor calculus.

General taxonomy of time varying phasor dynamics				
Formulation of dynamic model Assumptions on signal speed	Lines are boundary value problems on four terminal networks		Lines are approximated by lumped RLC or $Y-Z$ matrices	
	Include PDI's	Time delay equivalent	Nominal π or $Y-Z$ matrices	General Linear Network
General formulation	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t})$ Boundary values are typically the bus voltages, themselves states of the system. exact ①	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, T)$ Constraints are algebraic equations with line specific time delays, T . Precise equivalent to exact ②	$\dot{x} = f(x, y, p)$ $0 = g(x, y, p)$ constraints are approximated by algebraic equations Alternative: $\dot{x} = f(x, y, p),$ $y = g(x, y, p),$ if from π dynamics questionable because π dynamics is coded for lines. Approximate ③	$\dot{x} = f(x, y, p),$ $\dot{y} = A \begin{bmatrix} x \\ y \end{bmatrix}$ no $p = e.i$ terms, no power balance equations Not usable directly for power system ④
Steady state special case $\dot{x} = 0$	$0 = f(x, y, p, \frac{\partial x}{\partial t})$ boundary value problems in t only. exact	$0 = f(x, y, p),$ $0 = g(x, y, p, T)$ Alternative: "equivalent π " for the lines $\dot{x} = f(x, y, p)$ $0 = g(x, y, p)$ exact no time delays needed exact	$0 = f(x, y, p),$ $0 = g(x, y, p)$ Nominal π or $Y-Z$ matrix approximate	$0 = f(x, y, p),$ $0 = A \begin{bmatrix} x \\ y \end{bmatrix}$
Quasi stationary approximation Frequency band of signals ω less than ± 3 Hz	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, \frac{\partial x}{\partial t})$ variation of x slow (less than 3 Hz bandwidth) Questionable precise in t approximate in t approximate	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, T)$ Questionable although the accuracy in time is improved over ③ approximate	$\dot{x} = f(x, y, p)$ $0 = g(x, y, p)$ approximation in both t and ℓ widely used	NA
Unbalanced system Bandwidth limitations	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t})$ Time varying phasors produce precise result if the bandwidth of signal x is ± 60 Hz exact	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, T)$ Time varying phasors produce precise result if the bandwidth of signal x is ± 60 Hz	$\dot{x} = f(x, y, p),$ $\dot{y} = g(x, y, p)$ x, y bandwidth limited 0 to $+60$ Hz) π or $Y-Z$ model symmetrical components apply ⑤	NA
Balanced three phase system No limit on bandwidth of signals x	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t})$ Time varying phasors produce precise results on balanced three phase system exact	$\dot{x} = f(x, y, p),$ $0 = g(x, y, p, T)$ Time varying phasors produce precise results on balanced three phase system	$\dot{x} = f(x, y, p),$ $\dot{y} = g(x, y, p)$ No bandwidth limitation on x Approximation ⑥	NA
Note ① identifies cases discussed in text.				

3) Critical dynamic properties of the system should be assessed on physical models of the system derived from physical properties such as the distributed parameter time-delay model Σ_{td} . Introducing *arbitrary* perturbations of the type $\epsilon y = g(x, y)$ (e.g. introduction of the terms $\epsilon \dot{E}_i$ into the reactive power equations and the terms $\epsilon \dot{\delta}_i$ into the real power equations where $E_i / \angle \delta_i$ stands for the bus i voltage) for analyzing the component stability properties of the system would seem to be of questionable validity. Such models for studying local stability properties should be based on the physical properties of the system such as the transmission system dynamics, dynamic load models, etc. For instance, the dynamics of capacitor banks (or even parasitic capacitors) can be properly modeled into the load-flow equations using the time-delay phasor properties in Table II.1 and Table II.2. However when this is done properly, we showed in [117] that the voltage derivative term $\dot{E}_i$ appears in the real power balance equation and the phase derivative term $\dot{\delta}_i$ shows up in

the reactive power balance equation. The point is that these are not casual arguments but need proper analytical treatment such as introduced in this Chapter II. The component stability problem can indeed be tackled within the framework of the time-delay model with suitable models of the other relevant system elements such as the equipment, load etc.

II.7.2 The RLC circuit approximation (Column ③ in Table II.3)

The difficulties of analyzing the full distributed parameter model in either its partial differential equation ① or time delay ② representations (Table II.3) is conducive to seeking of approximations. Fortunately on the power system the typical line lengths of 100-200 miles produce small time delays or in other words the difference made by considering these is quite small and so it may be set to zero as an approximation.

When the time delay associated with transmission lines is neglected equation (151), (152) will collapse into a quite conventional RLC type circuit, a three terminal network representation (for balanced 3 phase systems). Conventional practice further approximates this to a simple nominal π circuit by simply lumping total line resistances capacitances and inductances in the three branches of the π .

This last step is questionable because it puts a lumped capacitor at all line ends at the bus which is not there. Remember that charge and energy passes freely into a capacitor but has to enter a line (like a pipe) at a fixed rate. This can cause erroneous results in certain problems as pointed out in the previous section.

The customary nominal π circuit approximation can be pretty crude even in the steady state when lines are relatively long. So when abandoning the time delay representation, it would be desirable to retain some correction terms. This can be done in the steady state [115] where an "equivalent" π circuit applied as follows

$$Z_{\pi} = Z \left[1 + \frac{Z}{6Z'} + \frac{Z^2}{120Z'^2} + \dots \right] \quad (155)$$

$$Z'_{\pi} = 2Z' \left(1 + \frac{Z}{12Z'} + \frac{Z^2}{120Z'^2} + \dots \right) \quad (156)$$

where $Z = \omega_c L$, $Z' = \frac{1}{\omega_c C}$ for the whole line and Z'_{π} are the columns and Z_{π} is the bridge of the π . The fully converged series will give the exact solution for a line of any length but unfortunately only in the steady case. The existence of such correction terms for the steady state raises some hope that some improved RLC circuit could be developed from the exact forms such as (152). Note for example that since the time-delays τ_{ij} are small, the correction terms of the form $\tilde{E}_i(t) \rightarrow \tilde{E}_i(t - \tau_{ij})$ can be very well approximated as

$$\tilde{E}_i(t) - \tilde{E}_i(t - \tau_{ij}) \approx \tau_{ij} \dot{\tilde{E}}_i(t)$$

under certain assumptions. In other words, it may be possible to rewrite the time-delay equations (139)–(140) (or (146)–(147)) as singularly perturbed dynamic equations with proper validity under certain assumptions. These issues will be discussed elsewhere.

Otherwise the conventional RLC or π circuit approximation is well-known.

Power balance equations in the general format of (154) (with $\tau = 0$) for the equipment $\dot{x} = f(x, y)$ and for the transmission system π circuits to define load flow on individual lines are used in the fashion of conventional power flow software.

Such equations although they are traditionally written for steady state conditions are still applicable for good approximate solutions for dynamic conditions as long as the dynamics is not faster than conventional "transient stability" analysis of electromechanical oscillations. This range is known as the "quasi stationary range" (see Table II.3).

II.7.3 Limitations imposed by the time varying phasors

It should be emphasized that the primary aim of this Section is the study of the effect of the time varying nature of the time varying phasors as defined in Theorems 10 and 11. This is a very vital feature when dynamic speeds on the system exceed those of historical transient stability.

At this point we should point out that the system equations whether in distributed constant form (125), (151), (152) or in an approximation, are written in time varying phasors which were defined in Theorems 10 and 11 (Properties 1 through 4). These are limited by one of two factors: 1) the modulating signals ($\tilde{E}$ and $\tilde{I}$) variations are contained in a frequency band of ω_c that is slower than the carrier (the low pass phasor), or 2) the three phase signal is balanced and the network is symmetric (the fast balanced phasors). Note that in the instantaneous time-domain, the power balance equations of the form (100) are not easy to work since the carrier is still present in the brackets $[1 + \cos 2(\omega_c t + \delta(t))]$ and $[\sin 2(\omega_c t + \delta(t))]$ in the power flow equation. On the other hand, by the time-varying phasor properties (Property 2 to be precise), it is possible to rewrite these equations purely in the form of real power $P(t)$ and reactive power $Q(t)$, hence the problem is analytically much simpler to handle. For instance, in (100), note that if the three phase system is balanced, then in the three phase power (which in per unit is indistinguishable from single phase power) the first bracket becomes 1 and the second 0 as a result of adding three symmetrically spaced phasors. This indicates the importance of studying the power flow equations in terms of the respective time-varying phasors. The rest of this section is devoted to studying some important special cases.

Table II.3 is attached to support the reader in developing an organized view of the various cases which occur. In this table the complete and precise model is located in the upper left corner ① In its general form (125), (151), (152), this model uses boundary value problems with partial differential equations for the lines, allows fast variation of modulating signals, considers line losses, treats fast time varying events along distributed constant transmission lines. Next to it on the right is located ② the time delay formulation which is equivalent to the PDE version ① The other columns depict various approximations and special cases which allow special solutions. In ③ an RLC linear approximation of the transmission system is introduced and this approximation, which includes the customary nominal π circuit, is used in the rest of the cases in columns ④–⑥ for simplicity of presentation, although the results also apply to the full distributed model (125), (151), (152).

The second row of Table II.3 discusses special features of each case for stationary conditions of the system. Finally the third row acknowledges possible usefulness of the stationary model under a "quasi-stationary" label for relatively slow events. The last two rows display short sketches of the cases connected respectively connected with Theorem 10 and 11.

A brief set of equations modelling the dynamics of the large power system is shown in Table II.4 where the G set is represented by two sets of equations for easy comparison. All the equations in Table II.4 are written under the assumption of a three phase balanced power system. In addition, if we impose the quasi-stationary phasor assumption, then using the nominal π circuit, the G set (the power balance equations) simplify to a simple set of algebraic equations shown on the left hand side of Table II.4. It is easily seen that these equations reduce to the traditional Y_{bus} formulation. On the other hand, by using the fast balanced time-varying phasors introduced previously in this Chapter, the power balance equations now possibly include derivative terms from the presence of capacitors (see the right hand side of the G set in Table II.4). Moreover when the distributed nature of the lines are considered, the algebraic

Table II.4. Illustrative example: a large balanced power system model.

F set: Dynamic equations (non-modulated non-phasor), $\dot{x} = f(x, y, p)$ (highly simplified to save space, detailed models can be readily used)	
1. Generator electromagnetics (simplification from Park-Concordia)	
$T'_{d0} \dot{E}'_i = -\frac{x_{d_i}}{x'_{d_i}} E'_i + \frac{x_{d_i} - x'_{d_i}}{x'_{d_i}} E_i \cos(\theta_i - \delta_i) + E_{fd_i}$	
2. Generator excitation control (simplified from IEEE Type 1 model)	
$T_i \dot{E}_{fd_i} = -K_i(E_i - E_{ref_i}) - (E_{fd_i} - E_{fd_i}^0)$	
3. Generator electromechanics (swing equation)	
$J_i \ddot{\theta}_i + b_i \dot{\theta}_i = P_{T_i} - \frac{1}{x'_{d_i}} E'_i E_i \sin(\theta_i - \delta_i)$	
4. Load:	
$P_{L_i} = P_{S_i} + P_{D_i}, \quad Q_{L_i} = Q_{S_i} + Q_{D_i}, \quad \bar{S}_{L_i} = P_{L_i} - jQ_{L_i}$	
Static $P_{S_i} = P_i^0 + M_i(E_i) + G_i(E_i)^2, \quad Q_{S_i} = Q_i^0 + H_i(E_i) + B_i(E_i)^2$	
Dynamic $P_{D_i} = P_{D_i}(x_{D_i}, E_i, \delta_i), \quad Q_{D_i} = Q_{D_i}(x_{D_i}, E_i, \delta_i)$	
$\dot{x}_{D_i} = f_{D_i}(x_{D_i}, E_i, \delta_i)$	
5. Notation: (see Figure II.2) $x_{G_i} = \omega_c L_{G_i}, \quad C_i = C_{\text{Bank}_i} + \dot{C}_{\pi_i}, \quad \bar{S}_{L_i} = \bar{E}_i \bar{I}_{L_i}^*$	
$\bar{E}'_i = E'_i \angle \theta_i, \quad \bar{E}_i = E_i \angle \delta_i, \quad \bar{I}_{G_i} = I_{G_i} \angle \varphi_{G_i}, \quad \bar{I}_{L_i} = I_{L_i} \angle \varphi_{L_i}, \quad \bar{I}_{ij} = I_{ij} \angle \varphi_{ij}, \quad i, j \in [1, \dots, N]$	
$E'_i \in x, \quad \theta_i \in x, \quad \delta_i \in x, \quad E_{fd_i} \in x, \quad x_{D_i} \in x$	
$E_i \in y, \quad \delta_i \in y, \quad I_{G_i} \in y, \quad \varphi_{G_i} \in y, \quad I_{ij} \in y, \quad \varphi_{ij} \in y$	
G set: Network equations (modulated-phasor) $g(x, y, p)$ The Power Balance equations (Figure II.2)	
Quasi-stationary equations $0 = g(x, y, p)$	Time-varying phasor time-delay equations $0 = g(x, y, x(t-\tau), y(t-\tau), p)$
Bus i:	Bus i:
$\bar{E}'_i \bar{I}_{G_i}^* - \bar{S}_{L_i} - j\omega_c C_i E_i^2 - \sum_{j=1}^m C_{ij} \bar{E}_i \bar{I}_{ij}^* = 0$	$\bar{E}'_i \bar{I}_{G_i}^* - \bar{S}_{L_i} - j\omega_c C_i E_i^2 - \sum_{j=1}^m C_{ij} \bar{E}_i \bar{I}_{ij}^* = C_i \bar{E}_i \frac{d\bar{E}_i^*}{dt}$
$\bar{E}'_i - \bar{E}_i - j\omega_c L_{G_i} \bar{I}_{G_i} = 0$	$\bar{E}'_i - \bar{E}_i - j\omega_c L_{G_i} \bar{I}_{G_i} = L_{G_i} \bar{I}_{G_i}^* \frac{d\bar{I}_{G_i}}{dt}$
Line ij:	Line ij:
$\bar{E}_i - \bar{E}_j - (R_{ij} + j\omega_c L_{ij}) \bar{I}_{ij} = 0$	$\bar{I}_{ij}(t) = -\bar{G}_{ij} \bar{\phi}_i(t) + \bar{I}_{ij}(t - 2\tau_{ij}) e^{-2(\bar{\alpha}_{ij} + j\bar{\delta}_{ij})}$
	$-2\bar{G}_{ij} \bar{\phi}_j(t - \tau_{ij}) e^{-(\bar{\alpha}_{ij} + j\bar{\delta}_{ij})}$
	$-\bar{G}_{ij} \bar{\phi}_i(t - 2\tau_{ij}) e^{-2(\bar{\alpha}_{ij} + j\bar{\delta}_{ij})}$
	$\frac{d\bar{\phi}_i}{dt} = -j\omega_c \bar{\phi}_i - \frac{R_{ij}}{2L_{ij}} \bar{\phi}_i + \frac{d\bar{E}_i}{dt} + j\omega_c \bar{E}_i$

nominal π equation on the left side of the line ij becomes a set of time-delay equations shown on the right side (which also includes certain flux quantities $\bar{\phi}_i$ from modelling the line losses, see (146)–(149)). If we set the derivative terms to zero and the time-delays to zero in the equations shown on the left side in the G set, then these equations precisely reduce to the corresponding equations for the nominal π representation after (after the well-known conversion (155)–(156) from the equivalent π to nominal π).

II.8 Summary

The time-varying phasor transformation (Tables II.1 and Table II.2) together with precise solutions for the transmission line wave equations (the time-delay equations) enable us to formulate the general problem (Table II.1) along with an overview of how the various approximations used in practice emerge under what assumptions (Table II.2). It is important to observe that moving between different models is quite easy, even though the analysis (naturally) becomes more and more challenging as

we approach the exact models (in the first two columns of Table II.3). The traditional wisdom is of course in choosing an adequate model for studying a specific phenomenon. However the underlying assumptions used in deriving a specific model should be remembered in its usage for guaranteeing the validity of results. As observed earlier, the general formulation provides means for analyzing difficult analytical questions such as the component stability problem and the behavior near the singularity where the traditional models lose their validity. Moreover in this age of fast thyristor controls and advanced satellite communication facilities, it is hoped that the detailed dynamic formulation of the network transients here along with the emerging insight on the behavior of fast dynamic behavior of the loads would encourage further studies on the *analysis* of fast dynamic phenomena, thus providing the foundation for fast and efficient voltage control.

III. THE TAXONOMY THEORY WITH HARD LIMITS FOR THE LARGE POWER SYSTEM DYNAMICS

Chapters I and II of this paper summarize the general theory for systems modeled by algebraically constrained ordinary differential equations Σ under the name of the *taxonomy theory*. It was observed that the complete state space of such systems is separated by *singular surfaces*, (where typically the algebraic constraint surface $g = 0$ cannot be eliminated), into open *components*. Within each component, the system behavior is smooth and the algebraic constraint admits unique local solutions everywhere. Thus the system behaves like a smooth unconstrained system governed by the dynamics of ordinary differential equations as long as the dynamics stays away from the singularity. Moreover recent results in [42, 43, 62] show that global solutions which are valid throughout a compact subset within a component can be constructed (theoretically), provided this subset is simply connected.

However as noted in Chapter II, in general, trajectories cannot pass through the singularity in a predictable manner since the underlying quasi-stationary assumption of traditional phasor models breaks down and unusual behavior, such as excessively fast motions, will occur near the singularity [86]. More detailed time-delay models based on the notion of fast time-varying phasors need to be used for analysis as introduced in Chapter II. Therefore *the singularity pin-points the region in the state space where traditional quasi-stationary models lose validity and additional modelling tools such as proposed in Chapter II.5 become necessary*. Moreover an analysis of the dynamic behavior near the singularity provides us with the insight on how this transition between different models takes place in reality. Therefore an analysis of *the singular dynamics of the algebraically constrained model Σ* together with the knowledge of a global model such as *the time-delay model that is valid outside the constraint* would enable us to develop a *global theory* of power system dynamics which is not limited by quasi-stationary speed limitations. Inherent problems in the power system dynamics connected with the uncertainty in defining the phase reference could introduce random phenomena for such global analysis [86].

In the general spirit of working towards such a global theory, in this chapter, we consider the effects of *system hardlimits* which are always present in modelling physical systems such as the power system. A different terminology of *saturation* is used by the control community to denote such limits (see the wide-body of recent literature on actuator saturation). In the power terminology, "saturation" is usually reserved for denoting *smooth* effects connected with saturating phenomena such as field saturation etc., and "hardlimits" are usually associated with *nonsmooth* effects such as actuation limits, tap changer limits, relay effects, etc.

In recent times, with the loads growing throughout the network, the effects of such hardlimits are becoming increasingly important in determining system dynamic behavior. The need for modelling these in voltage stability studies has been highly emphasized by working group reports etc. [130, 131], and there are many industrial reports of the conflicting effects of hardlimits on the evolution of dynamic instability mechanisms. While in some-case, tap changer limits have been shown to *enhance* system recovery, there are instances where they have been counter-productive during crises [131]. Most of the existing commercial software programs such as the EPRI programs: EMTSP, SSSP

and EMTP make elaborate provisions for modelling the effects of hardlimits. Detailed exposition on how to model hardlimits in dynamic analysis is being introduced into recent textbooks on the subject (for instance, see [51]). However analytical tools for handling the hardlimits in the general large system are lacking in the literature.

There have been a few recent papers in the literature which have analyzed the effect of hardlimits on system stability phenomena. In [32], it is shown that the operating point can undergo loss of small-signal stability when a generator reaches its VAR limits. The study concerns evaluating the eigenvalues at the operating point before and after the hardlimits are encountered in a power system model that includes a single generator connected to an induction motor load. In [72], some observations are made on the effects of excitation hardlimits in changing the structure of the system Jacobian, and the possible emergence of local bifurcations connected with hardlimits is explored. Apart from these two studies connected with certain types of hardlimits in specific power system models, we are not aware of any general theory (or a "toolbox") on how to analyze the effects of hardlimits in general power system models. In this Chapter, we report the essential core material for developing such a theory. This stability theory will then allow us to identify 1) what are the effects of hardlimits on existing results, and 2) what are the new types of stability related phenomena that can be expected directly related to hardlimits. The first question concerns how the hardlimits influence existing operating practices, and the second question is aimed at identifying potential "trouble-spots" connected with hardlimits.

In other words, the DAE taxonomy theory in Chapters I and II will be extended to a general setting, after adding the new element of hardlimits to the system models. As noted above, this is a significant step practically both as a concern in operation and as a potential source of disturbances. In this theory, the systems with hard limits also incorporate algebraic constraints but to isolate the new phenomena connected with hardlimits, we deliberately exclude the presence of singularity from the analysis, as stated in Section III.1.2.

From a conceptual view-point, there exist three types of hardlimits, namely windup limits or actuation limits, nonwindup limits or state limits, and relay type limits or switching limits. References to these types can be seen in IEEE committee reports [128, 129], in commercial software documents (EMTSP manuals), and in textbooks [8, 51]. The fundamentals of these different types is first introduced in Section III.1.1 which will then allow us to formulate a general dynamic model in Section III.1.2 that includes the effects of all the limiter types within the quasi-stationary phasor domain. As we will see, this model turns out to be a hard-constrained (from the presence of hardlimits) and algebraically constrained (from load-flow or power-flow equations) differential equation model.

We will then break up this problem into three subproblems, treating each of the three hardlimit types separately. That is, Sections III.1.2 (a), III.1.2 (b) and III.1.2 (c) each individually formulate the three hardlimit types, windup limits or actuation limits, nonwindup limits or state limits, and relay type limits or switching limits, respectively. Each of these subproblems is then treated in the following sections on state space analysis, and for parameter space analysis. Of course, in a realistic power system model, all three types of hardlimits are present, coupled and

mingled together. However the analysis in this Chapter reports on the study of the *individual* effects of the three types for easier visualization of the different behavior patterns. The combined effects of the three together with the inclusion of singularity for the general model will be reported elsewhere.

In the following sections, we will observe that the system with hardlimits does possess a two level cellular structure much like the algebraically constrained model Σ . However new features, rich in structural detail, appear now, because of the presence of hardlimits. In the parameter space, new bifurcations emerge which are directly connected with each of the three hardlimit types. These phenomena range from static bifurcations (e.g. loss of existence of the operating point) to dynamic bifurcations (e.g. emergence of hardlimit related oscillations). Transient stability properties also are affected by hardlimits as evidenced by changes in the structure and composition of the boundary of the region of attraction. It should be pointed out that the theory as presented here is a reporting of the current status of progressing research. In some cases, because of the richness of the problem considered (such as for the state space of nonwindups), we do not have a full theory at this point, and we state these results as conjectures with proper justifications.

III.1 Formulation of a power system model with hardlimits

In this section, we derive a general formulation for analyzing the effects of hardlimits within the quasi-stationary phasor domain. Then we “reduce” this problem to the study of three subproblems for the different hardlimit types separately. The analysis of these subproblems then follows in Sections III.2 and III.3 for state space and parameter space respectively.

III.1.1 Three types of hardlimits

Power systems as well as other physical dynamic systems have nonsmooth elements. Hardlimits are among the most important nonsmooth elements.

This section examines models for hardlimits typically used in power system stability studies.

The hardlimits described here are a) actuation limits or windup limits, b) state limits or nonwindup limits, and c) switching elements or relay type limits. The first two correspond to saturation phenomena and are therefore also called saturation type limits. Depending on how a system with saturation type limits comes back off the limit, windup and nonwindup limits are distinguished. Whereas nonwindup limits can come off the constraint at any moment, this is not possible for windup limits before the constrained variable has been released completely (see below). The relay type limit corresponds to a switching action taking place when some quantity related to the system states reaches a specified threshold value. Once the switching is done, the structure of the system is accordingly changed, and this structural change is irreversible.

In power system areas, examples of windup and nonwindup limits can be found in the models for excitation systems and steam/hydro turbines of synchronous machines [128, 129], SVC systems, and tap changers. In these models, windup/nonwindup limits are usually applied to the output of integrator, amplifier,

and lead-lag blocks [51]. The relay type limit is usually seen in relay protection systems (and that’s why it is so called). A detailed modelling study for incorporating the effects of relay switching into a power system dynamic model was recently presented in [59].

Note that in the power community, the word saturation is generally used when the limit develops with a smooth graduality. Saturation is sometimes used as an approximate assumption for actual hardlimits which could eliminate discontinuities in the analysis (sort of like smooth approximations for nonsmooth functions) but these approximations typically tend to make the analysis much more complicated.

(a) Actuation limits or Windup limits (IEEE)

Suppose we want to model the action of some physical device, in short *the actuation*. Let us denote this actuation mathematically by a function $\bar{h}(x)$. Then $\bar{h}(x)$ is a scalar master function of some set of states (possibly also parameters but this is ignored here for simplicity) of the system. For modeling the hardlimits of this device, mathematically we can distinguish the “free” action say $h(x)$ which when it reaches the limits becomes the respective hardlimit values. In other words, the true hardlimited actuation $\bar{h}(x)$ can be defined as

$$\bar{h}(x) = \begin{cases} h_\ell & \text{if } h(x) \leq h_\ell \\ h(x) & \text{if } h_u \geq h(x) \geq h_\ell \\ h_u & \text{if } h(x) \leq h_u \end{cases} \quad (157)$$

That is, $\bar{h}(x)$ is undisturbed if it is within the hard limits of which there can be an upper and lower one or only the limit itself. In fact, the limiting effect may be of a more general closed surface in the state space, but again this can be represented as above with suitable redefinitions.

Physically this can be thought of as representing a dependent state variable $z = \bar{h}(x)$ in the dynamic equations which will cause a dynamic effect in the system (including the states in $h(x)$). Examples can be 1) parts of control circuits which convey data such as control errors to the dynamic control process, or 2) internal variables which identify some internal link within the control, or 3) commands for the action of the actuators, or 4) $\bar{h}(x)$ can be the model of an actual actuator. Note that the use here of the word actuation should not be taken literally to imply a direct presence or involvement of an “actuator” in the servomechanism sense. For instance, windup limits (157) that appear in control sensors can be thought of as actuation limits in the sensing action.

Definition III.1 An *actuation Limit*, or in IEEE usage a *windup limit*, is a hardlimit applied to an actuation value defined by a (uni-directional) master function $\bar{h}(x)$ of a set of states. Non-invertibility here follows both, from the one directional information flow in the part (usually a link) of the system represented by $\bar{h}(x)$ and mathematically from the nonsmooth nature of $\bar{h}(x)$. Reaching this limit is not connected with any abrupt change in the parameter space or system structure. State will continue changing dynamically even after the limit is reached. $\square$

Actuation limits will be studied for both the state and parameter space in Sections III.2 and III.3.

(b) State limits, or Nonwindup limits (IEEE)

An important special case arises when the action variable of a physical device is directly one state variable and this state variable has limits. For example, the SVC impedance formulated as a state variable directly has upper and lower limits. When these limits are reached, the limits directly annihilate x_i as a state. That is, when the free system tries to go over the limit, the state x_i is stopped at the limiting value, and x_i simply becomes a constant (or $\dot{x}_i$ becomes zero). This can be thought of the system losing one degree of freedom while the state x_i is at its limit. So basically there is a different dynamics once the limit is reached. Note that in the actuation limited case, the same physical state space is simply driven in a different way. However in the state limit case, the only reason x_i stays at the limiting value is because the free (unconstrained value) dynamics tends to drive it past the limiting value (for instance when the control valve reaches its maximum opening and tries to move *past* the maximum value). There is nothing to keep x_i from reappearing as a regular state variable again once the free system returns to the unconstrained region.

When the limit is active, the free system is a kind of ghost in this case with no physical reality. However it should be remembered that the free system determines whether the state stays at the limiting value (if the dynamics is moving the state to go past the limit) or if the state returns to be a regular state variable (the converse).

The state hardlimit can be rather intuitively postulated as (see [128, 51])

$$\dot{x}_i = \begin{cases} 0, & x_i \leq x_{li} \text{ and } \dot{x}_{i,\text{free}} < 0 \\ \dot{x}_{i,\text{free}}, & x_{li} \leq x_i \leq x_{ui} \\ 0, & x_{ui} \leq x_i \text{ and } \dot{x}_{i,\text{free}} > 0 \end{cases} \quad (158)$$

where $\dot{x}_{i,\text{free}}$ denotes the velocity of the state x_i for the free system. The precise mathematical definition of this hardlimit will become evident when we analyze the state space dynamics for nonwindup limits in Section III.2.

There are plenty of examples. The generator field voltage E_{fd} will be a state (representing the main field in the generator) which also has a limit through the maximum excitation current.

Definition III.2. The *state limit* or in IEEE usage, the *nonwindup limit*, is a hardlimit applied to a single state x_i which becomes constant upon reaching its limit while the free dynamics is forcing the state to move past its limiting value. Physically x_i continues to exist at the limit value as a constant upon reaching its limit, and stays that way until the dynamics moves back into the section where the state can move back inside the limits. $\square$

Effects of state limits or nonwindup limits will be studied for both the state and parameter spaces. State limits will be studied in Section III.2 in the context of state space dynamics. Parameter space is treated in Section III.3. Particularly in both the actuation and state limit cases several new types of bifurcations arise and these are analyzed in Section III.3.

(c) Relay or switching type limits

Suppose the test actuation function $h(x)$ is again an implicit function of a set of states. Reaching the limit however will not

change either the actuation or the state directly. Instead it will initiate preprogrammed structural action which then in turn can change the whole system, its state and/or parameter space more or less thoroughly. These switching actions are initiated by relay mechanisms and protective limiters in the power system, which monitor certain states of the system and depending on their preprogrammed logic, take protective actions when certain (disruptive) events are sensed. Examples include overcurrent relays, distance relays, etc. [132].

Detailed models for including the effects of relays in system dynamic studies have been recently proposed in [59, 61]. Essentially the relay elements can be included in the analysis by modelling their internal logic mechanisms as state dynamic equations. However since these are switching devices by nature, the functional descriptions of these devices include discontinuities. It can be shown [59, 61] that these switching discontinuities can be modelled by introducing dependent state variables of the form w_j ,

$$w_j := \bar{v}_j(x, p) := \begin{cases} 0 & \nu_j(x, p) < 0 \\ 1 & \nu_j(x, p) \geq 0 \end{cases} \quad (159)$$

We will briefly indicate the effects of these switching mechanisms on state and parameter spaces in Sections III.2 and III.3 respectively.

III.1.2 A detailed power system model formulation

Combining the DAE model Σ with the three types of hardlimits introduced in the previous section, the power system model within the quasi-stationary phasor domain which includes hardlimits can be described by the equations

$$\Sigma_{\text{full}} : \dot{x} = f(x, y, z, w, p), \quad x \in \mathbb{R}^n \quad (160)$$

$$0 = g(x, y, z, w, p), \quad y \in \mathbb{R}^m \quad (161)$$

$$z = \bar{h}(x, y, p), \quad z \in \mathbb{R}^{n_z} \quad (162)$$

$$w = \bar{v}(x, y, p), \quad r \in \mathbb{R}^{n_w} \quad (163)$$

$$x_{li} \leq x_i \leq x_{ui}, \quad i = 1, \dots, n. \quad (164)$$

where

$$f : \mathbb{R}^{n+m+n_z+n_w+q} \rightarrow \mathbb{R}^n, \quad f \text{ is } C^\infty \quad (165)$$

$$g : \mathbb{R}^{n+m+n_z+n_w+q} \rightarrow \mathbb{R}^m, \quad g \text{ is } C^\infty \quad (166)$$

$$\bar{v} : \mathbb{R}^{n+m+q} \rightarrow \mathbb{R}^{n_r}, \quad (167)$$

$$\bar{h} : \mathbb{R}^{n+m+q} \rightarrow \mathbb{R}^{n_z}, \quad (168)$$

$$h : \mathbb{R}^{n+m+q} \rightarrow \mathbb{R}^{n_z}, \quad h \text{ is } C^\infty \quad (169)$$

$$\bar{h}_i(x, y, p) := \begin{cases} z_{li}, & h_i(x, y, p) \leq z_{li} \\ h_i(x, y, p), & z_{li} < h_i(x, y, p) < z_{ui} \\ z_{ui}, & h_i(x, y, p) \geq z_{ui} \end{cases} \quad (170)$$

$$\nu : \mathbb{R}^{n+m+q} \rightarrow \mathbb{R}^{n_r}, \quad \nu \text{ is } C^\infty \quad (171)$$

$$\bar{v}_i(x, y, p) := \begin{cases} 0, & \nu_i(x, y, p) < 0 \\ 1, & \nu_i(x, y, p) \geq 0 \end{cases} \quad (172)$$

The variables are of the following types: 1) the (independent) dynamic state variables x : these include generator state variables such as machine angle, frequency, flux quantities etc., control states, load dynamic states, etc. 2) the instantaneous (algebraic) state variables y : these are the load-flow variables such as bus voltages and angles, etc. (x and y are the same as in the DAE model Σ except here these may also include the dynamic internal states for relays and circuit breakers when protection is modeled [59]). 3) the (dependent) actuation variables z : these are the windup limit output variables, 4) the (dependent) switching variables w : these appear as the internal

dependent states in modeling protective relay type limits, and 5) the parameters p : these denote the system and operating parameters as before.

The functional equations in Σ_{full} describe the following: 1) dynamic equations (160) model the dynamics of the generators, controls, loads and possibly relay dynamics, 2) equality constraints (161) model the power-flow equations or the load-flow equations under the quasi-stationary assumption, 3) the actuation equations (162) together with (170) model the effects of the hardlimits on the actuation variables z_i , 4) the switching equations (163) together with (172) model the discontinuous nature of the switching functions appearing in protective relay type limits, and 5) the equations (164) imply the nonwindup limits on the state variables (for any state with no state limit, the limits can be set arbitrarily high say $\pm\infty$). For notational simplicity, all states are assumed to have nonwindup limits in (164)).

Therefore the problem becomes that of analyzing the dynamics of the model Σ_{full} . Note that models of the form Σ_{full} are routinely used in numerical transient stability simulation programs such as in the EMTSP even though the analytical studies so far have been mostly focussed on the algebraically constrained system model Σ of Chapter I.

The variables z and w in Σ_{full} are known as explicit (nonsmooth) functions of the state variables x , y and parameters p . Therefore the basic nature of the model Σ_{full} is still that of a constrained dynamics, with the constraint surface defined by $g = 0$. However note that now there arise special "hard-constraints" on the state variables because of the presence of the nonwindup limits or state limits (164).

Therefore by inspection of the formulation (160)–(172), it is clear that the overall dynamics for Σ_{full} involves the interaction of the following dynamic features: 1) the singularity of the constraint surface $g = 0$, 2) the nonsmoothness introduced by the windup or actuation limits for z (170), 3) the nonsmooth dynamics implied by the state limits or nonwindup limits in x (164), and 4) the discontinuities introduced by the switching limits w . To view the combined effects of these four different types of phenomena becomes quite technical all together.

Several special phenomena arising from the presence of the singularity in the model Σ have already been analyzed in Chapter I (e.g. new singular sections on the boundary of the region of attraction, and new bifurcation phenomena such as the singularity induced bifurcation). Therefore to simplify the problem in this Chapter, we will assume that the analysis stays away from the singularity. Precisely we can assume that the analysis is restricted to within a set say $\mathcal{R}$ such that 1) within $\mathcal{R}$, the Jacobian $D_y g$ is well-defined and is nonsingular everywhere ($\mathcal{R}$ does not intersect the singularity), 2) $\mathcal{R}$ is compact within the topological space defined by $g = 0$, and 3) $\mathcal{R}$ is simply connected, so that the equations $g = 0$ can be globally eliminated within $\mathcal{R}$ (see [43, 62]). Then the problem Σ_{full} reduces to a set of hard-constrained (in x , z and w variables) set of ordinary differential equations. For this problem, the boundary of the region $\mathcal{R}$ itself plays an important role in characterizing stability regions [62].

As a next step in simplifying the model, and for isolating the new phenomena connected with the nonsmoothness introduced by the three limit types, we restate the problem Σ_{full} by

ignoring the presence of singularity (for the reasons mentioned above) and by only considering one limit type at a time. These subproblems formulated below are studied in detail in Sections III.2 and III.3 respectively. It needs to be emphasized that the full model Σ_{full} does include the interactions of these different types together with the singularity, and the theory presented here provides the tool-box for understanding the overall behavior, and such a "full" theory for Σ_{full} will be presented elsewhere.

(a) Actuation limits or windup limits

The model Σ_{full} can be simplified to the model Σ_{al} defined below for studying the effects of wind-up limits:

$$\Sigma_{al}: \dot{x} = \bar{f}(x, p) = f(x, z, p), \quad z = \bar{h}(x, p) \quad (173)$$

where $z_i = \bar{h}_i(x, p)$ denotes the actuation function of certain device i , and has upper and lower limits z_{u_i} and z_{l_i} respectively. It is easily seen from (170) that the windup limit for z_i can be re-described by the composition of the "free action function" $h_i(x, p)$ with the scalar limiter function $\sigma_i(s_i)$ defined as follows:

$$\sigma_i(s_i) := \begin{cases} z_{l_i}, & s_i < z_{l_i} \\ s_i, & z_{l_i} \leq s_i \leq z_{u_i} \\ z_{u_i}, & s_i > z_{u_i} \end{cases} \quad (174)$$

In other words, we have $z_i = \bar{h}_i(x, p) = \sigma_i(s_i)$ with σ_i denoting the limiting effect where $s_i = h_i(x, p)$ is the smooth free action function. In (174), s_i is precisely the quantity to which the hardlimit is applied mathematically; it could be simply a state or a function of several states. The graph of $z_i = \sigma_i(s_i)$ is shown in Figure III.1.

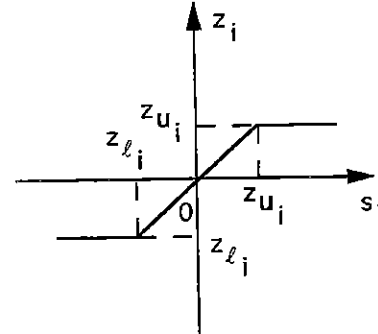


Figure III.1. The limiter function of a windup limit: $z_i = \sigma_i(s_i)$

An observation of (174) is that, although the real actuation variable z_i will never exceed the limits z_{l_i} and z_{u_i} , the "input" variable $s_i = h_i(x, p)$ itself can possibly take any value beyond these limits. In fact, after the state z_i reaches one of the limits, say z_{u_i} , the input s_i may continue to increase while z_i stays at z_{u_i} . If s_i begins to drop after increasing to some value beyond the limit, z_i will not come off z_{u_i} immediately. Rather, z_i will come off the limit only when s_i drops below z_{u_i} . The word "windup" appropriately characterizes the phenomenon described above.

Also note that $z_i = \sigma_i(s_i)$ is a piecewise affine function with three pieces glued together at z_{l_i} and z_{u_i} . This observation implies that $z_i = \sigma_i(s_i)$ satisfies the local Lipschitz condition. Since $h_i(x, p)$ is smooth, the composition $z_i = \bar{h}_i(x, p) = (h_i \circ \sigma_i)(x, p)$ is then also locally Lipschitzian, which guarantees local existence and uniqueness properties of Σ_{al} everywhere within the state space. We will see that this is not the case for the other

two types, i.e. for the state limits (nonwindup limits) and for the relay type limits. Therefore the problem of windup limits Σ_{sl} is the simplest of the three types of limits in this sense, and thus naturally will be the first to be analyzed in Sections III.2 and III.3.

(b) Nonwindup limits or state limits

A simplified dynamic model of Σ_{full} which is restricted to the study of nonwindup type limits or state limits say Σ_{sl} can be stated as

$$\Sigma_{sl}: \dot{x} = f(x, p), \quad x_{li} \leq x_i \leq x_{ui}, \quad i = 1, 2, \dots, n \quad (175)$$

From Section III.1.1(b), let us first recall the definition of nonwindup limit for x_i from (158). Then from (175), if we assume that $f_i \neq 0$ when either $x_i = x_{li}$ or $x_i = x_{ui}$, then the hard-constrained dynamics of x_i can be stated as

$$\frac{dx_i}{dt} = w_i f_i(x, p) \quad (176)$$

where

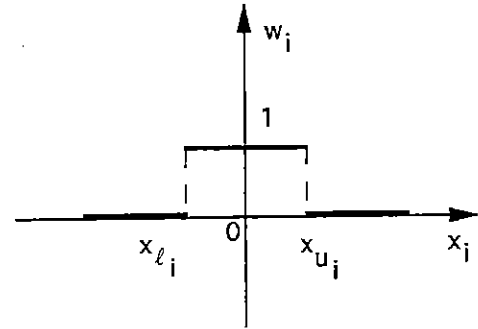
$$w_i = k_i(x, p) = \begin{cases} 1 & \begin{cases} \text{when } x_i = x_{li} \text{ and } f_i > 0, \\ \text{when } x_{li} < x_i < x_{ui}, \\ \text{when } x_i = x_{ui} \text{ and } f_i < 0, \end{cases} \\ 0 & \begin{cases} \text{when } x_i = x_{li} \text{ and } f_i < 0, \\ \text{when } x_i = x_{ui} \text{ and } f_i > 0 \end{cases} \end{cases} \quad (177)$$

The function $w_i = k_i(x, p)$ can be considered as a limiter function for the nonwindup limit. Figure III.2 (a) shows the graph of $w_i = k_i(x, p)$. In Figure III.2 (a) we assume that $f_i < 0$ when $x_i \leq x_{li}$ and $f_i > 0$ when $x_i \geq x_{ui}$. Figure III.2 (b) and (c) illustrate the differences between the windup limits and nonwindup limits when applied to the output of an amplifier block. The rudimentary power system which is used as an illustrative example in the paper includes the windup and nonwindup limits represented in the same way as shown in Figure III.2 (b) and (c). (see Figure A.2 in appendix for the block diagram of the excitation system for the rudimentary power system).

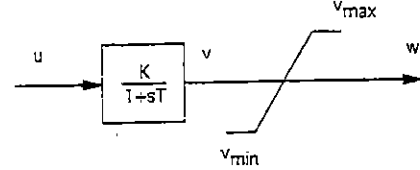
We remark that, if the initial point is such that $x_i = x_{li}$ or $x_i = x_{ui}$ together with the derivative $f_i = 0$, then at the initial point we could set either $\dot{x}_i = f_i$ or $\dot{x}_i = 0$. But since these points are points of discontinuity for the limited vector field, this does not determine the dynamics. For this higher order derivatives now need to be considered. We will discuss this issue in detail in Section III.2.2.

Compared to the observations made on the actuation limits, state limits are different in the following two respects: First, by (175), the state x_i , which has a nonwindup limit can never exceed its limits and x_i will leave the lower limit x_{li} or the upper limit x_{ui} as soon as f_i becomes positive or negative respectively. So there is no "windup" involved here. Secondly, from (176) and (177), we can see that the nonwindup limit actually introduces a discontinuous piecewise smooth function $w_i = k_i(x, p)$ into the dynamic model. The discontinuity of the function k_i induces more drastic changes in the solution behavior of the system Σ_{sl} .

For instance, the local existence and uniqueness of solutions for Σ_{sl} is not guaranteed everywhere. We will prove in Section III.2.2. that there exist points in the state space where solutions strictly exist in positive time and not in negative time (sort of like singular sources in the DAE theory, Section I.1.4), and there are also points where local uniqueness of solutions disappears. These cases of special behavior naturally arise when one or more of the states reach their limits since the dynamics Σ_{sl} is smooth when all the states are unconstrained. More details follow in Section III.2.2.



(a) The limiter function of a nonwindup limit.



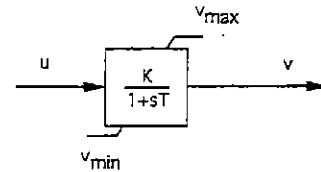
differential equation:

$$\frac{dv}{dt} = f(u, v) = \frac{1}{T}(Ku - v)$$

limiter function:

$$w = l(v) = \begin{cases} v_{min}, & \text{when } v < v_{min} \\ v, & \text{when } v_{min} \leq v \leq v_{max} \\ v_{max}, & \text{when } v > v_{max} \end{cases}$$

(b) Windup limit for an amplifier representation.



differential equation:

$$\frac{dv}{dt} = wf(u, v) = w \frac{1}{T}(Ku - v)$$

limiter function:

$$w = k(v) = \begin{cases} 1, & \begin{cases} \text{when } v_{min} < v < v_{max}, \text{ or} \\ \text{when } v = v_{min} \text{ and } f > 0, \text{ or} \\ \text{when } v = v_{max} \text{ and } f < 0 \end{cases} \\ 0, & \begin{cases} \text{when } v = v_{min} \text{ and } f < 0, \text{ or} \\ \text{when } v = v_{max} \text{ and } f > 0 \end{cases} \end{cases}$$

(c) Nonwindup limit for an amplifier representation.

Figure III.2 Differences between windup and nonwindup limits.

(c) Switching limits or relay type limits

The full dynamic model Σ_{full} can be simplified to the model Σ_{rl} stated below for analyzing the switching limit related phenomena:

$$\Sigma_{rl}: \dot{x} = f(x, w, p), \quad w = \bar{v}(x, p) \quad (178)$$

Inspecting the definition of the switching function $\bar{v}_i$, it is clear that this function takes only two values, zero or one, and is discontinuous when $v_i(x, p) = 0$. Therefore the overall function $f(x, \bar{v}(x, p), p)$ is generally not continuous in x at certain points in the state space (when one of the v_i 's is zero) (see Figure

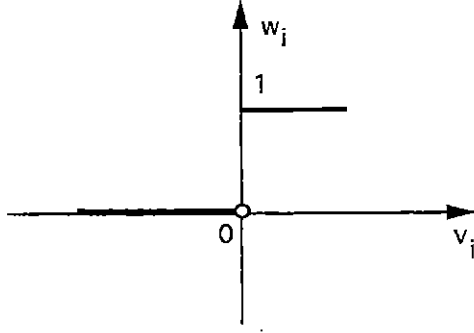


Figure III.3. Switching function for the relay type limit.

III.3). At these points then, even local existence of solutions is not guaranteed and local existence/uniqueness properties demand a careful study first. More details on the dynamics of $\Sigma_{\mathcal{I}}$ are presented in Section III.2.3.

III.1.3 Organization of the illustrations in Chapter III

Following this rather detailed introduction of the hardlimits written informally, the three hard limit types will be analyzed in precise mathematical fashion. This part of the text is sprinkled with illustrative examples. These examples are utilizing a simple rudimentary model which contains all four rudiments of the power system i.e. generation, transmission, load and control each modeled in the simplest possible way. So the entire system is only third order with two x and one y variables. So this system has a good measure of realism yet offers the tremendous advantage that everything can be drawn on a two dimensional sheet of paper. This later fact provides unimpeded special insight into what happens and so facilitates the formation of meaningful conceptual identities of the results.

The rudimentary model was used by our group earlier [123, 92] and its details are given in the *Appendix*. So the reader is advised to peruse the *Appendix* before reading the explanations of the various illustrative examples in this Chapter.

In Section III.3, two Tables full of pertinent case histories for respectively the actuation (i.e. nonwindup) and state (i.e. windup) limits are presented in the form of conceptual sketches. The illustrative numerical examples described previously are cross-referenced to the corresponding conceptual sketches in the Tables of Section III.3. Comparing the cases which have numerical examples with the sketches in the Table should also aid the reader in interpreting the additional cases in the Tables where numerical examples are not presented.

III.2 The State Space Structure

For the models given in Section III.1.2, we develop the basic elements for an analysis of the systems in the state space from definitions of stability properties of equilibria to descriptions of the regions of attraction of stable equilibria. While the model with windup limits only differs from the differential algebraic model Σ_{al} in the additional presence of nonsmooth actuation functions $z_i = \bar{h}_i(x)$, nonwindup models lead to piecewise defined dynamics which need to be established first. Similarly relay-type operations generate variable structure type models, and the problem is one of piecing together different smooth systems across the switching events.

III.2.1 Actuation limits (wind-up limiters)

In this section, we will analyze the state space structure of the dynamics which includes descriptions of the actuation limits. The mathematical formulation of this problem was introduced in Section III.1.2(b). The parameters p are assumed to be fixed in this analysis, therefore the parameter dependence is not shown in the equations. Essentially we can introduce some actuation variables z_i which are nonsmooth functions $\bar{h}_i(x)$ of the state variables for denoting the actuation limits or windup limits (the terminology, saturation of z_i , is more common in the control literature) at a maximum value z_{u_i} and a minimum value z_{ℓ_i} . Essentially the actuation variable z_i cannot exceed these limiting values, as defined next:

$$z_i = \bar{h}_i(x) := \begin{cases} z_{\ell_i}, & h_i(x) < z_{\ell_i} \\ h_i(x), & z_{\ell_i} \leq h_i(x) \leq z_{u_i} \\ z_{u_i}, & h_i(x) > z_{u_i} \end{cases} \quad (179)$$

The state space model that includes actuation limits can be represented as

$$\Sigma_{al} : \quad \dot{x} = \bar{f}(x) := f(x, z) \text{ where } z = \bar{h}(x) \quad (180)$$

$$f : \mathbb{R}^{n+n_z} \rightarrow \mathbb{R}^n, \quad \bar{h} : \mathbb{R}^n \rightarrow \mathbb{R}^{n_z} \quad (181)$$

$$x \in \mathbb{R}^n, \quad z \in \mathbb{R}^{n_z}, \quad f \text{ is } C^\infty, \text{ and } h \text{ is } C^\infty \quad (182)$$

where x denotes the state variables and z stands for the actuation variables. By inspection of the functions $\bar{h}_i$, it is clear that the model Σ_{al} is nonsmooth since the functions $\bar{h}_i(x)$ (which are arbitrarily cut-off at the limits z_{u_i} and z_{ℓ_i} by the saturation effect) is not differentiable at the limiting values when $h_i(x) = z_{\ell_i}$ and when $h_i(x) = z_{u_i}$. Therefore the standard tools of state space analysis are not readily applicable to this model. However we observe that each of the actuation functions, say $\bar{h}_i(x)$, is a piecewise smooth function, consisting of three such "pieces" (see 179), separated at z_{u_i} and z_{ℓ_i} . Moreover the limiting effect in (179) is such that the function $\bar{h}_i$ is locally Lipschitzian at any $x \in \mathbb{R}^n$. This guarantees that the individual smooth "pieces" in functions (179) "fit together nicely". Dynamically we say that given any $x_0 \in \mathbb{R}^n$, there exists a unique trajectory locally both forward (positive) and backward (negative) in time. Let us make this observation precise by the next Proposition.

Proposition 1 *There exists a well-defined flow say $\Phi_t^{al}(x)$ for the state differential equation model Σ_{al} . $\Phi_t^{al}(x)$ is C^1 in time t and is C^0 in state x .*

Proof: Note that $\bar{h}_i(x) = (h_i \circ \sigma_i)(x)$ where $\sigma_i(u)$ is the well-known saturation function,

$$\sigma_i : \mathbb{R} \rightarrow \mathbb{R}, \quad \sigma_i(u) := \begin{cases} z_{\ell_i}, & \text{if } u < z_{\ell_i} \\ u, & \text{if } z_{\ell_i} \leq u \leq z_{u_i} \\ z_{u_i}, & \text{if } u > z_{u_i} \end{cases}, \quad (183)$$

It is easily seen that σ_i is locally Lipschitzian in u , and $h_i(x)$ is smooth. As a composition, $\bar{h}_i$ is locally Lipschitzian in x which proves the result. Q. E. D.

The Proposition basically provides us with the fundamental result for carrying out the dynamic analysis. That is, even though the trajectories may lose differentiability at certain points (this will be made precise in the next section), they do not suffer loss of existence or loss of uniqueness along the way. Briefly it needs to be pointed out that when the state limits (nonwindup limiters) and switching limits (relay type limiters) are considered, this property breaks down. That is, there emerge

points on limit surfaces where trajectories can lose existence and uniqueness properties, and the resulting dynamics is richer in features. In case of state limits (Section III.2.2), we will prove that the flow exists only in positive time but not in negative time, whereas even this is not true for the switching limits (Section III.2.3).

The current problem of actuation limits introduces some of the nonsmooth features of the dynamics. Note that the existence of the flow sets up the global dynamic analysis, though the difficulty arises from the fact that the flow is only guaranteed to be a homeomorphism (note from Proposition that $\Phi_t^1(x)$ is only C^0 in x), and not a diffeomorphism. Standard tools in global analysis such as transversality of intersections lose their meaning. Stable and unstable surfaces for equilibria are not even guaranteed to be smooth manifolds, hence concepts such as λ -Lemmas are not immediately applicable to this problem. Therefore it becomes important to proceed step by step with caution, keeping in mind the strict topological (and nonsmooth) nature of the dynamics, introduced by the model itself.

Fortunately we will see in the following Sections that the analysis of the boundary of the region of attraction which is the principal problem of interest from the power system dynamic security view-point (see Section II.1 for DAE case), is inherently a topological problem (being an analysis of the *boundary*). Moreover since we have excluded the presence of singularity in this formulation (see Section III.1), special phenomena connected with singularity such as pseudo equilibria and semi-singular points do not appear here. Therefore it is indeed possible to carry out a rather complete analysis much like for a smooth system (see Morse-Smale theory: [74], [57], power-systems: [84], [114], [18]). These results are presented in the next few Sections, starting with an overview of the state space structure first.

The main result on the stability boundary analysis can be summarized as follows. *When the nonsmooth nature of the actuation limiters are included in the model, the structure of the stability boundary is topologically identical to the boundary structure of the corresponding result for smooth systems.* That is, the stability boundary under certain assumptions, again consists of stable manifolds (which are only topological manifolds, but no longer everywhere differentiable) of anchor points such as unstable equilibria and unstable periodic orbits. However this does not mean that these limiters can be excluded from the analysis because some of the anchor points occur *after the limits have been encountered*, hence in actual implementation, the stability boundary results would differ substantially when these “nonsmooth” anchor points are to be considered. Therefore energy function methods should consider the nonsmooth nature of the model Σ_{al} to accommodate the effects of windup actuation hardlimits on these anchor points. When we consider the state limits (nonwindup limiters) and switching limits (relay type limiters) in the next two sections, the problem becomes more complicated in the sense that those two limit types actually contribute additional boundary segments to the stability boundary.

A. Structure of the overall dynamics

In this section, we will decompose the state space into individual components such that the dynamics Σ_{al} is smooth when restricted to any of the components. The transitions then intro-

duce the nonsmoothness. We can formally define the transition boundaries as the sets ZL_i and ZU_i , denoting the points where the actuation $h_i(x)$ reaches its lower limit z_{li} and its upper limit z_{ui} respectively.

$$ZL_i := \{x \in \mathbb{R}^n : h_i(x) = z_{li}\} \quad (184)$$

$$ZU_i := \{x \in \mathbb{R}^n : h_i(x) = z_{ui}\} \quad (185)$$

In the following text, we use the terminology of *limit surfaces* or *transition surfaces* for denoting the lower and upper limit surfaces ZL_i and ZU_i respectively. By the implicit function theorem, it is clear that if $D_x h_i$ is nonzero at any $x_0 \in ZU_i$; then ZU_i is locally an $(n-1)$ -dimensional embedded submanifold in $\mathbb{R}^n$. Similar arguments hold for ZL_i . Moreover violation of this condition, that is, $D_x h_i = 0$ in ZU_i or ZL_i implies that the state x simultaneously satisfies $n+1$ equality conditions which is degenerate in $\mathbb{R}^n$. Therefore by Thom's transversality Theorem [22], we can assume that generically ZU_i and ZL_i are each $(n-1)$ -dimensional embedded submanifolds of $\mathbb{R}^n$.

Assumption (A0.1) ZU_i and ZL_i are $(n-1)$ -dimensional submanifolds of $\mathbb{R}^n$ for each $i = 1, 2, \dots, n_z$.

This guarantees a nice differential geometric platform for the analysis that follows. Let us introduce a notation for the collection of these upper limit sets ZU_i and lower limit sets ZL_i as ZU and ZL respectively,

$$ZL := \bigcup_{i=1}^{n_z} ZL_i, \quad ZU := \bigcup_{i=1}^{n_z} ZU_i \quad (186)$$

Note that even with the assumption (A0.1), the sets ZL and ZU are not manifolds in general, because they involve intersections of individual manifolds ZU_i and ZL_i . Once these transition surfaces ZL and ZU are left out of the state space $\mathbb{R}^n$, the resulting set, say SM , (SM for smooth) has the property that all the functions $\bar{h}_i$ are smooth in SM , therefore the dynamics Σ_{al} is smooth within SM .

$$SM := \mathbb{R}^n \setminus (ZL \cup ZU) \quad (187)$$

Lemma 21 The flow $\Phi_t^{al}(x)$ restricted to SM is smooth. $\square$

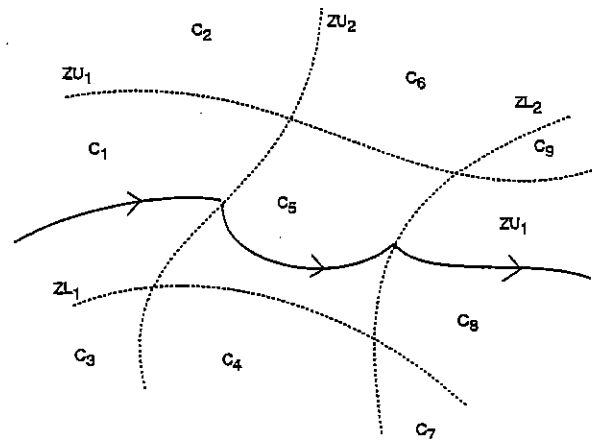


Figure III.4 Nonsmooth trajectories in the windup limit (actuation limit) model.

This provides us a link with the standard differential geometric arguments in that we can carry out such computations as long as we are in SM . Since the sets ZU_i and ZL_i are each closed in $\mathbb{R}^n$, it follows that SM is open in $\mathbb{R}^n$. It is convenient to break up the full set SM into connected components C_i so that $SM = \bigcup_{i=1}^{n_c} C_i$ where n_c is the number of such components. The whole state space $\mathbb{R}^n$ can now be written as the union

$$\mathbb{R}^n = SM \cup ZL \cup ZU = \left(\bigcup_{i=1}^{n_c} C_i \right) \cup \left(\bigcup_{i=1}^{n_z} ZL_i \right) \cup \left(\bigcup_{i=1}^{n_z} ZU_i \right) \quad (188)$$

The dynamics Σ_{af} restricted to each component C_i is smooth and is locally invariant since C_i is open. The trajectories within each component C_i are smooth (see Figure III.4), however when they cross the boundary surfaces ZL_i or ZU_i , moving into another component, smoothness is lost. Therefore as we will see later, the analysis is easy "most-of-the-time" whenever the trajectories stay in open components C_i . When the trajectories do reach one of the component boundaries in $ZL \cup ZU$, special care is necessary to determine where the trajectory is headed. Consistency while moving across the components is guaranteed by the global existence of the flow from Proposition 1. The construction so far can be summarized in the form of the next statement.

Corollary 2 *The flow $\Phi_t^{af}(\cdot)$ as a map is a homeomorphism in $\mathbb{R}^n$. Moreover the flow $\Phi_t^{af}(\cdot)$ when restricted to a component C_i is a diffeomorphism. $\square$*

Note that if the actuation limits were absent, i.e. for the smooth model, there exists a single component $C_1 = \mathbb{R}^n$ and the flow in that case is naturally smooth in the whole state space. With the limits present, "normal operation" corresponds to the components C_i where none of the limits has been reached, i.e. $z_i < h_i(x) < z_{ui}$ for all i , and the dynamics in all other components would be quite different compared to a model where these limits are ignored.

B. A technical assumption

From the decomposition (188), the analysis can be carried out by considering smooth dynamics Σ_{af} restricted to SM as long as the trajectories stay within open components C_i . The technicality arises when the trajectory reaches the surfaces $ZL \cup ZU$ since we need to know exactly where this trajectory is headed. Basically the trajectory upon reaching the boundary $ZL \cup ZU$, may either exit the component or may return to the component. There also exists the degenerate possibility that the trajectory stays on the boundary surface $ZL \cup ZU$ itself for a time interval, which would complicate the analysis. For instance, this scenario arises when an equilibrium of Σ_{af} happens to be on the boundary $ZL \cup ZU$. Note that by Assumption (A0.1), the set $ZL \cup ZU$ is at most of dimension $n - 1$, and generically an equilibrium condition can be expected not to occur on this lower dimensional set. This degeneracy can indeed occur and is important in parametric analysis, for questions of bifurcation study. We will see that these degenerate equilibrium points, which we will call *critical equilibria* (see Section III.3.2) play a fundamental role in understanding the feasibility boundary for system operation. However for state-space analysis, such points can be left out as degenerate cases by imposing a technical assumption detailed below.

Consider a point say $x_0 \in ZU_i$ on the boundary between two

components C_k and C_{k+1} . Assume that x_0 is not an equilibrium, i.e. $\bar{f}(x_0) \neq 0$. We are interested in looking at the solution $x(t)$ that starts from x_0 i.e. $x(t_0) = x_0$ (which is unique by Proposition 1), and impose some conditions to guarantee that $x(t)$ is locally smooth everywhere except possibly at x_0 itself. Note that from the problem formulation (by Corollary 2), this automatically follows provided that the solution stays in the open components C_k everywhere except at x_0 when $x(t)$ intersects the limit surface ZU_i . In other words, the degenerate possibility of a solution entirely lying on ZU_i over a time interval will be ruled out. This technical assumption is necessary to "carry over" the results for smooth systems valid in the open components "across" the transition surfaces such as ZU_i in global analysis of stability regions.

First assume that x_0 does not belong to any other limit surface i.e. $x_0 \notin \bigcup_{j \neq i} (ZL_j \cup ZU_j)$. Then it follows that among the

functions $\bar{h}_j(x)$, all the functions $\bar{h}_j(x)$ with $j \neq i$ are smooth at x_0 . For the function $\bar{h}_i(x)$, we have $h_i(x_0) = z_{ui}$. Therefore the point x_0 stands as sort of a transition in the description of z_i . $z_i = \bar{h}_i(x)$ changes from the actuation effected as $h_i(x)$ to the saturation value z_{ui} at x_0 . For analyzing the local dynamics at x_0 , it is natural to define the following two smooth functions $f_a^i(x)$ for actuation of z_i and $f_s^i(x)$ for saturation of z_i associated with ZU_i :

$$f_a^i(x) := f(x, z), \quad z_i = h_i(x) \text{ and } z_j = \bar{h}_j(x) \text{ for } j \neq i, \quad (189)$$

$$f_s^i(x) := f(x, z), \quad z_i = z_{ui} \text{ and } z_j = \bar{h}_j(x) \text{ for } j \neq i. \quad (190)$$

It follows that both functions $f_a^i(x)$ and $f_s^i(x)$ are locally smooth at x_0 since $x_0 \notin \bigcup_{j \neq i} (ZL_j \cup ZU_j)$. Now suppose there exists a

solution $x(t)$ which reaches x_0 at time t_0 , i.e. $x(t_0) = x_0$ with $h(x(t)) < x_0$ for $t < t_0$ (say the solution approaches from the actuation side of ZU_i), then the first order derivative of $h(x(t))$ at x_0 is easily seen to be the Lie derivative,

$$\lim_{t \rightarrow t_0} \frac{d}{dt} h(x(t)) = (D_x h_i f_a^i)_{|x_0} = L_{f_a^i} h_i(x_0) \quad (191)$$

If this derivative $L_{f_a^i} h_i(x_0)$ is nonzero, then it implies that $h(x(t))$ increases past z_{ui} at x_0 , hence there is a simple transition at x_0 .

We say that the solution $x(t)$ then has a first order of contact with the surface ZU_i where $h_i(x) = z_{ui}$. If the first Lie derivative $L_{f_a^i} h_i(x_0)$ is zero, then naturally we can look at the second Lie derivative $L_{f_a^i}^2 h_i(x_0)$ which equals the limit of the second order derivative $\frac{d^2 h_i(x(t))}{dt^2}$. When the first Lie derivative $L_{f_a^i} h_i(x_0)$ is zero and the second derivative $L_{f_a^i}^2 h_i(x_0)$ is nonzero, a second order of contact occurs at x_0 . In this case, from a Taylor expansion of $h(x(t))$, it is easily seen that the solution which reaches x_0 returns to the same component tangentially touching z_{ui} at x_0 and there is no transition. Carrying out this construction, it can be seen that if the first Lie derivative $L_{f_a^i}^k h_i(x_0)$ which is nonzero at x_0 is of order k , then two possibilities arise: 1) if k is odd, then the solution $x(t)$ crosses over z_{ui} to a different component at x_0 , and 2) if k is even, then the solution $x(t)$ returns to the same component. As long as there is at least one Lie derivative of any order that is nonzero, then these two are the only possibilities and the degeneracy of a trajectory entirely lying on z_{ui} is ruled out. We make this as a precise assumption next. Note that all the arguments so far have been based on the vector field f_a^i assuming that the solution originated from $h_i(x(t_0-)) < z_{ui}$. If this were not true, that is, for the case $h_i(x(t_0-)) > z_{ui}$, we get a similar condition with the function f_s^i replacing f_a^i as stated next:

(Assumption A0.2) $\bar{f}(x_0) \neq 0$ for any $x_0 \in ZU \cup ZL$. Consider any $x_0 \in ZL_i \cup ZU_i$ such that $x_0 \notin \bigcup_{j \neq i} (ZL_j \cup ZU_j)$. Then the solution $x(t)$ which satisfies $x(t_0) = x_0$ has a finite order of contact with the surface $ZL_i \cup ZU_i$ at x_0 . That is, $L_{j_0}^k h_i(x_0) \neq 0$ for some k , and $L_{j_0}^{k'} h_i(x_0) \neq 0$ for some k' .

Even though there appears to be a technicality in this assumption when the indices k and k' are not equal, note that consistency is automatically guaranteed by the existence of the flow in Proposition 1. Assumption (A0.2) gives us the following nice property which is crucial in our global analysis.

Lemma 22 Under Assumption (A0.2), suppose $x(t)$ is a solution of Σ_{al} with initial condition $x(t_0) = x_0$ such that $x_0 \in ZL_i \cup ZU_i$ and $x_0 \notin \bigcup_{j \neq i} (ZL_j \cup ZU_j)$. Then $x(t) \cap (ZL \cup ZU) = \emptyset$ for $t \neq t_0$ locally near $t = t_0$. $\square$

Therefore Assumption (A0.2) implies that trajectories which reach the transition surfaces $ZL \cup ZU$ strictly leave them immediately (for instance, see the trajectory shown in Figure III.4). This implies "transversality" of trajectories across $ZL \cup ZU$ in a very loose sense. But this is all that is required for later topological arguments.

C. Basic definitions and tools for stability boundary analysis

In this section, notions of hyperbolic equilibrium points and periodic orbits are developed along with an analysis of their stable and unstable manifolds, keeping in mind, the nonsmooth nature of the dynamics at certain points. We will see that much of the properties for smooth vector fields carry over to this case, provided the analysis is done "carefully" while dealing with the nonsmooth elements. A key technical tool that is proposed here is the notion of an "almost smooth manifold" which is a topological manifold that is also a smooth manifold almost everywhere (to be made precise later). This definition exemplifies the nonsmooth nature of the trajectories for the system Σ_{al} in a precise sense under Assumptions (A0.1) and (A0.2). We will go on to develop the notions of stable and unstable manifolds for hyperbolic equilibria which are then shown to be almost smooth manifolds for Σ_{al} . The fundamental concepts in stability boundary analysis such as λ -Lemmas for equilibria move over directly from the standard results to this formulation since all equilibria belong to open components. The corresponding results for periodic orbits are somewhat more technical.

1. Equilibria

For the dynamics Σ_{al} , the equilibrium points can be defined as usual. That is, $x_0 \in \mathbb{R}^n$ is an equilibrium point for Σ_{al} if $\bar{f}(x_0) = 0$. By Assumption (A0.2), equilibrium points do not occur on the transition manifolds $ZL_i \cup ZU_i$. Therefore an equilibrium point x_0 must belong to some open component say C_i . Since the flow is smooth when restricted to C_i (Corollary 2), the definition of hyperbolic equilibria directly follows from the definition for smooth flows. An equilibrium point $x_0 \in SM$ is said to be hyperbolic if all the eigenvalues of the Jacobian $J := D_x \bar{f}$ evaluated at x_0 have nonzero real parts. Let us define n_s (n_u) the number of eigenvalues of J with negative (positive) real parts. Then at a hyperbolic equilibrium, $n_s + n_u = n$. We can define the local stable and unstable manifolds in the usual sense.

$$W^s(x_0) := \{x \in \mathbb{R}^n : \Phi_t^{al}(x) \rightarrow x_0 \text{ as } t \rightarrow \infty\} \quad (192)$$

$$W^u(x_0) := \{x \in \mathbb{R}^n : \Phi_t^{al}(x) \rightarrow x_0 \text{ as } t \rightarrow -\infty\} \quad (193)$$

Again since $x_0 \in C_i$, the flow $\Phi_t^{al}(\cdot)$ is smooth within C_i and the local results follow immediately.

Lemma 23 Consider a hyperbolic equilibrium $x_0 \in SM$. Then locally near x_0 , $W^s(x_0)$ and $W^u(x_0)$ are n_s -dimensional and n_u -dimensional embedded submanifolds of $\mathbb{R}^n$ respectively. Moreover $W^s(x_0)$ and $W^u(x_0)$ intersect transversally at x_0 . $\square$

In extending this local result globally, the strength of Proposition 1 becomes apparent. Note that when the trajectories composing the stable (unstable) manifolds cross the transition surfaces $ZL \cup ZU$, they lose smoothness, therefore $W^s(x_0)$ and $W^u(x_0)$ cannot be expected to be smooth manifolds globally. However since the flow $\Phi_t(\cdot)$ is globally a homeomorphism by Corollary 2, it follows that the manifold property is preserved topologically.

Lemma 24 Given a hyperbolic equilibrium $x_0 \in SM$, $W^s(x_0)$ and $W^u(x_0)$ are n_s -dimensional and n_u -dimensional topological manifolds in $\mathbb{R}^n$ respectively. $\square$

Remarks: 1. When $x_0 \in SM$ is a hyperbolic equilibrium, since locally the dynamics is smooth near x_0 , all the local results for the dynamics near hyperbolic equilibrium points directly carry over from standard results. These include the concepts such as fundamental domains and tools such as λ -Lemma. However the notion of transversality of intersections introduces a technical difficulty in light of Lemma 24.

2. While characterizing the stability boundary, a fundamental assumption is the requirement on transversal intersections of stable and unstable manifolds to enforce "structures" which are preserved under small perturbations. However transversality (based on tangent spaces) is a geometric concept, and is not meaningful here for the topological manifolds $W^s(x_0)$ and $W^u(x_0)$. This problem can be "resolved" with the introduction of the notion of almost smooth manifolds as shown in subsection 3(b) below. Basically even though $W^s(x_0)$ and $W^u(x_0)$ are not smooth manifolds, in general, they are almost smooth in the sense that we can indeed carry out differential geometry almost everywhere.

2. Periodic orbits

The definition of periodic orbits can be again like in the case of smooth systems. That is, a closed orbit $\gamma \subset \mathbb{R}^n$ is defined to be a periodic orbit for Σ_{al} if γ is invariant under the flow $\Phi_t^{al}(\cdot)$. However the definition of hyperbolicity is more technical. When the periodic orbit γ intersects with the transition surfaces, i.e. $\gamma \cap (ZL \cup ZU) \neq \emptyset$, the local dynamics near γ will not be smooth for points in $\gamma \cap (ZL \cup ZU)$, and as such, it is not obvious how to handle these in our analysis. Moreover we cannot just assume that these cases do not arise because such intersections of γ with $ZL \cup ZU$ can arise generically. Note that unlike equilibrium points which are generically zero dimensional objects, periodic orbits are generically one-dimensional. Unlike in Assumption A0.2 which rules out equilibrium points intersecting with $ZL \cup ZU$, we cannot assume generically that periodic orbits stay away from the set $ZL \cup ZU$.

This technical difficulty can be obviated by resorting to the powerful notion of Poincare maps [37]. Precisely we can assume that there exists an $x_0 \in \gamma \cap SM$ and an associated smooth Poincare map so that standard definitions carry over. The point is that while for smooth systems, the existence of such smooth Poincare maps is a property of a periodic orbit, this is not true for the model Σ_{al} . Here, this needs to be assumed. For this reason, we introduce a notion of weak hyperbolicity of a periodic orbit γ .

Definition 10 A periodic orbit γ is weakly hyperbolic if 1) there exists a point $x_0 \in \gamma \cap SM$ such that the associated map $\phi(x) := \Phi_T^{\alpha}(x)$ (where T is the period of γ) is locally a smooth diffeomorphism, and 2) there exist a unique eigenvalue of the Jacobian $D_x \phi(x_0)$ on the unit circle. $\square$

The connection to the standard definition is immediate when we stay away from the transition sets $ZL \cup ZU$.

Lemma 25 If γ is a periodic orbit of Σ_{al} with $\gamma \subset SM$, then γ is weakly hyperbolic if and only if γ is hyperbolic for the SM restricted smooth flow $\Phi_t^{\alpha}(\cdot)$. $\square$

By exploiting the existence of the smooth diffeomorphism ϕ for which $x_0 \in \gamma$ is a fixed point (see definition above), we can define stable and unstable manifolds for the map ϕ at x_0 and locally these are guaranteed to be smooth manifolds.

$$W_{\phi}^s(x_0) := \{x \in \mathbb{R}^n : \phi^n(x) \rightarrow x_0 \text{ as } n \rightarrow \infty\} \quad (194)$$

$$W_{\phi}^u(x_0) := \{x \in \mathbb{R}^n : \phi^n(x) \rightarrow x_0 \text{ as } n \rightarrow -\infty\} \quad (195)$$

Suppose n_s^{ϕ} and n_u^{ϕ} are the number of eigenvalues of the Jacobian $D_x \phi(x_0)$ with modulus less than one and greater than one respectively. From Definition 10, it follows that $n_s + n_u = n - 1$. The next result directly follows from the smoothness of the map ϕ .

Lemma 26 Suppose γ is weakly hyperbolic. Let x_0 and ϕ be the corresponding elements in the definition of 10. Then locally near x_0 , $W_{\phi}^s(x_0)$ and $W_{\phi}^u(x_0)$ are n_s^{ϕ} and n_u^{ϕ} dimensional smooth embedded submanifolds of $\mathbb{R}^n$. $\square$

Next we can define the stable and unstable manifolds for the whole periodic orbit γ ,

$$W^s(\gamma) := \{x \in \mathbb{R}^n : \Phi_t^{\alpha}(x) \rightarrow \hat{x} \in \gamma \text{ as } t \rightarrow \infty\} \quad (196)$$

$$W^u(\gamma) := \{x \in \mathbb{R}^n : \Phi_t^{\alpha}(x) \rightarrow \hat{x} \in \gamma \text{ as } t \rightarrow -\infty\} \quad (197)$$

Note that locally near γ , $W^s(\gamma)$ and $W^u(\gamma)$ can be easily recovered from the respective manifolds W_{ϕ}^s and W_{ϕ}^u for the map ϕ at x_0 by extending these along the periodic orbit γ .

Lemma 27 Suppose γ is weakly hyperbolic. Locally near γ ,

$$W^s(\gamma) = \bigcup_{t \in (0, 2T)} \Phi_t(W_{\phi}^s(x_0)) \quad (198)$$

$$W^u(\gamma) = \bigcup_{t \in (0, 2T)} \Phi_t(W_{\phi}^u(x_0)) \quad (199)$$

$\square$

Combining Lemmas 26 and 27 along with the property of the global flow from Corollary 2, it follows that $W^s(\gamma)$ and $W^u(\gamma)$ are indeed topological manifolds. Note that we pick up a dimen-

sion by running along the periodic orbit γ in equations (198) and (199), hence define $n_s = n_s^{\phi} + 1$ and $n_u = n_u^{\phi} + 1$.

Lemma 28 Suppose γ is a weakly hyperbolic periodic orbit for Σ_{al} . Then $W^s(\gamma)$ and $W^u(\gamma)$ are n_s and n_u dimensional topological manifolds in $\mathbb{R}^n$ respectively. $\square$

Remarks. 1. Again for local analysis near γ , we can restrict our attention to the point $x_0 \in \gamma$ and the associated map ϕ in the definition of weak hyperbolicity for developing tools such as λ -Lemmas.

2. Like in the case of equilibria, the difficulty is in dealing with the transversality assumption. This can be handled using the notion of almost smooth manifolds to be developed later.

The concept of weak hyperbolicity introduced above, while powerful in analyzing actuation limited system Σ_{al} , is also a technical one in that the analysis becomes more complicated near periodic orbits. In fact, extensions of this notion need to be used in analyzing the other types of limits such as nonwindup limits and the switching limits. However for simplifying the presentation, the following assumption will be in effect. Every periodic orbit γ of Σ_{al} is assumed not to intersect with the transition surfaces $ZL \cup ZU$.

Assumption (A0.3). If $\gamma \subset \mathbb{R}^n$ is a periodic orbit of Σ_{al} , then $\gamma \subset SM$.

Essentially we assume that each periodic orbit γ entirely lies within a single component say C_i . By Lemma 25, the new definition reduces to the standard definition, and the standard local analytical tools are readily applicable.

3. Transversality conditions

We have noted in the previous sections that one of the main technical complexities associated with the model Σ_{al} is that the stable and unstable manifolds for equilibria (and periodic orbits) are only topological manifolds, and are not smooth globally. Therefore the notion of transversality needs to be revised in some sense to reflect on the possibility of nonsmoothness of the manifolds. Fundamentally the transversality requirements form the core of characterizing the boundary of the region of attraction. This is because transversality is an "open" property (roughly if this condition is satisfied everywhere in an open neighborhood in a suitable function space), and hence enforces a rigid structure on the composition of the stability boundary in terms of stable and unstable manifolds. In other words, the transversality assumption rules out the possibility of certain global bifurcations where the structure of the stability boundary undergoes changes. Therefore in order to characterize the stability boundary for the model Σ_{al} , it is absolutely essential that we have some notion of transversality though this is not obvious here since the manifolds are not smooth at points on $ZL \cup ZU$.

In our formulation, to check whether a stable manifold say W_1^s intersects an unstable manifold W_2^u transversally, we run into difficulty in the standard condition (the tangent spaces of W_1^s and W_2^u span the whole space at every intersection point) whenever there is an intersection point in $ZL \cup ZU$, that is, whenever $W_1^s \cap W_2^u \cap (ZL \cup ZU) \neq \emptyset$, because of nonsmoothness. To get around this difficulty, we need to indulge in a technical analysis of how the stable and unstable manifolds emerge glob-

ally while crossing the transition manifolds. This will allow us to prove that in fact, they are quite well-behaved generically in that they are smooth manifolds almost everywhere. Equipped with this knowledge of an almost smooth property of the stable and unstable manifolds, then we can indeed come up with a notion of transversality for the intersections of stable and unstable manifolds by restricting the definition to only the smooth counterparts within these topological manifolds.

3.(a) Transition boundary analysis

Without loss of generality, let us consider an upper level surface ZU_i . For other transition surfaces in $ZL \cup ZU$, the analysis is similar. First recall the definition,

$$ZU_i = \{x \in \mathbb{R}^n : h_i(x) = z_{u_i}\} \quad (200)$$

Also recall that by Assumption (A0.1), the surface ZU_i is a $(n-1)$ -dimensional smooth manifold in $\mathbb{R}^n$. It is useful to distinguish the following subsets within ZU_i by looking at the sign of the first order Lie derivative $L_{f_i} h_i$. As observed in Section B (see the construction of Assumption (A0.2)), when this derivative is nonzero, this implies a first order contact with ZU_i of the trajectory that reaches ZU_i , hence resulting in a simple transition at such points.

$$ZU_i^+ := \{x \in ZU_i \setminus \bigcup_{j \neq i} (ZL_j \cup ZU_j) : L_{f_i} h_i(x) > 0\} \quad (201)$$

$$ZU_i^- := \{x \in ZU_i \setminus \bigcup_{j \neq i} (ZL_j \cup ZU_j) : L_{f_i} h_i(x) < 0\} \quad (202)$$

$$ZU_i^0 := \{x \in ZU_i \setminus \bigcup_{j \neq i} (ZL_j \cup ZU_j) : L_{f_i} h_i(x) = 0\} \quad (203)$$

Naturally then we have divided the set ZU_i into these four mutually disjoint subsets,

$$ZU_i = ZU_i^+ \cup ZU_i^- \cup ZU_i^0 \cup \left(ZU_i \cap \bigcup_{j \neq i} (ZL_j \cup ZU_j) \right) \quad (204)$$

Among these subsets, note that the first two ZU_i^+ and ZU_i^- are special in the sense that the local behavior of the trajectories of Σ_{a_i} is well-understood. For instance, consider an $x_0 \in ZU_i^+$, and let $x(t_0) = x_0$ be a solution of Σ_{a_i} . Then simple calculation reveals that $h(x(t_{0-})) < z_{u_i}$ and $h(x(t_{0+})) > z_{u_i}$. Therefore the solution $x(t)$ crosses z_{u_i} from the actuation regime $h(x) < z_{u_i}$ to the saturation regime $h(x) > z_{u_i}$ as the time increases past t_0 . Similar arguments follow for ZU_i^- where the behavior is the converse. Observe that the sets ZU_i^+ and ZU_i^- are indeed smooth manifolds being open subsets within ZU_i .

Lemma 29 ZU_i^+ and ZU_i^- are $(n-1)$ -dimensional smooth embedded submanifolds in $\mathbb{R}^n$.

The sets ZU_i^0 and the higher order intersections in $\bigcup_{j \neq i} (ZL_j \cup ZU_j)$ are more difficult to analyze in the sense of local behaviors. However since these sets arise by imposing additional equality conditions within ZU_i , generically we can assume that these are lower dimensional objects as we state next. The assumption being a technical assumption in the process of developing transversality is denoted (T0.1).

Assumption (T0.1) The set ZU_i^0 is an $(n-2)$ -dimensional submanifold of $\mathbb{R}^n$. The set $\bigcup_{j \neq i} (ZL_j \cup ZU_j)$ can be written as a locally finite union of mutually exclusive smooth manifolds each of which is of maximal dimension $n-2$.

Since the sets are of maximal dimension $n-2$, it is not necessary to delve into a detailed analysis of these sets as we will see in the next section.

3.(b) Almost smooth manifolds

First we can make the define precisely what we mean by almost smooth manifolds.

Definition 11 A k -dimensional topological manifold L is said to a k -dimensional almost smooth manifold, if there exists a dense subset $M \subset L$ with $\bar{L} = M$ such that M is a k -dimensional smooth manifold in $\mathbb{R}^n$.

We claim here that the stable and unstable manifolds of hyperbolic equilibrium points and weakly hyperbolic periodic orbits which we established to be topological manifolds in Sections 1 and 2, are in fact, almost smooth manifolds generically.

Proposition 2 Suppose x_0 is a hyperbolic equilibrium. Then after imposing a countable number of transversal intersection requirements and by throwing out a countable number of strictly lower dimensional objects, it follows that the stable manifold $W^s(x_0)$ is an n_s -dimensional almost smooth manifold, and the unstable manifold $W^u(x_0)$ is an n_u -dimensional almost smooth manifold.

Outline of the proof: The proof is by construction. Without loss of generality, consider $W^u(x_0)$ and start with the local result that $W^u(x_0)$ is locally a n_u -dimensional smooth manifold from Lemma 23. Denote this local set $W_{loc}^u(x_0)$. Moreover the stable manifold $W^u(x_0)$ can be globally recovered by continuing this local manifold $W_{loc}^u(x_0)$ along the flow $\Phi_t^{a_i}$. We claim that $W^u(x_0) = W_{sm}^u(x_0) \cup J$ where $W_{sm}^u(x_0)$ is a n_u -dimensional smooth submanifold of $\mathbb{R}^n$ and J is a "junk set" of maximal dimension $n_u - 1$. The proof is by starting from $W_{loc}^u(x_0)$, continuing along the flow $\Phi_t^{a_i}$, and throwing out certain junk sets of maximal dimension $n_u - 1$ every time there arises an intersection with a transition surface in $ZL \cup ZU$. Basically the intersections with transition surfaces along the continuation are assumed to be transversal. Suppose we consider an intersection with a set say ZU_i . The smooth manifold is constructed by continuing along the set $ZU_i^+ \cap W^u$. The transversality implies that the intersection is of dimension $(n_u - 1)$, therefore continuation along the flow results in a smooth manifold n_u up to the next intersection. Along with this smooth manifold, two types of junk sets emerge. 1) intersections with the degenerate subset ZU_i^0 : by Assumptions (T0.1) and by assuming transversality, these intersections are $n_u - 2$ -dimensional, therefore flow extensions result in sets of dimension $n_u - 1$. 2) intersections with multiple transition sets in $\bigcup_{j \neq i} (ZL_j \cup ZU_j)$. Again by imposing a notion of transversality, we can guarantee that these sets are of maximal dimension $n_s - 2$ which under the action of the flow would still result in sets of maximal dimension $n_s - 1$. The process continues by considering only the smooth manifold part at each step. Note that there are countably many components of the transition surfaces ZU_i^+ and ZL_i^- . Since at each step, we continue only along the intersections with these sets (after throwing out the rest), there are countably many steps in the construction, hence the smooth counterpart $W_{sm}^u(x_0)$ can be constructed by leaving out a countable number of lower dimensional subsets in $W^s(x_0)$ and by imposing countably many transversal intersection requirements along the way. $\square$

The result for the periodic orbits is similar.

Proposition 3 Suppose γ is a weakly hyperbolic periodic orbit. Then generically the stable manifold $W^s(\gamma)$ is an n_s -dimensional almost smooth manifold, and $W^u(\gamma)$ is an n_u -dimensional almost smooth manifold. $\square$

3.(c) Definition of transversality

Since the stable manifolds and unstable manifolds in the analysis are almost smooth manifolds in the analysis, we can indeed introduce a notion of transversality of intersections by requiring this geometric property along the smooth counterparts.

Definition 12 Suppose x_0 and x_1 are two hyperbolic equilibrium points and $W^s(x_0) \cap W^u(x_1) \neq \emptyset$. Then we define this intersection to be transversal under the following three conditions: 1) The manifolds $W^s(x_0)$ and $W^u(x_1)$ are almost smooth manifolds with dense smooth manifold subsets $W_{sm}^s(x_0)$ and $W_{sm}^u(x_1)$. 2) The smooth counterparts intersect transversally, i.e. $W_{sm}^s(x_0)$ intersects $W_{sm}^u(x_1)$ transversally, and 3) Let C_0 and C_1 denote the respective components to which x_0 and x_1 belong. Then the connected components of $W^s(x_0) \cap W^u(x_1) \cap C_0$ and $W_{sm}^s(x_0) \cap W_{sm}^u(x_1) \cap C_0$ which include x_0 coincide. $\square$

Remarks. The justification for the nondegeneracy of the first requirement was given in the previous section in the form of Proposition 2. The second condition merely imposes a verifiable condition by restricting the geometric condition to the smooth subsets. The third condition is more technical and is crucial for stability boundary analysis. This implies that the intersections only occur along the smooth subsets in the component of interest. Note that since the smooth manifolds are dense in the respective topological manifolds, we claim that the intersection can be required (generically) to be only along these within one component. The proof of genericity for this condition would be highly technical since it demands a careful collection of the “junk sets” from the proof of Proposition 2 and is beyond the scope of this text.

The definition for transversality when the intersection involves periodic orbits is a direct modification of the definition above for equilibria. Conditions 1 and 2 remain identical while condition 3 needs to be restated as shown below since the periodic orbit may itself lie in multiple components (when the periodic orbit itself is nonsmooth). Without loss of generality, we state the definition when the intersections involve two periodic orbits.

Definition 13 Suppose γ_0 and γ_1 are two weakly hyperbolic periodic orbits and $W^s(\gamma_0) \cap W^u(\gamma_1) \neq \emptyset$. Then we define this intersection to be transversal under the following three conditions: 1) The manifolds $W^s(\gamma_0)$ and $W^u(\gamma_1)$ are almost smooth manifolds with dense smooth manifold subsets $W_{sm}^s(\gamma_0)$ and $W_{sm}^u(\gamma_1)$. 2) The smooth counterparts intersect transversally, i.e. $W_{sm}^s(\gamma_0)$ intersects $W_{sm}^u(\gamma_1)$ transversally, and 3) Let C_0 (C_1) denote the component to which the point $x_0 \in \gamma_0$ ($x_1 \in \gamma_1$) in the definition of weak hyperbolicity of γ_0 (γ_1) (for the existence of the smooth diffeomorphism ϕ , see Definition 10) belongs. Then the connected components of $W^s(\gamma_0) \cap W^u(\gamma_1) \cap C_0$ and $W_{sm}^s(\gamma_0) \cap W_{sm}^u(\gamma_1) \cap C_0$ that include $x_0 \in \gamma_0$ are equal. $\square$

D. Stability boundary analysis

In the previous two sections, the stable and unstable manifolds for periodic orbits have been constructed. In this section,

we are in a position to explore the structure of the stability boundary of the region of attraction of a stable operating point, and show that topologically the structure is identical to the case without the actuation limits. For the analysis, three sets of assumptions form the foundation. Essentially the analysis is motivated towards developing Lyapunov function type computational tools for the dynamic security assessment problem (e.g. [84, 114, 18, 43, 86, 62]). Here we want to establish the principal structure of the stability boundary so that this knowledge can be combined with Lyapunov theory results for developing practical and rigorous analytical tools.

It is well-known that for smooth systems, in general, (i.e. generically), this problem seems to be unsolvable (see the famous failed conjecture by Smale [75] on the genericity of Morse-Smale systems). That is, there exist generic dynamical systems where the stability boundary composition can be extremely complex (see the notion of Smale horseshoes [37]), hence indicating analytical intractability.

By imposing nongeneric but structurally stable assumptions (which are persistent under small perturbations), it is indeed possible to characterize the stability boundary as the union of stable manifolds of the limit points on the boundary, as was first demonstrated by Smale in his classical paper in 1967 [74]. In later mathematical works, a complete stratification of the stability boundary (as an “ordering” in [57], Section II.3 in [10], see also the notion of Morse indices) was established by Smale and Palis [57] under certain assumptions. Dynamical systems which satisfy these assumptions are known as Morse-Smale systems. We will introduce a modified version (to incorporate the effects of nonsmoothness) of these assumptions for our problem in Section D.1.

In other words, for a given problem, one of the main tasks in the boundary analysis is the identification of the required set of assumptions which guarantee that 1) a characterization is feasible and that 2) the structure remains persistent under perturbations. For a general smooth nonlinear system, whether it is possible to find such conditions in a generic sense, is an open question at the moment [57]. However possibly equipped with nongeneric but practical assumptions, we can possibly use extensions of geometric tools such as λ -Lemmas [57] of Morse-Smale theory together with topological arguments that were developed in [114, 86] for solving our problem.

One powerful topological operation is the novel concept of quasi-stability boundary that was introduced in [114] for removing the practically uninteresting interior segments in the stability boundary analysis. In the absence of periodic orbits, it has been proved in [114] that the quasi-stability boundary admits a much simpler structure since it is composed of the stable manifolds of only order one unstable equilibria (which have only one eigenvalue with positive real part) as compared to the full boundary which includes stable manifolds of unstable equilibria of any order. Hence this is a fundamental concept both mathematically and for practical implementations. In the previous sections, we have indulged in developing some of these conditions, with the notions of hyperbolicity of equilibria (and the concept of weak hyperbolicity for periodic orbits), and the definitions of transversal intersections. In this section, we will carry out the actual boundary analysis. Specifically the quasi-stability boundary analysis now also includes periodic orbits as shown for the DAE case in [86].

In the power system context, recent works on angle stability models include [84, 114, 18, 113]. For voltage stability models which include singularity in the state space dynamics, the results on the boundary characterization can be seen in [86, 91, 97, 62].

When the dynamics includes singular points, the analysis is more complicated, and standard Morse-Smale like geometric arguments are not directly applicable. Using topological arguments proposed in [86] together with special analytical tools such as pseudo equilibria and semi-singular surfaces, a stability boundary characterization was established in [86] (see Section I.2). Specifically standard Morse-Smale like induction arguments (ordering principle) are not applicable in this case and special topological tools are necessary [86, 91].

However for our problem, as mentioned in Section III.1.2, *we have assumed for simplicity that the actuation limited model Σ_{al} results from the hard-constrained DAE model Σ_{full} by staying away from the singular points.* The only technicality in Σ_{al} is the nonsmoothness introduced by the actuation variables z_i . Therefore with Assumptions (A0.1) and (A0.2) and with the "topological" definition of transversality as introduced above, most of the tools and concepts which we use in this section are extensions of standard geometrical properties of smooth dynamical systems, and a nice introduction can be seen in [58]. These include the concepts of invariance, λ -Lemmas, fundamental domains etc., and the standard induction arguments for proving the results.

1. Assumptions

We will introduce four types of assumptions for the stability boundary characterization in this section. The first set of assumptions denoted (A0) were introduced in the previous sections as (A0.1) in Section III.2.1.A and (A0.2) in Section III.2.1.C respectively. These two assumptions guarantee a nice structure of the nonsmooth transition manifolds so that component-wise smooth systems within individual components C_i fit together nicely in a global sense. This will allow us to "gather" component-wise analysis using standard results into the global results for the nonsmooth systems Σ_{al} . In addition, we also impose Assumption (A0.3) stated in Section III.2.1.B on the periodic orbits for technical simplicity. Note that Assumption (A0) is specifically connected with the transition manifolds $ZL \cup ZU$ introduced by the presence of the actuation limits. On the other hand, Assumptions (A1)–(A3) are direct extensions of the respective assumptions in Morse-Smale theory for smooth systems.

Assumption (A3) which is a crucial assumption in global analysis requires that the nonwandering set for all the trajectories consists only of a *finite* number of equilibria and a *finite* number of periodic orbits. This assumption virtually rules out the possibility of complex limit sets on the stability boundary (mentioned above in the introduction of Section D) by imposing the finiteness assumption. However this in itself does not guarantee a stable persistent structure since there may emerge either local bifurcations (connected with nonhyperbolic equilibria) or global bifurcations (from nonhyperbolicity of periodic orbits and from nontransversal intersections of stable and unstable manifolds). Such bifurcations if connected with the limit sets on the stability boundary again would disrupt the stability boundary structure upon small perturbations. Assumption (A1) requires that all equilibria and periodic orbits are hyperbolic. So any local property connected with the limit sets on the boundary is

preserved. (For our problem, we need to replace the requirement on the hyperbolicity of periodic orbits with the notion of weak hyperbolicity to accommodate possibly the nondifferentiable nature of the local flow near the nonsmooth periodic orbits.) Assumption (A2) enforces a transversality requirement that the intersections of all stable and unstable manifolds are hyperbolic, which is essential for preserving global properties. Together it follows that with Assumptions (A0)–(A3), the stability boundary of the region of attraction can be nicely characterized as the union of the stable manifolds of the unstable equilibria and unstable periodic orbits on the boundary.

First we will precisely state below Assumptions (A1)–(A3). Assumption (A0) has been introduced before.

Assumptions

(A0): Assumptions (A0.1), (A0.2) and (A0.3).

(A1): All equilibria are hyperbolic and all periodic orbits are weakly hyperbolic.

(A2): All stable and unstable manifolds of equilibria and periodic orbits intersect transversally (as defined in Section III.2.1.C).

(A3): All the trajectories remain bounded in forward time and the nonwandering set of Σ_{al} only consists of a finite number of equilibria and a finite number of periodic orbits.

Remarks: It can be shown that Assumptions (A0)–(A2) are generic but Assumption (A3) on the ω -limit set is nongeneric. However this assumption which rules out complex system behavior appears to be typically satisfied in power system models. Moreover as in Lyapunov standard theory, this requirement on the limit set can be restated as a condition on the existence of a suitable Lyapunov function [86] (see Section I.2.2 of this paper). Also Assumption (A3) can be weakened in some cases to the requirement that all trajectories converge to equilibria and periodic orbits within the region $\bar{A}$ where A denotes the region of attraction (see Section I.2.2). That is, the requirement on the nonwandering set in (A3) can be relaxed to an ω -limit property and the finiteness of the number of connected limit sets can also be removed. These are technical issues and will not be discussed here. Moreover for stability boundary analysis, it is sufficient to impose these conditions only within $\bar{A}$.

These assumptions (A0)–(A3) will be in effect throughout this section unless stated otherwise.

2. Basic structure of the stability boundary

The principal properties of the equilibria, the periodic orbits, their stable and unstable manifolds on the stability boundary, are treated in this section. The proofs are similar to the proofs in [57, 114, 18] for smooth systems and those in [86] for the DAE models. This Theorem does not require all the Assumptions (A0)–(A3). Only Assumption (A2) needs to be imposed for some of the statements as stated below. Also it should be pointed out that the three conditions in the definition of transversality in Section III.2.1.C precisely guarantee that the respective proofs for the DAE case in [91] nicely carry over to the nonsmooth model Σ_{al} .

Theorem 12 *Suppose the equilibria x_0 and x are hyperbolic, and the periodic orbits γ_0 and γ are weakly hyperbolic. Then the following equivalences hold true. For the statements marked with (*), also assume Assumption (A2) on transversal intersections.*

1. For an equilibrium point x_0

$$\begin{aligned}
x \in \partial W^s(x_0) &\iff (W^u(x) \setminus \{x\}) \cap \overline{W^s(x_0)} \neq \emptyset \\
(*) \ x \in \partial W^s(x_0) &\iff (W^s(x) \setminus \{x\}) \cap \partial W^s(x_0) \neq \emptyset \\
\gamma \in \partial W^s(x_0) &\iff (W^u(\gamma) \setminus \gamma) \cap \overline{W^s(x_0)} \neq \emptyset \\
(*) \ \gamma \in \partial W^s(x_0) &\iff (W^s(\gamma) \setminus \gamma) \cap \partial W^s(x_0) \neq \emptyset.
\end{aligned}$$

[If x_0 is stable, no transversality assumptions are required.]

2. For a periodic orbit γ_0

$$\begin{aligned}
x \in \partial W^s(\gamma_0) &\iff (W^u(x) \setminus \{x\}) \cap \overline{W^s(\gamma_0)} \neq \emptyset \\
(*) \ x \in \partial W^s(\gamma_0) &\iff |(W^s(x) \setminus \{x\}) \cap \partial W^s(\gamma_0)| \neq 0 \\
\gamma \in \partial W^s(\gamma_0) &\iff (W^u(\gamma) \setminus \gamma) \cap \overline{W^s(\gamma_0)} \neq \emptyset \\
(*) \ \gamma \in \partial W^s(\gamma_0) &\iff (W^s(\gamma) \setminus \gamma) \cap \partial W^s(\gamma_0) \neq \emptyset.
\end{aligned}$$

□

These technical results are the building blocks for establishing the composition of the stability boundary ∂A in the next section. Note that these results are not dependent on Assumption (A3). For convenience these results can be restated as a Corollary specifically for the boundary of the region of attraction ∂A . Basically we provide certain conditions when an unstable equilibrium x and an unstable periodic orbit γ can belong to the boundary ∂A . These are direct extensions of the results for the smooth models in [57, 114, 18].

Corollary 3 Suppose x is a hyperbolic equilibrium and γ is a weakly hyperbolic periodic orbit.

$$x \in \partial A \iff (W^u(x) \setminus \{x\}) \cap \overline{A} \neq \emptyset \quad (205)$$

$$x \in \partial A \iff (W^s(x) \setminus \{x\}) \cap \partial A \neq \emptyset \quad (206)$$

$$\gamma \in \partial A \iff (W^u(\gamma) \setminus \gamma) \cap \overline{A} \neq \emptyset \quad (207)$$

$$\gamma \in \partial A \iff (W^s(\gamma) \setminus \gamma) \cap \partial A \neq \emptyset. \quad (208)$$

□

From here onwards we define the set of all ω -limit points on the stability boundary ∂A as the set of anchor points denoted A . Specifically the unstable equilibrium points and unstable periodic orbits on the stability boundary ∂A will be called *anchors*. In the next section, we introduce all the Assumptions (A0)–(A3) for strengthening the results in Corollary 3 into a set of necessary and sufficient conditions for when an unstable equilibrium or an unstable periodic orbits is an anchor point on ∂A .

3. Analysis of the stability boundary ∂A

Note that the region of attraction by Corollary 24 is an open set in $\mathbb{R}^n$. Hence the dimension of its boundary ∂A is well-understood. Again recall that since we have defined the quasi-stability boundary as the boundary of the closure $\overline{A}$, the boundary $\partial \overline{A}$ does not contain any interior segments of possibly lower dimensions.

Lemma 30 ∂A is of maximal dimension $n - 1$. The quasi-stability boundary $\partial \overline{A}$ is of dimension $n - 1$. □

For the results that follow, all the four assumptions (A0), (A1), (A2) and (A3) are in effect. First we make an observation on the composition of the stability boundary ∂A which is a direct

consequence of Assumption (A3). Recall that the boundary ∂A is invariant by definition, hence only consists of trajectories. The nonwandering set of these trajectories by Assumption (A3) only consists of the anchors in ∂A . Moreover since the ω -limit set of these trajectories is a subset of the nonwandering set, we have proved the next result.

Lemma 31

$$\partial A \subset \bigcup_{x \in \partial A} W^s(x) \bigcup_{\gamma \in \partial A} W^s(\gamma) \quad (209)$$

□

We now state the Theorem which provides the complimentary result that the stable manifolds W^s of all the anchors belong to the boundary ∂A . Together Lemma 209 and Theorem 13 characterize the boundary ∂A , which is formally stated as a later Theorem. Note that in the statement of the next Theorem, the equivalences (211) and (213) stand as necessary conditions to verify when an unstable equilibrium or an unstable periodic orbit qualifies as an anchor. The other set of conditions (210) and (212) tell us that indeed the *whole* stable manifolds of these anchors lie in the boundary, so the composition can be nicely stated. As before, we formally define the dimension of the unstable manifold of the anchor as the *order* of the anchor and is denoted by a superscript (the notion, *type number*, is used in [18]). For instance, an unstable equilibrium x which possesses a one dimensional unstable manifold (i.e. the Jacobian evaluated at x contains exactly one eigenvalue with positive real part) is defined an order one equilibrium and is denoted x^1 . Order one equilibria and order two periodic orbits are significant in that their stable manifolds are of dimension $n - 1$, hence these are the only interesting anchors in the quasi-stability boundary $\partial \overline{A}$. These results on $\partial \overline{A}$ are presented later in Section D.4.

Theorem 13

$$x \in \partial A \iff W^s(x) \subset \partial A \quad (210)$$

$$x \in \partial A \iff (W^u(x) \setminus \{x\}) \cap A \neq \emptyset \quad (211)$$

$$\gamma \in \partial A \iff W^s(\gamma) \subset \partial A \quad (212)$$

$$\gamma \in \partial A \iff (W^u(\gamma) \setminus \gamma) \cap A \neq \emptyset \quad (213)$$

Proof: The proof is essentially an extension of the Morse-Smale ordering principle [74, 57]. First we can prove the necessary conditions (211) and (213). The proof starts from Theorem 12 and proceeds by an induction argument. The starting point for the induction is provided by the following Lemma on order one equilibria. The proof directly follows by Assumption (A2).

Lemma 32

$$x^1 \in \partial A \iff (W^u(x^1) \setminus \{x^1\}) \cap A \neq \emptyset \quad (214)$$

□

Note that order one periodic orbits by definition are locally attracting, hence do not belong to ∂A . For higher order equilibria and for periodic orbits, we can proceed by induction from Lemma 31 [57, 114, 18]. The induction is implied by the following Lemma which is an extension by the definition of transversality in Section III.2.1.3(c) of this paper from the ordering principle in [74].

Lemma 33 Assume (A1) and (A2). Suppose α and β are two anchors in $\bar{A}$. Suppose $W^u(\alpha) \cap W^s(\beta) \neq \emptyset$. Then the order of α is strictly less than the order of β if β is an equilibrium. When β is a periodic orbit, the order of α is less than or equal to that of β . $\square$

The induction argument is straightforward when equilibrium points are involved. However for periodic orbits, there exists the possibility that homoclinic and heteroclinic orbits can emerge on the boundary ∂A which can in fact result from transversal intersection of the stable and unstable manifolds of periodic orbits [74, 57] (hence not violating Assumption (A2)). However fortunately then the nonwandering set includes these complex orbits, hence Assumption (A3) would be disrupted. In other words, the existence of transversal heteroclinic cycles are precisely ruled out by Assumption (A3) as stated in the next Lemma. Note that this Lemma hinges on the requirement in terms of the *nonwandering set* in Assumption (A3) and on the finiteness of the number of anchors in ∂A .

Lemma 34 There does not exist a sequence of anchors $\alpha_k, k = 1, \dots, n_\alpha$ such that $W^u(\alpha_k) \cap W^s(\alpha_{k+1}) \neq \emptyset$ for $k = 1, 2, \dots, n_\alpha - 1$, and $W^u(\alpha_{n_\alpha}) \cap W^s(\alpha_1) \neq \emptyset$. $\square$

Using a technical argument based on λ -Lemmas, the induction can now be completed and this is not shown here because of space limitations. The proof of the equivalences (210) and (212) directly follow from the necessary conditions by another standard λ -Lemma type argument, exploiting the invariance of the various sets in the proof. This completes the proof of the Theorem. Q.E.D.

Remarks. A more straightforward proof of the statements (210) and (212) which are independent of (211) and (213) can be seen in [91]. Note that when the dynamics includes singular points such as for the DAE model in [86], there arise anchor points on the boundary ∂A which form high dimensional surfaces (semi-singular surface and pseudo equilibrium surfaces [86]). Therefore in that case, Assumption (A3) as stated here (the finiteness) is not applicable and Morse-Smale like induction arguments are not suitable for the boundary characterization (because of uncountably many limit sets). However interestingly by a direct modification of these Assumptions, it can be shown that the main results, namely (210) and (212), are still valid. However the necessary conditions turn out to be weaker compared to (211) and (213).

We end this section with the main result on ∂A which confirms that the boundary ∂A is "well-behaved" under Assumptions (A0)-(A3).

Theorem 14

$$\partial A = \bigcup_{x \in \partial A} W^s(x) \bigcup_{\gamma \subset \partial A} W^s(\gamma) \quad (215)$$

$\square$

Remarks. This Theorem guarantees that theoretically we can construct the boundary ∂A "completely" by first locating all the anchors, and "tracing out" their stable manifolds. In practical implementations, Lyapunov functions have been used in the past for approximating the stable manifolds by constant

level surfaces [64, 36, 112, 20, 43, 62] after locating the relevant anchor points (for instance, see Controlling equilibrium point methods [56, 36, 17]). The analysis in this section proves that these methods can be "easily" extended to include the presence of the actuation limits by constructing suitable Lyapunov functions. Note that since the model Σ_{al} satisfies local Lipschitz condition everywhere, standard Lyapunov theory is readily applicable. The actual construction of such Lyapunov functions which accommodate nondifferentiable nature of the actuation limits needs to be carried out. Research is indicated in this direction.

4. Analysis of the quasi-stability boundary $\partial \bar{A}$

By Theorem 14, for analyzing the stability boundary ∂A , technically we need to consider all the anchor points along their stable manifolds. However in reality, there may exist interior segments of this boundary [113] which complicate the analysis. The concept of quasi-stability boundary $\partial \bar{A}$ in [114] obviates this difficulty by providing a much simpler boundary structure which is more significant practically. Specifically note that by Lemma 30, the boundary $\partial \bar{A}$ is of dimension $n - 1$. We prove next that it is sufficient to consider only order one equilibrium points and order two periodic orbits for completely understanding $\partial \bar{A}$. These anchor points, namely order one equilibrium points and order two periodic orbits are defined the *principal anchors*.

First we state the necessary conditions for identifying principal anchors in the next Theorem. The proof is similar to the proofs in [114] (and [86]) based on another induction argument.

Theorem 15

$$x^1 \in \partial \bar{A} \iff W^u(x^1) \cap A \neq \emptyset \text{ and } W^u(x^1) \cap \bar{A}^c \neq \emptyset \quad (216)$$

$$x^1 \in \partial \bar{A} \iff W^s(x^1) \subset \partial \bar{A} \quad (217)$$

$$\gamma^2 \in \partial \bar{A} \iff W^u(\gamma^2) \cap A \neq \emptyset \text{ and } W^u(\gamma^2) \cap \bar{A}^c \neq \emptyset \quad (218)$$

$$\gamma^2 \in \partial \bar{A} \iff W^s(\gamma^2) \subset \partial \bar{A} \quad (219)$$

$\square$

The composition of $\partial \bar{A}$ can be stated in terms of the principal anchors as stated below.

Theorem 16

$$\partial \bar{A} = \bigcup_{x^1 \in \partial \bar{A}} \overline{W^s(x^1)} \bigcup_{\gamma^2 \subset \partial \bar{A}} \overline{W^s(\gamma^2)} \quad (220)$$

$\square$

Remark: Note that it is sufficient to consider only the principal anchors for the quasi-stability boundary analysis, that is, only the order one equilibria and order two periodic orbits. Since the quasi-stability boundary $\partial \bar{A}$ is the only practically significant subset of the full boundary ∂A , indeed in practical implementations, we can restrict our attention to order one equilibria and order two periodic orbits. Further research is indicated on developing Lyapunov function methods which permit the presence of periodic orbits in the analysis. The order two periodic orbits on the boundary $\partial \bar{A}$ would be highly significant for instance, when the operation is close (in the sense of operating parameter values) to the subcritical Hopf bifurcation boundaries [37, 1, 63, 100].

We have proved that the stability boundary results remain unchanged *qualitatively* when the actuation limits are included in the analysis even though the main constituents, namely the stable manifolds, are not smooth manifolds but topological manifolds. However it should be remembered that the unstable equilibria and unstable periodic orbits on the boundary ∂A , namely the anchors, (and principal anchors in $\partial \bar{A}$) which appear in the boundary analysis would be quite different (in physical position) when the anchors occur *after* one or more actuation limits have been encountered. With increasing industrial reports of the inter-plays between hardlimits and dynamics during system transients, the theory provided in this section leads to the feasibility of an extension (possibly even a direct extension) of the Lyapunov function methods to the actuation limited model Σ_{al} .

III.2.2 Nonwindup Limits or state limits

In this subsection we show that starting with a smooth vector field f defined in an open neighborhood of the constrained state space (with the hard limits) under generic assumptions there exists a unique dynamics $\bar{f}$ which is consistent with the paradigm imposed by the structure of nonwindup models as defined by IEEE. This dynamics generates a positive semi-flow, i.e. trajectories are unique forward in time. As will be seen, uniqueness backward in time has to be abandoned when transitions onto the surfaces defined by the hardlimits are involved. We will show that generically the dynamics is piecewise smooth in the sense that there exists a decomposition of the hardlimited state space into smooth embedded submanifolds, called here strata for convenience, with the property that the restriction of the dynamics $\bar{f}$ to each of these submanifolds is smooth. The overall dynamics involves transitions between these strata and these transitions are typically not smooth (very much like for a regular synthesis in optimal control theory). Nevertheless, the local structures of the phase portraits can be established and basic concepts like stability of equilibria, local stable and unstable manifolds can be defined. These definitions then allow to pursue the structure of regions of stability for stable operating points (see [86, 91]) as it was done in the previous section for windup limits.

A: Construction of the Dynamics

Consider a general differential-algebraic model of the form

$$\dot{x} = f(x, y) \quad (221)$$

$$0 = g(x, y) \quad (222)$$

suppressing the parameters in the notation. We first assume that the hard limits are given by explicit bounds on some of the dynamic state variables x . We will restrict the model to a compact simply connected subset say $\mathcal{R}$ of a component C where the Jacobian matrix $\frac{\partial g}{\partial y}$ is nonsingular. Then, by a global extension of the implicit function Theorem, we can eliminate the algebraic variable y from the equations globally over $\mathcal{R}$ [43, 62] and obtain a reduced system

$$\dot{x}_{red} = f_{red}(x) \quad (223)$$

on $\mathcal{R}$ with hard limits on some of the state variables. As noted in Section III.1.2, in order to keep the presentation as simple as possible, we drop the restriction to the subset $\mathcal{R}$ and relabel the reduced system simply as f . Then our state space takes the form of a set of differential equations

$$\Sigma_{sl}: \dot{x} = f(x), \quad x \in \mathbb{R}^n \quad (224)$$

with the hard-constraints

$$\bar{X} = \{x \in \mathbb{R}^n : x_{\ell_i} \leq x_i \leq x_{u_i} \text{ for } i = 1, \dots, n\} \quad (225)$$

(where we allow $x_{\ell_i} = -\infty$ and $x_{u_i} = \infty$) and we assume that f is smooth in a neighborhood of $\bar{X}$. We call f the *dynamics generating model* and our first aim is to modify f so that the overall system becomes consistent with the hard limits. The construction is based on the following guiding principles (for the precise construction, see Theorem 17 below):

- whenever possible retain the dynamics f ;
- if some components of the state variable x are at hard limits, then annihilate the dynamics of those components (by setting it equal to zero) which would lead to a violation of some of the hard limits;
- keep the number of components which will be annihilated to a minimum.

Let I denote a subset of $\{1, \dots, n\}$ and let J denote the complementary subset. Denote by f^I the vector field obtained from f by setting the components from the index set I equal to 0. For $I = \emptyset$ we do not write a superscript and this reduces to the dynamics generating model. Recall that a curve $\gamma : (-\epsilon, \epsilon) \rightarrow \mathbb{R}^n$ has order of contact k with a coordinate hyper-surface $N = \{x_j = \text{const}\}$ at a point p if $\gamma(0) = p \in N$ and the first $k-1$ derivatives of the x_j coordinate of γ vanish at p while the k^{th} derivative is nonzero. We make the following generic assumption:

(SL 0) All integral curves of the vector field f^I have finite order of contact with all the hard limit surfaces corresponding to complementary indices J .

Theorem 17 Under assumption (SL0) in the sense of the guiding principles set forth above there exists a unique vector field $\bar{f}$ which is compatible with the hard limits in the sense of the guiding principles set forth above. This vector field is piecewise smooth in the sense that there exists a decomposition of $\bar{X}$ into connected embedded submanifolds with the property that the restriction of $\bar{f}$ to these submanifolds are smooth vector fields. For every point $p \in \bar{X}$ there exists a unique solution γ_p forward in time to the ODE $\dot{x} = \bar{f}(x)$ which is defined over a maximal interval $[0, T(p))$. We say that $\bar{f}$ generates a positive semi-flow. $\square$

The construction of $\bar{f}$ is by induction on the number of variables which are at the hard limits. We shall call this the *level* of limitation. Level 0 corresponds to the trivial situation that p lies in the interior of $\bar{X}$,

$$p \in X_0 = \{x \in \mathbb{R}^n : x_{\ell_i} < x_i < x_{u_i} \text{ for } i = 1, \dots, n\}$$

and no hard limits are active. In this case, and according to the basic principles, we define the dynamics $\bar{f}$ by f . We describe in detail the construction of $\bar{f}$ for levels 1 and 2 and then indicate the inductive argument. For this we need to establish our notation first. We will denote the limiting surfaces by the letter H and we will write active upper and lower limits as superscripts and subscripts respectively. Our convention will be that we write all active variables and those variables which do not appear are assumed inactive, i.e.

$$H^j = \{x \in \mathbb{R}^n : x_j = x_{u_j}, \text{ and } x_{\ell_i} < x_i < x_{u_i} \text{ for } i \neq j\}$$

$$H_{r,s}^j = \{x \in \mathbb{R}^n : x_j = x_{u_j}, x_r = x_{\ell_r}, x_s = x_{\ell_s},$$

$$\text{and } x_{\ell_i} < x_i < x_{u_i} \text{ for } i \neq j, r, s\}$$

etc. We now define $\bar{f}$ for level 1 points p and without loss of generality (w.l.o.g.) we consider an active upper limit H^j . In this case the j^{th} component of f is essential for determining the dynamics $\bar{f}$. For simplicity of notation let us label this component α . Then decompose H^j further into

$$H^{j(+)} = \{x \in H^j : \alpha > 0\} \quad (226)$$

$$H^{j(0)} = \{x \in H^j : \alpha = 0\} \quad (227)$$

$$H^{j(-)} = \{x \in H^j : \alpha < 0\} \quad (228)$$

Integral curves of f starting at points p in $H^{j(-)}$ immediately enter the interior X_0 of the admissible state space and do not violate the constraint. Hence we define the dynamics $\bar{f}$ on $H^{j(-)}$ as f . The corresponding local flow is transversal to $H^{j(-)}$ directed into X_0 . We call points in $H^{j(-)}$ *inactive hard limits*.

On the other hand, integral curves of f starting at points p in $H^{j(+)}$ leave the the admissible state space $\bar{X}$ and violate the constraint. Hence we need to terminate the vector field f upon reaching $H^{j(+)}$. The local flow of f is transversal to $H^{j(+)}$ and directed away from $\bar{X}$. Points in $H^{j(+)}$ are called *active hard limits*. Here we define the dynamics $\bar{f}$ on $H^{j(+)}$ by setting the j^{th} component of $\bar{f}$ equal to zero while retaining all other components from f , that is by f^j .

This definition of the dynamics away from the zero set of α is trivial and is the only possible one to be consistent with the IEEE model for nonwindup limits (see [128, 51]). The interesting situations arise when $\alpha = 0$, i.e. when the vector field f is tangent to the hard limit surface H^j . We call these the *critical hard limits*. In this case the higher order derivatives of α along the integral curves of the vector field f determine how to select the dynamics. Following established terminology in differential geometry we call these derivatives the Lie-derivatives of α along f and denote the k^{th} Lie-derivative of α along f at the point p by $(L_f^k \alpha)(p)$. Under our generic assumption for every point $p \in H^{j(0)}$ there exists a smallest integer k such that $(L_f^k \alpha)(p) \neq 0$. If $(L_f^k \alpha)(p) < 0$, then the variable x_j decreases along the (forward) integral curve of f through p and the hard limit is not violated. Hence we define the dynamics $\bar{f}$ for such a point p as f . On the other hand, if $(L_f^k \alpha)(p) > 0$, then the variable x_j increases along the (forward) integral curve of f through p and the hard limit is violated. In this case we must terminate the integral curve of f through p and we define the dynamics $\bar{f}$ at p by f^j . That this definition is indeed consistent with our principles follows from the following result:

Lemma 35 Let γ^j and γ denote the integral curves of the vector fields f^j and f through p respectively. Then

$$\frac{d^r}{dt^r} \Big|_{t=0} \gamma^j(t) = \frac{d^r}{dt^r} \Big|_{t=0} \gamma(t) = 0 \text{ for } r = 1, \dots, k-1,$$

$$\frac{d^k}{dt^k} \Big|_{t=0} \gamma^j(t) = \frac{d^k}{dt^k} \Big|_{t=0} \gamma(t) = (L_f^k \alpha)(p),$$

i.e. γ^j and γ have order of contact equal to k with H^j . $\square$

The interpretation of the lemma is that the integral curve of f^j indeed enters the open submanifold $H^{j(+)}$ where $\bar{f}$ is defined as f^j . The proof of the lemma is a direct calculation which will be given elsewhere. We want to point out, however, that this

fact follows since $H^{j(0)}$ is precisely the set where the two competing dynamics f and f^j are identical (and 0). This is where smooth transitions from the interior X_0 to the hard limit surface H^j occur and where vice versa trajectories can enter into X_0 from the hard limit surfaces.

This argument establishes the dynamics $\bar{f}$ on H^j . Before we proceed with the construction of $\bar{f}$ for level 2, we briefly pause to establish the local phase portrait of the dynamics $\bar{f}$ for the least degenerate case (see Figure III.5). These phase portraits are trivial near $H^{j(+)}$ and $H^{j(-)}$: By the flow box theorem trajectories transversally cross H^j with the sign determining the orientation. For $H^{j(0)}$ consider the least degenerate subsets

$$H^{j(0+)} = \{x \in H^{j(0)} : (L_f \alpha)(p) > 0\} \quad (229)$$

$$H^{j(00)} = \{x \in H^{j(0)} : (L_f \alpha)(p) = 0\} \quad (230)$$

$$H^{j(0-)} = \{x \in H^{j(0)} : (L_f \alpha)(p) < 0\} \quad (231)$$

First consider a point $p \in H^{j(0-)}$: since the second derivative is nonzero, locally $H^{j(0)}$ is an embedded submanifold of H^j near p with $H^{j(+)}$ to one and $H^{j(-)}$ to the other side. Trajectories of f approach $H^{j(+)}$ transversally from inside X_0 and leave $H^{j(-)}$ transversally into X_0 . At $H^{j(0)}$ trajectories of f have second order contact. It follows from the lemma above that trajectories of f^j starting at $H^{j(0)}$ enter $H^{j(-)}$. Hence the flow of $\bar{f}$ on $H^{j(+)}$ is towards $H^{j(0-)}$ and at $H^{j(0-)}$ the dynamics switches to f . Here a smooth transition from the hard limit surface to trajectories inside the hard limits occurs. The same geometry, but with reversed signs is valid if $p \in H^{j(0+)}$. Now the lemma above implies that trajectories of f^j starting at $H^{j(0)}$ enter $H^{j(+)}$ and hence the flow of $\bar{f}$ is from $H^{j(0+)}$ into $H^{j(+)}$. These are points where a smooth transition from within X_0 onto the hard limit surface occurs.

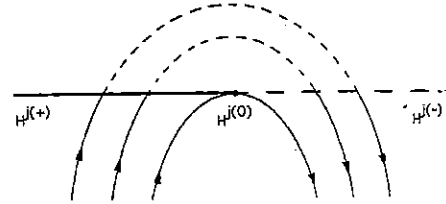


Figure III.5. Conceptual diagram: the dynamics at level 1 points for the nonwindup limit case (state limits).

We now return to the construction of the dynamics $\bar{f}$ for level 2. W.l.o.g. we consider a set $H^{i,j}$ of two variables which are at upper limits. Modifications for the other cases are straightforward and require only some changes in sign. In order to keep the notation slightly simpler we again rename the i^{th} and j^{th} component of the vector field f as α and β respectively. The dynamics $\bar{f}$ at a point $p \in H^{i,j}$ then depends on the 2-vector $(\alpha(p), \beta(p))$. A really new situation arises only if $(\alpha(p), \beta(p))$ is the zero vector. For, if $\alpha(p) > 0$, then the i^{th} component of the vector field f needs to be annihilated since otherwise the flow would violate the upper hard limit on x_i . Hence we are back to a level 1 situation, but for the reduced vector field f^i . Depending on the sign of β , the dynamics is then given by f^i if $\beta(p) < 0$ or $f^{i,j}$ if $\beta(p) > 0$. If $\beta(p) = 0$, the Lie-derivatives of β along the vector field f^i need to be considered. By our assumption not all of these derivatives are zero and analogous to the level 1 construction the dynamics $\bar{f}$ is then given by f^i if the first nonzero derivative is negative and by $f^{i,j}$ if this derivative is positive. Similarly, if $\alpha(p) < 0$, then the j^{th} component of the vector field f is tame because the flow is oriented into X_0 . Again we are

back to a level 1 situation, but now for the original dynamics generating vector field f . Thus, depending on the sign of β , the dynamics is given by f if $\beta(p) < 0$ or f^j if $\beta(p) > 0$. If $\beta(p) = 0$, the Lie-derivatives of β along the vector field f determine the dynamics $\bar{f}$ as f if the first nonzero derivative is negative and as f^j if this derivative is positive.

A true level 2 situation arises only if $(\alpha(p), \beta(p)) = (0, 0)$. In this case we need to look at the Lie derivatives of both α and β along f . By assumption there exist minimal integers k_i and k_j so that $(L_f^{k_i} \alpha)(p) \neq 0$ and $(L_f^{k_j} \beta)(p) \neq 0$. W.l.o.g. assume that $k_i \leq k_j$. If $(L_f^{k_i} \alpha)(p) < 0$, then the i^{th} component of the vector field f is tame and the Lie-derivative of β determines the dynamics as f if $(L_f^{k_j} \beta)(p) < 0$ and as f^j if $(L_f^{k_j} \beta)(p) > 0$. If however $(L_f^{k_i} \alpha)(p) > 0$, then the i^{th} component of the vector field f violates the constraint and must be annihilated. Now the higher Lie-derivatives of β along f become meaningless, but the Lie derivatives of β along f^j need to be considered. again there exists a smallest positive integer $\bar{k}_j$ so that $(L_{f^j}^{\bar{k}_j} \beta)(p) \neq 0$. An important observation is that

Lemma 36 $k_i \leq \bar{k}_j$ □

This lemma makes sure that the change in the i^{th} component of the vector field has no influence on the lower order terms. The proof is again a direct calculation which will be given elsewhere. Intuitively, f and f^j agree at p and corresponding integral curves have order of contact k_i with the i^{th} hard limit. Therefore changes in the Lie-derivatives only occur in higher derivatives. The dynamics $\bar{f}$ is then defined as f^i if $(L_f^{k_j} \beta)(p) < 0$ and as $f^{i,j}$ if $(L_f^{k_j} \beta)(p) > 0$. This concludes the definition of $\bar{f}$ for level 2.

The construction for level 2 presents the basic idea for the inductive step which we now only outline briefly. Assuming the dynamics $\bar{f}$ has been defined at all points of level $\leq k$, suppose $p \in H^{i_1, \dots, i_{k+1}}$ is a point of level $k+1$. If one of the components corresponding to one of these indices is nonzero, say $i_1 = i$, then depending on the sign we either keep the i^{th} component or replace it by 0. In any case, the problem of defining $\bar{f}$ has been reduced to a problem of order k which can be assumed solved by inductive hypothesis. Hence again the only new case arises when all these components vanish. In this case pick the component for which the lowest Lie-derivative of the component along f is nonzero and determine the corresponding component of $\bar{f}$ accordingly. Again then the problem becomes a problem of level k for either f or f^i . The details are easily written down precisely.

Remarks: 1. We carried out the construction for hard limits given by explicit bounds on the dynamic state variables x . More generally, smooth constraints in the form $\phi_r(x) \geq 0$, $r \in R$ can be allowed if the constraints are *regular* at every point p in the sense that the gradients $\nabla \phi_r$ for the at p active indices $r \in A$ are linearly independent. In this case, we can simply make a partial coordinate transformation $\tilde{x} = \phi_r(x)$ for the active indices and the situation is reduced to hard limits of the form $\tilde{x}_r \geq 0$ for $r \in A$. Of course, it is not necessary to actually perform this change of coordinates, but the constraints can be analyzed directly. In this case the relevant subsets of the constraint hypersurfaces are the submanifolds where the flow of f is transversal to $\phi(x) = 0$ (in or out) and where the flow is tangential. In the latter case the Lie-derivatives allow to determine the dynamics and the situation and notation are as described above. For our presentation we have chosen the notationally simpler framework.

2. The construction generates a decomposition of the state space $\bar{X}$ and the restrictions of the vector field $\bar{f}$ to one of these components is given by one of the f^I , I a subset of $\{1, \dots, n\}$. In particular, these restrictions have the same smoothness properties as the dynamics generating model f . Integral curves of the vector field $\bar{f}$ are unique when restricted to these components, but must be defined globally by piecing together the integral curves for different vector fields f^I . Nonsmoothness comes into play through the transitions between these surfaces. We obtain a unique piecewise defined integral curve for the vector field $\bar{f}$ forward in time, but it does not make sense to consider the backward history (see Figure III.6). At transitions onto the hard limit surfaces whole families of trajectories are channelled into the same trajectory on the constraint. On the hard limit surfaces we distinguish between components which support trajectories, which we call *flow strata*, and components which only serve as initial conditions for the forward flow of $\bar{f}$ which then immediately enters a flow stratum. We call these components *transient strata*. The forward flow here can be transversal to the stratum, like for $H^i(-)$, or tangential like for components of

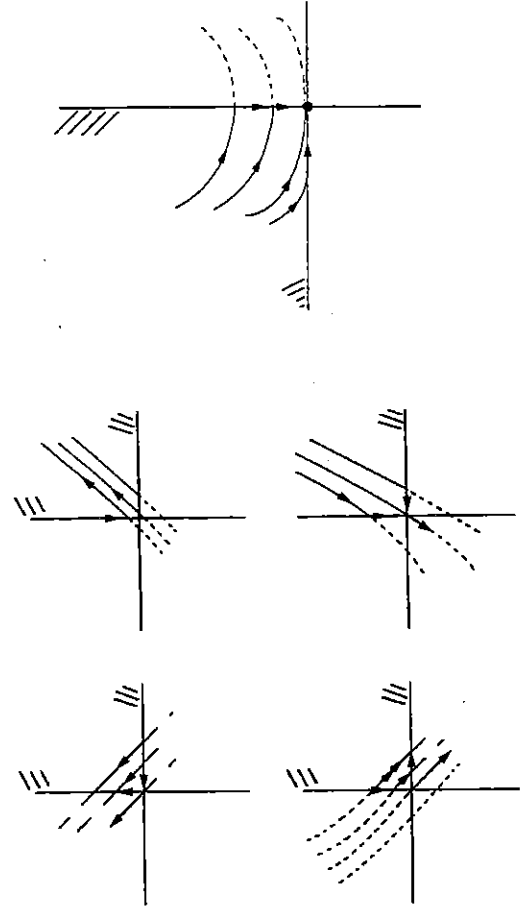


Figure III.6. Conceptual diagrams: the dynamics at level 2 points for the nonwindup limit case (state limits).

$H^i(0)$. The latter are the strata where smooth transitions between the hard limit surfaces and the interior of the state space happen (see Figure III.6). Note that it is not possible to have nonsmooth transitions from the hard limit surfaces into the interior of $\bar{X}$, while transitions from the interior of $\bar{X}$ to the hard limit surfaces are typically nonsmooth with smooth crossings the exception (only on sets of higher codimension). We call the corresponding components *entry* (into the hard limit surface) and *exit strata* (off the hard limit surface) respectively.

B: Equilibria and Local Stable and Unstable Manifolds

Equilibria which lie in the interior X_0 of the state space $\bar{X}$ will be called *ordinary* equilibria and all the local properties are the same as for the dynamical system given by f . In particular, stability and local stable and unstable manifolds are as defined in Chapter I. New phenomena arise due to equilibria on the hard limit surface. Without loss of generality we again consider only upper hard limits.

Lemma 37 *Under assumption (SL0) equilibria of the vector field f^I can only lie in the set*

$$H^{I(+)} = \{x \in H^I : f_i(x) > 0 \text{ for all } i \in I\}$$

of active hard limits.

PROOF: Observe first that the generic assumption (SL0) precludes that equilibria of the vector field f^I can lie on any of the hard limit hyper-surfaces corresponding to complementary indices J since the equilibrium trajectories would have contact of infinite order. Hence equilibria can only lie in H^I where no additional variables are at the hard limit. It is clear that no points p where at least one index $i \in I$ is inactive can be equilibria since the flow is oriented inward. If now I_c is the subset of critical hard limits at p , then let $I_a = I \setminus I_c$ be the set of active hard limits. Then p is also an equilibrium point for f^{I_a} and this is an equilibrium for which the variables corresponding to the subset I_c of complementary indices are at the hard limit. But again this is ruled out by assumption (SL0). Q. E. D.

So generically equilibria on the hard limits only occur at active hard limits. We call those equilibria for which exactly k limits are active *level k equilibria*. The definition of the *local unstable manifold* of a level k equilibrium p is straightforward. This simply is the local unstable manifold of p as equilibrium point of the dynamical system f^I which describes $\bar{f}$ on the component of $\bar{X}$ in which p lies. That is, if U is a neighborhood of p in $\mathbb{R}^n$, if C denotes this component, and if Φ^I denotes the corresponding flow, then

$$W_{loc}^u(p) = \{z \in U \cap C : \Phi_t^I(z) \in U \cap C \text{ for all } t \leq 0, \\ \Phi_t^I(z) \rightarrow p \text{ as } t \rightarrow -\infty\}$$

The definition of the local stable manifolds W_{loc}^s is more technical, though no less direct. Now the different flows have to be pieced together and strictly speaking, W_{loc}^s is no longer a smooth manifold, but still a topological manifold with boundary. However, it can be decomposed or stratified into finitely many connected embedded smooth submanifolds. Let p be a generic level k equilibrium and let U be a neighborhood of p in $\mathbb{R}^n$. Again denote by C the corresponding component and assume the restriction of $\bar{f}$ to C is given by f^I with flow Φ^I . Let S_0 denote the local stable manifold of p for f^I , i.e.

$$S_0 = W_{loc}^s(p) = \{z \in U \cap C : \Phi_t^I(z) \in U \cap C \text{ for all } t \geq 0, \\ \Phi_t^I(z) \rightarrow p \text{ as } t \rightarrow \infty\}.$$

For $j \in I$ let $I_j = I \setminus \{j\}$ and define S_j by integrating the vector field f^{I_j} backward from S_0 as long as it stays in U , i.e.

$$S_j = \{z = \Phi_t^{I_j}(q) \in U : q \in S_0, \Phi_s^{I_j}(q) \in U \text{ for all } t \leq s \leq 0\}.$$

Since the j^{th} component of f^{I_j} is positive (because p is a generic level k equilibrium), these trajectories will leave U within finite time. Then, for an ordered pair $(i, j) \in I^2$, let $I_{(i,j)} = I \setminus \{i, j\}$ and define $S_{(i,j)}$ by integrating the vector field $f^{I_{(i,j)}}$ backward from S_j as long as it stays in U , i.e.

$$S_{(i,j)} = \{z = \Phi_t^{I_{(i,j)}}(q) \in U : q \in S_j, \Phi_s^{I_{(i,j)}}(q) \in U \forall t \leq s \leq 0\}.$$

Since $f^i > 0$ again these trajectories leave U in finite time. Iterate this backward integration until one after the other all indices have been released. In the last step we integrate the vector field f backward into X_0 from $k!$ submanifolds. (Geometrically the situation is analogous to all the submanifolds meeting at a corner point of a k -dimensional cube.) We then define the *local stable manifold of a generic level k equilibrium point p of $\bar{f}$* , $\bar{W}_{loc}^s(p)$, as the union of all these submanifolds. Formally,

$$\bar{W}_{loc}^s(p) = \bigcup_{I \in \mathcal{I}} S_I \quad (232)$$

where $\mathcal{I}$ denotes the set of all ordered selections of r elements from the index set I for $r = 0, \dots, k$.

It is clear from the construction that the local stable manifold of a generic level k equilibrium point p is a stratified set. Topologically it's relative interior is a $k + \dim(W_{loc}^s(p))$ dimensional manifold, but the stratification properties are more relevant to the analysis of the dynamics $\bar{f}$. Global questions are considerably more difficult, even for ordinary equilibria, since they involve piecing together the flow Φ_t for $\bar{f}$ from the different flows Φ_t^I . Some examples of the complexities which can arise here can be seen in the figures given in this section and a discussion of these issues will be given elsewhere. For results about stability boundaries and in particular for bifurcation related phenomena, it are really only the local stable and unstable manifolds which are needed as long as global phenomena are excluded. The next section discusses the stability boundary of a stable ordinary equilibrium.

C: Outline of the Structure of the Stability Boundary

Here we only make some comments about the stability boundary of a stable ordinary equilibrium. This section reports on research which is very much in progress at the moment and therefore we restrict the presentation to an outline of what can be considered well substantiated conjectures. For this reason we assume that no periodic orbits are present. We do not expect significant problems with the incorporation of periodic orbits and their stable and unstable manifolds, but so far we have not looked into this question yet and therefore prefer to avoid this topic.

As was described in detail in section I.2 under Assumptions (A) in Section I.2.2, the stability boundary for a stable equilibrium for the differential algebraic system consists of stable manifolds of unstable equilibria (and unstable periodic orbits), stable manifolds of certain pseudo-equilibria and semi-singular points (which lie in the singular set) and other boundary sections from

the singular set. Here we also assume that the component stays away from the singularity so that out of these sections only the stable manifolds of unstable equilibria remain as obvious candidates. Also this simplifies the structure since it allows to focus on the phenomena introduced by the hard limits. Indeed, with a "correct" definition of (global) stable and unstable manifolds for equilibria of the system with hard limits, it will be true that under appropriate assumptions

$$\partial A = \bigcup_{x \in \partial A} W^s(x)$$

where the union is taken over all equilibria in the stability boundary. Essential hypotheses for results of this type are extensions of Morse-Smale like assumptions as before (see also [91, 86]):

- (A1) Equilibria x_0 in the boundary of the region of attraction, ∂A , are hyperbolic.
- (A2) Stable and unstable manifolds of equilibria intersect transversally. (For technical simplicity we also assume currently that intersections occur only along smooth sections like in the case of windup limits.)
- (A3) All trajectories on the boundary of the region of attraction converge to an equilibrium point.

In addition currently we also make some technical assumptions in order to simplify the analysis, like assuming that the dimension n is sufficiently large to allow for enough degrees of freedom to satisfy constraints generically. We also assume that not too many hard limits are active simultaneously.

Formally the statement of the structure of the stability boundary reads like for the unconstrained system [57, 114, 18], but now the system hard limits are incorporated into the definition of the stable and unstable manifolds and that is where the differences lie.

D: Examples and Illustrations

In this section we illustrate the structure of local stable and unstable manifolds, in particular of regions of attraction for stable operating points in some 2-dimensional examples. The illustrations are for the rudimentary system described in the appendix. The choices of parameters correspond to scenarios arising in situations resembling saddle-node, sub- and supercritical Hopf bifurcations. For these connections see Table III.2.

Figures III.7 to III.14 shown in this section provide an excellent overview of the state space and the parameter space phenomena for the hard-constrained DAE system. In this case, the examples are based on the rudimentary power system model studied previously by us in [123, 85, 92]. A comparison of the phase portraits given in this section with the phase portraits shown for the non-hard-limited DAE model in [92] would give valuable insight on how the windup and nonwindup hardlimits affect the state space stability regions and parameter space bifurcations. In this section, we provide a brief discussion of some of the salient features. Comparison with the conceptual illustrations in Tables III.1 (Section III.1) and Table III.2 (Section III.3) also would aid the reader in relating the concepts to the theory. Note that all these simulations are presented together

(from Figure III.7 to Figure III.14) for convenience and discussions of specific illustrations follow next.

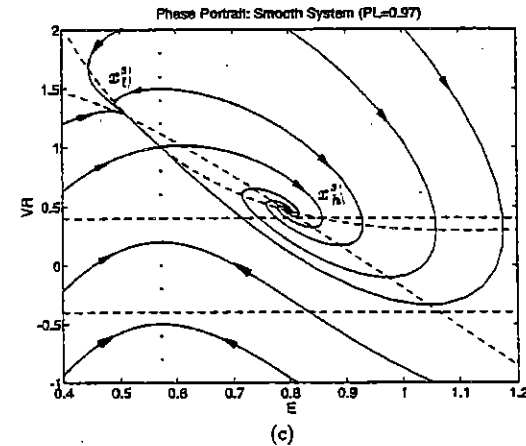
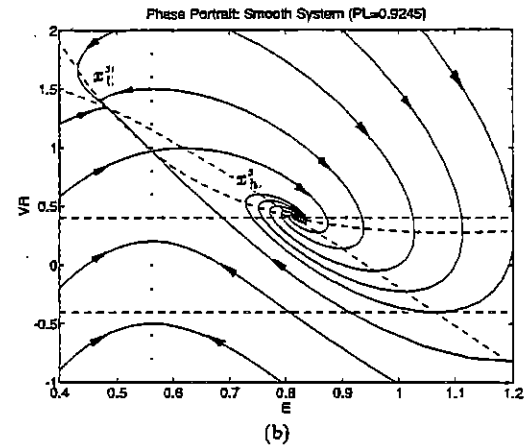
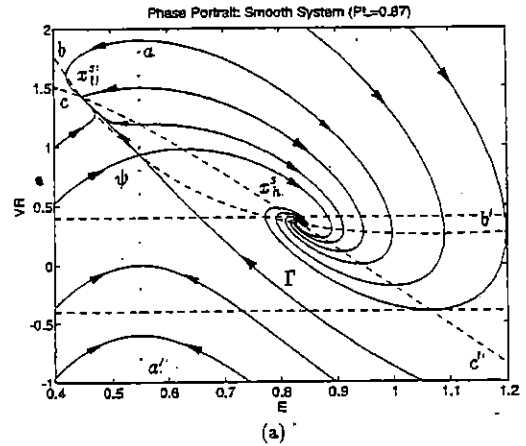


Figure III.7. Phase portraits for the non-hardlimit-constrained DAE system with parameters $x_d = 1.2$, $x'_d = 0.2$, $T'_{d0} = 10$, $E_{fd}^0 = 1.6$, $K = 3.5$, $T = 1.75$, $E_r = 1$, $x = 0.1$, $Q = 0.5$, $H = 0$, $B = 0$.

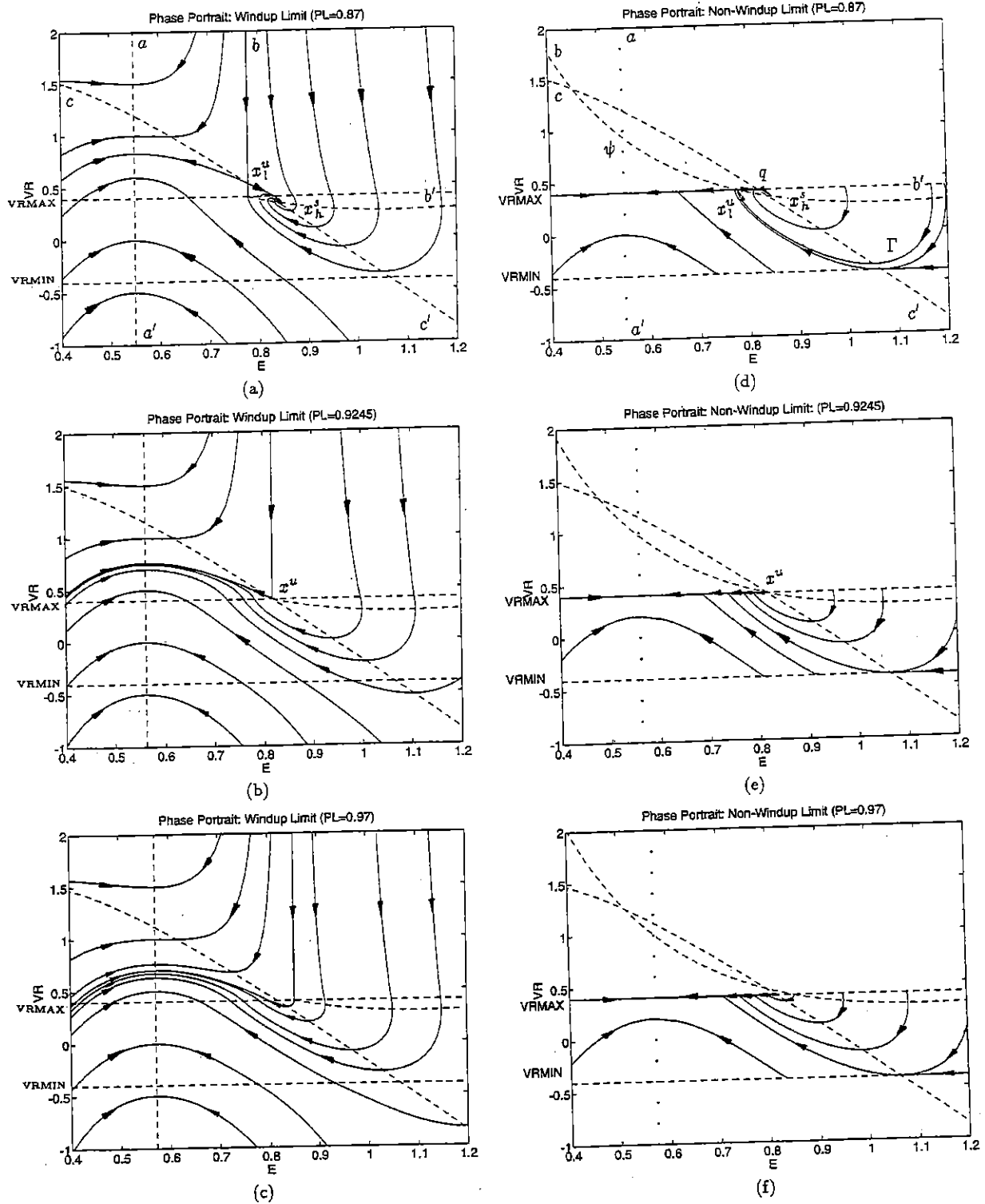


Figure III.8. Phase portraits illustrating a limit induced static bifurcation for the (actuation) windup limits ((a), (b) and (c)) and (state) nonwindup limits ((d), (e) and (f)) with $V_{R,MAX} = 0.4$, $V_{R,MIN} = -0.4$ and other parameters as listed in Figure III.7. A saddle and a stable focus meet and disappear.

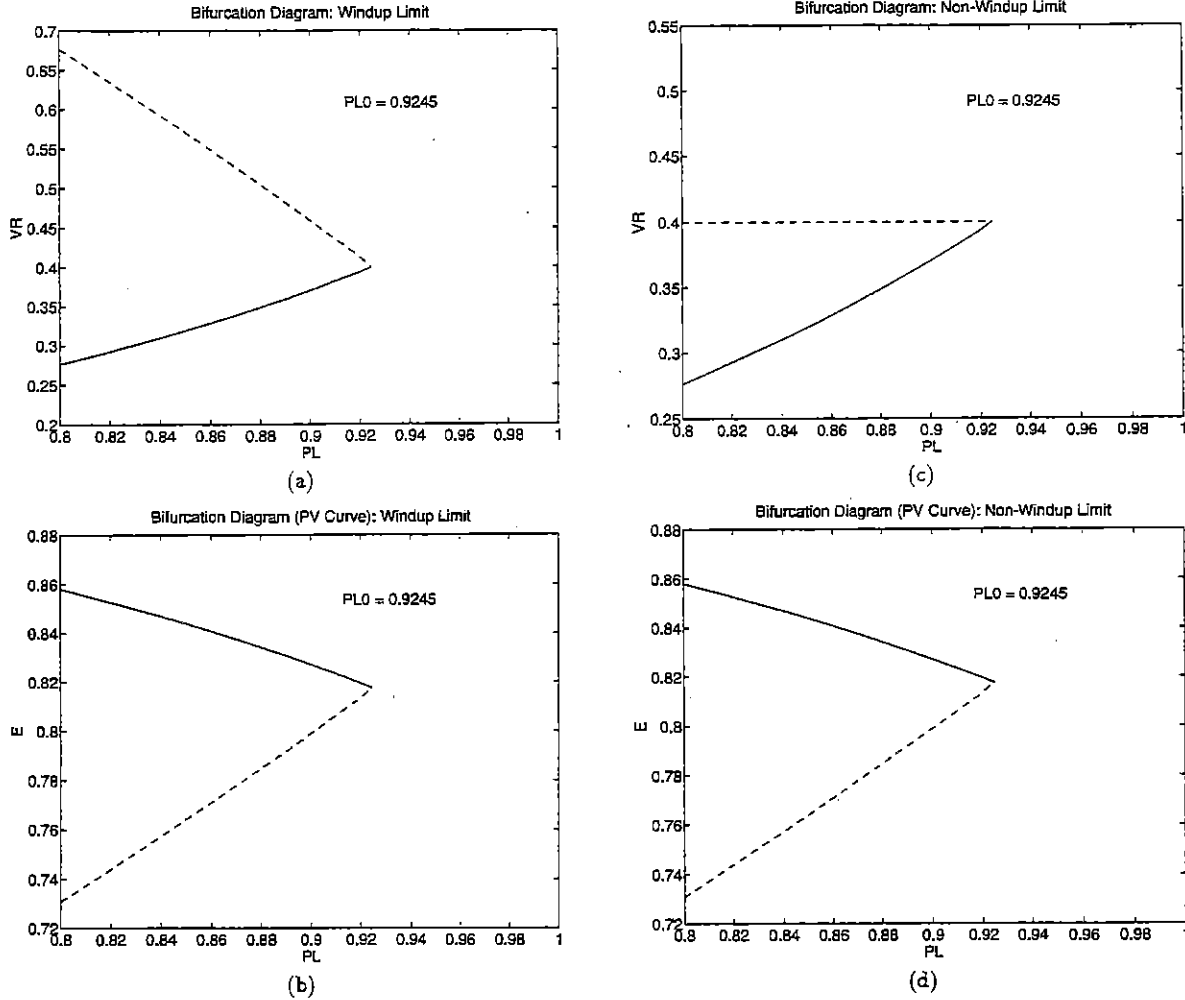


Figure III.9. Bifurcation diagrams for the limit induced static bifurcation illustrated in Figure III.8 for the windup limits ((a) and (b)) and the nonwindup limits (c) and (d)). Note the abrupt (nonsmooth) disappearance of the operating point at the static bifurcation.

Figure III.8(a) gives the phase portrait shortly before a level 0 operating point x_0 reaches the hard limit and is annihilated in a static limit induced bifurcation (see section III.3 and Table III.2). The figure also shows the zero sets for the dynamics generating vector field corresponding to control and generator equation. The operating point is a stable focus, labelled x_h^s . Its stability boundary consists of a line segment (1-dimensional stratum) C_1 on the upper hard limit surface and a trajectory Γ which lies in the interior of the state space. The trajectory Γ is the stable manifold of the level 1 equilibrium x_l^s on the upper hard limit surface and C_1 connects x_l^s with the exit point of Γ from the state space. All points inside the contour formed by C_1 and Γ form the region of attraction for x_h^s . Note that x_l^s is unstable under the dynamics induced on the hard limit surface and one part of C_1 is its local unstable manifold. The other part consists of inactive hard limits where the flow is oriented inward. The point q of separation is where the flow is tangent to the hard limit surface (and the order of contact is indeed 2). Here the trajectory enters into the interior of the state space smoothly at q . Changing the parameter the two equilibria x_l^s and x_h^s will meet precisely at a point corresponding to q and annihilate there (see Figure III.8(b)). The analysis of this bifurcation will be described in section III.3 (see Table III.2).

Comparing these figures with Figure III.7 which gives the corresponding phase portraits for the un-hard-constrained DAE system highlights another aspect of the changes which hard limits can introduce into the phase portrait. It is in general NOT correct that the stability boundary for the system with the hard limits can be obtained by simply cutting off the stability boundary for the unconstrained system at the hard limits. The reason is that the hard limits generate additional equilibria like x_l^s in this case (see Figures III.8(a)) which can cause a further reduction of the region of stability. For the state limited system the stability boundary is essentially given by Γ which is the stable manifold of this equilibrium. For the unconstrained system the corresponding equilibrium x_l^s lies in the low component and its stable manifold passes through ψ which is considerably to the left. The instability of the level 1 equilibrium x_l^s in the state limited system causes a reduction in the region of attraction of the operating point.

Also note that the original system shows no change in structure at all for these particular parameter values: a stable focus moves across the level corresponding to the hard limit. In the limited system the equilibrium x_l^s is "brought in" and this causes the annihilation of the two equilibria at the bifurcation. In a certain sense the hard limit generates a bifurcation which has the

same effect as a saddle node of the smooth system [37], but here the stable equilibrium is in fact a focus. Figure III.10 shows the corresponding sequence of events when the operating point x_1^* is a stable node.

Another interesting phenomenon is depicted in Figure III.12 which shows how an operating point (here a stable node) moves onto the hard limit surface and persists as operating point there. In this case stability is sustained. It is also possible that the equilibrium becomes unstable as it reaches the hard limit and this therefore leads to a loss of stability.

Figure III.14 describes a *hard limit oscillation* as it arises from what could be thought of as a nonsmooth version of the homoclinic bifurcation [37]. Starting with a stable level 0 equilibrium point, as the parameter is varied the equilibrium point becomes an unstable focus after a subcritical Hopf bifurcation (not shown in the figure) and at the same time moves towards the hard limit. In the unconstrained system the instabilities take over (the transient hits the singularity to be precise) and operation is not possible. But for the state limited system, this is not the case. The nonwindup hard limit on the state E_{fd} effectively “damps out” the instability thus inducing a stable sustained oscillation. In Figure III.14(a), a stable hard limit oscillation arises: at the point r the system reaches the hard limit and changes over to the induced dynamics. This dynamics is followed until again the point q is reached where the flows are tangential and the trajectory enters the inside smoothly at this point q . Then the oscillatory behavior of the trajectory makes it return to the hard limit surface at r . This stable concatenated trajectory thus absorbs the unstable transients diverging from the unstable focus x_0 . In fact, this phenomenon appears to be quite persistent even in more detailed power system models. However in large systems, the interaction of more than one hardlimits appears to be decisive on the transient behavior [24].

Figure III.14(b) shows the disappearance of the hard limit induced limit cycle when it touches the unstable equilibrium point x_1^* on the hard limit surface. This constitutes the formation of an exotic limit set, namely a hardlimit induced (nonsmooth) homoclinic orbit at the equilibrium x_1^* which is locally attracting from the interior. Since the system is two-dimensional, occurrence of complex behavior from perturbations of this stable homoclinic orbit is ruled out [37]. However the mere existence of such stable homoclinic orbits in hard-constrained power system models is significant for investigating the possibility of chaotic behavior. These will be discussed elsewhere. Note that as in smooth systems, the limit cycle disappears in Figure III.14(c) after the homoclinic bifurcation (also called the saddle connection bifurcation, see [37]) in Figure III.14(b).

III.2.3 Switching limits or Relay Type Limits

As formulated in Section III.1.2, the simplified state space model for analyzing the phenomena related to switching limits can be stated as

$$\Sigma_{rl} : \dot{x} = \bar{f}(x) := f(x, w), \quad w = \bar{v}(x, p) \quad (233)$$

where $x \in \mathbb{R}^n$ denotes the set of state variables and $w \in \mathbb{R}^{n_w}$ corresponds to the dependent states of the switching functions $\bar{v}_i$. Recall that the switching functions $\bar{v}_i$ have been defined as

$$\bar{v}_i(x) := \begin{cases} 0, & \nu_i(x) < 0 \\ 1, & \nu_i(x) \geq 0 \end{cases} \quad (234)$$

where $\nu_i(x)$ is a smooth scalar function (these ν_i will be called the *level functions*) that is input to a “level detector”, hence the output variable $w_i = \bar{v}_i(x)$ takes one of two values, either a zero or one, depending on the sign of the input function $\nu_i(x)$. Clearly in the ODE representation of the model Σ_{rl} , the function $\bar{f}(x)$ is not continuous in x and the discontinuities arise when one of the level functions ν_i become zero. In the following, we will show that much like the case of windup and nonwindup limits in Sections III.2.1 and III.2.2., the state space in this case can be decomposed into a set of piecewise smooth systems which when “pieced together” result in the global dynamics. The transitions between the various smooth subsystems occur precisely when one (or more) of the level functions hits the zero value.

Recall from Section III.2.1 that for the case of windup limits, flow exists for the model Σ_{sl} globally. Whereas for the case of nonwindup limits, in Section III.2.2, we proved that the flow exists for Σ_{sl} only in positive time (i.e. only positive semi-flow exists) and the structure is richer in detail. For the relay type limits, in this section, we will see that for the model Σ_{rl} , the flow need not exist globally in either positive or negative direction. In other words, for Σ_{rl} , generically there arise singularity like points in the state space where solutions undergo loss of existence/uniqueness. However we can show that generically *only a finite number of solutions reach or leave a point in the state space* in Σ_{rl} which hence is much simpler in the case of DAE models where an infinite number of solutions (for example, see pseudo sinks and pseudo sources, [86]) can originate at certain special singular points. In other words, the state space for Σ_{rl} is more technical to analyze than in the case of windup and nonwindup limits of the previous two sections, yet it is simpler to handle than the singular models of the DAE theory [86, 91].

In this section, because of space limitations, we will not indulge in a detailed state space analysis. In Section A, we propose a decomposition of the state space into open components separated by certain switching surfaces. Then the local dynamics near some of the generic switching surface points is investigated. Several interesting subcases arise as depicted by qualitative pictures of the local phase portraits. This local analysis would be the natural first step towards developing a global stability theory such as in [86], and this procedure would be similar to the analysis in [86].

A. A decomposition of the state space

Define the switching surfaces SW_i where the level function $\nu_i(x)$ becomes zero,

$$SW_i := \{x \in \mathbb{R}^n : \nu_i(x) = 0\} \quad (235)$$

Naturally the union of these can be defined as the switching set

$$SW := \bigcup_{i=1}^{n_w} SW_i \quad (236)$$

As in the case of windup limits (See Assumption (A0.1) in Section III.2.1), we can assume that these surfaces are nice manifolds individually.

Assumption (A0.1): SW_i is a $n - 1$ -dimensional embedded smooth submanifold in $\mathbb{R}^n$ for each $i = 1, \dots, n_w$. Specifically the function $\bar{f}(x)$ is not continuous on SW . However when restricted away from the switching set SW , the system Σ_{rl}

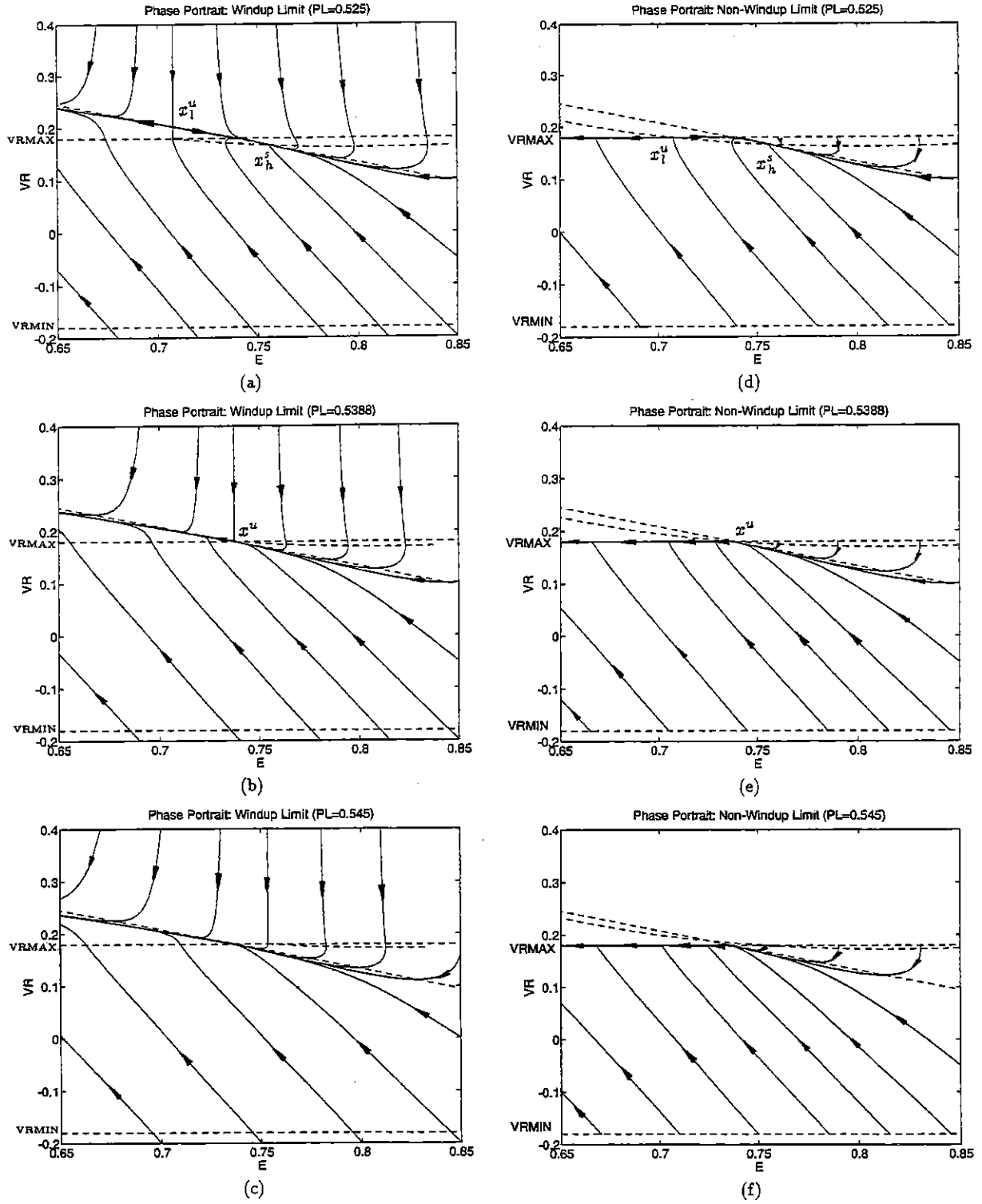


Figure III.10. Phase portraits illustrating a limit induced static bifurcation for the (actuation) windup limits ((a), (b) and (c)) and (state) nonwindup limits ((d), (e) and (f)) with parameters $x_d = 1.2$, $x'_d = 0.2$, $T_{d0} = 10$, $E_{fd}^0 = 1.2$, $K = 0.8$, $T = 1.75$, $E_r = 1$, $VR_{MAX} = 0.18$, $VR_{MIN} = -0.18$, $x = 0.1$, $Q = 0.25$, $H = 0$, $B = 0$. A saddle and a stable node meet and disappear.

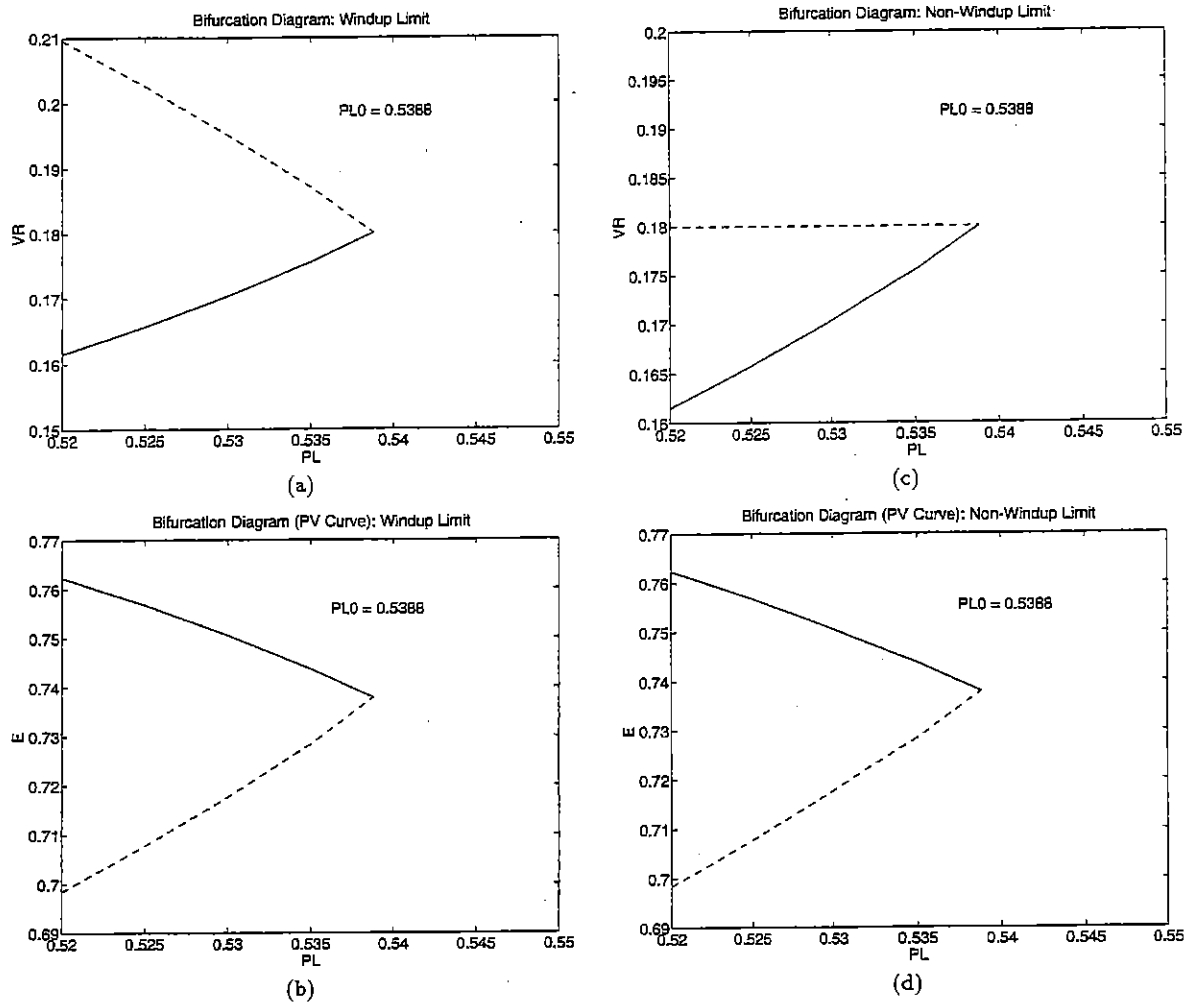


Figure III.11. Bifurcation diagrams for the limit induced static bifurcation illustrated in Figure III.10 for the windup limits ((a) and (b)) and nonwindup limits ((c) and (d)). Note the abrupt (nonsmooth) disappearance of the operating point at the static bifurcation.

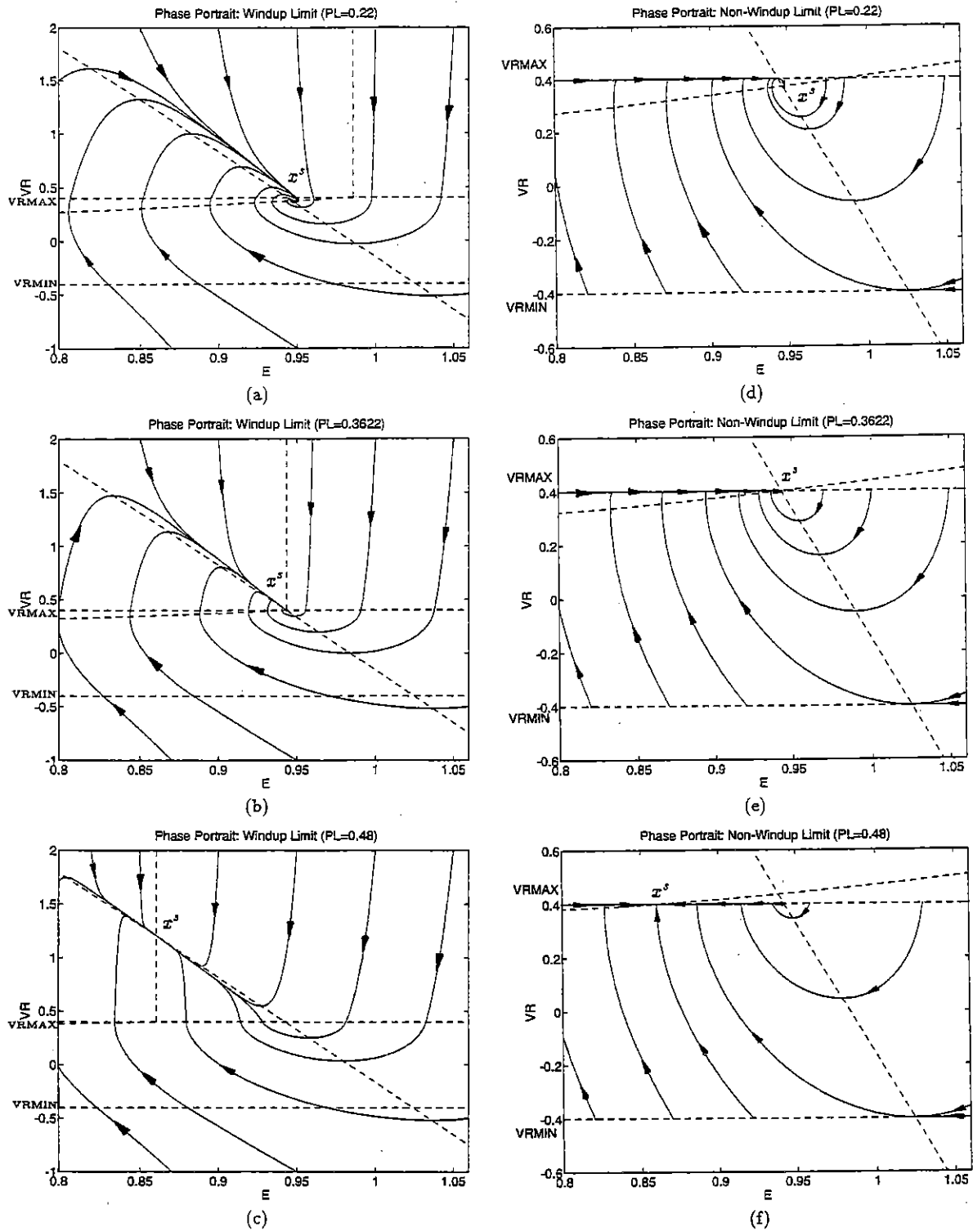


Figure III.12. Phase portraits showing a critical operating point for the windup limits ((a), (b) and (c)) and nonwindup limits ((d), (e) and (f)) with parameters $x_d = 1.2$, $x'_d = 0.2$, $T'_{d0} = 10$, $E_{fd}^0 = 0.8$, $K = 10$, $T = 1.75$, $E_r = 1$, $VR_{MAX} = 0.4$, $VR_{MIN} = -0.4$, $x = 0.1$, $Q = 0.15$, $H = 0$, $B = 0$. The system operation at the operating point continues even after reaching the respective hardlimit.

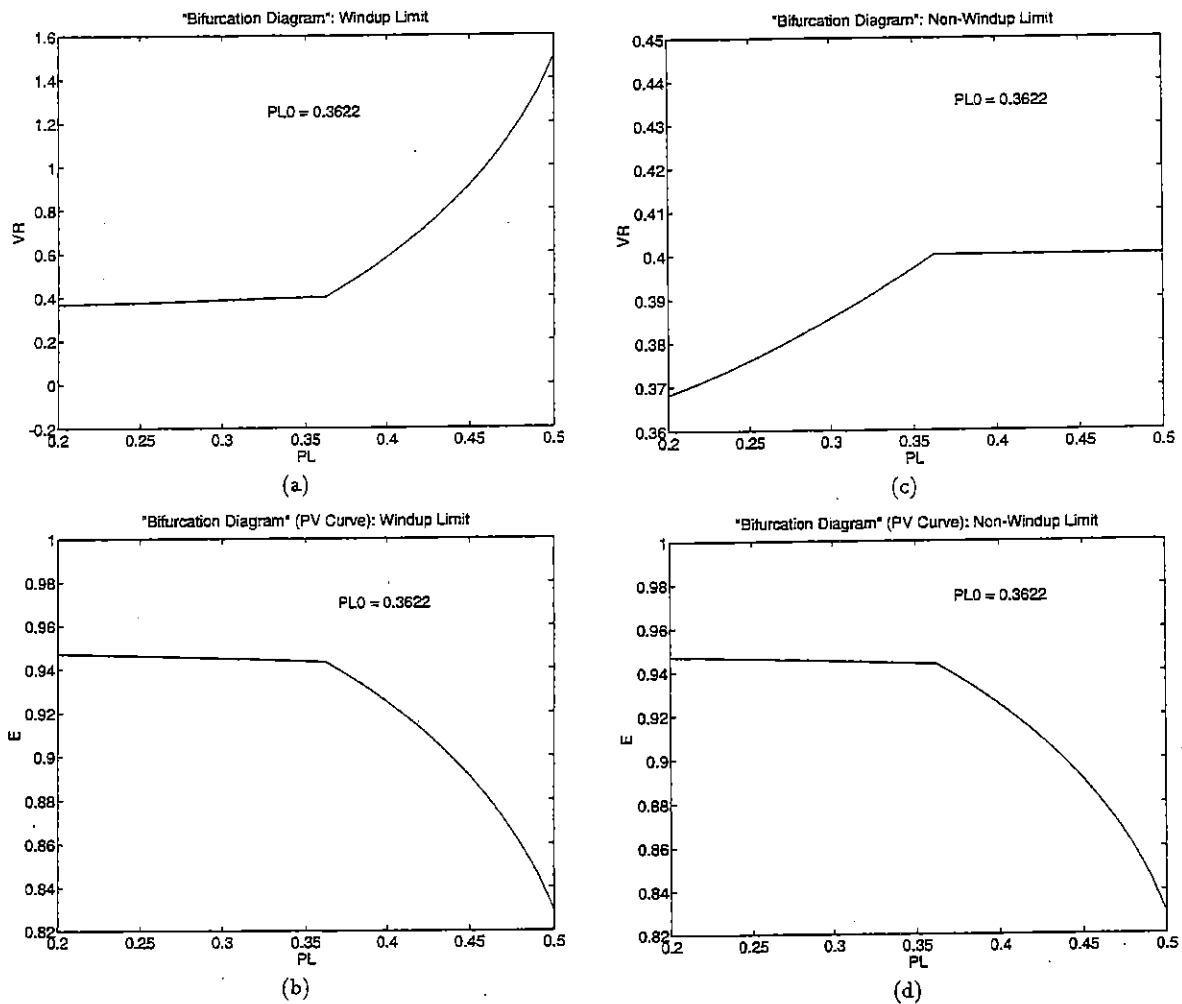


Figure III.13. Parameter space "bifurcation" diagram showing sustained stability at the operating point after reaching the hardlimits for the phase portraits in Figure 12. (see (a) and (b) for windup limits, (c) and (d) for nonwindup limits). Note the abrupt (nonsmooth) change in the PV diagram after the hardlimit is encountered, however the operation continues at the equilibrium.

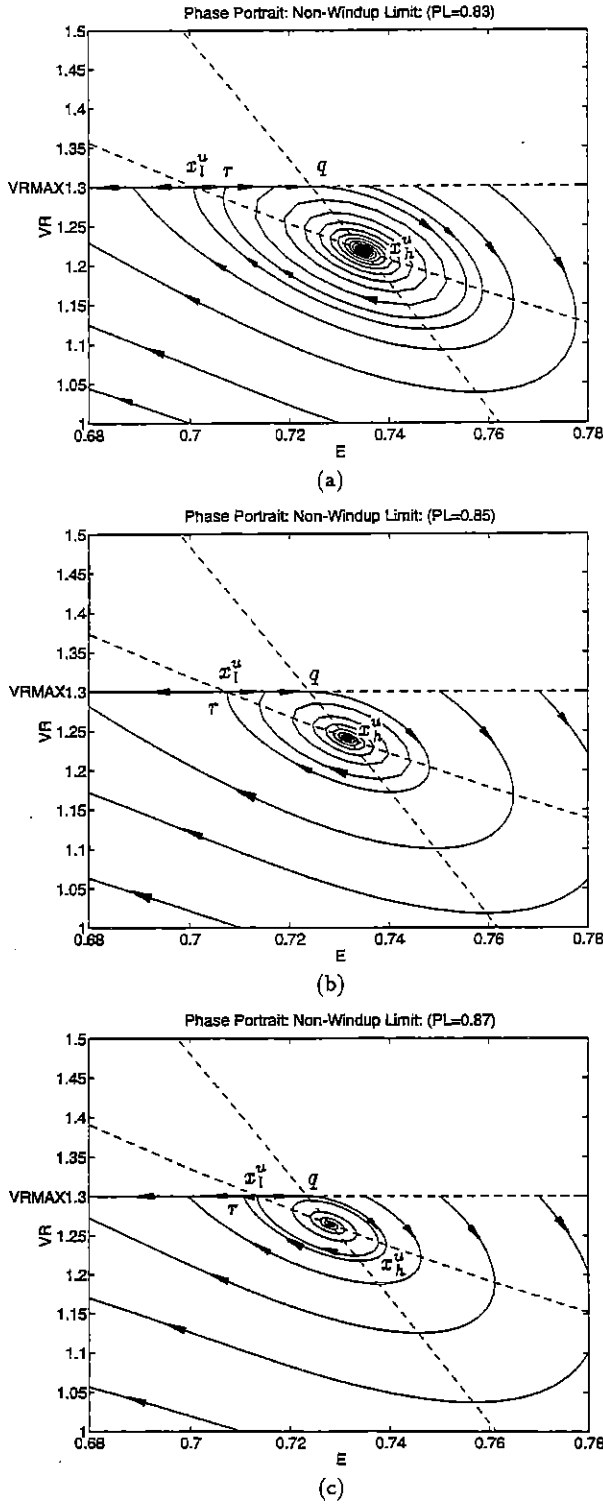


Figure III.14. A nonsmooth homoclinic bifurcation with parameters $x_d = 1.2$, $x_d' = 0.2$, $T_{d0} = 10$, $E_{fd}^0 = 1.6$, $K = 10$, $T = 1.5$, $E_r = 1$, $VR_{MAX} = 1.3$, $VR_{MIN} = -1.3$, $x = 0.1$, $Q = 1$, $H = 0$, $B = 0$. (a) stable hardlimit induced limit cycle. (b) the formation of a stable nonsmooth homoclinic orbit. (c) the stable limit cycle disappears after it touches the unstable equilibrium x_l^u .

is smooth, hence we can define the “smooth” subset SM of $\mathbb{R}^n$ as

$$SM := \mathbb{R}^n \setminus SW \quad (237)$$

It is easily seen that $\Sigma_{r,l}$ restricted to SM is smooth. Since SM is an open set by construction (note that SW is closed in $\mathbb{R}^n$), we can divide SM into open connected components say C_i so that the whole state space then decomposes into

$$\mathbb{R}^n = \left(\bigcup_{i=1}^{n_c} C_i \right) \cup \left(\bigcup_{i=1}^{n_w} SW_i \right) \quad (238)$$

where n_c is the number of components in SM . Since each component C_i is open, the dynamics $\Sigma_{r,l}$ restricted to any C_i is smooth and is locally invariant. These C_i restricted dynamics can then be thought of as n_c smooth subsystems which when “put together” determine the global dynamic behavior. The process of piecing these together demands that we understand how the transitions between these different subsystems occur across the switching surfaces SW_i . In the next section, we indicate briefly how such an analysis can proceed. The construction is seen to be mostly technical involving standard differential geometric computations.

1. Dynamic hierarchy in SW_i :

Without loss of generality, let us assume that we consider a point $x_0 \in SW_i$ such that $x_0 \notin SW_j$ for all $j \neq i$. When the latter condition is violated, x_0 happens to be a common intersection point between at least two different switching surfaces and these can be treated similar to the analysis here. Then we can define two functions $f_0^i(x_0)$ and $f_1^i(x_0)$ at x_0 depending on whether the variable $w_i = 0$ or 1 respectively:

$$f_0^i(x) := f(x, w), w_i = 0 \text{ and } w_j = \bar{v}_j(x) \text{ for } j \neq i \quad (239)$$

$$f_1^i(x) := f(x, w), w_i = 1 \text{ and } w_j = \bar{v}_j(x) \text{ for } j \neq i \quad (240)$$

Then it follows that both $f_0^i(x)$ and $f_1^i(x)$ are smooth locally at x_0 since $x_0 \notin SW_j$ for all $j \neq i$. Either of these is the actual ODE description $\bar{f}(x)$ depending on whether the level function $v_i(x) > 0$ or $v_i(x) < 0$. Note that suppose a trajectory $x(t)$ of $\Sigma_{r,l}$ satisfies $x(t_0) = x_0$ and $v_i(x(t_0^-)) < 0$. Then by carrying out simple Lie derivative type arguments at x_0 (like in the case of windup and nonwindup limits), it can be seen that the first order derivative of $v_i(x(t))$ at t_0 is given by the Lie derivative $L_{f_0^i} v_i(x_0)$. If this quantity say κ_0^i is positive, then this implies that along the trajectory, the level function exceeds zero value at time $t = t_0$, therefore the trajectory tries to cross over to the level value $w_i = 1$. Now repeating these computations assuming that we start with the functional description $f_1^i(x)$, we conclude that the Lie derivative $\kappa_1^i = L_{f_1^i} v_i$ determines what happens when the trajectory reaches x_0 from the situation of $w_i = 1$. Now depending on the relative signs of these two quantities, different possibilities arise as shown in Figure III.15. First let us define κ 's formally:

$$\kappa_0^i(x) := L_{f_0^i} v_i(x), \quad \kappa_1^i(x) := L_{f_1^i} v_i(x) \quad (241)$$

It follows that these quantities are nonzero, then the local phase portraits at x_0 can be drawn easily, hence the following set determines the first generic level of the dynamic structure within SW_i :

$$TR_i := \{x \in SW_i : x \notin \bigcup_{j \neq i} SW_j, \kappa_0^i(x) \kappa_1^i(x) \neq 0\} \quad (242)$$

These points in TR_i are denoted *transverse switching points*. Essentially the conditions $\kappa_0^i(x) \neq 0$ and $\kappa_1^i(x) \neq 0$ guarantee

that there exist two trajectories of $\Sigma_{r,i}$ from two different components (corresponding to whether $\nu_i < 0$ or $\nu_i > 0$) which reach x_0 . Now the directions of the trajectories at x_0 i.e. the signs of the κ quantities determine whether the trajectory of the overall system $\Sigma_{r,i}$ simply crosses over at x_0 , or whether there arises an impasse situation. Precisely the following three subsets emerge from TR_{SW_i} :

$$TR_i := TR_i^{pass} \cup TR_i^{source} \cup TR_i^{sink} \quad (243)$$

where

$$TR_i^{pass} := \{x \in TR_i : \kappa_0^i(x)\kappa_1^i(x) > 0\} \quad (244)$$

$$TR_i^{source} := \{x \in TR_i : \kappa_0^i(x) < 0, \kappa_1^i(x) > 0\} \quad (245)$$

$$TR_i^{sink} := \{x \in TR_i : \kappa_0^i(x) > 0, \kappa_1^i(x) < 0\} \quad (246)$$

The local phase portraits near each of these sets is well-understood and is as shown in Figure III.15. The proofs for the pictures are straightforward by simple chain rule type derivative computations. By construction, each of these subsets is an $n-1$ -dimensional submanifold in $\mathbb{R}^n$.

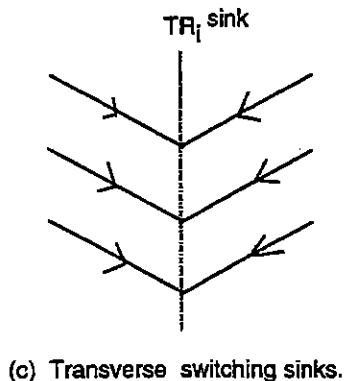
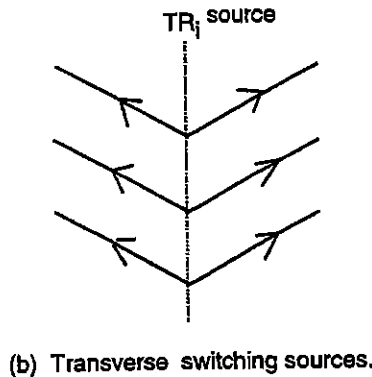
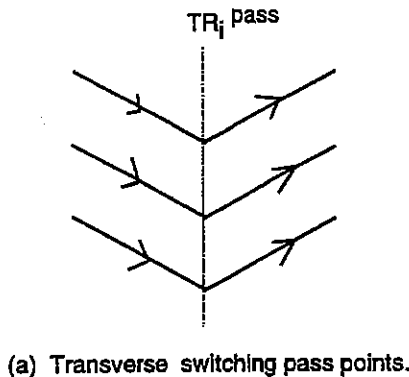


Figure III.15. The three types of local behaviors near transverse switching points.

Note that the points in the set TR_i^{pass} (Figure III.15a) correspond to the "normal transition" case when the switching occurs. The trajectories reach SW_i and leave immediately, however there is discontinuity in the state derivatives (the trajectory is continuous but not differentiable). The points in TR_i^{source} (Figure III.15(b)) behave like sources of trajectories where two trajectories emerge with finite velocities in positive time, and no negative time continuation possible. Similar phenomena have been analyzed in DAE circuit models as backward impasse points [23] and in our taxonomy theory, such points arise in the first level of the hierarchy as *transverse sources* [93] (see Section I.1.4). In the spirit of the notation in [93], we call the points in the set TR_i^{source} also as transverse sources.

There is a converse effect at the points in the set TR_i^{sink} (Figure III.15(c)) where two trajectories reach in finite positive time, but cannot be continued beyond. Such phenomena have been classified under *forward impasse points* in the circuit theory DAE analysis [23], and in our taxonomy theory, it can be shown that subsets of the singularity called *transverse sinks* are indeed forward impasse points. Again since this phenomenon is similar to the case of DAE models, we call these points as *transverse sinks*. Once a trajectory reaches a point in the set TR_i^{sink} , note that there does not exist a solution in positive time, that is, the model $\Sigma_{r,i}$ loses validity. Moreover this occurs without any connection to infinite speeds in the state variables. In the real system, such occurrences could correspond to a poor relay logic design resulting in repeated switching between two relay states. This could result in (hunting) a reduced order dynamics along the switching surface TR_i^{sink} or could lead to the action of another back-up protective relay mechanism. In our formulation, we merely assume that the model $\Sigma_{r,i}$ loses validity upon reaching these points, and the transients are required to stay away from these sets like in the case of the singularity in DAE models [86].

The analysis of the set TR_i completes an understanding of the first hierarchical level of the dynamics near SW_i . As a next step, we can look at the least non-degenerate subset of the remainder $SW_i \setminus TR_i$. These would proceed by considering the various subcases arising from 1) one of the κ 's become zero, or 2) there is an intersection with some SW_j for $j \neq i$. These case can be analyzed by straightforward if tedious Lie derivative computations, and the results are not shown here because of space limitations. Briefly a rich set of structural details emerge with local pictures motivating the names, "fork points", "U-turn points", and "cross-over points". These will be presented elsewhere.

From the topological analysis of the stability region in [86, 91], it appears that fortunately it is sufficient to consider the generic subsets up to this codimension two of the hierarchy. That is, like in [86], we can include the effects lower dimensional subsets within SW_i by taking suitable closures.

With this knowledge of the local structure near SW_i (up to codimension two), we can proceed with the stability region analysis for the model $\Sigma_{r,i}$ that includes the discontinuities. Essentially we can expect that under suitable assumptions, the boundary can be characterized as the union of the stable manifolds of anchor points which would include 1) unstable equilibria, 2) unstable periodic orbits, and 3) special subsets of the switching surfaces SW (such as the "fork points" mentioned above) along with sections of the switching surface SW itself (such as the set TR_i^{source} defined above).

In summary, it appears that using the tools from [91], and using the models for a unified formulation (of the quasi-stationary equipment dynamics and the relay dynamics) such as $\Sigma_{r,i}$ (see [59]), the stability boundary analysis of a power system dynamic model which explicitly includes the effects of relay switchings can be carried out. It is hoped that this analysis motivates an extension of Lyapunov stability theory techniques to such discontinuous cases, aided by the dynamic analysis from this section, so that detailed Lyapunov function analysis of the power system transient stability problem can be carried out which directly incorporate the effects of relay type limits (also fault clearings) and dynamic transients into a unified dynamic security formulation.

III.3 The Parameter Space Structure

In this section, we extend the concept of feasibility region in [86] to the three subsystems introduced earlier which involve the three types of hardlimits, namely actuation limits (section III.3.1), state limits (section III.3.2), and switching type limits (section III.3.3). In each of the three cases, we will observe that the standard approach of linearization around an equilibrium will not be applicable at certain points when the operation encounters the activation of one of such hard-limits. Precisely the functions describing the dynamic model lose differentiability at such transition points. Therefore the standard procedure of linearization based analysis is not directly applicable. Instances of loss of local stability upon reaching such hard-limits have been noted previously in the literature [32, 72]. Moreover industrial citings [131] point to the strong influence the hard-limits may have on the dynamic properties of the real system. However there does not exist any general methodology for handling the hard-limits in small-signal stability analysis. In this section, the theory base for such an analysis is developed. While smooth approximation of hard-limits may be used for specific case-studies, the tractability of such an approach in the large power system is questionable, hence the need for treating the hard-limits as analytical entities within the overall model.

Here we will prove that for each of the three types of hard-limits, new local bifurcations emerge on the feasibility boundary which are directly related to the presence of hard-limits. A precise theory is proposed here for analyzing the new bifurcations. These new bifurcations are largely classified into two types of phenomena: 1) Limit induced static bifurcation where the structure of the equilibria undergoes a change much like the well-known saddle node bifurcation (roughly like on the nose of the PV curve), and 2) Limit induced dynamic bifurcation which is a rich collection of local bifurcations where the local dynamic properties of the equilibria change. Either type of bifurcation boundary when encountered in system operation leads to loss of local stability, hence is practically significant. Moreover these are directly connected with the presence of hard-limits, i.e. these instability limits are encountered when the operation reaches the activation of some of these hard-limits.

Limit induced static bifurcation is similar to the classical saddle node type static bifurcation encountered in voltage stability studies. That is, in this case, as the parameter approaches the bifurcation point, the stable equilibrium approaches an unstable one (the nose of the popular PV curve) with the two annihilating each other at the bifurcation point. However the equilibrium locus encounters the bifurcation *transversally* here unlike in the case of the saddle node bifurcation where the limit is approached tangentially (the nose of the nose curve). Roughly this implies

that the system operation may end at the tip of a hard-limit induced “nose-curve” except that the “nose” would be more like a “beak” (for instance, see Figures III.9 and III.10) in this case. Locally there is no equilibrium past the static bifurcation. However the usual system Jacobian is not singular at the bifurcation point. A simple algebraic criterion to detect this phenomenon is proved here in Sections III.3.1 and III.3.2 for the actuation limits and state limits respectively. This bifurcation is therefore tractable even for the large system modeled by thousands of such hardlimits even though this condition is directly related to the effect of nonsmooth hard-limit models.

Limit induced dynamic bifurcation is a family of new dynamic phenomena that lead to loss of small-signal stability however the equilibrium structure is preserved at this bifurcation point. Basically when a hard-limit is encountered, this forces a change in the system Jacobian, hence on the eigenvalues rendering the system small-signal unstable. In reality, this phenomenon is very rich, with possible outcomes ranging from monotonic divergence (and possibly a collapse) to sustained oscillations. We analyze this phenomenon rigorously for two dimensional systems with several examples. The emergence of limit induced oscillations will be clearly demonstrated and explicit conditions derived for their existence. For instance, in the case of actuation limits (section III.3.1), Hopf like phenomena connected with the presence of hard-limit related stable (and unstable) limit cycles are analyzed. Extension to the large system of these two dimensional results requires the use of the center manifold theory and these will be presented elsewhere.

The organization of this section is as follows: we will consider each of the three cases, actuation limits (III.3.1), the state limits (III.3.2) and the switching limits (III.3.3) separately. In each case, the construction of the feasibility region is followed by the characterization of its boundary in the form of zero sets. The principal bifurcation properties of the bifurcation boundaries are established and algebraic conditions are derived for detecting them in the large system.

III.3.1. Actuation limits (Windup limiters)

The state-space model for actuation limits which includes the parameter dependence can be represented by

$$\Sigma_{at}: \quad \dot{x} = \bar{f}(x, p) = f(x, z, p), \quad z = \bar{h}(x, p) \quad (247)$$

$$z_i = \bar{h}_i(x, p) := \begin{cases} z_{\ell i}, & \text{if } h_i(x, p) < z_{\ell i} \\ h_i(x, p), & \text{if } z_{\ell i} \leq h_i(x, p) \leq z_{u i} \\ z_{u i}, & \text{if } h_i(x, p) > z_{u i} \end{cases} \quad (248)$$

$$x \in \mathbb{R}^n, \quad z \in \mathbb{R}^{n_z}, \quad p \in \mathbb{R}^q, \quad (249)$$

$$f: \mathbb{R}^{n+n_z+q} \rightarrow \mathbb{R}^n, \bar{h}: \mathbb{R}^{n+q} \rightarrow \mathbb{R}^{n_z}, \quad h: \mathbb{R}^{n+q} \rightarrow \mathbb{R}^{n_z}, \quad (250)$$

f is C^∞ and h is C^∞

Essentially the internal actuation variables z which are functions of (x, p) are bounded above by z_u and bounded below by z_ℓ . As noted in Section III.2.1, the vector field description $\bar{f}(x, p)$ in Σ_{at} is Lipschitz continuous, hence there exists a well-defined notion of dynamics and associated flow with this model. Therefore the problem of defining its equilibrium points is not technically difficult. Note that when the state limits are introduced (and for the switching limits in the later section), even these questions require precise development. For the current problem of actuation limits, the new feature compared to the standard theory for smooth systems is that the vector field $\bar{f}(x, p)$ may not be differentiable at those points when the functions h_i 's reach their limits. The notion of small-signal stability is more technical

since the local linearization arguments are no longer applicable. We will prove that such points where the variables z_i 's hit their limits can lead to bifurcations, both static and dynamic. Therefore the difference compared to the smooth model is that for the actuation limited model, the feasibility boundary has more boundary segments.

A: Equilibria and Operating Points

Let us start the analysis by defining the set of equilibria EQ .

$$EQ := \{(x, p) \in \mathbb{R}^{n+q} : \bar{f}(x, p) = 0\} \quad (251)$$

All the topological operations in this section are carried out with respect to EQ , which is then the full space for parametric analysis of equilibria and their local properties. Note that among these equilibria, we need to distinguish those where the local dynamics is smooth and the remaining ones where $\bar{f}(x, p)$ is not differentiable (i.e. some function h_i is at its limiting value). Recall the definitions of the sets z_{u_i} and ZL_{u_i} from Section III.2.1,

$$ZL_{u_i} := \{(x, p) \in \mathbb{R}^{n+q} : h_i(x, p) = z_{u_i}\} \quad (252)$$

$$ZU_{u_i} := \{(x, p) \in \mathbb{R}^{n+q} : h_i(x, p) = z_{u_i}\} \quad (253)$$

The collection of all these sets can be defined ZU and ZL respectively,

$$ZL := \bigcup_{i=1}^{n_z} ZL_{u_i}, \quad ZU := \bigcup_{i=1}^{n_z} ZU_{u_i} \quad (254)$$

Therefore special tools are necessary for analyzing the local dynamics near $EQ \cap (ZU \cup ZL)$. The equilibria with locally smooth dynamics is defined EQ_{smooth} , $EQ_{\text{smooth}} := EQ \setminus (ZU \cup ZL)$. The remaining equilibria are defined the critical equilibria $EQ_{\text{critical}} := EQ \cap (ZU \cup ZL)$. From among the set EQ_{smooth} , the ones with local stability can be defined the set of operating points say OP_{smooth} , since they possess small-signal stability and qualify as possible candidates for system operation.

$$OP_{\text{smooth}} := \{(x, p) \in \mathbb{R}^{n+q} : \sigma(D_x \bar{f}(x, p)) \subset \mathbb{C}^- \} \quad (255)$$

Here σ denotes the eigen-spectrum of the Jacobian and $\mathbb{C}^-$ stands for the complex open left half plane. By the implicit function theorem, we can immediately conclude that OP_{smooth} is a q -dimensional smooth manifold in $\mathbb{R}^{n+q}$.

Note that for the actuation limited model Σ_{al} , continued system operation is also possible even when some of the actuation functions h_i "hit" their limits, provided small-signal stability is not affected. Therefore the full set of operating points should also include those critical equilibrium points on the transition set $EQ \cap (ZU \cup ZL)$ which "preserve" local stability. If these transition operating points were left out of the set of operating points and are treated as instability limits, the analysis is overly conservative, and does not reflect on the actual system behavior. In other words, critical equilibria EQ_{critical} which occur as "interior points" within the set of operating points should again be formulated as operating points. We can capture all these transition points by using a simple topological operation, which is similar to a trick introduced by Zaborsky et. al. in [114] for the state space analysis. Define

$$OP := \text{int}(\overline{OP_{\text{smooth}}}) \quad (256)$$

It is easy to verify that $OP_{\text{smooth}} \subset OP$: note that OP_{smooth} is an open set in EQ , since the set OP_{smooth} is obtained from EQ by imposing a set of inequality conditions. The remaining points in OP are simply denoted the interior operating points,

$$OP_{\text{interior}} := OP \setminus OP_{\text{smooth}} \quad (257)$$

In general, we cannot say that all interior operating points lie on nonsmooth boundaries i.e. $OP_{\text{interior}} \not\subset (ZU \cup ZL)$ since there may also be smooth interior operating points (associated with nonhyperbolic equilibria). However recall from Section I.3.2 that generically the nonhyperbolic equilibria in EQ_{smooth} lead to bifurcations that effect changes in the local stability status [37, 86]. Hence generically these points in EQ_{smooth} do not occur as interior operating points. The interesting ones are the critical equilibria in the set EQ_{critical} . In the feasibility region analysis, the critical equilibria can show up either as interior operating points or as feasibility boundary points leading to instability. In the next section, we will develop results which allow us to distinguish between these two types of critical equilibria. Under some assumptions, we can prove that even in the large system, the distinction is easily made "half-the-time" provided certain constant $\alpha > 0$, otherwise more computations are necessary.

B. Feasibility Regions

As in the case of smooth systems, the feasibility region can be defined as shown below by identifying the connected component of OP that contains our operating point. Then the operation can "reach" all such points within the feasibility region by quasi-static parameter variations without losing small-signal stability.

Definition 14 (Feasibility region) Given an operating point $(x, p) \in OP$, the feasibility region F of (x, p_s) is the connected component of OP that contains (x, p_s) . The boundary of F (relative to EQ) is the feasibility boundary denoted ∂F . $\square$

We need to show that the definition is meaningful. For this, first note that OP by construction is open in EQ since it strictly consists of interior points of a closed set EQ (see (256)). Therefore it is clear that OP can be divided into open components within EQ . Now simply define the component that contains (x, p_s) the feasibility region F associated with (x, p_s) .

Intuitively it is clear that in the large system, there may exist more than one connected component of OP . However those operating points which are outside F can only be reached from (x, p_s) by crossing the feasibility boundary. We claim that the feasibility boundary crossing will always lead to loss of small-signal stability of the operating point thus inducing an instability event. Note that this claim is automatically satisfied at the "true" boundary points $\partial \bar{F}$ since these points strictly separate the feasible points F from the infeasible ones in $\bar{F}^c$. In general, there may be interior points of $\bar{F}$ (such as nonhyperbolic equilibria) which when crossed still preserve small-signal stability. For our problem, such degenerate cases are ruled out, because by construction F must contain such interior points (see (256)). We have thus proved the next result.

Proposition 4 (Loss of stability proposition)

$$\partial F = \partial \bar{F} \quad (258)$$

$\square$

By the Proposition, the feasibility boundary ∂F then precisely identifies the points where the operation encounters dis-

ruption of continued steady state operating conditions. What happens to the ensuing trajectory will depend on the specific type of feasibility boundary encountered. For this reason, it becomes important to study ∂F in more detail. First we can easily show that as in the case of smooth models, the boundary ∂F can be fully described by union of zero sets.

Theorem 18 *Feasibility boundary ∂F consists of the three zero sets C_Z , C_H and EQ_{critical} .*

$$\partial F = (\partial F \cap C_Z) \cup (\partial F \cap C_H) \cup (\partial F \cap EQ_{\text{critical}}) \quad (259)$$

where

$$C_Z := \{(x, p) \in EQ_{\text{smooth}} : \det(D_x \bar{f})(x, p) = 0\} \quad (260)$$

$$C_H := \{(x, p) \in EQ_{\text{smooth}} \setminus C_Z : \det(H_{n-1}(D_x \bar{f}))(x, p) = 0\} \quad (261)$$

and H_{n-1} denotes the Hurwitz matrix of order $n-1$ as usual.

Proof: Since the feasibility region F is a connected component of OP , it follows that $\partial F \subset \partial OP$. Again by construction (256), $\partial OP \subset \partial OP_{\text{smooth}}$. By definition of OP_{smooth} (255), we have $\partial OP_{\text{smooth}} \subset (C_Z \cup C_H \cup EQ_{\text{critical}})$. This completes the proof. Q. E. D.

A quick comparison of this Theorem with the respective result (Section I.3.1.1) for the DAE system reveals that the first two terms in the union (259), namely C_Z and C_H are direct extensions of the sets C_Z and C_H for the DAE model Σ (note that C_S is empty by assumption). Briefly recalling their character, the set C_Z corresponds to zero eigenvalues of the Jacobian $D_x \bar{f}$ from (260), whereas the set C_H stands for the case of purely imaginary eigenvalues for the Jacobian $D_x \bar{f}$. The third boundary segment EQ_{critical} is a new segment for the actuation limited model. These boundary points $\partial F \cap EQ_{\text{critical}}$ are significant because they precisely identify those points where encountering one of the actuation limits leads to loss of small-signal stability. However as observed earlier, this problem is not so easy because critical equilibria EQ_{critical} can either be feasible points in F or feasibility boundary points in ∂F , and the distinction is not straightforward. (For critical equilibria, both possibilities exist generically). On the other hand, the zero eigenvalue points in C_Z and the purely imaginary eigenvalue points in C_H generically do lead to loss of stability, hence are easier to handle. To make these arguments precise, we resort to local bifurcation analysis in the next section.

C. Feasibility boundary analysis - local bifurcations

Codimension one local bifurcations which occur in the sets C_Z and C_H are well-understood in the literature under the standard transversality assumptions. For the zero eigenvalue points C_Z , under certain transversality assumptions stated below as (SN1)–(SN3) [37], a saddle node bifurcation occurs. Note that saddle node bifurcation is a static bifurcation since locally there does not exist any equilibria on one side of the saddle node bifurcation surface defined below as B_{SN} ,

$$B_{SN} := \{(x, p) \in C_Z : (x, p) \text{ satisfies (SN1), (SN2) and (SN3)}\} \quad (262)$$

By Sotomayor's Theorem [76], these bifurcation points are generic for the zero eigenvalue problem. It is clear that these points lead to loss of stability since the operating point itself is annihilated

at the static bifurcation. They have been studied with much interest in the past decade related to the voltage collapse phenomenon [52, 31].

On the other hand, the boundary points in $C_H \cap \partial F$ where the Jacobian $D_x \bar{f}$ has purely imaginary eigenvalues lead to the dynamic bifurcations, namely the well-known Hopf bifurcations, under certain transversality assumptions (H1) and (H2)[37]. The Hopf bifurcation points can then be defined as B_H ,

$$B_H := \{(x, p) \in C_H : (x, p) \text{ satisfies (H1) and (H2)}\} \quad (263)$$

At the Hopf bifurcation boundary, a pair of complex eigenvalues crosses the imaginary axis transversally, hence there do not locally stable operating points on one side of the Hopf boundary. Again this implies the end of continued operation at the equilibrium point. Here it should be pointed out that if the Hopf bifurcation is of the supercritical type, then there emerge stable limit-cycles surrounding the unstable equilibrium upon crossing the feasibility boundary, therefore the system operation can be continued in the form of sustained oscillations. However as the parameter values change, these oscillations can grow in magnitude possibly leading to intolerable voltage and current magnitudes. Basically oscillations should be treated as global phenomena, and since the objective in this section is on the feasibility of operation at an equilibrium point, we will postpone this discussion on sustained oscillations and their properties. In this sense, it is clear that Hopf bifurcations, whether subcritical or supercritical, separate feasible operating points from the infeasible ones.

Next we turn our attention to the case of critical equilibrium points when the operation reaches one of the actuation limits. As in the case of the sets C_Z and C_H , we will show that local bifurcations can occur at these points. The problem then is to develop the necessary transversality conditions for guaranteeing the phenomena, and we need to understand whether these conditions are generic in some sense for our problem. We will prove that the critical equilibria can lead to both static and dynamic bifurcations. We call these phenomena, *actuation limit induced static bifurcations* B_{ASB} and *actuation limit induced dynamic bifurcations* B_{ADB} respectively. In some cases (especially in $\mathbb{R}^2$), the static bifurcation B_{ASB} closely resembles the well-known saddle-node bifurcation B_{SN} of the smooth model discussed above, even though the two are not related in general. The dynamic bifurcation B_{ADB} is similar to the Hopf bifurcations in some cases (proved below in $\mathbb{R}^2$), leading to the birth of actuation limited limit-cycles (both stable and unstable ones), even though in general, the new bifurcations B_{ADB} are much richer in character. To properly understand these phenomena, we need to proceed step by step in constructing the transversality conditions. To simplify the analysis, we make the following assumption.

(AB0) The actuation functions $h_i(x, p)$'s in Σ_{al} each depend on only one state variable and are parameter independent.

Consider a critical equilibrium $(x_0, p_0) \in EQ_{\text{critical}}$. By definition, critical equilibrium points arise when at least one of the actuation variables z_i is at its limit, either at z_{u_i} or at z_{l_i} . First we assume that only one such variable say z_i is at its limit. We will reserve the index i for denoting the index of this variable in the following discussion. This condition is common to both the static and dynamic bifurcations, hence is named simply (AB1) denoting a condition for actuation limit induced bifurcations.

(AB1) $(x, p) \in EQ_{\text{critical}} \cap (Z_{u_i} \cup ZL_i) \setminus \bigcup_{j \neq i} (ZU_j \cup ZL_j)$.

Without loss of generality, from here on, we will assume that $(x_0, p_0) \in z_{u_i}$. Next we need to impose some regularity assumption on the Jacobian at (x_0, p_0) for assessing the small-signal stability properties. For a critical equilibrium, this is not easy since the function $\bar{f}$ is not differentiable at (x_0, p_0) . However with the condition (AB1), it turns out that two Jacobians play a role in determining the local dynamics at (x_0, p_0) . As we show next, the two Jacobians arise because either the actuation ($z_i = h_i$) is in effect or saturation ($z_i = z_{u_i}$) is in effect.

Recall from the state space analysis that the function $\bar{f}$ is piecewise smooth with transitions (from one smooth functional description to another) occurring at ZU and ZL . Specifically the function $\bar{h}_i(x_0, p_0)$ changes from being the smooth function $h_i(x_0, p_0)$ to reaching the constant limiting value, z_{u_i} . Therefore for each $(x_0, p_0) \in z_{u_i}$, we can associate two functions 1) $f_a^i(x, p)$ and 2) $f_s^i(x, p)$ which stand for 1) $\bar{f}(x, p)$ with $\bar{h}_i(x, p) = h_i(x, p)$ (when actuation is in effect) and 2) $\bar{f}(x, p)$ with $\bar{h}_i(x, p) = z_{u_i}$ (when z_i saturates). Precisely given $(x, p) \in z_{u_i}$, we define

$$f_a^i(x, p) := f(x, z, p), z_i = h_i(x, p) \text{ and } z_j = \bar{h}_j(x, p) \forall j \neq i, \quad (264)$$

$$f_s^i(x, p) := f(x, z, p), z_i = z_{u_i} \text{ and } z_j = \bar{h}_j(x, p) \forall j \neq i, \quad (265)$$

In other words, the vector field description changes from the actuation function h_i for f_a^i to the saturation limiting value for f_s^i . We can formally define these two systems as Σ_a^i and Σ_s^i ,

$$\Sigma_a^i: \dot{x} = f_a^i(x, p) \quad (266)$$

$$\Sigma_s^i: \dot{x} = f_s^i(x, p) \quad (267)$$

Note that with the assumption that the remaining actuation functions h_j 's for $j \neq i$ are not at their limiting values, the two functions f_a^i and f_s^i are both locally smooth at (x_0, p_0) . Therefore near (x_0, p_0) , Σ agrees with Σ_a^i provided $z_{\ell_i} < h_i(x) < z_{\ell_i}$. When $h_i(x) > z_{u_i}$, saturation results and Σ is identical to Σ_s^i . Next we can define two Jacobians say $J_a^i := D_x f_a^i$ and $J_s^i := D_x f_s^i$ at (x_0, p_0) for the two functions f_a^i and f_s^i respectively. The Jacobians J_a^i and J_s^i can together be used to establish the local properties at (x_0, p_0) in some way, as we will see in the results that follow. First let us assume that each of these has all eigenvalues with nonzero real parts.

(AB2) All eigenvalues of J_a^i and J_s^i have nonzero real parts.

Since we are studying the parametric feasibility of small-signal stability, we next consider the parametric dependence of the functions and impose some regularity assumptions. Note that by Assumption (AB2), since the Jacobians J_a^i and J_s^i are both nonsingular, by the implicit function theorem, there exist equilibrium manifolds $\bar{EQ}_a^i$ and $\bar{EQ}_s^i$ for the two systems Σ_a^i and Σ_s^i respectively, locally at (x_0, p_0) . However as noted before, the system Σ_a^i is meaningful for our problem only when $z_{\ell_i} < h_i(x, p) < z_{u_i}$. So an equilibrium $(x, p) \in \bar{EQ}_a^i$ of Σ_a^i is indeed an equilibrium for Σ provided $z_{\ell_i} < h_i(x, p) < z_{u_i}$. To check this locally near (x_0, p_0) , we can compute the dominating term of an expansion of $h_i(x, p)$ along $\bar{EQ}_a^i$ at (x_0, p_0) . Since the zero-th order term vanishes, $(h_i(x_0, p_0) = 0$ by Assumption (AB1)), we can look at the first order derivative term and require this term to be nonzero. The first order term can be easily computed with a simple formula shown below using the chain rule of differentiation. Unless otherwise specified, all quantities

in the equalities to be stated in this Section are evaluated at (x_0, p_0) .

(AB3)

$$D_p h_i(\bar{EQ}_a^i) = -D_x h_i(D_x f_a^i)^{-1} D_p f_a^i \neq 0 \quad (268)$$

Since (268) in particular implies that $L_x h_i \neq 0$, by implicit function theorem, an immediate consequence of this Assumption is the fact that the limit surface z_{u_i} is locally a codimension one submanifold of $\mathbb{R}^{n+q}$ at (x_0, p_0) . Therefore a local neighborhood U of $\mathbb{R}^{n+q}$ at (x_0, p_0) divides into two components bounded by z_{u_i} , say U_a where $h_i(x, p) < z_{u_i}$ and U_s where $h_i(x, p) > z_{u_i}$. By Assumptions (AB1) and (AB2) and choosing a smaller neighborhood, if necessary, it follows that $\bar{EQ}_a^i \cap z_{u_i} = EQ_{\text{critical}}$. Note that depending on the signs of the derivative terms $D_p h_i(\bar{EQ}_a^i)$, either side of EQ_a^i may belong to the equilibrium set for Σ , i.e. $\bar{EQ}_a^i \cap N_a \subset EQ$ or $\bar{EQ}_a^i \cap N_s \subset EQ$ but not both.

Similarly, now resorting to the equilibrium manifold $\bar{EQ}_s^i$, it would seem that there should be a reciprocal condition on $D_p h_i(\bar{EQ}_s^i)$ to guarantee the first order dominance of the function h_i which will again allow us to pinpoint when $\bar{EQ}_s^i$ is meaningful for Σ . That is, if we assume that $D_p h_i(\bar{EQ}_s^i) \neq 0$, we can again conclude from the actual values for $D_p h_i(\bar{EQ}_s^i)$ whether $\bar{EQ}_s^i \cap N_a \subset EQ$ or $\bar{EQ}_s^i \cap N_s \subset EQ$ (but not both). Surprisingly this additional assumption $D_p h_i(\bar{EQ}_s^i) \neq 0$ is not necessary since we prove next that the two terms $D_p h_i(\bar{EQ}_a^i)$ and $D_p h_i(\bar{EQ}_s^i)$ are related by a nonzero factor α^i defined as

$$\alpha^i := \frac{\det J_a^i}{\det J_s^i} \quad (269)$$

Under Assumption (AB2), the Jacobians J_a^i and J_s^i are both nonsingular, hence the quantity α^i in (269) exists and is nonzero. The sign of α then directly relates which of the equilibrium segments of $\bar{EQ}_a^i$ and $\bar{EQ}_s^i$ are actual equilibria in EQ .

Lemma 38 Under Assumptions (AB0), (AB1) and (AB2), the following equality holds at (x_0, p_0) ,

$$D_p h_i(\bar{EQ}_s^i) = \alpha^i D_p h_i(\bar{EQ}_a^i) \quad (270)$$

Proof: First note that the derivative $D_p f_s^i$ can be computed as

$$D_p h_i(\bar{EQ}_s^i) = -D_x h_i(D_x f_s^i)^{-1} D_p f_s^i \neq 0 \quad (271)$$

To prove the result (270), it suffices to show that

$$D_x h_i \text{adj}(D_x f_s^i)^{-1} D_p f_s^i = D_x h_i \text{adj}(D_x f_a^i) D_p f_a^i \quad (272)$$

where adj denotes the classical adjoint of the matrix. This relationship between (270) and (272) can be seen from the definition of α^i (269) by multiplying (271) by the determinant $\det(J_s^i)$ and by multiplying (268) by the determinant $\det(J_a^i)$. Moreover from the definitions of f_a^i and f_s^i , we get

$$D_x f_a^i = D_x f_s^i + D_x f_s^i D_x h_i \quad (273)$$

By Assumption (AB0), there is only one nonzero entry in the row vector $D_x h_i$. This fact together with (273) proves that $D_x h_i \text{adj}(D_x f_a^i) = D_x h_i \text{adj}(D_x f_s^i)$. Finally to prove (272), note that $D_p f_s^i = D_p f_a^i$, and the proof is done. Q. E. D.

Summarizing the transversality conditions thus far, with Assumptions (AB0)–(AB3), we can guarantee the local existence of the equilibrium manifolds $\bar{EQ}_a^i$ and $\bar{EQ}_s^i$. Moreover each of

these is separated into two components by z_{u_i} of which only one segment can belong to EQ , as related by the derivative terms $D_p h_i(\tilde{EQ}_a^i)$ and $D_p h_i(\tilde{EQ}_s^i)$ respectively. By Lemma 38, the signs of the entries of these two are directly dependent of the factor α^i , hence the static properties of the equilibrium manifold EQ is completely determined by the sign of α^i as stated in the next Proposition.

Let us define $n_{u_a}^i$ and $n_{u_s}^i$ as the number of eigenvalues of J_a^i and J_s^i respectively, with positive real parts. Then since the determinant of a matrix is the product of its eigenvalues, it is easily seen that when α , the ratio of the determinants of J_a^i and J_s^i , is positive, we must have $n_{u_a}^i \neq n_{u_s}^i$. Therefore we have proved the following result, which nicely sets up the two Theorems on actuation limit induced static and dynamic bifurcations that follow.

Proposition 5 *Let $(x_0, p_0) \in EQ_{\text{critical}}^k$ and suppose conditions (AB0), (AB1), (AB2) and (AB3) are satisfied. Then the following statements hold locally near (x_0, p_0) :*

(AP0) EQ is a q -dimensional topological manifold.

(P1) z_{u_i} is a $n + q - 1$ -dimensional smooth manifold.

(P2) EQ and z_{u_i} intersect transversally along the $q - 1$ -dimensional smooth manifold EQ_{critical} .

(P3) $EQ \setminus EQ_{\text{critical}}$ consists of two connected components called EQ_a and EQ_s which satisfy $f_a^i(EQ_a^i) = 0$ and $f_s^i(EQ_s^i) = 0$ respectively. EQ_a^i and EQ_s^i are q -dimensional smooth manifolds.

(P4) If $\alpha^i \geq 0$, then (x_0, p_0) is a static bifurcation point and EQ_a^i and EQ_s^i both lie on the same side of the manifold z_{u_i} . Moreover $n_{u_a}^i \neq n_{u_s}^i$.

If $\alpha^i < 0$, then (x_0, p_0) is not a static bifurcation point and EQ_a^i and EQ_s^i lie on the two different sides of the manifold z_{u_i} . $\square$

Remarks 1. When α^i is positive, this implies that given a set of parameter values p near p_0 (on one side of z_{u_i}), the actuation limited system Σ has two equilibria in EQ_a^i and in EQ_s^i both near (x_0, p_0) . As p approaches p_0 , these two equilibria approach each other, meeting eventually at (x_0, p_0) . But for parameters on the other side of z_{u_i} , there do not exist any equilibria. This is precisely the condition for the occurrence of static bifurcations in the model Σ_{al} .

2. Note that the small-signal stability properties of two equilibrium manifolds EQ_a^i and EQ_s^i can be related to these properties at (x_0, p_0) since the Jacobians J_a^i and J_s^i satisfy Assumption (AB2). Essentially the local properties depend on the numbers $n_{u_a}^i$ and $n_{u_s}^i$. When $n_{u_a}^i = 0$, this implies that the equilibrium manifold EQ_a^i consists of operating points, i.e. $EQ_a^i \subset OP$ when $n_{u_a}^i = 0$. Similarly it follows that $EQ_s^i \subset OP$ if $n_{u_s}^i = 0$.

3. When $\alpha^i > 0$, the Proposition states that the numbers $n_{u_a}^i$ and $n_{u_s}^i$ must be different. That is, when one of the two say $n_{u_a}^i = 0$ is zero (that is, EQ_a^i are operating points), then the other $n_{u_s}^i$ cannot be zero (then EQ_s^i cannot be operating points). This is intuitively clear since otherwise there appears an "unpleasant situation" where two different small-signal stable equilibria in EQ_a^i and EQ_s^i appear arbitrarily close to each other for parameters p near p_0 . In other words, when $\alpha^i > 0$, then a static bifurcation occurs, and definitely there do not exist any stable equilibria near (x_0, p_0) on one side of z_{u_i} . This proves that when $\alpha^i > 0$, i.e. under condition (ASB4),

(ASB4) $\alpha^i > 0$.

the point (x_0, p_0) cannot be an interior point within the feasibility region, and we have proved the next Theorem.

Theorem 19 (Actuation limit induced static bifurcation Theorem) *Suppose the critical equilibrium $(x_0, p_0) \in EQ_{\text{critical}}$ satisfies the conditions (AB0), (AB1), (AB2), (AB3), and (AB4). Then a static bifurcation occurs at (x_0, p_0) . Moreover (x_0, p_0) cannot be an operating point, $(x_0, p_0) \notin OP$. Specifically (x_0, p_0) does not belong to any feasibility region, $(x_0, p_0) \notin F$. $\square$*

This proves that whenever α^i is positive at a critical equilibrium (x_0, p_0) , this immediately allows us to rule out this equilibrium from being an interior feasibility boundary point *without doing any eigenvalue computation*. Therefore this result stands as a powerful tool in computing the feasibility region of the actuation limited large system. It is even more surprising that we will develop a very similar criterion for the case of state limited model, even though at first sight, the two limits appear to be of a very different nature. Essentially the point is that for local computations such as in feasibility boundary analysis, there do exist similarities since conceptually the actuation limit is similar to the state limit in the sense of introducing nonsmoothness at the hardlimits.

We can formally define the actuation limit induced static bifurcation set B_{ASB} within EQ_{critical} by looking at the points where the transversality conditions (AB0)–(AB3) and (ASB4) are satisfied,

$$B_{ASB} := \{(x, p) \in EQ_{\text{critical}} : (AB0), (AB1), (AB2), (AB3) \text{ and (ASB4) are satisfied at } (x, p)\} \quad (274)$$

The previous Theorem 19 proves that $B_{ASB} \cap F = \emptyset$.

Note that by imposing the condition (ASB4), Theorem 19 is applicable only for the case of $\alpha^i > 0$. This is a restrictive assumption in the sense that from the transversality condition (SB2), we can only assume that $\alpha^i \neq 0$. This naturally takes to the next result which analyzes the problem for the case $\alpha^i < 0$. By replacing the static bifurcations condition (ASB4) by a dynamic condition (ADB4) stated below, we can conclude whether or not a critical equilibrium appears on the feasibility boundary ∂F or in the interior of F , when $\alpha^i < 0$.

(ADB4) $\alpha^i < 0$ and $n_{u_a}^i \neq n_{u_s}^i$.

From Proposition 5, we know that (x_0, p_0) is not a static bifurcation under the condition (ADB4) since the equilibrium structure does not change at (x_0, p_0) . Since EQ_a^i and EQ_s^i appear on the two sides of the limiting surface z_{u_i} , there exists only one equilibrium locally for parameter values p near p_0 on either side of z_{u_i} . The additional assumption on the eigenvalues that $n_{u_a}^i \neq n_{u_s}^i$ implies a change in the eigenstructure of the equilibrium at (x_0, p_0) , therefore under assumptions (AB0)–(AB3) and (ADB4) (x_0, p_0) emerges as a dynamic local bifurcation point. Since there is strictly a change in the eigenproperties, if one of the two, say EQ_a^i consist of operating points (i.e. $n_{u_a}^i = 0$), then the other one EQ_s^i cannot consist of operating points ($n_{u_s}^i \neq 0$). Therefore this point cannot lie in the interior of F . We have completed the proof of the next Theorem on dynamic bifurcations.

Theorem 20 (Actuation limit induced dynamic bifurcation Theorem) Suppose $(x_0, p_0) \in EQ_{\text{critical}}$ satisfies assumptions (AB0), (AB1), (AB2), (AB3) and (ADB4). Then (x_0, p_0) is a dynamic local bifurcation point. Moreover $(x_0, p_0) \notin OP$. In particular, (x_0, p_0) does not belong to any feasibility region. $\square$

Note that this result only enforces a necessary condition for the existence of actuation limit induced dynamic bifurcations. It does not provide any insight into the actual mechanism of the bifurcation (or an unfolding to be precise [37, 73]). Inspecting the relation between the Jacobians $J_a^i = D_x f_a^i$ and $J_s^i = D_x f_s^i$ in (273), we notice that the two are related by the factor $J_a^i = J_s^i + D_{x_i} f_s^i D_x h_i$. Here the difference $D_{x_i} f_s^i D_x h_i$ is a rank one matrix as the product of a column vector $D_{x_i} f_s^i$ and a row vector $D_x h_i$, but it is well-known that the eigenproperties of a matrix can be changed completely by adding even a rank one matrix (depending on the controllability criteria in the linear control theory). In other words, it is possible to construct generic examples where J_a^i has all eigenvalues with negative real parts, but J_s^i (upon reaching the limit) contains all eigenvalues with positive real parts. However in practice, this would seem unrealistic since the limit on z_i might affect only a few of the eigenvalues of the system Jacobian. More detailed investigation is necessary on the properties of the functions h_i and the equations f_i for deriving such results. Here we would like to point out that for the large system, there exist possibly a very large number of different possible scenarios associated with the actuation limit induced dynamic bifurcations, and therefore this phenomenon should be treated as a family of bifurcations. In two-dimensional systems i.e. by restricting to the case of $n = 2$, it is easier to derive precise results on some of the phenomena. For instance, the emergence of stable and unstable limit cycles near the limit induced dynamic bifurcations will be demonstrated in the next section.

Naturally as in the case of the previous cases, we can define the set B_{ADB} of limit induced dynamic bifurcations,

$$B_{ADB} := \{(x, p) \in EQ_{\text{critical}} : (x_0, p_0) \text{ satisfies (AB0)-(AB3) and (ADB4)}\} \quad (275)$$

As in the case of the unlimited model Σ , we can next impose some generic density assumptions to formally characterize the feasibility boundary as mostly consisting of the four bifurcations, namely the saddle node bifurcations B_{SN} (within the set $\partial F \cap C_Z$), Hopf bifurcations B_H (within the set $\partial F \cap C_H$) and the actuation limit induced static B_{ASB} and dynamic B_{ADB} bifurcations (within the set $\partial F \cap EQ_{\text{critical}}$).

$$(DA0) \quad \overline{\partial F \cap B_{SN}} = \partial F \cap C_Z, \quad \overline{\partial F \cap B_H} = \partial F \cap C_H \\ \text{and } \overline{\partial F(B_{ASB} \cup B_{ADB})} = \partial F \cap EQ_{\text{critical}}.$$

It can be seen that the assumption (DA0) is a generic one, since each of the bifurcations sets such as B_{SN} is obtained by imposing some inequality conditions (transversality conditions) within the respective zero sets.

Theorem 21 Under the assumption (DA0),

$$\partial F = \overline{F} = \overline{\partial F \cap B_{SN}} \cup \overline{\partial F \cap B_H} \cup \overline{\partial F \cap B_{ASB}} \cup \overline{\partial F \cap B_{ADB}} \quad (276) \\ \square$$

Before proceeding with the two-dimensional analysis in the next section, we state a simple result below which presents more information on the static bifurcation phenomena.

Corollary 4 Suppose $(x_0, p_0) \in EQ_{\text{critical}} \cap \overline{F}$. Assume that (AB0), (AB1), (AB2), (AB3) and (ASB4) are satisfied at (x_0, p_0) . Then (in addition to the statements of the Proposition 5), the following statements hold true locally near (x_0, p_0) :

(P5) Either $n_{u_a}^i = 0$ or $n_{u_s}^i = 0$, but not both.

(P6) $EQ = EQ_a^i \cup EQ_s^i \cup EQ_{\text{critical}}$. For the two cases:

$n_{u_a}^i = 0$: $EQ_a^i \subset OP$ and $EQ_s^i \cap OP = \emptyset$.

$n_{u_s}^i = 0$: $EQ_s^i \subset OP$ and $EQ_a^i \cap OP = \emptyset$.

(P7) $(x_0, p_0) \in \partial \overline{F}$. $\square$

D. Special results for two-dimensional systems

For the special case of two dimensional systems, i.e. with $n = 2$, the results characterizing the dynamic phenomena near the local bifurcations B_{ASB} (static) and B_{ADB} (dynamic) can be considerably strengthened. Note that by Assumptions (AB0)-(AB3), there exist two Jacobians J_a^i and J_s^i whose eigenstructure establishes how the bifurcation develops locally. Moreover since by (AB2), the two Jacobians have all eigenvalues away from the imaginary axis, the local dynamics for the two smooth systems Σ_a^i and Σ_s^i are completely determined topologically by their linear counterparts. Suppose we denote the eigenvalues of J_a^i to be λ_a^i (for an interesting case arises for the dynamic case B_{ADB} where hardlimit oscillations arise on one side of the bifurcations).

When one of the Jacobians say J_s^i has a pair of complex conjugate eigenvalues with positive real parts, and the other J_a^i has a pair of complex conjugate eigenvalues but with negative real parts. Then by direct local linear computations, the following two cases can be distinguished: 1) if the magnitude of the damping factor for the eigenvalues of J_a^i (the stable complex pair) denoted ζ_a^i is more than that ζ_s^i of the eigenvalues of J_s^i (the unstable complex pair), then the system is locally attracting at the bifurcation. 2) if the converse is true, that is, the damping factor of the right half-plane eigenvalues is more than that of the left half-plane eigenvalues, then the system is locally unstable. What is interesting is that there emerge hardlimit oscillations much like Hopf bifurcations when parametric perturbations are introduced. That is, in Type I) there emerge stable oscillations induced by the hardlimit surrounding the unstable focus on the other side: the bifurcation corresponds to a transition of operation from stable focus to a stable limit cycle, even though the operating point when returning from the limits has turned into an unstable focus (see Table III.1). On the other hand, in Type II), there emerges an unstable hardlimit induced nonsmooth limit cycle surrounding the stable focus on one side: the bifurcation here corresponds to a gradual annihilation of the stable focus leading to loss of stability. The distinction between the two cases can be seen intuitively from linear theory by explicitly computing the trajectories. As observed above, for the nonlinear problem Σ_{al} , this computation is sufficient to prove the result since the Jacobians J_a^i and J_s^i do not have any eigenvalues with zero real parts. The results is stated precisely in the form of illustrations in Table III.1. (the superscripts i have been left out for simplicity).

Table III.1 Actuation limit (windup) induced bifurcations in two-dimensional systems. (only three types are shown here. see Table III.2 for more details).

Type	Conditions	Bifurcation Diagram	$\mu < \mu_0$	$\mu = \mu_0$	$\mu > \mu_0$	Comments
I	$\det(J_a) > 0, \det(J_s) > 0,$ $\text{Tr}(J_a) < 0, \text{Tr}(J_s) < 0$ $\text{Tr}(J_a)^2 - 4\det(J_a) < 0$ $\text{Tr}(J_s)^2 - 4\det(J_s) < 0$ $\zeta_a > \zeta_s$					Transition from stable equilibrium to sustained stable hard-limit oscillations. resembles supercritical Hopf bifurcation.
II	$\det(J_a) > 0, \det(J_s) > 0,$ $\text{Tr}(J_a) < 0, \text{Tr}(J_s) < 0$ $\text{Tr}(J_a)^2 - 4\det(J_a) < 0$ $\text{Tr}(J_s)^2 - 4\det(J_s) < 0$ $\zeta_a < \zeta_s$					Gradual annihilation of the region of attraction. Unstable hard-limit periodic orbit shrinks onto stable focus making it an unstable focus. resembles subcritical Hopf bifurcation.
III	$\det(J_a) > 0, \det(J_s) < 0,$ $\text{Tr}(J_a) < 0,$ $\text{Tr}(J_a)^2 - 4\det(J_a) > 0$					Annihilation of the stable operating point. resembles saddle node bifurcation.

III.3.2. State limits (Nonwindup limiters)

For parametric analysis, the dynamic generating model for the state limits can be described by

$$\Sigma_{sl}: \dot{x} = f(x, p), \quad x \in \mathbb{R}^n, \quad p \in \mathbb{R}^q, \quad f: \mathbb{R}^{n+q} \rightarrow \mathbb{R}^n, \quad (277)$$

where p denotes the set of parameters under consideration. As before, the state limits can be described by requiring the inequalities

$$x_{li} \leq x_i \leq x_{ui}, \quad i = 1, 2, \dots, n. \quad (278)$$

at all times. The state space features of the dynamics generated by this constrained model was analyzed in Section III.2.2. Essentially a state variable (say x_i) loses its degree of freedom for its motion when it reaches its limiting value (say $x_i = x_{ui}$, the upper limit) provided the motion is pointing towards exceeding this limiting value (i.e. if $f_i > 0$). Recall that this hard-limit (the upper limit $H_{li}^{I_u}$) becomes active forcing the dynamics to stay at its limiting value, $x_i(t) \equiv x_{ui}$ until the time when the motion is pointing inwards again. As we observed in Section III.2.2, this unique feature of the state limited model introduces a rich complexity in the dynamics: the dimension of the constrained dynamics may range from n (no hard-limits active) to 0 (all hard-limits active) depending on how many limits are reached and are active. Therefore the problem of constructing the feasibility region in the parameter space becomes that of piecing together the relevant regions for several subsystems of varying system dimensions. As a first step, let us define the set of equilibria in the next subsection.

A. Equilibria

Define the index set $\mathcal{I}$ consisting of positive integers from 1 to n

$$\mathcal{I} := \{1, 2, \dots, n\} \quad (279)$$

Recall from Section III.2.2 the notation for level k hard-limit constraint surface $H_{li}^{I_u}$. Here I_u stands for the index set for the upper limits reached, while the index set I_ℓ corresponds to that for the lower limits, that is,

$$H_{li}^{I_u} := \{(x, p) \in \mathbb{R}^n : x_i = x_{ui}, \forall i \in I_u, x_i = x_{li}, \forall i \in I_\ell, x_{li} < x_i < x_{ui}, \forall i \in \mathcal{I} \setminus (I_\ell \cup I_u)\} \quad (280)$$

Note that compared to the definition in Section III.2.2, here the signs of relevant state derivative terms which appear within brackets (associated with each index) has been left out for convenience. From the set $H_{li}^{I_u}$, we can easily separate out the *active hard-limit set* defined below as $\hat{H}_{li}^{I_u}$ by enforcing all the necessary derivative rules:

$$\hat{H}_{li}^{I_u} := \{(x, p) \in H_{li}^{I_u} : f_i(x, p) > 0 \forall i \in I_u, f_i(x, p) < 0 \forall i \in I_\ell\} \quad (281)$$

As stated earlier, the dynamics on the active hard-limit sets $\hat{H}_{li}^{I_u}$ is described by the constrained model $\dot{x} = f^I(x)$ where $I = I_u \cup I_\ell$ with $x_i \equiv x_{li}$ for $i \in I_\ell$ and $x_i \equiv x_{ui}$ for all $i \in I_u$. Basically f^I is obtained by setting the derivatives $\dot{x}_i = 0$ for all those states in the index set $i \in I$. This naturally leads us to the g notions of a reduced set of state variables x_R^I and an associated reduced vector field $f_R^I(x, p)$ defined below. Let $J = \mathcal{I} \setminus I$ as before and let $j = \text{card}(J)$ denote the cardinal number of the index set J . Throughout this section, we will use the notation $I = I_\ell \cup I_u$, $J = \mathcal{I} \setminus I$ for denoting the index sets associated with the hard-limit set $H_{li}^{I_u}$ under consideration. Given any (x, p) and an associated index set I , we define

$$x_R^I := (x_{J(1)}, x_{J(2)}, \dots, x_{J(j)})^T \quad (282)$$

$$f_R^I(x_R^I, p) := (f_{J(1)}(x, p), f_{J(2)}(x, p), \dots, f_{J(j)}(x, p))^T \quad (283)$$

where $(\cdot)^T$ denotes the transpose operator (for easier representation of the vectors), and $J(i)$ stands for the i -th entry of the complementary index set J . Note that $\dot{x}_R^I = f_R^I(x_R^I, p)$ is a smooth dynamical system, and provides a convenient way for representing the limit-constrained dynamics. When $(x, p) \in \hat{H}_{li}^{I_u}$ (when

active), this smooth system generates the j -dimensional flow on the flow stratum $\hat{H}_I^{I_u}$ which is then supported by the trajectories transversally reaching (x, p) in the remaining $(n-j)$ -directions corresponding to the states x_i in the index set I . Naturally each of these smooth systems can possess equilibrium points, and depending on the size of the index set (say k), these can be collected together in the form of level k equilibrium points.

Here level 0 equilibrium points correspond to normal equilibrium points in X_0 where $f(x, p) = 0$ with no hard-limits reached. However for $k \geq 1$, note that $f(x, p)$ need not equal zero. For instance, consider a point say $(x_0, p_0) \in \hat{H}^i$ (i.e. $x_i = x_{u_i}$ and $f_i(x, p) > 0$), such that the other $n-1$ derivatives vanish ($f_j(x_0, p_0) = 0$ for $j \neq i$). Then (x_0, p_0) qualifies as an equilibrium point from a modelling view-point because in the model $f^i(x_0, p_0) = 0$. However such a point (x_0, p_0) exhibits a special character in that there exists a trajectory that reaches (x_0, p_0) in finite time in the x_i direction with nonzero velocity $f_i(x, p) > 0$. In the model, this velocity is then arbitrarily set to zero upon reaching (x_0, p_0) . Strictly speaking, we have to remember that the driving term f_i is still nonzero at (x_0, p_0) but only the state derivative $\dot{x}_i$ has been forced to become zero by the state limit. From a modelling view-point, in generating our constrained dynamics (in Section III.2.2), we have assumed that this derivative $\dot{x}_i$ becomes zero once upon H^i so that H^i is rendered invariant for the generating dynamics. In some implementations such as in overcurrent limiters for generator excitation systems, there is also a separate mechanism which switches off the whole unit when the operation remains near the maximum value over a long period of time (to prevent any permanent damage). Hence one may argue whether points such as (x_0, p_0) should even be modeled as equilibria or not. In this section, such switching effects are not modeled; therefore (x_0, p_0) will be treated as an equilibrium within the scope of this model. When such switching logic is included in the analysis, the concept of equilibrium is somewhat different and can be seen as formulated in Section III.2.3. However we do distinguish such limit induced equilibrium points from normal equilibria by explicitly specifying the number ($=k$) of hardlimits reached, that is, by using the notation of level k equilibria.

The set of level k equilibrium points can then be defined as EQ^k where

$$EQ^k := \{(x, p) \in H_I^{I_u} : \text{card}(I) = k, f_R^I(x_R^I, p) = 0, f_i(x, p) \geq 0 \forall i \in I_u, f_i(x, p) \leq 0 \forall i \in I_l\} \quad (284)$$

The collection of all these level k equilibria is defined EQ ,

$$EQ := \bigcup_{k=0}^n EQ^k \quad (285)$$

All the topological operations such as boundary, closure etc. which follow in this section are defined with respect to EQ . Among the level k equilibria, the points which belong to the active set $\hat{H}_I^{I_u}$ are distinguished as the active level k equilibria EQ_{active}^k while the remaining ones are simply denoted the critical level k equilibria EQ_{critical}^k ,

$$EQ_{\text{active}}^k = \{(x, p) \in H_I^{I_u} \cap EQ^k : (x, p) \in \hat{H}_I^{I_u}\} \quad (286)$$

$$EQ_{\text{critical}}^k = EQ^k \setminus EQ_{\text{active}}^k \quad (287)$$

Note that by construction, (by the inequality conditions in (284)), an equilibrium point cannot belong to the inactive hardlimit

sets. For convenience, we can divide the full set EQ also into EQ_{active} and EQ_{critical} as

$$EQ_{\text{active}} := \bigcup_{k=0}^n EQ_{\text{active}}^k, \quad EQ_{\text{critical}} := \bigcup_{k=0}^n EQ_{\text{critical}}^k \quad (288)$$

by taking the respective unions over all levels k .

B. Operating points

As discussed in Section III.2.2, the stability status of active equilibria can be readily determined by looking at is associated reduced order dynamics $\dot{x}_R^I = f_R^I(x_R^I, p)$. Therefore we can identify the small-signal stable equilibria among these equilibrium points by imposing an additional constraint on the relevant Jacobian. These small-signal stable equilibria are called the set of level k operating points OP^k ,

$$OP^k := \{(x, p) \in EQ_{\text{active}}^k : \sigma(J_R^I(x, p)) \subset \mathbb{C}^- \text{ where } J_R^I(x, p) := (D_{x_R^I} f_R^I)(x, p)\} \quad (289)$$

In other words, we require the familiar eigenvalue condition (that the real parts should be negative) only for the reduced system Jacobian J_R^I . The remaining directions are strictly attractive because the hardlimit set has been assumed to be active.

It needs to be pointed out that in typical stability problems, there are not more than ten (which is on the high side) hardlimits that may be reached during system operation. However with continuously growing loads across the network, and considering the recent industrial reportings of hardlimit associated instability events [131], it becomes important to understand the structure of such hardlimited system operation in general. Here again a cautionary note is in order on the long term implications of such limited operations. In this section, we will analyze the structure of the system operation in the parameter space in light of these hardlimits, which will be aimed at identifying the small-signal instability limits (feasibility boundary to be precise), and in the process discover a rich set of new phenomena directly associated with state limits being encountered. These range from limiter induced annihilation of equilibria to emergence of hardlimit induced oscillations.

First let us collect the level k operating points OP^k for all the levels starting from level 0 (what should be the normal operating points) to level n operating points (may not be practical since all limits need be encountered and be active, yet included here for the sake of completeness). Now for defining the set of all operating points say OP , this union $\bigcup_{k=0}^n OP^k$ is itself not complete because so far we have not considered the critical equilibrium points in EQ_{critical}^k . It is quite possible that some of these critical points act as interior points in OP or as transition points from one level operating point (say in OP^k) to an operating point of the next hardlimit level (such as in OP^{k-1} or OP^{k+1}). To include such points, we use a simple topological operation which is similar to a concept introduced by Zaborsky et. al. in [114] for the stability boundary analysis:

$$OP := \text{int} \left(\bigcup_{k=0}^n OP^k \right) \quad (290)$$

where the closure is taken with respect to EQ . At this point, a technical result is necessary which states that a level k operating point in OP^k is indeed an operating point in the sense of (290):

Lemma 39

$$\bigcup_{k=0}^n OP^k \subset OP \quad (291)$$

Proof: By the implicit function theorem, it follows that OP^k is a q -dimensional smooth manifold in $\mathbb{R}^{n+q}$ since the Jacobian J_R^I is nonsingular. The openness of OP^k in EQ follows, and hence the proof. $\square$

The additional operating points which have been picked up in the topological operation above are defined as the transition operating points OP_{transit} ,

$$OP_{\text{transit}} := OP \setminus \left(\bigcup_{k=0}^n OP^k \right) \quad (292)$$

We will see later that one of the main challenges associated with hardlimited operating points in the large system is the difficulty that arises in distinguishing the transition operating points OP_{transit} from the true transition boundary points which lead to loss of small-signal stability. However at this point, we will just make a claim that “half the time, this problem is easy to solve”. The claim is connected with a precise result called the “State limit induced static bifurcation Theorem” that will be stated later in this Section III.3.2.

C. The feasibility region

Briefly summarizing the construction so far, we have identified and defined the set of all equilibria EQ and the set of all small-signal stable operating points OP . The next task is the definition of the feasibility region F which is a direct extension of the concept in [127, 86] (see Section I.3.1).

Definition 15 (Feasibility region) *Given an operating point $(x, p) \in OP$, the feasibility region F of (x, p) is the connected component of OP that contains (x, p) . The boundary of F (relative to EQ) is the feasibility boundary denoted ∂F .* $\square$

Here note that by construction, OP is open in EQ since it is the set of interior points of the closed set $\left(\bigcup_{k=0}^n OP^k \right)$. Therefore OP can be divided into connected components: the one component that contains the operating point (x, p) under consideration then gives us the feasibility region associated with (x, p) . This definition implicitly permits the possibility of multiple operating points for the same parameter value p within a feasibility region F . Whether such cases arise in the real system is an open question at the moment. The main feature of the feasibility region (which has been repeated many times in the previous sections) is that the system operation (in the sense of slow quasi-static parameter variations) can move around freely within the feasibility region without an imminent danger of losing the small-signal stability at the equilibrium, whatever be the number of hardlimits encountered in the process (across various levels in our notation). The feasibility boundary then defines the parameter values which correspond to critical cases of small-signal stability at the equilibria. The characterization of the feasibility boundary ∂F is given first in the form of Theorem 22. In the next section, the boundary segments are analyzed in more detail.

Theorem 22 (Feasibility boundary Theorem with state limits)

$$\partial F = (\partial F \cap C_Z) \cup (\partial F \cap C_H) \cup (\partial F \cap EQ_{\text{critical}}) \quad (293)$$

where

$$C_Z := \bigcup_{k=0}^n C_Z^k, \quad C_Z^k := \{(x, p) \in EQ_{\text{active}}^k : \det(J_R^I)(x, p) = 0\} \quad (294)$$

$$C_H := \bigcup_{k=0}^n C_H^k, \quad C_H^k := \{(x, p) \in EQ_{\text{active}}^k \setminus C_Z^k : \det(H_{n-k-1}(J_R^I))(x, p) = 0\} \quad (295)$$

where $H_{n-k-1}(\cdot)$ denotes the Hurwitz matrix of order $n - k - 1$ as before.

Proof: Since $EQ = EQ_{\text{critical}} \cup EQ_{\text{active}}$, to prove the result, we only need to show that

$$(\partial F \cap EQ_{\text{active}}^k) \subset (\partial F \cap (C_Z^k \cup C_H^k)) \quad (296)$$

for each k . This follows from standard results since EQ_{active}^k is open in EQ . That is, since EQ_{active}^k is open in EQ , locally the boundary statement problem reduces to the problem restated for the smooth system $\dot{x}_R^I = f_R^I(x_R^I, p)$. As stated before, the feasibility boundary for this smooth system is completely exhausted by the union $C_Z^k \cup C_H^k$ which completes the proof. $\square$ E.D.

Remarks 1. This Theorem proves that the feasibility boundary for the large system model that includes state limits can be described explicitly as common zeros of a set of functions (called zero sets as before), hence is computationally manageable. The computational load appears to be comparable to the model without the state limits.

2. The first two sets in the union C_Z and C_H are direct counterparts of corresponding terms for the unlimited system (well-known in classical linear theory [124]), related to zero eigenvalues and purely imaginary eigenvalues respectively. The third term from EQ_{critical} is a special characteristic of the state limited model. More on this later.

3. Closer inspection of the first two terms in (293) reveals that even among these, the true counterparts of the smooth system are only C_Z^0 and C_H^0 which are the terms from level 0 (i.e. no state limits reached) in our construction. Note that for terms C_Z^k and C_H^k for $k \geq 1$, at least one hardlimit is involved and is active, hence the relevant system dimension of f_R^I is lower. In other words, these boundary points for level $k \geq 1$ whenever present are strictly different from those of the model that does not include the state limits. Of course, from operation view-point, the big question is, how far these boundary segments for the limited system are, in the sense of parameter variations, compared to a model that does not include the state limits. Whatever be the case, it is clear that since this state limited model is closer to reality, the results in Theorem 22 are more accurate than an unlimited state model.

4. Using standard results from bifurcation theory, we will prove in the next section that most of the boundary points in the first two terms in (293) do lead to loss of small-signal stability of the operating point. That is, mathematically $\partial F \cap C_Z \subset \partial \bar{F}$ and $\partial F \cap C_H \subset \partial \bar{F}$ under certain nondegenerate assumption which will be made precise in Section D. Therefore it is sufficient to simply solve for the relevant zero set conditions (294) and (295) to locate the feasibility boundary connected with C_Z and C_H since most of these are generically guaranteed to be instability points. However this is not true for the third boundary segment EQ_{critical} .

5. Generically there do exist critical equilibrium points in EQ_{critical} which are actually operating points (the transition operating points in OP_{transit}). Critical equilibria appear both in the interior and on the boundary of the feasibility region, hence we need more tools for distinguishing between the two in an efficient manner for the large system. In the next section, we develop one such powerful machinery, which is based on the intuitive argument that generically we cannot have two operating points arbitrarily nearby for the same parameter value.

D. Feasibility boundary analysis

In this section, we analyze the principal bifurcation phenomena associated with the three feasibility boundary segments, namely C_Z , C_H and EQ_{critical} respectively. The set C_Z related to zero eigenvalues will be shown to lead to saddle node bifurcations (for the level k reduced systems) under required transversality assumptions as in standard bifurcation theory [37]. Similarly we will see that the set C_H connected with purely imaginary eigenvalues generically corresponds to Hopf bifurcations for the reduced systems. The third segment EQ_{critical} is more interesting, owing its presence to the character of the state limits. These are here broadly classified into *state limit induced static bifurcation* (say SSB in short) and *state limit induced dynamic bifurcations* (say SDB in short), depending on the equilibrium structure of the system near the boundary points. As the names suggest, the equilibrium structure undergoes a change near the static bifurcations, whereas the dynamic bifurcations are related to changes in the local dynamic properties at the equilibrium point. A detailed analysis of the eigenstructure near these bifurcations is postponed to Section E. Here we focus on establishing the essential instability characteristics of the feasibility boundary.

We can easily recognize the set B_{SN} connected with the saddle node bifurcation within the zero eigenvalue set C_Z by enforcing the transversality conditions (SN1), (SN2) and (SN3) from Sotomayor's Theorem [76]. For our problem specifically, a technicality is that the conditions need to be associated with the reduced systems $\dot{x}_R = f_R^I(x_R, p)$ for each level set C_Z^k :

$$B_{SN} := \bigcup_{k=0}^n B_{SN}^k \quad (297)$$

where

$$B_{SN}^k := \{(x, p) \in C_Z^k : (x, p) \text{ satisfies (SN1), (SN2) and (SN3) for } \dot{x}_R = f_R^I(x_R, p)\} \quad (298)$$

In the definition, recall that $I = I_u \cup I_l$ denotes the index set associated with the level k hardlimit set $H_{I_l}^k$. Similarly the Hopf bifurcation set B_H can be constructed from the imaginary eigenvalue boundary set C_H by imposing the Hopf transversality conditions (H1) and (H2) [37]:

$$B_H := \bigcup_{k=0}^n B_H^k \quad (299)$$

where

$$B_H^k := \{(x, p) \in C_H^k : (x, p) \text{ satisfies (H1) and (H2) for the associated} \quad (300)$$

$$\text{system } \dot{x}_R = f_R^I(x_R, p)\} \quad (301)$$

The third segment $\partial F \cap EQ_{\text{critical}}$ is more difficult to analyze since it typically involves the transitions from one reduced order system to another. As noted in the previous section, the

difficulty is because of the fact that these transition points in EQ_{critical} can also be operating points. However the next Theorem tells us that we can directly conclude that some points in EQ_{critical} cannot be operating points (they must be boundary points if in $\bar{F}$) provided a certain constant α_s^I is positive and certain transversality assumptions (SB1), (SB2) and (SB3) are satisfied. More details on the structure of the equilibria near these points and the proof of the Theorem can be seen in the next section. In the statement of the Theorem, we need the following two quantities the significance of which will be evident from the proof in the next Section,

$$J^S := D_{x_R} f_R^S, \quad J^I := D_{x_R} f_R^I \quad (302)$$

Let us denote by n_u^I and n_u^S the number of eigenvalues of J^I and J^S which have positive real parts.

Theorem 23 (State limit induced static bifurcation Theorem) Suppose $(x_0, p_0) \in EQ_{\text{critical}}^k$ and assume that the following conditions are satisfied at (x_0, p_0) :

(SB1) $f_s(x_0, p_0) = 0$ for some $s \in I$ and $f_i(x_0, p_0) \neq 0 \forall i \in S$ where $S := I \setminus \{s\}$.

(SB2) All eigenvalues of J^S and J^I have nonzero real parts.

(SB3)

$$(D_p f_s - D_{x_R} f_s (D_{x_R} f_R^I)^{-1} D_p f_R^I) \neq 0 \quad (303)$$

(SSB4))

$$\alpha_s^I := \frac{\det(J^S)}{\det(J^I)} > 0 \quad (304)$$

Then (x_0, p_0) is a static bifurcation point and cannot be an operating point, i.e. $(x_0, p_0) \notin OP$. Specifically $(x_0, p_0) \notin F$. $\square$

A simpler criterion to evaluate α_s^I can be developed using Schur's Theorem. We can identify the set B_{SSB} as the bifurcation points where these transversality conditions (SB1), (SB2), (SB3) and (SSB4) are satisfied:

$$B_{SSB} := \bigcup_{k=1}^n B_{SSB}^k \quad (305)$$

where

$$B_{SSB}^k := \{(x, p) \in EQ_{\text{critical}}^k : (x, p) \text{ satisfies (SB1), (SB2), (SB3) and (SSB4)}\} \quad (306)$$

Basically the Theorem above guarantees that $B_{SSB} \cap F = \emptyset$. While the conditions (SB1), (SB2) and (SB3) are nondegenerate conditions, the assumption (SSB4) requires that certain quantity α_s^I is positive. This condition (SSB4) is not generic for our problem, and to complete the analysis of EQ_{critical} , we also need to look at points with $\alpha_s^I < 0$. (Note that by Assumption (SB2), α_s^I must be nonzero.) Here we state another result from the next Section on the dynamic nature of the boundary points:

Theorem 24 (State limit induced dynamic bifurcation Theorem) Suppose $(x_0, p_0) \in EQ_{\text{critical}}^k$ and assume that the conditions (SB1), (SB2) and (SB3) are satisfied at (x_0, p_0) along with the additional assumption that

(SDB4)) $\alpha_s^I < 0$ and $n_u^I \neq n_u^S$.

Then (x_0, p_0) is a dynamic bifurcation point and cannot be an operating point, i.e. $(x_0, p_0) \notin OP$. Specifically $(x_0, p_0) \notin F$. $\square$

A quick comparison of the static Theorem 23 and the dynamic Theorem 24 reveals that clearly the static Theorem is easier to check. This merely requires that $\alpha_s^I > 0$. However when this condition is violated, it also becomes necessary to compare the eigenstructure of the Jacobians of two reduced order systems which is more time-consuming for the large system.

Note that the conditions (SB1)–(SB3) are common between the two Theorems and these are nondegenerate assumptions for points in the set EQ_{critical} . As observed earlier, the big question in feasibility boundary analysis is how to distinguish whether a transition equilibrium say $(x_0, p_0) \in EQ_{\text{critical}}$ is an interior point in OP_{transit} or an exterior point. The two Theorems together provide us the answer generically. Also now we can substantiate our earlier claim that “half-the time, the problem is easy”. Assume that the nondegenerate assumptions (SB1)–(SB3) are satisfied. Then clearly the problem is easier “half-the time” when $\alpha_s^I > 0$. This directly implies that (x_0, p_0) is not an operating point, and there is no need for any eigenvalue computation. However when $\alpha_s^I < 0$, then there is an additional computation to be done, namely the comparison of n_u^I and n_u^S . The information that they are not equal suffices to rule out the possibility of (x_0, p_0) being an operating point by Theorem 24. An important property by definition (see (SB2)) is that the matrix J^I is a submatrix of J^S by deleting a row (corresponding to the index s) and a column (again the one with the index s). Therefore the structure and properties of the Jacobians J^I and J^S in the large system can be used to simplify this test, but this will not be attempted here. To complete the analysis, we provide a Corollary below, which provides the necessary conditions for the interior transition points in OP_{transit} under (SB1)–(SB3).

Corollary 5 Suppose $(x_0, p_0) \in EQ_{\text{critical}} \cap OP_{\text{transit}}$ and conditions (SB1), (SB2) and (SB3) are satisfied at (x_0, p_0) . Then $\alpha_s^I(x_0, p_0) < 0$ and $n_u^I = n_u^S = 0$. $\square$

Going to back to our construction of the principal bifurcation sets within ∂F , we can define a set B_{SDB} corresponding to the state limit induced dynamic bifurcations:

$$B_{SDB} := \bigcup_{k=1}^n B_{SDB}^k \quad (307)$$

where

$$B_{SDB}^k := \{(x, p) \in EQ_{\text{critical}}^k : (x, p) \text{ satisfies (SB1), (SB2), (SB3) and (SDB4)}\} \quad (308)$$

Theorem 24 proves that $B_{SDB} \cap F = \emptyset$. The two Theorems along with the Corollary imply that under the transversality conditions (SB1)–(SB3), the third boundary segment satisfies

$$\partial F \cap EQ_{\text{critical}} = \partial F \cap (B_{SSB} \cup B_{SDB}) \quad (309)$$

Since the conditions (SB1)–(SB3) are nondegenerate, we can assume that generically these conditions are satisfied in a dense subset of $\partial F \cap EQ_{\text{critical}}$. Similarly we can invoke genericity arguments to justify the following density assumptions on B_{SN} and B_H :

Assumption (DS0) $\partial F \cap C_Z = \overline{\partial F \cap B_{SN}}$, $\partial F \cap C_H = \overline{\partial F \cap B_H}$ and $\partial F \cap EQ_{\text{critical}} = \overline{\partial F \cap (B_{SSB} \cup B_{SDB})}$.

For the sake of completeness, we next state a Theorem that mirrors its counterpart for the (unlimited) smooth system. How-

ever remember that in this case, for the state limited system, the main result that $\partial F = \partial \bar{F}$ is not a surprise but actually a property of our definition because of the topological operation (290). Yet the Theorem states that most of the boundary points undergo one of four bifurcations: namely, the saddle node bifurcation, Hopf bifurcation, state limit induced static bifurcation and state limit induced dynamic bifurcation. A more detailed analysis of the latter two bifurcations is carried out in the next section.

Theorem 25 Under Assumption (DS0),

$$\partial F = \overline{\partial F \cap B_{SN}} \cup \overline{\partial F \cap B_H} \cup \overline{\partial F \cap (B_{SSB} \cup B_{SDB})} = \partial \bar{F} \quad (310)$$

$\square$

E. State limit induced bifurcations

In this section, we analyze the boundary set $\partial F \cap EQ_{\text{critical}}$ and develop the necessary transversality conditions. Suppose $(x_0, p_0) \in EQ_{\text{critical}}$. Then by definition, $(x_0, p_0) \in H_{I_u}^I$ for some index sets I_u and I_l with $\text{card}(I_u \cup I_l) = k$. Moreover since $(x_0, p_0) \in EQ^k$, by necessity, the reduced order vector field $f_R^I(x_R^I, p)$ must vanish at (x_0, p_0) . If in addition, all the state derivatives $f_i(x_0, p_0)$ were nonzero and positive (negative) for $i \in I_u$ ($i \in I_l$), then actually the equilibrium would be an active one, i.e. $(x_0, p_0) \in EQ_{\text{active}}^k$. However since we have assumed that $(x_0, p_0) \in EQ_{\text{critical}}^k$, at least one of these derivatives f_i for $i \in I$ must be zero at (x_0, p_0) . First we assume that exactly one of these derivatives vanish at (x_0, p_0) and denote this index by s ,

(SB1) $f_s(x_0, p_0) = 0$ for some $s \in I$. $f_i(x_0, p_0) \neq 0 \forall i \in S$ where $S := I \setminus \{s\}$.

The condition (SB1) implies that $f^S(x_0, p_0) = 0$, i.e. (x_0, p_0) is indeed an equilibrium for the level $k-1$ reduced system with associated index set S . Therefore (x_0, p_0) stands as the transition equilibrium from level $k-1$ (with index set S) to level k (with index set I). Therefore note that there are two reduced order systems involved at (x_0, p_0) namely for the level $k-1$ index set S and that for the level k index set I respectively. Let us denote these as Σ^S and Σ^I respectively:

$$\Sigma^S: \quad \dot{x}_R^S = f_R^S(x_R^S, p), \quad f_R^S: \mathbb{R}^{n-k+1+q} \rightarrow \mathbb{R}^{n-k+1} \quad (311)$$

$$\Sigma^I: \quad \dot{x}_R^I = f_R^I(x_R^I, p), \quad f_R^I: \mathbb{R}^{n-k+q} \rightarrow \mathbb{R}^{n-k} \quad (312)$$

The bifurcation point (x_0, p_0) then occurs as a transition in the reduced order system description from Σ^S to Σ^I at (x_0, p_0) . To understand this transition, we next impose an assumption that the transition equilibrium (x_0, p_0) is hyperbolic for each of the two levels:

(SB2) All eigenvalues of $J^S := D_{x_R^S} f_R^S$ evaluated at (x_0, p_0) have nonzero real parts. All eigenvalues of $J^I := D_{x_R^I} f_R^I$ evaluated at (x_0, p_0) have nonzero real parts.

Specifically the condition (SB2) implies that the Jacobians J^S and J^I are nonsingular. Therefore by the implicit function theorem, the reduced systems Σ^S and Σ^I each locally possess q -dimensional equilibrium manifolds say $\bar{E}Q^S$ and $\bar{E}Q^I$ respectively, at (x_0, p_0) . However these equilibria of Σ^S and Σ^I are meaningful for our problem (i.e. for EQ) only when the required inequality conditions in (284) are also satisfied. To carry out such an analysis, we need certain transversality assumptions on the parameter dependence.

Let us look at the equilibrium locus $\tilde{EQ}^I$ of the reduced system Σ^I . For instance, when $s \in I_u$ (i.e. the transition occurs at the upper limit of the state x_s), an equilibrium of the reduced system Σ^I say $(\tilde{x}, \tilde{p}) \in \tilde{EQ}^I$ will be an equilibrium for the limited system Σ_{sI} only when f_s is active, namely when $f_s(\tilde{x}, \tilde{p}) > 0$. Since $f_s(x_0, p_0) = 0$, to get the first order approximation of f_s along $\tilde{EQ}^I$, we impose the assumption that the first order derivative of f_s along $\tilde{EQ}^I$ is nonzero,

(SB3)

$$D_p(f_s(\tilde{EQ}^I))|_{(x_0, p_0)} = (D_p f_s - D_{x_R} f_s (D_{x_R} f_R^I)^{-1} D_p f_R^I)|_{(x_0, p_0)} \neq 0 \quad (313)$$

Similarly for analyzing whether $\tilde{EQ}^S$ belongs to EQ , we need to compute the first order derivative $D_p(x_s(\tilde{EQ}^S))$ of the state x_s along the equilibrium locus $\tilde{EQ}^S$. The next Lemma states that (SB2) and (SB3) together imply that this derivative $D_p(x_s(\tilde{EQ}^S))$ will be nonzero. Recall from the statements of the Theorems 23 and 24 that the following quantity defined α_s^I plays a fundamental role for this specific bifurcation problem:

$$\alpha_s^I = \frac{\det(J^S)}{\det(J^I)} \quad (314)$$

Note that by the condition (SB2), the quantity α_s^I must be nonzero. The next Lemma relates the two parameter dependences of f_s along EQ^I and x_s along EQ^S by the factor α_s^I ,

Lemma 40

$$D_p(f_s(\tilde{EQ}^I))|_{(x_0, p_0)} = -\alpha_s^I D_p(x_s(\tilde{EQ}^S))|_{(x_0, p_0)} \quad (315)$$

Proof: The proof follows from the definition of α_s^I , an application of Schur's Theorem, and by chain rule of differentiation. $\square$

Now we are in a position to state the main result on the relationship between the equilibrium structure and their eigenproperties near (x, p) as determined by the coefficient α_s^I .

Proposition 6 Consider $(x, p) \in EQ_{\text{critical}}^k$. Suppose (SB1), (SB2) and (SB3) are satisfied at (x_0, p_0) . Then the following statements hold true locally near (x_0, p_0) : EQ is a q -dimensional topological manifold. Define

$$W := \{(x, p) \in \mathbb{R}^{n+q} : f_s(x, p) = 0\} \quad (316)$$

(P1) W is a $(n+q-1)$ -dimensional smooth manifold in $\mathbb{R}^{n+q}$.

(P2) $EQ_{\text{critical}}^k = EQ \cap W$.

(P3) $EQ \setminus EQ_{\text{critical}}$ consists of two connected components called EQ^I and EQ^S which satisfy $f_R^I(EQ^I) = 0$ and $f_R^S(EQ^S) = 0$ respectively. EQ^I and EQ^S are q -dimensional smooth manifolds.

(P4) Let n_u^I denote the number of eigenvalues of J^I with positive real parts and n_u^S denote the number of eigenvalues of J^S with positive real parts.

$\alpha_s^I > 0$: (x_0, p_0) is a static bifurcation point. EQ^I and EQ^S both lie on the same side of the manifold W . Moreover $n_u^I \neq n_u^S$.
 $\alpha_s^I < 0$: (x_0, p_0) is not a static bifurcation point. EQ^I and EQ^S lie on the two different sides of the manifold W .

Proof: Note that by (SB3), at (x_0, p_0) , either $D_p f_s \neq 0$ or $D_{x_R} f_s \neq 0$, therefore W is a $(n+q-1)$ -dimensional smooth manifold at (x_0, p_0) by Implicit function Theorem. This proves (P1) and (P2). The rest of the proof is straightforward from the

Lemma 40. Basically the derivative $D_p f_s(\tilde{EQ}^I)$ in (315) determines which side of W (only one side since the intersection is transversal by (SB3)), the equilibrium locus $\tilde{EQ}^I$ satisfies the inequality constraint (284) to qualify as level k equilibria EQ^k . Denote this as EQ^I . By Lemma 40, the derivative $D_p x_s(\tilde{EQ}^S)$ is nonzero, and this derivative determines which side of W (only one side again by transversality), the locus $\tilde{EQ}^S$ satisfies the inequality requirement (284) to qualify as level $k-1$ equilibria. Define this qualifying segment of $\tilde{EQ}^S$ to be EQ^S . This proves (P3). By simple geometric arguments based on equation (315), (P4) follows. The stability result that $\alpha_s^I > 0$ implies $n_u^I \neq n_u^S$, directly follows from the definition of α_s^I , since the dimension of Σ^S is one higher than that of Σ^I . Q. E. D.

For simplicity, we can restate part of this result on the static properties as a Corollary for a single parameter variation say when $p = \mu$ is varied. First let us define a constant a_μ for the slope in (SB3):

(SB3)

$$a_\mu := (D_\mu f_s - D_{x_R} f_s (D_{x_R} f_R^I)^{-1} D_\mu f_R^I)|_{(x_0, \mu_0)} \neq 0 \quad (317)$$

The proof of the Corollary directly follows from the Proposition.

Corollary 6 Suppose $(x_0, \mu_0) \in EQ_{\text{critical}}$ and (SB1), (SB2) and (SB3) are satisfied at (x_0, μ_0) . Then the statements below follow locally near (x_0, μ_0) :

$\alpha_s^I > 0$: (x_0, μ_0) is a static bifurcation point.

(Case 1:) $a_\mu > 0$ and $s \in I_u$, or, $a_\mu < 0$ and $s \in I_l$: there exist two equilibria for $\mu > \mu_0$, one equilibrium at $\mu = \mu_0$, and no equilibria for $\mu < \mu_0$.

(Case 2:) $a_\mu < 0$ and $s \in I_u$, or, $a_\mu > 0$ and $s \in I_l$: there exist two equilibria for $\mu < \mu_0$, one equilibrium at $\mu = \mu_0$, and no equilibria for $\mu > \mu_0$.

$\alpha_s^I < 0$: (x_0, μ_0) is not a static bifurcation point. Given any μ near μ_0 , there exists a unique equilibrium point. $\square$

Remarks 1. The strength of the Proposition lies in the fact that it provides a simple algebraic criterion, namely that $\alpha_s^I > 0$ for guaranteeing the occurrence of a static bifurcation. The remaining conditions (SB1), (SB2) and (SB3) being nondegenerate assumptions are generically satisfied for points in EQ_{critical} .

2. For the case of $\alpha_s^I > 0$, it is also proved that the small-signal stability properties of the equilibrium loci of the two reduced order systems Σ^I and Σ^S must be different near the bifurcation since $n_u^I \neq n_u^S$. Suppose that $n_u^I = 0$ (i.e. $EQ^I \subset OP$). Then this implies that n_u^S must be nonzero, i.e. $EQ^S \cap OP = \emptyset$. Similarly if $n_u^S = 0$, then $EQ^I \cap OP = \emptyset$. Therefore by the Proposition, if $\alpha_s^I > 0$ and if conditions (SB1)–(SB3) are satisfied, then these points cannot be transition operating points in OP_{transit} . If $\alpha_s^I > 0$, then strictly the operation ends at the static bifurcation boundary.

3. Consider an $(x_0, p_0) \in EQ_{\text{critical}}$ with $\alpha_s^I(x_0, p_0) < 0$. Then by Proposition 6, we can conclude that there exist equilibrium manifolds EQ^I and EQ^S on both sides of the transition equilibria. That is, the equilibrium structure is locally the same (topologically) for any parameter value p near p_0 , hence (x_0, p_0) is not a static bifurcation point. Unfortunately in this case, the Proposition does not provide any information on the eigenproperties relating J^I and J^S . Note that even though J^I is a submatrix (by deleting a row and a column corresponding to index s) of J^S , in general the eigenvalues of J^I and J^S may be

quite different. Therefore an explicit condition such as $n_u^I \neq n_u^S$ in Theorem 24 becomes necessary in verifying local the stability properties. Again once the condition $n_u^I \neq n_u^S$ is imposed, the proof of Theorem 24 directly follows from Proposition 6.

4. For two-dimensional systems, i.e. when $n = 2$, a rather complete analysis of the generic (codimension one) bifurcation phenomena connected with the static bifurcations SSB and the dynamic bifurcations SDB can be carried out. These results are presented in the next subsection.

We end this section with a more informative Corollary on the static bifurcation points for the general problem. The proof directly follows from Proposition 6.

Corollary 7 Suppose $(x_0, p_0) \in EQ_{\text{critical}}^k \cap \bar{F}$. Assume that (SB1), (SB2), (SB3) and (SSB4) are satisfied at (x_0, p_0) . Then (in addition to the statements of the Proposition 6), the following statements hold true locally near (x_0, p_0) :

(P5) Either $n_u^I = 0$ or $n_u^S = 0$, but not both.

(P6)

$n_u^I = 0$: $EQ^I \subset OP$ and $EQ^S \cap OP = \emptyset$.

$n_u^S = 0$: $EQ^S \subset OP$ and $EQ^I \cap OP = \emptyset$.

(P7) $(x_0, p_0) \in \partial \bar{F}$. Recall that $EQ = EQ^I \cup EQ^S \cup EQ_{\text{critical}}$. $\square$

F. Special results for two-dimensional systems with $n = 2$

In this section, we will restrict the analysis to the case of two dimensional systems with $n = 2$. Moreover a single parameter variation say μ is considered for simplicity. We will analyze the transition from level 0 equilibrium point to a level 1 equilibrium point. Without loss of generality, let us assume that the state limit surface H^1 is of principal interest. The results in this section can be easily restated for other limit sets by using proper notation as shown in the previous section. Basically we want to understand the local structural changes around an equilibrium (x_{10}, x_{20}, μ_0) which satisfies

$$f_1(x_{10}, x_{20}, \mu_0) = 0, f_2(x_{10}, x_{20}, \mu_0) = 0, x_{10} = x_{u_1} \quad (318)$$

Whenever not specified, the partial derivatives are assumed to be evaluated at (x_{10}, x_{20}, μ_0) in the discussion that follows. In the notation of the previous section, we have $I_u = \{1\}$, $I_t = \emptyset$ and $s = \{1\}$. The reduced system for the index set I which is the constrained dynamics on $x_1 = x_{u_1}$ is given by

$$\Sigma^I: \dot{x}_R^I = f_R^I(x_R^I, \mu) := f_2(x_{u_1}, x_2, \mu) \text{ where } x_R^I = x_2 \quad (319)$$

Note that in this case we have $S = I \setminus \{s\} = \emptyset$. Therefore indeed the reduced system for index set S which should be level $k - 1$ dynamics is indeed level 0 dynamics, that is, the full dynamics,

$$\Sigma^S: \dot{x}_R^S = f_R^S(x_R^S, \mu) = f(x, \mu) \text{ where } x_R^S = x \quad (320)$$

The first condition (SB1) directly follows for this problem from (318). Let us quickly recognize that here we have $J^S = D_x f$ and $J^I = D_{x_2} f_2$ both evaluated at (x_{10}, x_{20}, μ_0) . Then the condition (SB2) can be restated for the two-dimensional case as follows.

(SB2) $\det(J^S) \neq 0$, $\text{trace}(J^S) \neq 0$, and $J^I \neq 0$.

By simple algebraic manipulation, it follows that

$$\alpha_s^I = (D_{x_1} f_1 - D_{x_2} f_1 (D_{x_2} f_2)^{-1} D_{x_1} f_2) \quad (321)$$

In fact, this formula directly extends to the general case of any n by Schur's Theorem. The constant a_μ is given by

$$a_\mu = (D_\mu f_1 - D_{x_2} f_1 (D_{x_2} f_2)^{-1} D_\mu f_2) \neq 0 \quad (322)$$

which is required to be nonzero by (SB3).

In summary, for this special case of a two-dimensional problem, we are investigating the dynamic structure around the three equality conditions (318) (hence a codimension one scenario), under the transversality assumption that the four quantities, namely $\det(J^S)$, $\text{trace}(J^S)$, $\det(J^I)$ and a_μ are all nonzero. Naturally we can expect that depending on the signs of these four quantities, and their specific values, different types of phenomena evolve. The problem is to characterize, if possible, exactly what they are. Table III.2 summarizes the answer in the form of a set of qualitative pictures, and we will see that in dimension two, indeed it seems possible to exhaust (almost) all the possibilities. The proof of these pictures should be straightforward by standard linear arguments since all computations are strictly local, and since both Jacobians J^I and J^S have been assumed to have all eigenvalues with nonzero real parts by (SB2). Here the full proofs are not provided because of space limitations. We only state the result for one particularly interesting case, Type II in the table, which establishes a transition from level 1 operating point to hardlimit oscillations under certain conditions. This phenomenon closely resembles the supercritical Hopf occurring in smooth models.

Proposition 7 (see Type II, Table III.2) Suppose that $a_\mu < 0$, $\det(J^I) < 0$, $\det(J^S) > 0$, $\text{trace}(J^S) < 0$ and $\text{trace}(J^S)^2 - 4\det(J^S) < 0$. Then for $\mu > \mu_0$, there exists a locus of non-smooth limit-cycles say $\Gamma(\mu)$ such that 1) $\Gamma(\mu_0) = (x_0, p_0)$, 2) $\Gamma(\mu)$ is locally attracting, and 3) $\Gamma(\mu)$ for each $\mu > \mu_0$ is a one-dimensional topological manifold but not a smooth manifold, and 4) $\Gamma(\mu) \cap H^{1(0-)} \neq \emptyset$ for each $\mu > \mu_0$. $\square$

From the Table, it is clear that even for $n = 2$, we can already observe the richness of new problems B_{SSB} and B_{SDB} as bifurcation phenomena, and at this point, it seems very doubtful whether such a qualitative characterization can be carried out for the static bifurcations B_{SSB} and dynamic bifurcations B_{SDB} in the large system.

III.3.3. Switching elements (Relay type limiters)

The parameter space structure for the case of switching limits can be developed much like the two limit types considered in the previous two sections. However the important distinction arises in the fact that the ODE description $\bar{f}(x, p)$ for $\Sigma_{r,t}$ includes discontinuities, hence standard definitions of equilibria are not applicable when the operation reached one of the switching surfaces SW_i (235). In the same spirit of the previous sections, this difficulty can be resolved by essentially carrying out the definitions within the smooth portion SM and then taking suitable closures to cover the switching surface degeneracies. Note that within the smooth counterpart SM (see (237)), the system $\Sigma_{r,t}$ is smooth, hence standard definitions apply for local stability, hyperbolicity etc. The main technicality arises in identifying the feasibility region (or the operating region) since by construction, the equilibrium surface in this case consists of several unconnected pieces corresponding to the various subsystems in the components C_i . For the operating region, these

separate pieces can be "connected" using the notion of transient stability from the state space. Again there emerge special bifurcation boundaries on the feasibility boundary when the operation reaches one of the switching limits possibly leading to loss of small-signal stability. The details of the analysis are more technical and will be presented elsewhere.

Table III.2 State limit (nonwindup) induced bifurcations in two-dimensional systems.

Type	Conditions	Bifurcation Diagram	$\mu < \mu_0$	$\mu = \mu_0$	$\mu > \mu_0$	Comments
I	$\det(J^1) < 0$, $\det(J^S) < 0$, $\text{Tr}(J^S) < 0$					Simple transition from level 1 operating point to level 0 operating point
II	$\det(J^1) < 0$, $\det(J^S) < 0$, $\text{Tr}(J^S) < 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) < 0$					Transition from level 1 operating point to hardlimit oscillations; resembles Hopf.
III	$\det(J^1) < 0$, $\det(J^S) < 0$, $\text{Tr}(J^S) < 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) > 0$					Transition from level 1 operating point to loss of stability.
IV	$\det(J^1) < 0$, $\det(J^S) < 0$					Annihilation of level 1 operating point; resembles saddle node bifurcation.
V	$\det(J^1) > 0$, $\det(J^S) > 0$, $\text{Tr}(J^S) < 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) < 0$					Annihilation of level 0 operating point; gradual annihilation of the region of attraction.
VI	$\det(J^1) > 0$, $\det(J^S) > 0$, $\text{Tr}(J^S) < 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) > 0$					Annihilation of level 0 operating point; resembles saddle node bifurcation.
VII	$\det(J^1) > 0$, $\det(J^S) > 0$, $\text{Tr}(J^S) > 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) > 0$					Transition from unstable to no equilibria; not of interest.
VIII	$\det(J^1) > 0$, $\det(J^S) > 0$, $\text{Tr}(J^S) > 0$, $v > 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) < 0$					Annihilation of hardlimit oscillations.
IX	$\det(J^1) > 0$, $\det(J^S) > 0$, $\text{Tr}(J^S) > 0$, $v < 0$, $\text{Tr}(J^S)^2 - 4\det(J^S) < 0$					Transition from unstable to no equilibria; no oscillations; not of interest.
X	$\det(J^1) > 0$, $\det(J^S) < 0$					Transition from saddle to "saddle"; not of interest.

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APPENDIX

THE RUDIMENTARY POWER SYSTEM MODEL WITH GENERATION, VOLTAGE CONTROL, TRANSMISSION AND MATCHED LOAD

The rudimentary case of a generator with load at the end of a transmission line as shown in Figure A.1 is used as a vehicle to illustrate various dynamical phenomena investigated in this paper. Since there is only one inertia in this system the assumption $P = P_G$ with no initial stored energy implies that $\dot{\theta} = 0$ and $\theta = \text{constant}$. Thus the voltage dynamics is isolated from the angle dynamics. So this assumption is a counterpart for voltage dynamics of the so-called "classical" model which isolates the rotor angle dynamics by assuming constant voltage behind the transient reactance (i.e. $\dot{E}' = 0$). Using a simplified one axis generator model and a first degree control dynamics, which is a simplification of the IEEE Type 1 excitation dynamics [128], the model for this rudimentary case can be stated as follows:

$$T'_{d0}\dot{E}' = -\frac{x_d}{x'_d}E' + \frac{x_d - x'_d}{x'_d}E_G \cos(\delta_G - \delta') + V_R + E'_{fd} \quad (323)$$

$$T\dot{V}_R = -V_R - K(E_G - E_r) \quad (324)$$

$$0 = \frac{E'E_G}{x'_d} \sin(\delta_G - \delta') + \frac{E_GE}{x} \sin(\delta_G - \delta) \quad (325)$$

$$0 = \frac{1}{x'_d} (E_G^2 - E_GE' \cos(\delta_G - \delta')) + \frac{1}{x} (E_G^2 - E_GE \cos(\delta_G - \delta)) \quad (326)$$

$$0 = \frac{EE_G}{x} \sin(\delta - \delta_G) + P \quad (327)$$

$$0 = \frac{1}{x} [E^2 - E_GE \cos(\delta - \delta_G)] + Q \quad (328)$$

$$Q = Q_0 + HE + BE^2 \quad (329)$$

The output V_R of the control block may be limited by either a windup limit or a non-windup limit. The block diagrams for the excitation system with a windup limit and a non-windup limit are shown in Figure A.2. (a) and (b) respectively. Note that the load model includes an algebraic dependence of the reactive load on voltage in the form of equation (329). This either implies a combination of a power source Q_0 , a current source HE and an impedance load BE^2 , or acknowledges the fact that the reactive load will be a smooth nonlinear function of the voltage as was shown experimentally [48]. Then a second degree expansion in E would be an appropriate approximation as given in (329). The quantity E_r is the set-point voltage, E'_{fd} is the nominal field excitation, and T and K are control coefficients. The other notations are conventional.

Note that the angle variables always appear in the form $\delta_G - \delta$ or $\delta_G - \delta'$ in equations (323)-(328) and hence using algebraic manipulations the model can be simplified as follows:

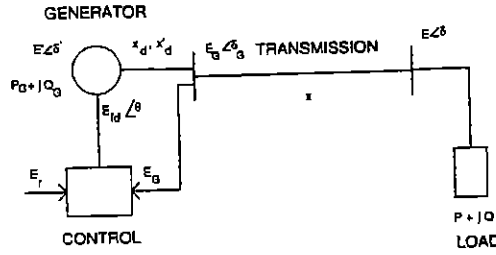
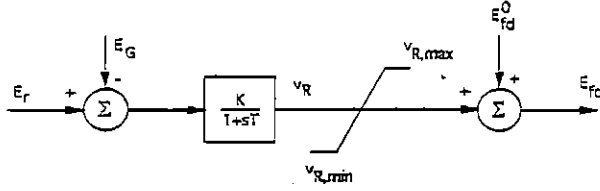
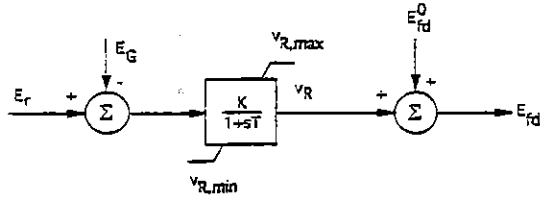


Figure A.1. The rudimentary power system.



(a) Windup limit.



(b) Nonwindup limit.

Figure A.2. Excitation system diagram with windup and nonwindup limits.

Dynamic equations:

$$\dot{E}' = \frac{1}{T'_{do}} \left[-\frac{x + x_d}{x'} E' + \frac{x_d - x'_d}{x'} \frac{(E^2 + x'Q)}{E'} + V_R + E'_{fd} \right] \quad (330)$$

$$\dot{V}_R = \frac{1}{T} \left[-V_R - K \left(\frac{1}{E} \sqrt{(xP)^2 + (xQ + E^2)^2} - E_r \right) \right] \quad (331)$$

Algebraic Equation:

$$0 = E'^2 E^2 - (x'P)^2 - (x'Q + E^2)^2 \quad (332)$$

The general form of the model is therefore a system of ordinary differential equations subject to algebraic constraints.

$$\dot{x} = f(x, y, p) \quad (333)$$

$$0 = g(x, y, p) \quad (334)$$

Here f and g are, respectively, n and m -vectors of smooth, for the power system actually real-analytic, functions; x and y are respectively the dynamic and instantaneous state variables which span the state space $\mathcal{X} \times \mathcal{Y}$. For the rudimentary system (Figure A.1) $x = \{E', V_R\}$, $y = \{E\}$, p denotes parameters and (secondary) controls which jointly span the parameter space $\mathcal{P} = \mathcal{P}_S \cup \mathcal{P}_O$ where $\mathcal{P}_S = \{x_d, x'_d, x\}$, $\mathcal{P}_O = \{P, Q_0, H, B, K, T, E_r\}$ are respectively the system parameters and the operating parameters.

Equation (332) defines the admissible points in the state-parameter-space for the power system. These are located in the *load-flow manifold* $0 = g(x, y, p)$ (a manifold for this simple case) for given parameters p . Load flow is used here in a general sense since voltages like E' , V_R may be present beside bus voltages E_G and E . The set of points (x, y, p) which satisfy $0 = f(x, y, p)$ is called the *stationary manifold* indicating the absence of dynamic events. Points which lie on both the stationary manifold and on the load-flow manifold are the equilibrium points. The state-space for the rudimentary system is three-dimensional with the variables E , E' and V_R . Accordingly, manifolds can be directly visualized. The load flow manifold, which here is diffeomorphic to a plane, is a vertical wall in V_R for this example because V_R does not appear in g . Since the state is constrained within the load flow manifold, all trajectories, as well as the stationary manifold, on the load flow manifold can be displayed in a planar view by projecting into the (E, V_R) sub-space. The set of points on the load flow manifold which also satisfy $0 = f_1(x, y, p)$ (resp., $0 = f_2(x, y, p)$) becomes a curve in the (E, V_R) sub-space, which will be denoted by $b - b'$ (resp., $c - c'$).

The load flow manifold is readily computed at or around regular points where the Jacobian of g with respect to E is nonsingular ($\partial g / \partial E \neq 0$) and $g = 0$ can be solved for E by the implicit function theorem. The remaining points are called singular here. (The terminology noncausal has also been used by other authors in this context [52, 43]). The singular points as defined are the solutions of equation (332) and the following relation:

$$(1 + Bx')^2 E^4 + Hx'(1 + Bx')E^3 - (HQ_0E + P^2 + Q_0^2)x'^2 = 0 \quad (335)$$

It is easily seen that these equations have a unique solution, and hence the singularity exists, unless both P and Q_0 are zero, which would represent an unrealistic model. For the special case considered later on, when $Q = Q_0$, a simple explicit solution for the singularity can be given as the positive square-roots of

$$E_{\text{sing}}^2 = x' \sqrt{P^2 + Q^2} \quad \text{and} \quad E_{\text{sing}}'^2 = 2x'(Q + \sqrt{P^2 + Q^2}) \quad (336)$$

Hence the set of singular points is precisely the line

$$LF_{\text{sing}} = \{(E'_{\text{sing}}, V_R, E_{\text{sing}}) : V_R \in \mathbb{R}_+\} \quad (337)$$

denoted by $a - a'$. This singular line divides the load flow manifold LF into two half-spaces LF_{high} and LF_{low} , called the *components* [86] or *causal domains* [43] of the state space. Each has its own dynamics which is identified by substituting the solution of (332) within the component into equations (330) and (331). As has been discussed extensively in [86], trajectories *cannot* predictably progress across the singular line, $a - a'$.

TRANSCRITICAL AND HOPF BIFURCATIONS IN AC/DC SYSTEMS

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Abstract: This paper illustrates transcritical and Hopf bifurcations in a realistic ac/dc power system model. These bifurcations are thoroughly analyzed based on bifurcation theory to understand and characterize their effect on the dynamic behavior of the system, particularly on voltage stability. A new technique to trace transcritical bifurcations diagrams is proposed, based on the generic characteristic of saddle-node bifurcations for constant P load changes, as demonstrated in the paper. Control mechanisms for avoiding the bifurcations are also presented. Finally, the paper discusses the computational requirements for studying possible chaotic behavior due to bifurcations on the system limit cycles.

Keywords: transcritical bifurcations, Hopf bifurcations, generic saddle-node bifurcations, chaos, voltage stability and control.

1. Introduction

Hopf and saddle-node bifurcations have been recognized as some of the reasons, albeit not the only ones, for stability problems in a variety of power system models [2]–[13]. These local bifurcations are detected by monitoring the eigenvalues of the current operating point. As certain parameters in the system change slowly, allowing the system to quickly recover and maintain a stable operating point, the system eventually turns unstable, either due to one of the eigenvalues becoming zero (saddle-node, transcritical, pitchfork bifurcations), or due to a pair of complex conjugate eigenvalues crossing the imaginary axes of the complex plane (Hopf bifurcation). The instability of the system is reflected on the state variables, i.e., frequency, angles, voltages and currents, by an oscillatory behavior or a continuous change that lead to voltage collapse and loss of synchronism. In some cases saddle-node bifurcations can be associated to the power transfer limit of the transmission system [6]; in other instances Hopf bifurcations appear due to voltage control problems, such as fast acting automatic voltage regulators (AVR) in the generators [12]. In all cases these bifurcations occur on very stressed systems, i.e., the region of stability for the current operating point (stable equilibrium point or s.e.p.) is small [6, 14], hence, the system is not able to withstand small perturbations and becomes unstable. Although there are reports of these bifurcations occurring in unstressed systems [13], this cannot be considered typical, since power system controls are designed so that eigenvalues of various operating points are well into the left half complex plane.

Some voltage collapse problems are not directly associated to bifurcations, but are generated by voltage control devices such as under-load tap changers (ULTCs) or AVRs [15]–[18]. In some of these cases the voltage controls force the eigenvalues to instantaneously jump into the unstable region, making

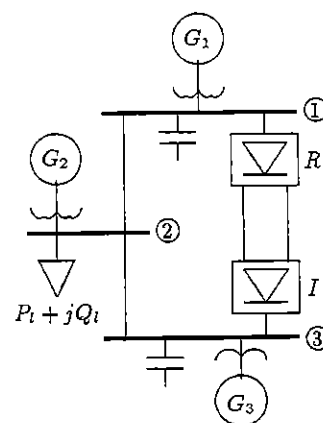


Fig. 1: Sample ac/dc system

the system immediately unstable. Although this particular phenomena is not directly associated to a bifurcation, since the eigenvalues do not go through zero or the imaginary axis, transcritical bifurcation theory can be used to explain the phenomena when AVR limits are assumed to apply gradually [16].

Hopf bifurcations have been detected in a variety of power system models [7, 11, 12, 13]. Particularly, in reference [11] a thorough study of this problem and period doubling bifurcations is presented for a simple dynamic model of 4 state variables and no algebraic constraints. On the other hand, there is little reference in the literature to transcritical bifurcations in typical power system models. This can be explained based on the fact that these types of bifurcations, as formally shown in this paper, are not generic and they only occur under certain assumptions that give the problem a special symmetry [19, 20].

This paper focuses on studying transcritical and Hopf bifurcations in a realistic model of the ac/dc system depicted in Fig. 1, with two different sets of data and system parameters to obtain the desired bifurcations. This network can be represented by a set of differential and algebraic equations [7, 21]. For each case, time simulations and eigenvalue analyses are carried out to study and depict the phenomena, and based on the results, control mechanisms are implemented to try to avoid the stability problems generated by these bifurcations. The paper also discusses the computational requirements and techniques needed for studying these bifurcations. Particularly, the last section is dedicated to discuss the problems of implementing known techniques to study bifurcations on limit cycles that lead to chaotic behavior.

2. Transcritical Bifurcations

The system model of references [7, 21] is used. This is a typical dynamic model useful for transient and voltage stability studies. Thus, a second order synchronous generator model is used, with constant terminal voltage, reactive power limits, and constant mechanical power that changes with system load level; this simulates some of the effects of the AVR and the governor. A II-equivalent circuit model is employed to represent all the elements of the transmission system; transformers and phase shifters are included as part of this transmission network. A ZIP load model with dynamic voltage and frequency dependent terms is used, since this simulates a wide variety of aggregated power system loads. It is important to mention that the location of the bifurcations depend significantly on the static load models used, as demonstrated in reference [6].

DC lines are simulated using R-L circuits, and HVDC controllers are modeled using saturable PI current controllers with K_P (Proportional) and K_I (Integral) gains. Although these control circuits are only approximations to the more complicated HVDC control structures, they recreate several of the main properties of the actual control systems, especially when close to the equilibria. Power controls and Voltage Dependent Current Order Limiters (VDCOL) are introduced into this model by representing the controller current order as a function of the converter voltages.

This system model can be summarized in the following set of equations [7, 21]:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \lambda) \\ \mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \lambda) \end{cases} \Rightarrow \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{0} \end{bmatrix} = \mathbf{F}(\mathbf{z}, \lambda) \quad (1)$$

where $\mathbf{x} \in \mathbb{R}^n$ is a vector of state variables, $\mathbf{y} \in \mathbb{R}^m$ is a vector of algebraic variables, $\mathbf{z} \triangleq [\mathbf{x}^T \mathbf{y}^T]^T$, and $\lambda \in \mathbb{R}$ is any parameter in the system that changes slowly, moving the system from one s.e.p. to another until a bifurcation is encountered. For this one-parameter dynamical system, non-singularity of the Jacobian $D_y \mathbf{g}(\cdot)$ along system trajectories of interest, guarantees a well posed system [22] or strictly causal system [4], i.e., for these trajectories

$$\begin{aligned} \mathbf{y} &\triangleq \mathbf{h}(\mathbf{x}) \\ \mathbf{s}(\mathbf{x}, \lambda) &\triangleq \mathbf{f}(\mathbf{x}, \mathbf{h}(\mathbf{x}), \lambda) \\ \Rightarrow \dot{\mathbf{x}} &= \mathbf{s}(\mathbf{x}, \lambda) \end{aligned} \quad (2)$$

If $D_y \mathbf{g}(\cdot)$ becomes singular, then the model represented by equations (1) breaks down. When this occurs, dynamic load models may be introduced to transform some algebraic constraints into differential equations to remove the singularity, which is similar to the singular perturbation approach proposed in [22].

For this particular model, bifurcations characterized by a singular dynamic Jacobian $D_x \mathbf{s}|_0$ at the equilibrium point $(\mathbf{x}_0, \mathbf{y}_0, \lambda_0)$ with nonsingular $D_y \mathbf{g}|_0$, can be detected using either the Jacobian of the full steady state equations $\mathbf{F}(\cdot) \triangleq [\mathbf{f}^T(\cdot) \mathbf{g}^T(\cdot)]^T = \mathbf{0}$, or the Jacobian of the power flow equations, which are a subset of the steady state equations including detailed static load models, as thoroughly demonstrated in reference [21]. Of these particular local bifurcations only saddle-nodes are generic [19, 20], i.e., they are expected

to take place under typical operating conditions and modeling assumptions. However, transcritical and pitchfork bifurcations could also occur when the system presents a special symmetry due to certain modeling assumptions [19, 20], as shown below for an ac/dc test system.

2.1. Background

The following conditions apply for a transcritical bifurcation of equations (1) at the equilibrium point $(\mathbf{x}_0, \mathbf{y}_0, \lambda_0)$ where $D_y \mathbf{g}|_0$ is non-singular [19]:

$$1. D_x \mathbf{s}|_0 \triangleq D_x \mathbf{s}(\mathbf{x}_0, \lambda_0) \text{ has a simple and unique zero eigenvalue, with normalized right eigenvector } \mathbf{v} \text{ and left eigenvector } \mathbf{w}, \text{ i.e., } D_x \mathbf{s}|_0 \mathbf{v} = \mathbf{0} \text{ and } \mathbf{w}^T D_x \mathbf{s}|_0 = \mathbf{0}^T \quad (3)$$

$$2. \mathbf{w}^T \frac{\partial \mathbf{s}}{\partial \lambda} \Big|_0 = 0, \text{ and } \mathbf{w}^T D_{\lambda}^2 \mathbf{s}|_0 \mathbf{v} \neq 0 \quad (4)$$

$$3. \mathbf{w}^T [D_x^2 \mathbf{s}|_0 \mathbf{v}] \mathbf{v} \neq 0 \quad (5)$$

Observe that from (1) and (2),

$$D_x \mathbf{s}|_0 = D_x \mathbf{f}|_0 - D_y \mathbf{f}|_0 D_y \mathbf{g}|_0^{-1} D_x \mathbf{g}|_0$$

Hence, these conditions have an equivalent representation for the steady state and power flow equations [21], i.e.,

$$1. D_z \mathbf{F}|_0 \triangleq D_z \mathbf{F}(\mathbf{z}_0, \lambda_0) \text{ has a zero eigenvalue, with unique normalized right eigenvector } \mathbf{v} \text{ and left eigenvector } \mathbf{w}, \text{ i.e., } D_z \mathbf{F}|_0 \mathbf{v} = \mathbf{0} \text{ and } \mathbf{w}^T D_z \mathbf{F}|_0 = \mathbf{0}^T \quad (6)$$

$$2. \mathbf{w}^T \frac{\partial \mathbf{F}}{\partial \lambda} \Big|_0 = 0 \quad (7)$$

$$3. \mathbf{w}^T [D_z^2 \mathbf{F}|_0 \mathbf{v}] \mathbf{v} \neq 0 \quad (8)$$

Observe that in general, $\mathbf{w}^T D_{\lambda}^2 \mathbf{s}|_0 \mathbf{v} \neq \mathbf{w}^T D_{\lambda}^2 \mathbf{F}|_0 \mathbf{v}$.

Parameterized continuation and direct methods can be used to trace the bifurcation diagrams and determine the exact location of these bifurcations, respectively [18, 20]. Furthermore, conditions (3), (4) and (5) can be used to prove that these methods have nonsingular Jacobians of the associated equations at the bifurcation point, by following similar steps to the ones used for saddle-node bifurcations in references [21, 23].

Continuation methods run into problems when constructing transcritical bifurcations diagrams, because in this case, as opposed to saddle-node bifurcations, there are two independent paths of equilibria to be traced, which cross transversally at the bifurcation point. Tracing one branch with this method gives the approximate location of the bifurcation point; however, it does not yield much information with regards to the location of the second path of equilibrium points. To find the direction that the second branch follows, researchers have developed techniques aimed at finding the tangent vectors of the two branches at the bifurcation point, which is a nontrivial problem [20]. Another approach is to find a point on the second branch of equilibria for certain value of the parameter λ , using then this point to apply the continuation method to trace the corresponding branch; this is done by finding an approximation of a perpendicular vector to one of the branches near the bifurcation [20]. Although the latter is computationally more efficient, it also presents several difficulties.

This paper presents a different and novel solution to the problem of tracing the second branch of equilibria in power

systems. The idea is also to find a point on the second branch for a fixed value of λ , but relying in this case on the generic characteristic of saddle-nodes for certain parameters in the system. Thus, for a particular operating point, i.e., for a given value of λ , one chooses a constant P load as the new system parameter, so that the system bifurcates through a saddle-node instead of a transcritical bifurcation, as demonstrated below. Hence, the saddle-node bifurcation diagram can now be fully traced around the bifurcation to find the desired point on the second branch for the chosen operating conditions.

To show that the saddle-node is a generic bifurcation when a constant P load change is chosen as the new system parameter, one starts by observing that for this particular choice of λ and for the system model described in [21, 23],

$$\frac{\partial s}{\partial \lambda}(x, \lambda) = k \quad \forall x \in \mathbb{R}^n, \lambda \in \mathbb{R}$$

where $k \in \mathbb{R}^n$ is a constant vector and λ represents the new load parameter. Hence, the second transversality condition (4) required in transcritical and pitchfork bifurcations [19], is not met since

$$D_{\lambda}^2 s(x, \lambda) = 0 \quad \forall x \in \mathbb{R}^n, \lambda \in \mathbb{R}$$

Thus, the local bifurcation associated to a singular eigenvalue in this system is a saddle-node, which is characterized by transversality conditions (3), (5), and

$$w^T \left. \frac{\partial s}{\partial \lambda} \right|_0 = w^T k \neq 0$$

2.2. Example

The data in Tables 1, 2 and 3 are used in the ac/dc test system of Fig. 1 to obtain a transcritical bifurcation. In this case, the impedance of transformer G_3 at the inverter side is used as the system parameter λ , so that the effect of Short Circuit Ratio (SCR) changes in the overall ac/dc system can be studied. Generator G_3 is modeled using a large inertia and damping, as shown in Table 2, to represent a large system on the inverter side. Moreover, this generator keeps V_3 at 1 p.u. when the SCR changes by using an open-loop control strategy, which is not a typical approach to remote voltage control; this particular modeling assumption yields the transcritical bifurcation, as shown below. The generator G_2 is modeled as a constant power source supplying the local load demand of $P_l = 0.1$ p.u. and $Q_l = 0$, and has no bearing in the dynamic behavior of the system for the transcritical bifurcation study. These generator and load at bus 2 are only used to transform the problem from a transcritical into a saddle-node bifurcation, by making P_l the new system parameter for a given value of the inverter SCR, so that the second branch of equilibria can be traced using the technique described above. Observe in Table 1 that the ac link between rectifier and inverter is very weak, so that ac voltage changes in one converter do not affect the other through the ac transmission system, which is the typical situation in long HVDC links. Finally, the HVDC is assumed to operate under power control in the rectifier and extinction angle control in the inverter, within the angle limits depicted in Table 3.

All the results depicted in this paper were obtained using a variety of tools and simulation packages, namely, PFLOW [18], SOLVER-Q [24], EES [25], MATLAB [26], and MAPLEV [27].

Element	G	B	$-\bar{B}_s$
Line 1-2	0	1.1513	0
Line 2-3	0	1.1513	0
Transf. G_1	0	146.63	0
Transf. G_2	0	100.00	0
Transf. G_3	0	λ	0
Capac. 1	0	0	13.75
Capac. 3	0	0	21.46

Table 1: AC transmission system data in p.u. for a 230 kV and 100 MVA base.

Variable	G_1	G_2	G_3
Inertia M	∞	—	1
Damping D	—	—	1
Terminal Voltage V_t	1.05	—	—
Mechanical Power P_m	—	0.1	27.72

Table 2: Generator data in p.u. for a 13.8 kV and 100 MVA base

Figure 2 shows the bifurcation diagram for both ac converter voltages V_1 and V_3 as the inverter SCR changes. The eigenvalues were calculated for all system equilibria; thus, the s.e.p.s are depicted in this figure with continuous lines, whereas the unstable equilibrium points (u.e.p.s) are represented by dashed lines. These curves were traced using a continuation method with the branch switching technique described above. Notice the characteristic $\times$ crossing at the bifurcation point, where the s.e.p. associated to $V_3 = 1$ p.u. changes into a u.e.p. Figure 3 depicts, for both equilibria branches, the real part of the eigenvalues of $D_x s|_0$ closest to the imaginary axis, showing how this matrix becomes singular at SCR=2.2123.

The exact bifurcation point was calculated applying a direct method to the full set of steady state equations $F(z, \lambda) = 0$, since the symbolic Jacobian $D_z F(z, \lambda)$ can be easily computed. This method cannot be applied in practice to the symbolic transient Jacobian $D_x s(x, \lambda)$, since this matrix is difficult to obtain symbolically (it takes several days using MAPLEV in a 32MB-RAM SPARCSTATION-LX with 200MB swap space). On the other hand, using numerical approximations to $D_x s(\cdot)$ significantly affect the convergence characteristics of the nu-

Variable	Rectifier	Inverter
K_P	1	1
K_I	43.5	43.5
Commutation reactance X_c	0.06654	0.06541
Tap a	3.7077	3.3731
Min. firing angle α_{min}	5°	$\sim 120^\circ$ [†]
Max. firing angle α_{max}	120°	$\sim 142^\circ$ [†]
Min. extinction angle γ_{min}	$\sim 40^\circ$ [†]	18°
Max. extinction angle γ_{max}	$\sim 155^\circ$ [†]	40°
Power order P_r	31.42	—
Extinction angle order γ_i	—	18°
DC resistance R_d	0.072892	
DC inductance L_d	0.008695	

[†] Assuming $\mu \approx 20^\circ$

Table 3: DC system data in p.u. for a 230 kV and 435 A base.

merical techniques used to solve the direct method equations.

Time domain simulations were carried out to study the effects of changing the SCR from a pre-bifurcation value to a post-bifurcation one, and see whether the system becomes unstable or recovers to a new equilibrium point on the second branch. The results of these simulations are shown in Figs. 4 and 5. In Fig. 4 the generator G_3 terminal voltage V_{tG_3} is changed after 0.5sec. to keep V_3 at 1p.u. In this case the system becomes unstable, with a clear voltage collapse of the inverter ac voltage and large changes in generator frequency and HVDC variables. Observe that some of the system variables oscillate in a stable limit cycle, which could be attributed to the HVDC controls. If V_{tG_3} is instantaneously changed, the system recovers and converges to the stable equilibrium point on the second branch of the transcritical bifurcation diagram, as shown in Fig. 5.

The results from the simulation demonstrate the importance of controlling the inverter ac voltage. Hence, a fast closed-loop control is introduced in the system to maintain V_3 at 1p.u. by instantaneously changing V_{tG_3} , i.e., $V_3 = 1$ p.u. for all system simulations. This is a more realistic control strategy than the open-loop control approach, and it eliminates the special symmetry responsible for the transcritical bifurcation, making all system equilibria stable for any value of SCR. Nevertheless, limits, gains, and time delays should be included for a more accurate representation of this type of remote voltage control.

3. Hopf Bifurcations

Hopf bifurcations are also generic bifurcations of dynamic systems represented by equations (2) [19, 20]. These types of bifurcations can be shown not to exist in a simple lossless power system model [28]; however, they have been observed in a variety of more realistic power system models [7, 11, 12, 13]. The following sections concentrate on discussing the theory of Hopf bifurcations and the methods used to study them, highlighting their computational requirements and limitations when applied to the ac/dc test system of Fig. 1.

3.1. Background

The transversality conditions of a Hopf bifurcation for equations (1) at the equilibrium point (x_0, y_0, λ_0) , where $D_y g|_0$ is non-singular, are [19, 20]:

1. $D_{xx}|_0$ has a simple pair of purely imaginary eigenvalues

$$\mu(\lambda_0) = \pm j\beta \quad (9)$$

and no other eigenvalue with zero real part.

2. These eigenvalues cross the imaginary axis with "non-zero speed", i.e.,

$$\frac{d\text{Re}\{\mu(\lambda_0)\}}{d\lambda} \neq 0 \quad (10)$$

This bifurcation gives birth to a zero-amplitude oscillation (limit cycle) with initial period

$$T_0 = \frac{2\pi}{\beta}$$

Hopf bifurcations can be detected using a continuation method to trace the system equilibria while monitoring the

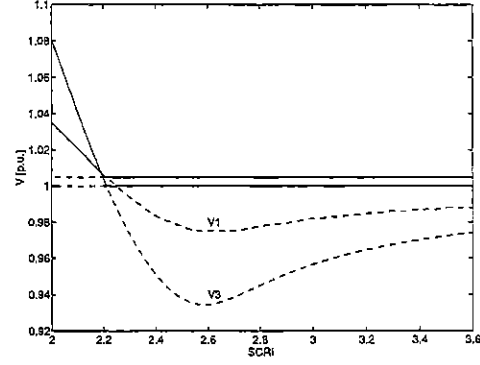


Fig. 2: Transcritical bifurcation diagram for ac converter voltages.

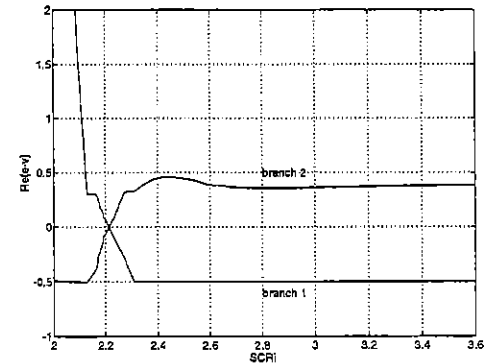


Fig. 3: Real part of the system eigenvalues closest to the imaginary axis on the complex plane for both branches of equilibria. The system presents a transcritical bifurcation at $SCR=2.2123$.

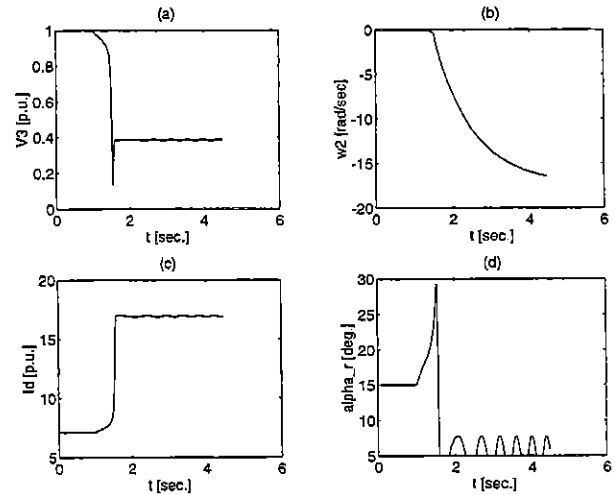


Fig. 4: Simulation of a SCR change from 2.24 to 2.20 at 1 sec., with a 0.5sec. delay on V_{tG_3} change to maintain V_3 at 1p.u. The (a) inverter ac voltage V_3 , (b) generator G_3 frequency ω_3 , (c) dc current I_d , and (d) rectifier firing angle α_r are shown. The system is unstable.

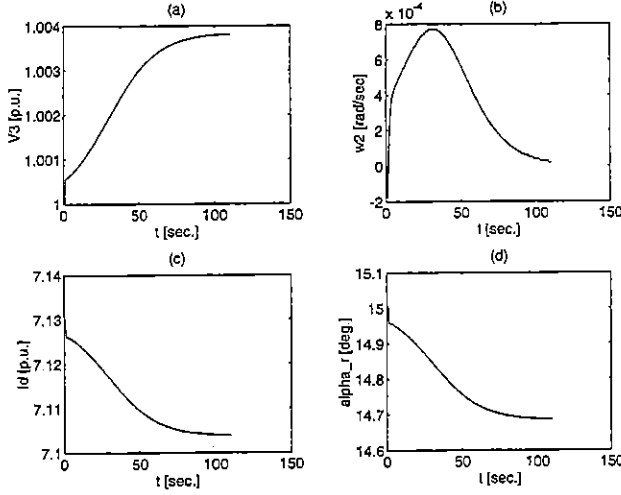


Fig. 5: Simulation of a SCR change from 2.24 to 2.20 at 1 sec., with an instantaneous change on V_{tG_3} to maintain V_3 at 1 p.u. The (a) inverter ac voltage V_3 , (b) generator G_3 frequency w_3 , (c) dc current I_d , and (d) rectifier firing angle α_r are shown. The system recovers, approaching a new stable equilibrium point.

eigenvalues of the dynamic Jacobian $D_x s_0$. In this case, the eigenvalues of $D_x F|_0$ do not change in any significant way that would allow to determine the location of this bifurcation by using this matrix. The system matrices, i.e., $D_x s|_0$, $D_x F|_0$, and the power flow Jacobian, do not become singular at the bifurcation point.

Two distinct direct methods can be used to find the exact position of the Hopf bifurcation point. The first method is based on a mathematical restatement of condition (9) [20, 29], thus, one has to solve

$$\begin{aligned} F(x, y, \lambda) &= 0 \\ D_x s|_0 v_r &= -\beta v_i \\ D_x s|_0 v_i &= \beta v_r \\ \|v_r\| &\neq 0 \\ \|v_i\| &\neq 0 \end{aligned} \quad (11)$$

where $v_r \pm jv_i$ ($v_r, v_i \in \mathbb{R}^n$) are the eigenvectors of the pair of complex conjugate eigenvalues $\pm j\beta$. The second method consists on solving equations (12) below, which can be easily obtained from equations (11) [20, 30],

$$\begin{aligned} F(x, y, \lambda) &= 0 \\ D_x s|_0^2 v &= -\beta^2 v \\ \|v\| &\neq 0 \\ p^T v &= 0 \end{aligned} \quad (12)$$

where $v \in \text{Range}\{v_r, v_i\}$, and $p \in \mathbb{R}^n$ is a known constant vector orthogonal to v . Of these two methods, equations (11) present better sparsity characteristics but requires of n more equations than (12). In both cases, good initial conditions for all system variables and unknown vectors can be obtained from an equilibrium close to the bifurcation point, produced by the continuation method. Notice that in (12), a vector $p \notin \text{Range}\{v_r, v_i\}$ must be chosen, hence, good approximations of v_r and v_i are required. Unless an exact location of

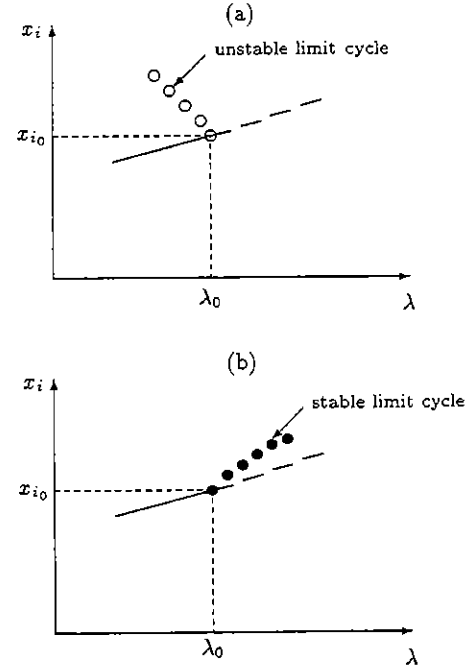


Fig. 6: Examples of (a) subcritical and (b) supercritical Hopf bifurcations. The symbols $\circ$ and $\bullet$ are used to depict half of the amplitude of the unstable and stable limit cycles, respectively.

the bifurcation point is needed, continuation methods usually suffice to determine the bifurcation. Furthermore, these direct methods are not useful in practice to locate Hopf bifurcations of systems represented by differential-algebraic equations (1), due to the computational difficulties of determining the symbolic Jacobian $D_x s|_0$. Although approximations to this Jacobian can be used, the convergence characteristics of the numerical methods used to solve equations (11) or (12) are significantly degraded.

Two types of Hopfs are possible depending on the stability of the limit cycle generated by the bifurcation, namely, subcritical and supercritical. Examples of these two types are depicted in Fig. 6 for a system modeled with equations (2). In this figure, the equilibrium points of a state variable $x_i \in x$ are depicted for several values of the parameter λ , and a Hopf bifurcation is shown at (x_{i0}, λ_0) . Half of the amplitude of the oscillation generated by the bifurcation is depicted using the symbols $\circ$ (unstable) and $\bullet$ (stable). For the subcritical bifurcation the corresponding limit cycle is unstable, whereas for the supercritical the limit cycle is stable [20].

The limit cycle generated by a Hopf bifurcation can be computed by solving the following Boundary Value Problem (BVP) for $t \in [t_0, t_0 + T]$ [20]:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{T} \end{bmatrix} &= \begin{bmatrix} s(x, \lambda) \\ 0 \end{bmatrix} \\ \text{s.t. } r(x(t_0), x(t_0 + T)) &= \begin{bmatrix} x(t_0) - x(t_0 + T) \\ p(x(t_0), \lambda) \end{bmatrix} = 0 \end{aligned} \quad (13)$$

where $p(x(t_0), \lambda)$ is a phase condition to set the initial time t_0 . This condition can be set to

$$p(x(t_0), \lambda) = x_i(t_0) - k = 0$$

where $x_i \in \mathbb{X}$, and $k \in \mathbb{R}^n$ is a fixed initial value of $x_i(t)$ that must be in the desired solution range. Another possible phase condition is

$$p(x(t_0), \lambda) = s_i(x(t_0), \lambda) = 0$$

with $s_i(\cdot) \in \mathbb{S}(\cdot)$, so that the initial t_0 is set to a local minimum or maximum of the oscillation of $x_i(t)$. The latter condition has the advantage of not requiring any previous knowledge of the solution range $x_i(t)$ in $t \in [t_0, t_0 + T]$; however, $s_i(\cdot)$ should be linear in order to apply simple known methods to solve BVPs with linear boundary conditions $r(x(t_0), x(t_0 + T))$ [31].

Normalizing the time interval to have a unit length, i.e., $t \in [0, 1]$, BVP (13) becomes

$$\begin{aligned} \begin{bmatrix} x' \\ T' \end{bmatrix} &= \begin{bmatrix} Ts(x, \lambda) \\ 0 \end{bmatrix} \\ \text{s.t. } r(x(0), x(1)) &= \begin{bmatrix} x(0) - x(1) \\ p(x(0), \lambda) \end{bmatrix} = 0 \end{aligned} \quad (14)$$

where x' and T' denote the derivatives of x and T w.r.t. the normalized variable t .

Current numerical techniques do not address the problem of solving differential-algebraic BVPs, and are limited in the number of states and equations that can efficiently handle. Moreover, due to the different nature of the BVPs that need to be solved, one has to use generic techniques proven to work in a variety of problems (e.g., the multiple shooting method [31]), rather than use more efficient methods which are model dependent (e.g., finite elements method [33]). Furthermore, BVP (14) introduces an additional zero equation that generates convergence problems if one does not start with a good initial guess for all system variables, particularly for the period T . An example to illustrate some of these problems is presented at the end of section 3.2.

Once the limit cycle is calculated for a given value of the parameter λ , one can proceed to address the issue of stability of these oscillations. The idea is simple, to determine the stability of a limit cycle one must answer the question of whether a system trajectory starting arbitrary close to the limit cycle converges to it (stable) or not (unstable). Hence, the distance between the system trajectory and the limit cycle at any point in time for a given λ can be defined as:

$$\|d_\lambda(t)\| \triangleq \|\varphi_\lambda(t; x(0) - d_0) - \varphi_\lambda^T(t; x(0))\| \quad (15)$$

where $x(0)$ is the initial point of the oscillatory system trajectory $\varphi_\lambda^T(t; x(0))$ of period T , i.e.,

$$x(0) = x(T) = \varphi_\lambda^T(T; x(0))$$

d_0 is the initial distance between this limit cycle and the system trajectory $\varphi_\lambda(t; x(0) + d_0)$ generated by the initial conditions $x(0) + d_0$. Then, the stability problem can be analyzed in two ways:

1. Use time simulations to solve an initial value problem with initial conditions $x(0) + d_0$, and see whether $\|d_\lambda(T)\| < \|d_0\|$ (stable limit cycle) or $\|d_\lambda(T)\| > \|d_0\|$ (unstable limit cycle). The limit cycle must be known beforehand, otherwise system trajectories that do not converge to it yield no information regarding the stability of the oscillations. The advantage of this method is that time simulations are relatively inexpensive.

Element	G	B	$-B_s$
Line 1-2	3.68	54.13	4.68
Line 2-3	3.68	54.13	4.68
Transf. G_1	0	166.67	0
Transf. G_2	0	100.00	0
Transf. G_3	0	100.00	0
Capac. 1	0	0	13.00
Capac. 3	0	0	13.68

Table 4: AC transmission system data in p.u. for a 550 kV and 100 MVA base.

Variable	G_1	G_2	G_3
Inertia M	0.1	0.016	∞
Damping D	0.001	0.001	—
Terminal Voltage V_t	1	1	1
Mechanical Power P_m	40	4.75	—

Table 5: Generator data in p.u. for a 13.8 kV and 100 MVA base.

2. Determine the monodromy matrix [20], which is part of the first term on the Taylor series expansion around $x(0)$ of $d_\lambda(T)$ in (15), i.e.,

$$M_\lambda = \Phi_\lambda(T) = D_x \varphi_\lambda^T(T; x(0)) \quad (16)$$

This matrix $M_\lambda \in \mathbb{R}^{n \times n}$ has one eigenvalue $\mu(M_\lambda) = 1$. Hence, based on Floquet theory, one can show that if $|\mu(M_\lambda)| < 1$ for all the remaining $n - 1$ eigenvalues (Floquet multipliers), the associated limit cycle is stable, otherwise, if only one eigenvalue $|\mu(M_\lambda)| > 1$, the limit cycle is unstable. If more than one eigenvalue $|\mu(M_\lambda)| = 1$, then the limit cycle bifurcates leading the system to chaotic behavior. A more detailed discussion of the latter condition is carried out in section 4.

Calculation of the monodromy matrix (16) is a nontrivial task; M_λ can be approximated, however, by the product of several matrices generated during the numerical solution of BVP (14) with the multiple shooting method [20, 31].

3.2. Example

Tables 4, 5 and 6 show the steady state and dynamic data used in the system of Fig. 1 to generate a Hopf bifurcation. The SCR in the rectifier is 5.6, whereas the inverter side SCR is 3.3. The load at bus 2 is modeled as a frequency dependent active power plus a voltage dependent reactive power term, i.e.,

$$\begin{aligned} P_l &= P_{l0} + dP + D_l \dot{\delta}_2 \\ Q_l &= \left(\frac{V_2}{V_{20}} \right)^2 Q_{l0} \end{aligned}$$

where $P_{l0} = 4.75$, $Q_{l0} = 1.561$, $D_l = 0.1$, and $V_{20} = 1.01965$, all in p.u. The active power load change at bus 2 is chosen as the slowly changing system parameter, i.e., $\lambda = dP$; thus, any local bifurcation associated to a zero eigenvalue is expected to be a saddle-node as discussed above. The infinite bus G_3 absorbs all losses and load power changes in the system.

Figures 7 and 8 depict part of the bifurcation diagrams, PV or “nose” curves, obtained with the continuation method

Variable	Rectifier	Inverter
K_P	1	1
K_I	75	75
Commutation reactance X_c	0.1345	0.1257
Tap a	1.7634	1.7678
Min. firing angle α_{min}	5°	$\sim 120^\circ$ [†]
Max. firing angle α_{max}	120°	$\sim 142^\circ$ [†]
Min. extinction angle γ_{min}	$\sim 40^\circ$ [†]	18°
Max. extinction angle γ_{max}	$\sim 155^\circ$ [†]	40°
Current order I_o	1.0	0.9
Min. current order $I_{o_{min}}$ [‡]	0.1	0.0
Max. ac voltage $V_{ac_{max}}$ [‡]	0.95	0.95
Min. ac voltage $V_{ac_{min}}$ [‡]	0.5	0.5
DC resistance R_d	0.06236	
DC inductance L_d	0.01981	

[†] Assuming $\mu \approx 20^\circ$ [‡] VDCOL

Table 6: DC system data in p.u. for a 550 kV and 2.5 kA base.

for the rectifier and inverter ac voltages when the parameter dP changes. These diagrams were traced up to the maximum value of dP , or maximum loading point, which corresponds to a saddle-node bifurcation at $dP_{max} = 4466$ MW for the ac/dc system with no VDCOL (Fig. 7), and $dP_{max} = 4683.6$ MW when VDCOL is included (Fig. 8). The continuous lines represent the s.e.p.s, whereas the dashed lines depict the u.e.p.s. Not all unstable equilibrium points were traced, since they are not relevant to this paper. Observe that in Fig. 7, the system is stable up to the saddle-node bifurcation, whereas in Fig. 8 the system loses stability due to a Hopf bifurcation before the saddle-node is encountered. Hence, from these results one can conclude that the VDCOL is the source of the Hopf bifurcation, but at the same time, this control mode increases the system loadability margin by about 5%, from $dP_{max} = 4466$ MW to $dP_{Hopf} = 4682.3$ MW. Although VDCOLs are typically designed to operate during transient conditions rather than in steady state, the VDCOL data used in this paper [32] enhances system stability by increasing the loadability margin.

The number of state variables in vector x is $n = 7$, and the number of algebraic variables in y is $m = 10$. By tracing the eigenvalues of the dynamic Jacobian $D_x s|_{(7 \times 7)}$ and the full system Jacobian $D_x F|_{(17 \times 17)}$ for all the equilibrium points depicted in Figs. 7 and 8, one can demonstrate that both become singular at the maximum loading point dP_{max} , as expected. However, the Hopf bifurcation depicted in Fig. 8 can only be detected by tracing the eigenvalues of $D_x s|_0$, as explained above. Figure 9 depicts the behavior of the eigenvalues for the system equilibria around the Hopf and saddle-node bifurcations. Figures 9(a) and 9(b) show a complex pair of eigenvalues crossing the imaginary axis with non-zero speed before the saddle-node bifurcation, which is represented by the zero eigenvalue depicted in Figs. 9(a) and 9(c).

Direct methods (11) and (12) cannot be applied in practice to the ac/dc test system, due to the computational difficulties of determining the symbolic Jacobian $D_x s|_0$. The continuation method and a trial and error approach was used here to successfully locate this bifurcation.

Figure 10 depicts the effect of the Hopf bifurcation in the ac/dc system. Observe the oscillations of all variables, partic-

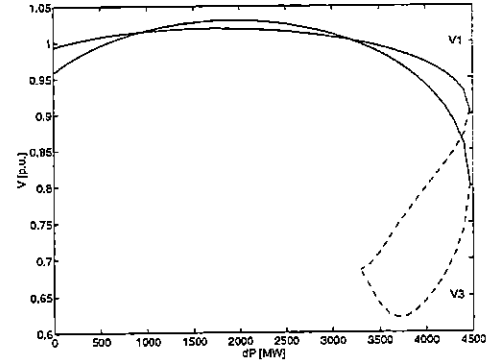


Fig. 7: Bifurcation diagram for ac/dc system with no VDCOL. There is a saddle-node bifurcation at $dP_{max} = 4466$ MW.

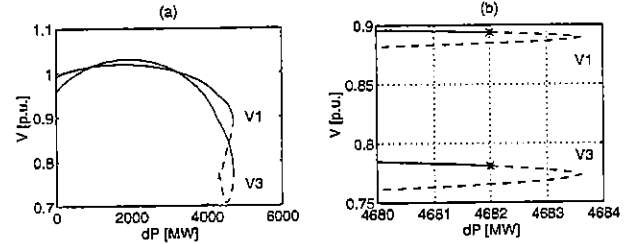


Fig. 8: Bifurcation diagrams for ac/dc system with VDCOL. Plot (b) depicts the region around $dP = 4682.3$ MW where a Hopf bifurcation occurs before the saddle-node is encountered at $dP_{max} = 4683.6$ MW.

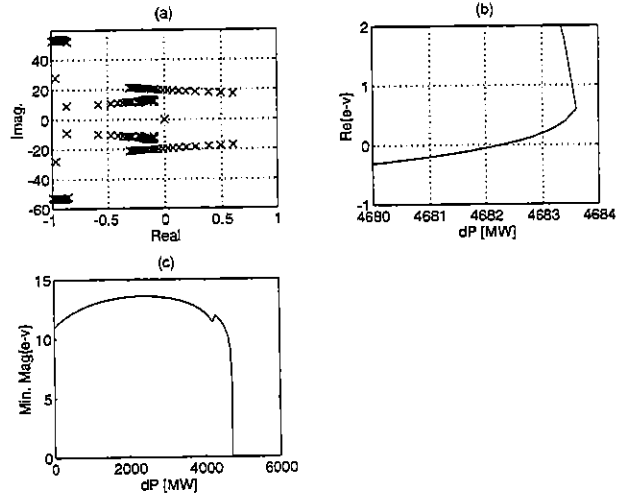


Fig. 9: Eigenvalues for ac/dc test system with VDCOL: (a) complex plane loci depicting a zero eigenvalue and the crossing of the imaginary axis of a pair of complex conjugate eigenvalues; (b) real part of complex eigenvalues closest to the imaginary axis showing the Hopf bifurcation at $dP_{Hopf} = 4682.3$ MW; (c) minimum absolute magnitude depicting the saddle-node bifurcation at $dP_{max} = 4683.6$ MW.

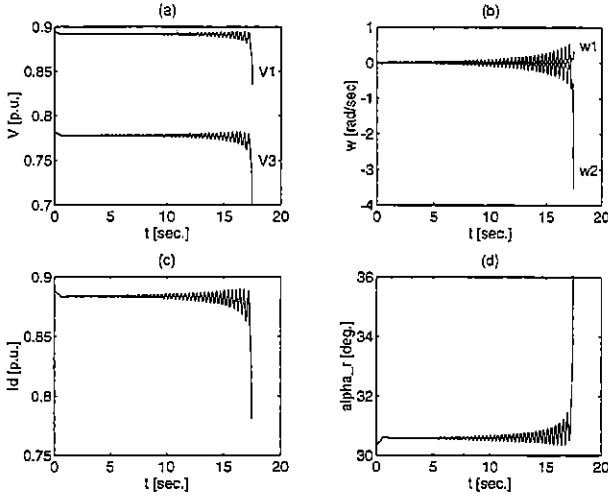


Fig. 10: Simulation of a change in dP from 4682 to 4683 MW at $t = 0.1$ sec. The (a) rectifier V_1 and inverter V_3 voltages, (b) generators frequencies w_1 and w_2 , (c) dc current I_d , and (d) rectifier firing angle α_r are shown. The system oscillates and becomes unstable.

ularly voltages and frequencies, before system collapse. From these simulations one cannot conclude whether the Hopf is subcritical or supercritical; for that type of analysis the limit cycles around the equilibria must be computed solving BVP (14), to then calculate the eigenvalues of the associated monodromy matrix, as previously explained. However, these types of studies could not be carried out on this example due to the limitations of the available numerical methods, simulation tools, and computer resources. To give the reader an idea of what is required to solve a BVP of the class represented by equations (14), a multiple shooting method with modified Newton-Raphson [31] was implemented in MATLAB to determine a stable limit cycle of a system of two state variables and no algebraic constraints. Depending on the initial guesses, this problem takes from 3-4 iterations to 15-20 iterations, with CPU times ranging from 10 minute to 3 hours in a dedicated 32MB-RAM SPARCSTATION-LX with a 50MHz MICROSPARC chip delivering a 21.0 SPECfp92. For an unstable limit cycle the method does not converge unless one starts from an initial guess very close to the final solution.

4. Computational Requirements to Study Chaotic Behavior

A system modeled by a set of one-parameter differential equations can display chaotic behavior through several mechanisms, all based on a series of bifurcations of equations (2) [20]. As the system parameter λ changes, the limit cycles born of Hopf bifurcations also bifurcate. These types of bifurcations can be detected by monitoring the eigenvalues $\mu(M_\lambda)$ of the monodromy matrix (16) for the various values of λ . Thus, the following bifurcations take place at $\lambda = \lambda_0$ when one or two of the $n-1$ eigenvalues $\mu(M_{\lambda_0})$ (one eigenvalue is always unity, say $\mu_n(M_\lambda) = 1$) cross the unit circle in the complex plane [20]:

1. If $\mu_j(M_{\lambda_0}) = 1$, for some $1 \leq j \leq n-1$, the limit cycle undergoes a saddle-node, transcritical or pitchfork bifurcation at λ_0 . In this case, limit cycles appear or disappear and gain or lose stability, depending on the type of bifurcation. This phenomenon is similar to the bifurcations of equilibrium points, and can be associated to a singularity of the Jacobian of the related Poincaré map equations.
2. If $\mu_j(M_{\lambda_0}) = -1$, for some $1 \leq j \leq n-1$, the limit cycle bifurcates through a period doubling bifurcation at λ_0 . This bifurcation can be characterized by the emergence of a second limit cycle; hence, this bifurcation becomes apparent in the time domain by the system trajectories jumping alternatively from one limit cycle to another. An infinite series of period doublings lead the system to chaos. An example of this behavior in a simple power system model of 4 state variables and no algebraic constraints is shown in references [11, 34].
3. If a pair of complex conjugate eigenvalues $\mu_j(M_{\lambda_0}) = \mu_k^*(M_{\lambda_0})$, for some $1 \leq j, k \leq n-1$, lie on the unit circle, i.e., $|\mu_j(M_{\lambda_0})| = |\mu_k(M_{\lambda_0})| = 1$, the system presents a torus bifurcation or generalized Hopf bifurcation at λ_0 . One can think of this bifurcation as the equivalent of the Hopf bifurcation in system equilibria. The torus bifurcation can be represented in the time domain as a torus like object, with system trajectories spiraling around it. Theoretical and experimental evidence point to a series of two consecutive torus bifurcations as another route to chaos.

To calculate the values of λ_0 where these bifurcations take place for differential equations (2), and thus study the mechanisms that might lead to chaos, continuation and direct methods can be applied to the BVP (14). This problem can be reduced for both methods to solve BVPs of the form [20]

$$\begin{aligned} \begin{bmatrix} \mathbf{X}' \\ \lambda' \end{bmatrix} &= \begin{bmatrix} \mathbf{S}(\mathbf{X}, \lambda) \\ 0 \end{bmatrix} \\ \text{s.t. } \begin{bmatrix} \mathbf{R}(\mathbf{X}(0), \mathbf{X}(1)) \\ \phi(\mathbf{X}(0)) \end{bmatrix} &= \mathbf{0} \end{aligned} \quad (17)$$

where $\phi(\mathbf{X}(0))$ is a scalar and linear boundary condition required to compensate for the introduction of the constant λ as a system variable, and $\mathbf{X}(t) \in \mathbb{R}^N$, $t \in [0, 1]$, represents the vector of the state variables $\mathbf{x}$ and T in BVP (14) plus the new variables introduced to solve the desired problem, which vary with $\mathbf{S}(\cdot)$ and $\mathbf{R}(\cdot)$ according to the method used.

For example, to find the location of a period doubling bifurcation using the direct method, one should solve the following $N = 2n + 3$ BVP:

$$\begin{aligned} \begin{bmatrix} \mathbf{x}' \\ T' \\ \lambda' \\ \mathbf{v}_x' \\ \mathbf{v}_T' \end{bmatrix} &= \begin{bmatrix} T\mathbf{s}(\mathbf{x}, \lambda) \\ 0 \\ 0 \\ T D_{\mathbf{x}}\mathbf{s}(\mathbf{x}, \lambda)\mathbf{v}_x + \mathbf{v}_T\mathbf{s}(\mathbf{x}, \lambda) \\ 0 \end{bmatrix} \\ \text{s.t. } \begin{bmatrix} \mathbf{x}(0) - \mathbf{x}(1) \\ p(\mathbf{x}(0), \lambda) \\ \mathbf{v}_x(0) - \mathbf{v}_x(1) \\ \mathbf{v}_x^T \frac{\partial p}{\partial \mathbf{x}(0)}(\mathbf{x}(0), \lambda) \\ \mathbf{v}_{x_k}(0) - 1 \end{bmatrix} &= \mathbf{0} \end{aligned} \quad (18)$$

BVP (18) comes from the linearization of the BVP (14) around the bifurcation point, i.e., for $\tilde{s}(\cdot) \triangleq [s^T(\cdot) \ 0]^T$, $\tilde{x} \triangleq [x^T \ T]^T$, and $\tilde{r}(\cdot) \triangleq [r^T(\cdot) \ p(\cdot)]^T$, the linearization of (14) yields

$$\begin{aligned} \tilde{v}' &= D_{\tilde{x}} \tilde{s}(\tilde{x}, \lambda) \tilde{v} \\ \text{s.t. } A\tilde{v}(0) + B\tilde{v}(1) &= 0 \end{aligned}$$

where

$$\begin{aligned} A &= D_{\tilde{x}(0)} \tilde{r}(\tilde{x}(0), \tilde{x}(1)) \\ B &= D_{\tilde{x}(1)} \tilde{r}(\tilde{x}(0), \tilde{x}(1)) \end{aligned}$$

A and B are constant matrices for a linear set of boundary conditions $\tilde{r}(\cdot)$. Observe that $\tilde{v} \triangleq [v_x^T \ v_T]^T$ could be viewed as the equivalent of the right eigenvector at a bifurcation of equilibrium points. These equations are similar to the ones used to find bifurcations of equilibria with direct methods.

Continuation methods for finding bifurcations of limit cycles is a two part process, similar to the continuation methods used to detect bifurcations in system equilibria. A predictor step is applied first to find an initial guess, which is then used in the corrector step to calculate the actual solution on the bifurcation branch. The corrector step requires the solution of an $N = n + 2$ BVP similar to (17) for x , T , and λ . On the other hand, the predictor step can be designed so that it uses by-products of the numerical solution of previous BVPs, particularly when multiple shooting methods are used [20].

Both continuation and direct methods need good initial guesses for convergence of the associated BVPs. Hence, this issue must be considered in the actual implementation of these methods.

For power systems represented by differential-algebraic equations (1), direct methods cannot be used in practice, since they require of $D_{\tilde{x}} s(x, \lambda)$ or a good approximation of it. Continuation methods also present difficulties due to the need of having to solve a series of BVPs with algebraic constraints. Although available numerical techniques, particularly multiple shooting methods, can be modified to solve these problems, they will require of rather large computer resources, as previously discussed.

5. Conclusions

Transcritical and Hopf bifurcations in differential-algebraic power system models are thoroughly discussed in this paper. Examples of these bifurcations are illustrated in an realistic transient stability model of an ac/dc power system, showing their direct link to particular voltage control mechanisms in the system. The paper also shows the computational requirements for the study of these phenomena, and a new technique for tracing transcritical bifurcations diagrams is proposed, based on the generic characteristics of saddle-node bifurcations in power systems.

This paper demonstrates the need for implementing and modifying existing numerical techniques to solve differential-algebraic BVPs, especially for the study of the mechanisms that could lead a system to chaos. However, the large amount of computational resources required for these types of analyses, significantly limits the practical application of these techniques to realistic power system models. More research on the solution of large BVPs is required before the methods discussed in this paper can be used in real environments.

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Discussion of papers presented by Glavitsch, Zaborszky, and Cañizares

Edited by L. H. Fink, ECC, Inc.

Fernando Alvarado (University of Wisconsin)

[re Glavitsch presentation]: I was a little puzzled by the fact that even though a lot of these indicators are most interesting, I think that we agreed that some of the most useful indicators are based on parameter space measures, minimum distance to collapse, etc. I find it a little surprising that they were really not mentioned. I believe that the parameter space, essentially what you could call the domain indicators rather than the range indicators, are really very useful, and I think they deserved a mention in the overview, particularly the worst direction indicators. The issue that you brought up very nicely, the idea of contingencies, must be brought in. Some work has in fact been done on bringing in contingencies, probabilistic techniques, to bear on the issue of voltage collapse. Again I think that needs to be a little supplemented a bit. Another comment: the issue of induction motors I thought was very good. I really think there is a lot more there, and I hope to make a minor contribution on Wednesday or Thursday. Finally I am going to read the title of a paper to be presented on Thursday. It is not mine. The paper is entitled "The irrelevance of load dynamics for the loading margin to voltage collapse and its sensitivities." That seems to run counter to some of the ideas that you were presenting and I would like to stimulate some really spirited debate on that issue here.

Hans Glavitsch (ETH, Zurich)

So far as the parameter issue is concerned, it seems to me that we have an example for this, and this is the study of line parameters, changes of line parameters and contingencies but again this is mixed with the loading. It is not just parameters alone; it also depends on the loading. So again we have this uncertainty of how the fault, or the whole dynamics, develops. Of course, we get an idea by studying, let's say, double lines, or single line in, or bus switching - then we have the effect of these parameters also. But again we have the effect of the loading, which is not quite well known; this may vary. We have also certain distribution level dynamics. But essentially, of course, we should consider this. Now as far as the sensitivity is concerned, I'm looking forward to the discussion and presentation of this paper.

Ian Dobson (University of Wisconsin)

Hearing the discussion so far, I think that Fernando, when he referred to parameters, was including among parameters things like loading. One of the common indicators that I think is accepted in industry and academia is the amount of loading that you put on the system till you reach the point of collapse. That can be regarded as one of the parameters. Now it is true that you can get various different loadings leading to collapse, but if you choose one of them and compute that loading margin, the distance until you reach instability, then you can compute the sensitivity of that to any number of other parameters. There are also techniques available for getting the minimum loading margin to collapse. In addition, I might mention that if you take another approach and look at energy function methods, they are to some extent independent of the loading. So there are several indicators that perhaps weren't emphasised in Professor Glavitsch's talk that do address some of the issues that he was concerned about for indicators.

Yakout Mansour (BC Hydro)

I think maybe we need even to define what is a good indicator. I do support Hans 100%. We've tried just about every one of those indicators. None of them is robust all the time, and none of them give you consistent results. They

give some results in small systems. Sometimes they give results under some loading conditions, and are not very good under others. Some are very nonlinear. We still do not have an indicator. Believe me, he is right.

Glavitsch: Just let me respond on the loading, because I think it is quite obvious what you can do and what you expect. Let's take a small system, let's say ten PQ nodes, and now you want to know what the limits are. It seems to me that you have an infinite number of solutions, and no single solution as to what the limit is.

Nelson Martins (CEPEL, Brazil)

I agree with Ian and Fernando. It is a nonlinear issue; you can only do it nonlinearly. Point of collapse, continuations, and other "minimum distance to collapse" techniques are a possibility. All these sensitivities, even modal sensitivities, provide just incremental information. They are very valuable to get you the trends and to do your control actions, but that is all they are meant for.

Nicholas Miller (General Electric Company)

As a decidedly minority representative of a company that actually makes some of the equipment that might be used for a hardware fix, I thought that Dr. Glavitsch's list of fixes was a little on the short side. Not all of them are brutally expensive. There are a lot of neat new ones out, besides static VAR compensators. I hope that in the course of the week we will talk about some of the other hardware fixes for moving these stability boundaries out a bit. There's a whole new world of equipment out there and not all of it is unreasonable to use.

Claudio Cañizares (University of Waterloo)

I will address not only Professor Zaborszky's comments and discussion, but also Professor Vournas' paper which is going to be presented soon. The ideas are the following. When you have differential-algebraic models, you have saddle node bifurcations, singular Jacobians, etc. Now I have a problem when I play with the system load. Regardless of how you model the system, you can define an energy function. You can discuss the Jacobians, you can find bifurcation diagrams, eigenvalues, eigenvectors, and everything makes sense in the Lyapunov stability sense. You can see energy functions as Ian Dobson mentioned. You monitor for different values of loading of your system, how these manifolds of energy start flattening out and disappearing, converging to points and drop out. Everything makes sense. But when you move to the condition that Professor Zaborszky mentioned, and that Prof. Vournas has mentioned in his paper, singularity induced bifurcations, we have a problem: the algebraic constraints become singular at a certain equilibrium point. Now, the question is: As Prof. Zaborszky defined in his paper, singularity induced bifurcations are basically related to a singular Jacobian, besides other conditions. This Jacobian is not well defined, but you can find how the eigenvalues move around this singularity point. So I move back to my system and work with this idea.

Now I have a particular problem here. When you go to analyse this problem with bifurcation diagrams, things do not make sense. You have an invisible region that you cannot enter, because there you have imaginary solutions to the algebraic constraints. You have a singularity point, the saddle node disappears, it is not there anymore. However, if you monitor only eigenvalues, all of them are stable, so that is a problem. However, when you try to model a saddle node bifurcation again, it behaves exactly the same way, voltage

collapses, and angles separate. However, the invisible region let's say was the singularity bifurcation condition of the singularity induced bifurcation. The system tends to recover and go to the stable equilibrium point because it cannot operate in the invisible region. So my question really is: whether differential-algebraic models are proper when the Jacobian is singular? Shouldn't we just ignore those cases, or model the load in a different way, so as to remove that condition? That happened not only in a small system; you can see it in larger systems with similar equations. They might be very simple equations, but you do get those problems.

The other question I have is: I couldn't see how the singularity induced bifurcation ends up in a voltage collapse. In my particular case I had this problem. Would that explain the weird behaviour of all the eigenvalues and the Jacobian, and in general of the dynamics of my system?

John Zaborszky (Washington University)

I seem to have the same opinion, because our point is largely that the differential-algebraic model is not the proper model, particularly for power systems where phasor notation is used. Regarding the comment that the dynamic behaviour of small and large systems does not make sense, well, we did not get that impression, but I presume that since you are discussing here, we will have to look at it and answer it. We do have the understanding that these singularity induced bifurcations can play a role in voltage collapse. We will take a look at it and study it, and then we can sit down and talk. Your discussion was very interesting, as much of it as I could understand.

Costas Vournas (National Technological University, Athens)

I would like to address one of the points raised by Claudio Canizares. The singularity induced bifurcation that Claudio is referring to is shown as point B on this [Figure]. Very similar results are reported in a paper by Sauer, Pai and Rajagopalan. The system consists of a voltage controlled generator feeding an isolated load. The equilibrium curve is shown in the Figure. We have solved for the equilibrium conditions with constant reference voltage.

Now, as the load varies, the load curve intersects the equilibrium curve, so you see that we have two solutions up to the saddle node bifurcation point, where the load curve becomes tangential to the equilibrium curve, as shown. Note that this is a P-V curve with constant reference voltage, not that of a constant voltage bus. Therefore, point C is clearly a saddle node bifurcation. Now, the line crossing the equilibrium curve at point A defines the condition for a Hopf, or Poincare-Andronov-Hopf, bifurcation. This means that the equilibrium points after point A are unstable in the form of oscillations. So by gradually increasing the load, you arrive at point A, then oscillations will start and the system will collapse. In Fig. 12 of my paper, you see the unstable oscillations starting exactly at point A (with eigenvalues having zero real part), how the oscillations grow, and then you have collapse. Now suppose that you have the load characteristic shown by the thick line in the Figure. How can this happen? This can happen only after a contingency.

Suppose that the system is initially operating at point I, and a contingency changes the network conditions so that from a stable and very well defined equilibrium point we go to this strange case of point B. What will happen at point B? We have the equilibrium condition satisfied, and we have the singularity of the algebraic equations satisfied. This is a singularity induced bifurcation. It is not a point where the system can operate. As I said, with a gradual load increase the system will collapse with oscillations before reaching it, but mathematically this point exists.

Robert Schlueter (Michigan State University)

A loss of causality occurs in a differential algebraic model when the Jacobian of the algebraic equations with respect to algebraic variables is singular. A loss of causality in a model that represents network and load as an algebraic model and generators as a differential equation model suggests that a bifurcation is actually occurring in the unmodeled induction motor dynamics. A loss of causality in a model that represents network and induction motor load as differential equations and generators as an algebraic model suggests that the bifurcation is occurring in the unmodeled generator dynamics. A loss of causality is an indication that the model is poorly suited to the stability problem being studied.

A recent paper by Sauer shows that if both generator dynamics and load dynamics are included in a model, loss of causality can't occur, since the network model is linear and always solvable with unique solutions. This result provides another reason why a loss of causality is an indication of improper modeling of the loss of stability phenomena observed.

A loss of causality, if it ever occurs, can result in two different phenomena. One loss of causality is a radial voltage instability, which does not affect generator dynamics in a differential algebraic model where generators are modeled by differential equations and network and load is modeled by algebraic equations. This class of loss of causality is called nonbifurcation loss of causality. A second class is one that causes poles of generator dynamics to go to infinity and cross from the right half to the left half S-plane or vice versa. This loss of causality is called singularity induced bifurcation. I believe nonbifurcation loss of causality or singularity induced bifurcation to be extremely improbable, if not impossible, events in an actual power system.

Impact of Modeling Accuracy on Type, Kind, and Class of Stability Problems In A Power System Model

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Abstract

A classification of a number of stability problems observed in the recent literature is performed in terms of the type of model used; the kind of bifurcation observed on that model (i.e. saddle node, Hopf, node focus, singularity induced, chaotic blue sky); and the class, which indicates specific generator and/or load dynamics that are associated with each kind of bifurcation on each model type. This classification is, of course, a first draft of a much more thorough classification that needs to be performed in the future.

Detailed simulation [1-3] of contingencies that result in voltage collapse have shown that assuring stable equilibria and a sufficiently large region of attraction surrounding the post contingency may not even be necessary, but is certainly not sufficient for any specific contingency to be asymptotically stable and avoid extremesis. This is true because discontinuous control actions, relaying, or operator action to protect equipment from damage cause loss of asymptotic stability or extremesis and cannot be addressed through assuring a stable post contingency equilibrium and a sufficiently large region of attraction. A bounded trajectory feasibility region is a subset of the set of all contingencies and operating changes specified by a utility's planning or operating criterion, which constitutes what the utility system is expected to meet if designed and operated properly. Contingencies in combinations with operation condition changes that satisfy all the conditions required for assuring asymptotic stability without extremesis belong to the bounded trajectory feasibility region. A bounded trajectory feasibility region can be used to directly assess security of a power system either deterministically or probabilistically.

Each kind of bifurcation is assessed in terms of retention or loss of bounded trajectory feasibility. An assessment is then made on the sequence of the kinds and classes of bifurcations of equilibria as well as closed orbits that have been observed on the different model types as some bifurcation parameter is continuously increased. Finally, an assessment is made on the

specific bifurcation in each sequence that causes loss of bounded trajectory feasibility. Two bifurcation sequences are identified as resulting in the specific uncontrolled spread of voltage decline associated with voltage collapse. The others cause extremesis, lead to equipment damage, or severely restrict the size of the region of attraction.

1. Introduction

Assuring that: (a) every contingency induced trajectory has a stable post contingency equilibrium point, and (b) that the precontingency equilibrium point lies within the region of attraction of the post contingency equilibrium point was viewed as sufficient to assure voltage stability of a power system. Since the requirement that the equilibrium point must be stable was considered to be the first requirement in assessing voltage stability, much research has been focused on computing equilibria [22] and attempting to assess whether they are stable [4,5,7,8-13,17-19,22-24,33,36,40-48,50-52,64,68-73]. The investigation of the region of attraction of post contingency equilibrium has received some attention [6,8,16,39,74], but far less than attempting to assess whether the post contingency equilibrium is stable.

Focusing on computation of equilibria and the feasibility region [16] does not appear to be appropriate for deciding whether the trajectory produced by a contingency is stable based on both simulation using EXSTAB [2] or EUROSTAG [1] and the theoretical results reviewed in this paper.

The first reason is that the EUROSTAG [1] and EXSTAB [2] simulation packages (along with the development of accurate models of the load, network, generators, excitation systems, power system stabilizers, governors, boiler turbine energy systems, under load tap changers, switchable shunt capacitors, SVC's, and synchronous condensers within these packages) have proven that a load flow is not generally a very accurate estimate of the steady state equilibrium point when the system is vulnerable to voltage instability. The load flow just does not capture the sequence of control actions inherent in the system's

response to a contingency that brings the system close to voltage instability. Even step by step load flow methods [3] are not accurate in capturing the contingency induced trajectory. The flurry of load flow modeling changes that include more accurate steady state generator and exciter modeling, inclusion of generator terminal buses in load flow models to better represent load drop compensators, use of actual field current limits rather than reactive limits, better under load tap changer control and switchable shunt capacitor controls have not yet made load flow a consistently accurate estimate of the equilibria produced by EXSTAB [2].

The second reason is EXSTAB [2] and EUROSTAG [1] simulation program results and the theoretical results quoted in this paper establish that even if the load flow solution were an accurate estimate of the equilibrium point produced by EXSTAB [2], existence of a stable or unstable equilibria would not accurately predict retention or loss of stability. In general, voltage instability doesn't generally occur because the steady state equilibrium lies outside a feasibility region defined in [16], but rather because the transient trajectory lies outside the region of attraction around each quasi equilibrium point [16] when the steady state equilibrium is indeed stable. EXSTAB [2] and EURSTAG [1] indicate the existence of several quasi equilibrium points, which occur after: (a) action by a field current limit controller to disable or over-ride an exciter; (b) action by a tap changer changing the tap; or (c) a switchable shunt capacitor switching a capacitor. A short transient occurs between each quasi equilibrium point and the next in response to the discontinuous control actions. The region of attraction around each quasi equilibrium is dictated by a different model due to the discontinuous control actions, and these regions of attraction at each quasi equilibrium along the trajectory would shrink as voltage instability is approached. Asymptotic stability of the entire trajectory is guaranteed as long as each quasi equilibrium is stable and its region of attraction contains the previous quasi equilibrium. Thus, the region of feasibility around each quasi equilibrium as well as the region of attraction of the quasi equilibrium points including the post contingency equilibrium must be sufficiently large if the trajectory is to reach an equilibrium. The existence of a stable post contingency equilibrium in general does not guarantee stability.

A third reason for not focusing on the stability of equilibria for assessing whether retention or loss of stability occurs is that stability or instability of limit cycles, the size of their orbit, as

well as the size of the region of attraction around stable equilibria or orbits can decide whether retention or loss of stability occurs. Evidence of small stable orbits around unstable equilibrium with sufficiently large regions of attraction would be viewed as being voltage stable and acceptable since such limit cycles exist as a precursor to voltage collapse and as low frequency interarea oscillations. When such small orbit stable limit cycle exist, no emergency actions are taken, operators do not trip generators or change operating conditions because of these oscillations, and power system stabilizers are not always retuned to damp these oscillations. Systems with a large stable orbit limit cycle, however, are not acceptable due to potential damage to equipment and station operator actions of tripping generator units experiencing such oscillations. Existence of large unstable orbits around a stable equilibrium, in contrast to large stable orbits, would have a large region of attraction around the equilibrium, potentially be never experience or observed, and allow the system to be considered voltage stable and operating acceptably. However, existence of a small orbit unstable limit cycle around a stable equilibrium would not be considered acceptable because the region of attraction around the stable equilibrium would be too small.

A fourth reason for not focusing on stability of equilibrium is that the trajectory may cause stalling or tripping of large motors causing an extremesis and yet have a stable equilibrium or small stable orbit and a sufficiently large region of attraction around that equilibrium or small stable orbit.

The above discussion of why specifying that a post contingency equilibrium is stable is not sufficient justifies defining a bounded trajectory feasibility region that insures: (a) the post contingency equilibrium is stable or that a small stable orbit limit cycle encircles the unstable equilibrium; (b) the region of attraction around the post contingency is sufficiently large to allow the trajectory to converge to the stable equilibrium or small orbit, stable limit cycle; (c) that the trajectory does not cause stalling or tripping of motors to the point where the system is in extremesis; and (d) that small orbit unstable limit cycles do not encircle the stable equilibrium. The bounded trajectory feasibility region is not just a region in state and parameters space guaranteeing that equilibria are stable for continuous parameter change, but a constraint on contingency induced or continuous or discontinuous operating change induced trajectories so that they are bounded and don't cause extremesis (tripping or stalling of several motors and loads), and lie within the region of attraction around each quasi

equilibria so that they are attracted to a stable equilibria or small stable closed orbit. The bounded trajectory feasibility region is a feasible set of trajectories in combination with a set of operating changes rather than a feasible set of equilibria and associated parameters.

It can be viewed that development of accurate models and simulation programs is the "experimental" aspect of investigating voltage instability problems. A "theoretical" approach to investigating voltage instability is to attempt to investigate and classify the types of models that can be used to study various aspects of voltage instability as well as classify the kinds and classes of stability problems associated with each type. This classification of model types performed in this paper is based on the current understanding of the level of modeling accuracy of specific components needed to accurately simulate a specific stability problem; understand where, why, and how it occurs; produce operating strategies for avoiding these stability problems; and determine feedback control strategies for stabilizing the dynamics associated with the stability problem. Classification of power system models and stability problems is in its infancy and is open to radical revision as: (1) the understanding improves on how modeling of each power system component affects each stability problem that can exist on a power system, and (2) the understanding of the great number of stability problems that have or will potentially occur in a power system improves. It will be quite clear from the theoretical work surveyed in this paper that the stability problems observed change radically with the type of model used. As models of a particular type used in theoretical papers have become more complex and accurate, it will also become clear from this paper that the kind of stability problems that occur change and new stability problems have been observed. If the model is complex but a totally inaccurate reflection of any actual power system, the stability problems that occur in the model may never actually occur in the system. The modeling complexity and accuracy possible in EXSTAB [2] or EUROSTAG [1] is so completely beyond the models used in theoretical studies that one can't necessarily expect current theory to explain what is or has occurred in a simulation of an actual contingency on a very accurate model of a specific system. The sequential development of theory on more complex and accurate models should begin to better explain measured and recorded phenomena as well as those produced from simulation of accurate complex power system models. A possible test for whether a simplified model, used in a theoretical analysis of a specific simulated or recorded stability

problem, is sufficiently accurate is to assess whether: (i) all observed behavior is explained; (ii) the observability and controllability properties of the stability problem is accurately captured; (iii) the modal structure is accurately captured; and (iv) the feasibility region and region of attraction is accurately captured compared to the most accurate complex model known. Once this has been established for each and every possible (many yet to be discovered) stability problem, one can produce a perfect classification of stability problem as to:

- type - simplest model that captures, observability, controllability, modal behavior, feasibility region, and region of attraction for one or more kinds and classes of stability problems;
- kinds - kinds of local or global bifurcations that occur for this model type;
- classes - the subset of generator and/or load dynamics which exhibit this specific kind of bifurcation in this model type. There can be several different classes of each kind of bifurcation on a particular type of model;
- location - a specific type, kind, and class of stability problem can occur at many different locations in a power system because most if not all of the type/kind/class stability problems are local in nature.

At this point, theoretical research must rely on: (a) measured and recorded voltage stability events, or (b) simulations of events using accurate complex models as the standard to judge adequacy of the theory. It would be hoped that the models used in theoretical investigation will catch up with simulation model complexity and accuracy and then help suggest: (a) model improvements for simulation and existing analysis tools as well as; (b) new analysis techniques for different type/kind/class of stability problems.

The paper is organized as follows: Three model types are proposed based on the existing literature on power system stability problems in Sections 2. In Section 3, a discussion of the definitions of structural stability, feasibility region, and region of attraction and their value in deciding when a system is operating acceptably are given. A bounded trajectory feasibility region is then defined as a means of: (a) better

assuring retention of stability; (b) avoiding extremesis where load are tripped or stall out due to unacceptable low voltages; (c) allowing small orbit stable limit cycles which are benign and yet cause loss of feasibility and structural stability; (d) preventing large orbit stable limit cycles that can damage boiler, turbine, or synchronous generator equipment; and (e) preventing small orbit unstable limit cycles that maintain structural stability and feasibility and yet reduce the region of attraction around the stable equilibrium; and (f) place lower limit constraints on the size of the region of attraction of each quasi equilibrium point, as well as the post contingency equilibrium, so that discontinuous control actions of field current limit controllers, tap changers, or switchable shunt capacitors do not cause loss of stability. The bounded trajectory feasibility region definition includes (a)-(f), which are not contained in definitions of structural stability region, feasibility region, or region of attraction; and yet are necessary constraints on any contingency induced trajectory so that the power system keeps operating. The bounded trajectory feasibility region is the subset of all contingencies in combination with all operating condition changes specified by a utility planning or operating criterion that satisfy (a-f). The combination of contingencies and operating conditions, which are within the utility planning and operating criterion, and not within the bounded trajectory feasibility region, indicates the security or insecurity of that system based on deterministic or probabilistic measures.

Section 4 also contains definitions and descriptions of saddle node bifurcation, Hopf bifurcation, period doubling bifurcation, blue sky bifurcation, 1:1 bifurcation, node focus bifurcation, chaotic blue sky bifurcation, and chaos. All of these bifurcations have been observed on power system models. The review of the definitions of these kinds of bifurcation allows an assessment as to whether this bifurcation causes loss of bounded trajectory feasibility, which also is contained in Section 4.

A description of Type 1 stability problems where extremesis occurs is given in Section 5.

In Section 6, a review of the Type 2 stability problems is given. Type 2 stability problems occur in a Type 2 model where generator inertial, flux decay, exciter, and boiler turbine governor dynamics are included, but all load dynamics are omitted because it is assumed that they are decoupled from the generator dynamics and do not contribute to the stability

problems observed on this model. A discussion of each kind of bifurcation found to date on Type 2 models is provided along with the dynamics within the model affected by that kind of bifurcation. Saddle node, node focus, singularity induced, and Hopf bifurcations have occurred on Type 2 models in the literature reviewed in this paper. Classes of a specific kind of bifurcation are those that affect different generator dynamics; i.e., generator inertial dynamics, generator exciter and flux decay dynamics, or both.

After each kind/class of bifurcation of Type 2 stability problems has been discussed, loss of bounded trajectory feasibility for Type 2 models is discussed. This subject is important because one can experience a sequence of bifurcations where some cause loss of structural stability or loss of feasibility, but where loss of bounded trajectory feasibility only occurs after several benign (no loss of bounded trajectory feasibility) bifurcations occur. Four possible loss of bounded trajectory bifurcations sequences have been identified:

- (a) unstable (subcritical) Hopf bifurcation or formation of an unstable small orbit limit cycle;
- (b) saddle node bifurcation in generator flux decay dynamics;
- (c) supercritical Hopf bifurcation followed by unstable node focus bifurcation;
- (d) formation of large stable closed orbit or development of a large orbit supercritical Hopf bifurcation;

These results contrast with the usual assumption that stability is lost or extremesis occurs due to load flow bifurcation or saddle node bifurcation in generator inertial dynamics if a Type 2 differential algebraic model is used. Saddle node bifurcation in generator inertial dynamics is never a cause of bounded trajectory feasibility in the research reviewed on Type 2 models if nonclassical generator models are used.

In Section 7, Type 3 stability problems on a Type 3 power system model is discussed. The Type 3 model include dynamic model of induction motors, under load tap changer and switchable shunt capacitor controlled loads; and thermostatic controlled loads. The generators are modeled by constant power sources because the generator dynamics are assumed to be decoupled and do not contribute to the stability problems on

a Type 3 model based on the literature reviewed. Saddle node and Hopf bifurcations are the only kinds of bifurcations that have been observed to date on this Type 3 model. These saddle node bifurcations can occur in either tap changer/switchable shunt capacitors or induction motor dynamics, and Hopf bifurcation can occur in tap changer controlled load dynamics. Thus, there are two classes of saddle node and one class of Hopf bifurcation in a Type 3 model based on the literature reviewed.

Section 8 provides a review of the kinds and classes of bifurcations that have been observed on a Type 4 model. A Type 4 model includes generator inertial, flux decay, exciter, power system stabilizer, and boiler-turbine-governor dynamics and a model of load dynamics. The load dynamics may be separate for real power and reactive power or may be a composite model such as that for induction motors. The dynamic load models used in Type 4 models would be similar to those used in Type 3 models.

Much of the early work on Type 4 models showed that saddle node, Hopf, period doubling, chaotic blue sky bifurcation, and chaos can occur on a single machine infinite bus model with a induction motor connected at a load bus between them. It is cautioned that there are features of this model that are not characteristic of models of actual power systems. For the sequence of bifurcations that occur on this model for an increase in a bifurcation parameter, the specific bifurcation that results in bounded trajectory feasibility is determined.

There has been only one study of a Type 4 model which has more than a single generator and a single induction motor dynamic model. The kinds of bifurcations and classes are identical to those on a Type 2 model. The sequence of bifurcations that lead to loss of bounded trajectory feasibility are the same. This result may suggest Type 2 models may well be sufficient to study voltage stability problems affecting dynamics (voltage control, reactive supply) of generators.

2. MODEL TYPES

A Type 2 model is perhaps the most common power system model used to study voltage instability [42]

$$\frac{dx}{dt} = f(x, I_d, I_q, V, \theta) \quad (1)$$

$$0 = I_d - g_d(x, V, \theta) \quad (2)$$

$$0 = I_q - g_q(x, V, \theta) \quad (3)$$

$$0 = P_L - g_P(x, V, \theta) \quad (4)$$

$$0 = Q_L - g_Q(x, V, \theta) \quad (5)$$

where the mp -dimensional vector x corresponds to generator dynamic states, (p is the dimension of the generators model used); I_d and I_q are the m -dimensional vector of generator stator currents, and P_L and Q_L are the N dimensional real and reactive load power. V and θ are the N -dimensional vector of bus voltage and angle at generator terminal buses, generator high side transformer buses, and load buses. If P_{L_i} and Q_{L_i} are functions of $|V_i|$

$$P_{L_i} = k_{P_i} V_i^p$$

$$Q_{L_i} = k_{Q_i} V_i^q$$

where either p or q is greater than or equal to 2.0, the equation (4) and (5) should be capable of being uniquely solved for V and θ locally so that V and θ can be eliminated [8] and the models (1-5) can be transformed into an equivalent differential equation model.

Venikov [4] was the first to study instability on such a Type 2 model. Kwatny [5] was the first to study saddle node bifurcation and loss of causality on such a model. Schlueter and Costi [11,43] was the first to add generator flux and exciter decay dynamics and relate the jacobian tests for saddle node bifurcation and loss of causality to load flow sensitivity matrices S_{VP} , S_{QLP} , S_{QQL} , and S_{QQL} . Sauer and Pai [17] also developed a jacobian test for loss of voltage stability. Recently, Schlueter [9,12,19,44] developed detailed models, jacobian tests, and tests for loss of causality, saddle node, and Hopf bifurcation based on these models. The most thorough theoretical treatments of stability problems for a Type 2 model is given in [8,16,74].

A Type 3 model assumes the generator dynamics are not affected or do not contribute to the voltage instability problems. A Type 3 model would be used to study radial voltage instability as defined in [29] and later in [34]. The state variable vector $\underline{x}$ of the generators is assumed to be fixed or nonexistent in the model equations. The Type 3 model has the form [42]

$$0 = P_L - g_P(x, V, \theta) \quad (6)$$

$$0 = Q_L - g_Q(x, V, \theta) \quad (7)$$

$$0 = h_v(P_L, Q_L, V, \omega, \frac{dV}{dt}, \frac{dP_L}{dt}, \frac{dQ_L}{dt}) \quad (8)$$

$$0 = h_\theta(\omega, \frac{d\theta}{dt}) \quad (9)$$

$$0 = h_\omega(P_L, Q_L, V, \omega, \frac{dV}{dt}, \frac{d\omega}{dt}, \frac{dP_L}{dt}, \frac{dQ_L}{dt}) \quad (10)$$

This model was derived from a third order induction motor model in the recent paper by Lesieutre, Pai and Sauer [42], V , θ and ω are each N dimensional state variables. P_L and Q_L can be replaced in (8-10) using equations (6,7). This results in an extremely nonlinear model expressed in an implicit form containing derivatives of state variables V , θ , ω . The implicit form makes solving the differential algebraic model more difficult.

The induction machine models (8-10) can be simplified to have the form of five different static and dynamic induction motor models that appear in the literature [58-62] as shown in [42]. The third order induction motor model used to derive (8-10) has the form [42]

$$\begin{aligned} \text{Stator} \quad V_{D_s} &= E'_D - X' I_{Q_s} \\ V_{Q_s} &= E'_Q + X' I_{D_s} \end{aligned} \quad (11)$$

Rotor Fluxes

$$T_o' \frac{dE'_Q}{dt} = -E'_Q + \frac{X_m^2}{X_r} I_{D_s} - s \frac{X_r}{R_r} E'_D \quad (12)$$

$$T_o' \frac{dE'_D}{dt} = -E'_D - \frac{X_m^2}{X_r} I_{Q_s} + s \frac{X_r}{R_r} E'_Q$$

Torque

$$2H \frac{ds}{dt} = T_L(\omega_r) - (E'_Q I_{Q_s} + E'_D I_{D_s}) \quad (13)$$

$$\text{where} \quad \omega_r = \omega_s(1-s)$$

$$X' = \frac{X_r X_s - X_m^2}{X_r}$$

$T_L(\omega_r)$ is load torque and a function of ω_r

$$E'_D = -\frac{X_m}{X_r} \psi_{Q_s}$$

$$E'_Q = +\frac{X_m}{X_r} \psi_{D_s}$$

$$T_o = \frac{X_r}{\omega_s R_r}$$

$$s = \frac{\omega_s - \omega_r}{\omega_s}$$

$$\begin{bmatrix} \psi_{D_s} \\ \psi_{D_r} \end{bmatrix} = \begin{bmatrix} X_s & X_m \\ X_m & X_r \end{bmatrix} \begin{bmatrix} I_{D_s} \\ I_{D_r} \end{bmatrix}$$

$$\begin{bmatrix} \psi_{Q_s} \\ \psi_{Q_r} \end{bmatrix} = \begin{bmatrix} X_s & X_m \\ X_m & X_r \end{bmatrix} \begin{bmatrix} I_{Q_s} \\ I_{Q_r} \end{bmatrix}$$

where stator equations are

$$0 = \psi_{Q_s} + V_{D_s}$$

$$0 = -\psi_{D_s} + V_{Q_s}$$

due to neglecting dynamics.

This model can be represented as a dynamic thevenin equivalent circuit with reactance X' and source $E'/\theta_E = E'_Q + jE'_D$. The main drawback of using a dynamic thevenin equivalent is that of obtaining data for X' and the internal source. Equations (11-13) can be helpful in that respect because parameter values that describe E'/θ_E may be estimated from measurements of bus voltage variables.

An advantage of viewing all loads and not just induction motor loads as dynamic equivalents circuits is that some of the "generic" loads (that have application in modeling thermostatic loads; static loads controlled by a series of distribution tap changing transformers; and multiple induction motors of similar size and at the same location) have the form of a dynamic equivalent circuit. Thus, one can write a far simpler nonlinear non-implicit Type 3 system model [42] using this generic load model dynamic thevenin equivalents:

$$0 = I_d(V) - g_{GP}(x, V, \theta) \quad (14)$$

$$0 = I_q(V) - g_{GQ}(x, V, \theta) \quad (15)$$

$$0 = I_{LD} - g_{LP}(V, \theta) \quad (16)$$

$$0 = I_{LQ} - g_{LQ}(V, \theta) \quad (17)$$

$$0 = I_{LD} - g_D(y, V, \theta) \quad (18)$$

$$0 = I_{LQ} - g_Q(y, V, \theta) \quad (19)$$

$$\frac{dy}{dt} = h(y, I_{LD}, I_{LQ}) \quad (20)$$

where I_{LD} and I_{LQ} are load currents, y is the vector of load state variables. Note that the dimension of both V and θ equals the sum of the dimension of I_{LD} and I_d (or I_q and I_{LQ}). If the load is fully modeled by the nonlinear differential equation (20), the remaining equations can be solved for (V, θ) knowing X , I_d , and I_q .

A Type 3 model must be capable of studying the saddle node bifurcations in load dynamics [30,31] and the saddle node and Hopf bifurcation brought on by under load tap changers and switchable shunt capacitors [16,53]. The assumption for Type 3 model to be valid is that generator dynamics are not affected and do not contribute or cause any of the stability problems that are properly studied using a Type 3 model. The radial voltage instability problem [29,34] is by definition a problem that should be studied using a Type 3 model because a radial voltage instability problems are defined as not affecting or being affected by generator dynamics. The literature on Type 3 stability problems [16,28,30,31,53] using a Type 3 model is not as extensive as the literature on Type 2 stability problems using a Type 2 model.

A Type 4 model must be used when load dynamics and generator dynamics are coupled and lead to stability problems that are observable, controllable, or caused by parameters in both load and generator dynamics. A Type 4 model has the form (1-5, 8-10) if a nonlinear implicit load dynamic model is used and (1-3, 14-20) if a dynamic equivalent circuit model for load is used. If all of the load is modeled using the nonlinear differential equation model and none is model as an arbitrary function of voltage and frequency then, the algebraic equations (2-3, 14-19) are linear and can be eliminated. This assumption would completely eliminate loss of causality as a cause of voltage instability. This will be discussed in more depth in Section 5 of the paper.

The recent literature on voltage instability suggests that the number of possible types, kinds, and classes of stability problems may be larger if load dynamics are included in the model. Moreover, many of the kinds and classes of the voltage instability problems have been recently discovered are Type 4

stability problems which are observable, controllable and caused by parameter changes in generator and load dynamics in a Type 4 model.

3. Feasibility, Structural Stability, and Bounded Trajectory Feasibility

The concern for system planners and operators has been that a system has a suitable large feasibility region of operating points and at each feasible operating point the system has a large region of attraction. The region of attraction is the set of all trajectories that converge to some stable equilibrium point. The feasibility region is defined in [16] as the set of operating points in model parameter and state space where the operating point is stable for continuous changes in parameters. The boundary of the feasibility region according to this definition is where bifurcation is a discontinuous change in stability properties when some parameter changes continuously. An alternative to a feasibility region is a structural stability region. Structural stability [46] requires that vector fields f

$$\dot{x} = f(x)$$

have qualitatively the same behavior for all nearby vector fields f . The system is both feasible and stable in a structural stability region as defined in [46]. This is true because no loss of stability bifurcation can occur when no qualitative behavior change can occur in a structural stability region even those that don't relate to stability.

A node focus bifurcation is an example where a change of stability properties is not associated with a qualitative change in behavior. A stable node focus bifurcation occurs when a complex pair of eigenvalues with negative real parts become two identical negative real eigenvalues as shown in Figure 5. An unstable node focus occurs when two complex eigenvalues with positive real parts become two positive real eigenvalues.

The feasibility region [16] is less restrictive, and thus larger than the structural stability region [46], but is of more pragmatic engineering value in terms of specifying where stability properties of the equilibrium point change. A feasibility region [16] is also too restrictive a definition because a system can experience a change in both qualitative behavior and stability properties at the equilibrium, and yet not experience unbounded trajectories that cause loss of synchronism, blackouts, and possible equipment damage.

Power systems experience onset of small stable limit cycles (supercritical Hopf bifurcations) where the equilibrium operating point is unstable without damage to equipment or without triggering emergency or extremesis controls. In fact, multiple supercritical Hopf bifurcations often occur as a precursor to voltage collapse and yet no control actions are necessarily taken. Supercritical Hopf bifurcation is an example of a change of stability property at equilibria without experiencing unbounded system trajectories. One could also have large orbit unstable limit cycles with a stable equilibrium where the region of attraction is large enough so that one might never recognize the evidence of the large orbit unstable limit cycle. Existence of large orbit unstable limit cycles retain bounded trajectory feasibility because the equilibrium is stable and a large region of attraction exists. Small orbit unstable limit cycles would cause loss of bounded trajectory feasibility if the region of attraction does not include a sufficiently large neighborhood around the equilibrium points (quasi equilibrium points and post contingency equilibrium points). Large orbit stable limit cycles also cause loss of bounded trajectory feasibility for trajectories because they illicit the tripping of generators to prevent equipment damage which would cause loss of reactive supply and voltage control of the generator at a time it is needed most. Finally, there should be a minimum constraint on the size of the region of attraction of the stable post contingency equilibrium point as a means of hopefully insuring that the region of attraction of each quasi equilibrium point (or closed orbit) contains the previous quasi equilibrium point. These issues are thus included in the definition of a bounded trajectory feasibility region as follows.

A bounded trajectory feasibility region is the subset of contingency and/or operation condition change induced trajectories from a set C on a power system model such that:

- (a) each trajectory converges to a post contingency stable equilibrium point or a stable closed orbit in the viability region V [14,16]. The viability region places constraints on voltage, current, real and reactive power flow, and steady state frequency;
- (b) each of the stable quasi equilibrium points or closed orbits along the trajectory lies in the region of attraction of the previous stable quasi equilibrium point or closed orbit. The quasi equilibria result from discontinuous control actions such as relays, under load tap changers, switchable shunt capacitors, or field

or stator current limit controllers. Since the model and the associated region of attraction change discontinuously after each control action, the only way to assure asymptotic stability is to assure asymptotic stability after the contingency and each discontinuous control action;

- (c) the size of any closed stable orbits in V must be small enough so that no equipment damage can occur or so that a station operator will not trip generators off the system;
- (d) unstable closed orbits that constrain the region of attraction around the stable (quasi or steady state) equilibrium or a stable closed orbit are not small;
- (e) each trajectory is contained in a region of attraction surrounding the post contingency stable equilibria or small stable closed orbit;
- (f) the trajectory lying within the region of attraction does not cause extremesis by tripping or stalling motors or tripping generators due to low voltage.

The set C is the set of all contingencies and in combination with the set of all operating changes that a particular utility must survive by its planning or operating criterion for voltage stability. Some utilities have different planning criteria and operating criteria while others have the same planning and operating criteria. If a utility has a separate planning and operating criterion, a separate bounded trajectory feasibility region would be needed for planning and for operation. The bounded trajectory feasibility region is a subset of C and must be determined via some security assessment procedure. The security assessment must include contingency selection for single generator outage, line outage, and combination generator and line outage contingencies. The set C also includes all realistic patterns of real and reactive load change and level, unit commitment change, generation dispatch change, and voltage setpoint change on generators and all other voltage control devices. The set of contingencies in combination with the set of operating changes constitute C . The bounded trajectory feasibility region is a subset of C and thus must be found by a rather sophisticated contingency selection procedure, such as [40], for equilibria that find the contingencies in combination with operating condition changes in C that do not belong to the bounded trajectory feasibility region.

The bounded trajectory feasibility region not only requires asymptotically stable trajectories, but requires the stable equilibrium or small stable closed orbits to lie within a viability region [14] specified by the utility as being the set of acceptable operating conditions. The trajectory must, in addition to not causing loss of viability, not cause extremesis in constraint (f). This constraint can be tightened to require no motors are stalled or tripped, but this may be too strict and may be too difficult to monitor from simulation or even from measurements after actual contingencies. The constraint (f) could be further strengthened to require that no generator exceed continuous field current limits during the first 20 seconds or during the entire trajectory as in the step-by-step procedure [3]. These additional requirements add additional robustness to the set of contingencies in combination with operating changes that are included in the bounded trajectory feasibility region since exceeding field current limit and disabling exciters can lead to immediate voltage collapse.

The feasibility region [16] would only assure that equilibria were stable; would not allow small orbit limit cycles to exist; would not prevent loss of stability due to unstable transitions between quasi equilibria that occur as a result of discontinuous control actions along the trajectory, would allow relay tripping of large motors and generators, and stalling of individual motors as long as they did not cause an unstable system bifurcation to occur. The feasibility region of [16] is defined for continuous parameter changes and yet equilibria for faults, line outage contingencies, and loss of generation contingencies are not due to continuous, but discontinuous parameter changes. One thus needs a bounded trajectory feasibility region which is a set of trajectories produced by continuous or discontinuous actions; do not have extremesis occurring; do not cause tripping of generating units; do not have small unstable closest orbits or large stable orbits; have a stable equilibrium or small stable closed orbits; and are asymptotically stable.

The description of a bounded trajectory feasibility region or just a feasibility region is not simple since:

- saddle node bifurcation
- node focus bifurcation
- supercritical and subcritical Hopf bifurcations
- blue sky bifurcation
- homoclinic bifurcation
- blue sky chaotic bifurcation
- chaos

have occurred in studies of bifurcations on power systems. Many of these kinds of bifurcations have affected different generator, induction motor, or both generator and induction motor dynamics depending on the model type and the complexity of the generator and induction motor model in that model type. A feasibility region [16] or the bounded trajectory feasibility region is impossible to establish because at the present time:

- (a) the number of different voltage instability problems keeps growing exponentially with rapid development of voltage stability theory, as will be evident from Sections 5-8 of this paper. Thus, it might not be possible to assure that the modeling is adequate to identify every bifurcation or closed orbit that can occur. Many of the improvements in modeling have brought new bifurcations. Without the ability to predict the bifurcations that occur and their sequence, it is impossible to assess a feasibility region or bounded trajectory feasibility region;
- (b) there is no classification of stability problems which prevents identifying what voltage instability problem actually occurred in a post mortem study or would occur given a particular operating change or contingency. Lack of classification based on the most accurate possible (Type 4) model prevents investigating the level of modeling accuracy needed to accurately simulate each stability problem, determine its modal structure, determine observability and controllability properties for it, and determine its cause, and assess its impact on the feasibility region and bounded trajectory feasibility region. One must ultimately establish every kind of bifurcation and class of bifurcation for the most complex Type 4 stability model to establish a classification of stability problems as to type, kind, and class. After such a careful classification is performed, some kinds and classes of bifurcations on each (simplified or nonsimplified) model type will be found to cause loss of bounded trajectory feasibility or loss of feasibility and some will not. The feasibility region and bounded trajectory feasibility region for each model type can't be found without this careful classification;
- (c) there is no systematic procedure for identifying a feasibility region or a bounded trajectory feasibility

region. Efforts for identifying a closest saddle node or Hopf bifurcation [50,51] in terms of the unit vector pattern of parameter changes p and the level of such change μ may be a start on such a procedure. There are possibly tens of different voltage stability problems of any one type, kind, and class in any subregion of a utility that is vulnerable to that specific type, kind, and class of stability problem. The closest saddle node or Hopf bifurcation algorithms [50,51] would only find the limit parameter vector p and level μ that would produce one particular bifurcation associated with one particular eigenvalue that in turn is one of several of that particular type, kind, and class of stability problem. There is currently no fast method for assessing asymptotic stability for each contingency, existence of stable or unstable limit cycles after each contingency, and whether a trajectory lies in the bounded trajectory feasibility region.

4. Types, Kinds, and Classes of Stability Problems on a Power System

The following definitions of types, kinds, and classes of stability problems is given below as a basis for the classification discussed in Sections 5-8 of this paper. The classification that is attempted in these sections is certainly a draft, that would be refined as the theoretical work on power system stability is applied to near perfect Type 4 models of actual systems to establish all of the stability problems that can be observed on such models. The classification can then be performed in terms of model type, kind, and class as described above.

Type: A type of power system stability problem is associated with a type of power system model. Classification of models into type must start by producing a system model where all system components are modeled with a level of accuracy beyond that may be needed to analyze a particular stability problem of any type, kind, or class. Hopefully, with improvements in load modeling, EXSTAB [2] and EUROSTAG [1] can accommodate such models, although one should never assume one has such a model since this is contrary to proper engineering practice. Such a complete and perfect model, if one can be found, would be the Type 4 model discussed in the previous section of the paper. A type of simplified model should include all power system components and each component modeled with sufficient accuracy so that: (1) all power system

stability problems of this type can be simulated with the desired level of accuracy; (ii) eigenvalue, eigenvector and participation factors associated with each stability problem be computed with a desired level of accuracy; (iii) observability and controllability properties of each stability problem of this type be computed with a desired level of accuracy; (iv) causal factor bifurcation properties that produce this stability problem be accurately assessed with a desired level of accuracy; and (v) the feasibility region and bounded trajectory feasibility region can be assessed with sufficient accuracy. If a subset of all possible stability problems on the perfect Type 4 model can be simulated with a particular simplified power system model and meet conditions (i)-(v), then: (a) one can properly call that simplified model one of a particular type; and (b) the stability problems that can be adequately analyzed on this simplified model can be called power system stability problems of this type. At this point in time it would be quite presumptuous to necessarily assume model 2 or model 3 are properly defined model types, since the requirements for being able to define a model type have yet to be carried out. This author believes the Type 2 and 4 models capture the classic voltage stability problems [29,34], Type 3 and 4 models capture the radial voltage instability problems [29,34], and only the Type 4 model capture the system transient voltage stability problem [29,34] or stability problems that are likely to occur as the system generation and load dynamics become more strongly coupled. These definitions of Type 2, Type 3 and Type 4 stability problems appear consistent with current theoretical literature on voltage stability and utility modeling practice.

It should be noted that all voltage instability problems could be simulated using the Type 4 power system model because it encompasses the generator modeling complexity of Type 2 models and the load modeling complexity of Type 3 models. Our choice is to define Type 4 stability problems as only those that require a Type 4 model rather than all stability problems that can be accurately analyzed using a Type 4 model.

Kind: Recent literature has demonstrated that the following kinds of bifurcation can occur on power system models:

- saddle node bifurcation
- Hopf {subcritical/supercritical} bifurcation
- loss of causality {algebraic/singularity induced/nonbifurcation}

- fold bifurcation
- node focus bifurcation
- 1:1 bifurcation
- period doubling bifurcation
- blue sky bifurcation
- chaotic blue sky bifurcation (transient chaos)
- chaos

Before determining and discussing each of these kinds of bifurcation, some important definitions are given.

Given a system model

$$\dot{z} = f_z(z, v, \mu) \quad (21)$$

$$0 = g(z, v, \mu) \quad (22)$$

an equilibrium point is defined for any value of $\mu = \mu_0$ such that (z_0, v_0) is a solution of (21-22) when $\dot{z} = 0$. The bifurcation parameter μ can vary over a range $[\mu_{\min}, \mu_{\max}]$ and produces equilibria $(z(\mu), v(\mu))$. A value of μ where the qualitative behavior of the time response of (21,22) changes is called a bifurcation value $\mu = \mu^*$ and $(z(\mu^*), v(\mu^*))$ is called the bifurcation point.

The jacobian of (21,22) is

$$J = \begin{bmatrix} \frac{\partial f_z}{\partial z}(z_0, v_0, \mu_0) & \frac{\partial f_z}{\partial v}(z_0, v_0, \mu_0) \\ \frac{\partial g}{\partial z}(z_0, v_0, \mu_0) & \frac{\partial g}{\partial v}(z_0, v_0, \mu_0) \end{bmatrix}$$

A solution $v_0 = v(z_0(\mu), \mu)$ locally exists and is unique by the Implicit Function Theorem when $\frac{\partial g}{\partial v}(z_0, v_0, \mu_0)$ is nonsingular. If $\frac{\partial g}{\partial v}(z, v, \mu)$ is nonsingular for all values z, v ,

and μ is some region, $v = v(z(\mu), \mu)$ can be found by locally solving (22) uniquely in this region and equations (22) can be eliminated to produce a differential equation model

$$\dot{z} = f_z(z, v(z(\mu)), \mu) = f^*(z, \mu)$$

in this region where $\frac{\partial g}{\partial v}(z, v, \mu)$ is nonsingular. A singularity is said to occur if $\frac{\partial g}{\partial v}(z, v, \mu)$ is singular. The

singularity can occur on a manifold called a frontier. Two types of points exist on the frontier; singular points and impasse points. If a trajectory of (21, 22) encounters a singular point, the trajectory stops because there is no solution $v(z, \mu)$ and thus no solution to (21, 22). If multiple solutions $v_i(z, \mu)$ exist, then there are multiple uncountable infinite numbers of trajectories along the impasse points on the frontier. Each trajectory appears like white noise with the possibility of discontinuities at all points in time. We will refer to the condition where singularities occur as loss of causality.

Loss of causality can never occur for the Type 4 model [42] where all load is modeled by differential equations (20) since all of the algebraic equations are linear and independent and can be solved uniquely. The assumption necessary to eliminate loss of causality as a possible stability problem is that all induction motor loads, thermostatically controlled loads, and all under load tap changer regulated loads satisfy the nonlinear differential equation (20) and thus there are no other loads to model. If an engineer were to assume any portion of the load at any bus in a model similar to that used in a Type 2 model

$$\begin{aligned} P_{L_i} &= k_{ii}|V_i|^{\zeta} \\ Q_{L_i} &= k_{di}|V_i|^{\eta} \end{aligned} \quad (23)$$

then one can not assume it is impossible for loss of causality to occur. Reactive and real power load is often modeled as in (23) in a transient stability model, because thermostatic and under load tap changer controls can be neglected due to the slow operation of these controls. The values are typically $\eta \geq 2$ and $\zeta \geq 1.5$. Thus, loss of causality should not occur (proven in [8] when either η or ζ is > 2 when there are 1 or 2 buses, but left as conjecture for 3 or more buses) even if static load models (23) are used for the portion of the load that: (a) is not induction motors; (b) is not controlled by thermostatic control; or (c) is affected by under load tap changer control and modeled by differential equations.

In a Type 2 model, loss of causality would be viewed as a saddle node bifurcation in the Type 3 model. Similarly, a loss of causality in a Type 3 model would indicate a saddle node bifurcation occurs in the generator exciter or inertial dynamics

of the Type 2 model. Thus, loss of causality may be viewed as occurring because the model type used is totally inappropriate since the dynamics where saddle node bifurcation occurs are neglected and the dynamics where no bifurcation occurs are preserved. Eliminating dynamics of the model where bifurcation occurs may distort or mask the true cause, electrical location, and bifurcation value μ^* , while preserving dynamics where bifurcation does not occur and likely has little or no effect on the cause or electrical location of the bifurcation in the system.

The definition of each kind of bifurcation will now be given along with a partial list of references where this kind of bifurcation has been found. Some of these bifurcations are local and can be analyzed by studying the vector field in the neighborhood of an equilibrium point or closed orbit. Global bifurcations can occur when a lack of transversality (transversality requires that if manifolds of dimension k and l meet along an n dimension space, then their intersection is a manifold of dimension $k + l - n$ for some $n > 0$) exists between stable manifolds and unstable manifolds of periodic orbits and equilibria. Homoclinic and heteroclinic orbits are examples where such lack of transversality occurs because stable and unstable manifolds coincide.

Definitions of local stable manifold, local unstable manifold, hyperbolic equilibrium point, saddle or node equilibrium point, orbit, and limit cycle are given in [32]. The discussion of some of the kinds of bifurcations is taken from Abed [47] where a very clear description of these bifurcations is given.

It is assumed that the basin of the region of attraction of complex power system models have the simple form shown in [39] where the boundary encircling a stable equilibrium or closed orbit is made up solely of stable manifold of unstable u.e.p. belonging to the boundary. The region of attraction could be more complex, but this simple form is used in assessing retention or loss of bounded trajectory feasibility for each bifurcation to be discussed. If future work on describing the boundary of the region of attraction establishes it is more complex and this complexity impacts the understanding of a bounded trajectory feasibility region, then the conclusions that follow on retention or loss of bounded trajectory feasibility for any bifurcation might change.

A saddle node bifurcation occurs at a bifurcation value where

J is singular, or where $\frac{\partial g}{\partial v}$ is nonsingular, and

$$A = \frac{\partial f_z}{\partial z} - \frac{\partial f_z}{\partial v} \frac{\partial g}{\partial v}^{-1} \frac{\partial g}{\partial z} \quad \text{has a simple zero eigenvalue as}$$

shown in Figure 5. A saddle node bifurcation occurs when an unstable equilibrium point (saddle) on the boundary of the region of attraction approaches the stable equilibrium point (node). The collision of the two equilibria results in disappearance of the stable equilibrium point [7]. Saddle node bifurcation can cause loss [47] of bounded trajectory feasibility for several trajectories if it is the only bifurcation in a system because the region of attraction would become quite small before it occurs and the trajectory becomes unbounded after it occurs.

A Hopf bifurcation occurs when the matrix $\frac{\partial g}{\partial v}$ is nonsingular and A has a simple pair of imaginary roots $\pm j\omega_0$

or when $\left[\begin{bmatrix} j\omega I & 0 \\ 0 & 0 \end{bmatrix} - J \right]$ has a pair of roots $\pm \omega_0$ as

shown in Figure 5. A Hopf bifurcation can be stable or unstable. In the former case, the bifurcation is called supercritical. In the later case, the Hopf bifurcation is called subcritical. Based on the bounded trajectory feasibility region boundary definition, a small orbit subcritical Hopf bifurcation would cause loss of bounded trajectory feasibility if it is the only bifurcation in the system because the subcritical Hopf bifurcation produces an unstable limit cycle with a small region of attraction around the stable equilibrium point. A large orbit unstable limit cycle does not cause a loss of bounded trajectory feasibility from its definition if the region of attraction around the stable equilibrium inside the limit cycle is sufficiently large. The supercritical Hopf bifurcation produces a stable limit cycle. The orbit size of the stable oscillation grows as μ increases for $\mu > \mu^*$. The supercritical Hopf bifurcation doesn't cause loss of bounded trajectory feasibility from its definition unless the orbits get large enough to cause equipment damage or operator initiated tripping of generation units. Stable limit cycle oscillations occur in a power system as a precursor of voltage instability and can also occur as interarea oscillations. Both supercritical and subcritical Hopf bifurcations have been observed on the same single machine infinite bus power system model [49] for different operating conditions. Supercritical and subcritical bifurcations can be observed in Figures 13-16.

A node focus bifurcation is a bifurcation in the topological structure of the system where a simple pair of complex roots become real at bifurcation value μ^* or vice versa [48] as shown in Figure 5. A node focus bifurcation can be stable if the real part of the eigenvalues are negative and unstable if the real part of the eigenvalues is positive. An unstable node focus bifurcation at $\mu = \mu_2^*$ for a system that previously experienced a small orbit stable Hopf bifurcation at $\mu = \mu_1^* < \mu_2^*$ causes loss of bounded trajectory feasibility if it was the only system bifurcation. An unstable node focus bifurcation at $\mu = \mu_2^*$ for a system that previously experienced an unstable Hopf bifurcation at $\mu_1^* < \mu_2^*$ would experience loss of boundary trajectory feasibility at μ_1^* when the unstable Hopf bifurcation has a sufficiently small orbit. A stable node focus bifurcation results in retention of bounded trajectory feasibility before and after the node focus bifurcation because the system is stable before and after the bifurcation.

The period doubling bifurcation occurs when a stable periodic orbit disappears at μ^* . The new periodic orbit is born which initially coincides with the original orbit, except that it has twice the period. The period doubling bifurcation can be seen by comparing Figure 1a and 1b. Cascaded period doubling bifurcation such as shown in Figure 1b and 1c can produce a strange attractor as shown in Figure 3a. Period doubling bifurcations are often a precursor of chaos. A period doubling bifurcation does not result in loss of bounded trajectory feasibility for stable orbits if the bounded stable orbits are small enough to prevent equipment damage or initiate operator tripping of generating units.

A 1:1 bifurcation occurs when two pairs of complex eigenvalues become two equal complex eigenvalues. The stability behavior may not change at a 1:1 bifurcation [48].

A global bifurcation may occur as part of voltage instability in a power system model when the basin of the region of attraction becomes small enough. A heteroclinic orbit is a global bifurcation where the unstable manifold of one equilibrium point coincides with the stable manifold of another equilibrium point. It can not be analyzed in a neighborhood of an equilibrium point. A homoclinic orbit is a global bifurcation which occurs when a branch of both the stable manifold and unstable manifolds of a saddle equilibrium point coincide.

Heteroclinic and homoclinic orbits generally lie in the vicinity of a saddle in the boundary of the region of attraction. As a saddle node bifurcation is approached for some contingencies

as it does for voltage instability, heteroclinic and for homoclinic orbits could sufficiently constrain the boundary of region of attraction to cause a loss of bounded trajectory feasibility for those trajectories.

A blue sky bifurcation is a third global bifurcation that occurs when a limit cycle disappears through collision with a saddle equilibrium point. Prior to the bifurcation parameter value μ^* , a saddle equilibrium exists with a limit cycle. At μ^* , the limit cycle and a branch of both the stable and unstable manifolds of the saddle equilibrium point coincide forming a homoclinic orbit as shown in Figure 2. For $\mu > \mu^*$, the limit cycle no longer exists which is also shown in Figure 2. Collision of a saddle with a limit cycle typically is a mechanism by which a limit cycle disappears. The blue sky bifurcation has two forms, disappearance of a stable limit cycle and disappearance of an unstable limit cycle. It results in unbounded trajectories. The blue sky bifurcation of a stable limit cycle is shown in Figure 2. Collision of a stable limit cycle with a saddle would occur after loss of bounded trajectory feasibility has occurred because if the stable orbit was large a loss of bounded trajectory feasibility already has occurred and if a small stable limit cycle collides with a saddle would indicate too small a region of attraction for bounded trajectory feasibility to hold for most contingencies. Collision of an unstable limit cycle with a saddle would not cause loss of bounded trajectory feasibility because a small orbit unstable limit cycle already causes loss of bounded trajectory feasibility. A large unstable limit cycle and its collision with a saddle may not cause loss of bounded trajectory feasibility if it occurs in the vicinity of the region of attraction boundary and does not limit the size of its region of attraction around the stable equilibrium or small stable orbit. A chaotic blue sky bifurcation is similar to a blue sky bifurcation in that collision of some strange attractor with a saddle results in blue sky chaotic bifurcation just as blue sky bifurcation is a collision of a limit cycle with a saddle and is shown in Figures 11 and 12.

Chaos is associated with:

- (a) divergence of trajectories no matter how close their initial conditions are;
- (b) containing periodic orbits of all periods. It contains infinitely many nonperiodic orbits;
- (c) there is an orbit which is dense.

Examples of chaos of a power system are shown in Figure 3. Chaos would generally be considered to cause loss of bounded trajectory because the oscillations are generally large and cause operator action to protect equipment. Chaos can be produced by several routes. Repeated period doubling bifurcation can result in chaos and is the means discussed in the recent papers on power systems being reviewed.

Class: Each kind of bifurcation can occur in the inertial or electrical dynamics of the generator or induction motor loads, the tap changer and switchable shunt controls, or in the thermostatic load control dynamics. The class is a third hierarchical level of designation below type (model type) and kind of bifurcation. The class of any specific type/kind of bifurcation can be found by finding the large state variable elements of the right and left eigenvectors and the causal factors associated with that kind of bifurcation in that model type. Possible classes of any type/kind of bifurcations are:

- generator inertial, flux decay, and exciter
- generator electrical
- induction motor inertial
- induction motor electrical
- tap changer and switchable shunt capacitor voltage controls
- thermostatic load control.

5. Type 1 Stability Problems

Type 1 is an extremesis condition where relays on large motor loads cause them to drop out due to low voltage. The relays often cause these motors, and even some generators, to trip out if voltage remains sufficiently low for a period of time. Low voltage without loss of asymptotic stability has been called voltage instability in [29,34], and radial voltage instability is such a type of stability problem. The extremesis condition of actually losing load (which can increase voltage) can be such a severe consequence of low voltage that it deserves a separate classification. A Type 1 voltage instability problem occurred on the Florida system [27] when voltage dropped due to a contingency causing large motor relays to trip. The tripping of the motor caused a recovery of voltage and resynchronization of the motor loads. The resynchronization of load caused

voltage to drop again. Voltage and load cycled for several hours due to the effects of relaying and resynchronization and was so severe that operators considered the system was in extremesis. If the relays had not tripped out the motor loads and they stalled, a Type 3 saddle node bifurcation of the Type 3 model would possibly have occurred because there were no generators near the load center experiencing this problem. Not only can a Type 1 voltage instability be brought on by low voltage, but also whenever extremesis conditions occur which could lead to blackout and result in significant loss of load or generation due to relays or operator action to prevent the loss of voltage stability.

6. Type 2 Stability Problems

6.1. Kinds and Classes of Type 2 Stability Problems

Type 2 stability problems occur on a Type 2 model where all generator dynamics are retained, but load dynamics are ignored and load is modeled by a constant power or static voltage and frequency dependent load model. Load modeling is often performed by aggregating the distribution system and assuming a constant power load model that reflects the effects of distribution system voltage controls. However, some utilities include parts of the distribution system and its voltage controls that not only restore load, but can cause a transient peak in load. Voltage and frequency dependent static load models can also be used as summarized in [42].

Substituting equations (2) and (3) for I_d and I_q , the jacobian of (1,4,5) is defined as

$$J^{***} = \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial \theta} & \frac{\partial F}{\partial V} \end{bmatrix}$$

$$= \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \\ A_3 & B_3 & D_3 \end{bmatrix} \quad (24)$$

6.1.1. Classes of Loss of Causality in a Type 2 Model. Loss of causality in a Type 2 model occurs when

$$J_c = \begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}$$

is singular as μ changes, which implies (4) and (5) have no solution or have multiple solutions

$$\begin{bmatrix} \theta_i(x_0) \\ V_i(x_0) \end{bmatrix} \quad i=1,2,\dots$$

in the neighborhood of an equilibrium $[x_0, \theta_0, V_0]$. If there is no solution

$$\begin{bmatrix} \theta(x) \\ V(x) \end{bmatrix}$$

to (4) and (5) which can be substituted back into (1), the trajectory terminates abruptly. If multiple solutions exist, randomly discontinuous trajectories can occur as the solution $[\theta_i(x), V_i(x)]$ changes from time point to time point [21]. Three classes of loss of causality are possible:

- (i) algebraic bifurcation [13] where

$$J_c^{***} = \begin{bmatrix} A_2 & B_2 & D_2 \\ A_3 & B_3 & D_3 \end{bmatrix} = \begin{bmatrix} J_{PL}^* \\ J_{QL}^* \end{bmatrix}$$

is row dependent;

- (ii) singularity induced bifurcation [16] where the eigenvalues of J cross from the right-half plane to the left-half plane or vice versa by going through infinity rather than across the $j\omega$ axis (Figure 5);

- (iii) nonbifurcation loss of causality [13] where no eigenvalue crosses the $j\omega$ axis or goes through infinity from the right-half plane to the left-half plane or vice versa. This bifurcation differs from algebraic bifurcation in that term

$$\begin{bmatrix} B_1 & D_1 \end{bmatrix} \begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_3 \end{bmatrix}$$

of J_D^{***} does not change significantly rather than J_c^{***} becoming row dependent as

$$\begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}$$

becomes singular. Nonbifurcation loss of causality of a differential algebraic model could be viewed as load flow bifurcation if all of the generator terminal buses are P-Q rather than P-V buses as they normally would be in a loadflow.

All three classes of loss of causality produce the truncation of trajectories or randomly discontinuous trajectories, but with

quite different affects on the generator dynamics (1). Each of the three classes of loss of causality can occur when

- (a) $\begin{bmatrix} B_2 & D_2 \end{bmatrix}$ is row dependent, which is called angle loss of causality;
- (b) $\begin{bmatrix} B_3 & D_3 \end{bmatrix}$ is row dependent, which is called voltage loss of causality;
- (c) $\begin{bmatrix} D_2 & D_2 \\ B_3 & D_3 \end{bmatrix}$ is row dependent but neither $\begin{bmatrix} B_2 & D_2 \end{bmatrix}$ nor $\begin{bmatrix} B_3 & D_3 \end{bmatrix}$ is row dependent, which is called voltage and angle loss of causality.

It can be viewed that angle loss of causality, voltage loss of causality, and angle and voltage loss of causality are subclasses of singularity induced bifurcation, algebraic bifurcation, and nonbifurcation loss of causality. It will now be proven that the voltage and angle bifurcation is the only generic subclass if the matrix J_c is nondecoupled, but that voltage loss of causality and angle loss of causality are generic if J_c is decoupled where D_2 and B_3 are null, which only occurs under light load conditions. Nongeneric stability problems can occur but have a zero probability, since the probability is proportional to that of a countable number of points on a real line. A generic stability problem has a nonzero probability.

The generic loss of causality is voltage and angle loss of causality if the model is nondecoupled [12,13] since loss of voltage and angle causality requires finding a vector v satisfying

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^T \begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix} = 0 \quad (25)$$

but loss of angle causality would require finding a v satisfying (25) where v_2 is required to be zero and loss of voltage causality would require finding v satisfying (25) where v_1 is required to be zero. Loss of voltage causality and loss of angle causality would thus be nongeneric zero probability events because they would be restricted to be bifurcations in a reduced dimensional space. If the load flow model were decoupled so that B_3 and D_2 are null, then: (i) loss of voltage causality and loss of angle causality would be independent, (ii) loss of voltage and angle causality, which requires both to occur simultaneously, would be nongeneric and (iii) both loss of voltage causality and loss of angle causality would be generic. Loss of causality is not likely to occur when a decoupled model

is valid, which occurs when since the system is lightly loaded. Loss of angle causality, it is to occur, will occur under light load conditions. Determining the percentage change in the magnitude of the singular value of D_3 or that of B_2 will help decide whether angle loss of causality or voltage loss of causality is the principal reason for loss of causality in a nondecoupled model. Usually, the singular value of D_3 experiences the largest change as shown in [25].

Nonbifurcation loss of causality in a Type 2 model is a radial voltage instability problem [29,34] that occurs on the distribution system. It occurs when the voltage collapse is electrically far from the generator dynamics so that: (a) the field current limits on generators are not reached, and (b) generator mechanical or electrical oscillations do not occur. Loss of causality occurs when motors stall [30,31] but are not tripped and after local generators are tripped out due to low voltages they are not affected by the low voltage. Stalling of the induction motor causes a runaway increase in slip, reactive power output, and voltage decline. Often the voltage level where nonbifurcation loss of causality could occur are quite low. When nonbifurcation loss of causality occurs, it reflects a static bifurcation in load dynamics in a Type 3 model. Nonbifurcation loss of causality in a Type 2 model is indication that a Type 3 or 4 model should be used since the Type 2 model approximates the very dynamics experiencing bifurcation and the approximation eliminates all dynamic bifurcations in this model.

Singularity induced bifurcation is a second important class of loss of causality and indicates that the network and load dynamic at $t = \infty$ after a disturbance causes an eigenvalue of the dynamic model to cross the $j\omega$ axis and approach infinity. The loss of bounded trajectory feasibility due to node focus bifurcation is to occur before the singularity induced bifurcation [18,19,36,57,64,71], which indicates the loss of feasible dynamic behavior of a trajectory occurs before loss of solvability of the algebraic equations. The fact that loss of bounded trajectory feasibility sometimes occurs just before singularity induced bifurcation as in [18,19,36,43,57,64,71], indicates the central role loss of causality (stress in the network and load) plays in bringing about loss of bounded trajectory feasibility that is measured in dynamic behavior.

Algebraic bifurcation is a nongeneric bifurcation because it requires that the differential algebraic bifurcation occurs in a

subset of the equations, which is a zero probability occurrence as argued above.

6.1.2. *Classes of Static Bifurcation in a Type 2 Model.* Type 2S is a static stability problem that occurs when

$$J_D^{***} = A_1 - [B_1 \quad D_1] \begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_3 \end{bmatrix}$$

exist where

$$\begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}$$

is nonsingular and J_D^{***} is singular as parameters μ change. Saddle node bifurcation is a special but generic case of static instability which occurs when a single eigenvalue of J_D^{***} is zero and certain transversality and nondegeneracy conditions hold [32]. Two classes of static bifurcation can occur in Type 2 model of a power system. A static stability problem can either occur in inertial angle dynamics [4-7] or in generator flux decay dynamics [9,19,33,44,52,54], or both. Static bifurcation in the generator flux decay dynamics has been called classical voltage instability in [29,34]. Static voltage instability in the generator flux decay dynamics is due to insufficient reactive supply in the EHV system. The French blackout [35] would most likely have occurred due to saddle node bifurcation in generator flux decay dynamics because it is the only known bifurcation that is known to explain the uncontrolled spread of a voltage collapse. If a single generator exciter is disabled by its field current limit controller, and the $S_{Q_{GE}}$ is not diagonally dominant, then a saddle node bifurcation in flux decay dynamics occurs which causes the generator's internal voltage and reactive generation to approach zero [33]. Matrix $S_{Q_{GE}}$ is a sensitivity matrix between reactive generation and voltage at generator internal buses. The matrix $S_{Q_{GE}}$ was studied in [11] and shown to be diagonally dominant if the system transmission network appears inductive and absorbs reactive power, but is not diagonally dominant if it appears capacitive due to line charging of long high voltage lines. If reduction of field current, generator internal voltage and reactive supply due to field current limit controller action is followed by a saddle node bifurcation, the generator dynamics are driving generator internal voltage to zero, reducing its reactive supply, and reducing the reactive supply from the line charging in the network as proven in [19,33]. Neighboring generators are thus driven to exceed field current magnitude/duration limits, causing field current limit controller

action and saddle node bifurcation on these generators. The field current limit controller/saddle node bifurcation action drive these generators internal voltage and reactive supply to zero spreading the voltage collapse to still other generators.

The condition that S_{QcE} is nondiagonally dominant generally occurs on systems vulnerable to voltage instability and will develop as a system begins losing control of voltage, which occurs long before the voltage collapse. Before voltage control is lost, the capacitive nature of the network is masked by the excellent control of voltage. Thus, the network condition for saddle node bifurcation in generator flux decay dynamics is established long before the saddle node bifurcation occurs. Occurrence of saddle node bifurcation in generator flux decay dynamics only requires a field current limit controller to disable an exciter. Thus, saddle node bifurcation in generator flux decay dynamics occurs long before node focus bifurcation in inertial and flux decay dynamics if a generator hits field current magnitude/duration limits and the field current limit controller completely disables the exciter. The theory and simulation results supporting this discussion are given in [19,33]. This discussion of saddle node bifurcation is supported by the observation in [25] that a generator's reactive generation is often near zero when field current limit controllers disable the exciter, and the description of voltage collapse as occurring once one single generator reached field current magnitude/duration limits.

$$\text{An algebraic measure } J_{VB} = \begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix} - \begin{bmatrix} A_2 \\ A_3 \end{bmatrix} A_1^{-1} [B_1, D_1]$$

was derived in [52,54] and was used to prove that loss of

causality (singularity of $\begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}$) occurs before saddle node

bifurcation in flux decay (J^{***} singular) and/or inertial dynamics when exciters are not disabled and saddle node bifurcation in flux decay dynamics occurs before loss of causality when exciters are completely disabled. These theoretical results are substantiated by computational results on the three-generator system of [52]. These papers [22,33,52] are the only known research on power system bifurcations where the effects of field current limit controllers totally disabling excitation controls is studied. In [57] the generators are assumed to have finite reactive limits, but a Maximum Excitation Limiter is used to avoid totally disabling the exciters. In most other papers it is assumed that infinite reactive capability exists or that voltage instability occurs

before reactive limits are reached. Classic voltage instability [25,29,34] is described as occurring when generators reach field current limits and field current limit controllers disable the exciters. If classic voltage instability is described as occurring when field current limit controllers disable excitation or reduce field current limits, omission of field current limit controller action is a major oversight.

A Maximum Excitation Limiter (MXL) [55], shown in Figure 4, is a field current limit controller that does not disable the exciter, but works through and maintains excitation control while observing field current limit magnitude/duration limits. The MXL reduces field currents rapidly if they exceed 1.6 times the continuous rating field current level. If field current exceeds 1.05 times the continuous rating limit, a delay proportional to the magnitude of the field current is used before the field current limit controller reduces and holds field current to its continuous rating. This delay can reach 30 seconds on the example shown in [55]. The exciter loop maintains excitation voltage control at all times, even when this Maximum Excitation Limiter control loop is adjusting the voltage setpoint after field currents exceed 1.05 of continuous rating. The retention of stability is greatly enhanced by this field current limiting over-ride of the exciter setpoint voltage as discussed in [25].

Theoretical results in [33] also cover the case where field current limit controller is an over-riding adjustment on exciter setpoint voltage. These results [33] indicate that the matrix S_{VE} , that relates voltage V at generator terminal buses, high side transformer buses, and load buses to generator internal bus voltage E'_q , must no longer be non-negative (no negative elements) with no zero rows (loss of P-Q controllability [11]) for a J^{***} to become singular. This will theoretically occur after a loss of causality occurs from [52] and after Hopf bifurcation occurs from computational results in [18,19,48,57,64]. Computational results also suggest that when a field current limit controller does not disable the exciter, but performs over-riding field current limiting both the generator inertial and generator flux decay dynamics become unstable simultaneously. This is supported by computational results in [18,19,36,48,57,64,71] as discussed later in this section after results on Hopf bifurcation in a Type 2 model are discussed. These results finally suggest loss of bounded trajectory feasibility occurs before either the loss of causality or this saddle node bifurcation due to a node focus bifurcation that

causes loss of stability in both generation inertial, and flux decay dynamics.

Loss of stability in both generator inertial and flux decay dynamics may also be associated with the loss of transient voltage stability that occurred on the Czechoslovakian system [29] after the tripping of lines in an important interface. Static bifurcation in flux decay dynamics has been proved to always occur before static bifurcation in inertial dynamics if the generator field current limits are reached [22,52,54]. Loss of voltage stability is likely to occur in both inertial dynamics and flux decay dynamics if the disturbances are very large and transmission interfaces or boundaries are weak so that loss of stability occurs soon after the contingency occurs and before the field current level/duration limits are reached. Loss of stability in inertial and flux decay dynamics is associated with transient voltage instability [34], such as occurred on the Czechoslovakian system, as described in [29] due to a trajectory exceeding the boundary of the region of attraction [6,39]. Static bifurcation in generator flux decay dynamics describes the classical voltage stability problem [34] that occurred on the French system [35] as discussed earlier. Loss of transient voltage instability and classical voltage instability both occur when the trajectory lies outside the region of attraction surrounding a stable equilibrium or closed orbit. The region of attraction surrounding a stable equilibrium has been described for transient voltage instability that results in a trajectory leaving the region of attraction in inertial flux decay and exciter dynamics [77]. A Lyapunov function has been developed [59] for measuring proximity to static bifurcation in both voltage and angle. The modeling required to study loss of classic voltage stability [34] and transient voltage instability due to outage of branches in an interface [29], is much more complicated than that used in the study of loss of stability due to faults [77] or due to voltage and angle stability [59] because it must include the affects of discontinuous control actions such as relays, underload tap changers, switchable shunt capacitors, and field current and stator current limiters. Methods for describing the region of attraction that include both generator flux decay, exciter dynamics, inertial dynamics, and the discontinuous control actions must be developed that are different than in [77], and Lyapunov methods for measuring proximity to either loss of transient voltage stability or loss of classical stability need to be developed. Such methods could check loss of stability after the contingency and each discontinuous control action.

6.1.3. Classes of Dynamic Bifurcation in a Type 2 Model.

Type 2D is a dynamic stability problem that occurs when J_D^{***} has complex eigenvalues with zero real parts where

$$J_c = \begin{bmatrix} B_2 & D_2 \\ B_3 & D_3 \end{bmatrix}$$

is nonsingular and nondegeneracy conditions hold as parameters μ changes.

Hopf bifurcation is the special but generic case where a single pair of complex eigenvalues has real zero parts and certain nondegeneracy conditions hold [32]. Three classes of Hopf bifurcation have been determined to exist in [56] for a single machine infinite bus model with single axis generator model and various exciter models. The first class is a Hopf bifurcation (Class 1) that occurs in generator inertial, flux decay, and exciter dynamics and is associated with low frequency interarea oscillations in a power system. The oscillation is often referred to as a power swing mode and can be associated with a single generator, two generators, or with groups of generators. A second class Hopf bifurcation (Class 2) occurs in generator flux decay and exciter dynamics only and often called a voltage mode. A third class of Hopf bifurcation (Class 3) occurs in generator flux and inertial dynamics and occurs when the exciter is completely disabled. These Class 3 Hopf bifurcations are likely to be the limit cycles that develop as a precursor to collapse and are thus called voltage collapse modes. The differences between Class 1 (power swing mode) Hopf bifurcation and Class 3 (voltage collapse mode) Hopf bifurcation is based on whether the exciter is present or disabled by field current limits. The Class 1 and 3 Hopf bifurcations often do not differ in the dynamics associated with the oscillation because in both cases the generator flux decay dynamics serve to reduce damping of the generator inertial dynamics. However, the actual cause of the two oscillations is quite different.

The Class 1 Hopf bifurcation (power swing modes) occurs when K_5 is negative on the single machine infinite bus model and are likely the low frequency interarea oscillations between groups of generators. Class 1 Hopf bifurcations appear to be supercritical Hopf bifurcations because they always appear as stable limit cycle oscillations in the literature reviewed in this paper, and they continually exist on the WSCC system without documented loss of stability. It has not been proven that Class 1 Hopf bifurcations are always supercritical and this is an important research task.

It is shown that Class 3 Hopf bifurcation in generator inertial and flux decay dynamics occurs when the generator exciter is disabled and single machine parameter K_4 is negative in [56]. It is proven that when K_4 is negative, that K_5 must be positive [56]. A further analysis establishes why K_4 and K_5 take on positive and negative values for a single machine infinite bus model. Hopf bifurcations are observed as a precursor to voltage instability when generator exciters are totally disabled. These Class 3 Hopf bifurcations are likely supercritical because they persist as a precursor to voltage collapse for long periods and at many different locations and frequencies. These Class 3 Hopf bifurcation (voltage collapse modes) have not been proven to always be supercritical and this is an important research task.

Class 2 Hopf bifurcation (voltage swing modes) [56] occur solely in the generator exciter/flux decay dynamics. These Hopf bifurcations have been shown to be a subcritical Hopf bifurcation on a two generator, three bus model for one operating condition [64]. It remains to prove that Class 2 Hopf bifurcations are always unstable or subcritical Hopf bifurcations. The Class 2 Hopf bifurcation has been shown to occur on a single machine infinite bus model when K_4 is positive and large or when K_5 is negative with sufficiently large absolute magnitude. The results in [56] also indicate that a Class 2 Hopf bifurcation only occurs on a single machine infinite bus model when the exciter compensator $\frac{K_F}{T_F S + 1}$ is not properly designed for a particular system.

A very interesting study of bifurcations on Type 2 models of increasing complexity was recently completed [57]. The bifurcation parameter was load at bus 5 on a three generator nine bus system. For a classical model a stable node focus bifurcation of the inertial mode of a generator occurred producing two real eigenvalues. The linearized model has the form

$$[M_s^2 + DS + K]\Delta\delta = \underline{AP}_m \quad (26)$$

where K approaches singularity and M and D are the fixed inertia and damping matrices. When $M^{-1}D$ is positive definite and $\underline{K}$ is positive definite the system is locally asymptotically stable. Once the $\det[K]$ is small enough, a node focus bifurcation occurs. After the node focus occurs, one positive real eigenvalue moves right and the other real positive eigenvalue moves left. The real eigenvalue that moves right results in saddle node bifurcation when K is singular. Since K is singular if and only if J^{***} is singular when the system is

causal for this simple system model, the system experiences saddle node bifurcation in generation inertial dynamics when the equilibrium equations (load flow) experiences bifurcation. For a Type 2 model with generator flux decay, exciter, and inertial dynamics [57], a Class 1 (power swing mode) Hopf bifurcation occurs in inertial/flux decay dynamics (E'_q, δ, ω) , followed by an unstable node focus bifurcation as shown in Figure 5. The node focus occurs as the frequency of the limit cycle produced by the Hopf bifurcation decreases to zero. The more positive of the two real eigenvalues is associated with inertial dynamic states (δ, ω) and the less positive real eigenvalue is associated with exciter/flux decay dynamics (E'_q, R_f) . The instability that occurs at the unstable node focus bifurcation reflects the equivalent of both a saddle node bifurcation induced loss of stability in both inertial (δ, ω) and in flux decay/exciter (E'_q, R_f) dynamics. A singularity induced bifurcation and finally the saddle node bifurcation follow the node focus bifurcation. The sequence of Hopf, node focus, singularity induced bifurcation, and saddle node bifurcation, shown in Table 1 and depicted in Figure 5 occurs in both the 3 generator 9 bus system and in the ten generator 39 bus New England system model. This same bifurcation sequence has been observed in [18,33,52,54,75,76].

It should be noted that loss of causality (singularity induced bifurcation) occurs after both the Hopf and node focus bifurcations in both the three and ten generator systems [57] and that the saddle node bifurcation in generator flux decay dynamics (E'_q, R_f) occurs after the loss of causality. This sequence of loss of causality occurring before the saddle node bifurcation in flux decay dynamics was proven in [52]. However, the sequence of loss of causality and saddle node bifurcation are irrelevant in terms of loss of bounded trajectory feasibility, that either occurred due to the Hopf bifurcation being subcritical or if not then due to node focus bifurcation.

Note that if the node focus causes the bounded trajectory feasibility, it occurs due to loss of stability in inertial dynamics and in flux decay/exciter dynamics as shown in Table 1. The loss of stability in exciter flux decay dynamics should cause the generator internal bus voltage and reactive supply to approach zero as was the case when the exciter was disabled. The action of field current limit controller and the node focus induced loss of stability of excitation/flux decay dynamics should cause the uncontrollable spread of voltage collapse throughout a system as indicated would occur in [33] when the exciter is disabled. The development of numerous undamped oscillations of various

frequencies and at various locations throughout the region where voltage collapse is about to occur is described theoretically by this sequence of supercritical Hopf bifurcation followed by the node focus bifurcation, which actually produces the voltage collapse. Thus, there are two possible bifurcation sequences that can produce the uncontrollable spreading voltage collapse; saddle node bifurcation in flux decay dynamics when generator exciters are completely disabled by field current limit controllers, and supercritical Hopf bifurcation followed by node focus bifurcation when exciters are not completely disabled. It should be noted that one might expect the orbit size of the supercritical Hopf bifurcation to increase as the frequency of the oscillation decreases to produce the node focus bifurcation. The size of these oscillations may be large enough to cause loss of bounded trajectory feasibility even before the node focus would cause loss of bounded trajectory feasibility.

It should be noted that if the Hopf bifurcation was subcritical, the loss of bounded trajectory feasibility would have occurred at the point where this subcritical Hopf bifurcation occurs. If the only subcritical Hopf bifurcation of those three that occur in generator inertial/flux/exciter dynamics is the voltage mode and if the role of the exciter compensator is to stabilize this voltage mode, then this cause of voltage instability can be corrected by redesigning the exciter compensator. This is rather a simple solution. Research is needed to prove the above assumptions.

If the Class 1 and 3 Hopf bifurcations are always supercritical, and if they always lead to node focus bifurcation; and if the node focus bifurcation causes the internal generator voltage to approach zero as has been proven to occur when the exciter is completely disabled; then possibly the most important cause of the uncontrollable voltage collapse is the supercritical Hopf bifurcation followed by node focus bifurcation. Research is needed to theoretically and experimentally verify the hypotheses and is an important subject for research.

It should be noted saddle node bifurcation that occurs when generator exciters are completely disabled by field current limit controllers is possibly the most frequent cause of the voltage collapse problems that were first encountered [35]. The recent use of maximum exciter limiters as shown in Figure 4 should prevent generators from exceeding continuous field current limits by continually readjusting the voltage setpoint on the exciter. This action should maintain a modicum of voltage

control by the exciter and thus quite possibly make the supercritical Hopf followed by node focus bifurcation the most frequent cause of voltage collapse currently. The discussion in [3] indicates that if the exciters are completely disabled, the system is very much more susceptible to voltage collapse and recommends that utilities avoid disabling the exciter completely and use maximum exciter limiters on most if not all generators. The greater susceptibility of a system to voltage collapse when exciters are completely disabled is thus verified theoretically [33,52].

The above discussion requires theoretical work as outlined and experimental validation on large systems. If the hypothesis prove correct, then the development of numerous oscillations at various locations and at various frequencies is not only a precursor, but a measure of proximity to voltage collapse. The magnitude of the oscillation should grow and frequency of the oscillation should decrease as node focus bifurcation and voltage collapse is approached for one of these supercritical Hopf bifurcations. This increase in amplitude and decrease in frequency of one specific limit cycle may be used as a proximity measure of approaching collapse for that voltage collapse problem.

Perhaps the most interesting hypothesis is that the node focus bifurcation may occur very close to that predicted by the nose of a P-V curve or Q-V curve. Note in Table 1 that the nose of the PV curve occurs just before the node focus bifurcation. Thus, the computation of a P-V or Q-V curve by an accurate equilibrium point solver may be a conservative, but accurate estimate of where node focus bifurcation occurs. If load flow is not a consistently accurate estimate of the equilibrium point, a new equilibrium point solver is needed similar to that available in EXSTAB [2]. This hypothesis also needs theoretical and experimental validation.

After this rather long discussion of the above results obtained in [18,33,52,54,57,75,75] on the sequence of supercritical Hopf followed by node focus bifurcation, the discussion of results in [57] is continued. This research study in [57] was repeated with finite reactive power limits rather than infinite reactive power limits. The eigenvalue movements are similar (Figure 5, Table 1) for the two cases and the bifurcations occur in the same sequence, but at much lower values of the bifurcation parameter μ which is a scaling factor on constant power loads at P-Q buses. Despite having finite reactive limits, the exciters were not disabled or otherwise the eigenvalues would have

changed. The results confirm our earlier analysis that if a field current limit controller does not disable the exciter, but uses a field current limiting loop to over-ride and reduce the exciter setpoint to a point where excitation control remains active, a simultaneous loss of stability in inertial dynamics and in flux decay dynamics occurs at an unstable node focus if the Hopf bifurcation is supercritical.

The research [57] was repeated for a Type 4 model using induction motor dynamics of [58], and the results are virtually identical to those for the Type 2 model. This confirms results in other papers [73] on Type 4 models when a full generator model with inertial, flux decay, and exciter dynamics is used.

A very similar result to that in [57] was observed [18] for the same three generator system studied in [22,33,52-54,48], but with a two axis generator model, a Type 1 exciter model with compensator, and no sensor dynamics. A Hopf bifurcation is followed by a node focus, singularity induced bifurcation and a saddle node bifurcation as shown in Figure 5. The results in [18] do not indicate whether the Hopf bifurcation is Class 1 or 2 and whether it is a stable or unstable Hopf bifurcation. The loss of bounded trajectory feasibility region could occur either due to the node focus (if the Hopf is stable) or due to unstable Hopf bifurcation and before loss of causality (singularity induced bifurcation) or saddle node bifurcation. This work [18] supersedes [57] and is thus less complete. A constant impedance load model was used and none of the bifurcations were observed that were found with a constant power load model. A more recent paper [46] contains the same example model as [18] and establishes that the Hopf bifurcation was a voltage mode (Class 2) and was subcritical. This indicates that the loss of bounded trajectory feasibility occurs at the very first bifurcation (subcritical Hopf) long before the node focus, loss of causality, or saddle node bifurcations occur as the Q_1 bifurcation parameter was increased.

Another result was on a two-axis generator model with a Type 1 exciter with a compensator, but no sensor dynamics were carried out on a two generator and one load bus model [64]. The bifurcation parameter was real power at the load bus. This system experienced Hopf bifurcation followed by node focus bifurcation in a similar sequence as in Figure 5. The Hopf bifurcation was proved to be subcritical and thus unstable, but it was not possible to determine from [64] results if it was in the power or voltage mode. The loss of bounded trajectory

feasibility occurs in this case due to subcritical Hopf bifurcation.

A very detailed study of a very simple single machine model with flux decay dynamics, exciter dynamics, and algebraic excitation controller is given in [65]. Inertial dynamics are neglected. It is shown that saddle node, singularity induced, homoclinic, transcritical, and Hopf bifurcations occur. Different bifurcation sequences were shown to occur for P-V curves computed when exciter gain is set at different values. This paper shows a very large number of possible bifurcations on a very simple generator flux/exciter model. Whether all of these bifurcations would occur on a model that has generator inertial dynamics is questioned since using too simple a model may result in bifurcations that cannot occur in a more complex model.

There are several important questions raised by the research on Type 2 models:

- (a) are there only five possible Type 2 kinds/classes that will cause a loss of bounded trajectory feasibility for specific contingencies?
 - (i) saddle node bifurcation in generator flux decay when the generator exciter is disabled [33,52,54];
 - (ii) node focus bifurcation that results in loss of stability of both generator inertial and flux decay/exciter dynamics [18,33,52,54,57, 75,76] when exciters are not disabled, but maximum exciter limiters are used. Is the positive real eigenvalue associated with inertial dynamics always more positive than the real positive eigenvalue associated with generator flux/exciter dynamics [48,57]? Does the positive eigenvalue, due to generator flux/exciter dynamics always result in generator internal voltage approaching zero as in the case where the exciter is disabled?
 - (iii) subcritical Hopf bifurcation [64];
 - (iv) large orbit supercritical Hopf bifurcation;

- (v) nonbifurcation loss of causality that indicates that Type 2 is the wrong model, but suggest bifurcation in the network and load dynamics is occurring.

The results in [65] suggest several other bifurcations are possible, but have yet to be observed on more complex power system models.

- (b) are Class 1 and 3 Hopf bifurcations in generator dynamics always stable (supercritical); do their orbital frequencies always decrease and result in unstable node focus bifurcations followed by loss of causality (J_c singular) and saddle node bifurcation (J^{***} singular) ?
- (c) are Class 2 Hopf bifurcations in generator dynamics always unstable (subcritical)? If Class 2 Hopf bifurcations are ever stable, can they lead to unstable node focus bifurcation, and will node focus bifurcation then always occur before loss of causality and saddle node bifurcation as they did in [18,46] for a subcritical Class 2 Hopf bifurcation?
- (d) do subcritical Hopf bifurcations in generator dynamics always result due to improperly designed excitation compensators; do saddle node bifurcations always cause a voltage collapse if the generator field current limit controller is one that disables the exciter; does node focus bifurcation always cause voltage collapse when field current limit controllers utilize a maximum exciter limiter to maintain excitation voltage control and only over-ride it to bring field current to within 5% of continuous rating at points in time when field current exceeds a field current magnitude/duration curve?
- (e) are there higher order bifurcations, strange attractors, chaos in a Type 2 model that is undetected because orbit tracking, Lyapunov exponents, etc. have not been applied to Type 2 models?
- (f) does the constant power load model adequately reflect the dynamic load response of a Type 4 model at saddle node bifurcation loss of stability which occurs infinitely slowly along the center manifold produced by a saddle node bifurcation?

- (g) does a constant power load model properly represent dynamic load response when an unstable node focus bifurcation occurs?
- (h) does a constant power load model properly represent the dynamic load response to Class 1, 2, and 3 Hopf bifurcations and regardless of whether these bifurcations are supercritical or subcritical.
- (i) is the nose of a P-V curve or minimum of a Q-V curve computed with an accurate equilibrium solver always an accurate conservative estimate of the bifurcation parameter value μ^* where node focus bifurcation occurs.

7. Type 3 Stability Problems

Type 3 stability problems have not received significant attention even though the stalling of motors due to low voltage was a recurrent and severe Type 3 stability problem on all early power systems. Stiffer and more redundant transmission and distribution systems and improved voltage control made these problems of less importance, although they still occur today [66]. A second reason for lack of study is the consequence of many Type 3 voltage stability problems is not as severe as Type 2 voltage stability problems that can black out a large region and even an entire country. Type 3 stability problems encompass the radial voltage instability problem as defined in [29,34] because by definition a radial voltage instability is not affected by generation dynamics because all generation is electrically distant and decoupled from the stability problem. The radial voltage stability problem that occurred in Florida [29] is an example of a potential Type 3 stability problem because the major generator near a load center were tripped off-line prior to the radial voltage instability. Voltage dropped to low levels after the local generator was tripped because loads were heavy and generation voltage control and reactive supply was not electrically close. Relays on very large motors tripped the motors, saving the motors from stalling, which is a saddle node bifurcation in this Type 3 model. Once the motor load was tripped, the voltage recovered and the motors were reconnected causing the voltage to drop again. This cycle of low voltage drop, motor tripping, voltage recovery, and reconnection of motors was repeated several times.

Earlier we discussed this same radial voltage stability problem as a Type 1 stability problem as it is most properly and most

often observed. We will discuss it as a Type 3 problem because the Type 3 model is generally the simplest model needed to study it. If relaying had not tripped loads, making it a Type 1 problem, then a Type 3 stability problem would have resulted.

We discussed this same radial stability problem as a nonbifurcation loss of stability problem for a Type 2 model. We noted that the Type 2 model is not the correct model to study stalling and tripping of motors and that a Type 2 model can't be used to find the kind and class of bifurcations that occur as radial voltage instability develops. Note that when a radial voltage instability leads to relaying and extremesis it is a Type 1 problem, but when it leads to bifurcation in network and load dynamics such as stalling induction motors it is a Type 3 problem.

Motor stalling or tripping is possible and likely for every Type 2 stability problem and if such effects severely affect generator dynamics, a Type 4 stability model should be used that would adequately portray how the load dynamics and motor stalling or tripping contribute to the overall stability problem.

It is important to study Type 3 stability problems in their own right as well as a precursor of studying Type 4 stability problems so as to distinguish those stability problems that are Type 2, those that are Type 3, and those that are neither and are Type 4 stability problems.

Although Type 3 stability problems have not received as thorough study as Type 2 and Type 4 stability problems, it is clear that there are at least two kinds of Type 3 problems:

- saddle node bifurcation
- Hopf bifurcation

At least two classes of Type 3 saddle node bifurcation have been studied. One is the stalling of induction motors for low voltage and heavy torque loads [31,66]. Analysis on how induction motors affect saddle node bifurcation on a two bus algebraic model is contained in [31]. In saddle node bifurcation in induction motor dynamics, slip keeps increasing and causes: (a) reduction of real power delivered to the motors; and (b) increase of reactive power delivered to the motor. Tripping and then stalling of very large motors can cause voltage collapse of a large remote region [67] of the B.C. Hydro system.

A second saddle node bifurcation occurs when tap changers run away attempting to control voltage [16,20]. The reactive losses within the tap changing transformer increase if the capacitive shunt secondary of a TT transformer model serves load and does not compensate the shunt inductor on the transformer primary. If too much of the capacitive shunt reactive is needed to meet reactive load, and the load is very large, the reactive losses can be enough to produce voltage instability.

There may be additional saddle node bifurcations in induction motor flux decay dynamics, thermostatic load dynamics, and combination of induction motor, under load tap changer, and thermostatic control load dynamics. This is a subject for further research.

There is only one known class of Hopf bifurcation which involved under load tap changing and switchable shunt capacitor controls [16,28,53]. Under load tap changers often compete for control of voltage so that increase in control of one bus by one device produces a reduction in voltage control by another device. This can produce orbits that look like Hopf bifurcation and would be if differential equations were used to model under load tap changer and switchable shunt capacitor control of load. There are likely to be additional Hopf bifurcation in or between induction motor, under load tap changer and thermostatic load controls, but at this point the author is unaware of literature on the subject.

There is no literature on bifurcations on a Type 3 model other than saddle node and Hopf to the knowledge of the author, although other bifurcations are probable.

8. Type 4 Stability Problems

Type 4 models should include very accurate generator exciter, power system stabilizer, and governor/boiler/turbine models. Accurate dynamic load models for effects of thermostatically controlled loads, under load tap changer, and switchable shunt capacitor controlled loads, and dynamic induction motor load should be included. If all load is modeled by differential equations, the algebraic equations are linear [42] and can always be easily eliminated to produce a differential model.

The anticipated results on accurate complex Type 4 models are that the static bifurcations observed on Type 2 and Type 3 models would also exist on Type 4 models because in many cases the generator dynamic model and load dynamic model are

weakly coupled. Thus, one would expect several classes of saddle node bifurcation in:

- (1) generator inertial, flux decay, and exciter dynamics;
- (2) generator flux decay and exciter dynamics;
- (3) tap changer dynamics;
- (4) thermostatic load dynamics;
- (5) induction motor inertial dynamics
- (6) induction motor voltage dynamics;

and combinations.

The classes of Hopf bifurcations that could be anticipated would be identical to classes of saddle node bifurcations (1-6) listed above. Each of the results [22,33,52-54,70] with Type 2 models where Hopf bifurcations were encountered assumed constant power loads. A constant power load may not be a proper load model for studying Hopf bifurcation or any other closed orbital phenomena when generator dynamics and load control dynamics are strongly coupled because the load dynamic controls do not respond fast enough to make the loads appear as constant power. The load dynamics and generator dynamics may generally be so weakly coupled that the load dynamics do not respond to Hopf bifurcation or other orbital phenomenon on generator dynamics with periods of seconds and not minutes or hours. In this case the steady-state model of load dynamics could be used, which is approximated by a constant power load model. An exception where Hopf bifurcation on generator dynamics may couple to load dynamics is when subcritical Hopf bifurcation occurs, when large orbit low frequency supercritical Hopf bifurcations occur, and when unstable node focus bifurcations occur. An indication that the results could be very different when constant power and constant impedance load models of the form (23) is contained in [71] where Hopf bifurcations occurred when a constant power load model was used, but no Hopf bifurcations occurred if a constant impedance load model was used. However, the results in [57] for an actual dynamic load model [58] indicate the results for an algebraic static constant power load model and that for the dynamic load were very similar. In that paper, Hopf bifurcation was followed in sequence by unstable node focus bifurcation. This result [57] is for a three generator

model and for a ten generator model and is possibly the best most accurate model used to date in studying power system bifurcations. Thus, one may infer that use of Type 4 models may not change results of Type 2 or Type 3 models substantially. The truth is yet to be determined. There has however, been a very large number of recent publications on voltage instability for a very simple Type 4 model. The results [7,47,69,72] indicate period doubling bifurcation, blue sky bifurcation, chaotic blue sky bifurcation, and chaos occur, and do not show results consistent with those on more complex Type 4 models. The conclusion might be drawn that voltage collapse can and will occur for these kinds of complex bifurcations in actual power systems even though the results were obtained for a simple Type 4 model [7]. This model contains one generator, an infinite bus, and a load bus containing an induction motor that lies between the generator and infinite bus.

There are several unrealistic features of the model [7] used to obtain the results in [7,47,69,72]:

- (1) The generator model was classical and thus no generator flux or exciter dynamics were included. From Type 2 model results discussed earlier [57], one can expect a stable node focus followed by a saddle node bifurcation if the generator model is classical and load and generator dynamics do not couple. Hopf bifurcations occur in generator inertial dynamics of a Type 2 model due to the negative damping effects of generator flux decay dynamics (Class 3) or generator flux/exciter dynamics (Class 1). The simple dynamic load model being a derivative of load voltage rather than generator internal voltage serves a similar function in producing negative or positive damping on inertial dynamics in [7,47,69,72]. As noted above, the kinds and classes of bifurcations in the Type 4 model is quite rich and thus there must be significant coupling of generator and induction motor dynamics to produce these kinds and classes of bifurcation;
- (2) The induction motor is electrically close to the generator in this model [7]. This is unrealistic because all dynamic loads are electrically distant from generator dynamics. There are at least two transformers and generally up to four or five transformers on all electrical paths connecting generators and induction motor loads. If motors are

electrically close to the generator, inductance is placed in the electrical path to provide isolation of motor and generator dynamics as in generating stations;

- (3) A constant power load model is used as the bifurcation parameter to produce bifurcation and from results on Type 2 models nonconstant power loads may not produce these bifurcations. If load is electrically close to generator dynamics, all load should be modeled by dynamic load models, no constant power load can exist, and thus constant power load cannot be increased to produce blue sky, period doubling, or blue sky chaotic bifurcation. Such bifurcation may no longer occur in these Type 4 models if another bifurcation parameter other than constant power load is used;
- (4) A very simple induction motor model of just induction motor electrical dynamics is used and thus all of the kinds and classes of Type 3 bifurcations could not occur in these studies [7,47,69,72] due to elimination of the inertial dynamics of the induction motor model.
- (5) The coupling of inertial and electrical dynamics of generators produce Type 2 bifurcations, and the coupling of inertial dynamics of induction motor, electrical dynamics of induction motors, thermostatic load control dynamics, and tap changer and switchable shunt capacitor control dynamics could produce Type 3 bifurcations when generator and load dynamics are poorly coupled. Such Type 2 and Type 3 bifurcations are likely to be the most probable bifurcations in the more complex Type 4 power system model if decoupling of generator and load dynamics is retained. The coupling of induction motor voltage and generator inertial dynamics that produce Type 4 bifurcations in [7,47,49,72] is the only coupling that has yet produced the bifurcations observed. If the generator dynamics were more complex, the induction motor dynamics were more complex, or both, this coupling of generator inertial and induction motor electrical dynamics would not be the only dynamics available to produce bifurcation and the strong natural coupling within dynamics of generators and within dynamics of induction motors would likely produce the bifurcations. This would most likely be true if generator and load dynamics were weakly coupled;

- (6) If the coupling of generator and induction motor dynamics is large as voltage instability is approached, the most complex generator and load models should be used to investigate the kinds and classes of Type 4 bifurcations that can occur. Certainly, all work on Type 4 kinds and classes of bifurcations using the simple power system model [7] must be viewed as projections of possible stability problems that might occur if complex generator and dynamic load models are used, and the coupling between generation and load is that of an actual power system.

A specialized set of programs have been used in studying Type 4 bifurcations that have the capability to follow equilibria or periodic orbits. The capability to follow periodic orbits allows one to establish existence of closed orbits and their bifurcations. The capability to follow periodic orbits has not been applied to Type 2 or 3 models and their stability problems. Thus, the orbital bifurcations observed for the simple Type 4 model may also exist for Type 2, Type 3, or complex Type 4 models, but have not been observed because orbits have not been tracked. Thus, one has a very distinct lack of knowledge on causes of loss of bounded trajectory feasibility on Type 2 and Type 3 models that needs to be addressed in future work.

8.1. Results on a Simple Type 4 Model

Two classes of saddle node bifurcation could possibly occur for the simple Type 4 model [7] as constant reactive power Q_L is increased. For the classical generator model used in [57], a stable node focus bifurcation was followed by a saddle node bifurcation when no induction motor dynamics were present. This would be the anticipated sequence of bifurcations if the series reactance between the generator and load was large enough so that little coupling occurred between generator and induction motor dynamics bifurcation in generator inertial dynamics. For this simple Type 4 model in [7], the center manifold is in the induction motor voltage dynamics rather than in generator inertial dynamics when the series reactance was small. It is not clear whether this saddle node bifurcation in induction motor dynamics would occur in the more complex model with an induction motor model with both inertial and generator dynamics, and a generator model with both inertial and flux decay/exciter dynamics. It is also not clear from [7] whether the saddle node bifurcation observed affects state

variables other than load voltage based on the eigenvectors at the bifurcation point.

The paper by Ajarapu and Lee [69] also uses the very simple Type 4 model developed in [7]. The paper tracks equilibria and periodic orbits and produces a bifurcation diagram shown in Figure 6. The bifurcation parameter is constant reactive power load at the load bus Q_1 . A very similar bifurcation diagram from [72] is shown in Figure 8. The results via tracking equilibria show that a subcritical Hopf bifurcation develops at S_1 far from the nose of the Q-V curve, and supercritical Hopf bifurcation occurs at S_2 close to the Q-V curve nose. The equilibria between S_1 and S_2 are unstable. A trace of the unstable eigenvalue between S_2 and S_3 is given in Figure 7. If Q_1 is increased above S_2 , the equilibria are stable again as shown in Figure 6, but an eigenvalue ultimately becomes zero at S_3 . The results in Figure 8 from [72] are identical in sequence, but have different magnitude. Note from Figure 7 that after the subcritical (unstable) Hopf bifurcation, no unstable node focus bifurcation occurs as in all studies on Type 2 models where stable (supercritical) Hopf bifurcations are followed by unstable node focus, singularity induced, and finally saddle node bifurcation.

The tracking of orbits, also shown in Figure 6 and Figure 8, starts at S_1 where subcritical (unstable) Hopf bifurcation occurs. For Q_1 decreasing from the Q_1 value at S_1 , unstable orbits of increasing frequency and voltage magnitude occur until P_t . At P_t , the orbit tracking changes direction so Q_1 is now increasing. At P_t the unstable orbits become stable orbits for a short interval. At P_{D_1} , periodic doubling bifurcation occurs with the orbit as shown in Figure 1b, and a second period doubling bifurcation occurs at P_{D_2} that ultimately result in chaos. From P_{D_2} to S_2 the orbits are stable with disappearance of the stable orbit at S_2 . The only difference in Figure 8 is that the stable orbits that start at P_t remain stable until S_1 rather than P_{D_1} .

The bounded trajectory feasibility region is difficult to establish solely from the branching diagram such as Figure 6. As Q_1 increases from 10.7 to P_t , a stable equilibrium exists. At P_t , one limit cycle exists. Between P_t and P_{D_1} , an inner unstable limit cycle and an outer stable limit cycle appear to exist. From P_{D_2} to S_1 both inner and outer limit cycles are unstable. The region of attraction between the stable equilibrium and the two unstable limit cycles might be quite small at P_{D_1} . The system may lose bounded trajectory

feasibility for $Q_1 = P_{D_1} \leq S_1$ where both inner and outer orbits are unstable. For $S_1 < Q_1 < P_{D_2}$, the system has both an unstable equilibrium and an unstable orbit and loss of bounded trajectory feasibility is maintained. At or near P_{D_2} , repeated periodic doubling bifurcations occur that lead to formation of a strange attractor and chaos. For $S_2 \leq Q \leq S_3$, stable equilibria with no closed orbits exist until at S_3 a saddle node bifurcation occurs.

The principal difference between Figure 6 from [69] and Figure 8 from [72] is that the stable outer and unstable inner orbit are retained from P_t and S_1 in [72] rather than to P_{D_1} in [69]. A cascaded period doubling bifurcation leading to chaos occurs at both points of period doubling bifurcation in [72], which is close to S_1 and P_{D_2} for the results of [69]. Chaos has a broad frequency spectrum and the chaos for both points have quite full spectra. The chaos shown in Figure 3b is from [72].

If this example system was predictive of the stability problems on actual power systems, the enormity of the task of assessing loss of bounded trajectory, feasibility, and the region of stability can be understood. There are no tools for assessing existence of stable and unstable Hopf bifurcation or tracking closed orbits in huge power system models. With each stable orbit and equilibrium one must assess its region of attraction, and for each unstable orbit one must assess its attractors for each value of bifurcation parameter and each bifurcation parameter pattern to assess both bounded trajectory feasibility and region of attraction.

The unstable Hopf at S_1 , stable orbits at P_t , period doubling at P_{D_1} and P_{D_2} , and chaos near P_{D_2} , and stable orbits above P_{D_2} are all in (δ, ω) power mode dynamics. It is not clear from results in [69] whether terminal voltage dynamics couple strongly to produce the complex dynamic behavior found in tracking orbits, but from results on bifurcation in Type 2, composed of the classical generator model, no Hopf bifurcation or complex orbits can occur in classical generator inertial dynamics without generator flux/exciter or induction motor dynamics being present and strongly coupled. In the Tamura paper [73], to be discussed later, the power mode (δ_1, ω_1) complex eigenvalue pair was shown to indeed be strongly coupled to voltage dynamics in both the q and d axis.

The Abed paper [47] studied the same simple system as that studied in Chiang [7] and Ajarapu and Lee [69], and the bifurcation parameter was again constant power reactive load

Q_1 as in [7,69]. In this case the shunt capacitive at the load bus is adjusted so that more realistic values of Q_1 (3.5 rather than 11.3) will produce bifurcation. The tracking of equilibria produces the Q-V curve shown in Figure 9. The voltage before the nose of the Q-V curve is realistic (< 1.10 rather than 1.3), but the nose of the Q-V curve occurs at .8 p.u. volts, which could cause motor tripping before this bifurcation can occur. In [7], the voltage before the Q-V curve nose was 1.3 p.u. which is unrealistic, but the nose occurs at 1.0 p.u. voltage which is more realistic. The orbital branching diagram in Abed [47] is totally different than in Ajjarapu [69] or Chiang [72] where the orbital tracking connects two nodes on the upper portion of the Q-V curve as shown in Figure 6, but the orbital path in Figure 10 connects a node on the upper portion of the Q-V curve with a saddle equilibrium point on the lower leg of the Q-V curve. The catastrophic bifurcation in Abed [47] is a chaotic blue sky bifurcation that occurs at the collision of the strange attractor or periodic orbit, produced through repeated periodic doubling bifurcations on the paths between the upper and lower Q-V curve, and the saddle on the lower Q-V curve. There are numerous orbital paths between the point on the upper portion of the Q-V curve and various saddle points on the lower portion of the Q-V curve. Along each such path, chaotic blue sky bifurcation occurs at the collision of the strange attractor with the saddle that exists at the point on the lower portion of the Q-V curve.

The bounded trajectory feasibility region is also very different than for Ajjarapu and Lee [69] where the shunt capacitance was larger. A supercritical Hopf bifurcation initiates both the orbital path in [69] and in [47]. The supercritical Hopf bifurcation (stable) orbits that occur at HB_1 up to P_{DB4} would produce bounded orbital trajectories and a flow from within a region of attraction to this stable orbit as an attractor. Beyond P_{DB4} the orbit quickly produces a strange attractor trajectory (as shown in a phase plot in Figure 11) due to repeated periodic doubling bifurcation. Since the strange attractor produces a large, stable complex orbit, a loss of bounded trajectory feasibility occurs before the chaotic blue sky bifurcation occurs at the saddle point on the lower portion of the Q-V curve. A phase plot of loss of stability of the chaotic strange attractor is shown in Figure 12. The chaos that results from this bifurcation is transient and slowly dies out at the disappearance of the strange attractor as shown in Figure 3a.

A paper [46] is now discussed that utilizes an even simpler network model than in [7] because it does not contain an

infinite bus, but solely a generator and load bus. The generator model is a two axis model and the induction motor at the load bus is a third order model. These generator and induction motor models are far more complex than in [7] and result in very different bifurcations. For the case where constant power P_L is the bifurcation parameter and the induction motor is not present, an unstable voltage mode limit cycle occurs in the generator dynamics as shown in Figure 13. As real power load increases, the unstable limit cycle disappears and the equilibrium is unstable as shown in Figure 14. No tracking of the orbit or equilibrium is performed so it is impossible to state whether an unstable mode focus bifurcation occurs as in Type 2 models in [18,33,52,54,57,75,76]. The unstable node focus would occur if the frequency of the limit cycle were to decrease to zero as the bifurcation parameter increased and two real positive eigenvalues exist at the equilibrium point. Loss of bounded trajectory feasibility would occur when the unstable small orbit limit cycle shown in Figure 13 first appears.

The study is repeated with an induction motor added at the load bus. At some level of parameter P_L , an inner limit cycle and outer limit cycle are observed. The outer limit cycle is stable and the inner limit cycle is unstable as shown in Figure 15. The inner limit cycle is a voltage mode and the outer limit cycle occurs in generator E_q and R_f dynamics from phase plane plots, but may also be due to induction motor dynamics. No eigenvector is given for either the inner or outer mode. The inner limit cycle disappears again as P_L increases, but the stable outer limit cycle remains as shown in Figure 16. The outer limit cycle has a large orbit.

The loss of bounded trajectory feasibility for the case of partial induction motor load occurs due to the very large orbit stable limit cycle because of potential damage to equipment. If this large orbit stable limit cycle had a small orbit, the loss of bounded trajectory feasibility might be at the saddle node of the P-V curve but, is certainly after the unstable inner limit cycle disappears, and should be viewed as being independent of its disappearance. The loss of bounded trajectory feasibility certainly occurs before the point where the unstable limit cycle disappears for the case where the induction motor is not present because the region of attraction inside the small orbit unstable limit cycle is not large, as shown in Figure 12.

The Tamura paper [73] discusses a complex generation (7-state) and induction motor (3-state) dynamic models, and a simple two bus network model as was the case in [46]. There is a

generator voltage mode (Class 2 Hopf bifurcation), a power swing mode (Class 1 Hopf bifurcation) and a high frequency (25 hz) third mode. The voltage swing mode in Type 2 models involves states (R_f, E'_q) or (E'_{fd}, E'_q) when sensor time constant $T_R = 0$. In this complex Type 4 model, the voltage mode includes the q axis and d axis voltage in the induction motor model. The power mode in a Type 2 model involves generator states (δ, ω, E') . In this complex Type 4 model the power mode includes $(s, V_d, \delta, \omega, E'_q)$. In fact, the induction motor slip(s) and d axis voltage (V_d) truly dominate this Type 4 model power mode where they do not even exist, and thus can't contribute to the power mode in a Type 2 model. The third pair of complex eigenvalues are not only high frequency, but poorly damped. No discussion of the right eigenvector for this pair of complex eigenvalues is given in [73].

Tamura [73] investigates the voltage mode Hopf bifurcation followed by a node focus bifurcation. The author shows the Hopf bifurcation is stable (supercritical) with increasing amplitude and decreasing frequency Hopf as the constant power load bifurcation parameter increases. It appears that this stable limit cycle disappears as a node focus bifurcation as in [18,33,52,54,57,75,76] for Type 2 models.

This paper by Tamura [73] does not have the inner unstable limit cycle as observed in [46] for a very similar model. This inner unstable limit cycle disappears with the increase in bifurcation parameter P_L in [46], and possibly this explains why it is not present in [73]. It is also interesting to question whether this stable limit cycle in [46] is the one observed in [73] and disappears due to node focus bifurcation as in [73]. Loss of bounded trajectory feasibility would occur when the small stable limit cycle orbit experiences unstable node focus just as in Type 2 models [18,19], except that this supercritical voltage node includes d and q axis voltage on the induction motor.

The most complete study of a Type 4 model has been performed in [57] for the three generator models of [22,33,52-54] and for a ten generator New England system model. The constant power load used in the cases presented as part of the Type 2 model results was replaced by a 1st order induction motor model in voltage dynamics. The results on the Type 2 and Type 4 models are identical in terms of sequence of bifurcations (Hopf followed by unstable node focus, singularity induced, and saddle node bifurcation). The results imply that

simple load modeling of voltage dynamics may make Type 2 and Type 4 model give very similar if not identical results.

A quite different Type 4 stability problem is discussed in [38]. It occurs due to slowly increasing load and produces a shrinking region of attraction.

9. Conclusions

The paper contains six principal themes:

- (1) that one cannot accurately assess proximity to voltage instability for line outage and generator outage contingencies by computing the post contingency equilibria and feasibility regions around each post contingency equilibrium because:
 - (a) extremesis can occur along the trajectory and could not be observed at the post contingency equilibrium;
 - (b) that the discontinuous control actions of field current and stator current limit controllers, under load tap changers, relays, and switchable shunt capacitors change the model, change the region of attraction, and produce transients between the quasi equilibria along the trajectory for any contingency. One must assess whether the previous quasi equilibrium is in the region of attraction of the next quasi equilibrium to assess asymptotic stability of the trajectory;
 - (c) stable and unstable Hopf bifurcations and closed orbits can occur which greatly impact the region of attraction and asymptotic stability and can't be determined from computing equilibria and their stability;
 - (d) existence of large stable closed orbits would cause station operators to trip generating units under their control and existence of a large stable orbit can't be computed by knowing an equilibrium point and its stability;

- (2) that one needs to assess whether the system is operating acceptably in terms of the contingencies and continuous or discontinuous operating changes and their combination. A bounded trajectory feasibility region is the subset of the set of all feasible contingencies and operating changes specified by a planning or operating criterion which:

- (a) produce trajectories that asymptotically converge to a stable equilibrium or small stable closed orbit;
- (b) do not cause extremesis to occur due to low voltage;
- (c) do not exhibit large orbit stable oscillations that cause tripping of generation units;
- (d) do not have unstable small closed orbits that restrict region of attraction.

The bounded trajectory feasibility region is not a mathematical measure of stability, but a power engineering measure of feasibility and one that could be used to evaluate security measures;

- (3) propose a procedure for classifying power system stability problems in terms of model needed, kind of bifurcation observed, and class of generator and load dynamics involved in that stability problem;
- (4) analyze each kind of bifurcation in terms of whether it would cause loss of bounded trajectory feasibility for specific contingencies or operating changes;
- (5) propose Type 2, Type 3, and Type 4 models and discuss the kinds of bifurcations observed on each model type as well as the classes of each bifurcation kind observed on each model type;
- (6) determine all of the possible kind/class bifurcation sequences that have been documented to occur on a particular model type and determine the specific kind/class of bifurcation that have been documented to cause loss of bounded trajectory feasibility for each kind/class bifurcation sequence based on the literature reviewed in this paper.

The paper points out that loss of causality should never occur in a power model if modeling is performed properly. Loss of causality can be associated with saddle node bifurcation in the network and load dynamics in a Type 4 model. It can cause singularity induced bifurcation in generator dynamics, but this singularity induced bifurcation was always observed to occur (in all results reviewed on Type 4 or Type 2 models) after unstable node focus, saddle node, or unstable Hopf. It has also been noted that small orbit unstable Hopf bifurcation, large orbit stable Hopf bifurcation, unstable node focus bifurcation, saddle node bifurcation, and extremesis, can cause loss of bounded trajectory feasibility. It is also pointed out that stable and unstable, small or large orbits can suddenly appear for a continuous change in bifurcation parameter. Bounded trajectory feasibility is retained if stable orbits are small enough and their region of attraction is large enough, even though the equilibrium point is unstable. Existence of unstable orbits may preserve bounded trajectory feasibility if the orbits are quite large because the encircled equilibrium point is stable and the boundary of the region of attraction is large. If unstable orbits are too small or stable ones too large compared to the region of attraction, a loss of bounded trajectory feasibility would occur.

The paper also asks several questions that currently appear to be unanswered that follow:

- (a) are there only seven possible kinds/classes that will cause a loss of bounded trajectory feasibility for specific contingencies?
 - (i) saddle node bifurcation in generator flux decay when the generator exciter is disabled [33,52,54];
 - (ii) node focus bifurcation that results in loss of stability of both generator inertial and flux decay/exciter dynamics [18,33,52,54,57, 75,76] when generator exciters are not disabled, but maximum exciter limiters are used. Is the positive real eigenvalue associated with inertial dynamics always more positive than the real positive eigenvalue associated with generator flux/exciter dynamics [48,57]? Does the positive eigenvalue due to generator flux/exciter dynamics, always result in

- generator internal voltage approaching zero as in the case where the exciter is disabled?
- (iii) subcritical Hopf bifurcation [64];
 - (iv) large orbit supercritical Hopf bifurcation;
 - (v) other more complex bifurcations in generator dynamics as suggested in [65] based on a generator flux decay/exciter model;
 - (vi) equilibrium and orbital bifurcations in load dynamics such as saddle node in induction motor and under load tap changer controlled load and Hopf bifurcation in under load tap changer controlled load;
 - (vii) saddle node, subcritical and supercritical Hopf bifurcations, in load and generator dynamics;
- (b) are there higher order bifurcations, strange attractors, and chaos that can occur on Type 2, and Type 3, and complex Type 4 models that have gone undetected because orbit tracking, Lyapunov exponents; etc., have not yet been applied to such models? Are the strange attractors, blue sky bifurcations, and chaotic blue sky bifurcations observed on simple Type 4 models valid on complex accurate Type 4 models of actual systems?
 - (c) does the natural decoupling of transformers between generators and load dynamics and the reactance inserted between generators and electrically close induction motors provide sufficient isolation between generator and load dynamics so that bifurcations involving both generator and load dynamics become improbable or in fact impossible?
 - (d) are Class 1 and 3 Hopf bifurcations in generator dynamics always stable (supercritical); do their orbital frequencies always decrease and result in unstable node focus bifurcations followed by loss of causality (J_c singular) and saddle node bifurcation (J'''' singular) ?
 - (e) are Class 2 Hopf bifurcations in generator dynamics always unstable (subcritical)? If Class 2 Hopf bifurcations are ever stable, can they lead to unstable node focus bifurcation, and will node focus bifurcation then always occur before loss of causality and saddle node bifurcation as they did in [18,46] for a subcritical Class 2 Hopf bifurcation?
 - (f) do subcritical Hopf bifurcations in generator dynamics always result due to improperly designed excitation compensators; do saddle node bifurcations always cause a voltage collapse if the generator field current limit controller is one that disables the exciter; does node focus bifurcation always cause voltage collapse when field current limit controllers utilize a maximum exciter limiter to maintain excitation voltage control and only over-ride it to bring field current to within 5% of continuous rating at points in time when field current exceeds a field current magnitude/duration curve?
 - (g) does the constant power load model adequately reflect the dynamic load response of a Type 4 model at saddle node bifurcation in generator dynamics, which occurs infinitely slowly along the center manifold produced by a saddle node bifurcation?
 - (h) does a constant power load model properly represent dynamic load response of a Type 4 model when an unstable node focus bifurcation in generator dynamics occurs?
 - (i) does a constant power load model properly represent the dynamic load response to Class 1, 2, and 3 Hopf bifurcations in generator dynamics, and regardless of whether these bifurcations are supercritical or subcritical.
 - (j) is the nose of a P-V curve or minimum of a Q-V curve computed with an accurate equilibrium solver is an accurate conservative estimate of the bifurcation parameter value μ^* where node focus bifurcation occurs.

A research program must not only answer the questions listed above, but also develop a security assessment methodology for finding the subsets of contingencies and operating condition

changes that belong to a bounded trajectory feasibility region. A proposed methodology for evaluating a bounded trajectory feasibility region is given:

- (a) develop a very fast EXSTAB [2] type simulation of quasi equilibria; and the equilibria for contingencies;
- (b) develop a fast method to determine whether small unstable orbits exist around any quasi equilibrium that would restrict the region of attraction. The fast method would be applied to EXSTAB [2] determined quasi equilibria and the post contingency equilibrium for each contingency;
- (c) develop a fast method of assessing when larger stable orbits exist around any equilibrium or quasi equilibrium that would be large enough for damage to occur or operators to trip units. The method would be applied to the fast EXSTAB [2] simulation of quasi equilibria and post contingency equilibrium for each contingency;
- (d) develop a method of assessing the region of attraction for any quasi equilibria or post contingency equilibrium;
- (e) develop two different fast assessments of asymptotic stability for contingencies:
 - (i) assess whether the precontingency equilibrium lies in the region of attraction of the post contingency equilibrium;
 - (ii) assess whether the EXSTAB [2] simulated sequence of quasi equilibria are associated with an asymptotically stable trajectory by testing whether each quasi equilibrium is in the region of attraction of the next quasi equilibrium;
- (f) determine the subset of all single line outage, single generator outage, and combination contingencies from the set C specified by the planning or operating criterion that meet the conditions specified as making a contingency part of the bounded trajectory feasibility region. This could be envisioned as requiring

application of (b), (c), and (e) for each contingency in C.

- (g) develop a method for assessing whether envisioned transfer, loading changes, unit commitment changes, generation dispatch changes, and voltage setpoint changes can cause any contingency in C to violate the bounded trajectory feasibility region constraints. The combinations of contingencies and feasible operating changes that do belong to (C) and do not belong to the bounded trajectory feasibility region is also noted as a means of assessing security for that power system.

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P_L	eigenvalue	participation factor
3.45	$-.08 \pm j4.26$	E_{q_1}, R_{f_1}
?	?	Hopf (Class 1)
3.51	$7.44 \pm j.45$	$E_{q_1}, \delta_2, \omega_2$
?	?	node focus
3.52	14.08 4.68	δ_2, ω_2 E_{q_1}, R_f
?	?	nose of P-V curve
3.49	910.98 2.87	δ_2, ω_2 E_{q_1}, R_{f_1}
?	?	SI
3.46	1.87	E_{q_1}, R_{f_1}
?	?	SN
2.76	.02	δ_2, R_3

Table 1. Bifurcation parameter, eigenvalues, and its participation factors showing sequence of bifurcations.

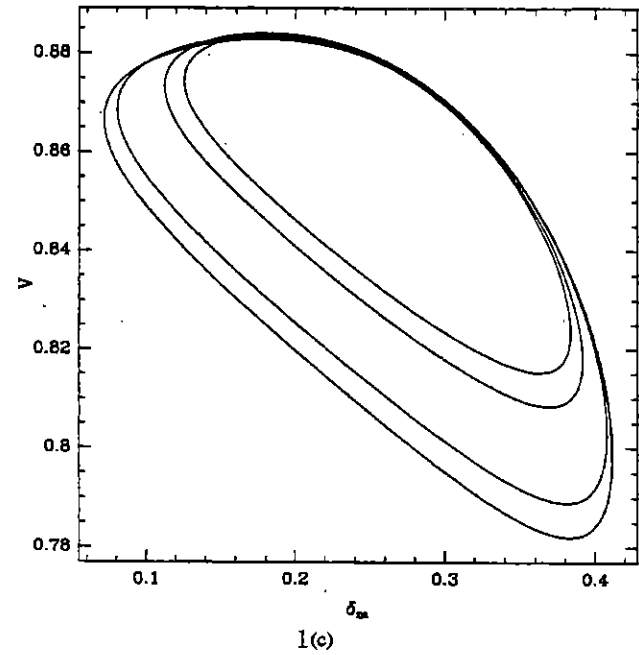
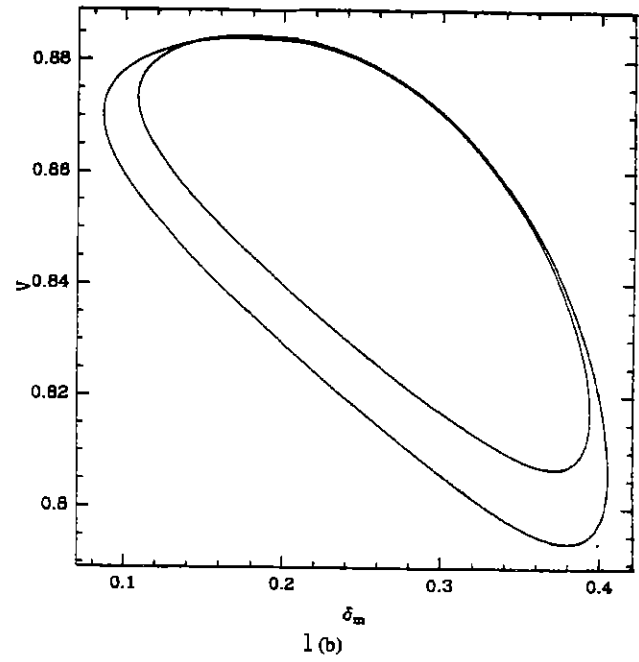
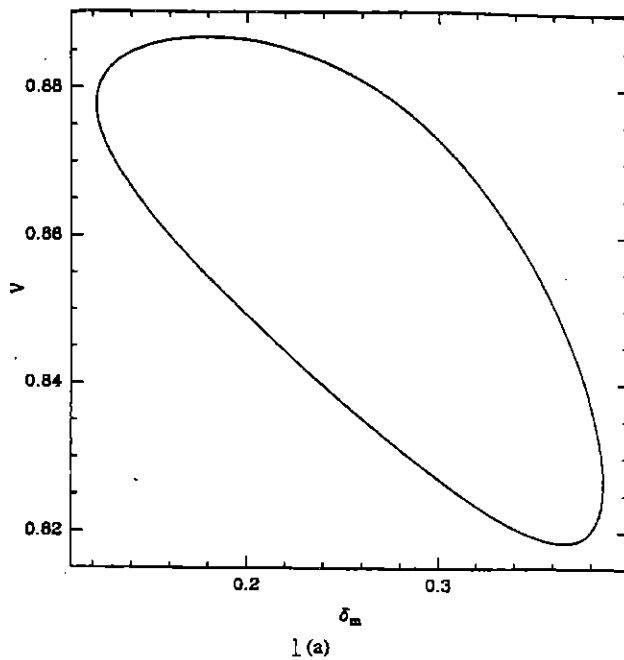


Figure 1. Example of Period Doubling Bifurcations [47].

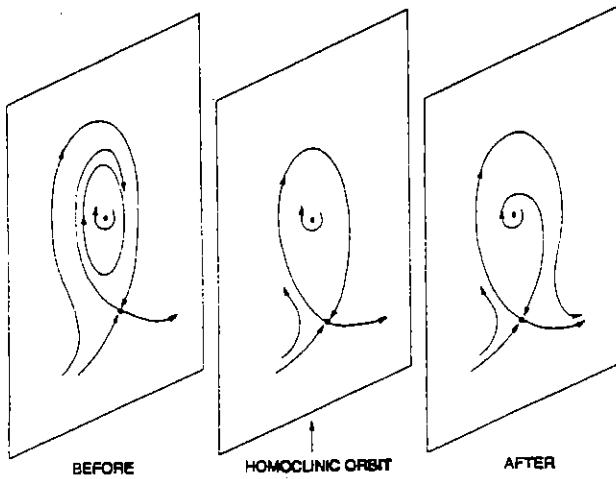


Figure 2. Example of blue sky bifurcation in which a collision of a stable homoclinic orbit and a saddle results in disappearance of the limit cycle [47].

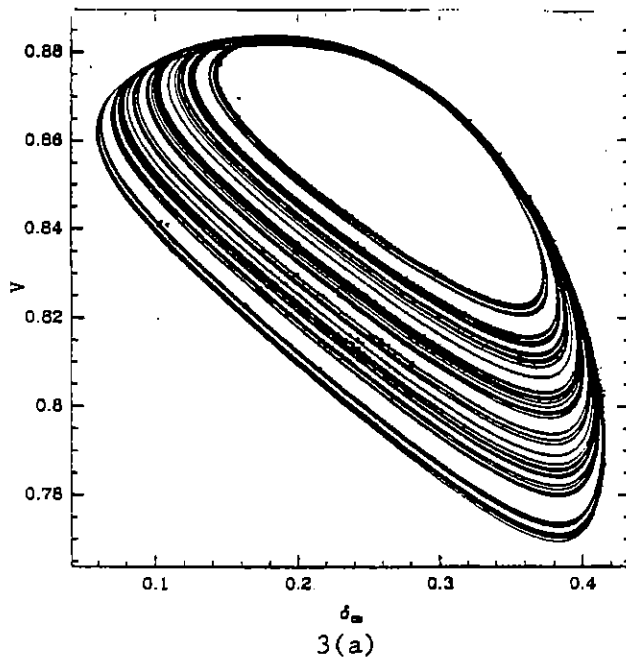


Figure 3a. Example of phase space plot of chaos in a power system [47].

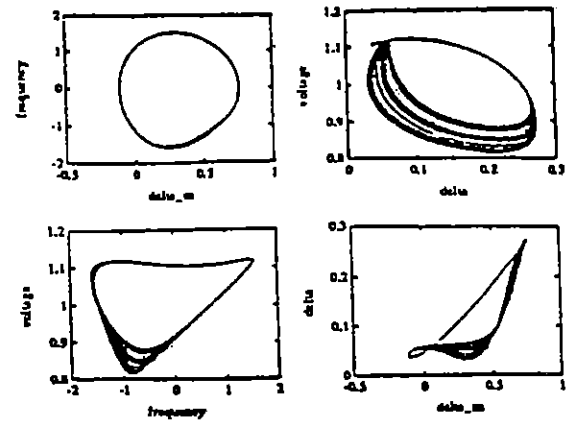
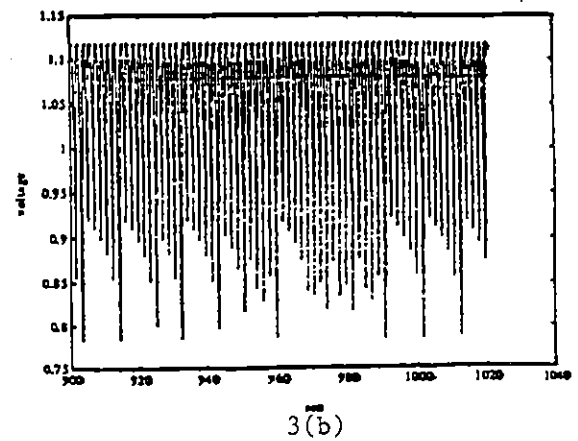


Figure 3b. Example of phase space and time recor plots of chaos in a power system [72].

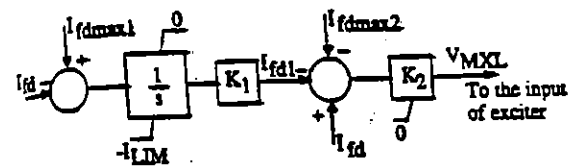


Fig. 2 : MXL Model

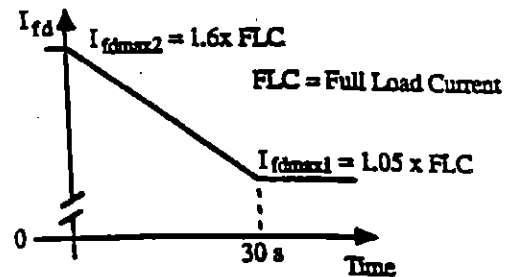


Figure 4. Example of an Maximum Excitation Limiter [55].

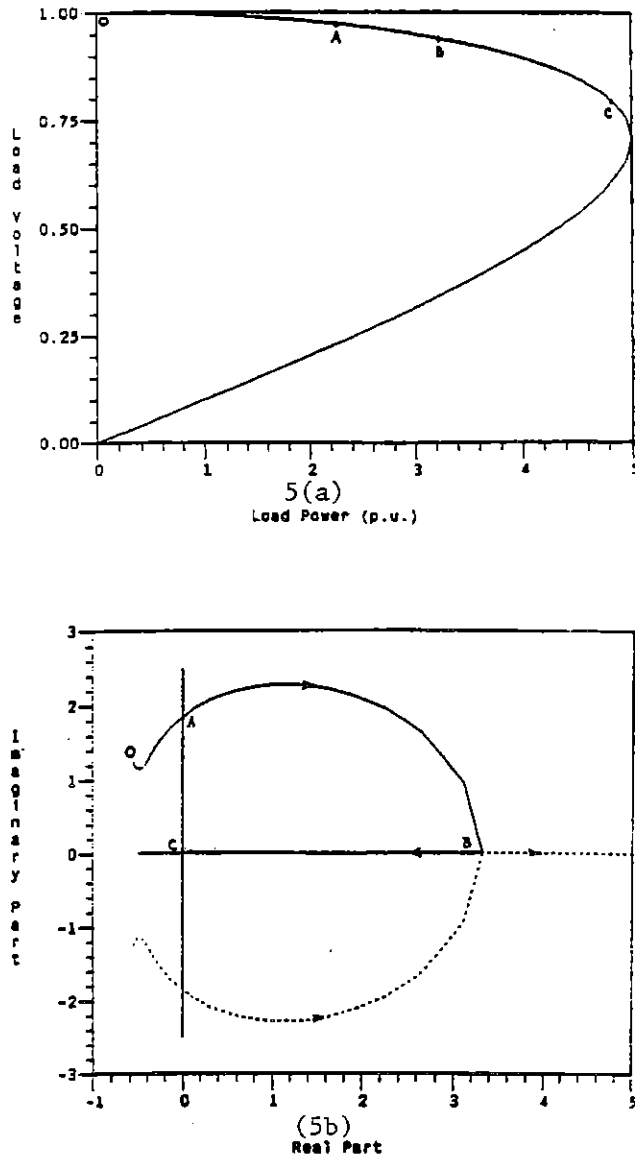


Figure 5. P-V curve (Figure 5a) and trace of eigenvalues (Figure 5b) showing Hopf bifurcation (A), node focus bifurcation (B), and saddle node bifurcation (C) [18].

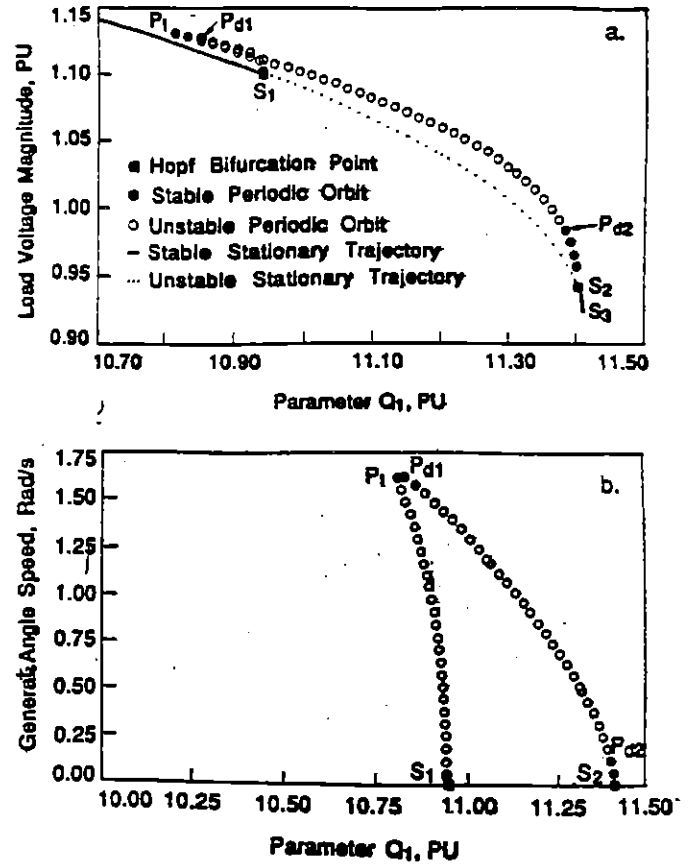


Figure 6. Bifurcation diagram of equilibria and periodic orbits [69].

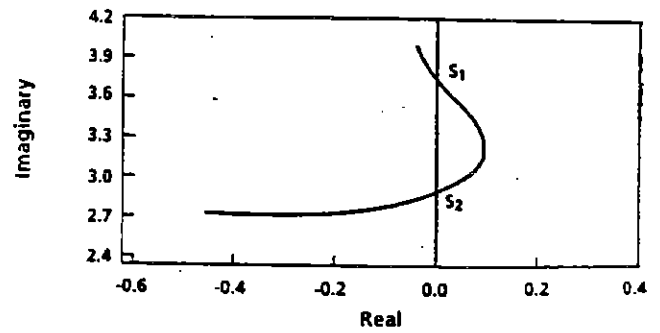


Figure 7. Movement of complex eigenvalue between S_1 and S_2 in [69].

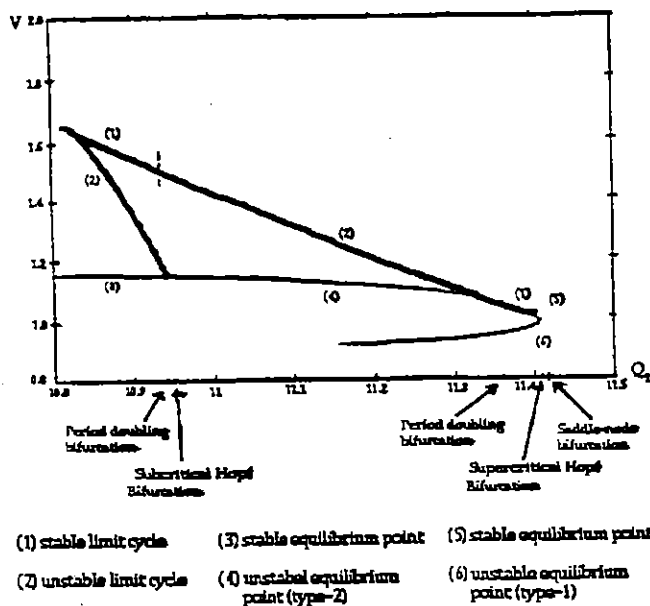


Figure 8. Bifurcation diagram for the same case as Figure 6, but with slight changes in operating conditions [72].

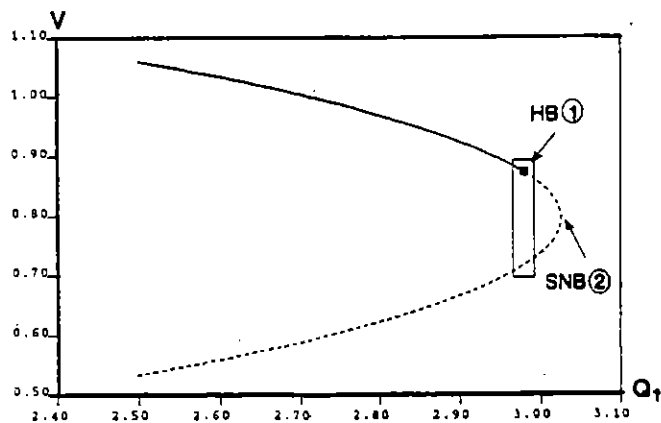


Figure 9. P-V equilibrium point curve for [47].

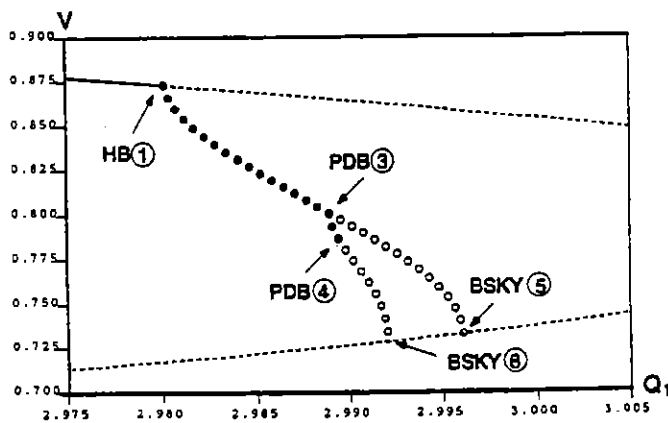


Figure 10. Bifurcation diagram for P-V curve for [47].

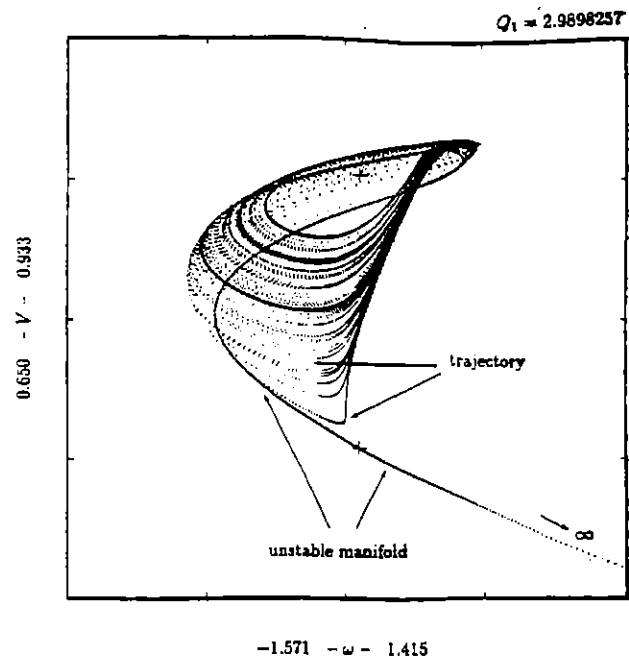


Figure 11. Strange attractor before chaotic blue sky bifurcation showing unstable manifold of saddle [47].

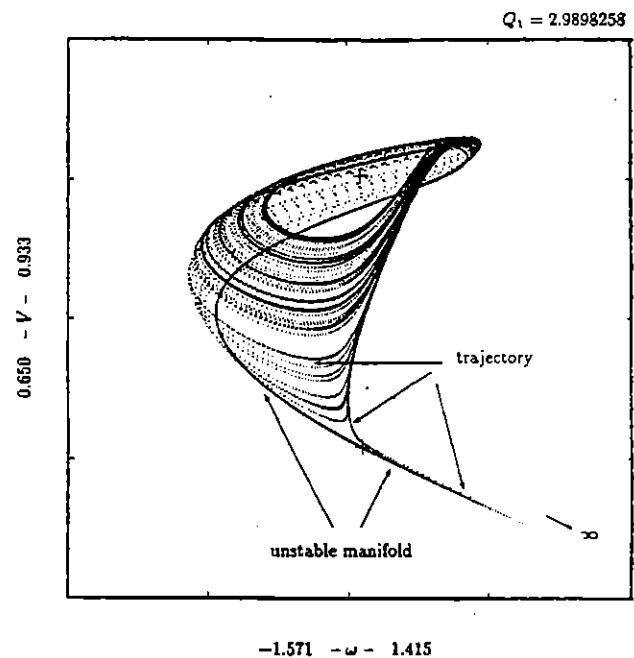


Figure 12. Chaotic blue sky bifurcation showing transient chaos and movement along unstable manifold of saddle [47].

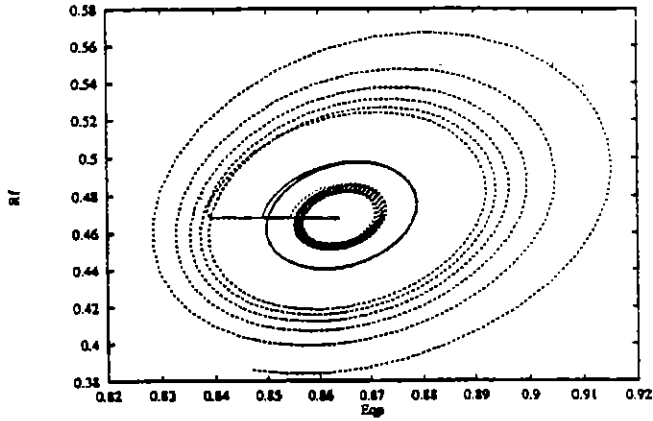


Figure 13. Unstable orbit for case with no induction motor load and $P_L = 2.35$ [46].

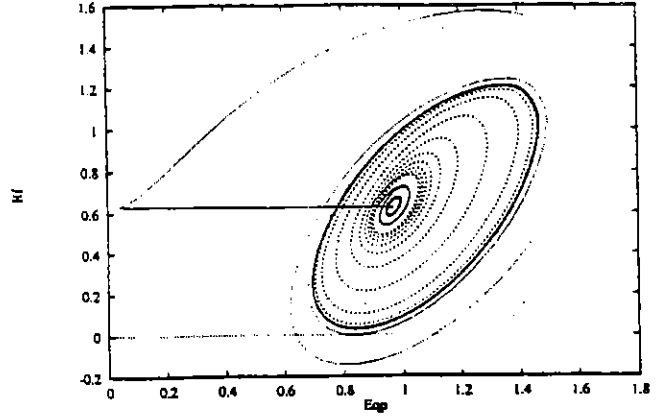


Figure 15. Stable and unstable orbit for case with induction motor load and $P_L = 3.15$ [46].

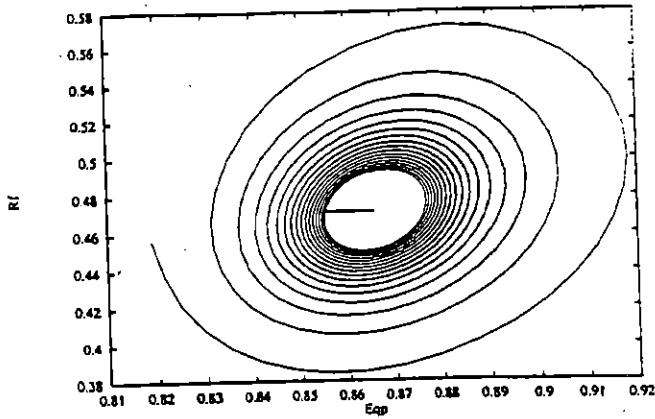


Figure 14. Unstable equilibrium that results after disappearance of unstable orbit for case with no induction load and $P_L = 2.37$ [46].

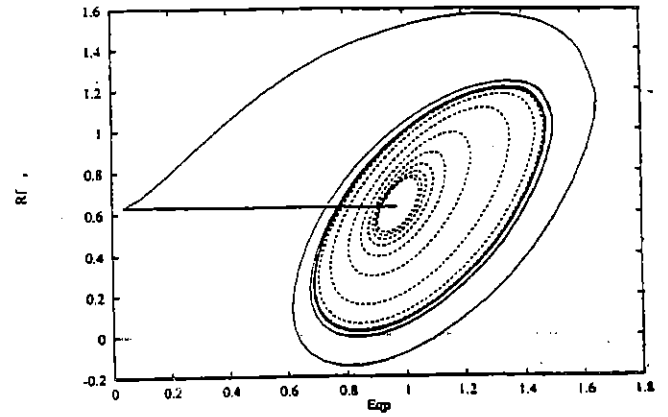


Figure 16. Stable orbit after disappearance of inner unstable orbit for induction motor load and $P_L = 3.17$ [46].

On the modelling of collapse in dynamical systems

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Abstract

In this paper the phenomenon of collapse is treated as a discrete transition occurring when the life time of a differential equation solution meets a finite upper bound. In the context of power systems the conditions of collapse are analysed and compared to the conditions of losing stability through a saddle node, Hopf, or singularity induced bifurcation. The relation between multiple time scales and the notions of stability and collapse is also investigated. Two case studies are included as illustrative examples of the concepts described in the paper. A further distinction between transient, and instability induced collapse is made with the help of these examples. Finally definitions for the concept of collapse are proposed in the conclusions.

1 Introduction

The recent (and the not so recent) literature on power system voltage stability and collapse is often based on power system models consisting of a set of algebraic and differential equations [1-9]. This paper is an attempt to summarize and unify (if possible in a compact and coherent form) some notions on the dynamic behaviour of systems described by differential equations subject to algebraic constraints. Part of this task is to propose a rather general definition of the concept of *collapse*, a term that is used often enough in conjunction with voltage phenomena, although the existing definitions of voltage collapse [10, 11, 12, 13] are not always clearly understood.

In Power Systems it is customary to classify voltage problems in different time scales [12]. In this paper, the modelling of collapse is at first treated generally, independently of the relative time scale of the model used. In a later section the time decomposition aspect in power system analysis is taken into account.

The paper is structured as follows: In Section 2, the concept of collapse is linked to the finite maximal interval of existence of an ordinary differential equation solution. Then, in Section 3, it is seen that in a system of algebraic and differential equations, the algebraic equation singularity points are points of collapse. Section 4 focuses on the loss of equilibrium point stability through saddle-node, Hopf, and singularity induced bifurcations. The time scale decomposition mentioned above is introduced in Section 5. In this section it is attempted to figure out the final outcome of a voltage collapse in a power system. In Section 6, two simple examples are presented to illustrate the concepts and ideas discussed in the paper. Finally, in the Conclusions, definitions of collapse, in terms of a mathematical model, as well as in the case of an actual physical system, are proposed as a modest contribution to the open discussion on this subject.

A general remark: The first part of the paper tries to be very general

and therefore the analysis remains somewhat abstract. This is done in order to obtain results that can be generalised to any type of system. On the other hand, the power system examples analysed in Section 6, may seem too simplified. These examples are included as an attempt to visualize the abstract ideas developed in the first part of the paper. Since our common visualisation abilities are limited to dimension three (being at their maximum at dimension two), the examples had to remain simple. In other works, some of the ideas presented here have been applied to medium size [14], or even large scale [9] power systems.

2 Maximal Interval of Existence

It is well known by now that a Power System may undergo voltage collapse in the form of a sudden catastrophic voltage drop, following usually a phase of rather slow voltage decrease. In practice, the sudden voltage drop can be limited by various factors (e.g. disconnection of loads, protection operation etc.) or it can lead to a complete system blackout. A sudden, instantaneous, self-initiated transition in the form of a collapse can be modelled even by a very simple nonlinear differential equation. Consider for instance the differential equation, taken (with a sign modification) from [15]:

$$\dot{x} = -x^2 \quad (1)$$

where the variable x could be in our case a bus voltage error:

$$x = V - V_{ref} \quad (2)$$

Although the quadratic control law (1) is a rather awkward attempt to regulate a bus voltage, it serves here as a means to introduce the concept of collapse in the context of ordinary differential equations. The dynamical system defined in (1) has the rare advantage of possessing an explicit time solution, given by:

$$x = \frac{1}{t + 1/x_0} \quad (3)$$

where x_0 is the initial condition for $t = 0$. Obviously when the initial error is positive, the error x will eventually reduce to zero (at infinite time). However, for negative initial error, the system will collapse before reaching the critical time t_{cr} , defined as:

$$t_{cr} = -1/x_0, \quad x_0 < 0 \quad (4)$$

The responses for positive and negative initial error, are plotted in Fig. 1. As seen in this Figure, the life time of a differential equation solution can be limited. Each solution of this kind collapses at a different time, depending on its initial condition. On the other hand, the solutions leading to the equilibrium point enjoy an infinite life time.

The remarks made above based on a simple example are generalized in the qualitative theory of ODE's. The implications of the well

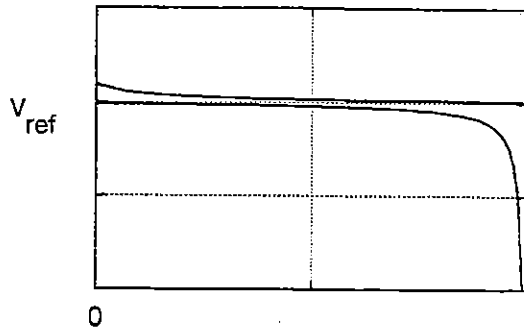


Figure 1: Time solutions of $\dot{x} = -x^2$

known 'existence and uniqueness' theorem [15] are briefly summarized below:

Consider a system of n differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad (5)$$

where $\mathbf{x}$ is a $n \times 1$ vector of state variables, and the n valued function $\mathbf{f}$ is defined in an open subset U of R^n . Then:

1. If $\mathbf{f}$ is continuous in U , there exists a solution $\mathbf{x}(t)$ for all initial conditions $\mathbf{x}_0$ in U . Each solution is defined on a maximal interval of existence $I_{\mathbf{x}_0}$, which depends upon the initial condition:

$$I_{\mathbf{x}_0} : \alpha_{\mathbf{x}_0} < t < \beta_{\mathbf{x}_0} \quad (6)$$

Of course, $\alpha_{\mathbf{x}_0}$, $\beta_{\mathbf{x}_0}$, or both can be infinite in some cases.

2. When $\mathbf{f}$ is differentiable with $k > 0$ derivatives, then the solution through $\mathbf{x}_0$ is unique and has also k derivatives.
3. When the maximal interval of existence is finite, the limit points of the solution $\mathbf{x}(t)$ for $t \rightarrow \beta^-$ or $t \rightarrow \alpha^+$ either belong to the boundary of U , when U is bounded, or go to infinity when U is unbounded.

In the example of Fig. 1, the maximal interval of existence is finite for negative initial error, and its upper bound is defined by (4). Intuitively, the collapse of a physical system can be described by a dynamical model consisting of differential equations, the solutions of which for specific initial conditions are defined in a maximal interval with finite upper bound. This finite upper bound defines the collapse time of the system for the particular initial conditions. In other words the life time of the system can be finite and it depends on the initial conditions. Inversely, the birth of a physical system can be modelled by a system of differential equations with a finite lower bound in its maximal interval of existence.

The phenomenon of collapse can be understood as a discrete transition occurring at time $t = t_c$, taking the physical system to another state, described by a different set of differential equations. This discrete event is brought about by the continuous dynamics of the system and is triggered internally, in contrast with other discrete events, such as line switching, machine limiting, short circuits etc., which are external events for the dynamics of the system.

Discrete events in a continuous time environment are characterised by instantaneous transitions, i.e. infinite slopes. The same is true for a collapsing system near its finite upper time bound. At this point it is evident that a finite increase of time results in an infinitely large

variation of the system variables. Such a transition is sometimes referred to as a catastrophe [16]. As discussed in Section 4, a linearized model close enough to a point of collapse will be characterized by an infinite eigenvalue corresponding to the instantaneous transition.

3 Algebraic Singularities and Collapse

In this section, one particular class of systems is analysed. These are the systems described by a set of n differential and k algebraic equations, which will be assumed smooth throughout this analysis:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{z}) \quad (7)$$

$$0 = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{z}) \quad (8)$$

where, $\mathbf{x}$ is again the vector of n state variables, $\mathbf{y}$ is the vector of k algebraic variables, and $\mathbf{z}$ are the p input and parameter variables. The algebraic equations (8) define a hypersurface of dimension $n+p$, which has been named *constraint manifold* [5], in the $n+k+p$ dimensional space of state, algebraic and input variables.

Systems in the form of (7), (8) have been characterized 'theoretically problematic' [1], since the system of k nonlinear algebraic equations (8) can have singular points where it cannot be solved for the k dependent algebraic variables $\mathbf{y}$. At these points the response of the system cannot be defined, which is contrary to engineering intuition that 'any physical system must have a time solution' [17]. A general analysis of systems in the form (7), (8) can be carried out with the help of the implicit function theorem. This theorem can be applied at all points $(\mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$ on the constraint manifold, for which the $k \times k$ algebraic equation Jacobian matrix $\mathbf{g}_y(\mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$ is nonsingular. The notation used for this Jacobian, as well as for all other similar matrices corresponds to the following definition:

$$\mathbf{g}_{y_i} = \frac{\partial \mathbf{g}_i}{\partial \mathbf{y}_j} \quad (9)$$

In the vicinity of such a point $(\mathbf{x}_1, \mathbf{y}_1, \mathbf{z}_1)$ there exists a unique, smooth function of the form:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{z}) \quad (10)$$

from which the algebraic variables have been eliminated.

Since $\mathbf{F}$ is smooth at all points in state space where $\mathbf{g}_y$ is nonsingular, there exists a unique time solution of the algebraic-differential system (7), (8) through all these points.

Let us consider now the points on the constraint manifold, for which the algebraic equation Jacobian is singular. These are given by the simultaneous solution of the k algebraic constraint equations (8) and the single equation:

$$\det(\mathbf{g}_y(\mathbf{x}, \mathbf{y}, \mathbf{z})) = 0 \quad (11)$$

The system formed by (8) and (11) has not necessarily a solution. When there are no points satisfying this system of equations, the algebraic constraints have no singularity points, and there is a unique solution of the differential - algebraic system (7), (8) through every point in state space. On the other hand, when algebraic equation singularity points exist, they form a $n+p-1$ dimensional surface, lying on the constraint manifold, since one more algebraic equation is added to the k initial ones. This surface of singularity points is called 'impasse surface' in [1, 5], because it cannot be crossed by the trajectories of the system. In the same references, the constraint manifold is decomposed into different 'voltage causality regions' separated by the 'impasse surface'.

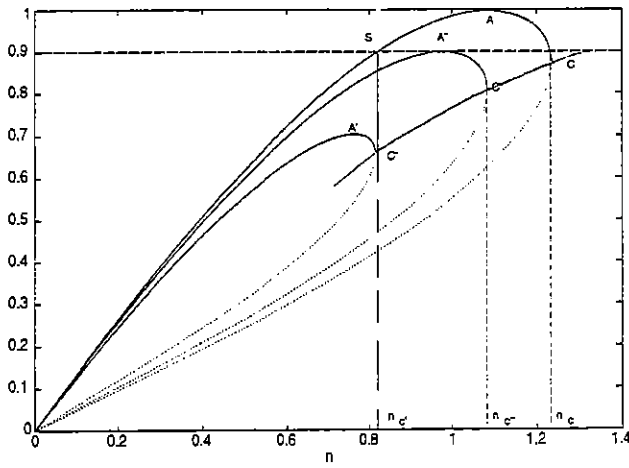


Figure 2: Constraint manifold for LTC system

A simple example of a constraint manifold is the one shown in Fig. 2 for a simple LTC system described in more detail in Section 6.1. The horizontal axis is the tap position n (state variable), the vertical axis is the secondary controlled voltage V_2 (algebraic variable), and the load demand (input variable) determines the size of the leaf shaped curves and can be visualized as the elevation dimension [18]. Each point of the two dimensional surface formed by the leaf curves satisfies the algebraic constraints (8). The impasse surface is the curve passing through points $CC'C''$. Obviously, this curve separates the constraint manifold in two regions: The upper part (solid lines) is the normal operating region.

For given inputs and parameters z , the projection of the impasse surface onto the state space, gives a $n-1$ surface S_z , consisting of algebraic equation singularity points. According to the above discussion this surface (if it exists) bounds the domain U_z on which a differential equation in the form (10) can be defined for the given value of z . The region U_z can shrink or expand depending on the variation of z . Geometrically, the singularity surface S_z can be viewed as the collection of points, on which the constraint manifold 'folds' with respect to the state space, having two projections before S_z and none after. This viewing of singularities is consistent with the 'fold' singularity, which is the simplest form of singularity in the theory developed by Whitney [16]. A simple example of fold singularity is shown in Fig. 3. More complex singularities such as 'cusps' are not considered in this paper, because they imply that the augmented Jacobian of equations (8) and (11) is also rank deficient, a condition not likely to encounter in a power system.

In the example of Fig. 2 the state space (on the horizontal axis) is bounded by the projection of the singularity point (C, C' , or C'' depending on the load demand level) on the tap axis n . The domain U_z in this case has dimension 1, and the singularity surface S_z is a single point. This system will be analysed in more detail in Section 6.1.

Since the constraint manifold can have multiple projections on the state space, let us consider an initial point (x_0, y_0, z_0) of the system, whose projection on the state space is $x_0 \in U_{z_0}$. Applying the existence theorem mentioned above, it can be seen that all solutions of (10) defined in a finite maximal time interval, either end on, or emerge from the boundary of U_z , which, in this case, is the singularity surface S_z . In other words, all collapsing trajectories end on algebraic equation singularities, if no other boundary of the domain

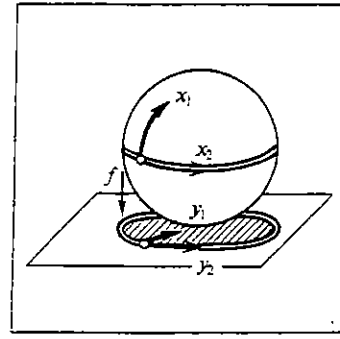


Figure 3: Fold singularity (projection of a sphere onto the plane)

of F exists. In this sense, S_z (if it exists) can be defined as the *collapse surface* of the system (7), (8), with initial conditions x_0, y_0, z_0 and input z .

What happens when a trajectory of the system reaches the collapse surface is not very clearly described in the existing literature. For instance, in [7] a stable equilibrium point is seen to pass through the singularity surface following a smooth parameter variation. The equilibrium point becomes unstable and the trajectories emanating from the transition either converge to a stable equilibrium point or return back to the singularity surface. In [1, 5] it is mentioned that 'unpredictable jumps' can occur between voltage causality regions, i.e. over the impasse or singularity surface. Also in [5] a trajectory encountering the singularity surface tangentially is allowed to cross this boundary entering another causality region.

In this paper the algebraic equation singularity is treated as a collapse, i.e. a discrete transition to another set of equations. As stated in [16]:

'singularities, bifurcations and catastrophes are different terms for describing the emergence of discrete structures from smooth, continuous ones'.

Some further properties of the algebraic singularity points (or simply for this paper *collapse points*) can be analysed by considering the linearized version of the system (8) around a point x_1, y_1, z_1 , of the impasse surface [19]:

$$0 = g_x \Delta x + g_y \Delta y + g_z \Delta z \quad (12)$$

When the algebraic equation Jacobian g_y is singular, this linear system has a nonzero solution Δy even when both Δx and Δz are equal to zero. More precisely, the solution Δy with the above conditions is the (right) eigenvector u_0 of g_y corresponding to its zero eigenvalue. This gives rise to the following incremental relations for all i and for the nonzero elements Δy_j of Δy :

$$\left. \frac{\Delta x_i}{\Delta y_j} \right|_{\Delta z=0} = 0 \quad (13)$$

$$\left. \frac{\Delta z_i}{\Delta y_j} \right|_{\Delta x=0} = 0 \quad (14)$$

These relations are necessary extremal conditions (sufficient also, when the second derivatives are nonzero, which is the general case) corresponding to local maxima or minima of all input and state variables with respect to (at least some) of the algebraic variables. This is another way to express the fact that at the algebraic singularity

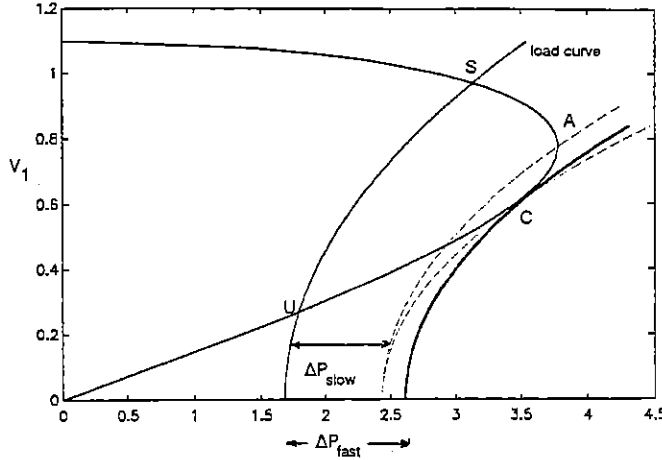


Figure 4: Maximum loadability on a P-V curve

points the constraint manifold 'folds', so that the state (or input) variable has to come back.

In the case of power systems the above singularity conditions have several practical implications. For instance, at a *maximum loadability* point the following sensitivity (incremental) relation holds:

$$\frac{\Delta P_{di}}{\Delta V_j} = 0 \quad (15)$$

where P_{di} is the load demand (and *not* the actual power consumed by the load). Since the load demand is an independent input variable, at a maximum loadability point the algebraic equation Jacobian is singular and the system collapses.

In order to avoid misunderstandings, the distinction between the maximum loadability point defined above and the maximum power transfer has to be made clear. The maximum power transfer refers to the actual amount of power transferred to the load, which is a dependent algebraic variable. A typical example of maximum loadability and its relation to maximum power transfer is shown in the P-V curve Fig. 4, taken again from example 6.1. For constant power load, the maximum loadability point is obviously point A, i.e. the tip of the P-V curve. However, for a composite load, having for instance the U-S characteristic of Fig. 4, loadability can increase up to point C, where the load characteristic becomes tangent to the network characteristic (P-V curve). Note that after passing through point A increased load demand results in reduced power transfer. This is clearly a critical situation, however the system can still operate on the points between A and C, whereas, as stated above, once point C (maximum loadability point) is reached the system collapses instantaneously.

4 Equilibria and Bifurcations

In this section, the properties of the equilibrium points of the differential - algebraic system are discussed. The equilibrium points are given by the simultaneous solution $\bar{x}, \bar{y}$ of equations (7), (8) with $\dot{\bar{x}} = 0$, for a given input and parameter vector $\bar{z}$. The equilibrium points $\bar{x}, \bar{y}$ form a p -dimensional surface in the $n + k + p$ space of state, algebraic and input variables. This surface lies on the constraint manifold since equation (8) is satisfied.

The linearized version of the system around an equilibrium point has the familiar state space form:

$$\Delta \dot{\bar{x}} = \mathbf{A} \Delta \bar{x} + \mathbf{B} \Delta \bar{z} \quad (16)$$

where the state matrix $\mathbf{A}$ can be defined using the Jacobian matrices of the system (7), (8):

$$\mathbf{A} = \mathbf{f}_x - \mathbf{f}_y \mathbf{g}_y^{-1} \mathbf{g}_x \quad (17)$$

The stability of the equilibrium point is determined by the real parts of the state matrix eigenvalues. When, following a smooth change of the inputs or parameters $\bar{z}$, a real eigenvalue of $\mathbf{A}$ becomes zero (without any higher order degeneracy), the system is said to undergo a 'saddle node bifurcation'. (A saddle node is a strange equilibrium point with the property seen in Fig. 1). At a SNB point the state matrix $\mathbf{A}$ becomes singular. The singularity of $\mathbf{A}$ implies also the singularity of the following $(n + k) \times (n + k)$ full Jacobian matrix of the system [2]:

$$\mathbf{J} = \begin{bmatrix} \mathbf{f}_x & \mathbf{f}_y \\ \mathbf{g}_x & \mathbf{g}_y \end{bmatrix} \quad (18)$$

Applying now to the full Jacobian $\mathbf{J}$ the same reasoning used in the case of the algebraic equation Jacobian, it is seen that at a saddle node bifurcation the following extremal conditions hold:

$$\left. \frac{\Delta \bar{z}_i}{\Delta \bar{x}_j} \right|_{\Delta \dot{\bar{x}}=0} = 0 \quad (19)$$

$$\left. \frac{\Delta \bar{z}_i}{\Delta \bar{y}_j} \right|_{\Delta \dot{\bar{x}}=0} = 0 \quad (20)$$

The above relation (19) is a necessary extremal condition for the input and parameter variables $\bar{z}$ for which equilibrium points exist. Considering again only first order singularities, this is also a sufficient condition giving for instance the maximum loading for which an equilibrium point can be reached. This is obviously different from the maximum loadability point discussed above. It can have the same form however, when the network P-V curve of Fig. 4 is replaced by an equilibrium curve. This will be further discussed in Section 6.2.

Note also the similarity and the difference between the conditions for algebraic singularity (14) and the conditions (20) for a saddle node bifurcation. Saddle node bifurcation occurs when an input variable or parameter reaches the extremal value (either maximum or minimum) of an equilibrium curve drawn in the state-parameter or the parameter-algebraic variable space. On the other hand, at the algebraic singularity the curve reaching an extremal point is the one drawn with the state and input variables 'frozen' at their current values.

Geometrically, the SNB points are the singularity points of the projection of the p -dimensional surface of equilibrium points on the input (parameter) space. Again this is a fold type singularity (Fig. 3). The projection of the SNB points onto the p -dimensional input and parameter space is a surface of dimension $p - 1$ which forms (at least partly) the boundary of the 'feasibility region' [4, 7] of the system, a region where equilibrium points exist. Going through a saddle node bifurcation the number of the projection points of the equilibrium surface onto the input space changes from two, to one (exactly at the SNB), and then to none.

The structure shown in Fig. 2 can be viewed as an equilibrium surface by considering the vertical axis as the reference voltage V_{ref} of the LTC. Obviously, at equilibrium the controlled secondary voltage has to be equal to this value. With this assumption the system consists of one state variable (the tap n) and two input variables (load demand and reference voltage). The equilibrium surface is therefore

two-dimensional and consists again of the leaf curves for different load levels. Clearly, for each load level there is a maximum V_{ref} for which equilibrium can be achieved, whereas for each reference voltage there is a maximum load after which no equilibrium points can be found. The saddle node bifurcation points are points A, A', A'' on which the equilibrium surface folds with respect to the parameter space.

In the context of Power Systems the parameter (input) space is usually the load or demand space. The projection of SNB points on this particular space is for instance the Σ surface defined in [8, 9] for the analysis of mid-term voltage stability and the assessment of corrective measures, such as load shedding. The infinite reactive generation sensitivity (algebraic variable) with respect to load demand increase (input variable) calculated at equilibrium, is a specific form of the saddle node bifurcation condition (20).

The saddle node bifurcation is not the only way that a system can lose stability. A stable operating point can become unstable also when a pair of complex eigenvalues crosses the imaginary axis of the complex plane. This condition is often referred to as 'Hopf bifurcation'. A case of oscillatory voltage instability is discussed later in the example of Section 6.2.

A third type of bifurcation occurs when the equilibrium surface intersects with the impasse (or collapse) surface. Technically the system cannot be defined on the crossing point of the impasse surface, however equilibrium points may exist arbitrarily close to both sides of a singularity point. Near this singular point, the determinant of G_y becomes very small and consequently the determinant of A very large. Therefore, one of the system eigenvalues tends to infinity as the system approaches the algebraic singularity surface. On the other side of the impasse surface, the equilibrium points will also have an infinite eigenvalue, but of opposite sign. This condition is a 'singularity induced bifurcation' [4, 7]. Therefore, the determinant of the state matrix A changes sign, either when it becomes singular, having a zero eigenvalue, or when it has an infinite eigenvalue going from minus infinity to plus infinity. (This corresponds to A^{-1} becoming singular).

From the point of view of this paper, the infinite state matrix eigenvalue of the system gives rise to an instantaneous (discrete) transition. As discussed earlier, this is equivalent to a collapse.

5 Multiple time scales

In this section some classical power system modelling assumptions are analysed in the general framework of the differential - algebraic equation representation (7), (8), in order to identify the inherently distinct time scales present in power system voltage stability analysis.

For reasons of simplicity, this discussion does not include the so-called long-term dynamics, although the same principles would hold for this time scale as well. The remaining phenomena can be classified in three time scales: mid-term dynamics, fast or transient dynamics, and instantaneous responses. Specifically:

1. *Mid-term* dynamics include the discrete dynamics of on-Load Tap Changers (LTC) at various voltage levels, the generator overexcitation limiters (OXL), as well as other similar protective devices. The OXL action effectively transforms the corresponding generator from a voltage controlled machine to a constant excitation one. It can be modelled therefore as a discrete tran-

sition in the system of equations without the introduction of additional state variables.

2. *Fast or transient* dynamics are associated mainly with generators, automatic voltage regulators (AVR), and other associated controls. Finally,
3. *Instantaneous* relations (typically the network constraints), which are usually modelled in voltage stability studies without any transients. Network transients can be handled with programs like EMTP (electromagnetic transients program).

In the above time scale enumeration, the load dynamics, which are the most important for voltage stability (though relatively less well known) have to be added. These span all the three time scales mentioned above:

1. *Mid-term* load behaviour due for instance to thermostatically controlled loads.
2. *Fast or transient* load dynamics associated mainly with induction (or synchronous) motors.
3. *Instantaneous* load response to voltage fluctuations. A typical load model implying instantaneous load response is the well known exponential representation.

All types of load behaviour can influence the voltage stability of the system. The instantaneous load response is the one that determines voltage collapse conditions. In fact it has been proved [5, 17] that algebraic singularities can be induced only by load models stiffer than constant current.

In the following two subsections the time decomposition between mid-term and fast dynamics on the one hand, and fast dynamics - instantaneous responses on the other are examined separately.

5.1 Mid-term vs. fast dynamics

A technique that can be used to simplify multiple time scale systems is the 'quasi steady state approach' [20], which is based on replacing the differential equations associated with fast dynamics with the corresponding equilibrium equations. An application of this approach to mid-term power system dynamics is described in [8, 9] as a method of successive transient equilibrium calculations. An alternative to the quasi steady state approximation is the iterative separation of time scales using singularity perturbation methods [20, 21, 22]. In this paper only the quasi steady state approach will be examined. Also, the mid-term load restoration due to thermostat controlled loads is not considered, so that the mid-term dynamics consist only of Load Tap Changers.

Let us consider such a mid-term power system model whose fast dynamics are modelled by equation (7). Using the quasi steady state approach mentioned above, the fast dynamics are replaced by their corresponding equilibrium equations giving:

$$0 = f(x, y, z) \quad (21)$$

The introduction of the mid-term state variables r , which are the variable LTC ratios does not change the fast state equations, but it affects the algebraic constraints which become:

$$0 = g(r, x, y, z) \quad (22)$$

The dynamic behaviour of the LTC's is determined solely by the secondary controlled voltages V_2 , which are part of the algebraic variables y . Following the notation of [9]:

$$V_2 = Uy \quad (23)$$

The stability of this mid-term dynamics system depends upon the eigenvalues of the following sensitivity matrix, which must be positive definite for stability [9]:

$$S = \frac{\partial V_2}{\partial r} = -[0 \quad U]J^{-1} \begin{bmatrix} 0 \\ g_r \end{bmatrix} \quad (24)$$

where J is the full Jacobian matrix defined in equation (18).

If following a contingency the mid-term dynamics become unstable, the system can arrive at a condition where all taps have reached their upper limits, or the trajectory of the system can hit the algebraic singularity surface, where the algebraic equation determinant J of the quasi steady state system becomes singular. Following the line of analysis presented in previous sections of this paper, this is a *collapse* of the quasi steady state model. It can be seen from equation (24) that in this case the sensitivity of the controlled voltages to the tap ratios becomes infinite. Since the LTC dynamics are already unstable before reaching the singularity of J , this would give rise to an infinitely fast instability (with a positive infinite eigenvalue), if the tap changers were continuously acting with a slope proportional to the voltage error. The discrete nature of LTC's saves an actual power system from this type of catastrophe.

Since, in this case the dynamics of the fast subsystem are known, an opportunity is given to explore the behaviour of a system *after* it has reached the collapse surface:

Since the taps are assumed discrete, the dynamics of the fast system are determined (between tap movements) by the differential equations (7) subject to the algebraic constraints (22) for constant taps. The full Jacobian of this system is still J , given by (18). Consider now an operating condition where, after successive taps, J becomes singular. As mentioned in the previous section, the singularity of J implies a *saddle node bifurcation* of the fast dynamics of the power system. Starting from this SNB, it is now the unstable fast dynamics that drive the system, before the LTC's have time to make their next move. Depending on the modelling assumption for the instantaneous load response, the unstable fast system trajectory will either fall to zero smoothly (as with constant impedance loads), or it will meet the instantaneous algebraic equation singularity, to collapse once more, this time in a different time scale.

Some remarks:

1. From the point of view of mid-term dynamics the instability of the fast dynamics subsystem is interpreted as a 'loss of equilibrium' [9] leading to collapse, regardless of the final outcome of the fast system trajectory.
2. The distinction between stability and collapse is relevant to the time scale of interest. What looks like a collapse in the framework of mid-term dynamics, can be characterised as an instability in the time frame of fast dynamics. It should be noted, however, that in the same time scale instability is normally met before the collapse.
3. In terms of system modelling, the collapse is linked to the singularity of the algebraic equations, which depend upon the initial time scale assumptions made.
4. Just before the saddle node bifurcation the 'fast' subsystem (7),(22), still stable, has a very small real negative eigenvalue.

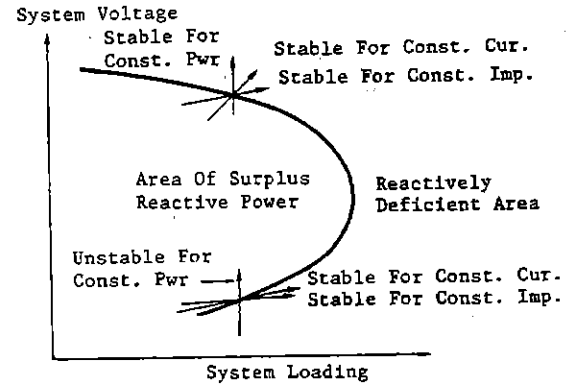


Figure 5: Implied stability on a P-V curve

Therefore, strictly speaking, the initial decoupling assumption of two time scales is no longer valid. However, when the system is close to the SNB, random load variations will inevitably trigger the instability of fast dynamics, i.e. the collapse of the mid-term dynamics.

5.2 Fast dynamics vs. instantaneous response

This time scale decomposition is rather robust, as far as the network equations are concerned. The same holds for the electromagnetic transients of generator stators, which are not likely to affect the voltage behaviour of the system. The major problem in this decomposition is the instantaneous load response assumption. Dynamic load modelling either in the form of explicit induction motor representation, or using generic load models [23, 24] would eliminate this problem by transferring the load behaviour entirely in the fast dynamics time scale. In the absence, however, of direct information concerning the actual load data, the instantaneous load behaviour should be also investigated.

An intuitive approach in dealing with fast vs. instantaneous responses, would be to follow the reasoning applied in the previous case: The collapse observed at the algebraic equation singularity could be explained as an instability of the faster (practically instantaneous) dynamics of the system.

One way to apply this approach is the reasoning given in [25] concerning the stability of an operating point on a classical P-V curve. This reasoning starts with the observation that outside of the P-V curve there is an area of reactive power deficit, whereas the area inside the P-V curve is characterized by a surplus of reactive power (see Fig. 5). Depending on the direction of the load characteristic for increased voltage, the operating point is considered stable or unstable. The underlying assumption is that the voltage will tend to increase in the area of reactive power surplus, and decrease in the area of reactive power deficit.

The suggestion of stable and unstable points on the network characteristic has the following implications: The network characteristic attracts the trajectories of the system up to the point where it becomes tangent to the instantaneous load characteristic (algebraic singularity, see Fig. 4). After this point the network characteristic loses attraction, since it consists of unstable points. Since network and load dynamics are assumed in this case as being instantaneous, the speed with which the trajectories stick to the stable part of the network characteristic is very large and no departure from the alge-

braic equations can be noticed. However, this attraction vanishes as the system arrives at the algebraic singularity, and the trajectories depart from the network characteristic driven by the implied 'instantaneous' dynamics instability. This is in fact a case of collapse, as described in previous sections of this paper.

A very similar result is obtained using a dynamic load model, for which the bus voltage derivative $\dot{V}$ is made proportional to the reactive power surplus [6, 10]. Referring again to Fig. 2, it is easily verified that the area inside each leaf curve is a voltage surplus region and the outside is a voltage deficit region. Using the above reasoning, the upper part of the constraint manifold shown in Fig. 2 with solid lines is attracting the trajectories of the system, whereas the lower part, drawn with dotted lines, is repelling and it cannot sustain system operation. After meeting the instability point A, a trajectory of this system will eventually reach the singularity point C, where it will depart from the constraint manifold along the dashed lines, causing voltage collapse.

Qualitatively the same results have been obtained by changing the instantaneous load modelling responsible for the algebraic equations singularity with a constant impedance load which restores to a stiffer load with a very small time constant. Using this approach it was shown in [3] that the algebraic singularity is transformed to a load instability driving the system voltage to zero, since the steady state power cannot be restored and the load admittance keeps increasing to infinity. In the same paper it is seen that the system trajectory passes through the gap left between the steady state network and load characteristics. This approach has the advantage that it allows the simulation of load behaviour at abnormal voltage levels explicitly, so that a mechanism (such as load rejection) that could stop or delay the collapse procedure can be modelled.

6 Applications and Examples

The case studies analysed in this section try to illustrate the abstract concepts introduced earlier in the paper. They also help to clarify some details of the mechanisms involved in the voltage collapse phenomena of power systems. Finally, they introduce some practical implications of rather academic distinctions such as that between stability and collapse.

6.1 A simple LTC: Transient and quasi steady state collapse

The system analysed in this first example is very simple, consisting of a constant voltage bus feeding a composite, unity power factor, constant power - constant admittance load through a Load Tap Changer (Fig. 6). The load demand is assumed to affect both constant power and constant admittance components simultaneously, so that the load depends upon a single input parameter z . The power actually consumed by the load is:

$$P = z(P_0 + GV_2^2) \quad (25)$$

The only state variable of this system is the variable LTC tap n .

The algebraic constraint manifold, for a given level of load demand, is a curve in the V_2 - n plane, already shown in Fig. 2. This type of curve has been named 'leaf' curve [26]. As stated before, a collection of leaf shaped curves can be visualized as a 2-dimensional surface in the 3-dimensional space, with the load demand z as the elevation dimension. A load increase will have the effect of shrinking the leaf

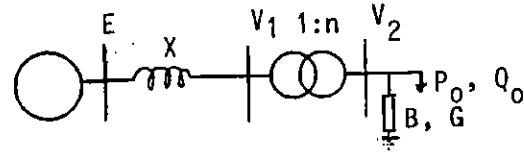


Figure 6: LTC with composite load

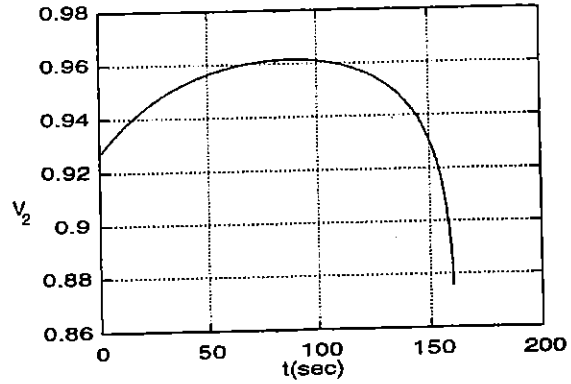


Figure 7: Collapse after reference voltage increase

shaped curve [6]. At the same time, the domain U_z in the state space is reduced.

The projection of each leaf curve on the tap n axis is the closed interval $[0 \ n_c]$. The dynamics of the system are not defined at the end points of this interval and therefore the domain U_z is the open interval $(0 \ n_c)$. Note that the dynamics of the system are different when the operating point is in the upper or the lower part of the leaf shaped curve.

It is easily seen from Fig. 2 that the stability limit, for which increased tap results in increased secondary voltage V_2 is point A, which was already identified as a saddle node bifurcation point. After this point the equilibrium points are unstable up to the point C, which is the algebraic equation singularity point. In accordance with equation (13) at this point the state variable has a maximum with respect to the algebraic variable V_2 .

An increase of the LTC reference voltage above the value corresponding to point A, initiates a voltage collapse sequence shown in Fig. 7. At the time of collapse the tap reaches the boundary value n_c [19].

A more realistic collapse mechanism is the load demand increase with constant secondary reference voltage. Consider that the system is initially at the operating point S with an initial tap n_c . It is interesting to see how the load margins change for a fast, or for a slow load increase:

1. **Fast load increase:** It is assumed that the demand increases so fast that the tap changer has no time to control the voltage. Therefore the tap remains constant throughout the load increase. In this case the system will collapse at the point C'. The secondary voltage V_2 is reduced along the curve SC'. When the critical load is reached, due to the infinite sensitivity of the voltage at the point of collapse a discrete transition will bring the voltage to zero (blackout), unless some loads are automatically disconnected, or the load behaviour changes at low voltage, in which case the transition may stop halfway. This is a case of

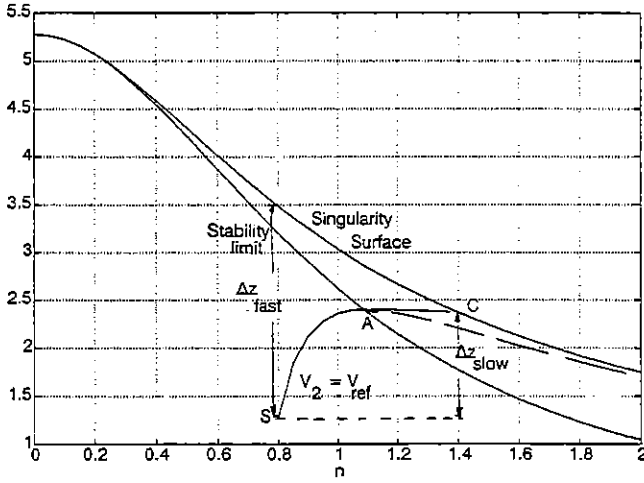


Figure 8: Stability and collapse limits of LTC system

transient collapse, i.e. a case where the dynamics of the system (in this case the LTC dynamics) have no time to react. This is clearly a very severe incident, for which there is no remedy.

2. *Slow load increase:* In this case it is assumed that the load increase is so slow that the LTC has time to reach equilibrium at each demand level (quasi steady state increase). Therefore, the operating point of the system will proceed along the SA'' (constant voltage) line of Fig. 2 as the load increases. At point A'' the stability limit is reached and before any further load increase the system trajectory will move along the curve $A''C''$, eventually collapsing at point C'' . This is a case of a quasi steady state, or instability induced collapse. The practical difference from the previous case is that the collapse is initiated by the LTC instability, and therefore the collapse procedure can be interrupted by blocking the tap changer, or when an upper tap limit is encountered. Such a situation, where the collapse process is stopped after the instability by the device limiters could be defined as a *partial collapse*.

The same sequence of events is depicted on the traditional power - primary side voltage ($P - V_1$) curve, already shown in Fig. 4. In the case of fast load increase the tap does not change and the system collapses at the point where the load characteristic becomes tangent to the network characteristic. For slow load increase, the tap has time to restore the secondary voltage up to the tip of the curve (point A). At this point the tap becomes unstable changing the slope of the load curve (without any further load increase), until it becomes tangent to the network characteristic (dotted lines in Fig. 4). Note how the LTC stability limit reduces the MW distance to collapse. Note also that the maximum power transfer point A is the stability limit in this case, whereas the collapse point is slightly different for the cases of slow and fast load increase.

A third different view of the same system can be achieved by looking at the parameter - state space shown in Fig. 8. This Figure comes from looking at the constraint manifold of Fig. 2 along the V_2 axis. In Fig. 8 the limits of stability and collapse are clearly shown. The outer curve is the singularity surface, whereas the inner curve is the stability boundary. Note that the singularity surface is the boundary of the projection of the constraint manifold of Fig. 2 on the state and parameter space. Inside this surface there are two projection points of the constraint manifold, and outside this surface there are no projections (non-feasible region).

Table 1: Generator, load and AVR data

Gen.	x_d	x_q	x'_d	T'_{do}
	0.8958	0.8645	0.1198	6 sec
AVR	Gain K_E		Time constant T_E	
	30.		0.5 sec	
Load	P_o	G	Q_o	B
	1.0	1.0	0.5	-0.5

For slow load increase the operating point will move along the equilibrium curve $V_2 = V_{ref}$ up to the point it reaches the stability boundary. At this point and without any further load increase the system will collapse on point C following the constant demand line with increasing tap. Obviously the collapse is avoided when the LTC action is blocked. On the other hand, for a fast load increase the collapse limit is reached following the constant tap line. The load demand margins to transient and quasi steady state collapse are shown in Fig. 8 as Δz_{fast} and Δz_{slow} respectively.

6.2 Generator with isolated load

This example involves a synchronous generator feeding an isolated load with a constant power and a constant admittance component, which depend upon a single load demand variable z as in equation (25) of the previous case. At nominal voltage, half the load is constant admittance, the other half being constant power. The load has a constant power factor corresponding to $Q/P = 0.5$. A lossless line connects the generator to the load through a total reactance of 0.1 pu. Since the system is isolated there are no angle dynamics involved and the generator field is modelled with the EMF E'_q , which is proportional to field flux. The excitation voltage E_f is an input variable when the generator is without voltage regulation, and it becomes the second state variable when the machine Automatic Voltage Regulator (AVR) is active. The machine, AVR and load data, in per unit on a common base, are shown in Table I.

As an illustration of the general modelling approach of equations (7), (8) the generator equations are written explicitly in this form. The two differential equations of the generator - isolated load system are related to the generator field and AVR dynamics:

$$\begin{aligned} T'_{do} \dot{E}'_q &= E_f - E'_q - (X_d - X'_d)i_d \\ T_E \dot{E}_f &= -E_f + K_E(V_r - V_t) \end{aligned}$$

The 4 algebraic constraints g determine in this case the real and reactive power balance, as well as the voltage magnitude at the generator and load bus:

$$\begin{aligned} 0 &= E'_q i_q + (X_q - X'_d)i_d i_q - z(P_o + GV^2) \\ 0 &= E'_d i_d - X'_d i_d^2 - X_q i_q^2 - z(Q_o - BV^2) \\ 0 &= (X_q i_q)^2 + (E'_q - X'_d i_d)^2 - V^2 \\ 0 &= (x_q i_q)^2 + (E'_q - x'_d i_d)^2 - V_t^2 \end{aligned}$$

Note: Reactances written with capital X include the external reactance $x_e = 0.1$ pu.

The 4 algebraic variables y in this case are the d- and q-axis machine currents, the load voltage V , and the controlled generator terminal voltage V_t . The input variables are the load demand z and the generator reference voltage V_r .

Let us consider first the collapse conditions (11) for this system. Since neither the excitation voltage E_f , nor the reference voltage

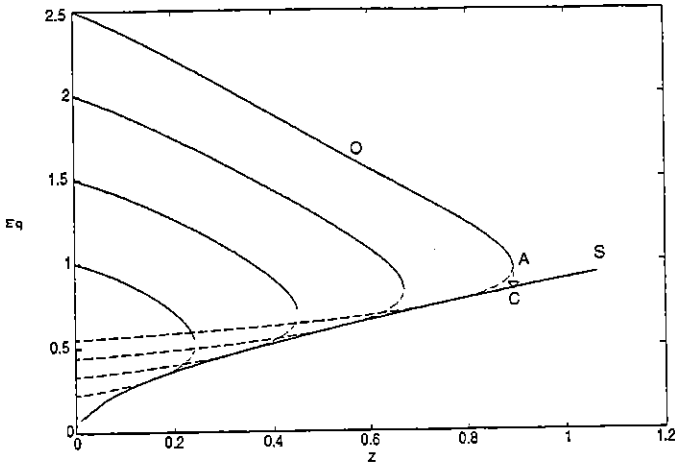


Figure 9: Equilibrium and singularity surfaces (constant excitation)

V_r appear in the above algebraic constraints, the algebraic equation singularity conditions depend only upon the transient EMF E'_q and the load demand. Therefore, equation (11) defines a curve of singularity points on the $z - E'_q$ plane. This singularity curve is shown in Fig. 9 as the solid line S. For a given level of demand, the transient EMF arrives at a minimum value when meeting the singularity curve, as suggested by equation (13) of this paper. There are no operating points below this curve, whereas there are two operating points (not shown in the Figure) for each point above the singularity curve. Note that the singularity conditions are the same when the machine operates with constant excitation, or when it is regulated.

When the generator is not regulated (if, for instance, it has been limited due to overexcitation), there is a single equilibrium equation:

$$0 = E_f - E'_q - (X_d - X'_d)i_d$$

which corresponds to replacing E'_q with E_f and X'_d with X_d in the algebraic equations.

The equilibrium surface $f = 0$ for the constant excitation case is also drawn in Fig. 9 as a set of curves for different values of E_f . These can be visualized as being in different elevations corresponding to the value of the excitation voltage E_f . According to the discussion in Section 4, the equilibrium surface is two-dimensional, since the system has two inputs (the load demand, and the constant excitation voltage).

The singularity curve discussed above corresponds also to a singularity or collapse surface which is perpendicular to the $z - E'_q$ plane. This is in accordance with the discussion of Section 2, since there is one state variable (E'_q) and two input variables (E_f and z). The singularity surface is therefore two dimensional ($n + p - 1$).

At the tip of each equilibrium curve the condition (19) for a saddle node bifurcation holds, since the demand (input) variable has a local maximum with respect to the equilibrium value of the state variable. Therefore, these points form a stability boundary on the $z - E'_q$ plane. The unstable part of the equilibrium curves is drawn in Fig. 9 with a dashed line.

Consider an initial point O on the upper part of one of the equilibrium curves of Fig. 9. For a slow load increase (quasi steady state) the operating point will move along the equilibrium (constant excitation) curve up to point A at the tip of the curve, which is a saddle node bifurcation point. At this point the stability of the equilibrium is lost, so the operating point departs from the equilibrium curve

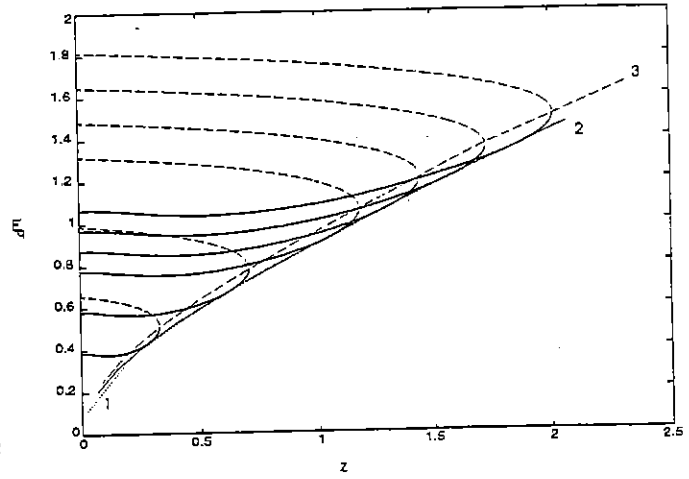


Figure 10: Equilibrium surface with AVR

along the constant demand line, and eventually collapses on point C of the singularity surface S. For a sudden, instantaneous load increase the system can also depart from the equilibrium curve along the constant E'_q line, and collapse on the singularity surface before the field dynamics have time to respond. This situation is a transient collapse similar to that of the previous section.

The effect of the AVR in increasing the stability region of the isolated generator is significant. To include this effect, the equilibrium equations have to be augmented by including the condition $\dot{E}_f = 0$:

$$0 = -E_f + K_E(V_r - V_t)$$

In general the following rules can be applied for the analysis of the voltage stability and collapse of systems that include synchronous generators:

1. The maximum loadability point with constant excitation is a SNB point of the unregulated system (e.g. point A in Fig. 9).
2. The maximum loadability point with constant generator fluxes is a point of collapse (e.g. point C and all points of curve S in Fig. 9).
3. The maximum loadability point with constant reference voltage is the SNB for the regulated system.

Obviously, the loadability of the system increases going from 1 to 3, since the equivalent generator reactance decreases from the steady state to the transient value, and then to an even smaller equivalent reactance [27].

By introducing the AVR reference voltage V_r as an input the state-parameter space becomes 4-dimensional. However, the equilibrium surface remains two-dimensional, since the inputs are still two (load demand and reference voltage). A 3-dimensional view of the equilibrium surface can be obtained by plotting the AVR equilibrium curves for different values of V_r on the $z - E'_q$ plane. This is done in Fig. 10, where the singularity curve is also drawn as curve 1. The maximum loadability points for each value of V_r form the saddle node bifurcation locus, shown as curve 3.

Comparing Figures 9 and 10 it is obvious that the AVR has significantly increased the loadability of the system. However, in the regulated machine case of Fig. 10, the normal operating region (low current) on each curve is the lower part, drawn with a solid line. The

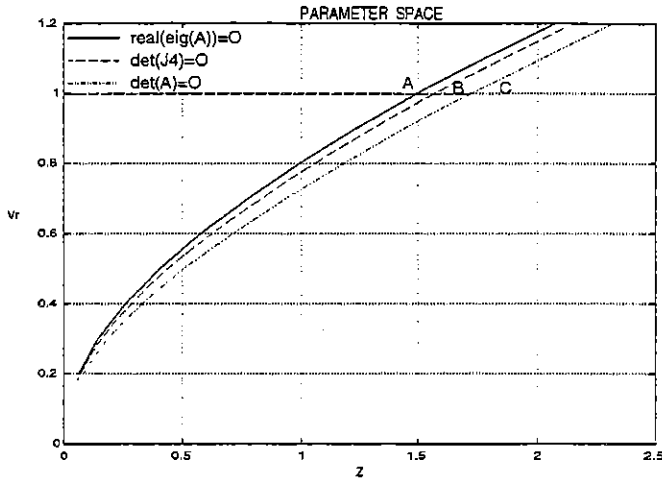


Figure 11: Parameter space

upper part, drawn with dashed line, corresponds to the high current solution of the algebraic and equilibrium equations. An equilibrium point, starting from no load and moving along the lower part of an equilibrium curve with increasing load will meet the singularity surface (curve 1) before the saddle node bifurcation. Since this point is an intersection of the singularity and the equilibrium surface it gives rise to a singularity induced bifurcation, occurring *before* the saddle node bifurcation. Between curves 1 and 3 the state matrix has one real positive eigenvalue, which changes sign at the saddle node bifurcation, so that the upper part of the equilibrium curves becomes stable again. However, an operating point starting in the normal operating region will exhibit unstable voltage oscillations for increasing load, eventually ending on a voltage collapse. The limit of unstable oscillations (Hopf bifurcation) is marked as curve 2 in Fig. 10, but it cannot be clearly distinguished from the collapse curve in this Figure.

The three curves of Fig. 10 are more clearly shown in the input variable or parameter space, which is the plane of the demand variable z and the reference voltage V_r (Fig. 11). The dotted curve shown in this Figure is the saddle node bifurcation locus, where the equilibrium surface folds with respect to the parameter space. Therefore, the system has two equilibrium points above this curve and none underneath. This can be understood as a minimum necessary reference voltage for each demand level, or as a maximum loadability for each level of voltage reference.

The singularity surface is shown in Fig. 11 as a dashed line. As stated above, this appears *before* the saddle node bifurcation. However, the permissible operating region in the parameter space is in fact limited by the third curve drawn in Fig. 11 as a solid line. This is the collection of equilibrium points with a pair of complex conjugate eigenvalues with zero real part. Therefore, when an equilibrium point moves (e.g. along the $V_r = 1$ line) following a slow (quasi steady state) load increase it will arrive first at point A where unstable voltage oscillations will start (Hopf bifurcation) and the operating point will depart from the equilibrium surface ending in a collapse. Therefore, the singularity induced bifurcation (point B) and the saddle node bifurcation (point C) cannot be experienced by a smooth load increase.

Figure 12 shows the simulated time response of the load bus voltage, starting from an unstable operating point between points A and B of Fig. 11. The nature of the unstable voltage oscillation has been

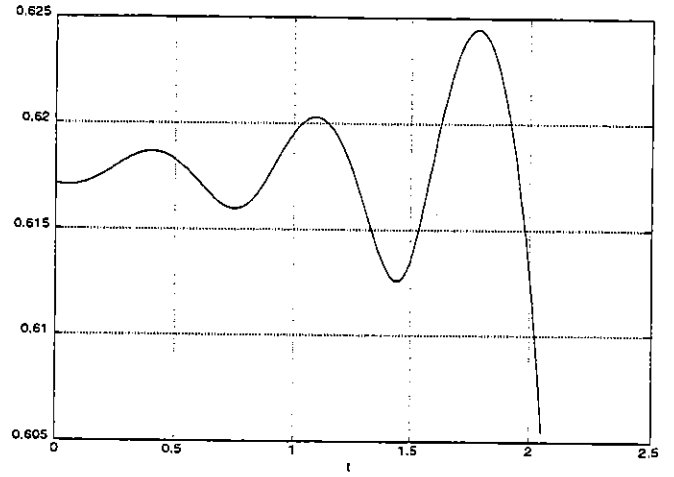


Figure 12: Load bus voltage

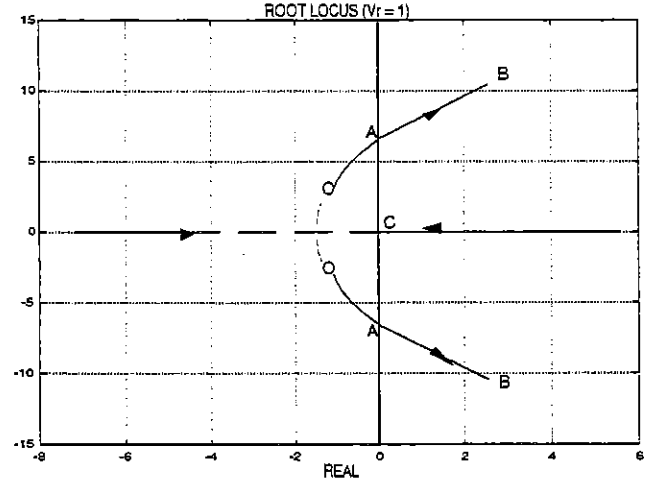


Figure 13: State matrix A eigenvalues

analysed in [28]. When the simulated trajectory reaches the singularity surface, the derivatives become infinite and the simulation stops. At this point the load voltage will depart from the constraint manifold falling to zero instantaneously (discrete transition).

The eigenvalues of the state matrix as an equilibrium point moves along a constant voltage reference line are plotted in the form of a root locus in Fig. 13. At point A the imaginary axis is crossed (Hopf bifurcation). At point B the eigenvalues become infinite (singularity induced bifurcation). After point B eigenvalues are real again coming from plus and minus infinity. At point C (saddle node bifurcation) the eigenvalue coming from plus infinity crosses the origin becoming negative again. After point C there are no equilibrium points with increased load. The rest of the root locus of Fig. 13 corresponds to the second equilibrium point for each load demand level.

Another view of the same process can be seen on the traditional $P - V$ curve drawn in Fig. 14 for constant reference voltage $V_r = 1$ (similar results have been reported in [29]). In this Figure, P is the actual power consumed by the load, and V is the load bus voltage. Points A, B, and C, corresponding respectively to Hopf, singularity induced, and saddle node bifurcation are identified on this curve. Note that none coincides with the tip of this curve.

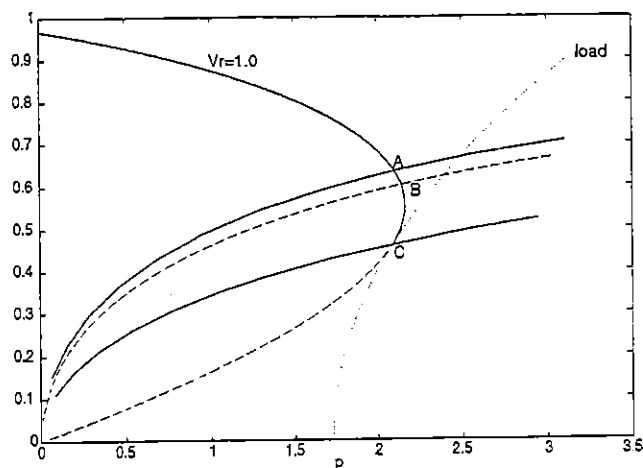


Figure 14: Load bus Power-Voltage curve of example 6.2

Actually, the saddle node bifurcation point C in this case is the point where the load curve becomes tangent to the *steady state* (or equilibrium) curve. The load characteristic is shown with a dotted line. This should not be confused with the collapse point C of Fig. 4, which shows the transient network characteristic.

7 Conclusions

Instead of conclusions, the following tentative definitions are proposed to facilitate further discussion on collapse problems:

- A time solution of a system of differential equations with or without algebraic constraints is said to *collapse* when it is defined in a maximal time interval with finite upper bound.
- The singularity of the algebraic equations in a system with differential and algebraic equations is a sufficient condition for collapse.
- A physical system is said to *collapse* when it undergoes a practically instantaneous, self-initiated catastrophic transition.
- The *collapse* of a physical system can be modelled as a discrete transition to another set of equations.

As mentioned in the paper, what is 'practically instantaneous' is relative to the time frame of interest. Therefore, the same phenomenon can look either as collapse, or as an 'ordinary' instability depending on the modelling assumptions or the detail of the available real time information. If a more precise distinction is required, this can be achieved by identifying the appropriate time scale.

Acknowledgement

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The case studies of Section 6 were prepared by Mr. George Manos, graduate student in the Department of Electrical and Computer Engineering of National Technical University, Athens, as part of his work for a Doctor of Engineering Degree.

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Discussion of papers presented by Schlueter and Vournas

Edited by L. H. Fink, ECC, Inc.

Nelson Martins (CEPEL, Brazil)

My comment [to Schlueter] refers largely to my experience, which is limited unfortunately to small signal analysis, but always working with large practical power systems. I think there are two points to be made:

First, I don't know if this exists, but it is high-time to define small, medium, and large bench-mark systems, with several scenarios, loading criteria, and contingencies, to assess the current level of non-linear tools for this area. This worked nicely for transient energy function methods in a CIGRE task force study, "Fast Transient Stability Assessment," and also for long-term dynamics. Such a procedure will identify the methods which currently can deal with large scale power systems.

All practical power systems are large scale, but theoretical work obviously is necessarily based initially on small systems. This work needs to proceed full steam to improve problem understanding and tools for practical large systems.

Now regarding something in the literature regarding another problem, power system oscillation damping. An approach was successfully used for damping control. The control of the oscillations was effected very successfully by cleverly solving the problem without devoting an enormous effort to attain a deep understanding it. Sometimes it's easier to correct a problem than to really find the sources of it. It is like a medical diagnosis. Dynamic systems are so complex that all bifurcations listed in this presentation, or any others still left - they will all exist, that is for sure.

Marija Ilic (Massachusetts Institute of Technology)

I have a question or comments related to the ones just made. I have not read Bob's paper, so I have to be careful, but I think that the message I hear is a very heavy emphasis on precise modelling of different components, which I think would require, for any actual application, a very serious approach to parameter and model identification. (By the way, that area was not mentioned by Prof. Glavitsch.) I think it is a relevant area if you take that approach. Particularly, the aggregate load model at the extra high voltage level offers a very serious challenge. I think, if I understand your paper correctly, there is some very wishful thinking there, that in order to catch all the phenomena we have to do our job of modelling right at the right time scales, and so forth. I think that problem is very serious, in terms of providing the data and the actual model identification. We've been working at MIT with quite a few utilities on practical projects, and utilities really don't have sufficient data on generators and on control devices.

That is just a question of parameter identification. I think the harder problem, the conceptual problem that I would like to refer to, is one that I have heard Dr Zaborszky's state which many times, that aggregate load modelling is one of the great unsolved problems that we have in power systems analysis, and I think we should very seriously think about that point as we advocate linking what we are doing in the theoretical setting with possible use by utilities.

The other approach, and again I've just glanced through Ian's paper, implies a quite opposite philosophy when he says maybe we can find models which are robust to identifying critical operating regions. So if I identify load models for one region at the very edge, then I'll know what is happening afterwards. I don't want to get into technical details of your message, but I think that is it. And that one is just the opposite idea to Bob's of actually working with robust models

which are fairly simple and maybe easier to identify. The only danger with that would be the conservativeness of controls required for models which are not very sensitive to critical operating regions. So I think the question that I'm posing here is a relevant one for practical applications.

Chair (Lester Fink, ECC, Inc.)

I might comment that on at least two different occasions I have tried to organise a seminar on load modelling problems and I never found enough people interested enough to get a critical mass together. But, I agree, I think it is one of the great unformulated problems.

Yakout Mansour (BC Hydro)

This is going to be part of my presentation on Wednesday, but it could open some healthy discussion right now. I have a mid-term full time simulation of a 6000-bus - 700-generator system. The study is produced by a technologist (two-year diploma after high school) with about 6 years of experience in system studies. He is given this large case to assess voltage stability, identify the key areas and interfaces, and investigate the effect of various countermeasures if needed. He starts with the step-by-step load flow modified to reflect the dynamics of equipment in the process of stressing the system up to its limit. At the limit, modal analysis is performed to see how extensive the problem is, and ranking of buses and lines according to their participation. At this point, the voltage stability limit is assessed and possible countermeasures are known. He then switches to mid-term simulation to assess the dynamic impact of countermeasures, and in the process other programs, like small signal stability, may be used to deal with side issues.

The interesting thing about that person is: He does not know the term "bifurcation", he does not debate whether it is Hopf or saddle point, he is not sure whether the leaf shrinks or expands. Yet, the job is done completely and very well, and our system didn't crash. Can we just open the discussion for a few minutes: What are we trying to achieve by spending more and more time defining things one way or another. What are we missing by not defining the outcome as one vs. another? Are we trying to explain things in 25,000 ways? The discussion is not necessarily with me only, but all of us.

Robert Schlueter (Michigan State University)

I'm going to answer Yakout's question first. The motivation for writing this paper partially was to bring up this question about the importance of bifurcations. My thought is that we have a debate going on in the voltage stability literature as to whether bifurcations are important and what is important about them. A power system engineer did not need to deal with the concept of bifurcations in the past. The reason is because most systems had a large region of attraction and a large feasibility region around a stable operating point. A region of attraction is the region where, given a stable operating point, if you start some place you will reach this operating point and if you go out far enough away from the operating point you will not return to the operating point. If you did that in every direction around the operating point you have determined a region of attraction.

In the "good old days" there were only two things an engineer was interested in, one was whether you had an operating point which was stable after some contingency, and the second was whether you could get to that operating point from where you started from. In effect, your precontingency operating point must be in the region of attraction of the post contingency equilibrium point. We checked for stable

postcontingency operation for each contingency by attempting to obtain a solution using loadflow, and we checked whether the precontingency equilibrium was in the post contingency region of attraction with a long term stability program. In most cases utilities never check whether the precontingency equilibrium lies in the postcontingency region of attraction but just check whether a unique local equilibrium exists. Checking whether a unique local equilibrium (load flow solutions) exists doesn't establish stability or instability of the equilibrium, existence of stable or unstable orbits and their size, and the size of the region of attraction. If the region of attraction was guaranteed to be large, and no stable or unstable orbits could ever exist, then checking existence of unique local equilibria (existence of saddle node bifurcation) would be sufficient. A long term stability simulation along with a load flow solution of any contingency also does not establish the region of stability boundary, the size of stable orbits and their proximity to the postcontingency region of stability boundary, nor the existence of unstable orbits. Thus, utilizing a long term stability simulation of a contingency does not provide the above information, it can only be provided through dynamical system theory based study of bifurcations. The following provides three reasons for studying bifurcations and the importance of the information obtained.

Hopf bifurcations are only oscillations! We have persistent interarea oscillations occurring in some utility systems. When voltage collapse is approached, we have oscillations developing all over the system, with several frequencies of oscillation. Are those oscillations harmful to us? Since there are systems with persistent oscillations, one might be tempted to say no.

If you detect an oscillation on a system, it is likely to have a stable orbit and be a supercritical Hopf bifurcation. If a supercritical Hopf bifurcation exists on your system, the size of the orbit will continue to increase as the system stress increases, possibly causing damage to turbine blading or generator windings. If system stress is increased further, the oscillation orbit becomes large, and at the same time the boundary of the region of attraction can shrink, so that they collide causing a "blue sky" bifurcation. This results in loss of the region of attraction, loss of feasibility, and a loss of stability. So how big the oscillation orbit is compared to the region of attraction boundary is going to decide the fate of your system. It is no longer how far the region of attraction is to the equilibrium that is always going to decide whether your system is stable or unstable. You had better monitor both the size of the oscillation orbit and the size of the region of attraction, using dynamical system theory based bifurcation analysis procedures to assess proximity to loss of stability.

Another example that is perhaps even more pertinent is if you have an unstable oscillation (subcritical Hopf bifurcation). You can't observe it on your system, or on a longterm stability simulation. You don't even know whether an unstable orbit exists, or its orbit size, without bifurcation analysis procedures. You can't find unstable orbits, because they are orbital knife edges in state space. You can never find a point that lies on the unstable orbit, and if you ever should find such a point, the trajectory will always move off it, either to a stable operating point or off to infinity. This unstable limit cycle is part of the region of stability boundary. The unstable limit cycle orbit is shrinking as the system is stressed, causing your stable equilibrium to disappear when the subcritical Hopf bifurcation occurs.

Both of these Hopf bifurcation problems indicate that one needs to know both the region of stability and the size of the orbit. Both of these properties are not currently known or assessed after any contingency, but they need to be assessed using bifurcation analysis procedures if we are to assess whether a utility system will remain stable after the contingency occurs. The three reasons we need to know the region of attraction of the postcontingency equilibrium as well as the

existence of and size of orbits associated with subcritical and supercritical Hopf bifurcations are to assess:

whether the precontingency equilibrium lies inside the region of attraction of the post-contingency equilibrium and thus whether the trajectory will ever reach the stable operating condition after any contingency;

whether a stable (supercritical) Hopf bifurcation orbit can cause a "blue sky" bifurcation that in turn causes the region of attraction to disappear; and,

whether an unstable orbit exists and subcritical Hopf bifurcation can cause the region of attraction to collapse around the equilibrium.

My view is that we must learn how to deal with these bifurcations because they can decide the issue of stability for power systems. It can happen in many different ways. We have just gotten started on figuring out how, where, and why.

I've heard Nelson Martins ask: Don't good engineers really solve problems they do not know about? and I agree. I have great faith in good engineering. It generally solves problems we do not know enough about. But voltage collapse is exactly a case where good engineering was not sufficient. It is one which brought down the French system, when we did not understand it. We have dealt with several of the issues that came out of that initial voltage collapse. But I would claim that there are some unknowns that can cause stability problems if we don't maintain our vigilance.

Marija Ilic

[Addressing Vournas] Again, I have not read the paper - I'm sorry. I think your starting point was that you are dealing with a finite escape time formulation. Is this what you are doing in the second part, where you propose time scale separation? Are you positing that you have to start with a finite escape time, and then that the transient part will actually be stable and you will not have a problem with the past dynamics, that you would only possibly have instabilities or finite escape because of discrete load tap changing actions and so forth? It was not so clear to me from the paper. In other words, I think the area of finite escape when you mix these different time scales should be clarified a little more.

Costas Vournas (National Technical University, Greece)

Well let me start by answering the question. What I did in the time scale separation is to show that the same system can be modelled in two ways. In one way the differential equations, or the part of the differential equations that I considered fast, are substituted for by algebraic equations corresponding to equilibrium conditions. Therefore this formulation goes back to the differential-algebraic system. But with differential-algebraic systems, part of the dynamics are replaced by equilibrium equations. So at the point that this system has an algebraic singularity, the actual system has a transient instability. That is the finding in this part of the paper. I don't know if I can answer the question any better.

Now I would like to make a comment considering the remark of Dr Mansour on bifurcations and engineering. I think that I agree with him that problems can be solved by people never having heard about bifurcations. What I fear is that engineers who are trained in linear systems for a long time in their formal education, might not be aware of what a non-linearity might mean in a system. If we train them with this new concept of bifurcation, we might enable them to deal better with the actual problems that are nonlinear. In my training years I had the feeling that the world was linear, so by finding the eigenvalues I said, OK, the system is linear. I'm comparing bifurcation theory with linear analysis. With bifurcation theory we see that a system loses stability when two equilibrium points coalesce. It is one step ahead of just

eigenvalue tracing. I'm agreeing with your remark that you can solve the problem without knowing about bifurcations. I'm also saying that you might get a wrong, overly optimistic idea of the world if you just compute eigenvalues.

Discussion of Impact of Modeling Accuracy on Type, Kind, and Class of Stability Problems In A Power System Model, by

John Zaborszky, Heinz Schättler, Washington University, St. Louis,

Vaithianathan Venkatasubramanian, Washington State University, Pullman

If one be allowed to inject a personal note in this process I am compelled to mention our really cordial reconciliation and the renewal of our old friendship one evening in Davos, on Bobs initiative, but warmly reciprocated by me. In this atmosphere then I hope that Bob will help me clear up some mistaken impressions (inadvertent I feel certain) left by this paper about the scope of our taxonomy work.

The paper roughly divides into three segments:

- 1 A sketchy survey of some theoretical matters in large system stability.
- 2 A survey of some power system models.
- 3 A survey of episodal literature dealing with limited and often isolated special cases arising from the rich inventory of stability problems. This literature is valuable, extensive and contains some very outstanding works. This paper presents a large and diverse assortment of loosely related items. Of course surveys of this kind can be useful under the usual limitations of sketchy information.

In an attempt to bring some organization into the survey, the paper introduces a classification grid by type, kind, classes and location. This serves the purpose of partitioning the material of the paper, yet this does not seem suitable for the basis of an overall theory because of its episodal nature.

In our Taxonomy approach the methodology is developed in a general way for nonlinear differential algebraic or differential equation type models. The episodal solutions then become special cases within the general setup. No limitations are set on the size (number of buses in the thousands or more) or the complexity of the model of whatever type. No particular types of model are postulated. (In some instances our papers use small low order systems to demonstrate some details for the reader, but there is no implication that taxonomy would or should be used for such small systems, or should in any way be limited to them, or that such system models are desirable for practical applications). There is strong current interest in developing realistic composite load models (though we are not active in this at the present time). Once such models emerge, then Taxonomy can be directly applied to them. In the meantime it can be used to study specific problems of any size, type, kind, class and location with precise and uniform perspective. In a related project we have demonstrated the use of this theory in analyzing a large real life disturbance in the Midwest with thousands of buses. A report on this application is now in preparation. This would represent a way taxonomy may be used in the future.

Unfortunately, a grossly truncated picture of our work is implied by this paper erroneously and we presume unintentionally. This is where we need Bob Schlueter's help to clear up this misimpression.

The quotation from the Introduction of this paper

Focusing on computation of equilibria and the feasibility region [16] does not appear to be appropriate for deciding whether the trajectory produced by a contingency is stable based on both simulation

using EXSTAB [2] or EUROSTAG [1] and the theoretical results reviewed in this paper."

is a gross misreading or misrepresentation of the contributions of our Taxonomy theory first introduced as such in [16] after several years of development and preliminary publications as stated in [16]. Page after page of this paper this misconception about our work is reiterating with [16] referenced some 20 times. Always the feasibility region is represented as the final result and typically ignoring not only the bulk of [16] itself which is not on feasibility, but also continuing developments in the last three years since [16].

The aim of the Taxonomy work is to establish a general theory and solution methodology for dynamic power systems which are representable by differential equations including practical constraints of either the algebraic (like load flow) or the hard limits (windup or nonwindup) kind. The latter contributions are presented in our paper in this volume.

The general aims of the Taxonomy theory are developed in the framework of a Region of Security.

The concept of a Security Region is defined in [16] as the intersection of three regions (that is a state and its parameter set must be within each one of these regions to belong to the Security Region). The three regions are:

- 1 *The feasibility region* where the operating equilibrium can be moved around by the operators or outside events while retaining local (small signal) stability for the operating point.
- 2 *A transient stability region* where the operating equilibrium point has a region of attraction (stability region) large enough to contain the post disturbance initial states (and any transition trajectories) of all contingencies specified by the appropriate industry organs (like NERC). Note: All trajectories originating within this region have trajectories internal to the stability boundary and thus inside the $\{\text{it "bounded trajectory feasibility region"}\}$. The latter then is covered by the transient stability region.
- 3 *A viability region* where the (stationary) values of the state variables voltage, current, frequency, active or reactive power are within prescribed limits (without however hard limit type cutoff).

This aim is clearly established in [16] where the transient stability region and the feasibility region are analyzed in great detail. Little attention is given in [16] to viability since this was extensively studied in our papers in the late seventies and early eighties and also since it is relatively straight forward.

Equilibrium points are precisely defined throughout [16] by $x' = f(x,y,p) = 0$ and $g(x,y,p) = 0$. The load flow equation $0 = g(x,y,p)$ by itself and as a function of x is used simply to establish the surface (locus) where the equilibrium must be located to satisfy the constraints.

Please observe that the feasibility region as defined in [16] covers only the small signal (i.e. local) stability aspect of security. It was never intended in [16] as the proposed answer to dynamic system security globally even though it is represented as such in this paper.

This misrepresentation leads to some specific criticisms of [16] in this paper which are easily dismissed once the proper role of the Feasibility Boundary in [16] is understood.

The Region of Security concept as defined in [16] was the, more or less conscious, aim of power system engineering throughout this century pursued by ad hoc and partial or episodal techniques for small systems in the past. Recent major developments in power system technology spurred by the computer age and in nonlinear system theory now make it

possible to have a broad perspective, precise comprehensive theory as the foundation of a comprehensive solution technology. This is what is offered for the first time in [16] and a number of our publications since [16] including two major contributions in this volume, one establishing a theory of hard limits (windup, nonwindup and relay type) and, the other clearing up the fundamental concept of phasors.

This paper creates a public misconception which

- 1 narrows down our contribution of the Taxonomy results to just "Feasibility Region" and
- 2 obscures the major contributions devoted to the Transient stability region and its foundations in the theory of the state space dynamics.

While ignoring our precise work in the areas of transient stability, state space dynamics, stability and the stability boundary, this paper introduces a "bounded trajectory feasibility region" concept which is never defined precisely but only loosely described verbally. The closest this paper comes to a definition is an (a) to (f) list amounting to a skimpy survey of a few episodal illustrations. These are directly included in the general concepts of the Transient Stability Region and the Viability Region. For instance (d) is directly covered by the definition of the Transient Stability Region and a very clear illustration of the development of a stability boundary consisting of an unstable limit cycle shrinking on a Hopf bifurcation as in (e) is given in the curves for Types 6 and B_1 in Figure 8 in [[1]]. The text explaining this illustration points out the small and decreasing size of this stability region, the interior of the limit cycle, which would restrict the region of attraction to a size too small to be included in the Transient Stability Region which is based on the lower limit described in (f). The same will handle the case of large limit cycles. Limit cycles in general are very much a part of Taxonomy theory.

While stable equilibrium points correspond to the most common type of stable limit sets for a dynamic system, stable limit cycles, and other stable attracting sets could also emerge as the operating conditions of a physical system [[3]] as noted and studied in Taxonomy Theory [[2]]. As pointed out in [[2]], the feasibility of operation at non-equilibrium point type limit sets would depend on whether these limit sets are viable among other factors such as possible damage to equipment from fatigue (in the case of long-term operations). The concept of feasibility region introduced for equilibrium points in the theory can be easily extended for other limit sets including stable limit cycles as has been observed in our publications such as [[4]].

We have retained the traditional concept of Transient Stability as accepted standard of the dynamic response to a contingency. This concept was used for most decades of the century, originally with numerical integration by hand calculations, later aided by "network analyzers", i.e. analog models for as much as perhaps 15 generators. In the 1960's, digital computers took over the numerical integration on steadily increasing systems recently leading to the development of large and effective software packages like EUROSTAG and EXSTAB. Our Taxonomy theory provides the theoretical foundation for the dynamics of the large power system and opens up an alternate way to numerical integration of trajectories for special cases to evaluate the adequacy of a Security Region and eventually the security margins connected with it. This route gives an insight into what actually happens physically rather than just giving a numerical knowledge for a single case simulation. We believe that keeping the traditional name of Transient stability region will give somewhat of a bridge to the successful practices under limited circumstances of the past.

In closing one should emphasize that the episodal (local in a geographic or structural sense) approach and the global large system approaches are not competitors but indispensable

complements on the large and challenging problem area of very large nonlinear systems and particularly the power system. They should be treated by both sides with respect and in the spirit of cooperation. The episodal approach treats problems at individual locations within the power system or its structure when they can be validly separated from the entire system for study. The size of the problem is then limited and the results physically tangible. The large literature of this approach is surveyed very well in this paper. However the system is a very large and complex entity where episodal events are coupled and the interaction of such events with the whole is often not separable. Hence the development of a global understanding, which is the aim of our recent work and which is surveyed and importantly expanded in our article in this volume, is essential. In this global perspective sizes and structural complexity are much larger so more complex mathematics needs to be used. But the results give a different kind of insight which also elucidates many of the episodal problems, and conversely episodal study leads to observations which in turn inspire the recognition of problems in the global setting. We certainly recognize the great value of the results of approaching the problem from the view of its subsystems (e.g. generator flux decay, various types of models etc.) and hope that a similar recognition of value and usefulness will prevail toward the accomplishments of the global approach. The mutual appreciation and utilization of results on both sides should result eventually in a merged and effective large dynamic power system theory and practice.

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Closure by Robert A. Schlueter

I am very happy that Dr. Zaborszky warmly reciprocates my desire for reconciliation. John Zaborszky's career is marked by numerous great accomplishments as well as contributions to our technical community. To have any barrier to our friendship, or to feel that I could ever have conveyed a lack of honor and respect for Dr. Zaborszky's accomplishments, gives me great pain.

I have, again, given an incorrect impression of the value and importance of Dr. Zaborszky's work from my paper at this conference. I acknowledged in my paper (page 121, column 2) that Dr. Zaborszky has laid out a theoretical foundation for a differential-algebraic model. I believe that Dr. Zaborszky's and his colleagues' contribution is extremely important.

I also desire to acknowledge the contribution of Dr. Zaborszky in defining feasibility regions and viability regions, as noted in his comments. Furthermore, my comments solely in the Introduction of the paper relating to feasibility regions were not criticism aimed at Dr. Zaborszky's work, but rather a criticism of the concentration of power utility engineering effort that is devoted to establishing a feasibility region using P-V or Q-V methods for post contingency equilibria and the relatively small amount

of engineering effort aimed at establishing a region of attraction surrounding the post contingency equilibria. I intended a compliment to Dr. Zaborszky's definition of feasibility region by my reference to it, because I viewed it as a more rigorous and complete description than obtained using current P-V and Q-V methods. I intended no criticism of Dr. Zaborszky's Taxonomy in these references, but rather a closer adherence to the taxonomy. I must confess that in preparing this paper I was unaware of the paper [2] recently published by Dr. Zaborszky, where stable closed orbits are included in his Taxonomy with warnings of possible equipment damage when the orbit sizes are large.

I believe several important points are made by the definition of bounded trajectory feasibility for a contingency and the bounded trajectory feasibility set of contingencies that impact the Taxonomy:

1. That a contingency can produce a system with a post contingency stable equilibrium and a feasibility region, but that unstable closed orbits can result due to the contingency that were not present prior to the contingency. These unstable closed orbits can reduce the size or the region of attraction and the feasibility region. The size of the orbit can shrink with additional stress, causing the region of stability to shrink. This additional stress can eliminate the feasibility region and the region of attraction if it is sufficiently large.
2. How existence of stable (supercritical) Hopf bifurcation orbits can increase and destroy the region of attraction; the fact that existence of stable orbits through (supercritical) Hopf bifurcations would eliminate the feasibility region associated with stable equilibria because the equilibria are unstable after Hopf bifurcation occurs; the fact that a feasibility region could be defined for stable closed orbits; and the fact that large orbit supercritical Hopf bifurcations can damage equipment.
3. The fact that regions of attraction and feasibility regions must change whenever discontinuities occur in the power system model either through disabling the generator exciter when field or stator current limits are reached; through discontinuous under-load tap changing action; or through switchable shunt capacitor actions. Each of these actions produce new quasi equilibria or associated closed stable orbits, and whether the previous quasi equilibrium or closed orbit lies in the region of attraction of the next quasi equilibrium.
4. The fact that the feasibility region and region of attraction are most applicable to each quasi or post contingency equilibrium rather than to the precontingency equilibrium because the precontingency feasibility region and region of attraction should be larger than each post contingency feasibility region and region of attraction and because stability is decided based on the post contingency feasibility for closed stable orbits and whether the precontingency equilibrium or orbit lies in the region of attraction of the post contingency stable (quasi) equilibrium or stable closed orbit.
5. The fact that one should never experience *in extremis* if the contingency is to belong to the bounded trajectory feasibility set even if it meets all other conditions (a)-(e) for stability.
6. That a number of bifurcation sequences exist for any model type and that several benign bifurcations can occur before one that causes loss of bounded trajectory feasibility.

One may desire the requirement that the post contingency equilibrium lies in a viability region or be able to lie in the viability region after changes in operation be included as part of bounded trajectory feasibility requirements. However, one may ignore the requirement that viability hold in the

bounded trajectory feasibility requirements. The imposition of the requirement for viability of the post contingency equilibrium or an equilibrium resulting from operating change depends on the intended use of the bounded trajectory feasibility set in assessing system security.

Based on Dr. Zaborszky's comments on the requirements for bounded trajectory feasibility, it should be noted that the Taxonomy could be expanded to include:

1. A feasibility region for stable closed orbits as noted in reference [2] of Zaborszky's comments.
2. A viability region for the post contingency equilibrium or for closed stable orbits. An expansion of the definition of viability would be needed to prevent large, stable, closed orbits from causing equipment damage.
3. A requirement that feasibility, viability, and region of attraction be assessed at each quasi equilibrium and not just at the post contingency equilibrium. Feasibility would be checked for the equilibrium or closed stable orbit at each quasi equilibrium. Viability would be checked to make sure that loss of viability would have resulted in load shedding or relay tripping of generators or large motor loads that would lead to *in extremis*. If viability were checked only at the post contingency equilibrium, voltages could conceivably be restored to viability and loads and generators would be reconnected achieving exit from *in extremis*. Such phenomena have been observed on radial voltage instability problems such as occurred in St. Petersburg, Florida. If loss of viability ever occurred along the trajectory and was detected by checking viability at each quasi equilibrium, *in extremis* and loss of viability would be detected that might not be present at the post contingency equilibrium. Region of attraction must also be checked at each quasi equilibrium to ascertain how close the region of attraction boundary was to any equilibrium or closed stable orbit. Increasing orbit size of supercritical Hopf bifurcation and/or shrinking of region of stability boundary can cause blue sky bifurcation, which results in loss of feasibility, loss of viability, and disappearance of the region of attraction. Region of attraction boundaries can change dramatically and can be affected by development of closed unstable orbits associated with subcritical Hopf bifurcation. The orbit sizes can decrease with increased stress and cause loss of viability, loss of feasibility, and disappearance of the region of stability. It should be noted that the region of stability, feasibility region, or viability region may not need to disappear at any quasi equilibrium to warrant calling a trajectory not bounded trajectory feasible.

Even if the Taxonomy is expanded as noted, one would desire to define a trajectory that meets all of the requirements imposed by the Taxonomy as acceptable and our paper proposes calling such a trajectory as bounded trajectory feasible. Furthermore, one would like to define the subset of contingencies or operating changes specified in a planning or operating criterion that produce bounded trajectory feasible trajectories. Our paper defined this subset as the bounded trajectory feasible set.

Our paper focuses on effects of modeling accuracy on types, kinds, and classes of bifurcation that have been observed. Although the literature review is episodal, skimpy, and sketchy, hopefully it is a start in a diagnostic procedure for understanding the generic bifurcation sequences that bring about loss of bounded trajectory feasibility. It may be optimistic to believe that a power system with all of its complexity has a finite number of bifurcation sequences that lead to loss of bounded trajectory feasibility, but it is certainly consistent with engineering practice to make such an assumption.

Session 2

Survey II

"Recent Developments and Researches at EdF for Improvement of Voltage Control"

P. Pruvot, C. Launay
Electricité de France

Utility Practice

"Voltage Collapse: Operating Philosophy at Pacific Gas & Electric"

Ziad Alaywan, Thomas C. Petrich, Theodore A. Reece, Jim G. McIntosh
Pacific Gas & Electric Company

"A Real-Time Voltage Stability Index for Bulk System With Secondary Voltage Regulation"

V. Arcidiacono, Sandro Corsi
ENEL

"Voltage Stability Impacts on Hydro-Québec Operations
and Simulation Tools Development"

Richard Mailhot: Hydro-Québec

"Modal Analysis for Voltage Stability: Application at Base Case and Point of Collapse"

H.J.C.P. Pinto, N. Martins, X. Vieira F^o, A. Bianco

CEPEL - Centro de Pesquisas de Energia Eletrica

P. Gomes, M. G. dos Santos

ELETROBRAS

"Low Voltages After a Disturbance in the Finnish 400kV Network"

Ritva Hirvonen, Liisa Pottonen
IVO Transmission Services Ltd.

"Determining Practical Transfer Limits from a Voltage Collapse Aspect"

M. F. Hadingham: ESKOM

RECENT DEVELOPMENTS AND RESEARCHES AT EDF FOR IMPROVEMENT OF VOLTAGE CONTROL

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SUMMARY

Management of the voltage profile of electrical networks is a complex problem, for which control centre operators do not always have appropriate tools. The complexity and size of the problem, the many control methods, the diversity of constraints and also the limitations of the tools used upstream in the string of calculations, all these show the difficulties to be resolved in order to control the network in the best way possible. Managing the electrical network voltage profile well also implies being able to control the phenomena which occur with very varied amplitudes and time constants. That is why it is important to analyse the problem as a whole, and to build a complete functional tool which can handle the various operating situations which the operators may encounter. For this, EDF has chosen a certain number of complementary research routes, covering secondary and tertiary voltage controls, monitoring of the voltage profile and emergency action. This article describes the main results obtained to date.

INTRODUCTION

The electrical networks which control-centre operators have to supervise today are becoming ever more complex. Modern networks are characterised by being of greater size, with grids becoming closely meshed, by the generating centres being located further from the consumers, and with overlapping interconnects across national boundaries. In addition, since it is becoming more difficult to build new installations (lines, generating units) network management is often more stretched. The task is therefore to be able to manage the network as close to the physical limits of the equipment as possible, without also sacrificing the standard of the customer's power supply. The situation is further complicated by the economic competition existing between the various electricity utilities for whom the search for profits becomes a condition of survival.

In this context, the best possible safety level for high volume power transmission must be ensured, whilst in particular, cutting down on the risks of voltage collapse. In order to achieve this aim, the requirements of the electrical system must be anticipated as far ahead as possible, in terms of reactive power compensation. Depending on the time scale, this anticipation may be implemented by investment decisions, modifications to operating structures, starting up supplementary generating units or emergency operations. The process begins far ahead with long-term planning over several years, and ends with real time control, where response time is very limited. The closer one gets to a real time situation, the less uncertain is the future, and the more the hypotheses may be refined. At each stage, a certain number of scenarios may be analysed according to specific criteria.

Once real time is reached, the main choices have already been made, and the operators only need to intervene to adjust certain parameters. These last minute actions are nevertheless vital to take account of unknown factors which cannot be predicted during the daily preparation phases. To implement these, the operator must have available a set of tools such as margin calculations, voltage profile optimisation and so on. These tools are normally integrated independently into control systems without seeking overall coherence in management of the voltage profile. Coherence at the various time scale levels as well as the various control levels

(national, regional, power stations and substations) is however vital to avoid contradictory or even inappropriate actions.

EDF has long been aware of the stakes resting on the control of the voltage profile, and has invested a great deal over the last few years, to build this overall view. Moving beyond simple scientific models, with more and more sophisticated algorithms, the ultimate aim is an integration of the various additional tools to be able to manage the various voltage-related aspects in real time. These processes do not only apply to the problem of voltage control and a comparable method was used for control of active power in real time.

This paper covers, in the first place, the characteristics of the French electrical system, especially as regards the management methods of reactive power. It then discusses the temporal and functional breakdown used for a coherent discussion of the control of the voltage profile. Finally it details, as regards the main components of this breakdown, the current state of the operational tools, as well as the current research in progress.

THE FRENCH ELECTRICAL SYSTEM

The overall organisation

The voltage control is organised in two main parts. The first one is the distribution part where the objective is to keep the voltages as near as possible from their nominal values. This is achieved through the on load tap changers of the EHV/MV and HV/MV transformers and with varmetric relays. This latter automatic device, installed in the HV/MV substations, computes the average reactive power absorbed by the MV level, compares it with given thresholds and finally switches on or off the shunt capacitors.

The second main part is the transmission one with a double decomposition, both in space and in time scale. This is performed within the so called primary, secondary and tertiary voltage control. The primary level reacts to fast random variations and is performed locally by the AVR of alternators. The secondary level deals with slower or high amplitude variations and is achieved on local areas including several units and static var compensators. Lastly the tertiary level optimizes the network status in terms of safety and economy and is applied on larger zones like regions or the whole EHV network. To ensure the time scale separation and avoid oscillations, the response times of this three levels are designed to be sufficiently different.

The equipment

These various items of equipment include :

the synchronous generators of the nuclear power stations, traditional thermal power stations and hydro-electric power stations : 20 nuclear generating units at 1300MW and 30 at 900MW are connected to the 400 kV network, 4 x 900MW nuclear units are connected to the 225 kV network, the various traditional thermal and hydro-electric units are connected to the 400, 225, 90 or 63 kV networks. The total capacities for supply and absorption may be calculated as being about 30000 and 26000 MVar respectively on the 400 kV network, 6000 and 5000 MVar on the 225 kV network, and 2500 and 2000 MVar on the HV network. This gives a total supply of 38500 MVar and

a total absorption of 33000 MVar. These generators are connected to the EHV or HV networks by a transformer with a 3-position off-load tap changer ; this regulator allows the nominal transformation ratio to be adjusted by $\pm 2.5\%$ (1300 MW nuclear generating units), $\pm 3.75\%$ (900 MW nuclear generating units) and $\pm 3.5\%$ (600 MW coal-fired generators).

- 2 synchronous compensators, representing total supply capacities of 580 MVar and absorption of 220 MVar. They are connected to the 225 kV network with a transformer fitted with a 3-position off-load tap changer ; this regulator allows the nominal transformation ratio to be offset by $\pm 2.5\%$. Several hydro-electrical units can also be converted to synchronous compensators at the various voltage levels. They represent a total supply capacity of about 7000 MVar on the 400 and 225 kV networks.
- 186 x 400/225 kV auto-transformers of 300 or 600 MVA, fitted with a off-load tap changer ; the 300 MVA auto-transformer changers have 2 positions for a $+5\%$ variation, and the 600 MVA auto-transformer changers have 3 positions for a $\pm 4.4\%$ variation.
- 517 x 225/63 kV transformers of 100 or 70 MVA, 182 x 225/90-150 kV transformers of 100 or 70 MVA, 23 x 400/90 kV transformers at 240 MVA and 15 x 400/63 kV transformers at 150 MVA. All these are fitted with an on-load tap changer with 25, 27 or 30 positions, for a variation of ± 12 or 15% .
- about 300 x 225/20 kV transformers at 40 or 70 MVA, 900 x 90/20 kV transformers at 12, 20 or 35 MVA, and 24 x 63/20 kV transformers at 12, 20 or 35 MVA. All these transformers are also fitted with an on-load tap changer with 17 or 25 positions, for a total variation of ± 12 , 15 or 16% .

The transformers with lower voltages have a 3 position, off-load tap changer for a total variation of $\pm 2.5\%$.

- 2 shunt reactances of 100 MVar connected to the 400 kV network, one 80 MVar shunt reactance on the 225 kV network, 36, 48 or 64 MVar shunt reactances on the 400/225 kV auto-transformers and finally 8 shunt reactances of 12.5 MVar on the tertiary winding of the 225/63 kV transformers ; giving a total absorption capacitance of about 2620 MVar.
- 8 shunt capacitors of 150 MVar connected to the 400 kV network, 3 shunt capacitors of 100 MVar connected to the 225 kV network and more than a hundred 10 to 30 MVar capacitors connected to the 63 and 90 kV networks ; giving a total supply capacitance of about 4428 MVar.
- shunt capacitors of 2.4 or 3 MVar connected to the MV bar set of the EHV/MV and HV/MV transformers ; giving a total supply capacity of about 13240 MVar.

THE CONTROL OF THE VOLTAGE PROFILE

The control of the voltage profile for power transmission networks is a very complex affair, characterised by the occurrence of phenomena with very variable amplitudes and time constants. It is a large scale problem, with numerous control variables as well as various constraints of different kinds which must be observed and with varying degrees of interaction between the different network areas under the control of operators.

In order to find a solution to this difficult problem, the best starting point is the skill and knowledge of the operators and the organisational management which implicitly incorporates the experience built up over many years of operation. A target

architecture has therefore been established, taking into account the operators' concerns with regard to the various operational situations which they have to face. The first level in the structural breakdown is given below, taking account of a progression through the network situations, from the worst case to the best one.

Power system restoration

In the worst breakdown situations, the operator has to restore the service. The physical phenomena which occur vary depending on the scale of the event, and it is normal to distinguish between generalised and localised black-out. The localised black-out is characterised by an area of power loss of variable size, close to which there will be an operational network which can restore power to the affected area. The generation-consumption balance is therefore little affected. By contrast, the generalised black-out is one where the production programmes and the automatic restart procedures no longer apply. The expected restoration horizon is a function of the event and will be between a few minutes and several hours. Confronted with these relatively unusual situations, the operator must take the necessary decisions to restore the power as quickly as possible to the customers, whilst avoiding the risks of transient over voltages, ferromagnetic resonance or auto-excitation of the alternators which are inherent in this type of situation. To do this, a real time evaluation of the results of the planned action is very useful.

Emergency actions

In situations of simple disturbance, the operator may decide to use emergency actions to avoid or block the incidents. The horizon is limited to only a few minutes taking account of the urgent nature of these type of actions. These actions are sometimes completely automatic. The most common actions are remote load shedding in several stages, and blocking of transformer on-load tap changers. Whichever method is chosen, it must be possible to determine the geographical extent of the action, its amplitude and the appropriate moment to implement it. This may be determined, for example, from the minimum volume and location of the load to be cut, or by deciding which tap changers should be blocked, together with their associated blocking thresholds.

Automatic control

It is better, when dealing with ordinary contingencies such as equipment tripping or natural fluctuations in consumption, that the operator should not need to carry out too many operations. For this reason, regulation must be carried out automatically. This is normally centralised, and supplements local regulation of the units where reactions are faster, but the system is left in a weakened state, without any margin to cope with the next contingency. The operating margins therefore need to be restored somehow for local regulation, by modifying the reference points. The scope of the actions must however be limited to ensure that the operating point does not deviate too far from the set optimal conditions established by the operator or arising from forecast management. These limits therefore provide double protection for safety and for operational economy.

Restoration of margins

During less common events, or those where a temporary deterioration in the system's functional parameters is acceptable, the adjustments may reach their limits locally and persistently. The situation then becomes potentially dangerous since the automatic system has no further margins to cope with new events. It is then important to restore the reactive power reserves by starting up new compensation sources, or looking for different balances between areas. The horizon for these actions falls between a few minutes and a few dozen minutes.

Optimisation

When the regulation systems operate normally, that is, without prolonged periods at their limits, the system trajectory can then be optimised, as regards both economy and safety. The horizon is broad, and may be from one quarter of an hour to several hours, depending on the required period. As regards voltage management, this is normally restricted to a few hours. Consequently, optimisation and margin restoration actions are normally merged together.

Monitoring

Finally, this structure is not viable unless coupled with the essential monitoring functions. Their role is to evaluate the system's current position, as well as its likely evolution, known respectively as real time and forecast monitoring. The first is by nature almost instantaneous, and may result in corrective actions such as emergency actions. The second is preventive by nature and may for example prompt the restoration of the margins or optimise the operation of the system. For voltage monitoring, two fundamental indications are sought, the margin in N , that is, additional consumption which the system may tolerate without collapse and the margin in $N-1$, that is the consumption which the system may tolerate following predefined incidents.

This macroscopic view of situations which the operators may encounter, allows their true need to be evaluated. It must be stressed however, that it does not give enough weight to the characteristics of the automatic voltage regulators of units whose performances are not fully neutral, in particular during hard events.

In order to deal with the various needs arising from this analysis, EDF has, over the last few years, followed several research routes. The aim has first of all been to find a function which allows the most common control situations to be handled. Control of events which involve more complicated phenomena, and where the active/reactive decoupling hypothesis is not longer valid is studied under another heading.

SECONDARY VOLTAGE CONTROL

In order to deal with slow voltage variations, and to mobilise reactive resources in the event of incidents (cf. fig. 1), the French EHV network has been equipped with secondary voltage control systems, in all the regional control centres. These systems automatically adjust the voltages at certain characteristics points on the network, around reference values, set by the operators. To do this, the regulators modify the reference values of the automatic voltage regulators and switch the static compensation of the reactive power located in the regulation areas.

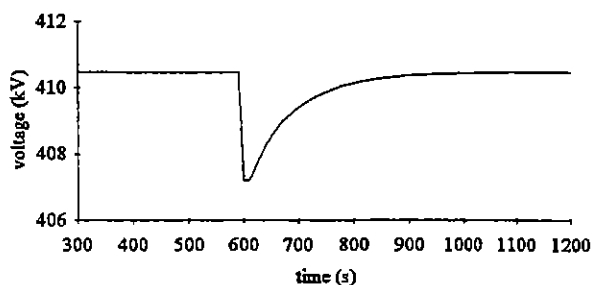


Figure 1 : Reaction of the Co-ordinated secondary voltage control (CSVC) when a 400 kV double line is tripping in a very meshed network

This secondary control level allows the operation of the automatic voltage regulators to be co-ordinated, and the working load of the operators to be alleviated, by automating EHV voltage regulation, with only a limited information requirement. It also allows reactive power reserves to be rapidly mobilised to deal with events. Finally, it helps the operators to estimate the reactive power margins available on the units.

Present SVC (secondary voltage control)

The 7 French regional control centres currently have available an automatic voltage control system, known as Secondary Voltage Control (SVC) [1][2]. It is based on the principle of dividing the EHV network into separate and electrically independent zones [3]. For each of these zones a "pilot" point is chosen, whose voltage alterations will be representative of the voltage profile of the zone as a whole.

From the deviation between the reference voltage value of the pilot point, and the measurement of voltage at a given moment, the SVC can develop a control signal, known as "level", using a proportional-integral type law. This level indicates the zone's requirement for reactive power. It is transmitted to all the regulation units in the zone and to an automatic centralised control unit for static compensation resources. The level is used on each set as a reference point for an automatic control loop for the reactive power, a loop which in turn can modify the reference point of the automatic voltage regulator. Each set takes part in the regulation process according to its reactive power capability.

On the French network, there are presently 27 regulation zones, 250 regulating units and 30 000 MVar available power. The SVC response time is approximately 3 minutes.

The new CSVC method : design

Faced with the obsolescence of the equipment used by the SVC, and the appearance of closer electrical interaction between certain regulation zones, a new secondary control method, known as Co-ordinated Secondary Voltage Control (CSVC) has been designed (cf. fig. 2) [4].

The purpose of the CSVC is identical to that of the SVC, that is, to adjust the voltages of the pilot points around reference values set by the operators. There are however, numerous innovations in this method of regulation.

The CSVC controls the voltage profile of an area consisting of several zones. The regulation of the voltages of the pilot points and the automatic control of all the regulation resources are therefore no longer independent. Consequently, this type of regulation is called co-ordinated and uses a linear model of the network based on sensitivity matrices which connect the control variables and the status variables of the system to be adjusted. The CSVC therefore takes into account the effect of each set on all the pilot points, and on all the other units.

The control signals calculated every 10 seconds by the CSVC are the reference voltage variations of the automatic voltage regulators. They may therefore be sent directly to these regulators, without using an intermediate automatic control loop for the reactive power. They are obtained using a control law, formulated as a multivariable quadratic function, to be minimised with a set of linear constraints. The response time at 63% of the asymptotic value is about 90 seconds.

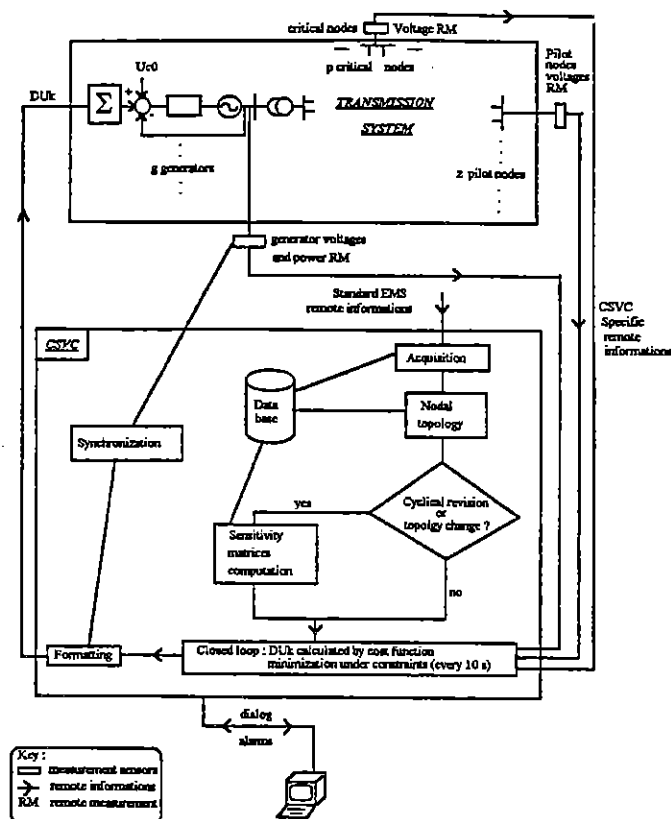


Figure 2 : Block diagram of the CSVC

The main purpose of this optimisation is to maintain the reference voltages at the pilot points. There is, however, another aim, of less importance, which is the automatic control of the reactive power of units to maintain a reference value. This additional aim allows benefits to be gained from the levels of freedom intrinsic to the system ; it minimises the reactive power provided, and gives reactive power margins which can be used to deal with any perturbations.

Many constraints are taken into account when calculating the control signals. These are minimum and maximum voltage limits on the EHV nodes for connection of units, at the terminals of the auxiliary transformers of units, and at certain "sensitive" nodes with regard to voltage regulation. The limits of the normal operating range of the generators are also respected.

Finally, algorithmic housekeeping will ensure, for example, the tolerance of the control system with regard to errors and delays of measurement, or the handling of exceptional conditions when the range of possible solutions to optimisation has been exhausted.

The new CSVC : implementation

The CSVC function, in order to begin properly, will need to be developed to the point where it can easily be integrated into the real time control environment and so that full use can be made of the operators' experience.

We have produced a CSVC prototype on a dedicated machine. This prototype will be connected to the telecommunications network, so that it can download data, and to the control computer so that it can retrieve more detailed information, such as the state of the network, and the topology. The prototype can then provide cyclic transmission of commands downline to the production units and the compensation devices.

The architecture used is clearly established in an open system logic environment. The CSVC function is built around a fully distributed task and data management system, which complies with Unix communication standards. It is thus guaranteed to be upgradable, and will be easy to integrate into the control systems.

The prototype has all the functions of the central CSVC system, from measurement downloading to transmission of commands, through filtering, sensitivity calculations, optimisation, load flow calculation, the Man-Machine Interface, on-line input of data and later off-line analysis of data stored in a relational DBMS, etc.

The new CSVC : general deployment

Taking account of the implementation problems inherent in the deployment of any new system, partial solutions have been investigated and validated by simulation. These involve connecting to the CSVC only those units which have the most profound effect on the region, and/or calculating a single control signal, called pseudo-level, for some hydro-electric units. This latter solution allows the present SVC transmission channels to be used and therefore reduces the investment required.

Following these various fields of research and development, EDF has gained full experience of the problems related to the deployment of this type of regulation method (configuration, transmission, data capture, etc.).

TERTIARY VOLTAGE CONTROL

From a global view, co-ordinated secondary voltage control mobilises the reactive power resources to minimise the gap between the network operating point, and a reference point. It seems logical to want to see this reference point at a level which produces the best compromise between the quality objectives (safety) and the economic objectives (cost). This optimisation is provided by tertiary voltage control.

Current practice for tertiary control

There is at present no facility in France for optimisation of the voltage profile, either at national dispatching level or at regional dispatching level. The operators take the real time decisions on the voltage profile to be used, that is, which reference voltage values should be applied to the secondary voltage control.

Normally, the regional control centres themselves choose the voltage profile for the EHV and HV networks, to provide the best integration of local constraints, especially for the 225 kV network. As for the national control centre, it monitors the 400 kV voltage profile, and the exchanges of reactive power between regions, modifies them if necessary, and co-ordinates the activities of regional centres.

The voltage profile sought by the operators depends on the power consumption level. With a heavy load, the aim is to maximise voltages, in order to prevent voltage collapse ; with medium or low load, the operator will prefer to maintain a medium voltage profile, in order to avoid activating high voltage constraints, and sufficiently stable to be held as long as possible. The operators also try to maximise the reactive power margins of units, both in supply and absorption, by frequent use of the reactive power static compensation resources.

The number of changes in the reference during the day is reduced. Regulation takes place mainly at the start of sharp variations in consumption and at the peaks and troughs of load.

Although the purpose of tertiary voltage control seems clear enough "to find the optimal operating point", it does still raise numerous questions.

What is the optimisation criterion which will provide the desired safety/economy compromise? The criterion used most often is that of minimising active loss. This is an excellent aim from the point of view of economy, but is not necessarily the best path to safety. Others choose to maximise reactive margins, which provides safety, but ignores economy.

Which section of the network should carry the optimisation in order to guarantee a good overall optimum? Optimisation may rely on a very full view of the network, requiring an enormous information input. Conversely it may be a largely decentralised optimisation process which needs less data but which may be significantly distant from global optimisation [5].

Which network situations should consider optimisation? This question relates to the dynamic aspect of the optimisation problem. Current practice is to rely on past situations arising from state estimation. It may however be of interest also to optimise short term forecast situations in order to anticipate variations in load, and to avoid limitation on generating units or transformer tap changers.

How may coherence between secondary and tertiary levels be ensured? This question obviously relates to the compatibility of the aims assigned to both levels but also to the resources which it controls. The resources for static compensation may for example be controlled at the tertiary or secondary levels or both, as required.

To answer all these questions EDF has initiated several research projects over the last few years, some with external collaboration.

Research status : optimisation criterion

Taking account of the size of the French network, and the transmission distances, the possibilities for gain from a minimisation of losses is not inconsiderable. The principle gains can be made from acting on a few pilot point references. It is not necessary to act very often, since only slight drift is noted from losses in natural load fluctuations. Only a few references need to be modified because of the presence of sensitivities sufficiently differentiated from losses with respect to voltages (cf. fig. 3). Conversely for limited load fluctuations, variations relative to sensitivities may be noted, which may induce a level of instability in the solution, without significantly modifying the values of the losses.

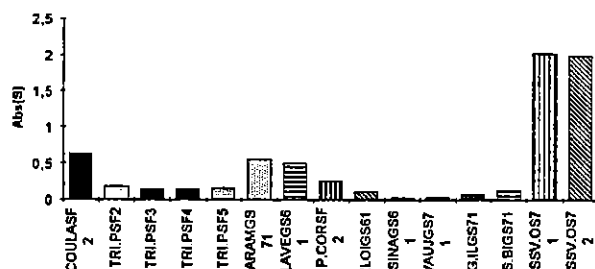


Figure 3 : Sensitivity from active losses with respect to voltages of different generating units

Research status : network view

A partial view of the network may be enough for secondary voltage control operating in a closed loop. This view is presently limited to a linear model of the network, as a function of control variables, which allows the main constraints to be respected, especially voltage limits at sensitive points. As far as tertiary voltage control is concerned, two options have been analysed.

The first is a decentralised tertiary control, based on respect for boundary conditions [6][7]. The optimisation criterion and the constraints taken into account must then be able to be expressed as a function of only the information available in the zone or at its boundaries. If the criterion is the reactive balance of the units, optimisation may be easily decentralised. If the criterion is minimisation of losses, the research conducted to date shows that optimisation cannot be completely decentralised. Conversely, it may be partially decentralised by seeking a less than optimal but stable solution using a Nash strategy. In addition, the results show that if the criterion can be dealt with at a local level, it is preferable to take it into account in the secondary control which has a sufficiently accurate view of this level. It has thus been shown that respecting a reference for reactive energy exchange between neighbouring regions is possible at the secondary control level, and that it even makes this level more independent of changes in neighbouring regions.

The second is national tertiary control, which only looks at the EHV levels of the network. In this case, the major balances of the network are ensured by tertiary control (losses, reactive margins, reactive exchanges with other countries). On the other hand, some constraints cannot be respected either because of their very local range, or because of imperfect modelling. This second point may be resolved by the potentialities of secondary control. Secondary control does actually monitor the constraints of the network very closely (stator nodes, EHV nodes of units, pilot nodes) thus ensuring safety. It also closely monitors a certain number of so-called sensitive nodes, the list of which is presently drawn up from the operators' experience. It may be possible for tertiary control to transmit to secondary control the list of sensitive nodes which may usefully be monitored.

Research status : network situations

While on a purely theoretical level, it is useful to anticipate the main load variations, at a practical level, everything depends on the activation cycle of the tertiary control. Thus, for an automatic tertiary control, whose cycle may be about 15 minutes, the problem does not arise. On the other hand, for an open loop tertiary control, left to the operator's initiative, the changes in reference must not be too numerous. It is therefore legitimate to consider some forecast situations such as the next trough or the next peak in load. This allows references to be suggested which would ensure that the limitations of each situation are respected while best satisfying the optimality criterion. There is, nevertheless a difficult problem, since the construction of a realistic future situation depends on the quality of the load forecasts, as well as an exact knowledge of planned outages and network schemes.

Research status : coherence between levels

Coherence between levels is partly ensured by the temporal disconnection of the two controls. One sets the aims, the other ensures mobilisation of resources to meet the aims (cf. fig. 4). The experience gained of the coupling of secondary control/tertiary control from simulation exercises shows that there are no major difficulties as long as the network does not experience severe hazardous events. When an event does occur, the references established when the tertiary control was last activated become temporarily unsuitable, and can only be updated when a new

network status has been provided by the estimator. During series of events, the situation may become even more critical, insofar as no further network state may be calculated until the situation has stabilised. This problem may be partly overcome by the intrinsic properties of secondary control. The linear model of the network may then be updated after each change in topology using the results of a network calculation based on the last inputs from the state estimator, as well as the last available voltages at the stator nodes. In addition, the closed loop, to some extent, allows the inaccuracy of the network model to be overcome.

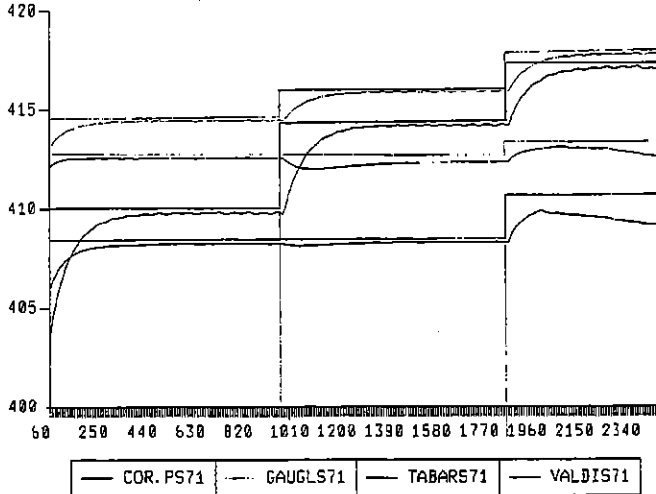


Figure 4 : Example of chaining of tertiary/secondary controls during load increase

As regards the facilities which must be controlled by the various regulations, it seems desirable that the continuous regulation facilities should be subject to secondary control. System security must be ensured by guaranteeing a rapid response to events. For discrete facilities (static compensation facilities) the question is less clear-cut. During significant events, it is useful to be able to mobilise static compensation facilities very quickly, harmonised with other actions, which favours control of the static compensation facilities by secondary control. On the other hand, to ensure more uniform participation for certain regions in terms of generation of reactive power, it may be useful to control these facilities from an overall viewpoint such as that of tertiary control. Control at two levels may also be envisaged, but to avoid the risk of oscillations, special precautions must be taken.

MONITORING THE VOLTAGE PROFILE

Evaluation of the safety of the voltage profile of the network is a major preoccupation for the operators. It is actually of primary importance to be able to evaluate the effect of perturbations on the network in order to be able to decide whether to restore the minimum reactive power reserve to cope with common events, or to undertake emergency actions to block any collapse following a succession of event events.

Current practice

The real time practice which presently prevails consists in monitoring breaches of the low voltage thresholds and guaranteeing that these are not violated following the loss of one or more installations. This is not, however, a very satisfactory approach since the momentary voltage values do not take account of the dynamic changes which the system may undergo in the near future. In addition, whether or not a network status is acceptable does not allow judgement to be made on whether or not the point of collapse is imminent. This difficulty is also amplified in the presence of the secondary control, which best mobilises the reactive power resources and also flattens out the famous PV curve (cf. fig. 5).

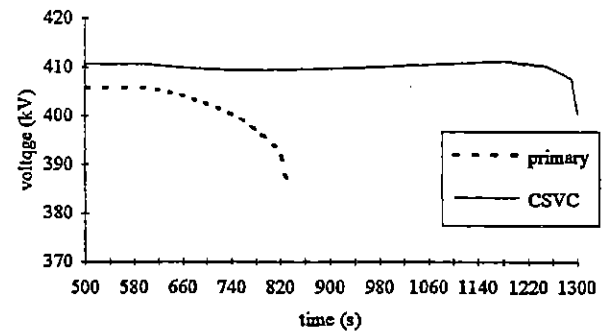


Figure 5 : Voltage collapse with and without CSVC

With this observation EDF chose to reason rather in terms of the power margin (MW) which the network may still consume without risking collapse. The calculation of this margin is entrusted to the EVARISTE software according to the following principles [8][9].

- a linear increase in active and reactive load is simulated ;
- at the same time, the restoration of the production/consumption balance is simulated ;
- the status variables of the network are supposed to have a linear variation but the variation gradients are updated each time a non-linearity is found.

Research status : a simplified dynamic simulation

One of the limitations of the EVARISTE software is that it does not allow for a fine processing of certain dynamic phenomena such as the transformer tap changing, the switching of static compensation resources or even the reaction of certain automatic emergency systems. In addition, it is very complicated to take account of the reaction of the co-ordinated secondary voltage control.

For this reason a method based on a simplified dynamic simulation, ignoring the fast transient phenomena has been investigated [10]. The behaviour of the network is reproduced from a succession of states of equilibrium whose calculations are initiated by asynchronous events such as the machines' arrival at their limits, the changing of transformer taps, or the application of new references from the CSVC. This simplified simulator allows a dynamic calculation of the margin with respect to the collapse to be carried out for a period completely compatible with real time.

The main limitation of these 2 methods is that they need a full network status. As long as the network does not undergo serious events, this does not actually cause problems, since the state estimators are activated in favourable conditions. On the other hand, following a more serious incident, or a series of incidents, the status calculation may be no longer operational and it is then impossible to recalculate the margin with respect to the collapse.

Research status : AI techniques

To compensate for the absence of network status, methods which can make use of raw measurement values or those which have been filtered only slightly, must be used, as long as they are still compatible with realtime requirements. Two methods, which only need remote measurement values (decision trees [11][12] and neural networks) have therefore been tested. The results achieved with these two methods meet expectations, since they have proved to be very fast, and by taking some precautions, the non-detection rate for dangerous situations is shown to be very low. The implementation of these methods in the context of control is still very delicate, however.

These methods do need a learning set containing several thousand network situations, with a sufficient proportion of these leading to voltage instabilities. If it is also necessary to calculate the margin in N-1, with respect to certain perturbations, the same number of learning sets must be constructed as there are perturbations. Even if it can be done outside real time, the calculation power required to generate these assemblies is still considerable, and must also be repeated for each structural development of the network.

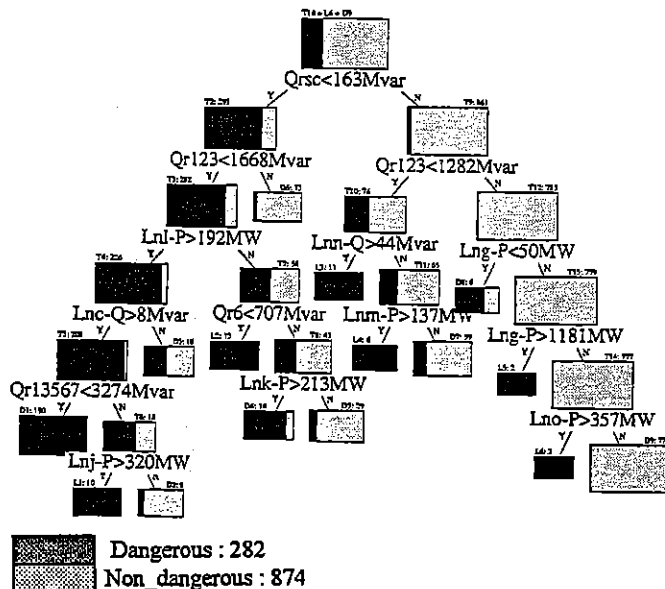


Figure 6 : Example of the decision tree for the Western network.

It may be noted that the results of these two methods are very similar as regards detection of dangerous situations. The decision tree method (cf. fig. 6) makes a more rudimentary distribution, compared to that of the neural network method, but the results are more easily interpreted, opening the path to proposal of corrective actions. In addition, if a criterion considered to be discriminatory, shown in the tree, becomes invalid, it would be possible to replace it with another measure with a closer score in the classification procedure.

EMERGENCY ACTIONS

However sophisticated the margin calculations with respect to the point of collapse, it is impossible to calculate margins for all possible combinations of event. In addition, even if it were possible to calculate these figures, it would not be economically viable to provide in advance for all the combinations of event, whose probability is infinitesimal. It is therefore vital to have facilities available to block such incidents and to restore the margins very quickly.

The characteristics of the monitoring function may significantly modify the method by which the network is controlled in the most common situations. On the other hand, to cope with exceptional situations, the presence of automatic or remote-controlled facilities able to respond within limited periods remains absolutely vital.

If margins are calculated regularly, the evolution of the margin may be followed and thus a respectable distance kept from limits, by for example, starting up supplementary units (gas turbines). If margin calculation is also preventative, that is, if it takes account of the network changes which can be forecast, the operator has more time to carry out the necessary actions. When the calculation is handling N-1 situations, the operation can also be forewarned against the

most common incidents. If this is not done, the risk of a more significant incident must be accepted. Finally if the margin calculation is made (or updated), after each tripping, the operator is still able to act, but the time is very restricted. The actions must then be automated to a greater extent, and be more abrupt, since it is not possible to refine the emergency actions.

Finally, there is the situation where a succession of events occur (N-K) for which no margin will have been calculated for preventative purposes, and for which it will be difficult to update the margin, because of the problem of obtaining a fixed point from nearly synchronous measurements. Complete blocking of the incident is then vital, either automatically, or with an on/off type action, which is by nature extremely crude.

Current practice

The emergency communication network : this was designed to allow orders to be transmitted for immediate execution, in order to accelerate implementation of a certain number of actions relating to emergency situations. This transmitter of commands connects national dispatching to the 7 regional dispatching as well as the regional dispatching to the various generation and distribution centres. The main orders regarding voltage management which may be transmitted by the emergency network are :

- request to the units to provide exceptional maximum reactive power with secondary voltage control put out of service ;
- direct blocking of on-load tap changers of some or all the EHV/HV transformers at a predetermined point ; this blocking may also be automatically active when the voltages of some stations drop below operator defined thresholds.
- request to distribution centres to drop by 5% the reference voltage values of the on load tap changers of the HV/MV and EHV/MV transformers.
- request to the distribution centres to shed part of the consumption ; in some cases, the shedding order sent by regional dispatching may be applied directly to consumption ; this is remote shedding.

Automatic network cutting-off : as soon as a prolonged loss of voltage appears, an automatic cut-off of the EHV and HV networks is necessary in order to prepare for staged energising of the network. Energising paths and consumption pockets are formed. The cut-off is made by automatons which remain selective, with the protection devices and some switch-overs which may lead to short power losses.

Automatic power system energising : in order to minimise the period of power supply loss for the customer, automatic energising functions are associated with the automatons described above. This automatic process is initiated by dispatching when it gives the order to restore voltage to one or more installations in the area of power loss. The restoration of the network when a generalised incident takes place is, on the other hand, taken position by position, manually, after disarming the voltage loss controls watch. National dispatching then defines the recovery strategy and each regional dispatching puts it into operation in its own area.

Research status : load shedding

Various methods have been investigated. The first uses the sensitivities which connect voltage variations to the load to be cut off. The aim is to achieve, by cutting off the power, voltages above a given threshold in order to move away from the critical point. The second method relies on a simplified indicator of voltage collapse risk [13]. Using the relationship between the variations of the

indicator and the loads, it is possible to calculate the necessary volume to be cut off and also to identify vulnerable network zones. The third method is based on the critical voltage which provides an estimate of the distance to the critical point. The purpose is then to determine the load to be cut off in order to reduce the critical voltage below a predetermined threshold. For all three methods, the calculation of the minimum load to be cut off may be determined by linear monovariable or multivariable programming.

Comparisons made between these methods show that the method based on critical voltages is faster and more accurate than the others. Overall, these methods do show some limitations, nevertheless. For example, it is not possible to take account of a secondary voltage control, especially co-ordinated secondary voltage control. The dynamic of the on load tap changers and the presence of any control systems cannot be dealt with. Finally, the modelling of the loads, which significantly affects the calculation of the cut-off volumes, is very brief (constant impedances, fixed injections).

To overcome these difficulties, a new method, based on a dynamic simplified simulation, has been investigated [14]. This method ignores the rapid transient dynamics and calculates a series of points of equilibrium, consecutive on the asynchronous events which affect the network (changing of the transformer taps, units reaching their limitations, new references calculated by the secondary control, ...). The volume and the location of the loads to be cut off following a hazardous incident are determined by simulation of the reaction of the network to this event and by a two-stage calculation.

First, the instant when the critical point was passed is determined. This is done by calculating at frequent intervals (50 sec) the sensitivities of the type dQ_g/dQ_0 with Q_g representing the overall reactive power, and Q_0 the vector of reactive loads. Once the critical point is identified (i.e. negative sensitivities) the second stage consists in calculating the minimum corrective action needed to remain within the range of admissible operating points for the load space. To do this, the hyperplan tangent to this space is determined close to the critical point, and the initial operational point (before the event) is brought within the hyperplan, if possible. It is also possible to determine the minimum volume to be cut off or the preferred loads to be cut off. This latter option is preferable from an operational point of view, since it is rare to be able to cut off only a part of the loads on a particular node.

In general, this method achieves a good compromise between accuracy and speed. It allows a rise in load to be simulated, and thus the power margin still to be consumed before reaching the critical point to be calculated. Through simulation of events, the final level may also be determined, and if required, when the critical point has been passed, a decision may be made on the cut-offs to be implemented. It is also able to take account of certain dynamic characteristics of the electrical system which are not available with more traditional methods, or with a cost in execution time which is incompatible with real time operation. It also offers the option of representing the loads in a more realistic way, as they vary with voltage levels. Finally, its ability to simulate the various events which affect the network opens new perspectives to compensate for the lack of state estimator results in situations of perturbation.

Research status : calculation of tap changer blocking thresholds

This is a simple but delicate operation, whose purpose is to protect the EHV voltage above certain thresholds. The problem lies in the fact that the ideal thresholds are heavily dependent on the network situation, and therefore on the perturbations which can affect the network. Thresholds set too high may alter the voltage quality at

the HV level ; conversely, thresholds which are too low may prove ineffective in blocking the collapse phenomenon.

To achieve better threshold values, it is important to have a solid statistical base. For this reason, a method for generating automatically the operational points of the network has been implemented. Since the added value of the statistical analysis depends heavily on the realism of the network situations generated, this latter point has been treated very carefully. Thus from the so-called basic situation, the main hypotheses of network operation (active and reactive loads, active generation, topology, reference installations, secondary voltage control references, etc.) are drawn at random, but in such a way as to produce predefined probability densities. For each viable network status obtained, a margin calculation with respect to the critical point is made, and all the information which will allow later analysis is stored. Various incidents may also be simulated on the various situations, in order to study the particular effect of a perturbation.

The statistical analysis carried out on the sets generated in this way, which may include up to 5000 operating points, is full of potential. Among other options, it allows the thresholds which provide protection against one or more perturbations to be determined. All that is required is to analyse the voltages obtained close to the critical point. It is also possible to analyse correlations between various sizes and/or various hypotheses and to deduce from them the operating rules for managing the network.

CONCLUSION

Management of the voltage profile in operation (and therefore the reactive resources available) is a complex problem, involving various timescales, various levels and various resources. In order to provide control centre operators with tools forming a coherent set and suitable for the various situations encountered, it is vital not to neglect any aspect of the problem. Above the co-ordinated secondary voltage control which constitutes an essential element for EDF to mobilise reactive resources, various researches have been pursued in order to build around it an overall functionality which can cover the main aspects of voltage profile management. A tertiary control system, intended to establish the optimal references for the pilot points of the secondary control will allow recovery of reactive reserves, or ensure more economical management of the network. A power margin monitoring function for reserves remaining before collapse will allow the operator to evaluate the moment at which corrective action must be taken. Finally, assistance in determining emergency action will complete the device for blocking events. For all these points, various methods have been investigated, taking care to analyse their relevance in the context of control. The methods presently available allow many difficulties inherent in this context to be overcome (accuracy/speed compromise) and their implementation is planned in the short term.

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Discussion

Edited by Nicholas W. Miller, General Electric Company

Carson Taylor (Bonneville Power Administration):

I thought I would say a few words about how Bonneville Power practices are quite different. Our approach is to try to keep the transmission voltage as high as possible through large shunt capacitor banks, and at the same time maintain a high level of spinning reactive power reserve. Regarding secondary voltage control, we haven't seen the need for it to be centralized, but through transformer drop compensation and local secondary loops, we can achieve good control. We do see a need for centralized control in more sensitive control of the large shunt capacitor banks, and one question is, in your secondary voltage control, are shunt capacitor banks controlled also, and how many banks are under control?

The only other thing I might mention is regarding load shedding. Our approach has been to use completely decentralized under-voltage load shedding. We evaluated both centralized and decentralized approaches, and the operators greatly favored the local approach. It's quite analogous to under frequency load shedding, and we found it to be much lower in cost, it provided good performance, and it would not be expected to be used very often, so since there was no need to try to optimize or minimize the amount of load, we provide a broad coverage of the entire area. We have had that system in service since 1991, with about fifty microprocessor-based under-voltage load shedding relays, controlling up to 1800 megawatts of load in the Seattle area.

Yakout Mansour (BC Hydro): Carsen, a question: Those controls, did you say the existing relays respond to just low voltage, that's all? don't they monitor anything else?

Taylor: No, they respond only to local voltage.

M. Pruvot (Electricité de France):

On the first point, we have decided to have such levels because, in our opinion, in dealing with the overall organization of a system, sometimes it is better to have a tertiary voltage control, a decentralized one. Nowadays we have some problems with some regions and sometimes, as I said during my presentation, the National Control Center is obliged to request some adjustments, because we do not have a good equilibrium of reactive reserve and generation in the various regions. So, in fact we are thinking more of a decentralized part of the tertiary voltage control, and perhaps another level at the National Control Center, just to be sure that there will be no problem with any deficiency of reactive power.

On the second point, about the under-voltage load shedding: generally with under-voltage load shedding the voltage threshold is very low. It is the last action you will take, so it's more than a defense plan or something like that. In my presentation, it was not exactly that point; it was more to be preparation of counter-measures when you have time. For example, if you see the margin to collapse is decreasing, if it's zero or a hundred megawatts, or something like that, you have time, and during that time you can compute the best way to restore the safety of your system. So it's not the last action, it's before the last one.

Sandro Corsi (ENEL):

If I understood, one of the differences of the new secondary voltage control is that you use only one control loop. I suppose that you are renouncing the unit reactive power control loop. So, I wonder what are the problems that you had in the past concerning that kind of unit control? The second question is, in what way do you operate to have at the same time, control over the unit voltage and the unit reactive power? It is not really clear in the paper.

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Pruvot: The second question is for the new control?

Corsi: Yes, you say the secondary loop relates to the reactive power, so you don't have this kind of reactive power control loop, but at the same time you control the unit voltage, the unit reactive power.

Pruvot: As to the first part of the question: in fact as we have two loops, one centralized loop and one local loop, sometimes when an event occurs, the local loop will act very fast because the information is there, and the centralized loop takes time to find the data to be transmitted, so the second loop is running after the first one. So we are sometimes going on the wrong way. So, because of that control limitation of the two loops with one centralized, and because it takes time to centralize the data, we have a delay for the acquisition of the data. With the previous structure, the problem was that the first loop, the local loop, was going the wrong way compared to what was optimal.

Discussion submitted subsequently by Julián Barquin

I have found this paper very interesting and very insightful. I would like to ask the following questions about it:

According to EDF strategy, secondary voltage controllers send set-points to every generator of every control power plant according to voltage changes at pilot buses. Has EDF thought about simplifying control tasks sending a unique set-point to the EHV bus of every control power plant rather than its actual strategy? (For that purpose there should be a local controller in every power plant to keep the voltage of the EHV bus of the plant at the reference value dictated by the secondary controller.)

Has EDF taken into account the possibility of carrying out dynamical coordination actions between 7 regional controllers considering the future increase of meshing in the EHV grid? These coordination actions also would allow reduction of the response time in driving the voltage at pilot buses to their optimal values dictated by tertiary control. Beside this, the electrical couplings between different areas of an electric power system may increase when the system becomes heavily loaded, so it seems that some kind of coordination should be implemented.

How does EDF carry out the trade-off between keeping an adequate voltage profile and maximizing the reactive power reserves in steady state at tertiary control level?

Closure by P. Pruvot and C. Launay

First, about local controllers in power plants:

Presently, our generators are actually equipped with local controllers. These controllers constitute an automatic control loop for reactive power. Each secondary controller (SVC located in regional control centers) calculates and sends a unique set-point to all generators of its control zone. This set-point indicates the zone's global requirement for reactive power. Then, each local controller regulates the reactive output of its generator according to its reactive power capability.

We know from experience that inserting any additional closed loop (in this case, the local controller) reduces the performance level and the steadiness of voltage control. In addition, each local controller operates without taking into account interactions with others. For these reasons, the next secondary controller (CSVC) will calculate a control signal for each generator and will transmit it directly to the automatic voltage regulators.

Second, about dynamical coordination of regional voltage controllers:

Secondary controllers (located in regional control centers) coordinate the use of more and more generators. The present systems (SVC) slave the generators of one or several power plants. The next system (CSVC) will slave most of the generators of a regional control center area. However, secondary controllers have to remain regional in order to provide the best integration of regional constraints and to exchange less data with the power system. Consequently, coordination of regional voltage controllers will be achieved by the Tertiary Voltage Control and its global optimization.

Third, about the objectives of tertiary voltage control:

The objective of the Tertiary Voltage Control is to achieve the best possible trade-off between safety objectives and economic requirements. These objectives can vary according to the power consumption level or differ from one part of the network to another. For instance, consider minimization of active losses to be the economic aim and maximization of reactive reserves the safety one. The first objective may be explicitly integrated in the optimization function. The second one may be implicitly reached by optimizing not only in N situations, but also in $N-k$ situations. In this way, reactive reserves are created in order to cope with contingencies, to respect operating limits, and to avoid voltage collapse.

VOLTAGE COLLAPSE : OPERATING PHILOSOPHY AT PACIFIC GAS & ELECTRIC

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Abstract: In recent years, voltage collapse has become an increasing concern in operating the electric power system at the Pacific Gas & Electric Company (PG&E). This is due to many factors that have made it more economical to operate around constraints than to eliminate them by reinforcing the system. In order to reliably operate the system and plan for daily operations under constrained conditions, PG&E has developed "Operating Instructions" or "procedures" for the System Dispatchers to use under normal and emergency conditions in each of PG&E's constrained areas. System Dispatchers rely heavily upon these procedures to operate the PG&E system.

I. INTRODUCTION

As the utility business faces unparalleled challenges and is forced to take additional risks to become more competitive, the emergence of a competitive energy marketplace, regulatory reforms, pressure for increased access to the bulk power transmission network, and strict environmental regulations for generation and transmission construction have made it more difficult to plan for system reinforcements. In some cases this has resulted in heavy utilization of the transmission network which exposes it to smaller security margins and operating patterns other than those for which it was originally designed. The result is increased operating complexity. The business environment has increased emphasis on real time system operations and operations planning to get the most, at the least cost, from existing equipment. This means that security and reliability concerns are becoming more important. One major aspect of the reliability issue is related to voltage instability.

The term voltage instability has been used to describe many situations which result in systems where voltages are not automatically restored to acceptable levels. These situations have virtually always been associated with reactive power capabilities attributed to shunt capacitor switching, transformer tap changes, transmission line loading, load power factors, load changes and voltage regulation. In general, PG&E's strategy is to operate smarter and be more effective by relying upon short term operational solutions to the existing transmission constraint problems. These operational solutions include

the development of operating instructions and the installation of Remedial Action Schemes (RAS) to initiate corrective action to mitigate overloads, stability and reliability concerns. Furthermore, PG&E, like several other utilities, continues to upgrade its Energy Management System to add much needed functionality through use of operational tools, such as security analysis, voltage collapse analysis, operator's load flow, state estimation, and transaction evaluation. In order to achieve the company's goals in this new competitive market, these tools must be used in real-time operations to help manage PG&E's existing generation and transmission assets, provide safe and reliable service, maintain acceptable voltage levels and improve transmission availability and reliability. The operating instructions, developed for that purpose, are designed to cover a wide range of operating conditions while at the same time reduce operating costs. Each set of instructions determines the envelope of safe operation for which a minimum generation requirement is specified to maintain reliable and economical operations while protecting against voltage collapse, system instability and thermal overloads. The instructions also are used to identify the worst single contingencies, worst bus voltages, adequate reactive margins and steady state voltage stability levels. This information is particularly important to the operators.

This paper will demonstrate how these instructions are helping PG&E's System Dispatchers maintain reliability and reduce operational costs, while protecting the system against voltage collapse and thermal overloads. Specifically, the paper: (1) describes PG&E's experience in planning, operating, and responding to voltage related problems, (2) provides an overview of the operating philosophy and guidelines PG&E has adopted to protect the system against voltage collapse, (3) identifies some of the transmission constrained problems the operators are presently experiencing and describes how PG&E is operating within the limitations imposed by these constraints, (4) presents the development of two operating instruction, and their corresponding Nomograms, which cover the operation of the transmission and generation facilities in the Fresno and San Francisco metropolitan areas, and (5) describes how PG&E operators implement

these procedures on a real time basis and how these procedures are helping them manage the operation of the system. This includes real time decisions such as ordering generation on-line, coordinating clearances, and adjusting power purchases and generation.

I.1. Brief Description of the PG&E Electric System:

PG&E is one of the largest electric utilities in the United States, with a peak load of 19,600 MW (1993) and a diverse resource base comprised of more than 160 various steam, hydro, nuclear, geothermal, combustion turbine, pumped storage resources, and more than 250 non utility power producers. PG&E's Power Control Department is responsible for the coordinated operation of these resources and PG&E's extensive 500, 230, 115, 70 and 60 KV transmission system. PG&E's electric transmission facilities form a grid over the northern two-thirds of California (93,000 square miles), and consist of an extensive interconnected network of 230 KV lines and an EHV transmission network of 500 KV lines which span from the California-Oregon border in the North to north of Los Angeles in the South. The 500 KV system includes three 70% series compensated lines, and has two interconnections with the neighboring utilities. The northern interconnection is used to transfer a significant amount of hydroelectric power (up to 4800 MW) to California and the Southwest region of the United States during the Spring and Summer periods. The southern interconnection is also used to transfer an increasing amount of energy (up to 3000 MW) from the steam and nuclear resources in southern California and the southwestern region of the United States. As the demand for electricity continues to increase, coupled with several consecutive dry years and much higher scheduling activities across PG&E tie lines, several transmission constrained corridors on both the 500 KV system and the 230 KV system within the PG&E system have become apparent.

I.2. PG&E Operating Philosophy

In the past, transmission constraints had a minimum impact on generation dispatch under most conditions. Today, the magnitude of power transfer between areas has increased to the point where transmission bottlenecks influence generation commitment and loading conditions more frequently. To deal with this problem, PG&E has implemented several operating procedures for each of the constrained areas in its service territory to protect against voltage collapse and thermal overload under normal, single and double contingency outage conditions. These operating procedures were developed using the following three step approach: a) Analysis of the constraints area under a wide range of operating conditions, b) identification of the worst single contingency within the

area (loss of a generating unit or a transmission line), and c) determination of the critical bus, the acceptable operating voltage level and the associated reactive margin requirement for the area. The identification and the determination of these limits determine the minimum level of local generation dispatch requirement to operate the system within the thermal, voltage, and reactive limits in the constrained area.

II. TRANSMISSION-CONSTRAINED SYSTEM OPERATIONS

This section, the development of the operating instructions, that prescribe the operations of the generation and transmission facilities in two major metropolitan areas in the PG&E system.. These are: a) The Fresno area and b) the San Francisco area.

II.1 Fresno Area Operations

The Fresno area is a major load center and one of the fastest growing areas in the State of California. It is the largest population center and the agriculture and business hub for the greater San Joaquin Valley. The electric supply needs for this area are normally supplied by a combination of the following generation and transmission resources:

- Three double circuit 230 KV transmission lines and one double circuit 115 KV line that transfers power to the Fresno area.
- Local Hydro generation consisting of:
 - a) Helms pump storage powerhouse - The powerhouse, located 50 miles from the city of Fresno, contains three identical reversible pump-turbine motor-generator units, each capable of generating or pumping more than 400 MW or 320 MW respectively. These units are housed in a chamber cut into granite that is over 1,000 feet below the surface. Water stored in the Courtright reservoir above the powerhouse flows at a maximum of 9,000 cubic feet per second (cfs) through a supply tunnel that is almost 22,000 feet long. The water drops a maximum of 1,744 vertical feet before it is discharged downstream from the power house into Wishon Reservoir. Although, the Helms Pump Storage facility provides valuable versatility in the overall operation of the PG&E system, it is crucial in supplying power to the local Fresno area loads.
 - b) Local hydro on the Kings River - Water released from Wishon Reservoir runs through several powerhouses on the Kings River system generating a total of 400 MW.

Transmission Constraints:

The current transmission system into the Fresno area is incapable of carrying loads greater than 1700 MW. With Fresno area loads reaching 2550 MW, local area and Independent Power Producers generation must be used to supply loads above 1700 MW. The service of all Fresno area loads depends on the availability of local area generation during the on-peak periods. The availability of local area generation is largely dependent on using Helms to pump enough water during the off-peak periods. Helms pumping capability is constrained by the same transmission system and is only possible when Fresno area load during the off-peak periods is below an acceptable minimum. Notwithstanding the possibility of dry water conditions and continued load growth, the only variable in the supply/demand equation in the Fresno area is the generating plant at Helms. Under the right circumstances Helms is used as a valuable system and area resource. Helms can produce generation during the on-peak periods and restore water used from the Courtright reservoir by pumping it back during off-peak periods. The increase in area load is such that there are fewer off-peak hours during which load is low enough to allow pumping at Helms without exceeding transmission constraints. Therefore, Helms may not have sufficient opportunity to restore water back to Courtright to provide for on-peak generation. In the absence of any Helms pumping and during a midsummer heat wave, Courtright Reservoir will be depleted to minimum storage in a matter of days. Under these conditions Helms can no longer be operated to cycle the water. Furthermore, any water released from the lower reservoir (Wishon) through the Kings River Powerhouses can no longer be used to generate power from Helms.

The transmission constraints during on-peak and off-peak times are due to thermal overloads and voltage stability problems that exist on key transmission lines and substations. In 1990, PG&E narrowly averted an area wide blackout when several 230, 115 KV lines in the area relayed during Helms pumping operations. As a consequence of this event, and because of existing transmission constraints and limited hydro availability, PG&E developed and implemented operating instructions to maintain reliable service for customer loads in the area. These procedures specify the minimum amount of local generation from Helms and Kings River generation that must operate during the summer peak loading periods in order to protect against single contingency outages and voltage collapse, thus providing reliable service to customers in the area. If adequate local generation is not maintained, a single outage of any major line in the area may result in severe low voltage and transmission line overloads. The following are the operating objectives that

are set forth in designing the operating instructions for the Fresno area:

(1) Identify the worst single contingency outage in the Fresno area that would result in the most severe overload on the transmission system. This is done by using a typical contingency analysis screening program. The results indicate that the worst single contingency outage during the on-peak and the off-peak periods are on two different 230 KV lines.

(2) Identify all transmission voltage constraints within the Fresno area under various operating conditions. This includes identifying the location of the most critical bus in the Fresno area during on-peak and off-peak periods. The analysis includes periods where Helms is used in both the generating and the pumping mode. This analysis is done using a steady state power flow sensitivity-based voltage collapse program that was developed in-house by PG&E's Electric Supply Systems Group. The program, called VCMARGIN, is used to determine the critical bus in the Fresno area and the exact operating margin under normal and emergency conditions for any operating point. The voltage stability margin is determined by measuring the distance from the critical point (nose curve) on the voltage vs. active/reactive power (v-p/q) curve using continuation methods. These methods avoid the singularity of the Jacobian near the collapse point by reformulating the power flow equations and incorporating a continuation parameter [1 - 5]. The result of the analysis indicates that the critical buses during the on-peak and off-peak periods are not the same.

(3) Identify the safe operating voltage of the critical bus under normal and single contingency conditions [6]. This is done by simulating the single contingency outage on the line identified in (1), incrementally increasing Fresno area load (MVA) and monitoring the bus voltage at the critical bus identified in (2). To assess the reactive margin at the critical bus, reactive power is incrementally increased at the critical bus identified in (2) and the voltage profile at the critical bus is monitored. Operational and planning studies indicate that, for most of PG&E's constrained areas, 150 MVAR of reactive margin is an adequate margin to protect against voltage collapse under different operating conditions in the Fresno area. The reactive margin availability is determined by increasing the reactive load at the critical bus and identifying the nose collapse point. The point that is 150 MVAR away from the nose point is where the minimum acceptable operating voltage is determined.

Applying the above three operating objectives for the Fresno area, Figure 1 below shows the voltage profile of the critical bus under normal condition (line in service). The 207 KV level was found to be the minimum

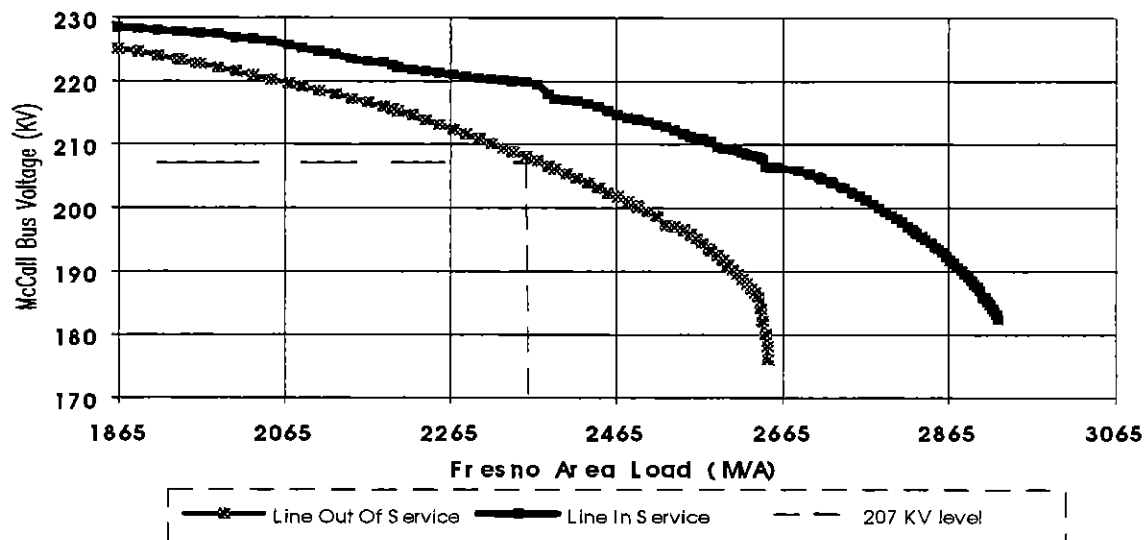


Figure 1. Voltage Assessment

operating voltage at the critical bus where a reactive margin of at least 150 MVAR is maintained. In other words, the distance between the 207 KV voltage level and the nose point (line out of service) is 150 MVAR. The 207 KV level (line out of service) correspond to 220 KV under normal condition (line in service). Figure 1 indicates, under certain operating conditions, the level of voltage under which we should operate to protect the system against voltage collapse. In this case we should maintain at least 220 KV at the critical bus at all times so that if we lose a single line, the voltage, at the critical bus, drops to 207 KV or above and will still allow for at least 150 MVAR of reactive margin. As load increases generation must be adjusted to maintain the voltage above 220 KV.

Once the acceptable operating voltage level is identified at the critical bus, the last step is to: (4) Identify the acceptable operating region. This final step identifies the minimum required local generation under which it is safe to operate and protect the system against thermal overloads and voltage collapse. This is done by simulating the worst contingency outage, increasing the Fresno load and finding the minimum required local generation without violating the thermal and voltage constraints.

II.1.a On-Peak Hours: Helms and Kings River Generation

Figure 2 shows the acceptable operating region, for the Fresno area, for 1992. The Nomogram in Figure 2 is used to determine the level of Helms and Kings River generation that must be maintained in Fresno whenever the area load is greater than 2100 MW. Each operating point in Figure 2 determines the minimum Helms and Kings River generation required to maintain the voltage level specified in (3) under different operating conditions and protect against thermal overload. Helms and Kings River generation must be operated to ensure the Fresno area load is less (on the left) than the applicable Nomograms line in Figure 2. This will provide enough local generation to serve the Fresno area load:

- Without overloading the 230 KV and 115 KV transmission system.
- Maintaining at least 220 KV, under normal conditions, at the critical bus in the Fresno area.
- Providing at least 150 MVAR reactive margin at the critical bus under a single contingency outage.

The recommended operating envelope is always to the left of the curve for each of the four operating regions (A, B1, C and D). As shown in the Figure 2, in order to supply additional load in the area, either Helms or Kings River generation should be increased. Depending on water availability, utilization of the Kings River generation is preferred as opposed to Helms generation because the Kings River generation shifts the Nomogram more to the right thereby increasing the operating margin.

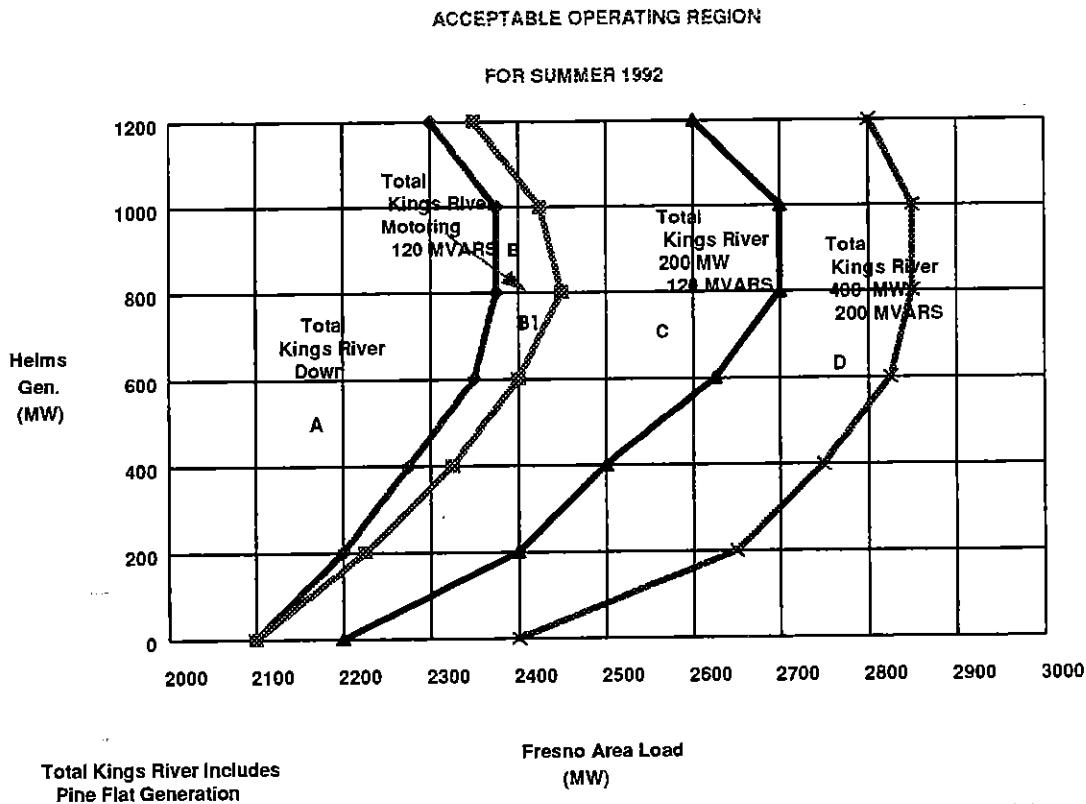


Figure 2. On-peak Fresno Area Operating Nomogram for Summer of 1992

Region "A" in Figure 2, defines the acceptable operating region with no Kings River generation on-line. As the Fresno area load approaches 2360 MW (which is the maximum load that can be served in Region "A") and continues to increase, the Nomogram indicates that Helms provides less benefit than Kings River, thus the Kings River units should be placed on-line and operated within Regions "B1", "C" or "D" to meet additional load demand and provide adequate reactive margin. Motoring the Kings River units results in a slight increase in reactive margin in the Fresno area as shown in Region "B1" of Figure 2. However, there is a little or no increase in the amount of Fresno area load that can be supplied when Helms generation is above 600 MW because of the voltage constraints. When Fresno loads reach 2100 MW, no additional power can be transferred from the outside system via the transmission network due to thermal constraints. Helms generation between 0 mw and 600 MW is needed due to the thermal constraint on the transmission system. Also Helms generation between 600 and 800 MW is needed since the constraints on the transmission system are both thermal and voltage. However, Helms generation at 800 MW or above deteriorate the voltage problem since the constraint on the transmission system is related to the voltage problem only.

With the Kings River units generating approximately 200 MW (Region "C"), the additional load that can be served in Fresno is still dependent on the level of Helms generation. On an average, for every 1 MW increase in Kings River generation (up to 200 MW) an additional 1.5 MW of Fresno area load can be supplied when Helms generation is above 400 MW. With the Kings River generation at approximately 400 MW and 200 MVAR (Region "D"), the maximum amount of Fresno area load that can be supplied is 2850 MW providing Helms generation is between 600 to 1000 MW.

II. 1.b Off-peak Hours- Helms Pumping

Depending on the number of Helms units pumping (up to three units) and the total on-line Fresno area generation at the time, the Gregg and McCall 230 KV bus voltages (the two critical bus voltages during on-peak and off-peak periods respectively) can drop significantly (up to 5 to 15 KV) from scheduled voltage levels for single contingencies. Also, thermal constraints such as overloads beyond the summer emergency rating can occur for single contingencies. Table 2 was developed following the same methodology used in Figure 1 to develop

Worst Contingency outage	No Kings River Generation	Kings River Generation 150 MW and above 100 MVARs
Loss of the Panoche-Kearney 230 KV line.	<i>OK to pump with One unit (320 MW) if Fresno Load is less than 1650 MW</i>	<i>OK to pump with One unit (320 MW) if Fresno Load is less than 1750 MW</i>
Loss of the Panoche-Kearney 230 KV line.	<i>OK to pump with Two units (628 MW) if Fresno Load < 1050 MW Gregg Bus Voltage must be 230 KV or above</i>	<i>OK to pump with Two units (628 MW) if Fresno Load is less than 1250 MW</i>

Table 2. Off-peak Fresno Area Operating Table for Summer of 1992

instructions concerning safe dispatch of Helms pumps. During the summer periods, off-peak Fresno loads and the thermal and voltage constraints on the transmission system do not permit pumping with all three Helms units. Temperature adjusted ratings on the 230 KV system were taken into consideration for these studies to increase pumping capability. One pump can remain on line as long as the Fresno area load is less than 1650 MW and normal scheduled voltages are being maintained in the area. Two pumps can remain on line as long as the Fresno area load is less than 1350 MW and normal scheduled voltages can be maintained in the area.

Summary of action taken in Fresno to protect against voltage collapse:

- Operate within the "Fresno Area Operating Nomogram" in Figure 2 and Table 2 during on-peak and off-peak hours.
- Installed undervoltage relay schemes able to trip 200 MW of firm load in the Fresno area when voltages at key stations dip below 207 KV.
- Installed Dynamic System Monitor (DSM) at a key 230 KV station in Fresno to monitor various voltage characteristics.
- Maintain adequate storage in Courtright and Wishon vis-a-vis water releases and irrigation requirements as indicated in the Water Management Procedures for Courtright, Wishon and the Kings River system.

The increasing summer loads in the Fresno area are requiring greater dependence upon local generation throughout the summer loading periods. As the load continues to grow, there could be insufficient off-peak pumping time and/or capacity, or insufficient stored water (especially during drought conditions) to allow for the

local hydro powerhouses to provide the required level of local generation throughout the entire summer. In the next few years, Fresno area loads are expected to increase up to 3164 MW, depending on forecast assumptions. Based on this forecast, Fresno area loads would exceed 1700 MW over 67% of the time during the summer months. PG&E is currently pursuing the deployment of a Remedial Action Scheme (RAS) in the Fresno area. The new RAS will detect any critical single line outage in the Fresno area and will automatically trip a Helms pump. This will allow us to substantially increase Helms pumping capability which in turn will increase Helms generating capability during the on-peak period. This short term solution is a low cost alternative to maintain reliability in the Fresno area until new transmission reinforcements are made. The Fresno area Operating instructions have been in operation since Summer of 1991. The dispatch computer monitors the Fresno area loads and alerts the System Dispatchers of any violation, allowing the Dispatchers to adjust the level of generation and/or to commit additional Hydro generation. The Fresno area operating instructions have been proven to be reliable, and very cost effective by utilizing local hydro generation, optimizing transmission capacity within the constraints, and dynamically dispatching to meet Fresno area loads.

III. San Francisco Area Operations:

San Francisco has a concentrated metropolitan load that is supplied by a single transmission source and local generation. The single source transmission system is not capable of supplying all loads in San Francisco during the on-peak periods. Unlike other metropolitan areas that have multiple transmission sources, a single contingency transmission outage can cause cascading outages, voltage collapse and total blackout unless minimum levels of local generation are maintained. The San Francisco Operating

Criteria (SFOC) are designed to maintain reliable service and reduce operational cost. These instructions specify the amount of local generation that must be dispatched to compensate for the transmission constraints to the city of San Francisco. Minimum San Francisco generation levels are determined for *normal operating conditions* and during *scheduled clearances*. Furthermore, these instructions contain *emergency procedures* to respond to severe transmission and/or generation outages in the area.

The SFOC consists of three reliability levels: Required Generation, Required Operating Capacity and Required Operating Capacity within 30 minutes. The Required Generation, following the same logic for voltage assessment used for the Fresno area (section II. 1), is the minimum amount of San Francisco generation which must be dispatched to (1) protect San Francisco against voltage collapse under normal and single contingency, (2) maintain at least 150 MVAR reactive margin at key stations and (3) protect against thermal overloads.

Dispatching San Francisco generation at or above the levels shown in Figure 3 will protect San Francisco customers against the worst single contingency outage (loss of the San Mateo-Martin 230 KV cable) and still supply all San Francisco loads. Figure 3 is comprised of three curves: (1) The "Required Generation" curve is the minimum amount of San Francisco generation which must be dispatched during all hours of the day; (2) The "Required Operating Capacity" curve is the on-line capacity of San Francisco steam generation that must immediately be available to respond to generation and

transmission outages; and (3) The "Required Operating Capacity within 30 minutes" curve is the minimum amount of San Francisco generation that must be dispatched within 30 minutes following the transmission or generation outage. It is comprised of the "Required Operating Capacity" plus a maximum of 150 MW of gas combustion turbines. The System Dispatcher will commit and operate the necessary number of San Francisco generation plants so that their generation output exceeds the "Required Generation" and their capacity exceeds the "Required Operating Capacity" level in Figure 3 during all hours of the day. Since bringing units on-line requires between four to eighteen hours of lead time, the System Dispatcher must estimate the peak load in advance and start up additional steam units to meet or exceed the "Required Operating Capacity" during the peak hours of the day. This will minimize the need to commit combustion turbines to meet or exceed the "Required Operating Capacity" levels. The "Required Operating Capacity" also provides a much needed voltage support from local generation. The "Required Operating capacity in 30 minutes" in Figure 3 is the total local generation capacity that must be dispatched within 30 minutes to respond to a generation or transmission outage. It consists of the operating steam units and assumes that three out of four combustion turbines will start and operate at full output (each combustion turbine is capable of generating 52 MW). If there are any combustion turbine capacity limitations the "Required Operating Capacity" must be increased by a corresponding amount to make up for this capacity loss.

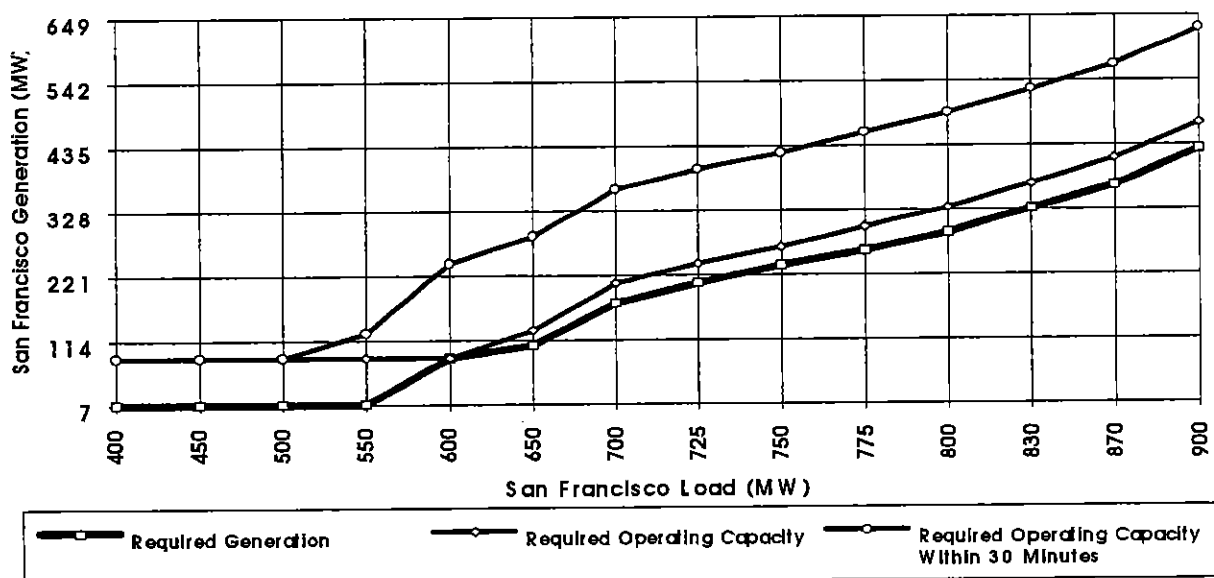


Figure 3 - San Francisco Operating Nomogram

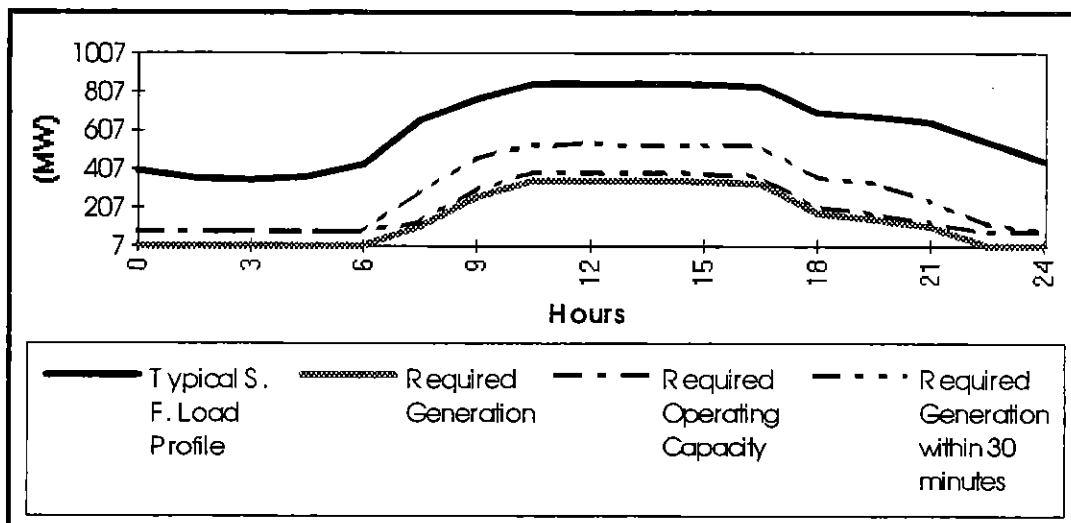


Figure 4 - Typical San Francisco loading curve

Summary of action taken in San Francisco to protect against voltage collapse:

- Operate within the "San Francisco Operating Nomogram" in Figure 3.
- The required San Francisco generation for any given San Francisco loads is below 50%. This means that if San Francisco became an islanded area from the rest of the PG&E system following a local and/or system disturbance, the required generation will not be enough to serve all of the San Francisco loads. Studies have indicated that by increasing the under frequency load shedding in San Francisco by 70 MW, the remaining essential San Francisco load would not be interrupted and will be served from the "Required Generation" level immediately followings an outage or a wide area disturbance. As more generation reaches the level "required generation within 30 minutes" in Figure 3, more loads can be restored. This will ensure that not all San Francisco load will be lost following a major disturbance.
- Raise the voltage regulators set points as needed especially during peak loading conditions.

This SFOC criteria has been in operation since April of 1992. The dispatch computer monitors the San Francisco loads and dispatches the local generation to meet the minimum generation requirement in accordance with Figure 3. The dispatch computer alerts the System Dispatchers of any violation of Figure 3, allowing the Dispatchers to adjust the level of generation and/or to commit the gas turbines. The

SFOC have been proven to be reliable and very cost effective as local generation is dynamically dispatched to meet San Francisco loads.

Figure 4, shows a typical daily loading curve for San Francisco loads. It also shows how the amount of minimum required generation, required operating capacity and required operating capacity within 30 minutes specified in Figure 3 are dispatched. Note that operating capacity was utilized (which can provide needed local reactive support) rather than actual megawatt output to protect against single contingency and voltage collapse. Relying on operating capacity to recover in the event of single contingency outage, voltage collapse or major disturbance enabled us to reduce the megawatt output under normal operating conditions from the "Required Operating Capacity Within 30 minutes" to the "Required Generation" level (which amounts to 70 MW per hour reduction in generation).

IV. CONCLUSION

This paper describes effective ways to deal with increasing operating complexity in the PG&E system necessitated by transmission constraints. The analysis requires consideration of several different system limitations, including voltage stability and first contingency limits under a wide range of operating conditions. System Dispatchers can reliably operate the system within these constraints at minimum cost when they are provided a clear description of the safe operating window and a range of control actions that they can take to stay within that window.

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Biography

Ziad Alaywan, received the B. S. and the M. S. degrees in Electrical Engineering from Montana State University (MSU), Bozeman, Montana, in 1986 and 1987 respectively. From 1986 to 1988 he was a research engineer at MSU's Electrical Engineering Department in the area of Computer Applications for Power Systems. In 1988, he joined Pacific Gas & Electric Company as a Power System Engineer in the Power Control Department. His major responsibilities were involved in the general area of providing technical and engineering support to System Dispatchers regarding the operation of the Generation and Transmission system. He was recently

promoted to Supervisor of Power System Dispatchers at the Energy Control Center (ECC) responsible for the operation of PG&E 's Generation and Transmission system. He is a Registered Professional Electrical Engineer in the State of California.

Thomas C. Petrich, received the B. S. and the Master degree in Electric Utility Management in Electrical Engineering from California Polytechnic State University (San Luis Obispo) and New Mexico State University (Las Cruces) in 1965 and 1971 respectively. In 1971 he joined Pacific Gas & Electric Company as a Transmission Planning Engineer in the Transmission Planning Department. Since 1978 he has been with the Power Control Department. He is currently the Supervisor of Grid Operations support for the southern PG&E system. His major responsibilities involve providing technical support to System Dispatchers regarding the operation of the Generation and Transmission system. He is a Registered Professional Electrical Engineer in the State of California.

Theodore A. Reece, was born in California in 1947 and received the B. S. degree in Electrical Engineering from the University of California at Davis in 1969. He has been with Pacific Gas & Electric Company since 1969. He was responsible for the Company's EHV and high current test facilities at the Department of Engineering Research until he transferred to Electric Operations, Substations Department in 1976. Since 1984 he has been with the Power Control Department and is now the Director of Operations Support responsible for providing technical and computer assistance to the System Dispatchers regarding the operation of PG&E bulk Transmission and Generation system. He is currently a member of the NERC Operating Committee. He is a Registered Professional Electrical Engineer in the State of California.

Jim G. McIntosh, Joined Pacific Gas & Electric Company in 1971. Since 1973 he has been with the Power Control Department responsible for operating the Generation and Transmission system. Currently, he is the Supervisor of Power Operations in the Energy Control Center (ECC). He hold a degree in Business Management from Saint Mary's College, Moraga, California.

Discussion

Edited by Nicholas W. Miller, General Electric Company

Claudio Cañizares (University of Waterloo):

Regarding the discussion we had yesterday, this is a good example that bifurcation analysis after all is not all that bad. I understand from what I read that you ended up putting your continuation power flow to work, right? That's good. For those of us who are working on that, it is encouraging - seeing this put to some use.

Now, I have a couple of questions. Have you considered using other tools all the tools available now? They include more detailed dynamic models, like VSTAB for instance that EPRI has, which just might make some difference between the results you are getting and the actual dynamics of operation of your system. I wonder whether those static tools that you are using are actually giving you good enough results to operate the system.

The other question is, are these tools, continuation or whatever, that you developed to create these curves, are they available to the operator on line, or do the operators work with curves given by other people?

Ziad Alaywan (Pacific Gas & Electric):

I'll answer your second question first, which is easier. The operators worked with the curves; that's all they see. Now, there's been a change in our culture at PG&E in California. We haven't had any engineers in the whole control center - I am the first. It's a different culture, so we try to give them very simple tools. Sometimes it's difficult, but these guys have a lot of experience, so we are trying to insert some engineers in the control center. Right now there is one; I'm the first. I've been there six months. So, to answer your question, they are only using these curves. Some people know the background of how we develop these, other people have no idea. We sit down for an hour or two, and discuss this in detail with everybody. We see if anybody has questions. This is how we're going to operate.

Let me answer the first question quickly. There are a lot of tools we have, very few are on line. It's probable that every utility suffers from the same thing. We have not tried any other tool. This tool was developed in house, because we have expertise in house to use that kind of technology. It seems to work. The numbers are very good.

Mansour: Is the tool you're talking about the P-V curve? Is that what you refer to as bifurcation analysis?

Alaywan: Yes. I'm talking about the results, and how the results match real life. From what we have seen, it seems to be very good. But, the optimal thing of course is to have these tools on line, and operators who will run them and look at them as we go.

Mansour: How many engineers are there in control centers? We have seven control centers in BC Hydro: one central and six area control centers. However, there is only one center that has just one engineer shift, and he is not for calculating things; he is just to monitor things. So what you're hearing about the expertise in control centers is very typical.

Marija Ilic (M.I.T.):

Were you just considering the most expensive nearby generation to as the candidates to cope with voltage collapse? Did you try any less expensive generation, a little further out?

Alaywan: Yes and no. For some areas, we try to deal with this problem by focusing on the area where the problem is. One area has only a few local generators, and the next genera-

tion is hundreds of miles away. So for this area, this is it. But there are other areas like San Francisco, where we try to use outside generation.

Ilic: Actually, it's a very hard question when you want to do the trade-offs between economics and technical effect on the problem. We played a little bit with that, they are almost exclusive, so it is hard to answer, but I think it would be good. Maybe the one which is a little further out would have better effect for less money.

Alaywan: Well, our number one criterion is security, and then economics basically. You deal with these problems as you can. The San Francisco example was a unique one; we have a lot of restrictions on local generation. We use outside generation, even outside gas turbines, about fifty, seventy-five miles away from the city.

A REAL-TIME VOLTAGE STABILITY INDEX FOR BULK SYSTEM WITH SECONDARY VOLTAGE REGULATION

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Abstract: A real-time control system of transmission network voltages such as the one under development at ENEL for applying the *Secondary and Tertiary Voltage Regulations* also allows to define a voltage stability deterministic index based on real-time measurements and signals at disposal at the Regional Dispatcher Centres, related to the unit working conditions, overexcitation limits and terminal voltage limits.

In fact the *Secondary Voltage Regulation* seasonably knows instant by instant, by a Regional Voltage Regulator (RVR), the proximity to the overexcitation limits and to the terminal voltage limits of all the units controlling the voltage of an important EHV bus bar close to them, named "Pilot Node".

This estreme situation happens when that network area (the bus set of the network in which the voltages are close to the pilot node voltage value defines an area) is operating very close to voltage instability.

To prevent this critical working condition RVR computes a real-time index based on the measurement of the area control effort to subtain the pilot node voltage, and according to it operates in time to avoid the saturation of the area voltage regulation, switching on the local capacitors banks and modifying the set-points of the area on-load tap changer (OLTCs) automatically, following definite laws.

Through this way the area units are automatically discharged, their control margins increase and the area voltage stability limits are enlarged. The local OLTC are automatically blocked when the saturation of the area *Secondary Voltage Regulation* is unavoidable.

Keywords - transmission network, voltage stability, voltage collapse, voltage stability index, real-time control, secondary voltage regulation, tertiary voltage regulation.

1. INTRODUCTION

During the last years it is grown the interest of the utilities on the problem of transmission network voltage instability/collapse and moreover on the research of a representative proximity indicator of the power systems or network area to the voltage stability limit.

The growing lack in the power systems of reactive power resources for transmission network voltage support is one of the reasons of this interest. The voltage degradation phenomenon here considered mainly concerns "long term voltage instability" [1]

having a time frame of about 0,5+30 minutes. It is usually due to a wide variety and combination of different conditions such as high load, failure of a part of the power system, control actions of OLTCs, units operation at the overexcitation limit, etc.

The phenomenon arises with a gradual reduction of transmission network voltages, which progressively becomes without recovery: the reduction of voltage profile when protracted till to reach unreversible working conditions, can determine the consequent and terminal phenomenon of "voltage collapse, which is characterized by uncontrollable transient instability with unit step breaking and the intervention of the power system protections.

On this topic the literature is very wide: one reference for all is the report [1] which includes the description of basic phenomena, terms and definitions, modelling requirements, simulation methods, description of incidents, and an exstensive list of references.

The literature [1, 2, 3, 4, 5] also shows the growing trend to study the phenomenon taking into account the system dynamics, through detailed mathematical models inclusive of the unit regulation and limitation loops, of the OLTCs, and of the network voltage control loops (i.e the existence of a *Secondary Voltage Regulation*).

These dynamic aspects contribute to take into account the non-linearities effects when the control loops reach their own limits, and the system protections operate.

The deterioration in the voltage profile can lead to network collapse so it is important on one hand to evaluate voltage profile security and on the other hand to take in time control actions of preventive type. Experience has shown that monitoring voltage levels and/or using load flow calculations when low voltage threshold values are reached for checking the consequence of a possible outage of one or more equipment, is an insufficient strategy. Voltage levels or load flow divergence are not sufficiently accurate indicators of the proximity to voltage instability.

Over recent years a lot of indices and criteria for voltage stability have been studied and proposed by the literature [6]. The objective of all the indices is to give a measure of the margin between the point of operation and the point where the system becomes instable and thereby provide an early warning of a potential critical situation. The attributes of the indices in use or under development are very different and there are some attempts for their classification. From the examination of the subject it is possible to draw that the proposed indices mainly concern the off-line preventive analysis related to the system

planning while their computation usually needs of system state condition, load-flows and algorithm which, according to the model used, fall within either the small-variations analysis around a given state or the large-perturbations analysis.

The computing of these indices is often onerous due to the iterative methods with network complex model elaborations, repeated load-flows and/or computation of Jacobian/Sensitivity matrices and/or the use of optimization algorithms, etc.

Moreover it is possible to note that a lot of the voltage stability indices till now proposed refer to power system models without dynamics and non-linearities of the system controllers, so they are hardly credible because of the previous considerations.

On the other hand a detailed dynamic simulation cannot be performed at the present time if the objective is the network real-time control, because the computation time is too high, the computing resources are too big, and because not enough it is known about a number of network parameters that are inaccessible in real-time.

However simplified power system simulations are possible and, on principle, it could be pursued the computation for predicting impending voltage collapse indices with an acceptable computing time lasting from 5 minutes to one hour.

The forecasts referred to the previous half an hour are surely useful for the network operation. Indices of this type related to short term operation are under study, design, development and test at EdF [7] and Enel [8].

On the contrary the literature does not give, as far as we know, information about examples of voltage stability indices representing the margin, at the present time, of the current point of network operation with respect to the point where the power system could become unstable.

Such an index must be necessarily based on real-time measurements of the power network and its controls, and must be computed in a short time (few seconds) in comparison with the dynamic of the network voltage reduction process. Such an index could pull the trigger for an automatic network control and protection action when overcomes proper thresholds. The objective of this automatic action would be the delaying and/or the avoiding the network to overtake voltage instable operating conditions.

Such an index, being representative of the real current network situation is completely different, both from the conceptual and computational points of view, from the others which conversely are conceived for off-line forecasting studies or for giving indications with a consistent accepted delay (at least many minutes).

The present paper will show that a real-time dynamic indicator of the proximity of the current network operating point to voltage instability, can be, under certain conditions, computed in a simple way, in a short time (few seconds) and can turn out very reliable. The necessary condition to obtain this result is the application and then the availability of an automatic control system of the transmission network

voltages (i.e the *Secondary and Tertiary Voltage Regulations*).

In the following a preliminary short description of the Enel project for the automatic regulation of the EHV power system is given, as necessary and useful introduction to the proposed real-time index.

2. THE ENEL'S PROJECT OF SECONDARY AND TERTIARY VOLTAGE REGULATIONS

The ENEL project for *Secondary and Tertiary Voltage Regulation* (see ref. [9]) requires an effective coordination of the reactive power resources through a composite on-line automatic control system, to meet the objective of keeping up a suitable voltage profile in the EHV network in the face of the continuously changing reactive power demanded by loads and the possible network perturbations [10]. The chosen control system (Figure 1) regulates, in closed loop, the voltage of some busses (pilot nodes), selected among the strongest ones, through controlling the reactive powers of the generators which most influence those busses.

The project is based on a hierarchical control structure which provides a network subdivision into areas around the pilot nodes. The areas are controlled by signals of "reactive power level" (one for each area) supplied by *Regional Secondary Regulators (RVRs)*. At higher hierarchical level a *Central Tertiary Voltage Regulator* coordinates, in closed loop, the action of the regional controllers, establishing the voltage pattern of the pilot nodes and effecting slow corrections in order to have a better balance of reactive power generation among the areas.

The National (Tertiary) Voltage Regulator (NVR) operates in real-time on the basis of the actual pilot node voltages and area reactive power levels compared with a day before (short term) or some hours in advance (very short terms) forecasts of optimal voltage profiles.

Owing to the very satisfactory results obtained in the first implementations [10], [11], ENEL has planned the general application of the *Secondary and Tertiary Voltage Regulations* to the overall EHV power system, to be completed before '96.

Secondary Voltage Regulation (SVR): The update studies for the choice of pilot nodes and respective control power plants provide the subdivision of the power network into 13 areas. The voltages of the 13 pilot nodes will be regulated by *Regional Regulators* installed at the 8 *Regional Control Centres*. The regulators of the Rome, Naples and Cagliari Centres will each control two pilot nodes, while the Sicily Centre will control three pilot nodes.

The overall plant involves all the major power plants connected with the 400 and 220 kV network, for a total reactive power capacity of about 16,000 MVAR. Figure 2 shows the subdivision into areas of the Italian network, pointing out the nodes and the corresponding control power plants.

According with this plan ENEL is putting in operation the *Power Station Reactive Regulator* named

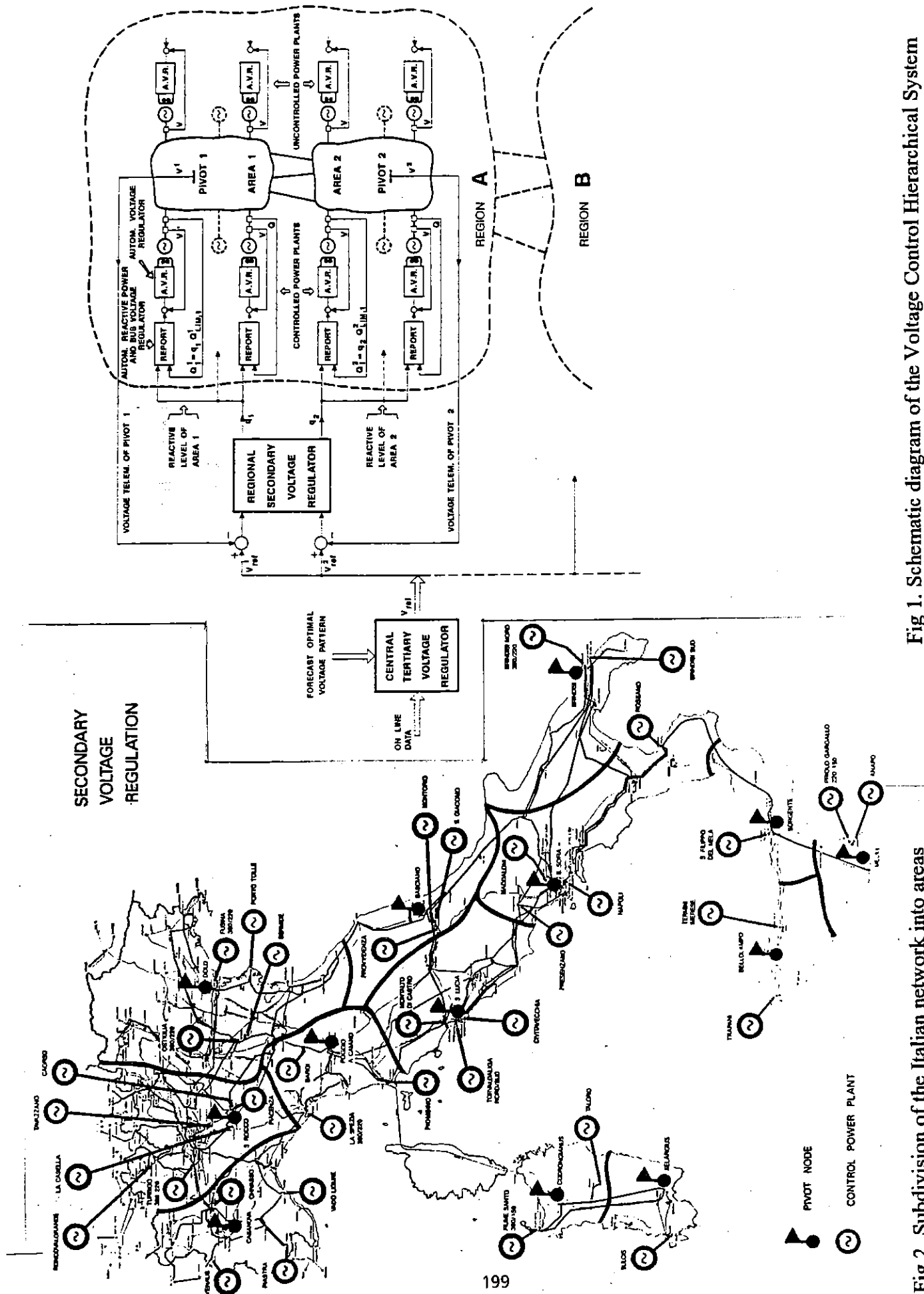


Fig 1. Schematic diagram of the Voltage Control Hierarchical System

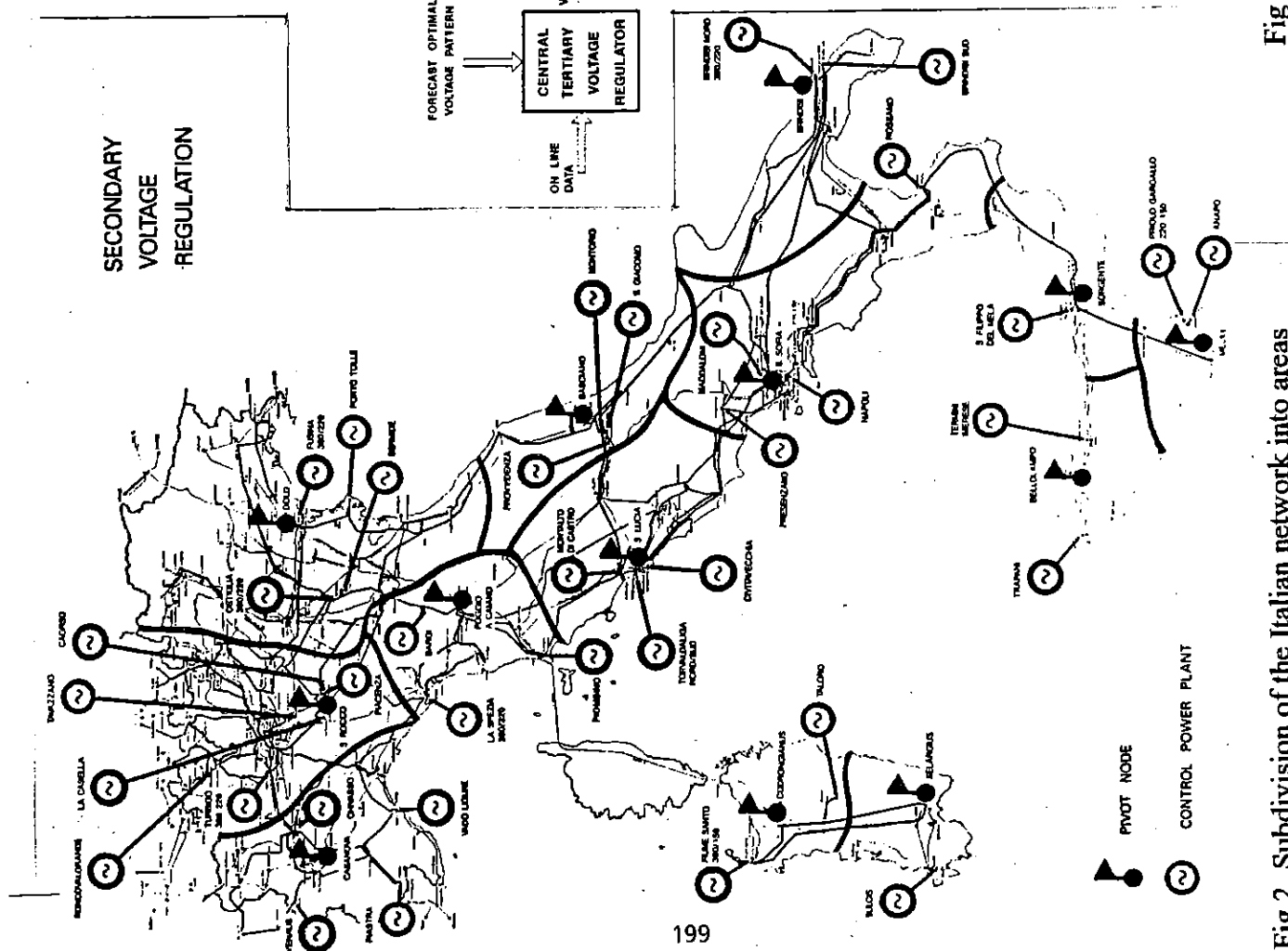


Fig 2. Subdivision of the Italian network into areas

REPORT. This is an innovative and advanced apparatus as regards both its sophisticated functionalities, and its technology. REPORT allows two different control modes:

- Reactive power Level Teleregulation, in which the power plant participates to the EHV pilot node *Secondary Voltage Regulation*;
- Local EHV Bus Voltage Regulation, in which the power plant does not directly participate to the *Secondary Voltage Regulation*, but supports the local EHV voltage.

Moreover ENEL is designing and developing the final version of the *Regional Secondary Voltage Regulator* as well as the modification of the existing regulators and control systems to be interfaced with the new equipments.

Tertiary Voltage Regulation (TVR): At the higher hierarchical level a Central Computer coordinates, at national level, the actions of the regional controllers, defining in real-time for a secure and economic operation, the more appropriate voltage set-points of pilot nodes on the basis of the actual state of the network and the forecasted optimal voltages and reactive powers.

More precisely the *Tertiary Voltage Regulator* minimize the following objective function:

$$OF = [V - V_{ref}]^T Q^2 [V - V_{ref}] + [q - q_{ref}]^T R^2 [q - q_{ref}]$$

where:

$[V]$, $[q]$ are the vectors of the real-time measurements of pilot node voltages and the area reactive power levels.

$[V_{ref}]$, $[q_{ref}]$ are the vectors of the optimal forecasted pilot node voltages and the area reactive power levels.

Q^2, R^2 are weight diagonal matrices of the pilot voltages and the area reactive power levels deviations from the optimal-forecasted values.

With this objective function an optimal compromise is reached between the need to have a desired voltage pattern and the need to maximize the reactive power margins in respect to the control variable bounds.

3. A REAL-TIME INDEX

3.1 Generalities

From the above mentioned considerations on the voltage stability problem and taking also into account the main results shown by the recent studies on this subject, we can syntetically reach to the following conclusions:

About Voltage Instability Phenomenon

- The power system voltage degradation begins when, growing the load, the units of a network area reach their own overexcitation limits [1, 12];

- A timely OLTCs shut-down is an unanimously acknowledged very effective control action for stopping the voltage degradation;
- An automatic voltage regulation of the transmission network allows benefits on voltage stability [7, 12, 13].

About Models for Index Computing

- The reliable reconstruction of voltage instability phenomenon requires to consider the power system dynamics (that is the effects of the units and network automatic control loops) and the power system non-linearities (such as the dependence of the loads by the voltage, the limiting control loop effects and the protection operation, the saturation of the regulating control loops, etc).

About Network Simulation and Time Computing Required by Voltage Stability Indices:

- Real-time precise dynamic simulation for the network management cannot be performed in practice, due to the lack of network real-time parameters and also because they are needed a large amount of computing resources and high computing time.
- Simplified simulations are possible. They are mainly devoted to the computing of voltage collapse proximity indicators to be used for off-line planning studies. In a few cases these simulations allow the computing of indicators tied to very short-term operation [7, 8].

About Real-Time Informations Given by the Secondary Voltage Regulation

- The basic principle of *Secondary Voltage Regulation* is the transmission network subdivision into areas. A single area consists of an amount of busses having each other an high electrical coupling, therefore their voltages change in unison in front of local loads variation or network perturbations, according to the trend of the area pilot-node voltage [11],
- An automatic control system of network voltages needs to operate, of a very fast updating interval (a few seconds) of the main measurements coming from the power plants and of the control signals sent to the plant regulators,
- Referring to the *Secondary Voltage Regulation*, the area-i control signal, named reactive power level q_i represents instant by instant the control effort under-way at area-i and therefore it is the real-time reactive power load for the area-i control units.

More precisely q_i value stands for the percentage of the area-i units reactive power with respect to these units overexcitation limits. When q_i reaches the 100% value then the area-i voltage regulation begins to be saturated because the operating points of all

the area-i control units are fixed by the units overexcitation limits.

All the points above mentioned define a syntetic precise description of the relevant aspects linked with voltage instability phenomenon and with the mathematical models and computing time required by off-line voltage stability indicators.

Furthermore it is pointed out the availability given by the *Secondary Voltage Regulation* of real-time informations on the power-system operating state and on the regulation itself. These informations are strongly representative of the reactive power reserves at disposal for each network area, to be used for real-time control of the corresponding pilot nodes.

From these last observations it is possible to reach to the following conclusions:

- With the *Secondary Voltage Regulation*, the pilot node voltage of a given network area is regulated, growing the load, to the desired value unless all the area control units reach their overexcitation limits;
- For a given area, the approaching of this extreme operating condition determines the achievement of the area voltage instability limit.

According to these conclusions and also considering that the voltage degradation usually takes a lot of minutes for developing from the initial voltage instability up to reaching the irreversible voltage collapse condition, it seems reasonable, simple and very effective to compute, inside the *Secondary Voltage Regulation*, a real-time indicator of the area proximity to voltage instability. Obviously such an index will be mainly based on the actual value of the area reactive power level: $q(t)$.

3.2 The proposal of a deterministic index

With reference to the instantaneous value $q_j(t)$ of the Area-j reactive power level, the proposed proximity indicator of that area to the voltage stability limit is given by:

$$VSI_j(t) = q_j(t) + \rho \frac{\partial q_j(t)}{\partial t} \quad (1)$$

where:

ρ is a weight coefficient

while $-1 \leq q_j(t) \leq +1$

In case some of the units of Area-j can be transiently overloaded with respect to the overexcitation limits, then:

$$-1 \leq q_j(t) \leq 1 + \mathcal{E}q_j(t)$$

$\mathcal{E}q_j(t)$ is a parameter usually kept at 0 value which, under RVR consent, can transiently take positive growing value according to the pilot node voltage regulation effort. It is timely brought back to 0 by RVR according to a control logic which takes into account either the setting of the transient overloading of Area-j units overexcitation limits, or the number of the overexcitation transient overloads already made by those units.

In case, for supporting the pilot node voltage to the optimal value defined in real-time by the *Tertiary Voltage Regulation*, the level $q_j(t)$ reaches +1 value, then all the Area-j control units begin to operate on their overexcitation limits. In this condition, if the transient overloading is allowed, then the overexcitation limits could be transiently exceeded.

Usually RVR operates for avoiding the units operation at their overexcitation limits controlling in advance all the possible reactive power, resources (capacitors banks, synchronous compensators, etc.) installed in

the Area-j, in such a way to reduce the $q_j(t)$ operating value. RVR acts automatically on the reactive power resources under its control and moreover sends signals to the regional operator asking the manual switching of the remaining resources.

When all these control actions concerning Area-j voltage are accomplished and the area control units don't allow additional excitation overloadings, then the

difference $q_j(t) - 1$ clearly represents the distance of Area-j from its voltage stability limit.

The proposed index includes the derivative term

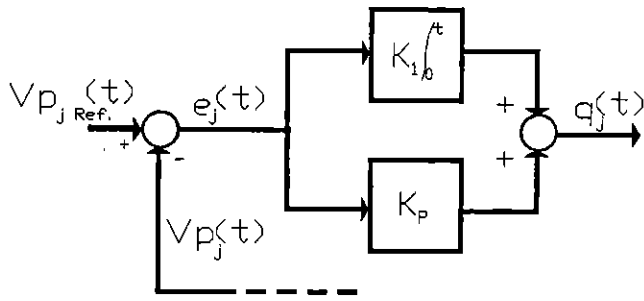
$\rho \frac{\partial q_j(t)}{\partial t}$ which takes into account in real-time the

$q_j(t)$ dynamics and therefore its expected short-term trend.

This "before time" contribution is particularly useful in relation with the possible real-time control action

based on $VSI_j(t)$:

- the greater is the positive value of the derivative term the more $q_j(t)$ is speedily reaching the maximum limiting value;
- when the derivative term is negative, its contribution avoids the prolongation of unusefull control action because of the actual network and loads evolutions which will allow the *Secondary Voltage Regulation* to keep distance from the limiting operation.



$V_{Pj}(t)$ = pilot node voltage of area-j

Figure 3: PI Control low of the Area-j pilot-node voltage regulator

The weight ρ allows an effective setting of the derivative term contribution inside $VSI_j(t)$

3.3 Computing Modality and Computing Time

With reference to the equation (1) related to Area-j index $VSI_j(t)$, and taking into account that the control low of pilot-node voltage regulator is of PI type (see Fig. 3), it follows that:

$$q_j(t) = K_{Ij} \int_0^t e_j(t) dt + K_{Pj} \cdot e_j(t)$$

therefore

$$\frac{\partial q_j(t)}{\partial t} = K_{Ij} \cdot e_j(t) + K_{Pj} \cdot \frac{\partial e_j(t)}{\partial t}$$

and

$$VSI_j(t) = \left[K_{Ij} \int_0^t e_j(t) dt + (K_{Pj} + \rho K_{Ij}) e_j(t) \right] + \rho K_{Pj} \frac{\partial e_j(t)}{\partial t}$$

The last equation shows that $VSI_j(t)$ is given by two terms: the one between brackets is suddenly computable by RVR because it is a combination of variables already present on RVR and updated with a time interval of about a few seconds. The derivative

term $\partial e_j(t) / \partial t$ conversely needs of an elaboration which can be contained to a single step (τ) computation or enlarged to a finite number of steps (mobile mean of the $e_j(t)$ single step changes). In the

second case the derivative term $\partial e_j(t) / \partial t$ is filtered as often needed.

In conclusion the $VSI_j(t)$ index computing can be carried out by RVR with an updating time of τ seconds.

3.4 The Control Function

The $VSI_j(t)$ index represents an useful real-time information for the Regional Dispatcher which can be drawn by it, when a given threshold is exceeded, to look over the critical state of the area voltages.

According with this area-allarm the operator could decide to put into operation the remaining reactive power resources of Area-j, carrying out remote controls or asking by phone.

Besides, reaching Area-j in proximity of high risk condition for voltage instability, the operator could shut down manually the area OLTCs. Nevertheless,

because $VSI_j(t)$ is a real-time updated variable (the updating time interval lasts τ seconds) it can be effectively used for real-time automatic control

actions. More precisely exceeding $VSI_j(t)$ a given threshold value which, even if not critical, is representative of a consistent voltage control effort as well as of Area-j small voltage control margin, then the control logic will order the automatic switching of the capacitors banks, reactors and SVCs of the concerned area.

An on-line check of the commutation frequency of those reactive power equipments will restrict automatically the number of their commutations according to the project prescriptions and always considering the network voltage criticism.

After working out all the Area-j reactive power resources and still persisting a network working condition with small reactive power control margins for the *Secondary Voltage Regulation* of Area-j, then

the $VSI_j(t)$ index could reach critical thresholds which allow the RVR to modify automatically the voltage set-points of the local OLTCs for obtaining a reduction of the load seen by the transmission network.

After these preliminary RVR control actions based on $VSI_j(t)$ value, the mentioned control logic could point out either:

- the overcoming of a considered critical threshold by the $VSI_j(t)$ index:

$$VSI_j(t) > VSI_{jMax};$$

- or the reaching and the permanence of all the Area-j control units at the upper terminal voltage limit;

- or the pilot-node voltage reduction under the lowest allowable value.

$$V_{P_j}(t) < V_{P_j \min}$$

Happening at least one of these three extreme working conditions, the control logic will automatically shut-down the Area-j OLTCs.

After this last control action the Area-j secondary voltage regulator RVR will have operated, in a coordinate way, all the possible automatic control actions at his disposal for supporting the pilot node voltage $V_{P_j}(t)$. Besides RVR will have drawn the

operator attention monitoring the $VSI_j(t)$ value and asking for possible manual controls.

When the critical operating conditions will be leaved then $VSI_j(t)$ value will decrease and RVR will be able to release the Area-j OLTCs and to reduce the operating reactive power resources.

4. THE CRUCIAL ROLE PLAYED BY THE TERTIARY VOLTAGE REGULATION

The Secondary Voltage Regulation carries out the optimal voltage plan defined and updated in real-time by the Tertiary Voltage Regulation. In other terms, considering the Area-j, RVR carries out the set-point $V_{P_j \text{ref}}$ trend which is computed and updated with a slow dynamics (a few minutes) by the National/Central Voltage Regulator (NVR) on the basis of the optimal forecasting voltage plan and the network actual operating condition.

Therefore the pilot-node voltage set point $V_{P_j \text{ref}}$ is continuously updated to values which allow the Area-j *Secondary Voltage Regulation* to maintain, instant by

instant, an adequate control margin $(1 - |q_j(t)|)$ for

the $V_{P_j}(t)$ voltage regulation.

The reactive power control margin will be strongly reduced by very critical network situations, when the network voltage degradation becomes considerable. In other works NVR attends the transmission network voltage degradation progressively renouncing to the optimal short-term planned voltage profile (defined according to economic reasons) and therefore reducing

the $V_{P_j \text{ref}}$ value to retain the Area-j controllability and security. Consequently, with the exception of fast transients, the *Secondary Voltage Regulation* operation on the limits can happen only when the transmission network voltages are very low notwithstanding all the network reactive power resources are in operation for voltage support.

From all these considerations it comes out that, the $VSI_j(t)$ index is really representative of Area-j

proximity to the voltage stability limit only when operating the *Tertiary Voltage Regulation*.

Conversely, $VSI_j(t)$ assumes a different meaning of that till now attributed to it when the set-point $V_{P_j \text{ref}}(t)$ can take values either setted by the operator through a calibrator, or memorized into RVR according to optimal forecasting studies. In both the cases the *Secondary Voltage Regulation* of Area-j can reach the extreme operating condition with the units

on their limits $(|q_j(t)| = 1)$ only because the set-point

value of $V_{P_j \text{ref}}(t)$ is inadequate for the present network situation.

In consequence of this consideration the index $VSI_j(t)$ when the *Tertiary Voltage Regulation* is out of operation, is not representative of the Area-j distance from the voltage stability limit but only means the distance of the Area-j *Secondary Voltage Regulation* from its operative limits. This distance given by the $VSI_j(t)$ index is shown in real-time to the Regional Dispatcher operator who will evaluate, according to the index value the need of $V_{P_j \text{ref}}(t)$ set-point changes for putting the Area-j *Secondary Voltage Regulation* away from its operative limits.

5. CONCLUSIONS

The paper presents a real-time dynamic indicator of the power system proximity to voltage stability limit.

The proposed index is strongly tied to the existence of a Secondary and Tertiary Voltage Regulation system. *Secondary Voltage Regulation* is based on the transmission network subdivision into areas, each area being substantially independent from the others as concern voltage and reactive power control.

The availability given by SVR of real-time measurements of pilot node voltages and area-reactive power levels allows the on-line computing of area-voltage stability indices based on the actual measurements of the area-reactive power levels and their derivatives.

Considering the time interval required by the telecommunications and the computing time needed by RVR, the updating time interval of the area-indices is of a few seconds.

The area index represents instant by instant the actual proximity of a network area to its voltage stability limit, provided that the *Tertiary Voltage Regulation* is operating. Otherwise the proposed real-time index is only representative of the area SVR proximity to the limiting operating conditions.

This real-time index is under study at Enel and will be tested on the new, microprocessor based, RVR prototype under development at Enel-Centro Ricerca di Automatica. The new RVR will be put in service at

the Milan and Naples Regional Dispatchers at the beginning of 1996.

It could be useful to point out that the proposed index is computed on the base of the real-time trend of the real power system variables and of the real network voltage automatic regulation variables and not on the base of a power system mathematical model.

Furthermore the considered variables are very few and very important for the SVR, therefore the index computing is very reliable and accurate.

Lastly it is to be pointed out that the real-time voltage stability index here proposed, based on real-time measurement, represents a novelty with respect to the other proposals given by the literature which are mainly devoted to off_line planning and short-term operation and therefore to different time frames.

In conclusion, considering a hierarchy in time from the present instant to the next future, the various indices proposed by the literature are not each-other exclusive but they cohabit facing different needs from those tied to long-term forecasting, to those for short-term operation, till up to those, here particularly stressed for real-time control and network security. In fact the proposed index is a tool for real-time operation, which gives to the operator real-time informations on the network voltage criticism, and moreover can be powerfully employed for timely automatic control actions facing unexpected contingencies.

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Discussion

Edited by Nicholas W. Miller, General Electric Company

Nelson Martins (CEPEL):

Secondary voltage control is analogous to automatic generation control, which has tie line bias, area control error, and in which we have various automatic control loops, generator on voltage control, generator speed control, power station control, AGC control. They work well together, because they operate and are active in different time frames: AVR is fast, speed governor control is slow, one decade apart; AGC is very slow, another decade apart. Now, secondary voltage control and tertiary voltage control probably will be acting in the same time frame as AGC, and so there may be oscillations, interactions. These are problems that we must be alert to.

Sandro Corsi (ENEL):

The system is structured in such a way that all of the different dynamics are well defined. So, for a simple example, a short circuit on a unit is handled in the network without any problem in the secondary voltage regulation loop, because there is a decoupling in time between these two different loops. And this decoupling time goes from some hundreds of a milliseconds for the AVR, for example, going to the unit reactive power control loop. I refer to the previous question. We realize this kind of reactive power control loop, just to avoid the exchange of the reactive power between the units, because it is well known that when you set the set point of the AVR, in that case you have exchange of reactive power between the units. And the reason why I asked before, why EdF renounced this kind of control? For us it was very important. After that, we have the second consideration, which is time decoupled. The constant time with this is about one minute, even if the control loop is of a proportional-integral type, in such a way that the phase of a large variation, the secondary voltage regulation is also able to phase fast after the first step of the transient. It will give an important contribution to the system.

Hans Glavitch (ETH):

I was wondering about the determination of your reference voltage profile. This also refers to the presentation of Mr. Pruvot. Is it economic? Is it security constrained? Is it contingency constrained? We have to have some objective in the beginning in order to adjust an optimum, or is it a mixture?

Corsi: We consider both economy and security, because we consider these needs at a different level. The tertiary voltage regulation is mainly devoted to security, but the tertiary voltage regulation is simple. The tertiary voltage regulation acts as an input to the real time measurement of the system at one end, and in second end, the ultimate for the forecast voltage plan. The optimal forecast voltage plan is computed considering economic requirements. So, the tertiary voltage regulation tries to reach a compromise between the economics and the needs of the system in real time, taking into account the real situation in your method.

Haria Crisciu (Tractabel):

Secondary voltage control, based on splitting the network into a number of interdependent areas, and also on the control of the generator voltage, real power, and the reactive power (Q) regulator, has at least two major inconveniences in my opinion.

The first one - if a disturbance occurs in the system, the secondary voltage control will be active because of the different time frame. The generators are under the control of the Q regulator - that is to say, a Q set point and not a V set point. This is not a good thing because it favors a voltage

collapse. The behavior is contrary to what is optimally necessary to do.

The second inconvenience, in my opinion, is that in the modern power system, because the modern power systems are more and more meshed, it is more and more difficult to find a number of independent areas. If the areas are not independent, the secondary voltage control may not converge; that is to say, it is not able to realize the voltage set point on the pilot mode. And my question is: how, in the Italian system, is it possible to resolve this inconvenience, because in Belgian system we had very great difficulties avoiding these two inconveniences?

Corsi: You said that the reactive power control loop is a problem as concerns voltage instability. Well, you have to take into account that the problem in the power system is a coordination of all the reactive power that you have at your disposal, and the way to coordinate is to move all these other sources. The reactive power control loop does not pose any problem, because your problem begins only when all the units in your area have reached their extreme working condition. So, your problem is, first, to move all your reactive power sources to support the voltage in your system and, second, when you have reached that point, your problem is to check if the voltage that you are trying to sustain is good for that moment. Now this is the role of the tertiary voltage regulation. So, in other words, the coordination between the secondary and tertiary voltage regulation is a good approach. I don't understand the problem of the reactive power control loop. There must be some kind of a misunderstanding, or some difference in your realization or studies. In our realization, we don't have any problem at all concerning reactive power.

About the areas aspect, you have to take into account that the main topic of the secondary voltage regulation is the subdivision of the network into areas. And the more they are decoupled, the better, you don't have a problem of coupling between these two areas. But, you have also to take into account that you can improve your control law. In some cases in Italy, we have some areas that are very decoupled, but others that are coupled. In that case you can use a more advanced control law that tries to decouple the regulation of the two areas in such a way to avoid this kind of coupling.

Discussion submitted subsequently by Julián Barquín

I have found this paper to be very interesting and insightful. I would like to ask the following questions about it:

- A I would like to know whether ENEL carries out some coordination between the eight regional secondary voltage controllers. We noticed that there exist strong couplings in the matrix which relates changes in reactive power injections with voltage changes at pilot buses, so it would seem to be very difficult to establish a decentralized secondary voltage control strategy when reactive power injections are selected as control signals rather than set-point voltages of AVRs.
- B Apart from the above remark, I would like to know how ENEL takes into account the electrical couplings between intermediate decentralized reactive loop controllers. For instance, the cited electrical couplings between different control generators belonging to the same power plant may become important.
- C I don't understand very well how tertiary control calculates the pilot voltage and reactive reference values that it sends to the secondary voltage controllers. How does tertiary control achieve the trade-off between keeping an appropriate voltage profile and keeping adequate reactive power reserves?

Closure by Sandro Corsi

In response to question A:

At ENEL the tertiary voltage regulation coordinates the regional secondary voltage controllers through optimizing in real time the pilot node voltage plan.

Regarding the electrical coupling among the pilot nodes, it is normally very small because they are chosen according to a criterion which guarantees a given electrical distance between them. So, in principle, each pilot node can be controlled in a decentralized way without affecting strongly the voltage of the neighboring pilot nodes. In other words, referring to the Spanish transmission network, the voltage regulation at Barcellona has a very small interaction with the voltage control at Madrid or Bilbao or Sivilla or in Galicia. Nevertheless it could happen that for particular network conditions, or political or administrative reasons, two pilot nodes are chosen with electrical coupling not insignificant.

The last part of the question is not clear, because the decentralization of the secondary voltage control is not a matter tied to the choice of reactive power injections as control signals rather than the set-points of the AVRs. The reasons that suggest regulating the pilot node voltages through control of unit reactive powers instead of AVR set points are tied to some basic criteria of the "automatic controls," and mainly to considerations of the practical effectiveness and quality of the control solutions.

In response to question B:

The electrical coupling among the units in a power plant is a very important subject to be considered when designing a real-time voltage control system. One of the main tasks of the "Power Plant Reactive Power and Voltage Regulator" (REPORT), designed and developed by ENEL, is to avoid dynamic interactions in a power plant among the unit reactive power control loops. In other words, each unit reactive power control loop operates as if it were alone in the plant.

In response to question C:

The tertiary voltage regulator (TVR) optimizes the pilot node voltage profile on the basis of the optimal forecast plan and real-time measurements of the pilot node voltages and area reactive power levels. The secondary voltage regulators (regional controllers) in their turn compute in real-time, on the basis of the pilot node voltage set points received from the TVR, the area reactive power levels to be sent to the power plants for regulating the pilot node voltages. The weighting matrices Q and R of the TVR objective function OF (chapter 2 of the paper) allow reaching a compromise between the economic optimization of the pilot node voltages and the need to have reactive reserves for network control.

Voltage Stability Impacts on Hydro-Québec Operations and Simulation Tools Development

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Abstract

The gradual increase of line-loading in the past 10 years has significantly affected the voltage control on the Hydro-Québec system. In 1988, an overvoltage incident confirmed the necessity to study the long term voltage behaviour following contingencies. Consequently, appropriate tools and methodologies were developed and used to revise operating strategies and implement automatic control devices.

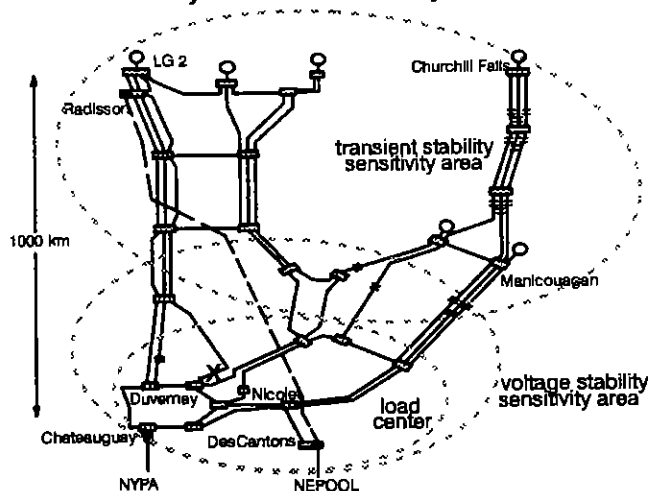
This paper summarizes the Hydro-Québec experiences in this field. The description of actual practices and expected developments is included. Although the HQ system presents some particularities, we believe that this overview of its voltage control problematics may be of interest to other utilities.

1.1 System evolution

Maximum power transfer limits on the Hydro-Québec system have traditionally been determined by transient stability. With the gradual improvement in system robustness, the maximum power transfer has gradually been increased, particularly during the last 10 years. Consequently, the characteristic of heavily loaded systems has become increasingly capacitive over time and voltage stability has become a concern. For this reason, the power transfers are now limited by transient (angular) stability in the northern regions of the system and by voltage stability in the southern regions (figure 1).

The Hydro-Québec system is characterized by long distances between the principal generation centers and load areas. Its long EHV lines present high series reactance and shunt capacitance values. Due to the distant locations of the principal power plants, the voltage control near the load areas is mainly insured by static var compensators and synchronous condensers (referred to henceforth as compensators).

Figure 1
Hydro-Québec 1994 system

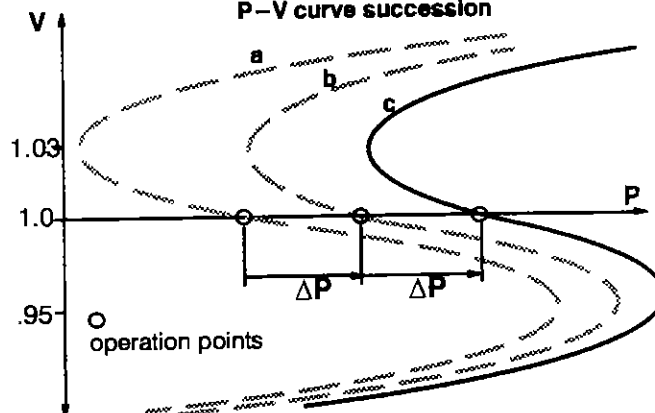


1.2 Voltage control on the HQ system

With the exception of heavy load periods (in winter), the excess reactive power generated by EHV lines is mainly compensated by switchable 330 Mvar shunt reactors located at the 735 kV substations. When the system is heavily loaded, the series reactive losses increase and the shunt reactors are gradually disconnected. Consequently, operation of the EHV system

consists of a succession of reactive corrections to maintain the voltage profile. Therefore, the voltage level doesn't follow the P-V characteristic of a fixed configuration as the power transfer and load are increased but it remains quite constant as illustrated by the P-V curves of those successive configurations (figure 2).

Figure 2
P-V curve succession

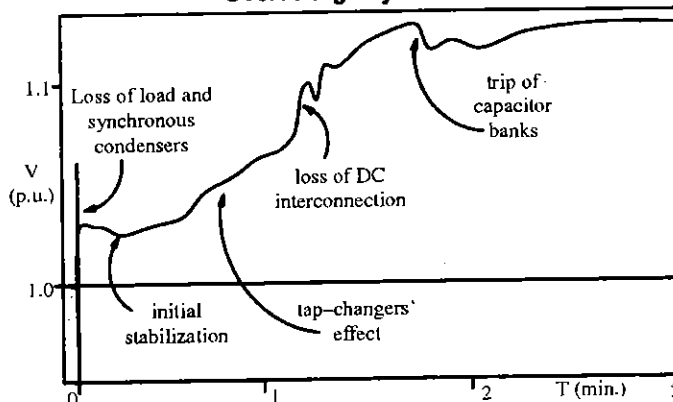


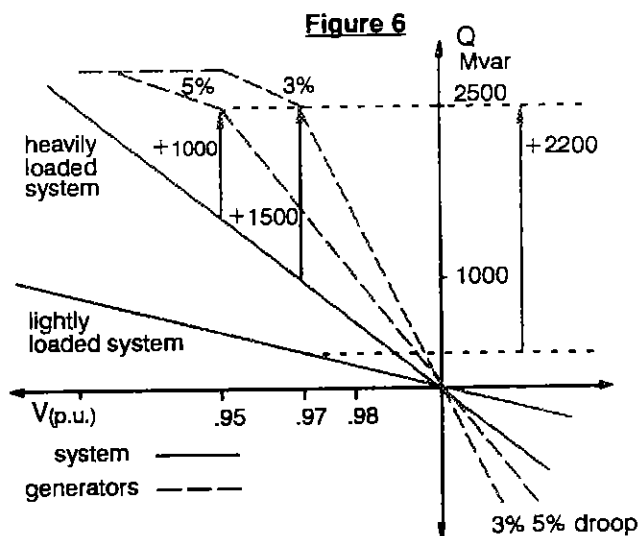
Each of these curves indicates that a fixed topology cannot support more or less than certain power transfer levels without causing voltage destabilization. Furthermore, as can be seen in figure 2, the stability margin on each side of the operating point is reduced as power transfer and capacitive characteristic are increased (from curve a to c). Since the reactive generation capacity of compensators is greater than their absorption capacities, we can also observe that the stability margin on each side of the normal operation point is not symmetric. Consequently, the system can more easily withstand a rapid load increase (or a loss of generation near the load centers) than a load trip. This fact, characteristic of the Hydro-Québec system configuration, will be illustrated in the following section.

1.3 Overvoltage system event

In 1988, an event occurred which confirmed the overvoltage sensitivity of heavily loaded systems. Essentially, a fault occurred on the Montréal 315 kV system causing the loss of approximately 350 MW of load and the tripping of 2 nearby compensators. The voltage behaviour is illustrated in figure 3.

Figure 3
Overvoltage system event

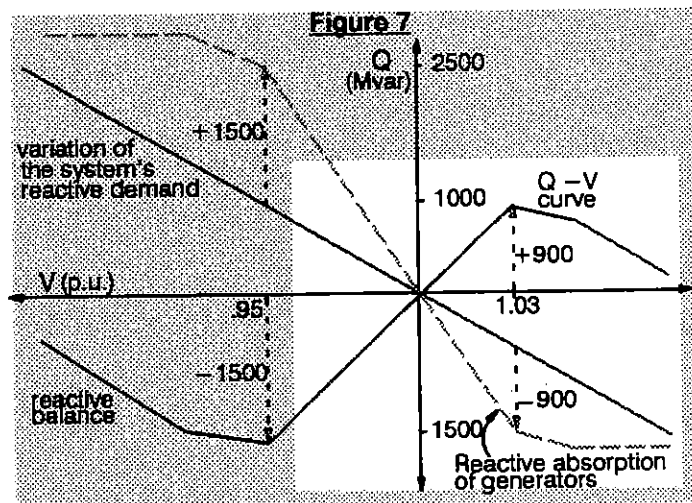




This kind of "supply and demand" curve is useful in illustrating the impact of the voltage regulation settings and of the system characteristics. The reactive balance and the total reactive contribution of generators for a voltage deviation are easy to obtain. The reactive losses on the system are simply the difference. The example, in figure 6, illustrates two different system conditions: heavy and light load periods, and two different settings for the voltage regulation droop of generators (as seen at a given bus). The individual and cumulative effects of these parameters on the Q-V curves appear very clearly.

First, we can observe their effects on the maximum reactive power margin. With the heavy load condition and the 5% droop this margin is 1000 Mvar. The improvement of these generators' regulation, without changing their ceilings, results in a 50% larger security margin (1500 Mvar). Consequently, it's possible to conclude that a tighter control of the voltage increases the transmission capacity. However, the critical voltage level is also increased (from .95 to .97 in this example). Finally, the light load system curve simply indicates that the security is significantly improved out of the peak load periods.

Finally, if we now complete figure 5 to illustrate both sides of the Q-V curve, we obtain in figure 7 the confirmation of the observation made on the S shaped P-V curves illustrated in figure 2. The lesser capacity of generators (mostly compensators) in controlling overvoltages results in a non-symmetric security margin on both sides of the operation point. In this example, the reactive absorption capacity is approximately 1600 Mvar and the critical voltage is 1.03 p.u. (refer to figure 2).



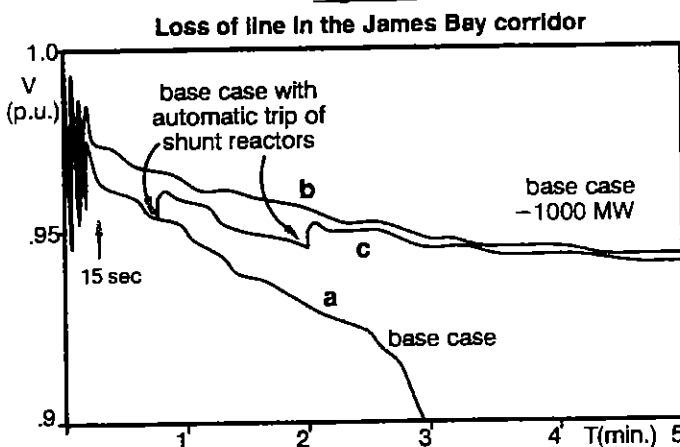
3. LT STABILITY IMPACTS ON POWER TRANSFER LIMITS

3.1 Voltage collapse prevention

Since 1989, exhaustive use of the long term stability program has greatly improved system security. For example, the improvement in overvoltage protection insures a more reliable operation of voltage sensitive interconnections. However, the most important aspect is the revision of the power transfer limitations to prevent voltage collapse.

Figure 8 illustrates the voltage behaviour at the Duvernay substation following the simulated tripping of the LaVérendrye-Duvernay line (indicated by an X in figure 1). As can be seen (although the time scale was chosen to illustrate LT behaviour), the transient stability response is excellent and the voltage after 15 seconds is close to 0.97 p.u.. At this moment, the tap changers begin to gradually restore the secondary voltage, and consequently the load, causing a constant reduction of the voltage on the high voltage system. Two minutes after the line trip, voltage collapse is unavoidable (curve a).

Figure 8

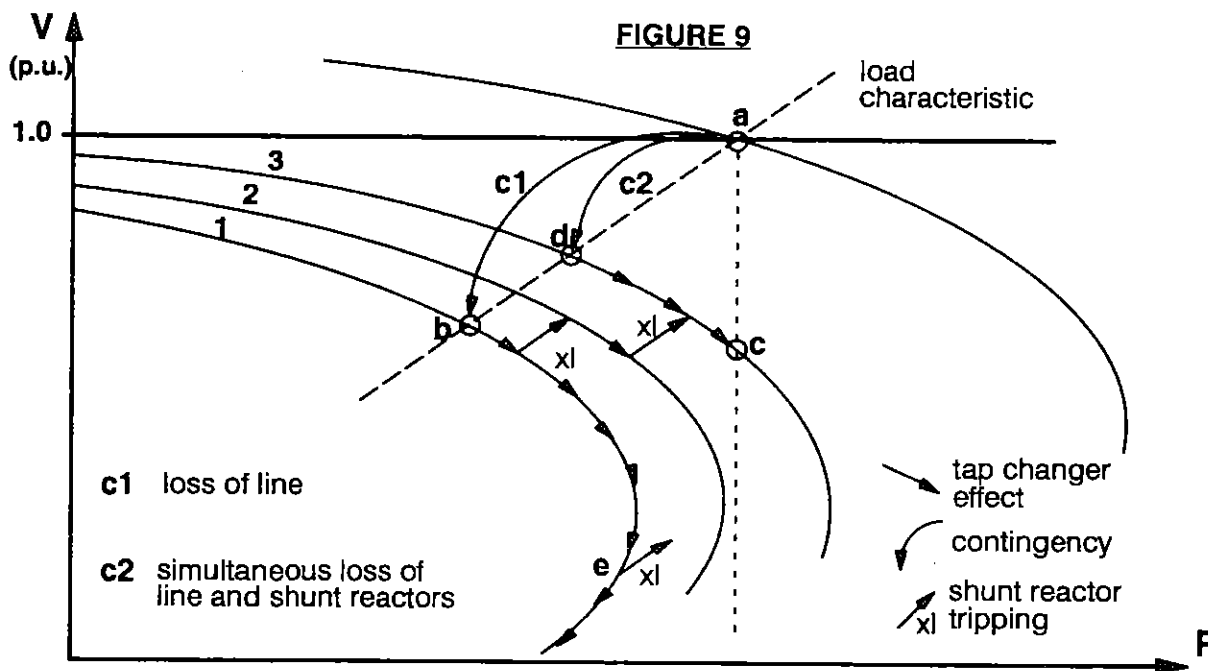


3.2 Strategies to Restore Power Transfer Limits

A 10 % reduction (1000 MW) of the generation carried on this corridor results in the stable case illustrated by curve b. However, to avoid this limitation and securely transmit all the available generation, corrective measures had to be found. After further analysis, an appropriate solution, inexpensive and rapidly feasible, consisted in associating the tripping of shunt reactors in the load areas to the reactive power output of nearby compensators. Because of the relatively small variation in system voltage, this variable was found to be more reliable than a voltage measurement.

When a preset level of reactive power generation is reached and maintained for a given period, a shunt reactor is automatically tripped. Therefore, the compensator returns immediately to its normal operation range and continues to support voltage. The compensators' reactive power threshold is typically 200 Mvar (approximately two thirds of the maximum output) with a 10-20 second delay before reactor tripping.

We proceeded to implement this solution and, by the end of 1991, twelve 330 Mvar shunt reactor switching systems were available. Each of the shunt reactors is associated to a specific compensator and, consequently, all the switching systems are locally controlled and independent from one another. Of course, these systems permit automatic reclosing of shunt reactors under certain overvoltage conditions. It should be noted that the automatic switching of equivalent capacitor banks would have the same effect on voltage stabilization.



The system behaviour following the previous incident (figure 8), but with the operation of 2 of the shunt reactor tripping systems, is shown by curve **c**. This time, the fully-loaded system stabilizes with a more comfortable reactive power reserve still available. Once again, it may be useful to illustrate the voltage behaviour using a P-V curve representation.

Figure 9 indicates the evolution of the voltage following the P-V characteristic of each successive configuration. Point **a** indicates the pre-contingency operation point from which the contingency **c1**, the loss of line, is applied. After the contingency, and before the tap changers' operation, the voltage decreases and so does the power transfer level **P** (point **b**) in relation to the load characteristic (typically $P \approx V^{1.0}$). After approximately 20 seconds, the first tap change occurs, the load begins to increase and the voltage (on the HV side of the transformer) to decrease. This process continues until the lower voltage level reaches 1.0 p.u. and the load is restored to its pre-contingency level. In this first case, this power level is higher than the maximum that can be obtained on the post-contingency curve 1 and the voltage collapses.

However, if we allow shunt reactor devices to operate before critical voltage level is reached, the system characteristic changes (curves 2 and 3) and this time the voltage can stabilize when the precontingency load level is reached (point **c**). This result is essentially equivalent to the simulation of a contingency involving the loss of the same line and, simultaneously, the two shunt reactors. This pattern is illustrated by the displacement from point **a** to point **d** and finally to point **c**.

It should be noticed that, to insure shunt reactor automatic trip efficiency, the operation setting must allow operation before the critical voltage level is reached (the nose of the P-V curve). For example, a shunt reactor trip at point **e** is almost totally inefficient.

4. METHODOLOGIES

4.1 Automatic power transfer limit assessment

The Hydro-Québec operations planning group determines the optimal strategies for a large number of probable system configurations given security criteria. This requires extensive use of simulation tools and computer facilities. To meet our objective,

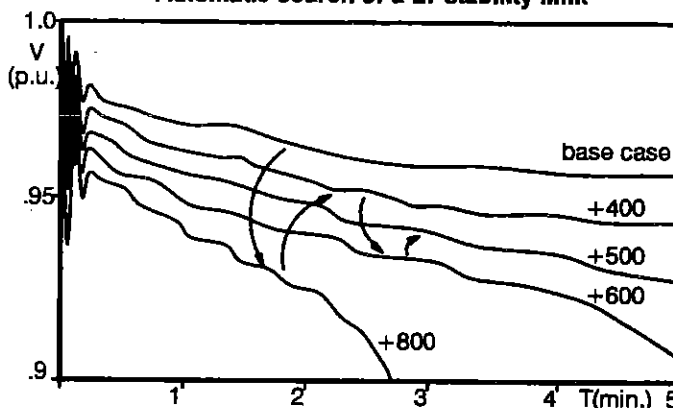
we have developed a program to automate time-consuming routines and optimize the use of the workstation system [6].

Since 1990, we have been using this program, called ELISA, to automatically assess the maximum power transfer of given topologies. An example of the results obtained, for an incident on the James Bay corridor, is shown in figure 10. Five LT stability tests were automatically produced: the first, the base case, is stable; the second, the base case +800 MW, is unstable; the binary search continues until a power transfer limit is reached with a precision of 100 MW.

Each long term stability simulation (5 minute time frame) takes 4 hours of simulation on a SUN Sparc 10. As a result, an automatic search of a power transfer limit, using the LT version of the stability program, takes approximately 20 hours of CPU time.

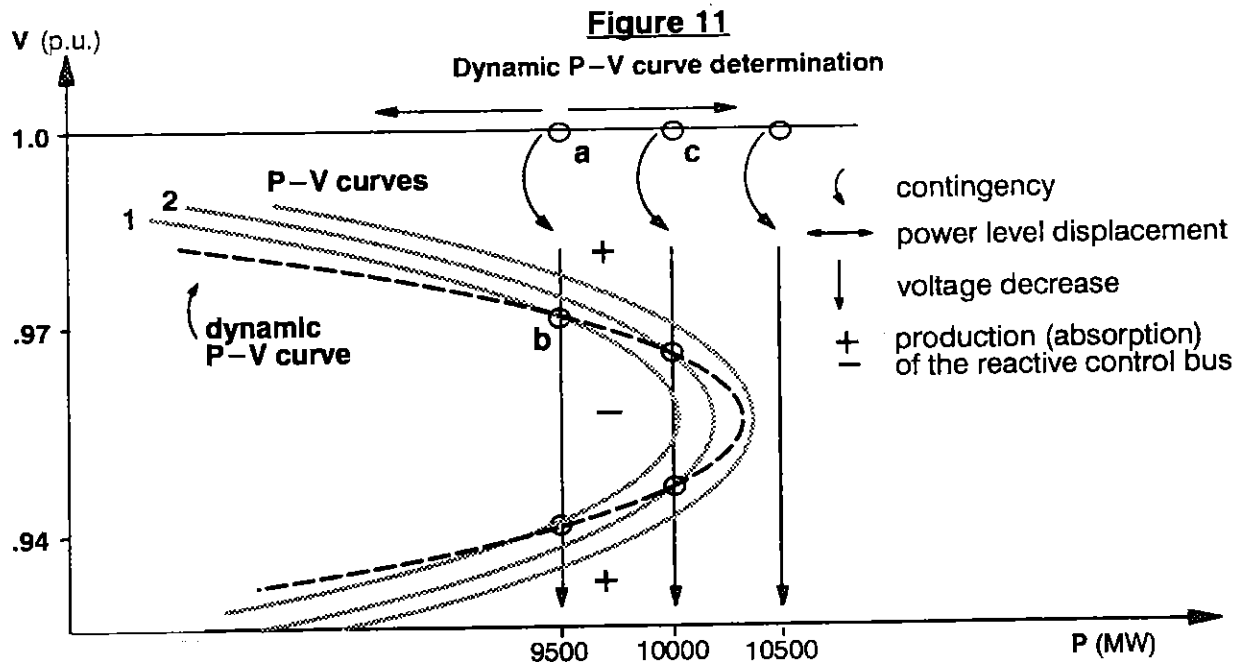
FIGURE 10

Automatic search of a LT stability limit



4.2 The PCLF methodology

To accelerate the power transfer limit assessment based on LT voltage support, we also developed and automated a post-contingency load-flow (PCLF) methodology. This approach, using the Hydro-Québec RP600 Load Flow



program, was completed, tested and compared with the LT stability program [5].

An example of this approach is shown in figure 11. Starting from a pre-contingency configuration at nominal voltage and a power transfer level of 9500 MW (point a), a contingency is applied. Our objective is to find an operation point (if any exist) on the post-contingency configuration. Successive load flows are run at different voltage levels, from .98 p.u. to .93 p.u., imposed on a chosen bus. This automatic determination of the Q-V diagram (not illustrated), where Q is the reactive power generated at a given control bus and V the voltage at this bus, is readily performed by the PC load flow procedure. The voltage control is assumed by a fictitious compensator located at a chosen bus. The voltage corresponding to zero production from this generator is the operation point on the P-V curve (point b). This simple technique insures load flow convergence near, or above, the maximum acceptable P.

If the first case is stable, we must return to the pre-contingency topology, increase the power transfer level (from a to c in figure 11) and make appropriate corrections to maintain voltage profile. By doing so, and applying the same contingency, we switch from post-contingency P-V curve 1 to P-V curve 2. That's why we call the resulting P-V curve, determined by the succession of operation points on different PC topologies, the dynamic post-contingency P-V curve.

In this process, load representation is constant MVA, the tap-changers regulate the low-voltage systems and the steady-state reactive power limits of the generators are used. One important point: voltage regulation droop must be adjusted to realistic values. This means that the voltage regulator settings are taken into account. Another important point: the modifications needed to maintain voltage profile after the redispatching of power on the system must not affect the initial values (reactive power reserve and voltage).

The combination of this methodology and the binary search used by ELISA gives the results illustrated in figure 12. This time, the complete search of a power transfer limit, with a precision of 100 MW, takes only 5 minutes of CPU time on a SPARC 10 (approximately 40 PC load flows were performed).

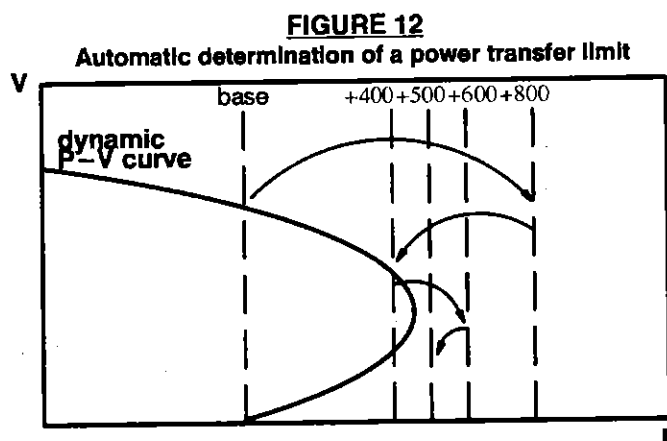
4.3 LT Stability vs PC Load Flow

The main advantage of the power transfer limit assessment using the PC load flow methodology is evident. This approach requires only 5 minutes (without particular efforts on optimization) compared to 20 hours with LT stability. In simple cases, the comparisons revealed that the PC load flow results are accurate but approximately, and consistently, 200 MW lower (2% of a corridor's transmission capacity) than those using LT stability, due to the constant load model. In comparison, the LT stability's final secondary voltage is slightly lower than 1.0 p.u. due to the tap changers' dead band of 1.25 %.

However, the direct PCLF approach described in section 4.2 may be less accurate when delayed automatic devices are used. To insure accuracy of results we must use this technique with caution and perform validation tests using LT stability simulations as a reference. In some cases, such as determining automatic device settings, LT stability simulation is the appropriate method.

4.4 Expected developments

Between these two extremes, LT stability simulation including complete modelization of equipment and direct PCLF methodology, other techniques combining speed and accuracy may be implemented. We have demonstrated, in figures 4 and 9, that a static PCLF representation can adequately reproduce the step by step voltage dynamics following contingencies [2].



Consequently, appropriate tools must be able to simulate this step by step quasi-dynamic evolution in time. This can be done using a mechanized succession of power flows representing the intermediate states following each switching event until final stabilization. The representation of: load characteristic, tap changer effect, automatic switching devices and even governors is necessary to represent this quasi-dynamic voltage behaviour.

To accelerate the establishment of accurate power transfer limits in operation planning or, to confirm that on-line assessment of voltage stability limits is a realistic objective, improvements on current PCLF methodology have been made a priority. Because interesting approaches have already been developed [1][3][4], collaboration with other utilities or Universities to test and validate have already been concluded. Our experience is that such collaborations are often beneficial to both partners.

5. CONCLUSION

The constant yearly increase in line-loading has resulted in greater voltage sensitivity of the Hydro-Québec system. In addition to transient stability studies it is now necessary to ensure LT voltage security. Since LT stability is a relatively slow phenomenon, implementation of reliable and inexpensive delayed shunt reactor tripping systems has resulted in a significant increase of the available reactive power reserve on the system. Additional measures such as dynamic blocking of the tap changers and undervoltage load shedding to ensure stability under extreme contingencies are still under study.

New study methodologies were developed to assist the Hydro-Québec operations planning group in the process of security limits assessment. These methodologies include a fast and automatic PCLF search process to determine the P-V curves. Occasionally, LT stability is used to determine the accuracy of the load-flow parameters and results and to ensure correct operation of the delayed automatic devices.

These simulation tools are integrated in a single environment in which time-consuming routines are automated. We believe that this approach, by reducing the time required to establish strategies, will eventually contribute to an on-line optimization of the power transfer limits. We believe that PCLF methodologies have the potential to allow the required fast and accurate evaluation of voltage stability margins.

6. REFERENCES

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Discussion

Edited by Nicholas W. Miller, General Electric Company

Claudio Cañizares (University of Waterloo):

I am curious about how you obtain the curves in Figures 2 and 4 of the paper; will you explain to me how you obtained those curves.

Richard Maillhot (Hydro Quebec):

With a load flow.

Cañizares: And P in this case means what?

Maillhot: P is the power in the system. It's the load. We can reproduce this behavior with the long term stability program, too. We had an event in 1988 that had this behavior, and it is possible to simulate that event with the long term stability program and with continuation flow methodology.

Mansour: This is basically constant MVA load, right? That is, based on studies with constant MVA load?

Maillhot: Yes, there is a little variation with constant load.

Mansour: In reality, this is one of the things that you really can make use of: the real characteristic of loads and how they change with change of voltage. We ran some tests as recently as two weeks ago. (We are planning to do this every three months, in various parts of the system, where we can characterize the load.) We managed to drop the voltage by seven and one-half percent, and to keep it there for fifteen minutes, to see how loads react. The reactive power dropped by the power 3, and stayed there for the whole time. Active power dropped initially, by something that's a bit better than constant current, and it recovered slowly to something still less than constant power. That's for a residential load. In an industrial load, as you would expect, there are lots of induction motors on; it does not drop as much as the residential loads.

In another test, the voltage was dropped six and one-half percent. Again when you look at the residential load - this is really interesting - it again dropped by better than V to the power three, somewhere between V to the power three and power four. Megawatts, again, dropped in the beginning - the transient characteristic - then it recovered, but it has a characteristic again closer to constant current, and then it stabilizes. This by the way was in summer, so we didn't expect thermostatic loads to play a major part.

I could go on and on and on, but the fact of the matter is, the actual load in a real system is a lot more forgiving than we actually model.

Maillhot: Yes, but when I present a first contingency, with constant power load, then I suppose that time has passed, that the transformer has had time to correct the situation, the voltage near the load, so that's why we use constant load. After two or three minutes, the load is restored to its initial value.

Mansour: A brief point that you were talking about earlier, about blocking tap changers: if you make use of that, you see that now you assume that, that is not assumed, but if you see that the load characteristics are like this, then you can enter from the high side, actual recovery would be primarily because of tap changers. You would have a lot more time for emergency control, slower controllers. It would affect the way you would control these things, not to say that that's an answer, but the way you would deal with it knowing that it does give you a lot of time to actually react.

Maillhot: It is dependent on the load, because we have the same tests on the Hydro Quebec system, but with electric heating, when we reduce the voltage, the load quickly restores to its initial value, because of the thermostatic control. After fifteen or twenty minutes it begins to reach the initial value, even if the voltage is lower. So, it depends on the type of the load you have, because electric heating has to do the job and heat the house.

Mansour: On the megawatt side, but not the reactive power side, necessarily. Because of effects of transformer saturation, when we have a reduction of the voltage, transformers go way beyond saturation, when we drop the voltage, the reactive power consumption of those transformers can go down quite a bit. It's something that you can make use of.

Thomas Overbye (University of Illinois):

This is just a brief follow-up question to Claudio on your Figure 2 where it loops back to the right for increasing power. What happens after it goes off the graph, where you've got it going off to the right? Do you get a maximum power way far out?

Maillhot: When you have an event, like we did in 1988, when the voltage passes above that point, it becomes unstable and increases until, in that event, we automatically tripped the condenser at 1.15 per unit. That's the only thing that stopped the increase. The same way that here the voltage level decreases. It's not stable here. You're still moving down and here you're still moving up. That's exactly the same thing fundamentally. If one can understand this side, you can understand that this other side is the same thing. You cannot control the voltage increase and create a lot of reactive power on the network and you cannot control this reactive power. So the voltage goes up until something happens - the line trips, the condenser trips and that's what happens. This is interesting to study on our network, because we can have lots of load like that, one thousand megawatts, and we have implemented some automatic devices to reduce this elevation of the voltage, to control it. But it is exactly the same, same behavior.

MODAL ANALYSIS FOR VOLTAGE STABILITY: Application at Base Case and Point of Collapse

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1. INTRODUCTION

Voltage stability has become an important issue to many power systems around the world, the large Brazilian interconnected system being no exception. There is a great interest in the development and application of computational tools and methodologies to voltage stability problems detected in power system planning and operation studies [1-31]. Various methodologies have been proposed in the last decade, together with system models to properly simulate the voltage stability phenomena.

Considering the length of this paper, it is appropriate to list its contributions at this initial stage:

- A multivariable control framework is provided to the modal analysis methodology established in [1].
- Specialized use of modal sensitivities for better problem identification and proposed solutions.
- First reported use of the concept of transfer function zeros to the voltage stability area.
- A method to determine suitable locations for Static VAR Compensators (SVC) and their required droops (gains in pu/pu) to help preventing voltage collapse.
- Voltage stability (modal) analysis of a large system containing over two hundred induction motor loads, each represented by a first-order motor model.
- Determination of required minimum size for a system model to properly simulate the behaviour of a given voltage collapse mode.
- Effective use of modal analysis at (or near to) the point of collapse [9].
- Report on the use of Point of Collapse and Continuation Power Flow programs.
- Describes the stage of development of the Brazilian voltage stability tools and the future R&D work to be jointly carried out by CEPEL, ELETROBRÁS and other Brazilian utilities.

The results described in the paper are for a large practical model of the South-Southeast Brazilian

system. The operating point considered is intended to duplicate real system conditions of March 1993, when a blackout, due to voltage collapse, took place in the Rio de Janeiro area.

Keywords: Voltage Stability - Modal Analysis - Large Scale Systems - Continuation Methods - Power Flow - Point of Collapse Methods - Transfer Function Residues - Eigenvalue Sensitivities - Maximum Loadability - Static VAR Compensators - Induction Motor Loads.

2. VOLTAGE STABILITY METHODS

2.1. Modal Analysis of Small Signal Voltage Stability

Modal analysis can be used to determine which areas are most vulnerable to voltage stability problems, to select the best sites for installing new reactive compensation equipment, and to determine the most effective actions such as generation redispatch and load shedding to alleviate voltage conditions.

The user must always recognize that modal analysis results are rigorously valid for incremental changes only. The power of this methodology, therefore, lies in its ability to provide information on system trends rather than estimate the actual numerical values of system variables following changes.

The work reported in this paper gives a multivariable control framework to the methodology established in [1]. The methodology of [1] performs modal analysis on the state matrix $\begin{pmatrix} \mathbf{J}_R^Q \end{pmatrix}$. One can alternatively obtain the same information from the eigensolution of $\begin{pmatrix} -\mathbf{J}_R^Q \end{pmatrix}^{-1}$, which can be expressed as:

$$s\mathbf{I} + \begin{pmatrix} \mathbf{J}_R^Q \end{pmatrix}^{-1} = 0 \quad (1)$$

where $\mathbf{I}$ is the identity matrix and $(\mathbf{J}_R^Q)$ is the reduced Jacobian matrix defined in [1], and also known as the Q-V sensitivity matrix.

Good insight is obtained by recognizing that equation (1) describes the dynamics of the system model represented by the block diagram of Figure 1:

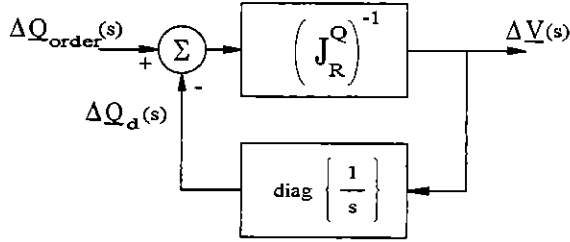


Figure 1. Simple Dynamic Load Model for Voltage Stability Analysis of Large Systems

The elements of the diagonal matrix in the feedback loop of Figure 1 are pure integrators. The symbol $\Delta \mathbf{V}(s)$ denotes the vector of bus voltage magnitudes and $\Delta \mathbf{Q}_{order}(s)$ is the input vector of specified changes in reactive power for the various loads in the system. State vector $\Delta \mathbf{Q}_d(s)$ contains the incremental changes in reactive demand of the load buses, whose active powers are independent of voltage deviations (constant MW loads).

This model has some analogy to the linearized form of the classical multimachine electromechanical stability problem, described by equations:

$$s^2 \mathbf{I} + \text{diag} \left\{ \frac{\omega_o}{2H_i} \right\} \mathbf{K}_s = 0 \quad (2)$$

where $\mathbf{K}_s$ is the matrix of synchronizing torque coefficients [32].

The eigenvalues of the dynamic model of Figure 1 are those of matrix $(-\mathbf{J}_R^Q)^{-1}$. A system eigenvalue therefore goes to infinity at the voltage stability limit where $\det(\mathbf{J}_R^Q) = 0$.

The dynamic load model of Figure 1 is very useful despite not being meant to reflect actual load dynamics. The authors of [28] made a comment on this model in their closure, which is here quoted: "The development of this simple dynamic load model has made it possible to relate the eigenvalues of $(\mathbf{J}_R^Q)$ to the poles of the closed-loop transfer function of the dynamic system".

2.1.1. Identifying Critical Buses, Branches and Generators Via Residues and Eigenvalue Sensitivities (Fast Ranking)

Small signal stability assessment of large power systems can be efficiently computed when using an augmented form of the system state-space description [48,49,50]. For the sake of brevity let us here simply define $\mathbf{A}$ as the system state matrix and $\mathbf{b}$ and $\mathbf{c}^t$ as the vectors for a given input/output pair.

Once the eigenvalues of matrix $(\mathbf{J}_R^Q)^{-1}$ are calculated, valuable information on corrective measures can be obtained through transfer function residues [32], participation factors [1,39] and eigenvalue sensitivities. Residues can be used to determine best locations for dynamic voltage devices among other uses.

A scalar function $F(s)$ can be expressed as [32]:

$$F(s) = \frac{N(s)}{D(s)} = \mathbf{c}^t \cdot (s\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{b} = \sum_i^n \frac{R_i}{s - \lambda_i} \quad (3)$$

where integer n is the dimension of $\mathbf{A}$ and R_i is the residue of $F(s)$ associated with the i -th eigenvalue of $\mathbf{A}$. The residue can be expressed as

$$R_i = \tilde{\mathbf{b}}_i \cdot \tilde{\mathbf{c}}_i \quad (4)$$

where $\tilde{\mathbf{b}}_i = \xi_i^t \cdot \mathbf{b}$ is the mode controllability factor and $\tilde{\mathbf{c}}_i = \mathbf{c}^t \cdot \eta_i$ is the mode observability factor. Note that ξ_i and η_i are, respectively, the left and right eigenvectors associated with λ_i .

Once the desired right and left eigenvectors are calculated, the residues for a large number of transfer functions can be efficiently obtained through the use of the corresponding augmented input/output vectors [32].

A general formula for eigenvalue sensitivity coefficients applied to the generalized eigenvalue problem was described in [37].

Useful information can be obtained through eigenvalue sensitivity analysis when α is chosen to be a network impedance, a dynamic component parameter, etc.

The differences between eigenvalue sensitivities $\frac{\partial \lambda}{\partial \alpha}$, residues and participation factors are described in [3,5]. Participation factors have been the ones mostly used in the voltage stability area [1,9,24]. The authors of this paper support a more specialized use of modal sensitivities, such as that proposed in Table 1. Various functions are listed in Table 1 together with their recommended uses when attempting to stabilize a critical voltage mode. The computation of these sensitivities can be made very fast through clever sparsity programming. Scaling problems should be

recognized and properly dealt with when producing ranking lists out of modal sensitivities.

2.1.2. Transfer Function Zeros

The zeros of the transfer function of equation (3) can be obtained by solving the generalized eigenvalue problem [42]:

$$\begin{pmatrix} \underline{A} & \underline{b} \\ \underline{c}^t & 0 \end{pmatrix} \cdot \begin{pmatrix} \underline{x} \\ u \end{pmatrix} = \lambda \cdot \begin{pmatrix} \underline{I} & \underline{0} \\ \underline{0}^t & 0 \end{pmatrix} \cdot \begin{pmatrix} \underline{x} \\ u \end{pmatrix} \quad (5)$$

where $\underline{I}$ is the identity matrix, $\underline{0}^t$ is a row vector with all elements equal to zero and u is the input variable. A

standard QZ routine [51] can be used to obtain the complete set of transfer function zeros, as long as the system is of moderate size.

Subspace iterations methods can be used to efficiently calculate several zeros at a time of large power system dynamic models. These methods must be applied to the augmented generalized eigenvalue problem, which is sparse, and is described in [52]. An application of the concept of transfer function zeros to the voltage stability problem is described in section 5.3.2 of this paper.

<i>Residues</i>	
$\frac{\Delta V}{\Delta B_{sh}}(\lambda)$	Find best buses to place Static VAr Compensators
$\frac{\Delta V}{\Delta Q_{sh}}(\lambda)$	Find best buses to place synchronous condensers, generators and advanced Static VAr Compensators
$\frac{\Delta V}{\Delta B_{ij}}(\lambda)$	Find best lines to place Thyristor Controlled Series Compensation
$\frac{\Delta V}{\Delta \phi_{sh}}(\lambda)$	Find best lines to place static phase shifters
<i>Participation Factors of ΔV</i>	
Define the component buses of reduced size system model to properly simulate a given voltage mode	
<i>Eigenvalue Sensitivities</i>	
$\frac{\partial \lambda}{\partial B_{sh}}$	Find best buses to connect fixed shunt compensation
$\frac{\partial \lambda}{\partial B_{ij}}$	Find best lines to install fixed series capacitors
$\frac{\partial \lambda}{\partial tap}$	Find OLTC transformers where tap changes are most effective
$\frac{\partial \lambda}{\partial V}$	Find generators where voltage level changes are most effective
$\frac{\partial \lambda}{\partial P_g}$	Find generators where redispatch is most effective
$\frac{\partial \lambda}{\partial P_{load}}$	Find most critical active loads
$\frac{\partial \lambda}{\partial Q_{load}}$	Find most critical reactive loads

Table 1: Modal Sensitivity Functions for Voltage Stability Analysis and Their Recommended Uses.

2.1.3. How to Stop Missing Unstable Eigenvalues in Partial Eigensolutions

In large scale power flow models, only partial eigensolutions of $(-J_R^Q)$ are possible. The computer program described in [1] computes a group of eigenvalues which have the smallest moduli, and therefore are of interest. However, sometimes, it might exist (in voltage unstable cases) positive eigenvalues (unstable) with not so small a modulus and, therefore, this approach may fail to detect these instabilities in a moderate number of computer runs.

A practical solution to this problem is to operate on matrix $(-J_R^Q)^{-1}$. A zero eigenvalue of $(-J_R^Q)$ is therefore mapped into infinity, while negative small values become large negative and small positive eigenvalues become large positive ones. The strategy is to perform partial eigensolutions on the spectral transformations $(-J_R^Q - \alpha I)^{-1}$ and $(-J_R^Q + \alpha I)^{-1}$, where I is the identity matrix and the real shift α is usually given a value around 5 or larger for near collapse situations. This solution leads to an apparently larger number of computer runs, but the chances of missing unstable eigenvalues are practically eliminated.

2.1.4. Recommended Use for Modal Analysis

Some voltage stability experts [9,24], advocate the use of modal analysis mainly for highly stressed system conditions. The authors' experience confirms that modal analysis is most effectively used for system reinforcement studies when made at the point of collapse (maximum loadability point) or near it, as will be shown in a later section of this paper. Modal analysis can not be used to predict system loadability margins. Such function must be left to Continuation and Point of Collapse Power Flow programs. Voltage stability margins as functions of the nature of dynamic loads can however be assessed by modal analysis based even on a single power flow condition (see Table 7 of this paper).

2.2. Continuation and Point of Collapse Power Flows

These two methodologies, both of which enhance the power flow analysis area, are detailed described in [17,18,38,31]. The Continuation method makes the Newton Power Flow very robust, allowing convergence in all cases that have a solution in the real domain. The Point of Collapse method obtains, on a single run, the maximum system loadability point for a specified pattern of load changes. The latter method is specially useful when used in conjunction with a modal analysis tool (see section 2.1.4).

A recently developed function to Point of Collapse Power Flows is the computation of the locally closest bifurcations in power systems. The distance in load parameter space to this closest bifurcation is a good index of voltage collapse and a minimum load power margin [14].

3. SYSTEM MODELING FOR VOLTAGE STABILITY ANALYSIS

3.1. Induction Motor Model

The load model of Figure 1 is only meant to be used in connection with steady state power flow problems: maximum loadability; maximum transferability between system areas; transmission and generation reinforcements; allocation of VAR compensation; assessment of voltage critical areas in large system models; etc.

In order to have a more realistic simulation of the voltage dynamic behavior, more detailed models for load and voltage control equipment are needed [2,4,5,7,8,25,26]. Aperiodic voltage stability was shown to be correctly assessed from first-order load models [16]. Slip from synchronous speed is the state variable adopted in the first-order induction motor model of this paper. The block diagram for a linearized system comprised of a generic number of first-order induction motor loads is shown in Figure 2.

The symbol H_i in Figure 2 denotes the "i-th" motor inertia constant, while ΔP_{mec} and ΔP_{el} are vectors containing the motor mechanical and electrical powers in synchronous watts. Matrix $[\partial P_{el}/\partial S_{lip}]$ describes the incremental relationships, at a given operating point, between electrical power requirements and rotor slip for all motors in the system.

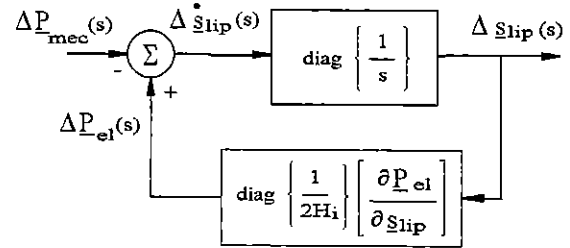


Figure 2. Block Diagram Model for Linearized System Comprised of a Generic Number of Induction Motor Loads

The voltage stability limit occurs at the condition of maximum power transfer to the induction motor. Therefore, the voltage stability limit for a motor (or group of motors) occurs exactly at motor (aggregate motor model) pull-out torque conditions considering the entire transmission network. The latter fact, was

initially described in [44] under section "Stability of Loads".

Some of this paper's results consider induction motor loads and the action of static VAR compensators (SVC). Table 5 of this paper presents the critical eigenvalues associated with the voltage collapse phenomena in the RIO AREA as a function of the SVC location. Voltage stability results for a large system having 239 induction motors are given in Table 7 of this paper.

3.2. Making a Conventional Small Signal Electromechanical Stability Program Suitable to the Analysis of Voltage Stability Problems

Eigenanalysis tools for voltage stability have been developed around the power flow Jacobian matrix, with voltages expressed in polar coordinates, calculated at specified operating points. This is the easiest route for the code development of a first generation program, with the same level of modeling as a conventional power flow. Load, generator and voltage support equipment models of higher complexity are, however, needed in a more accurate simulation of voltage stability phenomena, and the conventional power flow Jacobian matrix is no longer the most convenient network model to use. A more familiar network model to power system dynamics specialists is the nodal admittance matrix, used in all production type programs for electromechanical stability analysis.

The major advantage of using a comprehensive small-signal electromechanical stability program for voltage stability software development is the ready availability of models for the elements which drive the system voltage response (loads, induction motors, HVDC links, etc.) and for those which provide the voltage support (Generators and their excitation control, Synchronous Condensers, Static VAR Compensators, FACTS devices, etc.).

The connection of system devices (Generators, Loads, etc.) to the electrical network, in small-signal electromechanical stability programs, is expressed through current injections to the associated buses in the nodal admittance matrix.:

$$\Delta \mathbf{I} = \mathbf{F} \Delta \mathbf{x} + \mathbf{Y} \Delta \mathbf{V} \quad (6)$$

where $\Delta \mathbf{I}$ is comprised of individual injections $\Delta \mathbf{I}_{\text{real}}^k$ and $\Delta \mathbf{I}_{\text{imag}}^k$, $k=1, \dots, n_{\text{devices}}$, $\Delta \mathbf{x}$ is the system state vector, and

$$\mathbf{F} = \frac{\partial \mathbf{I}}{\partial \mathbf{x}} = \text{block diag} \{ \mathbf{F}_i, i = 1, \dots, n_{\text{devices}} \} \quad (7)$$

Small-signal voltage stability analysis usually deals with models based on the incremental active and reactive power changes to the system buses. The modeling of the system of Figure 1, on a small-signal electromechanical stability program, can be done through the use of the incremental relationships below:

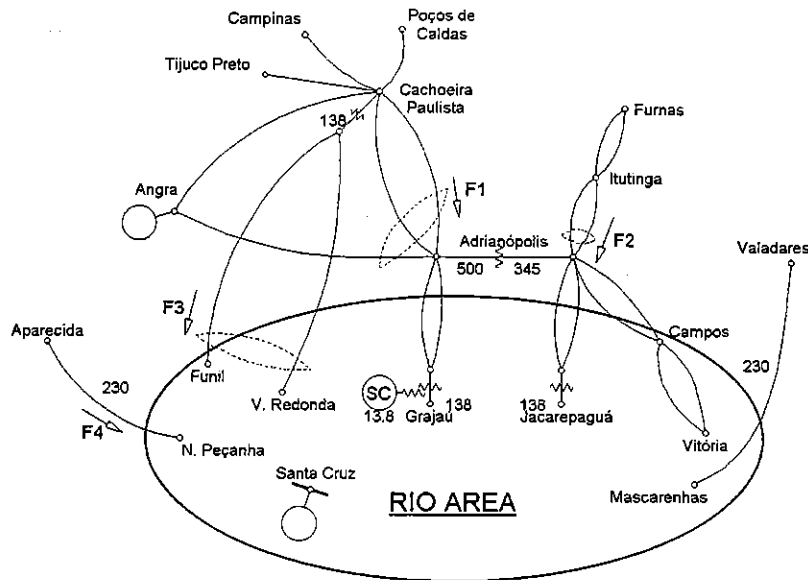
$$\begin{pmatrix} \Delta \mathbf{I}_{\text{imag}}^k \\ \Delta \mathbf{I}_{\text{real}}^k \end{pmatrix} = \begin{pmatrix} \frac{\partial \mathbf{I}_{\text{imag}}^k}{\partial \mathbf{P}_d^k} & \frac{\partial \mathbf{I}_{\text{imag}}^k}{\partial \mathbf{Q}_d^k} \\ \frac{\partial \mathbf{I}_{\text{real}}^k}{\partial \mathbf{P}_d^k} & \frac{\partial \mathbf{I}_{\text{real}}^k}{\partial \mathbf{Q}_d^k} \end{pmatrix} \begin{pmatrix} \Delta \mathbf{P}_d^k \\ \Delta \mathbf{Q}_d^k \end{pmatrix} \quad (8)$$

where $\Delta \mathbf{P}_d^k$ and $\Delta \mathbf{Q}_d^k$ are the incremental changes in the active and reactive load demands at the k -th bus.

4. LARGE SYSTEM MODEL UTILIZED IN THE ANALYSIS OF A REAL INCIDENT OF VOLTAGE COLLAPSE

The main results presented in this paper are for a 1663-bus model of the South-Southeast interconnected, predominantly hydro-powered, Brazilian system. The base case operating point attempts to duplicate real system conditions of March, 1993, when a blackout, due to voltage collapse problems, took place in the Rio de Janeiro area. The voltage collapse was induced by the permanent loss of two 500kV lines (from C. Paulista to Adrianópolis substations). These two major lines remained disconnected for several days and the power utilities of the RIO AREA, which encompasses Rio de Janeiro and Espírito Santo States, held an intensive campaign for voluntary reduction in power consumption. This campaign was quite successful and the power flow into the RIO AREA fell to presumably safe values.

A schematic diagram of the area in focus is presented in Figure 3. The total power flowing into the RIO AREA, denoted by FRJ, is given by the summation of flows F_1 , F_2 , F_3 and F_4 which are indicated in Figure 3. The total active load in the area is about 5000MW, while the local hydro generation is about 900MW. There is one thermal power station (Santa Cruz) in the Rio area with a generating capacity of 600MW (2x220 plus 2x84), which can not be operated in a synchronous condenser mode. It is desirable, due to the high fuel costs, to operate the thermal plant at a minimum level. Dynamic security studies have established a minimum thermal generation of 45MW under heavy load conditions.



Note: Numbers shown denote transmission voltages in kV

Figure 3. Major Transmission Delivering Power to the RIO AREA

5. RESULTS

5.1. Conventional Power Flow Results

The base case system configuration described in section 4 together with some credible contingencies (line outages and generation outages) were considered. The conventional power flow results are displayed in Figure 4, in the form of P-V curves, where V denotes the voltage magnitude of a critical system bus (Jacarepaguá) and P is the total active power flowing into the RIO AREA (FRJ). All these curves were obtained by keeping the Grajaú synchronous condenser generation below its MVar limit (400 MVar). This synchronous condenser group (2 x 200 MVar) is a key element in ensuring safe voltage stability margins to the RIO AREA.

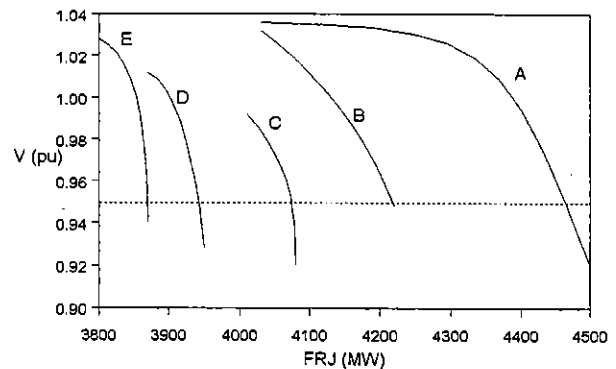
The maximum power import values, obtained from the analysis of Figure 4, are shown in Table 2 for the various conditions simulated.

Case	FRJ (MW)	System Condition	
		Angra Nuclear Station	500kV Line Outage
A	4450	ON	NONE
B	4230	OFF	NONE
C	4070	ON	Angra-Adrianópolis
D	3950	OFF	Angra-Adrianópolis
E	3860	OFF	C. Paulista-Adrianópolis
F	4200	ON	C. Paulista-Adrianópolis

Note: 1) Values determined by assuming 0.950 pu as the minimum allowed voltage level at critical bus Jacarepaguá 345kV (pilot node)

2) The Santa Cruz thermal station generates 45MW in all cases

Table 2: Approximate Maximum Values of Power Import by RIO AREA (FRJ)



System Conditions A, B, C, D and E are described in Table 2

Figure 4: Voltage Level at Pilot Node (Jacarepaguá bus) as a Function of MW Power Imported by RIO AREA

5.2. Modal Analysis at a Given Operating Point

The 1663-bus system has a state matrix of order 1554, and therefore the same number of eigenvalues. The QR eigensolution of such a large matrix is not practical and one must use sparsity programmed partial eigensolution routines. As mentioned in section 2.1.3, the state matrix used in the authors' program is $(-J_R^Q)^{-1}$ and, therefore, the eigenvalues of interest are those of largest moduli and negative sign, together with all those which are positive (unstable).

Some computer runs utilizing a lop-sided simultaneous iteration algorithm [1,33] with different shifts produced, in a few minutes of CPU on a PC 486, all of

the most critical system eigenvalues/eigenvectors. Three of the most critical ones are depicted in Table 3. From eigenvector information (mode-shape of bus voltage magnitudes), the critical eigenvalues are seen to be associated with voltage modes in clearly defined areas of the large interconnected system. Two of these areas are known to power system operations engineers as problem areas regarding power flow convergence. Actually, this base case power flow problem only converges when modeling a good part of the loads in these two areas as voltage dependent (constant I and constant Z load models). Note that the power flow Jacobian is practically singular ($\lambda=-19.784$ and $\lambda=-41.896$), since the Corumbá and Serra da Mesa loads were modeled as constant P in the voltage stability analysis program.

It is worth reminding that this paper's results are obtained from the inverse of the Q-V sensitivity matrix of [1], and therefore its eigenvalues are the inverse of those obtained with the methodology of [1].

Modal analysis helps developing voltage control strategies to better converge other power flow cases derived from the base case analyzed. The results of Table 3 make one very much aware of the multiple and simultaneous quasi-voltage collapses contained in a base case power flow of a large interconnected system.

Critical Eigenvalues	Problem Area *	D** (km)
-19.784	Serra da Mesa (Celg/Furnas)	1000
-41.896	Corumbá (Enersul)	1500
-3.5686	RIO AREA	0

*System Area where Mode is Mostly Observable

**Approximate Distance of Problem Area from RIO AREA

Note: All system eigenvalues (total of 1554) are stable

Table 3: Critical Eigenvalues of $(-J_R^Q)^{-1}$ Matrix for Base Case Power Flow

5.2.1. Determining a Model of Minimum Size for the Voltage Stability Analysis of a Given System Area

The Q-V sensitivity (state) matrix of the power flow Jacobian contains as many lines as the full set of PQ buses in the system. A critical mode, in voltage stability analysis of practical systems, is generally associated with a specific system area rather than being of a global nature (note that, in the electromechanical stability problem, the inter-area modes are of a global nature). This suggests the possibility of reducing the state matrix utilized in the analysis of phenomena associated with a given system area. The reduced Q-V matrix will then only contain the equations for the buses with significant participation factors in the critical modes of the area (internal system). One can perform elimination (Kron's reduction) on the equations for the external

system buses, but this should not be done explicitly for the sake of preserving sparsity. The practical solution is to always work with the full size system Jacobian. If different shifts are specified to find all critical eigenvalues, routines for sparse partial refactorization should be applied to operate on the internal system factorization tree [40].

Table 3 showed the three most critical modes identified from the partial eigenanalysis of the 1554th-order state matrix of the 1663-bus system. Table 4 shows that these modes can be obtained and further analyzed from highly reduced Q-V state matrices. The quality of the approximate model is assessed from the bus voltage mode shape or participation factors information and not from the numerical value of the eigenvalue. The quality of the reduced model is, above all other factors, determined from its ability to present about the same maximum area loadability point as the full size model. One must ensure that there exists connectivity among all buses represented in the Q-V state matrix. Otherwise, the dynamic models in isolated portions of the network will interact through the algebraic part of the network, producing some spurious eigenvalues, which may bring difficulties to the interpretation of results.

Eigenvalue	Utilities Represented in State Matrix	No. of States	Quality of Approximate Model
RIO AREA Mode			
-3.5686	All Utilities	1554	-
-3.5626	Escelsa, Furnas, Light, Cerj, Cernig	448	Excellent
-3.5209	Escelsa, Furnas, Light, Cerj	331	Excellent
-2.8175	Escelsa, Furnas, Light	279	Good
-3.5970	Escelsa, Furnas, Cerj	216	Excellent
-2.8197	Escelsa, Furnas	164	Good
-1.8430	Escelsa	53	Fair
Serra da Mesa Mode			
-19.784	All Utilities	1554	-
-20.113	Celg, Ceb, Furnas	227	Excellent
-13.342	Celg, Furnas	169	Good
+8.1381	Celg, Ceb	116	Fair
+9.4093	Celg	58	Fair
Corumbá Mode			
-41.896	All Utilities	1554	-
-41.693	Enersul, Eletrosul, Cesp, Copel	369	Excellent
-37.879	Enersul, Eletrosul, Cesp	232	Excellent
+1029.7	Enersul, Eletrosul	125	Excellent
+16.71	Enersul	60	Good

Table 4: Analysis of Critical Area Modes Using Reduced Size State Matrices

A practical advantage of using reduced dynamic models is that full eigensolution becomes then possible through use of a standard QR library routine [41]. Another advantage is the reduction in the numbers of eigenvalues per cluster in large Q-V matrices, since the constituent eigenvalues of a single cluster may belong to numerous voltage areas, which can be entirely decoupled from that one under focus.

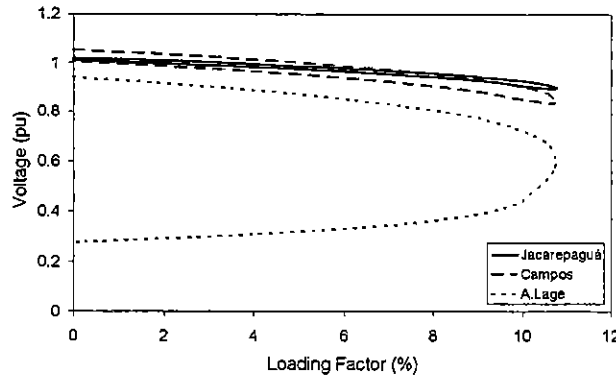


Figure 5: System with no Q -limits

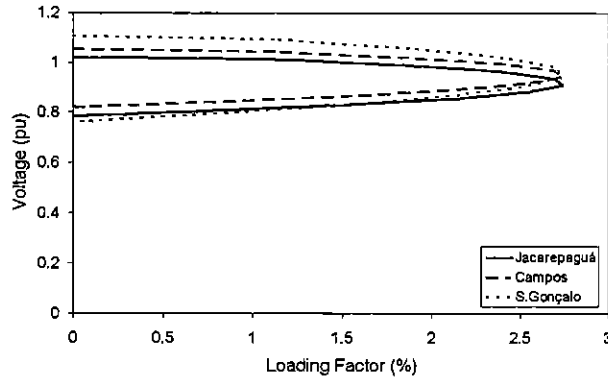


Figure 6: Q -limits active only at Grajaú Synchronous Condenser (Grajaú limited at +400MVar)

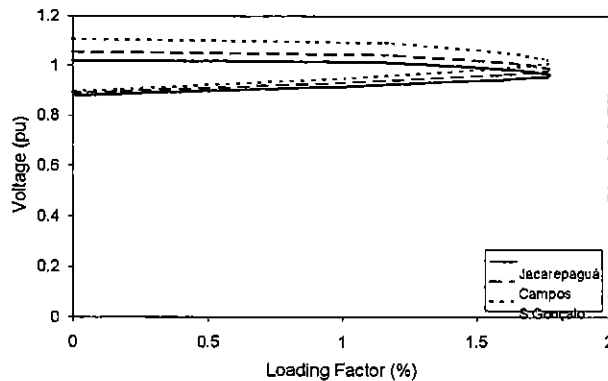


Figure 7: Q -limits active at all PV Buses of the System

5.3. P-V Curves Produced by a Continuation Power Flow

The Continuation Power Flow and Point of Collapse programs used in the work of this paper were developed at the University of Wisconsin by Dr. Fernando Alvarado and associates [18,14,38,31].

Figures 5, 6 and 7 show the P-V curves for the voltage magnitudes at two major 345kV substations in the RIO AREA (Jacarepaguá and Campos), with the thermal

power station (Santa Cruz) disconnected. Voltage profiles are also included for two of the weakest buses in the RIO AREA: A.Lage (Escelsa) and S. Gonçalo (Cerj). The horizontal axis in these curves is the extra loading (in percentage of the base case MW-MVar loads) imposed to the base case system load. Power interchange controls, LTC action and maximum VAr limits were not considered, to obtain the nose curves of Figure 5.

Note that the system maximum loadability point is about 111% (base case loading is 100%) in Figure 5, with Grajaú Synchronous Condenser generating 1300MVar, well above its rated continuous maximum limit of 400MVar. Figure 6 nose curves were generated by applying the 400MVar limit to the Grajaú condenser, and the maximum loadability drastically dropped to 102.7%. Actually, a different mode of collapse occurred at 102.7% loading as a result of the loss of voltage control action at the Grajaú bus. Figure 7 curves were obtained by considering proper MVar limits in all PV buses of the system, and the maximum loadability dropped further to 101.7% loading. These results indicate that the synchronous condenser at Grajaú bus is a key element for the voltage stability of the RIO AREA.

5.3.1. Determination of Suitable Locations for New Voltage Control Devices (Fast Ranking Lists)

The bus with the largest magnitude in the ranking list of residues $\frac{\Delta V^i(s)}{\Delta B_{sh}^i(s)}$, for $s = \lambda_{critical}$, is initially taken as

the best one for installing an SVC in order to move $\lambda_{critical}$ (see Table 1).

The system test chosen to validate the methodology proposed in Table 1 was to assume, in the base case condition, the Grajaú synchronous condenser, here generating 340MVar, to be at VAr limits and, therefore, considered as a PQ bus.

The system, as seen from the comparative analysis of Figures 5 and 6, has its voltage stability margins significantly reduced. The most critical RIO AREA eigenvalue jumps from -3.5626 to -11.764 when the Grajaú condenser bus is changed from PV to the PQ type. The ranking list of $\frac{\Delta V(s)}{\Delta B_{sh}(s)}$, shown in Table 5, is

used to define the best locations for a dynamic voltage controller device to improve the most critical RIO AREA mode (-11.764). Note that the high-voltage bus (Grajaú 500 kV) of the synchronous condenser comes first in this ranking list among all other 500 kV buses. Table 5 also shows the location of the least stable system eigenvalue after installing, one-at-a-time, a purely algebraic model of an SVC (SVC gain is

30 pu/pu at all buses) at each one of the listed buses. Note that, as expected, the SVC action improves the stability of the RIO AREA mode. This validates the methodology proposed for the location of voltage-controlling devices in the system based on modal sensitivities.

Bus	kV Level	Residues of $\Delta V/\Delta B_{sh}$	RIO AREA Eigenvalue	Zeros of $\Delta V/\Delta B_{sh}$
none	-	-	-11.764	
Grajaú	500	0.30580	-5.5843	-3.1072
Adrianópolis	500	0.28540	-5.7093	-3.0510
Angra	500	0.22820	-6.7907	-3.9419
C. Paulista	500	0.16100	-7.4144	-3.7988
Taubaté	500	0.10380	-8.7119	-5.5181
Tijuco Preto	500	0.07507	-9.3180	-6.0962
Campinas	500	0.05644	-9.9089	-7.3855
P. Caldas	500	0.05831	-9.8482	-7.2441
Araraquara	500	0.02640	-10.7713	-8.8341
Marimbondo	500	0.00417	-11.5683	-10.6369
Jacarepaguá	345	†	-5.1186	-2.9985
Campos	345	†	-6.8121	-4.6254

Table 5: Location of Critical RIO AREA Eigenvalue After SVC Installation at a Given Bus

5.3.2. Assessing the Maximum Impact to System Stability of a Voltage Control Device at a Given Bus

This assessment can be done through the calculation of the critical system zeros [42] of the transfer functions associated with the various voltage control devices. The analysis results presented in this section are concerned only with the zeros of the $\frac{\Delta V^i(s)}{\Delta B_{sh}^i(s)}$ transfer function.

Table 5 displays the most critical zero of this transfer function for various buses of the system. These zeros represent the location in the complex plane, for one of the system poles when installing an SVC of infinite gain to a given system bus. If a zero is sitting on top of a given eigenvalue, it means that an SVC at that bus is incapable to control such voltage mode.

The critical zeros and system eigenvalues after SVC addition, all shown in Table 5, should be compared with $\lambda = -11.764$. This comparative analysis allows the determination of adequate gain values for the SVC's, as described below for the Grajaú 500kV bus. The eigenvalue is -11.764 and -5.4666 before and after the SVC addition, respectively. As the -5.4666 value is comparatively close to $\lambda = -3.1072$ which needs an SVC of infinite gain, one can conclude that $K=30$ is an adequate SVC gain for the 500kV Grajaú bus.

5.4. Report on the Use of the PoC Power Flow Program

Once a given scenario of load and generation increase is defined, the Point of Collapse Power Flow program can find in a direct manner the maximum loadability

point, where the power flow Jacobian matrix becomes singular. The solution time needed to find the PoC (running in a modern UNIX Workstation) for the 1663-bus system may be as high as 20 times that needed for a single load flow solution. The results obtained were very precise and will be shown in conjunction with the modal analysis in the next section. This algorithm needs, however, to be made faster so as to be more effectively used in multiple scenario situations and minimum distance to collapse studies [15].

Reference [30] points out that Point of Collapse power flows, apart from being computationally very expensive, depend greatly on a good initial guess of the desired saddle-node bifurcation point. Otherwise, the method may diverge or converge to an undesired saddle-node bifurcation point. We had sometimes to use, out of convergence difficulties, the Continuation Power Flow program to obtain good enough initial values to the PoC program. These problems, which will certainly disappear with future enhancements, do not reduce the merits of this direct solution method.

5.5. Point of Collapse Results

The authors, like others [24], support the use of modal analysis mainly for highly stressed system conditions. The drawback of using modal analysis for medium-load conditions is the probable existence of not just one but several eigenvalues of equivalent moduli associated with the same system area. This fact impairs the practical assessment of the impact of alternative system changes to a base case operating point. This problem comes from the difficulty in keeping track of several voltage stability modes within a given area as the operating conditions are varied.

In the Point of Collapse Power Flow computer program, the scenario of load/generation increase is defined and kept fixed until a point of maximum loadability (singularity of the power flow Jacobian) is reached. The maximum loadability point represents a saddle-node bifurcation having a single zero eigenvalue associated with this load increase scenario. Attention is therefore focused on the bus voltage mode-shape, transfer function residues (or, alternatively, participation factors) associated with this single zero eigenvalue. Short-term remedial actions, system operation strategies and long-term system reinforcements can be ranked by these functions.

The base case condition ($\lambda = -11.764$) of Table 6 is very close (only 1.5% below) to the PoC condition. The residues, shown for the 500kV buses also in Table 6, provide a ranking list for the best buses to become voltage controlled ones (PV). A more powerful ranking information (though much more expensive) is provided

from the zeros of $\frac{\Delta V(s)}{\Delta B_{sh}(s)}$, which are shown in Table

6. The maximum system loadability (PoC) were calculated by the PoC Power Flow program and are given, in Table 6, in percentage loading above the base case condition. The generated MVar at these buses, at the PoC solution, are presented in Table 6 and also

have good correlation with the transfer function zero locations.

Note that the transfer function zero analysis at a given operating point is of practical use to area loadability studies, as long as no PV buses within the area reach Q-limits. When this happens, another transfer function zero analysis should be carried out.

Bus Converted to PV Type	kV Level	Zeros of $\frac{\Delta V}{\Delta B_{sh}}$	Residues of $\frac{\Delta V}{\Delta B_{sh}}$	PoC % Loading Above Base Case	Generated MVar at Bus
none	-	-	-	1.5	-
Grajaú	500	-3.1072	0.30580	17.534	1551.8
Adrianópolis	500	-3.0510	0.28540	20.006	1772.47
Angra	500	-3.9419	0.22820	11.125	1123.52
C. Paulista	500	-3.7988	0.16100	12.624	1544.7
Taubaté	500	-5.5181	0.10380	5.8857	709.88
Tijuco Preto	500	-6.0962	0.07507	4.7813	623.35
Campinas	500	-7.3855	0.05644	3.7430	321.41
P. Caldas	500	-7.2441	0.05831	4.0176	351.55
Araraquara	500	-8.8341	0.02640	3.2631	224.51
Marimbondo	500	-10.6369	0.00417	2.8745	154.58
Jacarepaguá	345	-2.9985	†	18.4343	1718.0
Campos	345	-4.6254	†	6.7160	495.32

Table 6: Critical Zeros of $\frac{\Delta V_b^i(s)}{\Delta B_{sh}^i(s)}$ for Base Case Power Flow with $\lambda = -11.764$

The State Matrix is of Order 448

5.6. Voltage Stability Results Considering Induction Motor Loads

The RIO AREA, as described in Section 4, has a total load of 5000 MW which is distributed among 239 load buses.

The results presented on the previous sections were all related to the load model of Figure 1, which yields the state matrix $(-J_R^Q)^{-1}$.

This section results were obtained including first-order induction motor models (Figure 2) to all the 239 load buses of the RIO AREA. The same motor parameters (typical values) were adopted for in all motors, except for their ratings, which were made compatible with the MVA values of the individual loads.

The voltage stability limit, for the load model of Figure 2, is reached at pull-out torque conditions where the critical eigenvalue becomes equal to zero (note that, for the previous model, the critical eigenvalue would go to infinity at the voltage collapse point).

Table 7 shows the system critical eigenvalues, obtained for the same power flow condition (base case), as the induction motors contents of the area load are gradually

increased. The remaining part of the area load is treated as constant impedance.

Motor Contents in RIO AREA (% of Total Load)	Critical Eigenvalues	
60%	-14.242	λ_1
	-15.979	λ_2
	-16.612	λ_3
80%	-4.426	λ_1
	-6.632	λ_2
	-7.445	λ_3
85%	-1.103	λ_1
	-3.593	λ_2
	-4.479	λ_3
86%	-0.2416	λ_1
	-3.014	λ_2
	-3.825	λ_3
87%	+0.892	λ_1
	-2.416	λ_2
	-3.142	λ_3

Note: Bus with largest participation:

λ_1 : Cachoeiro (#2617)

λ_2 : A.Lage (#2611)

λ_3 : J. Ne. (#2647)

Table 7: Voltage Stability as a Function of Motor Load Contents in RIO AREA for Base Case Condition

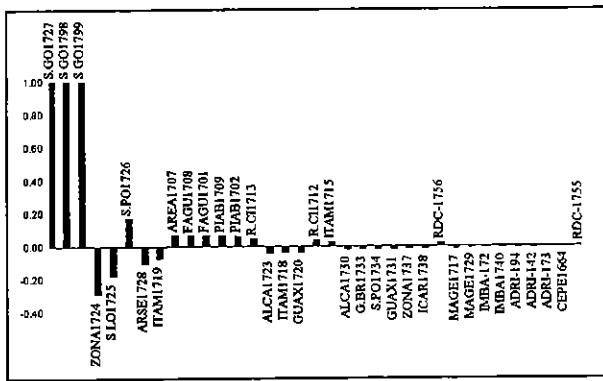


Figure 8: Mode Shape for $\lambda = -20$ or -30
(Denotes local behaviour only)

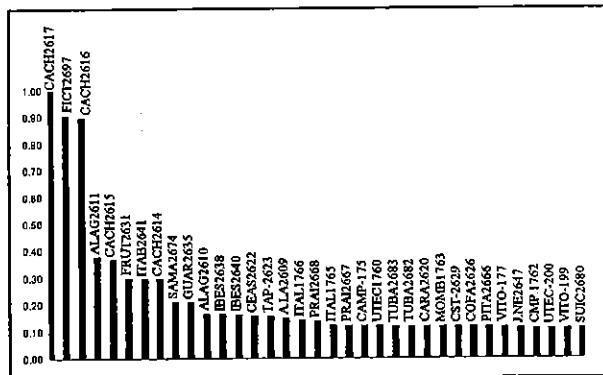


Figure 9: Mode Shape for $\lambda = -0.204$
(Denotes global area participation)

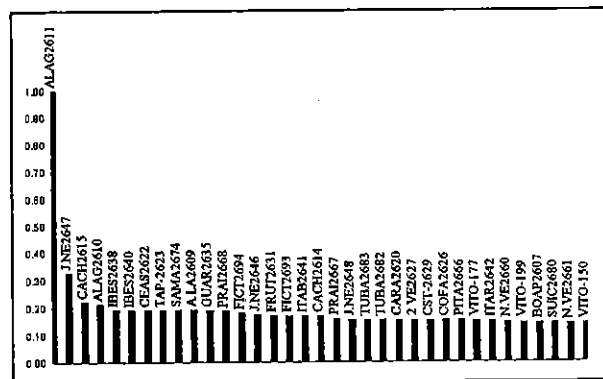


Figure 10: Mode Shape for $\lambda = -3.014$
(Denotes global area participation)

Most of the 239 system eigenvalues have values smaller than -20 and are related to local motor (or group of motors) behavior, as can be seen from the mode-shape of Figure 8. Only the 3 eigenvalues of Table 7 merit attention regarding voltage stability. Their mode-shapes (Figures 9 and 10) are similar to those obtained with the other load model (Figure 1). The area load stability limit occurs with 87% of motor content. The use of modal analysis results like these may be valuable in the determination of aggregate load models for voltage stability.

6. NEW HORIZONS IN VOLTAGE STABILITY ANALYSIS

- A conventional power flow program does not contain adequate generator, load and voltage control equipment models to produce realistic steady state solutions. Enhanced modeling is needed, as discussed in [2,3,5,7,8,23,25,26], to obtain better initial values for detailed large scale dynamic simulations, snapshots for modal analysis and more realistic point of collapse conditions.
- The interest in the on-line assessment of voltage stability problems has been expressed in technical publications [1] and in international IEEE & CIGRÉ activities. Parallel eigensolution algorithms were applied to the electromechanical problem [33], and will next be applied to voltage stability. These parallel algorithms will render feasible the inclusion of on-line eigenanalysis functions, for voltage stability assessment purposes, in the Energy Management Systems of the future.
- The application of probabilistic methods to the adequacy assessment of the bulk system transmission network is already well established in Brazil and abroad. Cepel and other Brazilian utilities have jointly developed a state-of-the-art generation/transmission reliability model. This package does not, however, directly consider the voltage stability margins of the system under analysis. Cepel is studying ways to add a voltage stability function to the generation/transmission reliability software for the purposes of adequacy studies. The action of emergency control aids, such as undervoltage load shedding to prevent voltage collapse [12], can be considered in the adequacy analysis.

The probabilistic approach described in [11], which introduces the supply interruption cost into the analysis, determines the tradeoff point between the risk of voltage instability and the cost of implementing preventive and corrective measures. The concepts expressed in [11] will be used to the further development of CEPEL's probabilistic tools. Parallel processing will play a major role in the integrated probabilistic tools of the future [34].

- Existing optimum VAR allocation software are reported [9] to sometimes produce poor results from the voltage stability viewpoint. The strategy adopted in [9] was to determine preferred buses for shunt compensation based on modal analysis (participation factors of critical voltage modes) and then to split this compensation among these buses using an optimum VAR allocation software. Cepel is pursuing the idea of integrating the following existing tools into a single package: PoC and Continuation Power

Flows, Optimum VAR Allocation [36], Optimum Power Flow and Modal Analysis.

The integration of all these programs into a generation/transmission reliability framework will yield a new software for the optimum probabilistic VAR allocation considering voltage stability margins.

- e) Use of multivariable control theory, modal analysis included, in the design of secondary voltage and reactive power control strategies of large power grids [10]. Robust control techniques, if made computationally efficient to tackle large scale problems, will be valuable to this work.
- f) Selective modal analysis [53] has always attracted great interest for its practically sound objective: finding only those system eigenvalues which are relevant to the problem under focus. The AESOPS algorithm was designed to only find the dominant oscillation modes in the torque-angle loops of the disturbed generator [45,46,42,47]. This concept has been recently extended in [40], which describes a most powerful algorithm to find the dominant poles of high order transfer functions between any pair of system variables. The results of [40] are related to the electromechanical stability problem, but the method has already been applied to the modal analysis of voltage stability. Once the system area under study is specified, the algorithm produces only the dominant modes of voltage collapse for this area. The further refinement of this class of algorithms will enhance the practical use of modal analysis and make it more amenable to non-expert engineers.

7. FINAL CONCLUSIONS

A range of methods aimed at voltage stability problems has been addressed, as well as their best forms of use. Some of the methods involving modal analysis are being presented for the first time in this paper. The results included indicate the power of the methods proposed.

The combination of Point of Collapse Power Flow and Modal Analysis proved to be synergetic as foreseen by others. The authors have obtained excellent results regarding system reinforcements or operative measures so as to maximize system loadability.

There is an urgent need for an advanced methodology to assess probabilistic voltage stability margins, to be applied both in expansion and operational planning stages [11]. Cepel is committed to the development of a comprehensive software package for this purpose.

The previous section, entitled "New Horizons on Voltage Stability Analysis", summarizes some of the

future research and development work to be jointly carried out by Cepel, Eletrobrás and other Brazilian utilities.

8. ACKNOWLEDGMENTS

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Discussion

Edited by Nicholas W. Miller, General Electric Company

Yakout Mansour (BC Hydro):

Side Two of tape begins in mid-sentence:

.....your judgment based on the results, but just from those results, I think you still have to look to make sure at least that observability and controllability of the original system is maintained when you reduce the number of states. You could get the same result, but observability and control does not mean that you still maintain . . . do you do that or if you say yes, then we can open . . .

Nelson Martins (CEPEL):

Yes.

Christopher DeMarco (University of Wisconsin)

Nelson, I am curious about your induction motor models. Certainly, as you say, it is a linear model, but as you update the linearization at different operating points, are you taking into account the shape of the slip curve, and therefore can you capture transition to stalling, for example?

Martins: Yes. It's the same model as used in a transient stability model, which is a third order model. But then using the concept that Ian Dobson has made available in the literature, but which I knew before (laughter) - I used it for electromechanical stability; aperiodic stability was proved by Anam Mohammed Ali Hamdam, from Jordan University, my previous colleague at U. Mist. He was probably for one year or two, but he proved that, through control, aperiodic instability could be foreseen exactly, using a reduced model, a first order model -- it has to be second for the generator because of the double integration, but I got these results for large systems, for synchronous stability, but then I said "it is to no use!" because our problems in electromechanical stability are all oscillatory, so there is no use, it's useless. But now after so many years I found the application to it. And then they started understanding what Ian Dobson is talking about.

Antonio Silva (University of Waterloo):

When you reduced the number of states in your system, I could not understand which criteria have you used to choose the part of the system you want to analyze.

Martins: Participation factors. The buses that participate strongly in that critical mode which I am investigating.

Silva: Can you analyze for the whole system?

Martins: Yes, I got the solution for the whole system. I have that critical mode for the Rio area. The system is 1600 buses. What is valuable to represent, what I need to represent, to catch this mode is about 200 buses, so when I get those 200, and this information I get from participation factors.

Thierry Van Cutsem (University of Liege):

You have shown this local mode and in this scheme you have some buses with negative participation. Maybe participation is not the right terminology. Can you give an explanation of this? My understanding would be that a load bus would be injecting power instead of consuming, maybe I'm wrong. Can you give a physical interpretation?

Martins: In this case, I have all induction motors represented, and all modes are real. But they may have different polarity. They don't need to be all positive. Actually, out

of this work, I am working now on a voltage power flow problem, okay? And, by analogy, this could be a convergence mode. You have your discrete problem, non-stationary, Jacobians change during iterations, etc. For power flow, just power flow, your mismatches have different signs for different buses. They don't need to be all the same.

Mansour: I think I note that part of the confusion is that Nelson has invented his own convention for signs and numbers; he wanted to go along with dynamic stability, so that's why you have negative consistent with that concept. You are also introducing the negative participation now which is unconventional compared to what we know as participation. So it is your own convention, I think.

Martins: Yes, that's not participation. This is mode shape, okay? So mode shape can be positive and negative.

Mansour: So that's a vector, then, an eigenvector, not participation?

Martins: Yes, that's the eigenvector.

Robert Schlueter (Michigan State University):

Nelson, in this paper you indicated that the sensitivity or the modal analysis is very good down at the point of collapse, but not very good as an indicator of the distance to collapse. Also, in previous discussions, you have pointed out that there are many modes, there are many points of collapse, and you've done it again today. Our experience is that, despite the fact that point of collapse sensitivity analysis is very accurate, the movement up to the point of collapse has discontinuities, or nonlinearities, which really is the issue that brings on collapse. We get very different kinds, when we compare modal analysis results. What our techniques say, from modeling and understanding the discontinuities that come from generators hitting field current limits, is that the enhancements that we would do are totally different from what is done by modal analysis. I think that it is really betting on whether you're looking at the point of collapse or you're looking at how you got to collapse. Do you have any comment on that?

Martins: Yes, I sort of missed the point. I have many points of collapse, and you say that points of collapse change when they hit reactive limits. Of course, I mean modes change and the critical modes change; that's what linear analysis is about. You have to get many pictures of the system and build your global picture, out of analysis of different conditions. That's a deficiency of linearized analysis. With linearized analysis you can go very far into understanding and into controlling, and then you go to the non-linear world and you hope for the best, hope that the sort of different pictures you have for your system will be suitable in the real life world.

LOW VOLTAGES AFTER A DISTURBANCE IN THE FINNISH 400 KV NETWORK

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IVO Transmission Services Ltd, Finland

Abstract

The Finnish power system is a part of the synchronous Nordic Power System (Nordel). In August 1992 a disturbance occurred, where the voltages decreased heavily in 400 kV network and the system was near voltage collapse. Disturbance was initiated by a tripping of a 735 MW generator in south-western Finland. This and the decrease of reactive power in another 735 MW generator at the same power station caused the decrease of voltage in southern Finland. The lowest voltage in 400 kV grid was 344 kV. The voltages were restored to 400 kV level in 30 minutes by starting gas turbines, by some load shedding and by increased reactive power production.

Introduction

The Finnish power system is a part of the synchronous Nordic Power System (Nordel), which includes Finland, Sweden, Norway and the eastern part of Denmark. The annual energy consumption in Nordel is about 345 TWh and installed capacity is about 85 GW. Most of the interconnections between the countries are realised with 400 kV AC lines. Transmission distances in Nordel are long, which implies that the power transmission capacities are mainly determined by stability criteria.

Finland has transmission line interconnections to Sweden, Norway and Russia. The AC connections to Sweden, two 400 kV lines and one 220 kV line, are located in the northern Finland and the DC interconnection to Sweden, the Fenno-Skan HVDC link, is located in south-western Finland. The AC transmission capacity from Sweden towards Finland is about 1000 MW and the DC transmission capacity of the Fenno-Skan HVDC link is 500 MW. A 220 kV transmission line connects the Norwegian transmission network to the Finnish system in northern Finland. The power from Russia is transmitted to Finland by means of an asynchronous direct current connection of 1000 MW (back-to-back HVDC station in Vyborg). Figure 1 presents the main Finnish grid of 400 kV and 220 kV.

The annual energy consumption in Finland is about 65 TWh and installed production capacity is about 13.5 GW. The main consumption areas are situated in the southern part of the country. The main part of the energy is generated by hydro and thermal (nuclear, coal and gas) power plants (hydro 20 %, thermal 60 %). Hydro power plants are situated mainly in the northern Finland and thermal power plants in the coastal area of southern Finland. There are three big thermal plants, namely Olkiluoto (2 * 735 MW), Inkoo (4 * 250 MW) and Loviisa (4 * 220 MW). About 13.1 % of energy is imported, mainly from Sweden and Russia.

Owing to the concentrated power production, the import from Sweden in the north and the production of northern hydro plants, the operational conditions of the Finnish network are typical for a system with long-distance transmission, in which stability problems, especially voltage stability, dominates when the power transmission increases.

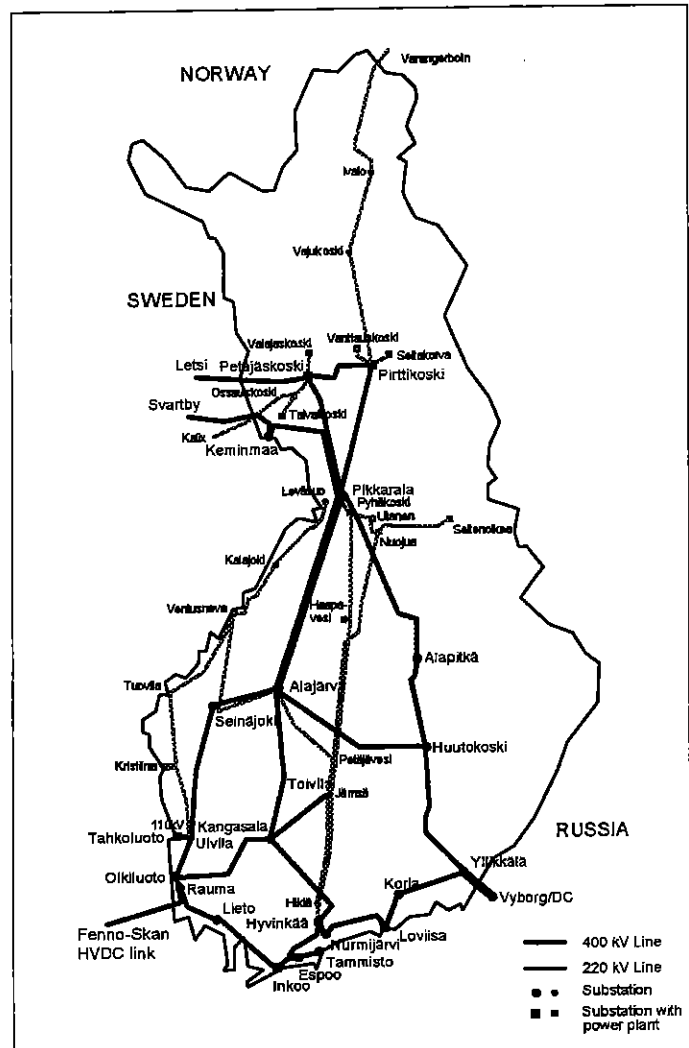


Figure 1. Finnish grid of 400 kV and 220 kV.

The transmission capacity is determined using the mainly deterministic $n - 1$ design criteria worked out by the Nordel countries [1]. According to criteria, no overloading nor loss of stability may occur as a consequence of dimensioning faults. These faults include, e.g. sudden outage of the largest production unit and three-phase short circuit at a station busbar without reclosures. The criteria must be fulfilled for the intact network and also for a weakened system with planned outages owing to maintenance of generators, transmission lines or station equipment.

It has been found out (from experience and from calculations) that the behaviour of the Finnish network depends mainly on two transmission parameters; AC transmission between Finland and Sweden (RAC) in the north and the north-south transmission in Finland (P1). The transmission capacity of the Finnish network can thus be presented with an area seen in Figure 2, where the vertical axis

presents the AC import/export Finland - Sweden (RAC) and the horizontal axis presents internal north-south transmission (P1) in Finland. The power system withstands all the faults defined by Nordel's dimensioning criteria when the operation point (transmission RAC and P1) is inside the area presented in Figure 2.

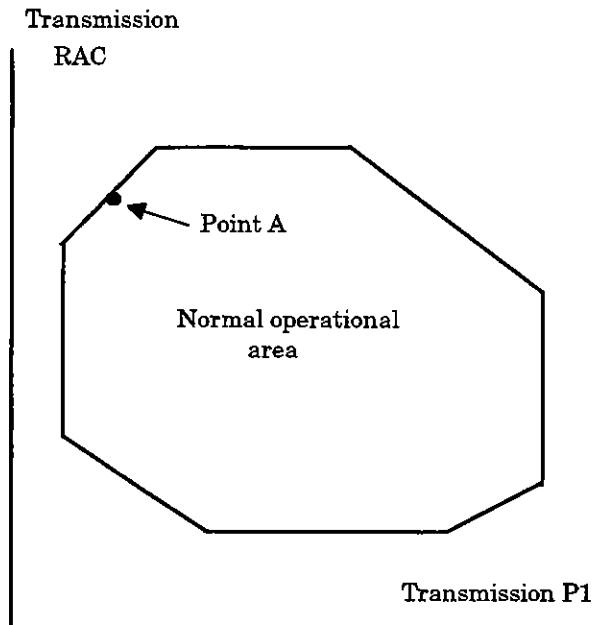


Figure 2. The transmission capacity of the Finnish network defined by Nordel's dimensioning criteria. Inside the area the normal operation of the network is maintained according to the dimensioning criteria. Transmission RAC is the AC transmission between Sweden and Finland in the north. Transmission P1 is the internal north - south transmission in Finland.

Network before the disturbance

Before the disturbance the transmission from northern to southern Finland (P1) was near its maximum and there were also high AC import from Sweden in the north (RAC). About 500 MW was imported from Sweden with Fenno-Skan HVDC link. Point A in Figure 2 presents the transmission before the fault. The transmission was near the border of the normal operational area, where the system tolerates all the dimensioning faults.

It was summertime and due to low loading there was only a few generators in production in the southern Finland. The load was supplied mainly by northern hydro power, communal power production and import from Sweden. From generators directly connected to the 400 kV network only 2 units in Olkiluoto (2 * 735 MW) and one unit in Loviisa (2*220 MW) were in production. Thus the voltage support from generators in southern Finland was considerably weak. However, the transmission capacity of the network was such that it could withstand the loss of one generator and its reactive support (n-1 criteria) without any voltage or stability problem (point A in Figure 2).

In the 400 kV grid there were planned maintenance outage on 400 kV Loviisa - Nurmijärvi line, which is the 400 kV transmission line westwards from Loviisa power plant. This maintenance work decreased the voltage support from Loviisa power plant to the Helsinki district in southern Finland and the generators of Olkiluoto in south-western Finland were the main generators to deliver the reactive power to the south and south-western part of Finland.

Disturbance

The incident was started by the tripping of one 735 MW generator in south-western Finland (Olkiluoto) on 14.8.1992 at about 9.50 a.m.. After the tripping the lacking active power was supplied from the northern hydro resources in Finland and in Sweden. This increased the transmission from Sweden and north-south transmission in Finland. With increase of the active power transmission the reactive power consumption also increased but the 400 kV voltages were decreased immediately after the tripping of generator only about 10 kV due to the activated reactive reserve of the generators. Figure 3 presents the north-south transmission in Finland (P1) during the disturbance.

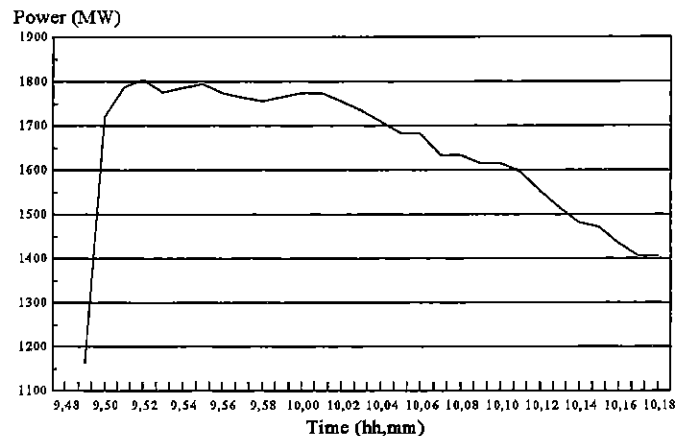


Figure 3. Transmission P1 (north-south transmission in Finland).

After about 3 minutes the reactive power of the other 735 MW generator in the same station (Olkiluoto) in south-western Finland was decreased manually due to the false interpreted alarm to the value it had before the tripping of the another generator. The alarm in the generator was due to increased shaft currents caused by increased reactive power production in generator and not indication of serious fault in generator. Because this generator was the main generator connected to the 400 kV network in south-western Finland the voltages decreased drastically in southern Finland after the decrease of the reactive power supply in 735 MW generator. The minimum voltage in 400 kV network was 344 kV and the network was near the voltage collapse. This low voltage level remained about 10 minutes. The voltage started to increase when the staff at aforementioned 735 MW plant increased the reactive power production to nominal value (about 400 Mvar) requested by National Control Centre and when some gas-turbine generators were started in southern Finland to decrease the transmission from northern to southern Finland. Figures 4 - 7 present the voltages in Finnish 400 kV stations during the disturbance.

Figure 4 show that the voltages stayed in the northern Finland over 370 kV during the incident. Near the production sites (as Petäjäskoski in Figure 4) the voltages were at acceptable level (over 380 kV) during the whole disturbance. Also in eastern Finland the voltages began to increase quite soon after the start-up of gas turbines at Huutokoski 400 kV station at about 9.58 a.m. as seen in Figure 5.

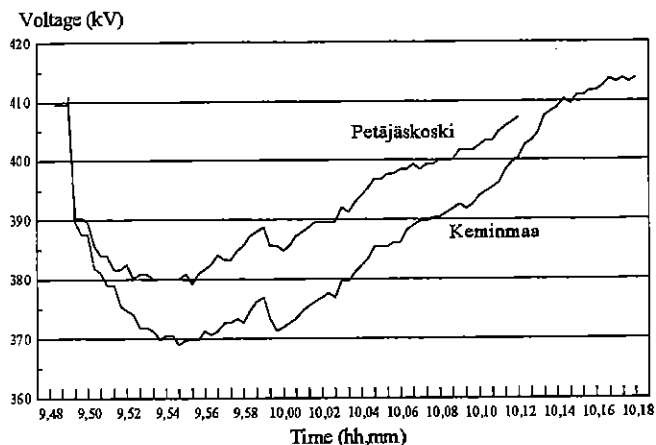


Figure 4. 400 kV voltages in northern Finland (Petäjäskoski and Keminmaa 400 kV station). Time is in hours and minutes (hh,mm).

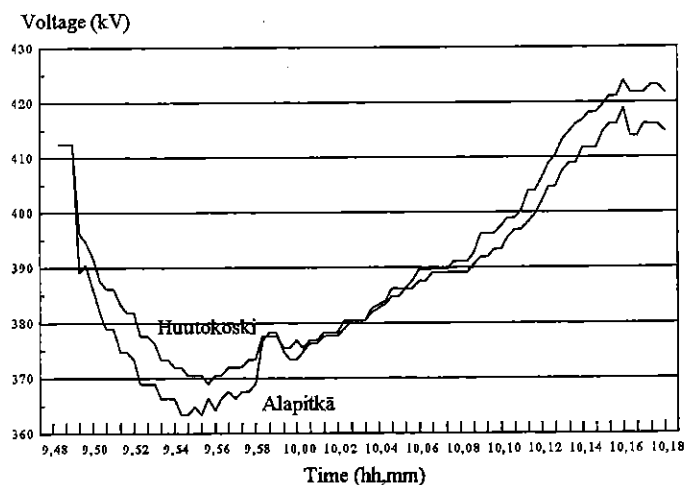


Figure 5. 400 kV voltages in eastern Finland (Alapitkä and Huutokoski 400 kV station). Time is in hours and minutes (hh,mm).

In western and southern Finland the voltages were dependent on the reactive power production of Olkiluoto generator as seen in Figures 6 and 7. The increase and decrease of reactive power in Olkiluoto generator are clearly seen in Figure 6.

Causes

Due to increased transmission from north to south Finland after the tripping of 735 MW generator in south-western Finland (Olkiluoto) the reactive reserve to maintain the voltages in southern Finland was decreased drastically after the manual decrease of reactive power in another remaining 735 MW generator in Olkiluoto. Because of the planned maintenance work on 400 kV line westwards from Loviisa power plant in south-eastern Finland the reactive reserve of Loviisa power plant could not be used to increase the 400 kV voltages in southern Finland.

This low voltage level should have been avoided if there had not been that manual reactive power decrease in 735 MW generator. This shows how important it is to take into account the actual reactive reserves when defining the transmission capacity in the network and to observe online the reserve requirements defined for the generators.

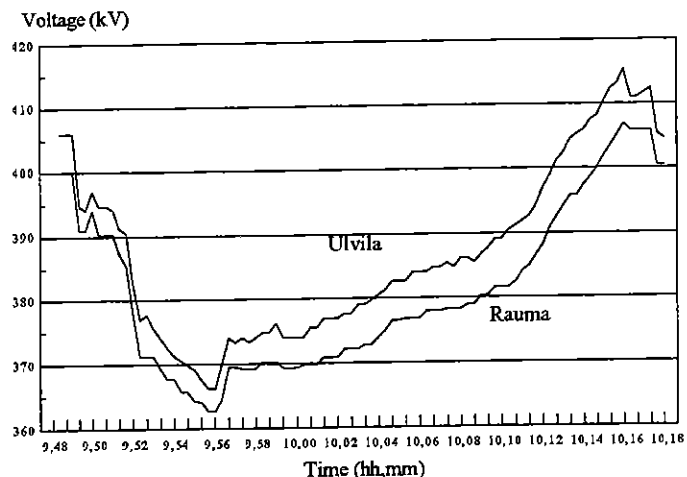


Figure 6. 400 kV voltages in western Finland (Ulvila and Rauma 400 kV station, which are nearest 400 kV stations to Olkiluoto power station). Time is in hours and minutes (hh,mm).

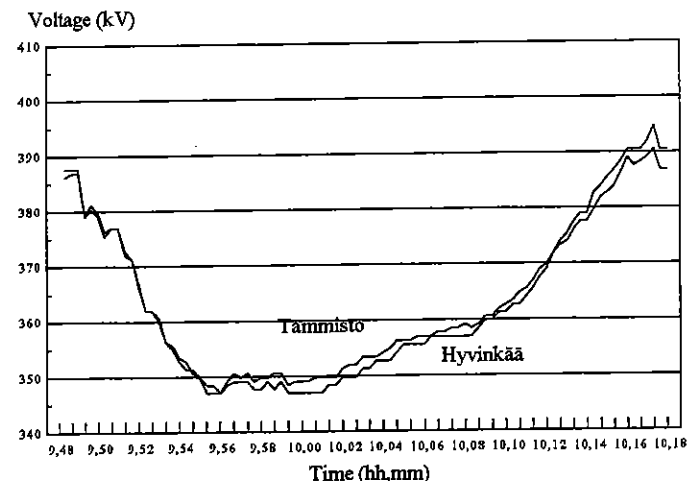


Figure 7. 400 kV voltages in southern Finland (Tammisto, and Hyvinkää 400 kV station). Time is in hours and minutes (hh,mm).

The decrease of the north-south transmission in Finland (P1) took long time because the start-up of gas turbines and increase of power production in southern Finland was delayed, mainly due to delayed power production in gas turbines. Transmission couldn't be decreased by import with Fenno-Skan HVDC link because the link was at its full import already before the disturbance. These problems with the power production caused that the voltages stayed low so long time. The normal level at 400 kV voltages was reached after 30 minutes the incident began.

Also the operation of transformer tap-changers after the voltage decrease increased the voltage dependent load to the pre-fault value thus increasing the transmission P1. From measurements it can be estimated that the amount of voltage dependence in load was about 120 MW.

Simulations

The incident gave new information about the behaviour of the Finnish power system when voltage stability is considered. By analysing the disturbance with power system simulation program the system model can be tested and verified.

The disturbance was analysed with PSS/E program. Table 1 shows the measured and calculated voltages at some 400 kV stations before the tripping of the 735 MW generator and immediately after the tripping. As seen from Table 1 the simulation model is a little bit conservative but otherwise the calculated values agree with measured values.

The significance of reactive power production at the 735 MW generator in south-western Finland (in Olkiluoto) was also evaluated with PSS/E program. Right after the tripping of another 735 MW generator and until the manual decrease of reactive power production the generator delivered about 300 Mvar. If the generator should have been able to produce 300 Mvar (nominal reactive power is about 400 Mvar) during the whole disturbance the voltages should have remained over 370 kV also in southern Finland. After the manual decrease the reactive power output of the generator was about 180 Mvar. The restriction of reactive power to this value in analysis caused that the system collapsed and the power flow didn't converge anymore.

References

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Discussion

Edited by Nicholas W. Miller, General Electric Company

Thomas Overbye (University of Illinois):

I was particularly interested in the last point you mentioned in your presentation, that you have a situation where the power flow didn't converge, and presumably did not have a real solution any more, but yet the system still existed, it was just undergoing a very slow voltage collapse. That was a very interesting situation to occur, that in actual practice we had a situation where a power system no longer had a powerful solution. Did you check any other solution methods to see whether the power flow just did not converge or whether there was no real solution?

Liisa Pottonen (IVO Transmission Services):

Well, actually no. We simulated it so that immediately after the disturbance we perceived a solution that was really near what we had measured. And then, step by step, we reduced the reactive power of the overloaded generator, about ten megavar per step, and then we checked the voltage and we could see the nose curve decreasing. But of course, we couldn't reach the end. But, I think for us it's not of importance since we have now tested that the network has some implemented reserves, so we have some kind of safety margin in it. So if you can simulate, we can be sure that the network can stand it.

Overbye: When you go down to minus one hundred and eighty megawatts you couldn't get that to converge?

Table 1. Measured and calculated (by PSS/E program) voltages before and immediately after tripping of 735 MW generator (in Olkiluoto) at some 400 kV stations in Finland (location of station see Figure 1).

400 kV station	Measured voltages (kV)		Calculated voltages (kV)	
	Before fault	After fault	Before fault	After fault
Keminmaa	411	389	411	385
Petäjäskoski	409	390	412	389
Huutokoski	412	396	415	395
Ulvila	406	396	407	393
Rauma	400	390	405	393
Inkoo	391	384	391	381
Hyvinkää	387	379	387	374
Tammisto	386	379	388	378
Espoo	388	380	389	378

Pottonen: No, the worst situation, the minimum production of reactive power, we could not simulate.

Yakout Mansour (BC Hydro):

And we assume that the load is constant?

Pottonen: Yes, we used constant load.

Mansour: That's why the real system slows itself, because it's not constant load.

Pottonen: Yes. Ultimately we used different kinds of loads in ordinary load flows, but in this situation the voltage at the lower levels was not decreased very much, so I think that's not so important in this case.

Marija Ilic (M.I.T.):

I was particularly intrigued by your comments about forcing reactive power manually. That point was brought up this morning in context of secondary control: that you either have the Q loop at the generators, which controls Q in addition to controlling the voltage of the generator, or you don't have it. So my question to you is: when you said that Q was reduced manually, what did that mean? Is there a knob on the generator to reduce Q on the Finnish system?

Pottonen: Ordinarily, this nuclear power station on auto load will regulate the voltage at the level. I don't know exactly what button the pushed, but I suppose it was something in the excitation system. That's not interesting actually for us.

Ilic: Could you ask the operators what they actually did?

Pottonen: Okay, I can do that.

DETERMINING PRACTICAL TRANSFER LIMITS FROM A VOLTAGE COLLAPSE ASPECT

M F Hadingham
(Eskom)

1.0 ABSTRACT

The paper develops a way to determine the steady state operating limits on a transmission system whose load is approaching the voltage collapse limit. Criteria including the load at which collapse occurs, the voltage at which it occurs, actual network voltages and reactive reserve on the main reactive generator are examined and limits set. Limits using these criteria are determined for an actual system with and without an SVC installed at the load centre.

2.0 INTRODUCTION

The ESKOM network includes a 1500 km radial transmission system from the main generation centre which is approximately 100 km from Johannesburg to Cape Town. At Cape Town there is a two set Nuclear power station at Koeberg. The loading of the top half of the transmission system from the main generating centre has not reached any limits even under credible contingencies. This is not true of the other half of the transmission system especially if the two Koeberg units are off and if there is an outage of a transmission line. Figure 1 shows the part of the transmission system of interest.

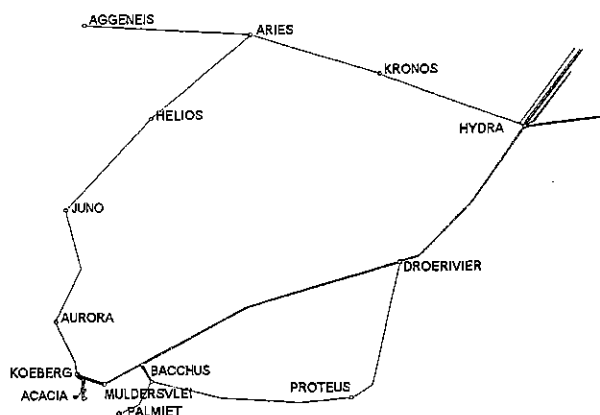


Figure 1

Besides the nuclear power station, there is a 400 MW pump storage scheme at Palmiet, at Hydra there are two SVCs and at and around Muldersvlei there are switched shunt capacitors. All lines longer than 150 km are series compensated.

With no generation from the nuclear power plant, the power flow is from Hydra towards Aries and Droërvier. From Droërvier the power flows towards Proteus and Muldersvlei and from Proteus towards Muldersvlei.

The three critical lines are the Bacchus-Muldersvlei, the Aurora-Juno and the Kronos-Aries lines. A loss of one of these lines with no Nuclear generation trends to put the system at risk. Loss of the Kronos-Aries line, reduces the number of parallel lines from three to two and adds an additional load to that in the Cape Town area. Loss of the Bacchus-Muldersvlei line reduces the number of lines into the Cape Town area from three to two and moves the Palmiet generation source away from the Cape Town area significantly.

The network being close to its transmission limit, it is necessary to determine operating limits beyond which switching out of less critical load must be considered. In order to determine the limits it is necessary to set out the criteria for the limits. At the present the network with no nuclear generation tends to a classical radial network with very limited local voltage control at the end. In the future a SVC is proposed at Muldersvlei. It is necessary to determine the limits of the system with and without the SVC.

3. CRITERIA FOR TRANSFER LIMITS

The transfer limits of a transmission system are based on thermal overload of lines and equipment, limits due to voltage collapse and limits due to transient stability. Computing of limits due to thermal overload and transient stability are well known. Determining limits due to voltage collapse is not as well documented. The general factors, which can affect transfer limits in the voltage collapse area are :

- $\frac{dV}{dP}$ - the rate of change of voltage with load.
- the safety margin between the operating point and the point of maximum power transfer.
- the voltage at which voltage collapse would occur.

- the safety margin between the output of the most important VAr source and its maximum output if applicable.
- adequate and reasonable voltages to supply the load.

The dV/dP criteria is required to prevent excessive tap changer operations especially in the lower voltage networks. Step changes in voltage due to network and generation changes should be relatively infrequent and it is accepted that there will be tap changer operations after such events. However the load level is generally fluctuating and too frequent tap changer operations to accommodate the change in voltage due to load fluctuation are not acceptable. A limit of 5 % voltage change for a 10 % load variation is suggested.

The safety margin between the operating point and the point of maximum power transfer is almost evident. This caters for transient load pulls and to allow a margin of error, a safety margin of 10 % is proposed.

The addition of shunt capacitors raises the level at which voltage collapse occurs and in the extreme, voltage collapse could happen at voltages greater than those during normal operation. If voltage collapse occurs close to normal operating voltages it is still dangerous and under these conditions a voltage collapse could occur without the operators knowing they had even been near voltage collapse. It is suggested that the voltage at the voltage collapse point or maximum transfer point be no greater than 90 % of U_n . Typically the 400 kV transmission system is run at 103% to 104% of the U_n giving a 13% to 14% margin between normal operating voltage and the voltage at voltage collapse. The issue of the voltage at which voltage collapse occurs is seen as important if no major voltage regulating MVar sources are in service. If there is a major voltage regulating MVar source then this issue is not as important as long as the MVar source is not operating beyond or even close to its limit.

If a significant voltage regulating MVar source is installed in a strategic position in the transmission system the transfer of the transmission system is increased. The greatest danger from a voltage collapse point of view occurs if the MVar source should reach its limit of generating capacitive VARs. This is particularly true of a SVC which, when it reaches its limit, becomes a plain capacitor. At the point, where the limit is reached, the incremental change of voltage versus power become very large and could even be negative. Under these conditions voltage collapse may be more than a possibility.

An adequate and reasonable voltage to supply the load is an obvious requirement and is a necessary

consideration for a transmission system when the voltage at voltage collapse is much lower than an adequate and reasonable operating voltage. The limit for the operating voltage would be determined by the tapping range of the transformers supplying the distribution system from the transmission system. Typically a limit of 95 % for the transmission voltage would be allowed.

From the above discussion, it can be seen that the various limits are not always applicable in all cases.

4. TRANSFER LIMITS

Under normal conditions, that is with nuclear generation available in the Cape Town area, there are no transfer limits with the system healthy and with a line out of service for the present loads. Without any nuclear generation, the transmission system is adequate without any lines out of service but transfer limits are necessary to limit the load under critical line outages. Initial studies identified the Aries-Kronos, the Koeberg-Aurora and the Bacchus Muldersvlei as the critical line outages for the present loads. For the studies, it was decided to consider all lines except the critical ones in service and all reactive VAr sources in service since the nuclear generation is reasonably reliable and the chances of have no nuclear operation, a critical line out and a reactive VAr source out together is thought to be too unlikely.

The studies were conducted using Q-V curves. These curves allow the limits of the system with an without the addition of the 300 MVar SVC planned for end 1995 without doing additional studies.

4.1 TRANSMISSION LIMITS UP TILL END 1994 WITH NO SVC

Figures 2,3 and 4 show the Q-V curves for no nuclear generation with no Aurora-Juno line, no Aries-Kronos line and no Bacchus-Muldersvlei respectively.

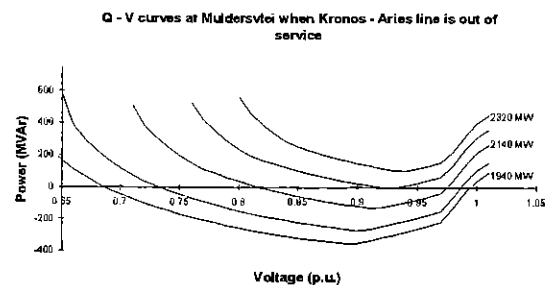


Figure 2

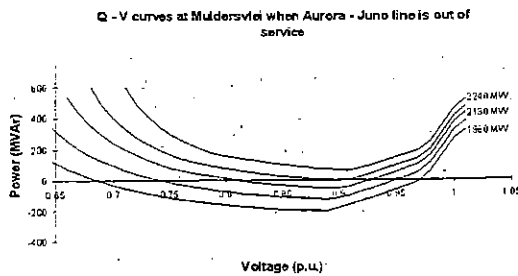


Figure 3

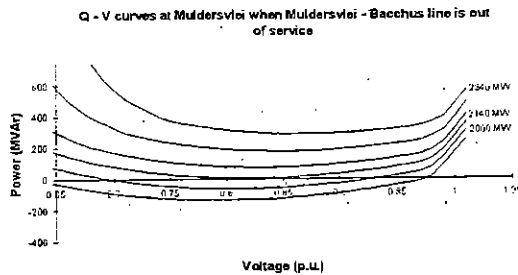


Figure 4

Table 1 - Kronos Aries Line Out

Load MW	1980	2020	2060
Criteria			
dV/dP	2%/10%	2,03%/10 %	2,3%/10%
Load/ P_{max}	,883	0,90	,918
V Collapse %	90,1	90,1	91%
V Working %	99,0	98,6	98,3

From this table it is seen that the critical factors are the load margin to collapse and the voltage at which collapse occurs.

No rapid changes in dV/dP , Load/ P_{max} , V collapse or V working are seen for a 2% change in load. For this reason and the fact the load of 2020 MW meets the criteria a transfer limit of 2020 MW is satisfactory.

Table 2

Line Out	Aries Kronos	Aurora Juno	Bacchus Muldersvlei
Criteria			
90% load at voltage collapse	2020	1960	1860
dV/dP	2,03%/10%	4,7%/10%	4,8%/10%
V Collapse	90,1	89,2	78%
V Working	99	97,3	98,6%

Table 2 has been drawn up by assuming the transfer limit is determined by a Load/ P_{max} of 0.9 and checking that the other factors are within specified limits. In all cases the critical factor was the ratio of load to peak power and the transfer limits will be those given by that criterion.

4.2 TRANSMISSION LIMITS FROM END 1994 WITH SVC

Figures 5,6 and 7 are the same as Figures 2,3 and 4 with the addition of the Muldersvlei SVC characteristic.

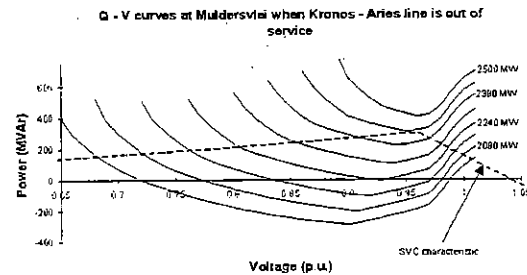


Figure 5

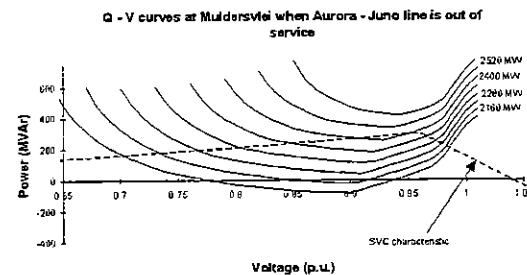


Figure 6

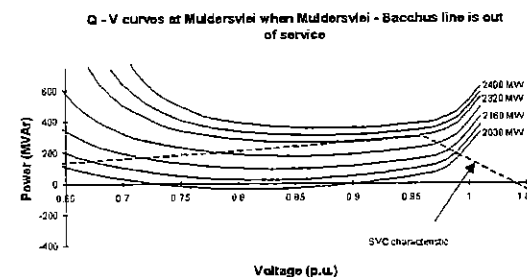


Figure 7

It is obvious from these figures that the voltage droop characteristic of the SVC in the region where it is able to control the voltages is very important as far as the transfer limits are concerned. The greater the droop, the greater the power transfer can be. The limit on the droop is that the SVC must be able to go from one end of its range to the other without the terminal voltage exceeding U_m or going below 95% U_n and the dV/dP should not exceed 5%/10%.

Another consideration is that the voltage of the SVC at its full capacitive output should be to the right of the minimum of the Q-V curve. This will at least keep the system on the stable side of the Q-V curve if the output of the SVC is exceeded. It is necessary to first determine which set of Q-V curves has its minimum at the highest voltage. This, with a margin, sets the limit on the voltage at the maximum SVC output. For this study the minimum voltage of the Q-V curves at the maximum Q of the SVC occurred on outage of Aries-Kronos. The voltage of the SVC was set to 96% at maximum output. The limit at the minimum output of the SVC, zero in this case was set to 104%, just below the U_m of the system. These voltage limits will give the maximum safe power transfer on the system. Table 3 was drawn up assuming the output of the SVC should not go above 200 MVar or 66,6% of its rating and checking the other criteria at this load. For voltage collapse criteria, this was taken to be the voltage at which the SVC reaches its capacitive output limit.

Table 3

Line Out Criteria	Aries - Kronos	Aurora - Juno	Bacchus - Muldersvlei
Load, SVC @ 200 MVar	2220	2100	2120
dV/dP	2,2%/10%	1.9%/10%	1.4%/10%
V Collapse %	96	96	96
VWorking %	98.6	98.6	98.6
Pworking/ Pmax	92.6%	88.6%	91.8%

All the criteria are met except P working/Pmax <90%. Ensuring that this criteria and the criteria that the SVC should not exceed 200 MVar at the transfer limit gives the following limits, Table 4.

Table 4

Line Out Criteria	Aries - Kronos	Aurora - Juno	Bacchus - Muldersvlei
Transfer limit MW	2190	2100	2080

5. CONCLUSIONS

For determining the steady state voltage collapse Q-V curves are relevant as the exact point of voltage collapse can be determined.

The settings of the voltage droop characteristic is important in order to get the most benefit from the

installation of a SVC installed to extend the capability of a transmission system.

From an operating point of view the point of voltage collapse is not the only aspect that should be investigated. The rate of change of voltage with load, the voltage at which collapse occurs, adequate system voltages and sufficient margins of Q generation on reactive power sources are also important. For this investigation, the important factors were the load at the point of voltage collapse, the voltage at which collapse occurs and the margin of Q generation on reactive power sources.

The SVC increases the transfer limits by between 140 MW and 220 MW. If there is no nuclear generation the transfer limit would be 1860 MW before the SVC is installed and 2080 MW after it has been installed. This will also be the limit if the Bacchus-Muldersvlei line is out of service and only one nuclear set is running. If the Aurora-Juno line is out instead of the Bacchus-Muldersvlei the limits without and with the SVC are 1960 MW and 2100 MW and if the Aurora-Koeberg line is out instead of the Bacchus-Muldersvlei the limits are 2020 MW and 2190 MW.

The effect of the SVC is to increase the overall transfer limit by 220 MW. This is just over 1 MW of transfer capability for 1 MVar of reactive power generated by the SVC.

6. ACKNOWLEDGEMENT

To assistance of Mr G Hurford in executing the studies and preparing the results is gratefully acknowledged.

REFERENCES

1. "Modelling of Voltage Collapse Including Dynamic Phenomena" Electra No 147 April 1993
2. "Planning Against Voltage Collapse" Electra No 111 March 1987.

Discussion

Edited by Nicholas W. Miller, General Electric Company

Nick Miller (General Electric):

I was interested to note early in your talk that you said that at least under one condition you were adopting a policy of self-load shedding, where if you don't have the nukes on line you just let the load sort of take care of itself. We have observed in a number of systems that the line protection, overcurrents and/or second or third zone distance relaying, gets into the picture before the loads take care of themselves. Have you been keeping an eye on the relaying, to see whether or not you end up inadvertently having your line protection in on the voltage load shedding?

Mike Hadingham (ESKOM)

We are very fortunate: we don't have any overcurrent protection on our transmission lines. Whether we would pick this up on the distance protection, I am not sure, but I agree we should look at that.

Session 3

Security Analysis - I

"Some Optimality Notions of Voltage Profile
for the Steady-State Operation of Electric Power Systems"
Marija D. Ilic, Xiaojun Liu: Massachusetts Institute of Technology
Christine Vialas, Electricité de France

"Extensions and Applications of a Mid-Term Voltage Stability Analysis Method"
Thierry van Cutsem: University of Liège
Y. Jacquemart, J.-N. Marquet, P. Pruvot
Electricité de France

"Power System Modeling for Coupled Voltage/Power Dynamics"
J. W. Chapman, Marija D. Ilic
Massachusetts Institute of Technology

"Considerations on a Practical Neural Network for Voltage Stability Assessment"
Yutaka Tada, Yasuhiro Hayashi, Shinichi Iwamoto
Waseda University

"Assessment of Transient Voltage Stability
Based on Critical Operating Time of Emergency Control Using Neural Networks"
Tasumi Maeda, K. Takigawa, Y. Minato
Shikoku Research Institute
A. Yokoyama: University of Tokyo

Some Optimality Notions of Voltage Profile for the Steady-State Operation of Electric Power Systems

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Abstract

In this paper the issue of optimal steady-state voltage in electric power system operation is analyzed on the simplest topology of a power network.

The importance of this topic is far reaching, for both meaningful use of static type optimization tools, as well as for defining performance criteria in designing automated secondary and tertiary level voltage controllers. Related open questions for systems of realistic size are stated.

1 Introduction

The motivation for this paper is two-fold. First of all, historically in the power industry scheduling of real power generation has been the prime goal, while maintaining voltages within the prespecified limits is assumed. With the increased number of technologies for direct voltage control it is becoming important to develop a full understanding of optimal voltage profile in support of the main task of delivering real power.

Secondly, it is necessary to develop a theoretical framework for defining optimal voltages under often qualitatively different operating modes, which do not lend themselves to the same objectives.

While the problem of optimal voltage performance in power systems has been studied in the past [1], [2], [3], no definite conclusions are readily available.

We have recently re-visited this problem in context of performance criteria for tertiary voltage coordination of the large interconnected system under a joint MIT/EDF research project [4]. The results of this study are presented in this paper only for the simplest network topology. The findings for large systems are presented in [4], [6].

It is concluded that the topic of optimal voltage profile still poses many theoretical challenges.

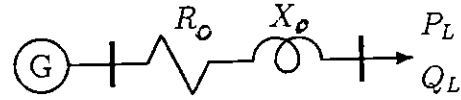


Figure 1: Simplest Topology of a Power Network

2 Voltage "Optimality" on the Simplest Power Network Topology

Consider an electric power network consisting of one generator and one load, given in Figure 1.

The generator is connected to the load through a transmission line whose parameters are line resistance R_0 , and line reactance X_0 . The steady state generator model is assumed to be an ideal voltage source, whose magnitude V_G and angle are fixed.¹ The real power limits on the generator output are given as

$$P_G^{min} < P_G < P_G^{max} \quad (1)$$

The load model used is given as

$$P_L = P_0 + \frac{V_L^2}{R} \quad (2)$$

and

$$Q_L = Q_0 + \frac{V_L^2}{X} \quad (3)$$

In this section first the theoretical question of optimal V_L^{opt} is studied.²

We suggest here a notion of voltage optimality by recalling that the basic function of the network given in

¹In order to have a PV representation of generators, one needs at least two generators.

²It is straightforward to compute the corresponding optimal voltage at the generator V_G^{opt} , with V_L^{opt} known. Depending on the control technology either of the two could be useful to know for control purposes.

Figure 1 is to deliver required average (real) power to the load within the given constraints on generation (1). The voltage operating constraints on both load and generator terminal voltages have to be met also, as

$$V_G^{min} < V_G < V_G^{max} \quad (4)$$

and

$$V_L^{min} < V_L < V_L^{max} \quad (5)$$

Definition of optimal network operation

A power network given in Figure 1 is defined as operating optimally if it is capable of meeting the largest range of real power load variations P_L within the pre-defined operating constraints (1), (4) and (5).

This definition is particularly justifiable for unusual system operation when load varies significantly away from its nominal value. Further reasoning for introducing this definition is described in context of mathematical derivations in this paper.

We also observe here that the theoretical problem of optimal operation does not lend directly itself to any circuit theoretic results [2], [5] which are unconstrained (global) optimization problems, in contrast to the problem of constrained optimization defined here for operation of power systems. Moreover, these results are stated mainly in context of optimal load matching on linear networks with ideal sources and impedance type loads. The problem of optimal voltage profile in power systems is concerned, in addition to optimal load matching, with the issue of optimal voltage settings for controllers which directly regulate voltage magnitude, such as excitation systems on generators. Loads are generally modeled as non-linear static devices, and generators are not independent voltage sources. This makes the issue of optimality much more complex.

Another intriguing issue is that sufficient conditions for the optimal power network operation as defined above are likely to be operating mode dependent. This is well recognized in practice. Particularly well studied are problems of peak-load conditions in [1]. Optimal voltage strategies for medium and low load conditions are not well supported by the theory. Such optimality conditions in support of strategies for medium and low load demand are derived in this section for the first time.

2.1 The Problem of Optimal Load Voltage Setting

To start with, the load flow equations for the network given in Figure 1 take on the form

$$V_L V_G \cos \theta - V_L^2 = a \quad (6)$$

$$V_L V_G \sin \theta = b \quad (7)$$

where

$$a = R_0 P_L + X_0 Q_L \quad (8)$$

$$b = X_0 P_L - R_0 Q_L \quad (9)$$

Note that variables a and b are quadratic functions of load voltage V_L and can be re-written as

$$a = a_0 + a_1 V_L^2 \quad (10)$$

$$b = b_0 + b_1 V_L^2 \quad (11)$$

where

$$a_0 = R_0 P_0 + X_0 Q_0 \quad (12)$$

$$b_0 = X_0 P_0 - R_0 Q_0 \quad (13)$$

$$a_1 = \frac{R_0}{R} + \frac{X_0}{X} \quad (14)$$

and

$$b_1 = \frac{X_0}{R} - \frac{R_0}{X} \quad (15)$$

An implicit relationship between the load voltage V_L and generator voltage V_G which directly follows from the above load flow constraints is

$$V_G^2 = \frac{c_0}{V_L^2} + c_1 V_L^2 + 2[a_0(a_1 + 1) + b_0 b_1] \quad (16)$$

with the notation for c_0 and c_1 as

$$c_0 = a_0^2 + b_0^2 \quad (17)$$

and

$$c_1 = (a_1 + 1)^2 + b_1^2 \quad (18)$$

It is straightforward to show that the extremum of this function occurs at

$$V_L^4 = \frac{c_0}{c_1} \quad (19)$$

The load voltage corresponding to this minimum is denoted as $V_{L,N}$, since it corresponds to the same value on the "PV" (nose) curve [1].

Two other conditions relevant for understanding optimal load voltage profile of the given network are related to the values of load voltage $V_{L,P}$ and $V_{L,Q}$ which correspond to the minimum generation real and reactive power output P_G and Q_G , respectively.

They directly follow from stating sufficient conditions for optimizing real power generation

$$P_G = \frac{R_0}{X_0^2 + R_0^2} (V_G^2 - V_L^2 - 2a_0 - 2a_1 V_L^2) + P_0 + \frac{V_L^2}{R} \quad (20)$$

and reactive power generation

$$Q_G = \frac{X_0}{X_0^2 + R_0^2} (V_G^2 - V_L^2 - 2a_0 - 2a_1 V_L^2) + Q_0 + \frac{V_L^2}{X} \quad (21)$$

with respect to V_L^2 .

Minimum of P_G occurs at

$$V_{L,P}^4 = \frac{c_0}{c_1 - (1 + 2a_1) + \frac{X_0^2 + R_0^2}{R_0 R}} \quad (22)$$

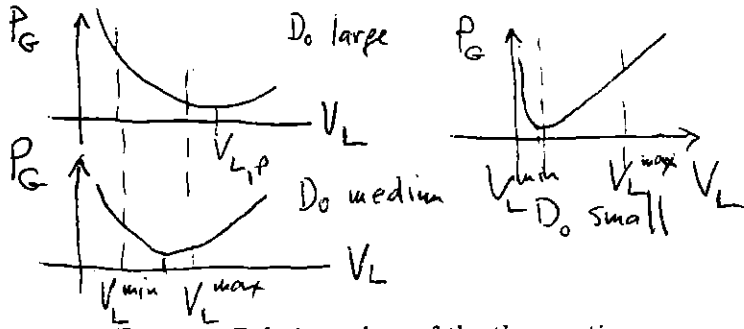


Figure 2: Relative values of the three optima

The minimum of Q_G occurs at

$$V_{L,Q}^4 = \frac{c_0}{c_1 - 2(1 + 2a_1) + \frac{X_0^2 + R_0^2}{X_0 X}} \quad (23)$$

In order to analyze relative values of $V_{L,N}$, $V_{L,P}$ and $V_{L,Q}$, we define next

$$r = \frac{R_0}{X_0} \quad (24)$$

$$q = \frac{X}{R} \quad (25)$$

For typical power systems both r and q are smaller than one in magnitude.

The optimum load voltages $V_{L,N}$, $V_{L,P}$ and $V_{L,Q}$ take on the form

$$V_{L,N}^4 = \frac{X^2(1+r^2)(P_0^2 + Q_0^2)}{(\frac{X}{X_0} + qr + 1)^2 + (q-r)^2} \quad (26)$$

$$V_{L,P}^4 = \frac{X^2(1+r^2)(P_0^2 + Q_0^2)}{(1+rq)^2 + (q-r)^2 + \frac{q}{r}(1+r^2)} \quad (27)$$

and

$$V_{L,Q}^4 = \frac{X^2(1+r^2)(P_0^2 + Q_0^2)}{(1+rq)^2 + (q-r)^2 + (1+r^2)} \quad (28)$$

First of all, we observe that the right hand side of all three expressions is always positive, implying that these minima always exist. To study their relative values, let us consider three different cases:

Case 1: $q < r$

For this case it can be shown that

- $V_{L,Q} < V_{L,P}$.
- It is slightly more involved to show that also $V_{L,N} < V_{L,Q}$, for the typical design of $\frac{X}{X_0}$ larger than one in magnitude.

Case 2: $q = r$

It can be shown that in this case

- $V_{L,Q} = V_{L,P}$
- $V_{L,N} < V_{L,Q}$

Case 3: $q > r$

In this case

- $V_{L,P} < V_{L,Q}$ and
- $V_{L,N} < V_{L,P}$

To interpret these relations in terms of loading levels, define next a variable

$$D_0^2 = \frac{P_0^2 + Q_0^2}{1 + q^2 + \frac{q}{r}} \quad (29)$$

This quantity is referred to as the net power demand as seen by the load which has a constant power component P_0 and Q_0 according to the load model given above in Equations (2) and (3). Given the linear parameters r and q of the transmission line and the load, respectively, D_0 is small for small P_0 and Q_0 , medium when P_0 and Q_0 are intermediate in size and large, in the other extreme.

It follows from the above relations that

$$V_{L,P}^2 = X D_0 \quad (30)$$

This leads us to a possible strategy for choosing optimal V_L^{opt} .

2.1.1 Optimal Strategy for V_L^{opt}

In order for the simple power system to operate optimally according to the definition proposed in this paper, it is necessary to operate at the conditions when P_G is minimized. This corresponds to the load voltage $V_{L,P}$ introduced above. It follows from (30) that when D_0 is large $V_{L,P}$ is large. This quantity decreases as D_0 , i.e. net load, decreases.

This allows one to establish an operating strategy for setting V_L^{opt} within the prespecified operating limits on load voltage (5). The three plots representing dependence of P_G on V_L in Figure 2 indicate a strategy for choosing optimal V_L^{opt} , which is summarized as follows:

- Choose $V_L^{opt} = V_L^{max}$ for heavy (peak) load.
- Choose $V_L^{opt} = V_{L,P} = \sqrt{X D_0}$
- Choose $V_L^{opt} = V_L^{min}$ for low load.

These results are directly related to the present practice. At high load (peak) operating at the highest possible load voltage is considered to be the best.³ Quantification of the optimum load voltage at medium and low load conditions is derived theoretically for the first time in this paper.

2.2 Constrained (Local) Maximum Power Transfer for Optimal Load Compensation

A second important problem related to the optimum voltage /reactive power management in steady state is the choice of optimum use of devices which directly effect parameters of transmission network and/or load

³In practice this thinking is more pronounced in terms of having minimum transmission loss for high voltage.

characteristics. Typical devices are series capacitors and shunt capacitors/inductors. Often these devices are used for load power factor compensation or for changing impedance properties of transmission lines. The process of changing load and line properties by these devices can be directly related to the well-known circuit theoretic problem of load matching so that the system delivers maximum power to the load [5]. In attempting to use the matched load conditions result for this purpose on the simplest power network topology, it is important to recognize two fundamental obstacles. The first is the distinction from the problem of interest in power systems in terms of operating constraints on acceptable voltages (5) and (4). The second problem is that the maximum power transfer theorem does not apply directly to linear networks with non-linear loads, as is the case with typical load models in power systems.

This requires development of new theoretical results. In this section we introduce a definition of constrained (local) maximum power transfer for the case of constant power load model. This load model is a degenerate case of the model given above in (2) and (3), with only P_0 and Q_0 .

The approach assumes given voltage magnitude V_G at the generator terminal. Starting with the same load flow equations as in the previous subsection, solution for load voltage V_L in terms of a specified generator voltage V_G can be found as

$$2V_L^2 = V_G^2 - 2a \pm \sqrt{V_G^4 - 4(aV_G^2 + b^2)} \quad (31)$$

In order for a real-valued solution to exist, one must require

$$V_G^4 > 4(aV_G^2 + b^2) \quad (32)$$

This inequality is the fundamental system constraint, and is seen as the starting relationship for defining constrained maximum power transfer. Obviously, when $R_0 = X_0 = 0$, a solution always exists. When these quantities are non-zero, only a limited amount of power can be delivered to the load, for a specified V_G .

Let us consider V_G to be fixed and convert the inequality in (32) into an equality to study the extreme case of interest. This equality determines a relationship between P_0 and Q_0 .

The global (un-constrained) maximum operating point for P_L is the absolute power the network can deliver to the load, under specified V_G . This maximum is easily obtained from the sufficient condition

$$\frac{dP_L}{dQ_L} = 0 \quad (33)$$

leading to values at this optimum

$$P_L = \frac{V_G^2}{4R_0} \quad (34)$$

and

$$Q_L = -\frac{XV_G^2}{4R^2} \quad (35)$$

When the load is modeled as a constant impedance load $\hat{Z} = R + jX$, the maximum occurs at the load impedance values $R = R_0$ and $X = -X_0$.

The load voltage corresponding to this operating point is

$$V_L = \frac{\sqrt{R_0^2 + X_0^2}}{2R_0} V_G \quad (36)$$

and it is typically much higher than the maximum load voltage defined in (5).⁴ Because of this inherent over-voltage occurrence, operating at the global maximum power transfer is unacceptable. A more practical approach to defining local optima under a given load power factor results in acceptable solutions. In this subsection we further analyze operating conditions under a local optimum, after formally introducing a general definition for useful constrained (local) maximum power transfer in power networks.

2.2.1 Constrained (Local) Maximum Power Transfer

To extract fundamental characteristics of the power system, and also make the analysis general, we use dimensionless quantities defined as⁵

$$r = \frac{R_0}{X_0} \quad (37)$$

$$q = \frac{Q_0}{P_0} \quad (38)$$

$$v = \frac{V_L}{V_G} \quad (39)$$

and

$$p = \frac{X_0 P_0}{V_G^2} \quad (40)$$

In this notation the load power factor becomes a function of variable q only, and it is of the form

$$\alpha = \frac{1}{\sqrt{1 + q^2}} \quad (41)$$

Constant load power factor implies constant q , and large q corresponds to a small power factor.

Using this notation, the inequality constraint (32) becomes

$$(r + q)p + (1 - rq)^2 p^2 < .25 \quad (42)$$

This explicitly defines limits on achievable p as

$$p_m < p < p_M \quad (43)$$

where the limits are

$$p_M = \frac{1/2}{\sqrt{(1 + r^2)(1 + q^2) - (r + q)}} \quad (44)$$

⁴This is generally true when $\frac{R_0}{X_0} < 1$.

⁵The notation for q is identical to q used in context of optimal load voltage problem for the case of constant impedance load.

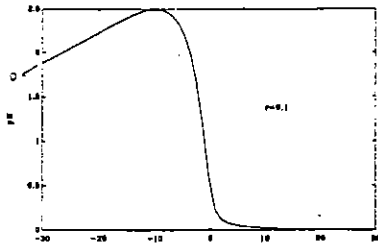


Figure 3: Dependence of p_M on q

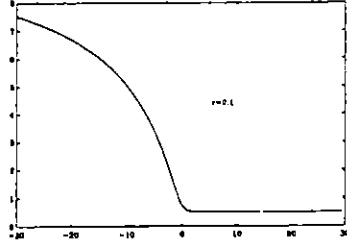


Figure 4: Dependence of v_M on q

and

$$p_m = \frac{1/2}{\sqrt{(1+r^2)(1+q^2) + (r+q)}} \quad (45)$$

Equation (43) clearly indicates that, for a given r and q , p is bounded by both a minimum and a maximum value. To distinguish these extrema from the global maximum power transfer, we refer to them as the local optima.

Observe that the constraint (43) is just a load flow feasibility constraint. Optimality of p_m and p_M with respect to transmission line and load compensation is studied next.

The limits on load voltage achievable at the p_m and p_M , respectively is

$$v_m = \sqrt{1/2 - (r+q)p_m} \quad (46)$$

and

$$v_M = \sqrt{1/2 - (r+q)p_M} \quad (47)$$

To further analyze the dependence of these extrema on line and load parameters r and q , we plot p_M and v_M as a function of load characteristics q , for a fixed line parameter r , as illustrated in Figures 3 and 4, respectively.

It can be further seen from Figure 5 that overvoltage is inevitable at the global maximum denoted as GM . It can be avoided at a local maximum by choosing appropriate q . For example, it can be seen from the same Figure that near $q = -.72$ $v_M = 1$ for $r = .1$.

It is interesting to observe further theoretical relations between the global maximum power transfer and the local maximum p_M . The global maximum can be "re-discovered" from the condition

$$\frac{dp_M}{dq} = 0 \quad (48)$$

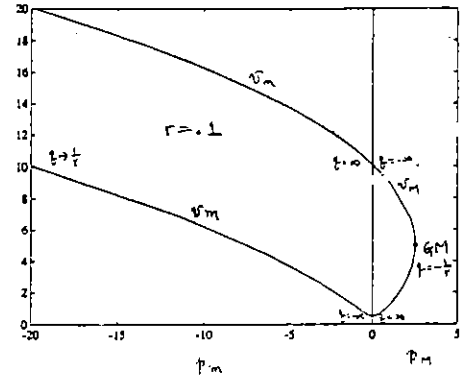


Figure 5: Parametric Dependence of Achievable Voltage and Power Limits on Load Compensation, Given Line Parameters

In other words, the global maximum is the maximum feasible (possible) power delivered to a load, further optimized with respect to load parameter q !

It can be shown that the optimum defined in (48) occurs when

$$rq = -1 \quad (49)$$

For $r = .1$ in Figure 5 it is sufficient to have $q = -10$ to achieve global maximum power transfer. Recalling that q corresponds to the ratio of load reactive and real power, it is clear that such q leads to a strong overcompensation (therefore high load voltage).

It is interesting to note that at the global maximum the real power loss on the transmission line is equal to the real power delivered to the load, and the reactive power line loss is equal to the negative reactive power of the load. This allows for defining the same optimality objective in operating modes when generation constraints (1) are critical, as when attempting to minimize transmission losses for economic reasons.⁶

2.2.2 Impedance Load Model

For the impedance load model represented as

$$\hat{Z} = R + jX = Z \exp^{j\psi} \quad (50)$$

and the complex valued line impedance represented as

$$\hat{Z}_0 = R_0 + jX_0 = Z_0 \exp^{j\psi_0} \quad (51)$$

the real power and voltage at the load take on the form

$$P_{load} = \frac{m}{(1+m^2)\sqrt{(1+r^2)(1+q^2) + 2m(r+q)}} \quad (52)$$

and

$$v_{load} = \frac{m\sqrt{(1+r^2)(1+q^2)}}{(1+m^2)\sqrt{(1+r^2)(1+q^2) + 2m(r+q)}} \quad (53)$$

⁶This condition is not generalizable to the local optima of a simple system, nor of large systems. This is the fundamental theoretical reason for conflicting optimality criteria in different modes of operation.

where $m = \frac{Z}{Z_0}$. One can prove [6] that:

- The local maximum occurs if and only if $Z = Z_0$ (magnitude matching).
- The global maximum occurs if and only if $Z = Z_0$, and $\psi = -\psi_0$ (phase matching), i.e. $\hat{Z} = \hat{Z}_0^*$.

2.3 Summary of the results for the simplest network topology

Based on the above analysis, we suggest that a great caution should be exercised in applying different reactive power devices for achieving optimal operation of an interconnected system.

The results derived in Subsection 2.1 are useful for justifying strategies in setting voltages at the controllers designed to respond to voltage magnitude directly. Most representative of such devices are generators.

The conditions derived are loading conditions dependent, and strategies for setting optimal load voltage are, therefore, different. Through an analysis presented here this fact is justified for the first time. Moreover, some rationale for quantifying these optimal values is presented.

On the other hand, analysis presented in Subsection 2.2 is directly relevant for using reactive devices which change parameters of a transmission line and/or load. It is emphasized that the general circuit-theoretic results do not apply to the constrained problem of interest in operating power networks. This has led us to a notion of local maximum power transfer, illustrated in context of load compensation.⁷ A general condition for achieving this local optimum for the case of constant impedance load model is established, in terms of matching magnitudes of line and load impedances. A fairly narrow range of changes in load characteristics, for specified line parameters, is possible to improve power delivered to the load within the prespecified limits (5). This indicates inherent limitations in line and load compensation for increased power transfer.

3 Open Theoretical Issues for Large Networks

One of the hardest issues in generalizing fairly definite, quantifiable measures of optimality for the simplest topology described above is the issue of the most meaningful definition of “optimal” operation in large scale power networks.

While at the global maximum power transfer in the simple network described above problems of optimizing P_G , P_{loss} and P_L all lead to the identical operating point, this no longer holds true in the general network. This raises up the fundamental issue of optimal voltage

support. An interesting reference illustrating in a numerical way these problems can be found in [3]. The voltage optimality issue is addressed in this reference as the question of selecting “best” objective function for the voltage/ Var management.

Using two qualitatively different test systems (a “stretched” type and a “meshed” type) the following different candidate objective functions were studied [3].

- Min ΣQ_i^2
- Min $\Sigma \frac{Q_i^2}{Q_{nominal,i}}$
- Min $\Sigma P_{i,loss}$
and
- Min $\Sigma (V_i - V_{nominal,i})^2$

It was shown that under peak load conditions the choice of objective function did not effect the actual solution significantly. Under low- and medium loading conditions, the objective function in terms of losses leads to quite unbalanced reactive power generation, particularly for the long systems. Indeed, the generators located near the loads tend to generate reactive power, while the generators located further are sometimes absorbing. Thus some generators reach their power limits before the others which causes a drastic change in the generator operation when the load increases. This phenomenon is also observed on a meshed network, however, to a lesser extent. This paper also reports difficulties in using the performance criterion in terms of voltage limits. It is also documented numerically in this paper that the solutions using loss minimization and performance criterion in terms of voltage limits lead to a different solution. This result should not be suprising, given the overall complexity of the underlying theoretical issues.

We have recently reported our experience with different choices of performance criteria on the French power network in [4]. In context of hierarchical voltage control designs this issue becomes complicated even further by the questions of regional contributions to a specified system-wide performance criterion. It has been documented in [4] that certain classes of performance criteria could be optimized regionally, while this is not generally the case.

We close with a positive note regarding possible generalization of the result on constrained (local) optimum in terms of matching impedance magnitudes. This result has been shown to be generalizable to any linear networks with independent voltage sources terminated by constant power loads [6]. It is important to observe, however, that generators can not be modeled as ideal voltage sources, instead they are “PV” buses. This calls for more theoretical developments on networks with dependent sources.

Various definitions of network optimality attempted in general circuit theory in [7] have to be studied as applied to electric power networks. This is an extremely challenging theoretical area.

⁷Variations of a local optimum can be analyzed in a similar fashion, such as the effect of series capacitors on transmission lines.

4 Conclusions

In this paper a seemingly straightforward topic of voltage optimality for electric power systems is studied on the simplest network topology. Two different types of results are obtained. First, the results on optimal load voltage setting, and second, the results on line and/or load compensation. A definite strategy, which supports present practice is derived rigorously for the problem of optimal load voltage for different loading levels. A notion of a constrained (local) maximum power transfer for line and/or load compensation is proposed. Its relationship to the global maximum power transfer condition in the case of constant impedance load model is derived.

The detailed theoretical derivations presented in this paper for the simplest possible power network are intended to illustrate the difficulties in generalizing these results for realistic size power networks.

A better understanding of voltage /reactive power related optimality issues is fundamental to a meaningful use of a variety of controls available for direct regulation of voltage magnitudes, as well as for the line/load compensation. We are continuing work in this direction.

5 Acknowledgement

The authors are grateful to MM. B. Heilbronn and P. Pruvot of Electricité de France for their encouragement and support of this work.

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Discussion submitted subsequently by Paolo Marannino

The authors must be congratulated for the successful attempt of providing a full understanding of the complex problems inherent in the definition of the optimal voltage profiles for an electric power system. The method, adopted in the paper, for the evaluation of the optimal load voltage setting in the simple 'two busses' network clarifies the dependence of the values on the transmission system parameters and on the load power factor. How many actual constraints on generator and load voltages influence the optimal setting V_L^{opt} for different load conditions is stressed, when the generated real power P_G is minimized.

It can be of particular interest to know if the authors investigated the influence of the reactive power production constraints on V_L^{opt} and if they derived easy to use formulas for the simple model network.

Besides, some statements of the paper relight the never declined interest of the power system engineers in investigating the performances of the network under different alternative reactive control actions or in selecting the 'best' objective function for the voltage VAR scheduling. In addition to reference of the paper, other researches have been carried out at ENEL in this field. In particular, the adequacy of economy and/or security oriented objective functions for the reactive power scheduling has been tested in large and real systems [1r] with the help of the NEWREA program. Two objective functions have been implemented in the procedure proposed in 1983. The first one, based on security, distributes the reactive power generation among the units as proportional as possible to their ratings. It guarantees, essentially at heavy load hours, a suitable distribution of the reactive power in order to cope with critical contingencies and emergency situations. The second one minimizes the real losses. It allows the system operators to obtain an important electric energy saving at medium and low load levels, when they manage the system for the larger operating margins at their disposal. The influence of the network structure and of the load levels on the choice of the best objective function is underlined. The conclusions of [1r] seem to agree in part with the results sketched in Fig. 2 of the paper and discussed in paragraph 2.1.1.

More recently a hierarchical architecture has been adopted at ENEL with the addition of the secondary voltage regulation (SVR) and of the tertiary voltage regulation (TVR) [2r] to the well known primary automatic voltage regulation (AVR) of the synchronous generators. At the highest hierarchical level the TVR gives, in a real time environment, the most appropriate voltage set points of the pilot nodes on the basis of the actual state of the electric system and of the optimal voltage profiles defined by the short or very short-term reactive scheduling. The reactive power scheduling procedure, based upon off-line forecasts, carries out the optimal targets for the pilot node voltages, for predefined time intervals, one day (short term) or a few quarter of hour (very short term) in advance. These optimal values are determined by satisfying a minimum real loss criterion, a set of security constraints [3r] and the SVR requirements for the reactive power productions. The adequacy of the adopted choices to the primary objectives of security and economy is evaluated with the help of a suitable simulator of the steady-state behavior of the system submitted to the planned voltage control functions [4r]. The authors in the paper emphasize the value of their research in the context of examining the performance of the TVR function that will be in operation at EDF. It is of great interest for the readers to have some anticipation on the first results obtained in the joint MIT/EDF project.

In any case, the authors are encouraged to continue the work in this direction.

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Closure by Marija D. Ilic

The authors greatly appreciate the written discussion of this paper. The first author also is thankful to the discussor for the long discussion of this topic during the meeting in Davos.

To answer the specific question regarding the impact of reactive power production constraint on V_L^{opt} , it is interesting to observe that at V_L^{opt} the reactive power out of the generator Q_G is also minimized in the case of a simple network under consideration. Therefore, at V_L^{opt} the reactive power reserve is also utilized in the best possible way. To summarize, in the case of the simple network the three performance criteria (maximizing P_L , minimizing P_{LOSS} , and minimizing Q_G) lead to an identical solution V_L^{opt} .

It is also important to recognize that this is strictly speaking true only as long as all operating inequality constraints stated in the paper are met. Even in the case of the simplest network, as different inequality constraints are reached, hierarchical-type decision making is needed to reach adequate trade-offs. This point was made during the meeting by Mr. Fink and is well taken.

An interesting twist to the problem even for the simplest case is in a distributed (competitive) environment, a point made by Dr. Knight during the meeting. The first author is presently working on the formulation of the optimal voltage profile in a competitive market. Understanding this is essential to establishing (economic) value of voltage support under competition.

The hope of the first author is to exchange interim results with our informal "working group" (Dr. Marannino, Dr. Costi, Mr. Fink, and Dr. Knight) established in Davos, and possibly report joint findings at the next Seminar.

The first author has found reference no. 4r of the discussor extremely stimulating to the thinking relevant for generalized notions of optimal voltage profile in a multi-bus network

Extensions and applications of a mid-term voltage stability analysis method

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Abstract

This paper describes an approach to mid-term voltage stability analysis which combines mid-term simulation, stability analysis and corrective action evaluation. The system time response is obtained with the transient (fast) dynamics replaced by its equilibrium equations. A saddle-node bifurcation analysis is applied to the mid-term dynamics of tap changers and constant energy loads. The evaluation of adequate locations and amount of corrective actions relies on an intuitive view of the problem in the load power space. These complementary aspects are combined in a coherent and fast analysis tool, dealing with both load increases and equipment outages. The method is illustrated on the French Western system, using a 1200-bus modelling which includes sub-transmission levels and distribution feeders.

1. Introduction

The occurrence of numerous voltage incidents throughout the world has motivated the development of adequate voltage stability analysis methods. This paper reports on a multi-faceted analysis and its application to a real-life system.

Reference [1] classifies voltage instability into *transient* and *longer-term*, according to the time taken by the catastrophe to develop after a triggering event.

This paper focuses on longer-term instability, called here mid-term for simplicity, coherency with [2, 3], and because we do not take into account some long-term dynamics (e.g. boiler, operator actions, thermal overloads, etc.). The main mid-term dynamic phenomena to be taken into account are [1] :

- load power restoration by transformer On-Load Tap Changers (OLTCs) [4, 5, 6, 7]
- load power restoration by "constant energy" (or "thermostatic") loads [4, 8, 9]
- limitation of generator reactive power outputs, mainly by Overexcitation Limiters (OXLs)
- automatic capacitor/inductor switching
- secondary voltage control, if any [10]
- possibly, centralized frequency control.

The proposed method combines mid-term simulation, stability analysis and corrective action evaluation into a coherent and fast analysis tool, dealing with both load increases and equipment outages.

The system time response is obtained with the transient (fast) dynamics replaced by its equilibrium equations, an idea which can be traced back

to early voltage stability investigations [11]. In the general case, integration of the mid-term differential equations alternates with discrete event handling while no numerical integration is needed when mid-term dynamics results from discrete events only (tap changing, etc.).

For stability diagnosis purposes, a saddle-node bifurcation analysis [12, 13, 14] is applied to the mid-term dynamics of tap changers and constant energy loads. In practice, fast sensitivity calculations are performed along the system trajectory in order to identify the critical point, the large-disturbance equivalent to bifurcation point.

Corrective control evaluation aims at automatically suggesting the adequate location and amount of corrective actions like capacitor switching, decrease in OLTC voltage set-points, load shedding, etc. It relies on an intuitive view of the problem in the load power space, in which corrective control consists in bringing the point representative of demand back into the feasible region. The boundary of the latter is linearly approximated using the geometrical concepts of Refs. [12, 14].

This paper is the continuation of the previous work reported in Refs. [2, 15] and follows the more recent Ref. [3]. With respect to the latter, it considers the inclusion of constant energy loads, provides some simple interpretations of mid-term instability and sensitivities, proposes an improved criterion for corrective action ranking, and offers some additional practical examples. Sections 2, 3 and 4 of the paper relate to respectively the above three main aspects of the approach. Section 5 gives results obtained on the French Western system, using a 1200-bus modelling which includes sub-transmission levels and distribution feeders. Some openings are given by way of conclusion.

2. Fast simulation

2.1. Full system modelling

A general modelling relevant to voltage stability studies is shown in Fig. 1 along with examples of system components relevant to each equation.

The traditional equations (1) and (2) involve the transient state variables (x) and network voltages (y). Unless represented dynamically through Eq. (2), each load is involved in Eq. (1) through its active and reactive power :

$$P_l = z_P P^0 \left(\frac{V}{V^0}\right)^{\alpha_T} \quad \text{and} \quad Q_l = z_Q Q^0 \left(\frac{V}{V^0}\right)^{\beta_T} \quad (5)$$

where α_T , β_T relate to the transient load behaviour and z_P , z_Q will be described later on. These relationships can be straightforwardly extended to incorporate multiple exponents and sensitivity to frequency f , e.g. :

$$P_l = z_P P^0 (1 + D_P \Delta f) \sum_i a_i \left(\frac{V}{V^0}\right)^{\alpha_{Ti}} \quad \text{with} \quad \sum_i a_i = 1 \quad \text{etc.}$$

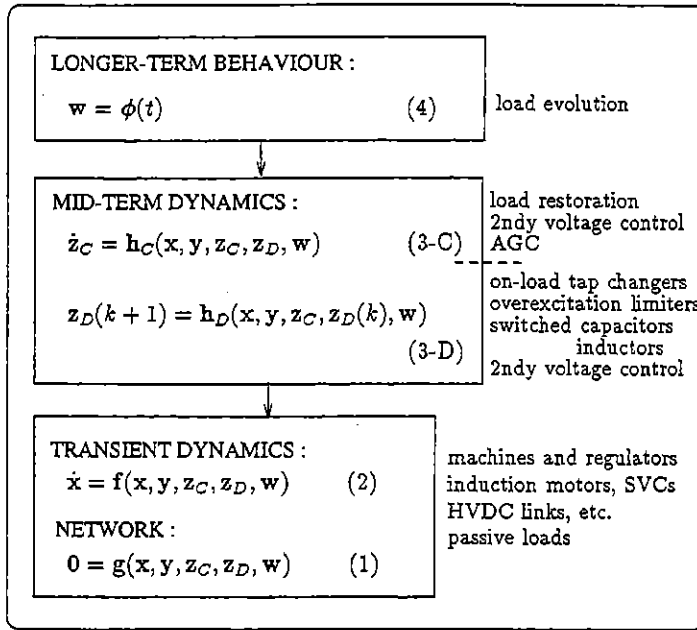


Figure 1: full system modelling

The mid-term dynamics is captured by discrete and/or continuous-time variables. The discrete-type variables z_D relate to controls and protecting devices. OXL operation can be reasonably considered as a discrete event. OLTC transformer ratios are discrete by nature : at transition times k dictated by internal delays, one has :

$$r(k+1) = \begin{cases} r(k) - \Delta r & \text{if } V_2 > V_2^0 + \epsilon \\ r(k) & \text{if } |V_2 - V_2^0| < \epsilon \\ r(k) + \Delta r & \text{if } V_2 < V_2^0 - \epsilon \end{cases} \quad (6)$$

$$\text{with } r_{min} \leq r(k+1) \leq r_{max}$$

where V_2 is the secondary, controlled voltage, V_2^0 the corresponding set-point, ϵ half the dead-band and Δr the ratio step size. Load power restoration by OLTCs is thus taken into account through Eq. (3-D).

The continuous-time state variables z_C originate from centralized voltage / frequency (e.g. PI-type) control representation. They also originate from constant-energy (or thermostatic) load modelling. The latter effects are captured by z_P and z_Q in Eq. (5), with power restoration described by :

$$T_P \dot{z}_P = \left(\frac{V}{V_0}\right)^{\alpha_S} - z_P \left(\frac{V}{V_0}\right)^{\alpha_T} \quad \text{with } z_P < z_{Pmax} \quad (7)$$

$$T_Q \dot{z}_Q = \left(\frac{V}{V_0}\right)^{\beta_S} - z_Q \left(\frac{V}{V_0}\right)^{\beta_T} \quad \text{with } z_Q < z_{Qmax} \quad (8)$$

where α_S, β_S refer to the steady-state behaviour (i.e. after restoration). Equations (7, 8) are slightly adapted from [8, 9] and generalize the case $\alpha_S = \beta_S = 0, \alpha_T = \beta_T = 2$ considered in [4]. "Passive" loads merely correspond to $z_P = z_Q = 1$ and no equations (7,8).

Finally, the w vector includes the above P^0 and Q^0 "demand" coefficients and the corresponding generation rescheduling. It is taken as a "smooth" function of time.

2.2. Mid-term system modelling

As mentioned in the Introduction, we focus on mid-term behaviour and therefore we replace Eq. (2) by the equilibrium equation :

$$0 = f(x, y, z, w) \quad (9)$$

References [2, 3] outline how a simple - although accurate - set of equilibrium equations (9) can be derived.

2.3. Mid-term time simulation

The time simulation is characterized by numerous discontinuities corresponding to Eq. (3-D), the possible presence of differential equations (3-C) and the algebraic constraints (1,4,9).

Figure 2 depicts the simulated time evolution of an x or y component. The continuous change from (x_{k-1}^+, y_{k-1}^+) to (x_k^-, y_k^-) results from the evolution of w and z_C . The transition from (x_k^-, y_k^-) to (x_k^+, y_k^+) is caused by discrete changes (3-D), based on internal delays of the various discrete-type components.

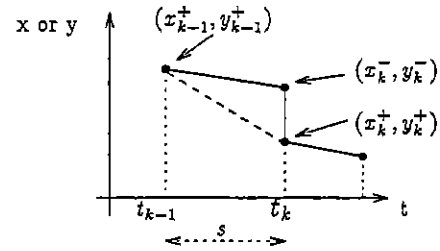


Figure 2: time evolution due to z_D, z_C and w

In practice, there is no point in detecting very exactly (e.g. by interpolation) when these discrete transitions (3-D) take place, considering that transient dynamics has been ignored anyway. Rather, discrete transitions are "synchronized" at times dictated by the step size s , which yields high computational efficiency in large-scale systems (e.g. for a large number of OLTCs). Decreasing s tends to cancel this synchronization but may be of interest for a finer analysis of numerous interacting OLTCs. Increasing s obviously delays some transitions.

A variant, considered in [3], is to have the time steps dictated by the discrete transitions, i.e. (x_{k-1}^+, y_{k-1}^+) is used to predict the next transition time of each component and the nearest is taken as the next time step. Because z_C and w are assumed to vary smoothly, their effects are not considered when predicting transition times, except that care is taken to sample their variations often enough.

The discontinuity at time t_k is dealt with by solving Eqs. (1,9) with respect to (x, y) with (w, z_C) fixed. This is done through "very dishonest" Newton iterations ($j = 1, 2, \dots$) :

$$\underbrace{\begin{bmatrix} \partial f / \partial x & \partial f / \partial y \\ \partial g / \partial x & \partial g / \partial y \end{bmatrix}}_{J_{xy}} \begin{bmatrix} x_k^{+(j)} - x_k^{+(j-1)} \\ y_k^{+(j)} - y_k^{+(j-1)} \end{bmatrix} = \begin{bmatrix} -f(x_k^{+(j-1)}, y_k^{+(j-1)}) \\ -g(x_k^{+(j-1)}, y_k^{+(j-1)}) \end{bmatrix} \quad (10)$$

starting from (x_k^-, y_k^-) (or possibly (x_{k-1}^+, y_{k-1}^+) : see hereafter).

The sparse Jacobian J_{xy} is updated and factorized following events like equipment outages or OXL operation. On the contrary, the numerous changes in transformer ratios, shunt susceptances and (possibly) generator voltage set-points as well as the smooth changes in w and z_C do not cause a Jacobian update, unless the right-hand-side vector exhibits large components or slow convergence is experienced.

As regards the integration from t_{k-1} to t_k , the well-known implicit trapezoidal rule :

$$(z_C)_k = (z_C)_{k-1} + \frac{s}{2} [(\dot{z}_C)_{k-1}^+ + (\dot{z}_C)_k^-] \quad (11)$$

is well suited to the numerous discontinuities, as it relies on values at time t_k only. The corrector can be initialized with the explicit Euler prediction :

$$(\dot{z}_C)_k = (\dot{z}_C)_{k-1}^+ + s (\dot{z}_C)_{k-1}^+$$

Moreover, it is appropriate to use a partitioned scheme, i.e. to alternatively evaluate the corrector (11) and solve the algebraic constraints (1,9), for this uses the same linear system (10) for both integrating and solving the discontinuities.

A particular case of special interest, considered in [3], is when the mid-term dynamics consists of discrete-time Eq. (3-D) only. This is the case when no Eqs. (7, 8) are included in the modelling and the centralized controllers are of discrete type. In this case, to go from (x_{k-1}^+, y_{k-1}^+) to (x_k^+, y_k^+) , no numerical integration is needed: rather, (x_k^+, y_k^+) can be obtained by solving the algebraic constraints with the updated w . Moreover, this is even not required: savings in computational efforts are obtained by "jumping" from (x_{k-1}^+, y_{k-1}^+) to (x_k^+, y_k^+) (see the oblique dashed line in Fig. 2).

A similar mid-term simulation without numerical integration was described in [16]. It also corresponds to one mode of operation of the software described in [17].

3. Stability analysis

3.1. Instability mechanisms

Let us first analyze the stability of a set of tap changers and/or constant energy loads, as well as its relationships with faster instability.

Since the discrete-time modelling (6) used for simulation does not allow a standard linearized analysis, it is conveniently replaced, for stability analysis purposes, by the continuous one:

$$\frac{d}{dt}r = K(V_2^0 - V_2) \quad (12)$$

or, in matrix form, for all OLTCs:

$$\frac{d}{dt}r = K(V_2^0 - Uy) \quad (13)$$

where $r = [\dots r_i \dots]^T$, $K = \text{diag}[\dots K_i \dots]$, $V_2^0 = [\dots V_{2i}^0 \dots]^T$, and U is a matrix of 0's and 1's relating transformer ratios to controlled bus voltages.

Similarly, we group all load restoration eqs. (7,8) into:

$$\dot{z} = T^{-1}h(y, z) \quad (14)$$

where $z = [\dots z_{Pi} \ z_{Qi} \dots]^T$, $T = \text{diag}[\dots T_{Pi} \ T_{Qi} \dots]$ and $h_i(y, z)$ is the right-hand-side of Eq. (7) or (8).

A mid-term equilibrium point is the solution of:

$$h(y, z) = 0 \quad (15)$$

$$V_2^0 - Uy = 0 \quad (16)$$

$$f(x, y) = 0 \quad (17)$$

$$g(x, y, r, z) = 0 \quad (18)$$

Around such a point, the system is linearized into:

$$\begin{bmatrix} \dot{\Delta z} \\ \dot{\Delta r} \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} T^{-1}\partial h/\partial z & 0 & 0 & T^{-1}\partial h/\partial y \\ 0 & 0 & 0 & -KU \\ 0 & 0 & & \\ \partial g/\partial z & \partial g/\partial r & & J_{xy} \end{bmatrix}}_J \begin{bmatrix} \Delta z \\ \Delta r \\ \Delta x \\ \Delta y \end{bmatrix} \quad (19)$$

Eliminating Δx and Δy yields:

$$\begin{bmatrix} \dot{\Delta z} \\ \dot{\Delta r} \end{bmatrix} = A \begin{bmatrix} \Delta z \\ \Delta r \end{bmatrix} \quad (20)$$

$$\text{with } A = \begin{bmatrix} T^{-1}\partial h/\partial z & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & T^{-1}\partial h/\partial y \\ 0 & -KU \end{bmatrix} J_{xy}^{-1} \begin{bmatrix} 0 & 0 \\ \partial g/\partial z & \partial g/\partial r \end{bmatrix} \quad (21)$$

A saddle-node bifurcation of mid-term dynamics occurs when two mid-term equilibria coalesce and then disappear. At the bifurcation point, matrix A is singular. The whole J matrix of Eq. (19) is also singular since:

$$\det J = \det A \cdot \det J_{xy}$$

Both matrices have a zero eigenvalue. Practical criteria based on J singularity are derived in §3.3.

A singularity of the mid-term time response occurs when, along the mid-term trajectory, matrix J_{xy} becomes singular (i.e. A evaluated at this point has an infinite eigenvalue) making the subsequent trajectory "undefined". This is mainly a matter of having replaced Eq. (2) by Eq. (9); indeed, a singular J_{xy} corresponds to a saddle-node bifurcation of the faster dynamics (1,2) for (z, r) "frozen" at their current value and, as the latter evolve, the faster dynamics loses its equilibrium. Singularities thus reveal situations where the transient dynamics cannot be neglected any longer. Reconsidering this neglected dynamics would suppress the singularity, but the system behaviour would be found unacceptable anyway.

Clearly, this interpretation requires Eq.(9) of the mid-term modelling to be the equilibrium of the reference full modelling (2). For instance, using simplified PV/PQ-bus representation of generators would not guarantee such interpretations. Similarly, as singularity is revealed by the impossibility to solve Eqs. (1,9), care must be taken that a pure numerical problem is not the cause of a diverging iterative scheme (10).

An oscillatory instability (Hopf bifurcation) of the transient dynamics is obviously not revealed by the equilibrium equation (9). However, the mid-term simulation provides "snapshots" around which the stability of the transient dynamics can be analyzed, using the eigenvalues of:

$$J_{tr} = \partial f/\partial x - \partial f/\partial y (\partial g/\partial y)^{-1} \partial g/\partial x$$

This assumes full time decoupling between the transient and mid-term dynamics, which is reasonable taking into account the discrete nature of z_D and the slow response of z_C .

3.2. Illustrative examples

Consider the simple system of Fig. 3, in which a passive load is fed through an OLTC.

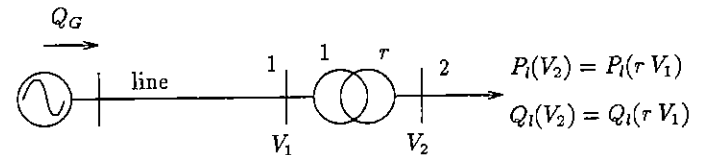


Figure 3: 2-bus system with OLTC

Example 1: constant impedance, active-only load: $P_1 = P^0 (\frac{V_2}{V_2^0})^2$

Figure 4 depicts a typical instability scenario where a large disturbance causes the system to lose its mid-term equilibrium. All curves refer to bus 1. The dotted lines are the transient loads characteristics for various values of r . The solid lines are the pre- and post-contingency network PV curves (drawn for $Q_1 = 0$). The vertical dashed line is the steady-state, constant power load characteristic due to the tap changer restoring voltage V_2 , which does not cross the new network PV curve. In its attempt to restore $V_2 = V_2^0$, the tap changer makes the operating point move from B to C to D.

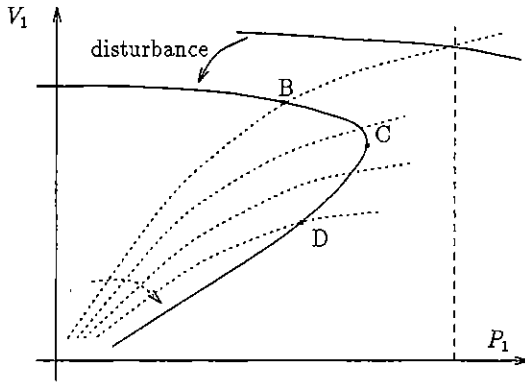


Figure 4: loss of mid-term equilibrium

In this case, A is scalar : $A = -K \frac{\partial V_2}{\partial r}$. It is negative (stable) above point C, positive (unstable) below. The sign of A has to be interpreted as follows. A negative value at point B, for instance, means that if B were a mid-term equilibrium point, it would be stable. In the shown large-disturbance scenario, B is however not an equilibrium : there is no equilibrium any longer, as already mentioned. One can figure out B as an equilibrium, with all system variables left at their current values, by merely lowering the set-point V_2^0 (this would shift the vertical dashed line to the left up to crossing the PV curve at B). The stability can be easily checked by slightly disturbing the state variable r .

A is zero at point C. This is a saddle-node bifurcation point in the sense that for a smaller demand P^0 or set-point V_2^0 , the steady-state load characteristic (a vertical line) is tangent to the PV curve at point C. In the sequel, we rather call C the critical point. In this example, C is the maximal load point because the tap changer restores the load to constant power. Clearly, this would no longer hold if there was some voltage dependent load in parallel with the transformer or if the tap was replaced by a constant energy load with nonzero α_S or β_S .

Assume that the system settles down at point D due to the OLTC hitting its limit. Since voltage V_2 is no longer controlled, one might think of discarding the corresponding Eq. (16) for stability analysis. As a result, point D would be found mid-term stable. This is not acceptable as it would hide the fact that the critical point C has been crossed. Besides, an increase in demand at D (or some load restoration) could not be met by the system. Please also refer to §3.4 for further discussion of tap changer limits.

Example 2 : active-only load $P_l = P^0[a + (1-a)(\frac{V_2}{V_2^0})^2]$ ($0 < a < 1$)

Consider the effect of disturbance D_1 . In this case, the initiating instability is again the loss of mid-term equilibrium. However, as system degradation takes place, a singularity is met, at point S (see Fig. 5). At this point, the transient load characteristic is tangent to the PV curve. Hence, for any further increase in tap, both curves do not intersect any longer. This means that the transient dynamics (of the generator in this case) loses its equilibrium, since the PV curve is a collection of equilibrium points of this dynamics. Full dynamic simulation would reveal a plunging down of voltages to zero (with the system representative point sliding down along the load characteristic) [2, 7].

The value of A is negative (stable) above point C and below point S; it is positive (unstable) in between. Stable and unstable portions of the PV curve are thus separated by either the critical point C or the singularity S. Refer to [18] for a rigorous and general treatment.

Through a corrective action like load shedding, the vertical dashed line

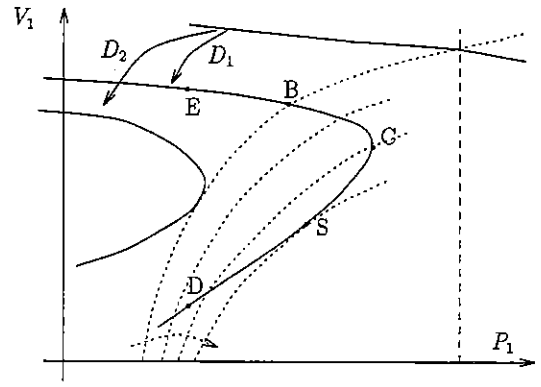


Figure 5: loss of mid-term equilibrium and singularity

can be moved to the left of point S; this would restore two stable mid-term equilibria, e.g. points D and E on Fig. 5. However if, at the time the action is taken the system operates above S, it will be attracted towards E. To be attracted towards D, the system must first enter the singularity then the corrective action must be (promptly !) taken. Beyond this theoretical discussion, the practical problem of concern will be to take some corrective action before the singularity is reached, since the latter could trigger fast collapse and hence uncontrollable system situations.

Disturbance D_2 (see Fig. 5) corresponds to a limit case where a singularity occurs right after the disturbance [16]. If it is more severely stressed, the system will experience transient (instead of mid-term) voltage collapse.

Note finally that singularity may also correspond to the loss of steady-state angle stability, especially for field current limited generators [7, 2].

Example 3 : constant active power, capacitor compensated reactive load of the type $Q_l = -B_c V_2^2 + Q_0(\frac{V_2}{V_2^0})^\beta$

A large enough B_c along with $\beta < 2$ can yield a negative dQ/dV slope as quoted in [1] (p.102). In Fig. 6, following disturbance D_1 , the tap changer brings the operating point back from B to D, which is the stable post-contingency mid-term equilibrium. Disturbance D_2 is a limit case where a singularity occurs at point S. For stronger disturbances, the system experiences transient (instead of mid-term) voltage collapse. Unlike to case 2, the critical point C is "hidden behind" the singularity S.

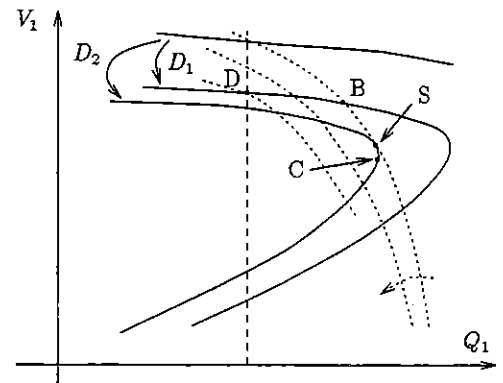


Figure 6: critical point hidden behind singularity

3.3. Practical mid-term stability analysis

A few remarks on $\mathbf{J}$ are of interest. We have shown that at a critical point, both $\mathbf{A}$ and $\mathbf{J}$ are singular, i.e. they have a zero eigenvalue. $\mathbf{J}$ has the computational advantage of being very sparse, while $\mathbf{A}$ is dense. For stability analysis, a smaller - although still very sparse - formulation can be obtained by eliminating $\mathbf{z}$ from Eqs. (15,18), before linearization. Indeed, setting $\dot{z}_P = \dot{z}_Q = 0$ in Eqs. (7,8) yields :

$$z_P \left(\frac{V}{V_0} \right)^{\alpha_P} = \left(\frac{V}{V_0} \right)^{\alpha_S} \quad z_Q \left(\frac{V}{V_0} \right)^{\beta_P} = \left(\frac{V}{V_0} \right)^{\beta_S}$$

and the active and reactive load power expressions (5) become :

$$P_i = P^0 \left(\frac{V}{V_0} \right)^{\alpha_S} \quad Q_i = Q^0 \left(\frac{V}{V_0} \right)^{\beta_S}$$

In other words, Eq. (15) can be discarded provided the steady-state characteristics of constant energy loads are substituted for the transient ones in Eq. (18) (this is equivalent to anticipating the restoration effects). The corresponding Jacobian has the structure of the (3×3) lower right submatrix of $\mathbf{J}$.

The critical point identification does not require an explicit eigenvalue (or singular value) computation of the dense state matrix $\mathbf{A}$. Rather, sensitivities computed from the sparse Jacobian $\mathbf{J}$ can be used.

We consider the sensitivity of the total reactive power generation Q_G to the various reactive demands Q_i^0 ($i = 1, 2, \dots$). These are easily interpretable sensitivities, positive in normal operating conditions; they have been proposed in [19] as voltage stability indicators and are usually computed from the transposed load flow Jacobian [20]. We derive them from the mid-term equilibrium equations (15-18) as follows :

$$\partial Q_G / \partial Q^0 = - \begin{bmatrix} 0 & 0 & 0 & (\partial g / \partial Q^0)^T \end{bmatrix} (\mathbf{J}^T)^{-1} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \partial Q_G / \partial \mathbf{y} \end{bmatrix}}_{\mathbf{b}} \quad (22)$$

The zero sub-matrices come from the fact that Q_G is a function of $\mathbf{y}$ only and Q^0 appears in Eq. (18) only.

It is proved in [3] that, as the critical point is approached, these sensitivities have larger and larger positive values and right after crossing they all change to large negative numbers, thus providing a simple identification criterion.

The computational effort involved in using Eq. (22) is quite moderate. It mainly consists of computing and factorizing the sparse $\mathbf{J}$ matrix (or preferably its smaller equivalent discussed above), then solving $\mathbf{J}^T \mathbf{a} = \mathbf{b}$ for $\mathbf{a}$, the remaining calculations being trivial.

The physical meaning of these sensitivities is discussed on the following

Example. Consider a 2-bus system with the QV curve shown in Fig. 7. The generator production Q_G can be easily shown to increase monotonically as one goes down along this curve (i.e. from no-load to short-circuit).

Assume here that the load is constant energy with the steady-state characteristic shown by the dashed line S . Point A is above the saddle-node bifurcation C and is thus mid-term stable; point B is unstable.

Impose a small increase ΔQ^0 in demand and consider the change in mid-term equilibrium. The steady-state load characteristic changes from S to S' . Therefrom we obtain the sensitivities :

$$\left(\frac{\partial Q_G}{\partial Q^0} \right)_A \simeq \frac{Q_G(A') - Q_G(A)}{Q^0(A') - Q^0(A)} > 0 \quad \text{since } Q_G(A') > Q_G(A)$$

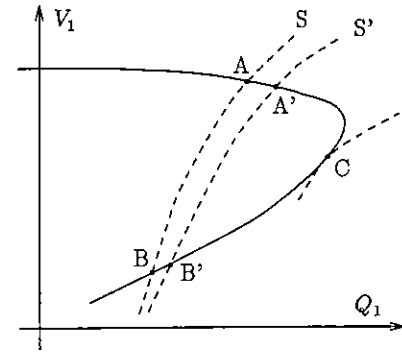


Figure 7: evaluation of $\partial Q_G / \partial Q^0$

$$\left(\frac{\partial Q_G}{\partial Q^0} \right)_B \simeq \frac{Q_G(B') - Q_G(B)}{Q^0(B') - Q^0(B)} < 0 \quad \text{since } Q_G(B') < Q_G(B)$$

which confirms the stability behaviour.

3.4. Analysis of multi-level OLTC structures

In many power systems, sub-transmission and distribution are connected through several levels of OLTCs "in cascade". There are motivations for representing these lower levels in some detail. First, sub-transmission participates significantly in the reactive power balance. Second, this representation allows to characterize the system stability at voltage levels where most corrective actions have to be evaluated (see Section 4).

We discuss hereafter how the sensitivity analysis of the previous section applies to a multi-level OLTC structure. We consider the simple example of Fig. 8 in which T1 and T2 regulate their secondary voltages.

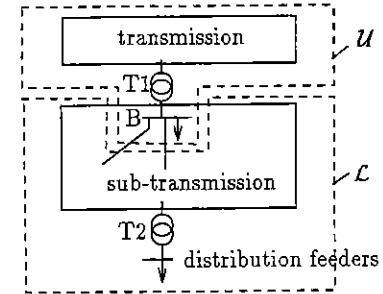


Figure 8: multilevel OLTC structure

Consider an increase in demand at some bus of the upper sub-system $\mathcal{U}$ (see Fig. 8). Sub-system $\mathcal{L}$ can be thought of as a voltage dependent load connected to bus B and hence, once T1 has restored the voltage at bus B, this equivalent load is restored to its previous power level and does not influence Q_G . Hence, at a given operating point, the sensitivities at buses of $\mathcal{U}$ are not influenced by the modelling of $\mathcal{L}$.

Consider now some bus k of $\mathcal{L}$. As the required power has to come through transformer T1, the sensitivity at bus k is :

$$\partial Q_G / \partial Q_k^0 = \partial Q_G / \partial Q_B^0 \cdot \partial Q_B / \partial Q_k^0$$

where $\partial Q_G / \partial Q_B^0$ is the sensitivity at bus B and $\partial Q_B / \partial Q_k^0$ is the sensitivity of a (fictitious) reactive power generation located at bus B. Instability can thus appear in two ways :

(i) $\partial Q_G / \partial Q_B^0$ changes sign : this corresponds to voltage instability at the upper level;

(ii) $\partial Q_B / \partial Q_k^0$ changes sign : this corresponds to a local instability, namely a voltage instability of $\mathcal{L}$ with the voltage at bus B kept constant. This is less likely to occur, owing to precisely this voltage control; it might however result from contingencies in $\mathcal{L}$.

This discussion shows the importance of taking into account the effect of OLTCs, if lower voltage levels are represented. The effect of a transformer like T1 is to decrease the electrical distance between upper level generation and loads.

As a corollary, it is important to take into account a limit on this tap changer (by simply removing the equation (16) of concern). This contrasts with the situation of transformers like T2. For the latter, closest to the loads, we have explained in Example 1 of §3.2 why tap changer limits should not be taken into account.

4. Determination of corrective actions

4.1. System portrait in the load power space

An intuitively appealing view of the problem is the system portrait in the load power space. This is the space of P_i and Q_i powers (5) of loads subject to restoration effects.

Rigorously speaking, the derivations hereafter apply to loads restored to constant power. This is the case for passive or constant energy loads fed through OLTCs, as well as for constant energy loads with $\alpha_S = \beta_S = 0$.

For loads fed through OLTCs, we neglect the OLTC dead-band ϵ and take the V_2^0 set-point as the V^0 reference of Eq. (5). With this assumption, all the loads of concern have the asymptotic behaviour :

$$S(t) = [\dots P_{li} Q_{li} \dots]^T \longrightarrow S^0 = [\dots P_i^0 Q_i^0 \dots]^T$$

In the load power space, there is a feasible set $\mathcal{F}$ outside which no mid-term equilibrium exists. This set is limited by a (hyper)surface Σ , each point of which corresponds to a saddle-node bifurcation of the mid-term dynamics, i.e. to singular $\mathbf{A}$ and $\mathbf{J}$ matrices.

A voltage instability caused by a prohibitive increase in demand corresponds to the "target point" S^0 going out of the feasible set $\mathcal{F}$.

A voltage instability following a large disturbance is sketched in Fig. 9 (a very simplified 2-D representation where S_1 and S_2 are active or reactive powers of the same or different loads). Here, a generation or transmission equipment outage causes Σ to shrink so much that S^0 ends up outside the new feasible set $\mathcal{F}$. The load power jumps from its pre-disturbance value to $S(0^+)$, which is necessarily in the new feasible set. From there on, restoration mechanisms drive $S(t)$ towards S^0 . However, the trajectory cannot escape from $\mathcal{F}$ and after "hitting" Σ at point C, it goes backwards. The solid line part of the trajectory corresponds to points at which matrix $\mathbf{A}$ has all stable eigenvalues while along the dotted part $\mathbf{A}$ has one positive real eigenvalue.

The instability scenario of Fig. 9 corresponds to that of Fig. 4. In both figures, C is the critical point. This figure is typical of many practical situations. However, more complex trajectory plots in the load power space cannot be ruled out. This is illustrated in Figs. 10 and 11. These figures relate to overshoot phenomena, which may result from interactions between several layers of OLTCs or from combined effects of OLTCs and constant energy loads. In the stable case of Fig. 10 there is a post-contingency mid-term equilibrium but overshoot makes the system hit Σ twice, with an unstable $\mathbf{A}$ matrix in between (to be interpreted as in §3.2). In the unstable case of Fig. 11, there is a lack of attraction towards the stable post-contingency equilibrium. Both cases are likely to result from small post-contingency security margins.

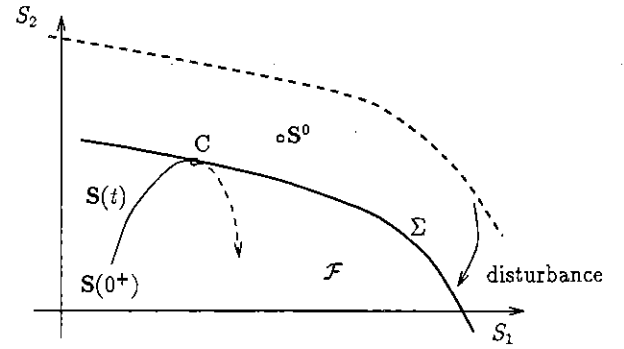


Figure 9: load power space : instability caused by loss of equilibrium

Note that the case of Figure 10 also depicts a situation where a corrective (emergency) action has brought point S^0 into the feasible domain $\mathcal{F}$ but too late : when the system enters its final configuration, its operating point does not lie in the domain of attraction of the corresponding stable equilibrium [4, 9].

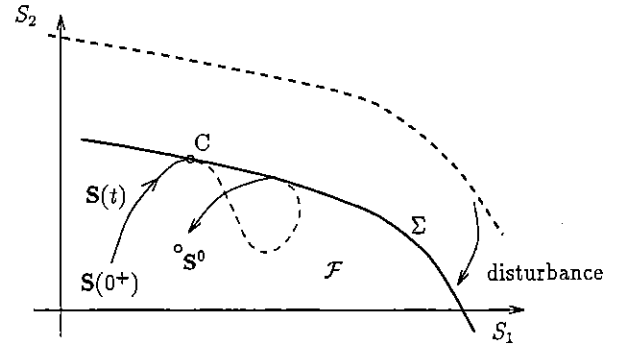


Figure 10: load power space : stable case with overshoot

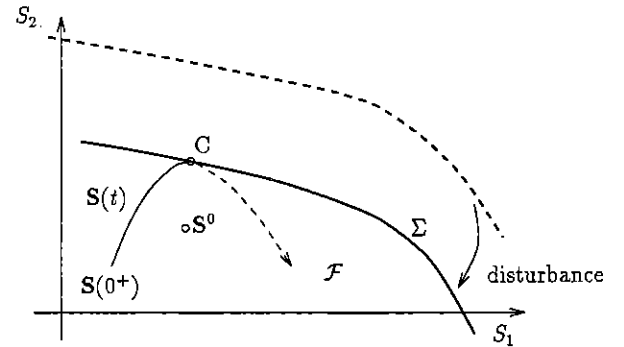


Figure 11: load power space : instability due to lack of attraction

4.2. Corrective control determination

The preceeding section has shown the relationship between voltage collapse and a demand S^0 located outside the feasible set $\mathcal{F}$ in the load power space (see Fig. 9).

Within this context, corrective control can be viewed as the action of bringing S^0 back within $\mathcal{F}$, by decreasing the demand or providing active/reactive power support. A problem of much interest is obviously to determine the minimal control required to recover a stable operation.

The approach we propose can be simply summarized as follows. Since the critical point C can be identified along the system trajectory and the normal vector to hypersurface Σ can be easily computed, Σ can be approximated by its tangent hyperplane $\mathcal{H}$ at point C. Therefrom, the minimal corrective control corresponds to the shortest - while feasible - path from S^0 to this hyperplane.

The corrective actions which the method can evaluate relate primarily to changes in bus injections as provided by :

- (i) capacitor switching
- (ii) decrease in voltage set-point of OLTCS
- (iii) fast generator starting up
- (iv) load shedding.

Each of these actions is equivalent to a load decrease $(\Delta P, \Delta Q)$. In case (iv), one merely has $\Delta P = P^0$ and $\Delta Q = Q^0$. In case (ii), for a passive load of the type (5) in parallel with a compensation capacitor of susceptance B_c , a decrease ΔV_2^0 results in :

$$\Delta P = P_0 - P_0 \left(\frac{V_2^0 - \Delta V_2^0}{V_2^0} \right)^\alpha \quad (23)$$

$$\Delta Q = Q_0 - Q_0 \left(\frac{V_2^0 - \Delta V_2^0}{V_2^0} \right)^\beta - B_c [(V_2^0)^2 - (V_2^0 - \Delta V_2^0)^2]. \quad (24)$$

Consider then the load power space projection of Fig. 12, in which the axes represent the active and reactive power of the same load. $(\Delta P, \Delta Q)$ defines the direction d along which the load could be modified, as depicted by the dashed line in the figure. This line intersects hyperplane $\mathcal{H}$ at point S^* . The magnitude of the needed correction, along this direction, is given by :

$$\|S^0 - S^*\| = \frac{|\mathbf{n}^T(S^0 - S^c)| \|\mathbf{d}\|}{|\mathbf{n}^T \mathbf{d}|} \quad (25)$$

where $\mathbf{n}$ is the normal vector to Σ , S^c the vector of load powers at the critical point and $\|\cdot\|$ denotes the Euclidian norm. In the space of the active and reactive powers, this distance is expressed in MVA. The smaller this distance the more appropriate a corrective control along direction d .

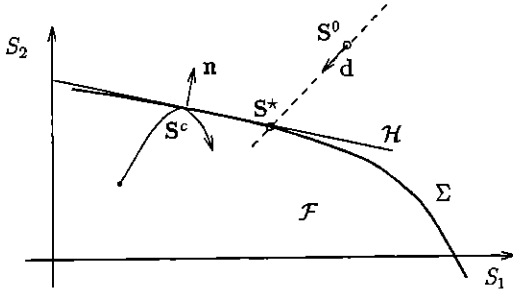


Figure 12: load power space : corrective action along direction d

A practical procedure is simply as follows. $\|S^0 - S^*\|$ is computed for all candidate actions (e.g. for all buses where capacitor/inductor can be switched, for all loads which can be shed, etc.). Then, the actions are taken successively by increasing order of the above distance. Each selection is merely followed by an update of S^0 and the procedure is repeated until $\mathbf{n}^T(S^0 - S^c)$ becomes negative. Within the limits of validity of linearization, this means that S^0 has come back into the feasible region (in practice, has crossed $\mathcal{H}$).

Note that in the cases of Figs. 10 and 11, $\mathbf{n}^T(S^0 - S^c)$ is directly found negative, indicating that the demand point is inside the feasible set.

4.3. Computing the normal vector $\mathbf{n}$

Adapting the derivation made in [12], the analytical expression of $\mathbf{n}$ is

easily obtained as :

$$\mathbf{n} = \begin{bmatrix} 0 & 0 & 0 & (\partial g / \partial S^0)^T \end{bmatrix} \mathbf{w}_c \quad (26)$$

where $\mathbf{w}_c$ is the left eigenvector relative to the zero eigenvalue of $\mathbf{J}$:

$$\mathbf{w}_c^T \mathbf{J} = 0 \quad (27)$$

In practice, a close estimate of the critical point is obtained along the simulated trajectory, using the sensitivities discussed in §3.3. At this point, $\mathbf{J}$ has an almost zero eigenvalue whose left eigenvector is very close to $\mathbf{w}_c$. In [3], it is proposed to compute this eigenvector using the Inverse Iteration algorithm, which merely requires several substitutions on the already computed and factorized sparse $\mathbf{J}$ matrix.

5. Simulation results

The proposed approach has been extensively tested on the Western region of the E.D.F. (Electricité De France) system, where voltage security is a major concern [21]. The corresponding peak load is 7150 MW. Examples are given hereafter for the purpose of illustrating the various aspects of the method.

5.1. System modelling

The system modelling covers EHV (380, 225 kV) transmission, HV (90, 63 kV) sub-transmission and MV distribution feeders, connected together through two levels of transformers with OLTCS. Figure 13 shows the one-line diagram of the EHV system in the region of interest along with the boundaries of the HV areas. The overall modelling includes 1244 buses (out of which 500 are at HV level and 341 at MV level), 1090 lines and 541 transformers.

Referring to Fig. 1, the modelling mainly consists of the following :

- algebraic equations (1,9) :

- 341 loads modelled as in Eq. (5) at the MV distribution feeder buses, most of them in parallel with a compensation capacitor;
- 36 detailed generators in or close to the region of interest. Each generator is described by 3 equilibrium equations (9) involving 3 internal state variables x , namely the rotor angle, the e.m.f. behind saturated synchronous reactance, and the e.m.f. proportional to field current. Farther generators are represented as PV buses with governor effects;

- discrete-type dynamics (3-D) :

- 431 OLTCS on EHV/HV, HV/MV and EHV/MV transformers
- 47 switched capacitors at the HV level (totaling 1750 Mvar)
- OXLS on the 36 detailed generators
- the Coordinated Secondary Voltage Control (CSVC) [10] : this centralized controller issues each 10 seconds new generator voltage set-points in order to keep the voltage profile constant while maximizing the generator reactive reserves within a region;

- no continuous-time dynamics (3-C);

- explicit functions of time $w(t)$ to represent the evolution of the demand.

Some more details are given in [3, 21].

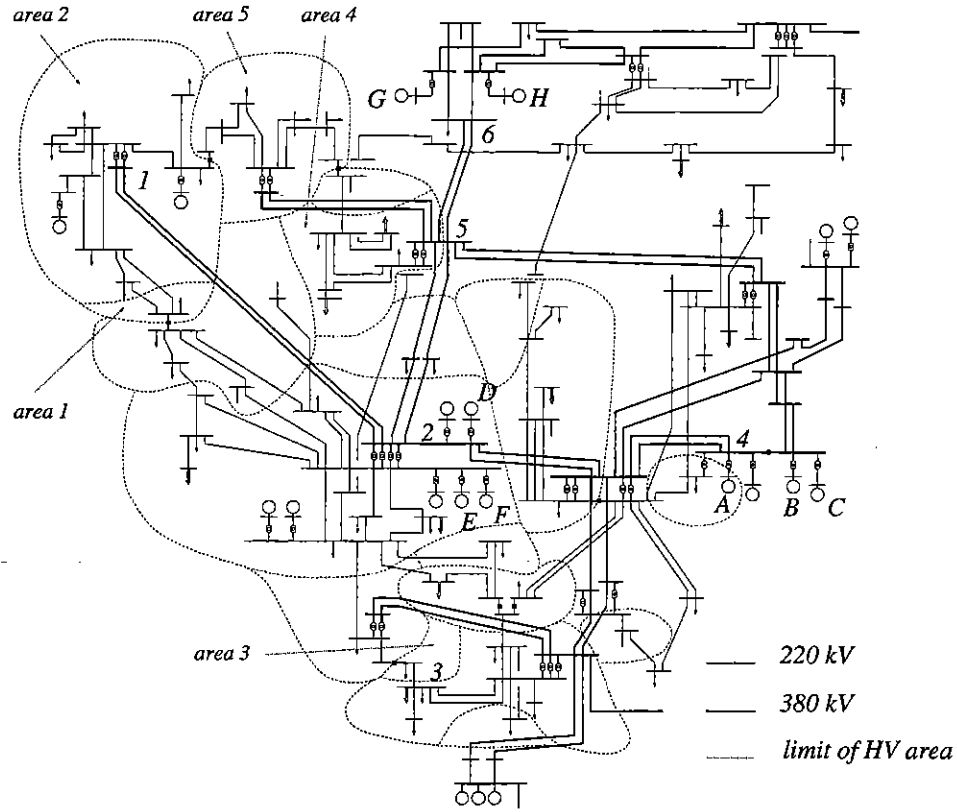


Figure 13: one-line diagram of the French Western EHV system

As the system mid-term dynamics consists of discrete-time equations (3-D) only, the technique described at the end of Section 2.3 has been used to speed-up calculations (see the dashed line in Fig. 2).

5.2. Example 1 : load power margin calculation

Load power margins (LPMs) are used to preventively assess the system robustness in pre- and post-contingency configurations. LPMs are computed by imposing a ramp-type variation of the load demand S^0 and simulating the system response up to crossing of the critical point. Stability is checked at regular time intervals through the sign of the $\partial Q_g / \partial Q^0$ sensitivities given by Eq. (22). The demand ramp is chosen steep enough to speed up calculations but mild enough for the load powers to follow the desired pattern of increase.

We consider a high load situation with two 380-kV lines and some generators not in service (or previously tripped). Figure 14 shows the increase in regional load demand and load power, obtained by summing the increases in P^0 and P_i (see Eq. (5)) over the 341 loads of the region..

Sensitivities are computed each 100 s. As the system modelling does not include constant energy loads, J amounts to the (3×3) lower right submatrix in Eq. (19). A change in sign at $t = 1200$ s stops the simulation. The LPM (390 MW/ 102 Mvar) is taken as the increase in load power (not demand) obtained at $t = 1100$ s.

Figures 15 and 16 show sensitivities evaluated at all trajectory points. As expected, they tend to plus infinity and then switch to minus infinity as the critical point is crossed. Figure 15 refers to four EHV buses while Fig. 16 refers to three HV (sub-transmission) buses. At the critical point, all sensitivities change sign together [3].

Even if the demand stopped increasing before reaching instability, the

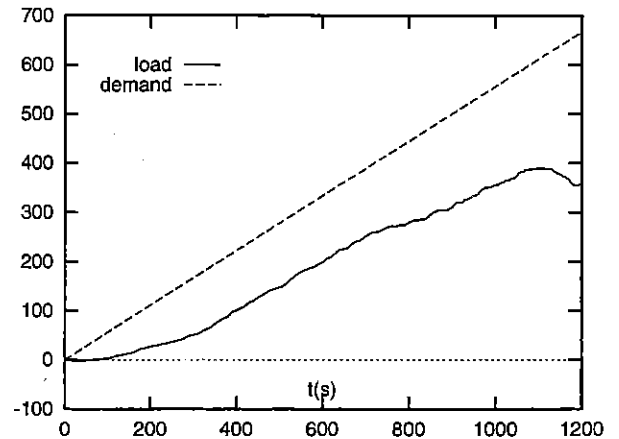


Figure 14: increase in active demand and load power (MW)

load power curve would not catch up the demand curve, owing to OLTC dead-bands effects. These effects cumulated over all loads are not negligible. This might be a motivation for considering constant energy load effects, which would restore some part of the load power not restored by the discrete tap changers.

5.2. Example 2 : simulation of incident and stability analysis

We consider the loss at $t = 400$ s of generators B and C (see Fig. 13), in the following three cases :

- (i) load level as in Example 1
- (ii) demand increased by 200 MW/ 64 Mvar before generator tripping (i.e. in between $t = 0$ and 400 s)

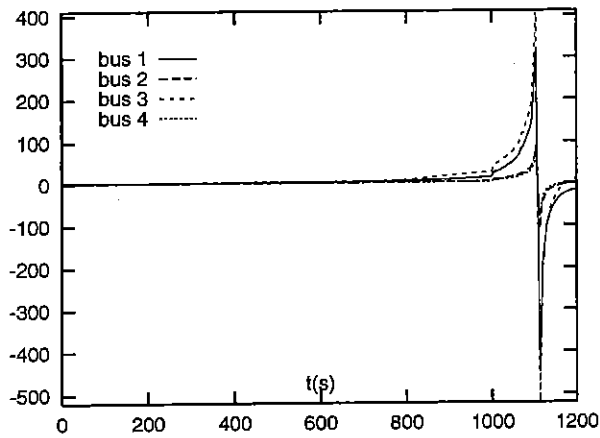


Figure 15: $\partial Q_g / \partial Q_i^0$ at some EHV buses during demand increase

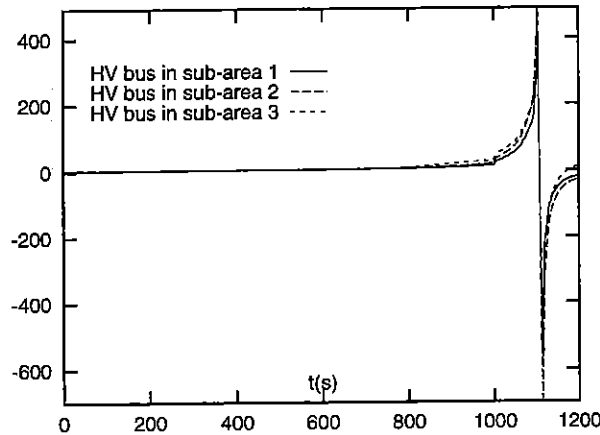


Figure 16: $\partial Q_g / \partial Q_i^0$ at some HV buses during demand increase

(iii) demand increased by 300 MW/ 96 Mvar before generator tripping.

Figure 17 shows the time evolution of the voltage at bus 1, the western extremity of the 380 kV grid. Clearly, case (i) is stable and the other two unstable. Case (iii) collapses faster than case (ii) because - referring to Fig. 4 - the gap between point C and the mid-term equilibrium (vertical) line is larger. This is a well-known behaviour of saddle-node bifurcation (see e.g. [7] for an illustration in the transient time frame). Note that it is observed here with the discrete-time system of Eqs. (6) !

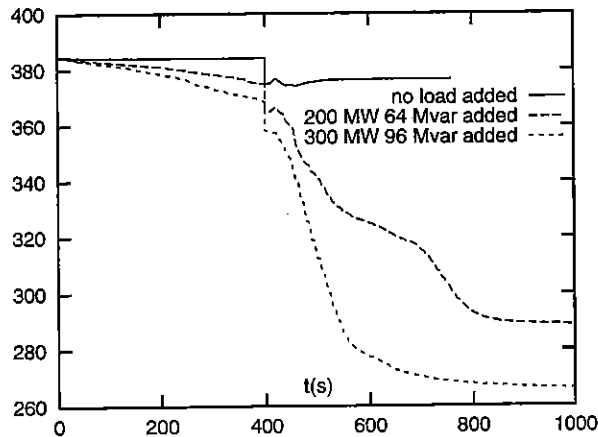


Figure 17: voltage at bus 1 (kV) for 3 load levels and the same contingency

Figure 18 illustrates the influence of load modelling. The curves refer to respectively $(\alpha = 1, \beta = 2)$ and $(\alpha = 1.5, \beta = 1.8)$ for loads at MV

level. The $(\alpha = 1, \beta = 2)$ case collapses faster. It results in a mid-term singularity at $t = 630$ s (loss of transient equilibrium). This case qualitatively corresponds to Fig. 5 while the case $(\alpha = 1.5, \beta = 1.8)$ relates to Fig. 4 with the OLTCs reaching their limits. In both cases the critical point has been crossed well before reaching these extreme situations, as shown in the sequel.

A singularity right after a contingency has been experienced in some (rare) severe cases, e.g. the loss of two circuits between buses 5 and 6 (see Fig. 13; this prevents the production of generators G and H from being imported into the region of interest).

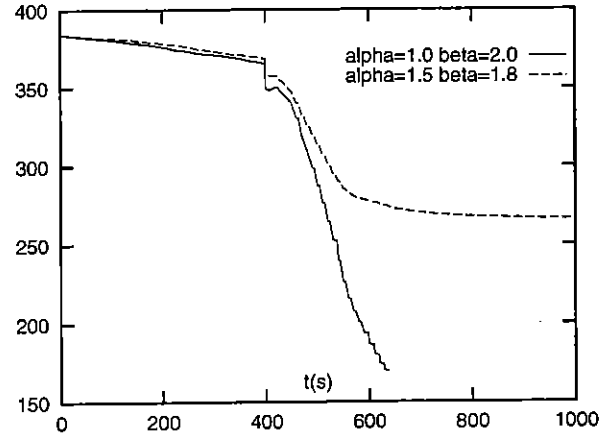


Figure 18: voltage at bus 1 (kV) for 2 load modellings. Case (iii).

Figure 19 recalls the important role of reactive power limitations. It shows the field currents of four generators in case (ii) above. The OXLs of generators A and D act shortly after the contingency while generators E and F (connected to the 225 kV grid) stay under AVR control.

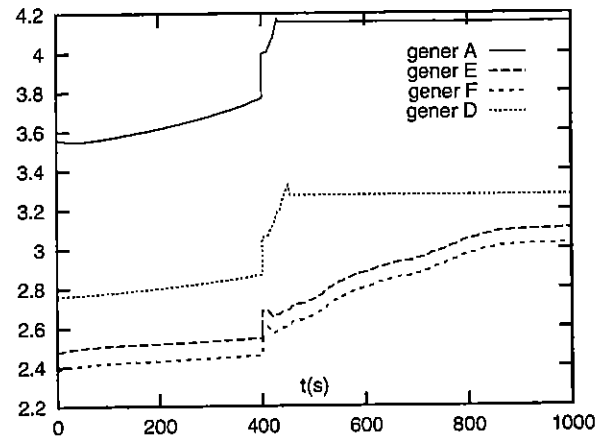


Figure 19: field currents (pu) of 4 generators. Case (iii).

We check the system stability along the simulated trajectory of case (ii). Figure 20 shows the $\partial Q_g / \partial Q_i^0$ sensitivities at four EHV buses. The critical point is crossed at $t = 530$ s. In practice, sensitivity calculation is performed - say - each 100 s only and the first unstable point, corresponding to $t = 600$ s, is taken as an estimate of the critical point, for the subsequent corrective control analysis.

5.3. Example 3 : corrective control through OLTC set-points

We consider the possibility of containing the instability of case (ii) by decreasing the voltage set-points of OLTCs controlling the MV buses (OLTCs of EHV/HV transformers are left unchanged). We adopt $\Delta V_2^0 = 0.05$ pu, a usual value. The corresponding load decreases ΔP

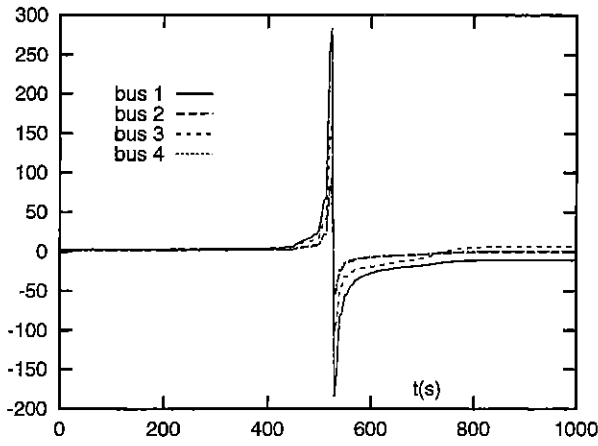


Figure 20: some $\partial Q_g / \partial Q_i^0$ along the trajectory of case (iii)

and ΔQ are computed according to Eqs. (23,24) and the distances $\|S^0 - S^*\|$ are obtained from Eq. (25).

MV buses are ranked in Table 1 by increasing order of this distance. The results relative to bus 10 - for instance - mean that, along the direction d defined by $\Delta P = 1.5$ MW and $\Delta Q = 0.5$ Mvar, it would be necessary to decrease the load by 70.16 MVA to bring the demand back into the feasible region (see Fig. 12). Since the 0.05 pu voltage decrease yields a $\sqrt{1.5^2 + 0.5^2}$ MVA decrease only, it is necessary to consider the next load in the list, and so on until coming back in the feasible region. Clearly, the further one goes down in the list, the larger the distance and the less effective the action. It is required to act on about 80 OLTCs; the total decrease in load is 109 MW / 15 Mvar (note that $\sqrt{109^2 + 15^2} > 70.16$). The OLTCs of concern are almost all in areas 1, 2, 3 (see Fig. 13) and a few of them in areas 4 and 5.

Clearly, there is a level of system disturbance beyond which acting on all OLTCs would not be sufficient to contain the instability. A similar remark holds if the sensitivity of loads to voltages decreases. Load behaviour is obviously critical for the success of this technique.

Note the negative value of ΔQ for some loads. This originates from compensation capacitors whose production decreases with the MV voltage, as quoted in [1] (p.102). However, the proposed distance takes into account both the active and reactive power effects : as can be seen, in this system, the beneficial effect of ΔP compensates for the detrimental effect of (negative) ΔQ .

To validate the above analysis, a simulation was performed with the generators tripped at $t = 400$ s and the OLTC set-points modified as above at $t = 460$ s. Fig.21 shows the time evolution of EHV voltages. As can be seen, voltages come back to normal values.

5.4. Example 4 : corrective control through load shedding

We finally consider load shedding to counteract instability in case (iii).

Table 2 below is similar to Table 1, except that the change in each load is now the whole load itself. Note that buses are not ranked in the same order because the direction of corrective action is different. The seven listed loads are those which need to be shed according to the method.

A validation is shown in Fig. 22. This corresponds to the following sequence of events : generator tripping at $t = 400$ s, decrease in OLTC voltage set-points at $t = 500$ s (identified as in previous section), shedding of the above identified loads at $t = 600$ and 620 s, release of OLTCs at $t = 1000$ s. As can be seen, voltages recover normal values and the system is able to face the load recovery, once the tap chang-

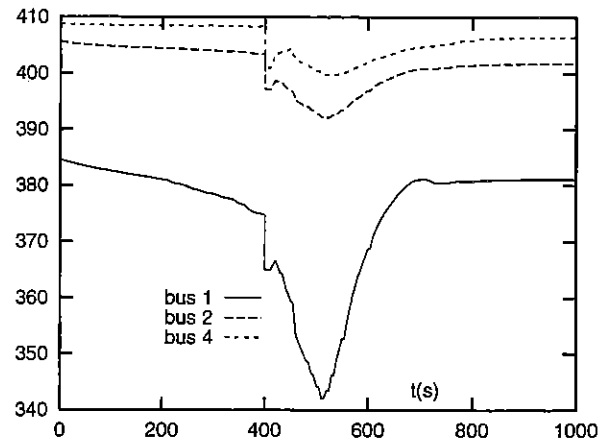


Figure 21: voltages (kV) at EHV buses. Case (ii) with action on OLTCs

Table 1. Ranking of MV buses candidate for a change in OLTC voltage set-point

MV bus	area (in Fig. 13)	ΔP (MW)	ΔQ (Mvar)	$\ S^0 - S^*\ $ (MVA)
10	1	1.5	0.5	70.16
11	1	0.8	0.5	74.58
12	2	1.1	0.3	75.85
13	2	2.0	0.2	75.87
14	2	0.6	0.3	78.80
15	2	1.0	0.3	79.47
...	...	...	...	...
16	2	2.3	0.4	85.27
17	1	0.7	0.1	86.12
18	2	1.5	-0.1	87.67
19	2	1.9	-0.2	89.59
20	2	0.8	0.3	92.65
21	3	3.4	0.6	94.17
22	2	1.3	0.3	95.22
23	2	0.7	0.0	95.23
...	...	...	...	...
24	2	1.0	-0.2	101.16
25	1	1.1	-0.1	105.50
26	2	2.8	0.4	106.19
27	2	1.2	-0.3	106.94
28	2	1.6	-0.1	107.47
29	3	1.5	0.6	109.30
30	3	5.8	2.7	110.40
31	2	0.1	0.1	111.13
32	2	1.9	-0.3	112.97
33	3	0.7	0.3	113.20
...	...	...	...	...
34	4	2.3	0.7	142.29
35	2	0.8	-0.1	143.43
36	3	1.6	-0.1	144.87
37	4	0.9	0.4	153.29
38	3	0.4	-0.1	158.16
39	5	0.2	0.1	161.41
40	3	2.1	0.2	162.37
41	5	0.9	0.3	163.04
42	5	1.2	0.4	165.10
43	5	1.9	0.7	169.30
44	3	0.7	-0.1	171.19
45	5	1.3	0.2	175.05

ers are released. The action on OLTCs aims at slowing down the fall of voltages and allowing the system to be attracted towards the new mid-term equilibrium after load shedding. The high voltages are due to

Table 2. Ranking of MV buses candidate for load shedding

MV bus	area (in Fig. 13)	$\Delta P = P^0$ (MW)	$\Delta Q = Q^0$ (Mvar)	$ S^0 - S^* $ (MVA)
10	1	21.8	11.8	126.48
13	2	27.7	11.5	129.65
19	2	26.8	7.6	135.72
18	2	21.5	9.9	141.20
11	1	11.8	5.8	143.35
17	1	10.1	6.0	145.37
27	2	16.9	6.9	145.82

cumulative effects of tap changer control and load shedding.

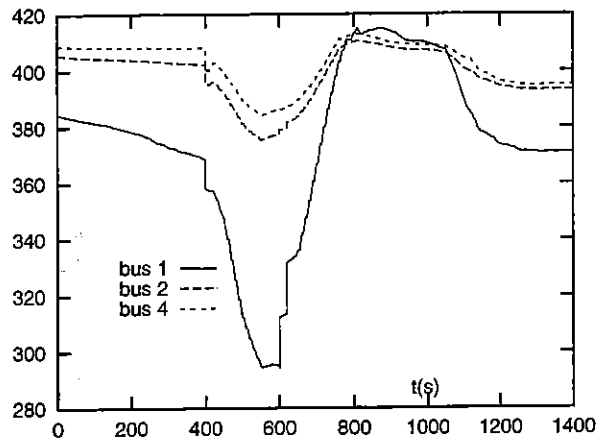


Figure 22: voltages (kV) at EHV buses. Case (iii) with load shedding

5.5. On the computational efficiency

The high computational efficiency of the overall approach has been shown in Ref. [3].

More recently, the method has been extensively used within the context of a research on "learning-by-example" methods. This required to build off-line large data bases of scenarios. Within this context, 26 disturbances have been studied on 5000 operating points of the EDF system. Using a SUN SPARCstation 10 equipment, it has been found that :

- simulating a disturbance (with stability check) takes on the average 10 s CPU;
- computing a load power margin (demand increase of 2000 MW over 1 hour, stability check each 100 s) took on the average 30 s CPU. Note that this includes many cases where the margin is high, in which case the simulation could be stopped before reaching collapse.

The method is thus very well suited to extensive simulations as well as to an interactive use. The current rise in computer power allows to envisage shorter time responses along with larger system modellings.

6. Perspectives

This paper has proposed extensions and shown real-life applications of a method combining several facets of mid-term voltage stability analysis.

The computational efficiency of the method makes it suitable for :

- extensive simulations, typical of planning and operational planning,

- interactive use, as an operator tool,
- real-time use, in emergency conditions.

An operator tool appears to be the most promising application in the short term. Let us quote non exhaustively :

- the computation of load power margins, as indicators of the system robustness with respect to load increases;
- the simulation of incident scenarios and the determination of post-contingency margins;
- in vulnerable operating conditions where the system is found to be insecure with respect to credible load increases or contingencies :
 - the instability can be analyzed and corrective actions can be suggested in order the operator to be prepared to face such situations. Depending upon the time left, actions may range from fast unit start up to preventive load shedding. In any case, taking such important decisions require evaluation criteria which the proposed method could provide;
 - the automatic actions implemented by protecting devices (tap changer blocking, undervoltage load shedding, etc.) can be simulated in order to evaluate their effect and benefit.

Truly real-time applications would be more ambitious. However, the possibility to simulate the system mid-term response faster than real-time combined with an automatic identification of system outages open some perspectives in automatic voltage security monitoring.

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Figure to accompany discussion by Alvarado on facing page

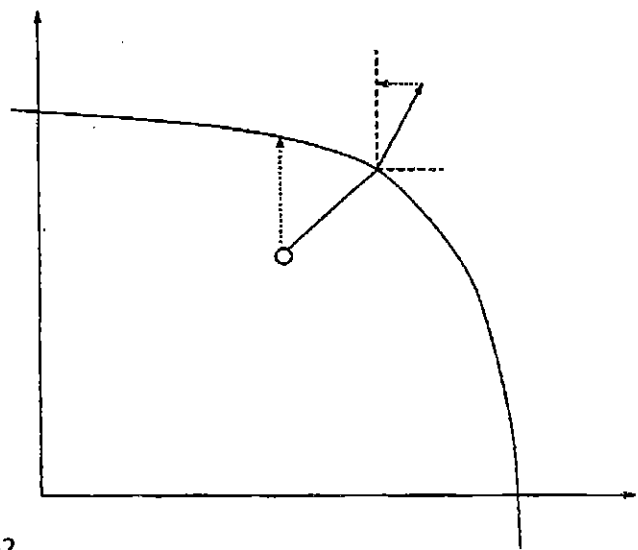


Figure 1: Determination of minimum L^1 norm distance.

Thomas J. Overbye (University of Illinois at Urbana-Champaign)

I would like to compliment the authors on a very interesting paper concerning the longer-term aspects of voltage instability. Of particular interest is the determination of corrective control actions for cases where a disturbance changes the boundary Σ so that following the disturbance S^0 is outside of Σ . For such cases, the power flow equations (using appropriate mid-term models) no longer have a real solution. Thus, if corrective control action is not taken, the system will never reach an acceptable equilibrium point (since none exists). Rather, the state will vary as dictated by system dynamics, with the usual consequence of either sustained voltage decay and collapse, or angular instability.

Therefore, it is important to determine the best control actions to restore the case to solvability. An alternative to calculating S^0 through a dynamic simulation is to directly calculate the closest boundary point (in a Euclidean sense) through the iterative power flow based method of [a]. The Euclidean distance between these two points, D , then provides a measure of the unsolvability of the case. The advantage of this approach is that it only requires computation equivalent to several power flow solutions.

The left eigenvector associated with the zero eigenvalue, w^* , is then parallel to the normal vector to Σ at this point, and shows the "optimal" direction in parameter space to return to solvability. However, this direction is seldom practical since it commonly involves changes in power injections at a number of system buses, and in its determination neither cost nor the availability of such controls has been considered (for example, w^* may have nonzero components at junction buses which have no associated load, generation, or controls). The sensitivity of D to practical controls is then given by [b]

$$D_u = -w^* J_u$$

where J_u is the Jacobian of the power flow equations with respect to the controls. An LP based algorithm for using these sensitivities, along with the dynamic consequences of unsolvability, is discussed in [c]. I would again like to commend the authors for an interesting paper highlighting the longer term aspects of the voltage instability problem.

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Fernando Alvarado (University of Wisconsin)

The author is to be commended for an outstanding paper full of practical insight. The recognition of time frame of interest as an explicit consideration in the stability analysis model is most welcome. This discussion addresses two points raised by this paper:

In all the figures considered, there was a feasible operating point right after a contingency. This point $S(0^+)$ is shown as always inside the feasibility boundary Σ in active and/or reactive demand parameter space. The

paper considers then several cases concerning the slower dynamics and eventual stable operating point (figures 9 to 11). However, it is possible that even the initial operating point would be outside of Σ . This is illustrated by transition D2 in Figure 5 of the paper: a case of "fast collapse." If this were to be the case, the system would be immediately and unavoidably in an unstable situation. This would demand the recognition that such an event would result in loss of load, or it would require preventive action to forestall the loss of load. Many of the same techniques described by the author to measure distance to security boundary would be directly applicable to this case. For example, one could design a minimum disruption preventive load curtailment program based on these ideas.

The left eigenvector associated with the zero eigenvalue at (denoted by w^*) illustrates a direction normal to Σ . If the parameter space corresponds to active and/or reactive powers at one or more buses, this direction gives an indication of a direction of demand modification that is most effective in increasing the Euclidean distance from the operating point to the boundary. However, the Euclidean distance may very well not be the best direction to consider, for the following reasons:

A minimum Euclidean distance does not mean minimum Megawatts and/or Megavars. This would be accomplished by an L^1 norm, not an L^2 norm.

A minimum distance does not take into consideration the relative importance of the bus and its context within the system (high voltage versus low voltage buses). It may perhaps be best to determine these distances only within a voltage class. For a further discussion of issues associated with weighting of demands, refer to the paper by Makarov and Hiskens in these Proceedings.

Just because a number of MW would help at a given location, it does not mean that these may be available for load shedding.

Shedding is likely to take place in discrete steps. As a result, a Euclidean norm may prove too conservative. An L^2 norm favors shedding of smaller amounts of load at more locations. An L^1 norm favors shedding larger amounts of load at fewer locations.

Shedding active load almost invariably entails shedding reactive load.

The notion of using an L^1 norm was briefly described by the author in reference [15] of the paper. In this reference it was pointed out that, if the boundary was a hyperplane, the computation of the minimum distance would be trivial. Even if the boundary Σ is not a hyperplane, an algorithm for the computation of a minimum L^1 norm would benefit from the knowledge of the normal direction. The algorithm would assign all the load to be shed to a single bus until the ability of that bus to shed load was exhausted. This would be done based on the largest direction cosine component of the normal vector. Once one vector component is exhausted, the next direction would be used. The enclosed figure illustrated the idea. Convergence issues remain to be explored, but our limited experiments using this method over the past few years indicate that the method works very well.

An L^1 norm is not without shortcomings. However, it may, for some cases, prove to be a better norm than an L^2 norm.

Thanks to Ian Dobson for comments on this discussion.

DISCUSSION ON LONG-TERM DYNAMIC MODELING AND LINEAR INDICES

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In [1], the authors propose a methodology for evaluating voltage stability that can very accurately model many long term dynamics, while yielding considerable computational savings relative to a *full* dynamic simulation. The results they display indicate the efficacy of the method in many applications. However, while the authors provide some qualifications as to the range of application of their technique, the goal of this discussion is to highlight these points further, and to seek feedback from the authors as to whether their numerical experience is consistent with the limits one might expect from an analytic standpoint. Based on our interpretation, it appears that their work rests on several key assumptions, and we would welcome their comments as to whether our observations match their own.

First, the approach assumes one can isolate a subset of components of the state equation that may be associated with a stable fast time scale. Certainly, this is often the case in power system models, especially when dealing with automatic transformer tap changers, as the authors propose in the paper. In particular, the technique requires the trajectories of the true system (including these “fast” state equations) to be well approximated by a system obtained when these state equations are replaced by algebraic constraints. To illustrate this concretely, consider a set of system equations of the form:

$$\dot{z} = h(z, x, y, \lambda(t)) \quad (1a)$$

$$\dot{x} = f(z, x, y, \lambda(t)) \quad (1b)$$

$$0 = g(z, x, y, \lambda(t)) \quad (1c)$$

where $z \in Z \subseteq \mathbb{R}^p$ represents the set of “slow” system dynamic variables such as transformer taps, the vector $x \in X \subseteq \mathbb{R}^n$ stands for “fast” transient variables like those of generator dynamics, and $y \in Y \subseteq \mathbb{R}^m$ corresponds to the variables associated to the algebraic constraints $g(\cdot)$ and are assumed to change instantly. $\lambda(t) : \mathbb{R} \mapsto \mathbb{R}^l$ represents the various system parameters that slowly change with time t , such as system loads. By eliminating the fast transient dynamics in equations (1), i.e., equating the right hand side of (1b) to zero, one could approximate the actual system trajectories when “small” changes take place in the network; tests to establish whether the approximate trajectory remains close to the full trajectory are well treated in the singular perturbation literature. We will comment further below.

Second, the authors wish to confirm stability of the system by examining eigenvalues of a linearization about this approximate trajectory. Alternately, one might use various linear sensitivity indicators as computationally efficient approximations to stability information obtained from an eigen-analysis. In this context, the motion of the system along the trajectory is being driven by time variations of one or more system parameters or exogenous inputs, i.e., $\lambda(t)$ in (1). The linearization that describes local behavior of deviations of the state from

this trajectory is itself a time varying linear system. As the authors no doubt realize, even a full eigen-analysis that displays all eigenvalues with strictly negative real parts is not adequate to rigorously *guarantee* stability of such a time varying system.

To translate these general theoretical observations to a practical context, one might expect that several requirements need to be met for this technique to be applicable. First, the dynamics of the “fast” states must indeed have their time scales well separated from the dynamics that are assumed slow, and these fast dynamics must be stable and converge rapidly to the quasi-equilibrium predicted by setting their right hand sides to zero. A heuristic test of these properties might be to check that the eigenvalues of the sub-block of the linearized dynamics associated with these states has all eigenvalues “far” into the left half plane, with all complex pairs showing good damping. Hence, the analysis that the authors proposed would provide good indication that this requirement is indeed met along the approximate trajectory.

Assumptions to guarantee stability of a time varying system from a sequence of “quasi-equilibrium” linearizations has been the subject of many works in the control literature; work by Dr. DeMarco may be found in [2]. In brief, one needs one of two conditions on the time varying inputs driving the evolution the system: their time rate of change must be sufficiently slow (where slow is measured relative to time constants of all the system dynamics), or they must be sufficiently small in magnitude (where the size of the allowable perturbation relates to the region of attraction about the nominal trajectory). In our opinion, it is these conditions that are not checked within the method as described, and need to be imposed as a priori assumptions. To illustrate the potential difficulties, the following examples for the simple system of Fig. 1 illustrate two possible types of difficulties, as shown below. This network can be simply model using equations

$$\begin{aligned} \dot{\delta} &= \omega \\ \dot{\omega} &= \frac{1}{M} \left[P(t) - \frac{V}{X} \sin(\delta) - D\omega \right] \\ \dot{V} &= \frac{1}{\tau} \left[-kP(t) - \frac{V^2}{X} + \frac{V}{X} \cos(\delta) \right] \end{aligned} \quad (2)$$

The third equation in (2) could be viewed as a particular representation of a dynamic load model, and is necessary for the system to “make sense” from the point of view of Lyapunov stability, as discussed in detailed by Dr. Cañizares in previous comments to papers [3, 4]. In these equations, the loading level $P(t)$ represents the exogenous input, i.e., $P(t) \equiv \lambda(t)$. The results shown here were obtained for the following p.u. parameters values: $M = 0.1$, $X = k = 0.5$, $\tau = 1\text{ms}$. Different values of the damping D were used in each example in order to reduce computational burden.

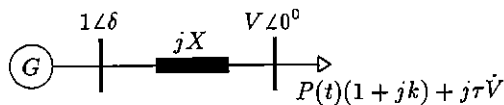


Fig. 1: Sample system.

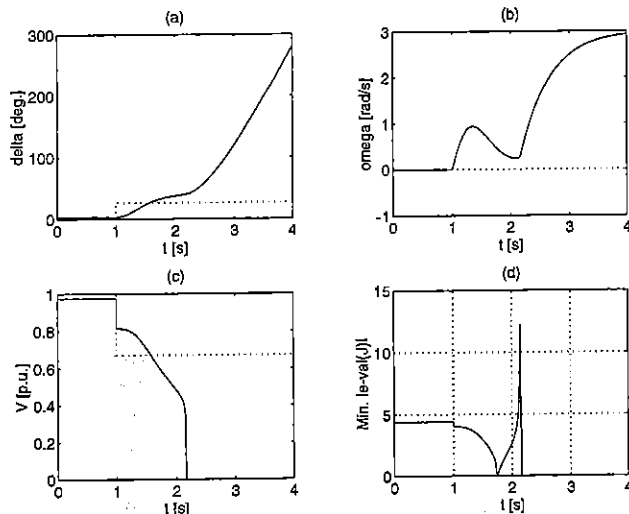


Fig. 2: Simulation of a large change in $P = 0.1 \rightarrow 0.6$ at $t = 1$ s. All state variables (a) δ , (b) ω , and (c) V are depicted, as well as the (d) minimum magnitude of the Jacobian J eigenvalues. The continuous lines are used to represent the actual system trajectories, whereas the dotted lines are used to depict the equilibrium points. The voltage collapses due to the large system perturbation.

In the first case, the exogenous input $P(t)$ undergoes a step change of relatively large magnitude. Hence, the time variation is obviously neither slow nor small in magnitude, and the state diverges from the approximate trajectory that might have been predicted as stable, as shown in Fig. 2 ($D = 0.2$). Observe that the linear indicator in this case, i.e., the minimum magnitude of the Jacobian eigenvalue, goes to zero, erroneously suggesting the presence of a system bifurcation, if one were to use this particular value as a gauge of proximity to a fold bifurcation.

The second example is more subtle, as illustrated in Fig. 3 ($D = 1$). In this case the loading level was slowly increased to a value slightly larger than the corresponding bifurcation point value of $P \approx 0.618$. Observe that the rate of change of the input does appear slow in an absolute sense; however, an eigenvalue predominately associated with the fast dynamics of voltage approaches zero. Hence, relative to the time scale of the dynamics, the rate of change of the input is no longer slow. This is clearly illustrated in the system trajectories of Fig. 3, where they follow the corresponding equilibrium points, up to $t = 65$ s when a saddle-node bifurcation is encountered resulting in the disappearance of the corresponding equilibrium points. Due to the highly nonlinear behavior of the system Jacobian eigenvalues, i.e., an eigenvalue converges to zero only when the system is very close to the bifurcation point (similar behavior has been observed for other linear sensitivity indicators [5, 6]), the indicator fails to detect the precise moment when the system has reached the bifurcation condition, con-

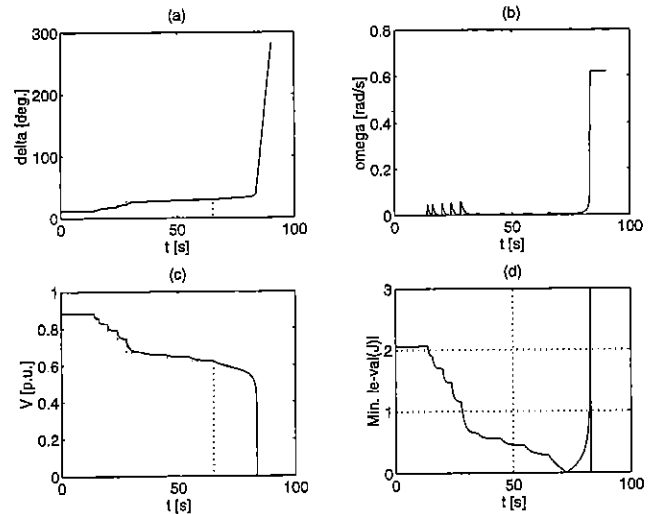


Fig. 3: Simulation of a gradual change in $P = 0.3 \rightarrow 0.62$. All state variables (a) δ , (b) ω , and (c) V are depicted, as well as the (d) minimum magnitude of the Jacobian J eigenvalues. The continuous lines are used to represent the actual system trajectories, whereas the dotted lines are used to depict the equilibrium points. The voltage collapses due to a saddle-node bifurcation problem.

verging to zero at about 72s.

We would be interested in the authors opinions on the cases shown, and whether their experience with practical computations in larger systems displayed similar challenges. In particular, have they observed difficulties in cases with large magnitude changes in inputs, or cases where there is not sufficient separation between rate of change of input and the time scale of the system dynamics? What are the authors observations on the behavior of the suggested sensitivity measure in these cases, particularly when the system approaches a bifurcation point?

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Closure to discussions of the paper “Extensions and applications of a mid-term voltage stability analysis method”

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We thank all the Discussers for their interest in our work.

To Prof. Overbye, about the approximation of Σ :

The first point raised by Prof. Overbye is about the approximation of the boundary Σ of the feasible set $\mathcal{S}$.

The key idea is to find just one point of Σ , compute the normal vector there and then (trivially) obtain the tangent hyperplane. Refs. [a,c] propose to determine the boundary point closest to S^0 in a Euclidian sense, by solving an optimization problem. We propose to identify the point where the system trajectory (provided by simulation) hits this boundary.

In fact, as far as the linear approximation is acceptable, all points in some “reasonable” neighbourhood are equally valid to build this tangent hyperplane. In our approach, there is no advantage in determining the one closest to S^0 in a Euclidian sense, since we do not use this criterion for corrective control (see hereafter).

Moreover, we see several advantages in using dynamic simulation :

- in practice, different parts of Σ correspond to different generators under reactive power (e.g. rotor current) limitation. Through simulation, the generators of concern are automatically identified. Although we have no experience with the method of Refs. [a,c] we suspect that the effort of identifying the (a priori unknown) limited generators at the closest boundary point is not negligible;
- intuitively, the point where the system trajectory hits Σ is a good candidate since load power restoration naturally tends to make it as close as possible to S^0 ;
- using the sensitivity techniques described in the paper, it is very easy to detect the hitting of Σ along the system trajectory. More generally, we believe that combining dynamic simulation and sensitivity analysis is a desirable evolution of voltage stability analysis tools;
- dynamic simulation takes into account controls which cannot be easily incorporated in a power flow formulation. This is particular true for controls whose activation depends on the system time evolution (e.g. capacitor/inductor switching triggered by protections against voltage collapse, secondary voltage control, etc.).

On the other hand, one might argue that dynamic simulation is computationally demanding. While still true for full numerical integration, this does not apply to the simplified simulation we use, which offers a good compromise between efficiency and accuracy.

To Profs. Overbye and Alvarado, about the L_1 vs. L_2 -norm :

We agree with Profs. Overbye and Alvarado that the (Euclidian) L_2 -norm does not provide a practical direction to return to solvability. We would like to offer the following comments.

The L_2 -norm criterion introduced by Dr. Ian Dobson is a reasonable choice to compute the closest bifurcation point in the load power space, in the solvable case, i.e. in a preventive context. Even in this case, it is necessary to add “safeguards” preventing the algorithm from converging to unrealistic load patterns.

On the other hand, for corrective control purposes, the procedure we propose in the paper is a *improved* L_1 -norm criteria, as clarified hereafter.

Assume we can act on the injections of N buses, hence having a $2N$ -dimensional load power space. Consider the tangent hyperplane $\mathcal{H}$ with normal vector $\mathbf{n}$, on which we want to bring the point S^0 in some optimal way.

- Assume first that active and reactive powers can be changed independently, so that we have $2N$ degrees of freedom. The criterion initially proposed in Ref. [15] of the paper, consists in finding the point S of $\mathcal{H}$ which minimizes :

$$\sum_{i=1}^{2N} S_i^0 - S_i \quad (a)$$

In the case of load shedding for instance, (a) is the amount of unrestored load, which it is desirable to minimize. The same holds for other controls. Since we limit ourselves to $S_i \leq S_i^0$, (a) can be rewritten as the conventional L_1 -norm criterion :

$$\sum_{i=1}^{2N} |S_i^0 - S_i| \quad (b)$$

Finding the point of $\mathcal{H}$ which minimizes (b) is trivial and does not require to use optimization methods. Indeed :

- without limits on the S_i 's : the optimum is the projection of S^0 on $\mathcal{H}$ parallel to the axis along which the normal vector $\mathbf{n}$ has the largest component. This corresponds to acting on a single power, the one with the lowest distance D_i to $\mathcal{H}$, where D_i is measured along the S_i -axis.
- with limits of the type $S_i \geq S_i^{\min}$: the optimum is obtained by changing the power with the smallest D_i as far as possible (i.e. up to having $S_i = S_i^{\min}$), then considering the power with the second smallest D_i , and so on until coming back on $\mathcal{H}$. This corresponds to summing up the actions, as described in the paper.

- The above strategy applies to capacitor/inductor switching directly. For other types of control however, the active and reactive powers cannot be changed independently. In this case, for each bus j , we measure the distance D_j to $\mathcal{H}$ along the direction corresponding to changes in both active and reactive powers (see d in Fig. 12 of the paper where S_1 is the active and S_2 the reactive power). We are thus left with N (instead of $2N$) distances D_j and again, we take actions at successive buses by increasing order of D_j .

As can be seen, the simple idea of bringing the point S^0 back on $\mathcal{H}$ accommodates many practical constraints.

To Profs. Cañizares and DeMarco, about decoupled time scale modelling :

The first point raised by the Discussers deals with the consequences of assuming time decomposition and setting $\dot{x} = 0$. Briefly stated, these simplifications do not allow to detect all instability mechanisms but they are reasonable for mid-term voltage stability analysis, which is the scope of our paper and the practical concern for many utilities. We now elaborate on this viewpoint.

The existence of some time separation between the transient and mid-term dynamics is a well-known fact. The former typically spans over a few seconds and the latter over a few ten seconds to a few minutes. As far as voltage stability is concerned, the mid-term dynamics comes from controls which were generally designed so as not to interact with the transient dynamics. Obviously, in practice, the two time scales are not totally separated and discarding the transient dynamics results in some approximation. We do not expect and - in fact do not require - the so approximated trajectory to match perfectly the "exact" response given by full numerical integration. It renders however the average trends of the system.

Assuming a good separation between transient and mid-term time frames, we can classify the unstable system responses to a large disturbance as follows (the following material is borrowed from Refs. [2, 7] and our closure to the discussions of Ref. [3]).

1. *Instability of the transient dynamics before mid-term dynamics has time to act.* This may correspond to :

- 1.1. the loss of a Transient Equilibrium Point (TEP), i.e. a solution to Eqs. (1,9) with z at its initial value;
- 1.2. or the lack of attraction towards a stable TEP, which in turn is a matter of domain of attraction and disturbance severity.

If the above instability does not occur, then the system evolves due to the mid-term dynamics. This evolution can be seen as the response of the (faster) transient dynamics (1,2) to the (slower) sequence of perturbations $z(k+1) - z(k)$ stemming from mid-term controls. The following two instability scenarii may result :

2. *Instability of the transient dynamics caused by changes in mid-term variables z .* This may correspond to one of the following :

- 2.1. the TEP disappears;
- 2.2. the TEP still exists but becomes unstable (e.g. growing oscillations appear);
- 2.3. the domain of attraction of the TEP shrinks.

3. *Instability of the mid-term dynamics, with stable transient dynamics.* Again, this may correspond to :

- 3.1. the loss of a mid-term equilibrium point;
- 3.2. the lack of attraction towards a stable mid-term equilibrium point.

At this point, it is appropriate to distinguish between transient and mid-term (or "longer-term") voltage instabilities [1].

Transient voltage instability may result from loads which try to restore their power in the transient time frame (e.g. induction motors) and/or from strong disturbances (e.g. D_2 in Figs. 5 and 6). It corresponds to instabilities of type 1 above.

On the other hand, mid-term voltage instability corresponds to a great majority of experienced incidents and to load power restoration within minutes. The *cause* is an instability of type 3. The *result* is a sagging voltage profile, which *may or may not* yield instabilities of type 2 (see e.g. the two cases of Fig. 18).

The methods described in our paper primarily address Cases 3.1 and 3.2. This analysis is not affected by the $\dot{x} = 0$ simplification, which yields considerable savings in computing time and serves our objective of real-time application.

Let us nevertheless discuss the consequences of setting $\dot{x} = 0$ on the detection of the other instabilities.

Instabilities of types 1.1 and 2.1 can still be detected, through the loss of a TEP. In Ref. [7], instability 2.1 was shown to be responsible for the plunging down of voltages or the loss of synchronism concluding a more progressive voltage profile degradation.

On the other hand, we are unable to detect instabilities of the types 1.2, 2.2 and 2.3.

If a contingency is suspected to yield type 1.2 instability, a full dynamic check is required. An elegant solution proposed in [17] consists in checking the transient period with full dynamic simulation, then switching to $\dot{x} = 0$. This would nevertheless remain heavy for real-time applications.

As mentioned by the Discussers, Case 2.2 can be detected by checking the eigenvalues of the transient dynamics, considering the mid-term dynamics as infinitely slow (incidentally, this is exact in between step changes of tap changers). This precisely was done in Ref. [7] on a simple but realistic example.

Case 2.3 may relate to a saddle-node bifurcation (Case 2.1) as follows. The changes in mid-term variables z can be considered as changes in parameters of the faster transient dynamics. If these parameter changes make the transient dynamics approach a saddle-node bifurcation, the domain of attraction of the stable equilibrium shrinks. Hence, the changes in z which can be considered "small" in normal circumstances, may become sufficient to make the system operating point leave the domain of attraction.

At this point, it is interesting to proceed to the examples given by the Discussers.

We understand that these examples have been designed to illustrate situations where $\dot{x}$ cannot be neglected. However, before discussing this aspect, we would like to stress the following.

1. The Discussers' equations (2) look erroneous. δ cannot be at the same time the rotor angle with respect to a synchronous reference and the phase angle difference along the reactance. This would require the load bus to be an infinite bus...
2. Undoubtedly the $\dot{V}$ term in the load modelling makes the analysis easier (V is then a state instead of an algebraic variable) but the physical meaning is obscure.
3. The load active power is modelled as instantaneously constant while the reactive part is restored to constant power with a time constant of 1 millisecond. Consequently, there is nothing like a slow vs fast

dynamics in the authors' example : only transient (and even faster) dynamics is involved.

In the system shown, a saddle-node bifurcation of the transient dynamics (loss of TEP) occurs for (the load parameter) $P \simeq 0.618$ pu.

In the first example (Fig. 2), the load is changed from 0.1 to 0.6 in a single step. The equilibrium point corresponding to $P = 0.6$ has a little domain of attraction since it is close to bifurcation. Expectedly, the system is not attracted towards this equilibrium : an instability of type 1.2 occurs. The latter cannot be detected with $\dot{x}$ set to zero, although one could easily check that the system has a little post-disturbance load power margin ($0.618 - 0.6 = 0.018$ pu compared to the 0.5 pu load increase).

In the second example (Fig. 3), a pseudo-mid-term evolution is rendered by increasing the demand from 0.3 to 0.62 in several steps over one minute. Up to $t \simeq 65$ s the transient dynamics follows closely the trajectory approximated by successive equilibria. The last step change in demand causes the TEP to disappear : instability of type 2.1 occurs. As this saddle-node bifurcation is approached, an eigenvalue of the transient dynamics (with fixed z) approaches zero and hence the time decoupling assumption is less and less valid. This explains the final discrepancy between the dotted and the solid lines in Fig. 3. However, we do not share the Discussers' pessimism about the successive equilibria approach : after all, without simulating the system dynamics, the plunging down of voltages is *anticipately* detected at $t \simeq 65$ s through the loss of TEP. Also, with smaller load steps, one would determine more precisely the bifurcation value 0.618.

The Discussers' examples do not offer the possibility to illustrate what our method has been designed for : mid-term voltage stability analysis. Therefore, we would like to briefly address this point, through an example on the EDF system.

In Fig. A hereafter, we reproduce the system evolution shown in Fig. 18 (case $\alpha = 1, \beta = 2$). The dramatic voltage decline ends up with a loss of TEP, at $t \simeq 640$ s. This corresponds to the saddle-node bifurcation of the transient dynamics as in the Discussers' second example. Probably, close to this point, the successive equilibria trajectory does not closely match the exact system trajectory... This is however of little concern. Indeed, consider now Fig. B which shows a $\partial Q_g / \partial Q_i^0$ sensitivity computed along the trajectory. The change in sign experienced at $t = 460$ s identifies the critical point of the mid-term dynamics. *This is the point of concern for security analysis.* Observe how earlier it is met than the saddle-node bifurcation of the transient dynamics. Moreover, we expect this point to be stable from the transient dynamics viewpoint.

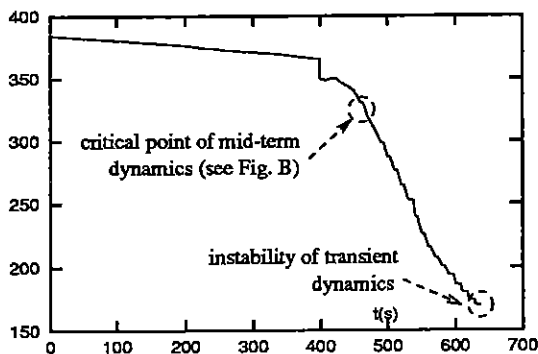


Figure A: Voltage at bus 1 (kV) in Example 2 of the paper

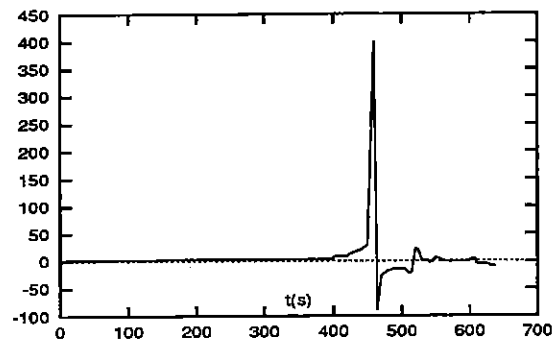


Figure B: a sensitivity $\partial Q_g / \partial Q_i^0$ along the trajectory of Fig. A

To Profs. Cañizares and DeMarco, about linear indices :

The second point raised by these Discussers deals with the interpretation of sensitivities computed along the system trajectory.

We fully agree that linearizing a nonlinear system driven by some variable inputs yields a time-varying system of which the standard eigenvalue criterion does not guarantee stability. We also conceive easily that some conditions must be imposed on the magnitude and/or rate of change of the inputs in order the stability to be guaranteed by the standard eigenvalue criterion.

In fact, we do not see anything contradictory to the Discussers' remarks in what we do. Indeed, we compute and interpret sensitivities (equivalent to eigenvalues) in the following way :

- *once the system has reached mid-term equilibrium point* : for instance, to confirm that the final point of the solid (resp. dotted) line in Fig. 17 is stable (resp. unstable). Clearly, positive sensitivities can be interpreted as stable mid-term equilibrium.
- *in load power margin calculations* : a ramp increase in demand is imposed to the system and the sensitivities are checked at regular time intervals. Speaking of this load increase, we indicate in [3] that "the rate of change, while steep from system operation viewpoint, can be considered "smooth" with respect to the system dynamics". This means that the system evolution can be considered close to a succession of mid-term equilibria. Hence, positive sensitivities can again be interpreted as stable mid-term equilibrium.
- *Note incidentally that our voltage collapse proximity index is the load power margin. The sensitivities are only used to determine the local stability while stressing the system in the large.*
- *in large-disturbance scenarios (e.g. Figs. A and B) : we compute sensitivities along the system trajectory with the sole aim to identify the (critical) point where they change sign, which corresponds to the point where the trajectory hits the boundary Σ of the feasible set $\mathcal{S}$. In this case, we do not interpret positive sensitivities as the stability of a mid-term equilibrium, for the simple reason that this mid-term equilibrium does generally not exist any longer.*

However, in Section 3.2 of the paper (and in the conference oral discussions) we mentioned a possible, careful interpretation of the sign : for the particular mid-term dynamics of concern here, each point of the collapsing trajectory can be seen as the mid-term equilibrium point corresponding to modified tap changer voltage set-points.

Transposing the above considerations to the Discussers' examples of transient dynamics :

- in the second example, from $t = 0$ to 65 s, the load increase can be considered smooth. The linear indicator (in this case, the minimum magnitude Jacobian eigenvalue) can be interpreted as a stability condition. Rigorously speaking, this indicator should be computed at the successive equilibria although it does not make a big difference to evaluate it along the system trajectory;
- at $t = 65$ s, we detect the loss of equilibrium. Hence, the linear indicator cannot be subsequently interpreted (stability condition of something inexistent ?!);
- in the first example, the large disturbance makes the system trajectory depart significantly from equilibrium. Hence, the linear indicator evaluated along the trajectory cannot be interpreted as a stability condition of this equilibrium. The indicator should be evaluated at the equilibrium point, and would indicate that the latter is stable (although marginally as already quoted);
- in Fig. 2 and after $t = 65$ s in Fig. 3 : the only point of interest is where the system has a zero eigenvalue. This point belongs to hypersurface Σ , which is the parabola $PX^2 + QX - 0.25 = 0$ in this simple example.

To Prof. Alvarado, about the curvature of Σ :

As briefly quoted by the Discusser, we suspected in Ref. [15] that the curvature of hypersurface Σ might cause some difficulties. There are some indications however that it is not too pronounced in practice. For instance, using a CIGRE test system, it was observed in Ref. [A] (pp. 69-71) that a load power margin computed for reactive power *only* decreases *linearly* as active *and* reactive loads are increased up to the critical point.

To Prof. Alvarado, about transient voltage collapse :

Transient voltage collapse offers similarities with transient angle instability in the sense that it has to be counteracted either preventively, as Prof. Alvarado suggests, or by fast controls or protections. Dealing with these two aspects through the techniques we devised for mid-term voltage stability analysis would require some caution. However, this extension deserves attention.

Again, we thank all the Discussers for their questions and valuable comments.

[A] J. Van Hecke (convener), N.D. Hatziaargyriou, T. Van Cutsem (editors), "Indices Predicting Voltage Collapse including Dynamic Phenomena", Final Report of CIGRE TF 38.02.11, July 1994

Power System Modeling for Coupled Voltage/Power Dynamics

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1 Introduction

Voltage phenomena on the bulk power system are typically studied in the context of the commonly-used decoupling assumption that separates the behavior of the electric power transmission network into two independent subsystems. The first involves the electromechanical dynamics of the generators and the real power transmitted, while the second is concerned with system voltages and the generation and transmission of reactive power. This assumption is adequate for many purposes, but under certain conditions that may be considered as unusual operating conditions or as short-term transients following system disturbances, the decoupling assumption does not hold. Under conditions of heavy transmission line loading, depressed load-bus voltages and large phase angle differences across the lines, real power/voltage coupling is a significant factor in the behavior of the system, and must be considered for accurate analysis or prediction of system behavior. In particular, some instances of inter-area oscillations seem to also involve large deviations in both generator rotor angles and load-bus voltages, creating a situation in which the most effective means for preventing undesirable oscillations may be a reactive-power device such as a static VAR compensator, located so as to mitigate the load-bus voltage response. The widespread control of rotor-angle oscillations via power system stabilizers, which modulate the field excitation and therefore can be thought of as acting directly on the generator voltage, is another indication that the voltage/real power link cannot always be ignored.

In the field of short-term phenomena, the classical area of concern is so-called "first swing" stability, limited to the first cycle of generator rotor acceleration following a disturbance, but it is becoming more common to see loss of generator synchronism on the second swing or later following a disturbance [1]. Moreover, inter-area oscillations may exhibit negative damping that leads to loss of synchronism, which is not predicted via small-signal analysis. Although it is true that properly-applied generator excitation control can produce a significant improvement in the first-swing stability of the system, when the system

is at risk over a longer time period, excitation control (and other forms of voltage-type control) can be even more effective. This means that the traditional decoupling assumption as applied to the short-term stability problem may yield overly conservative results.

Energy-function methods have been successful in predicting first-swing stability in some cases, however, in spite of the fact that some effort has been made to include voltage dynamics in energy function methods, a generally-applicable method does not appear to be available. So-called controlling unstable equilibrium point (uep) and more recently, the boundary controlling unstable equilibrium point (BCU) analysis, have refined this approach, but typically ignore the voltage coupling [2, 3, 4].

In this paper, a formulation of the system equations is presented that preserves the voltage and power/angle dynamics, and results in a state-space-like structure in which the impact of the generator voltage and field excitation on the output power of the generator is captured in a state-dependent matrix. The machine real and reactive power output is defined in terms of the new model, as well the total system real power and the reactive power and its time derivative. Finally, an application is suggested for including an energy term based on the total system reactive power as part of the total system energy.

2 Modeling the Network

In all that follows, loads are assumed to be constant impedance. This allows the admittance matrix that represents the transmission network to be reduced via Kron's reduction, so that a system with m generators utilizes an $m \times m$ reduced admittance matrix. It is well-known that neither the constant-impedance nor the constant-power load model accurately reflects the actual behavior of system loads. In situations where voltage is declining over a period of several minutes, in which the electromechanical behavior of generators is not significant and discrete-acting control devices such as tap-changing transformers and switched capacitor banks have time to react, a constant load

model may well be justified. On a shorter time scale, when generator dynamics are indeed significant, and voltage deviations take the form of large oscillations, it is not clear that this is still the case, and for the purposes of transient analysis, the constant-impedance model may be adequate.

In the following development, a two-axis flux-decay model is used for the generators. This is a standard model, and is presented in Appendix A. Some current work in power system modeling is to be found in [5, 6].

The solution for the Park-transformed machine currents i_d and i_q may be expressed most simply using a block-matrix form. Initially, the state variables are grouped into block form, so that for a system with m machines, using underbars to denote vector quantities, we have

$$\begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix} = \begin{bmatrix} E'_{d_1} & \dots & E'_{d_m} & E'_{q_1} & \dots & E'_{q_m} \end{bmatrix}^T. \quad (1)$$

We now define the well-known transformation from the network frame of reference (subscripts capitalized) to the machine frame of reference (lower-case subscripts) in block form as:

$$\begin{bmatrix} \underline{V}_D \\ \underline{V}_Q \end{bmatrix} = T(\delta) \begin{bmatrix} \underline{V}_d \\ \underline{V}_q \end{bmatrix}, \quad (2)$$

where

$$T(\delta) = \begin{bmatrix} S(\delta) & C(\delta) \\ -C(\delta) & S(\delta) \end{bmatrix} \quad (3)$$

$$T^{-1}(\delta) = \begin{bmatrix} S(\delta) & -C(\delta) \\ C(\delta) & S(\delta) \end{bmatrix} \quad (4)$$

$$S(\delta) = \text{diag}(\sin(\delta_1) \dots \sin(\delta_m))$$

$$C(\delta) = \text{diag}(\cos(\delta_1) \dots \cos(\delta_m)).$$

Here δ_i is the rotor angle of the i^{th} generator in the Park-Blondel transformed coordinates, measured relative to an arbitrary systemwide reference. Since this is just a coordinate rotation, $T^T T = I$. Now, assuming that loads may be represented as constant admittances, the complex-valued admittance matrix $\hat{Y}_r$ (the circumflex denotes a complex-valued matrix) can be ordered such that all non-machine busses occupy the bottom of the full admittance matrix, so that

$$\hat{Y} = \begin{bmatrix} \hat{Y}_{GG} & \hat{Y}_{GL} \\ \hat{Y}_{LG} & \hat{Y}_{LL} \end{bmatrix}, \quad (5)$$

which reduces to

$$\hat{Y}_r = \hat{Y}_{GG} - \hat{Y}_{GL} \hat{Y}_{LL}^{-1} \hat{Y}_{LG}, \quad (6)$$

from which the block-form reduced matrix is formed:

$$\begin{aligned} Y_r &= \begin{bmatrix} \Re\{\hat{Y}_r\} & -\Im\{\hat{Y}_r\} \\ \Im\{\hat{Y}_r\} & \Re\{\hat{Y}_r\} \end{bmatrix} \\ &= \begin{bmatrix} G_r & -B_r \\ B_r & G_r \end{bmatrix}. \end{aligned} \quad (7)$$

Note that G_r and B_r are symmetric.

The currents $\underline{i}_d$ and $\underline{i}_q$ are now expressed in terms of the voltages $\underline{V}_d$ and $\underline{V}_q$ and matrix Y_r as follows:

$$\begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} = T(\delta)^{-1} Y_r T(\delta) \begin{bmatrix} \underline{V}_d \\ \underline{V}_q \end{bmatrix}. \quad (8)$$

The scalar relationships

$$V_d = E'_d + x'_q i_q \quad (9)$$

$$V_q = E'_q - x'_d i_d \quad (10)$$

can be written for an m -machine system as the vector equation

$$\begin{bmatrix} \underline{V}_d \\ \underline{V}_q \end{bmatrix} = \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix} + X \begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix}, \quad (11)$$

where

$$\begin{aligned} X &= \begin{bmatrix} \mathbf{0} & \mathcal{X}'_q \\ -\mathcal{X}'_d & \mathbf{0} \end{bmatrix} \\ \mathcal{X}'_d &= \text{diag}(\underline{x}'_d) \\ \mathcal{X}'_q &= \text{diag}(\underline{x}'_q). \end{aligned} \quad (12)$$

Note that here and in all that follows, capital calligraphic letters are used exclusively to denote diagonal matrices (e.g., in (12), $\mathcal{X}_q$ is diagonal, but X is not). Substituting (11) into (8):

$$\begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} = T^{-1} Y_r T \left\{ \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix} + X \begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} \right\} \quad (13)$$

or

$$\begin{aligned} &[I - T^{-1} Y_r T X] \begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} \\ &= T^{-1} Y_r T [T^{-1} Y_r^{-1} T - X] \begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} \\ &= T^{-1} Y_r T \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix}, \end{aligned} \quad (14)$$

where the dependence of the transform upon δ has been suppressed for notational convenience. Finally, premultiplying by $[T^{-1} Y_r^{-1} T - X]^{-1} T^{-1} Y_r^{-1} T$ and factoring, we have

$$\begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} = T^{-1} [Y_r^{-1} - T X T^{-1}]^{-1} T \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix}. \quad (15)$$

This expression may be simplified if $\mathcal{X}'_q = \mathcal{X}'_d$, i.e., when saliency is ignored. Specifically, when $\mathcal{X}'_q = \mathcal{X}'_d$, the matrices T and X commute, so that $T X T^{-1} = X T T^{-1} = X$, and

$$\begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} = T^{-1} (Y_r^{-1} - X)^{-1} T \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix}. \quad (16)$$

Denoting this matrix by $\bar{Y}_r$, we have

$$\begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} = T^{-1} \bar{Y}_r T \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix}. \quad (17)$$

This can be expressed in a more general form, where the context will determine whether saliency has been ignored.

$$\begin{bmatrix} \underline{i}_d \\ \underline{i}_q \end{bmatrix} = Y_m(\underline{\delta}) \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix} \\ = \begin{bmatrix} Y_{m11} & Y_{m12} \\ Y_{m21} & Y_{m22} \end{bmatrix} \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix}. \quad (18)$$

It can be shown that $(Y_r^{-1} - X)^{-1}$ from (16) is precisely the reduced admittance matrix that is calculated when the machine transient reactance is included as part of the network, and the generator terminal bus is eliminated by matrix reduction. To see this, we note that in including the machine transient reactance, it must first be converted to a susceptance. The $m \times m$ block matrix of machine susceptances is $-X^{-1}$, so that the augmented matrix takes on the block form

$$Y_{aug} = \begin{bmatrix} -X^{-1} & X^{-1} \\ X^{-1} & Y_r - X^{-1} \end{bmatrix}. \quad (19)$$

This matrix may again be reduced via (6). Of course, in practice the reduction would be accomplished in one step, but the results are the same in either case. Applying (6), we have

$$\begin{aligned} Y_{rr} &= -X^{-1} - X^{-1}(Y_r - X^{-1})^{-1}X^{-1} \\ &= X^{-1}[-I - (Y_r - X^{-1})^{-1}X^{-1}] \\ &= X^{-1}(Y_r - X^{-1})^{-1}[-(Y_r - X^{-1}) - X^{-1}] \\ &= X^{-1}(X^{-1} - Y_r)^{-1}Y_r \\ &= (I - Y_r X)^{-1}Y_r \\ &= (Y_r^{-1} - X)^{-1} \quad \square \end{aligned}$$

3 The Structure of Y_m

In the case in which saliency is ignored, we can proceed to derive a form for the matrix Y_m . Performing the block-form multiplication in (17),

$$Y_{m11} = S(\underline{\delta})G_r S(\underline{\delta}) - C(\underline{\delta})B_r S(\underline{\delta}) + S(\underline{\delta})B_r C(\underline{\delta}) + C(\underline{\delta})G_r C(\underline{\delta}) \quad (20)$$

$$Y_{m12} = S(\underline{\delta})G_r C(\underline{\delta}) - C(\underline{\delta})B_r C(\underline{\delta}) - S(\underline{\delta})B_r S(\underline{\delta}) - C(\underline{\delta})G_r S(\underline{\delta}) \quad (21)$$

$$Y_{m21} = -Y_{m12} \quad (22)$$

$$Y_{m22} = Y_{m11}. \quad (23)$$

Recognizing that $S(\underline{\delta})$ and $C(\underline{\delta})$ are diagonal matrices, and applying trigonometric identities leads to a compact formulation. First, we require some new notation. We define the state-dependent matrices

$$\widetilde{BC} : \widetilde{bc}_{ij} = b_{ij} \cos(\delta_i - \delta_j) \quad (24)$$

$$\widetilde{BS} : \widetilde{bs}_{ij} = b_{ij} \sin(\delta_i - \delta_j) \quad (25)$$

$$\widetilde{GC} : \widetilde{gc}_{ij} = g_{ij} \cos(\delta_i - \delta_j) \quad (26)$$

$$\widetilde{GS} : \widetilde{gs}_{ij} = g_{ij} \sin(\delta_i - \delta_j), \quad (27)$$

using b_{ij} and g_{ij} to represent the ij^{th} elements of the B_r and G_r matrices from (7). This leads to

$$\begin{aligned} Y_{m11} &= \widetilde{GC} + \widetilde{BS} \\ Y_{m12} &= \widetilde{GS} - \widetilde{BC} \\ Y_{m21} &= -Y_{m12} \\ Y_{m22} &= Y_{m11}. \end{aligned} \quad (28)$$

Now, making the following definitions,

$$\mathcal{E}'_d = \text{diag}(\underline{E}'_d) \quad (29)$$

$$\mathcal{E}'_q = \text{diag}(\underline{E}'_q), \quad (30)$$

and recalling the scalar expression for generator real and reactive power when saliency is ignored,

$$P_e = \underline{E}'_d i_d + \underline{E}'_q i_q \quad (31)$$

$$Q = -\underline{E}'_d i_q + \underline{E}'_q i_d \quad (32)$$

one can express the vector of generator power outputs in terms of these matrices as follows:

$$\underline{P}_e = \begin{bmatrix} \mathcal{E}'_d & \mathcal{E}'_q \end{bmatrix} \begin{bmatrix} Y_{m11} & Y_{m12} \\ -Y_{m12} & Y_{m11} \end{bmatrix} \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix}. \quad (33)$$

Similarly, the reactive power output is given by

$$\begin{aligned} \underline{Q} &= \begin{bmatrix} \mathcal{E}'_q & -\mathcal{E}'_d \end{bmatrix} \begin{bmatrix} Y_{m11} & Y_{m12} \\ -Y_{m12} & Y_{m11} \end{bmatrix} \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix} \\ &= \begin{bmatrix} \mathcal{E}'_d & \mathcal{E}'_q \end{bmatrix} \begin{bmatrix} Y_{m12} & -Y_{m11} \\ Y_{m11} & Y_{m12} \end{bmatrix} \begin{bmatrix} \underline{E}'_d \\ \underline{E}'_q \end{bmatrix} \quad (34) \end{aligned}$$

4 Total Network Power Consumption

Note that in (16), the fact that T is orthogonal and X is skew-symmetric means that Y_m preserves the definiteness property of the reduced network admittance matrix, which in turn depends only on the definiteness of G_r in (7). When saliency is present, this is not so straightforward, since X is not antisymmetric. We may decompose X into a symmetric and an antisymmetric part, $X = X_a + X_s$, and note that X_a factors through T , which gives

$$Y_m = T^{-1}(Y_r^{-1} - X_a - T X_s T^{-1})^{-1}T. \quad (35)$$

It can be seen that the symmetric part of X affects the definiteness of Y_m in a state-dependent way. It is interesting, however, to look at the expression for the total power dissipated in the network, since the passivity of the network implies that this must be a positive definite function.

Starting from the scalar expression for the power transmitted by a single generator,

$$P_e = E'_d i_d + E'_q i_q + (x'_q - x'_d) i_d i_q, \quad (36)$$

it can be seen that since

$$X_s = \begin{bmatrix} 0 & \frac{1}{2}(X'_q - X'_d) \\ \frac{1}{2}(X'_q - X'_d) & 0 \end{bmatrix}, \quad (37)$$

the total network power dissipated can be written as

$$P_{tot} = \begin{bmatrix} E'_d & E'_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} i_d & i_q \end{bmatrix} X_s \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad (38)$$

from which the block-form expression arises:

$$P_{tot} = \begin{bmatrix} E'_d & E'_q \end{bmatrix} (Y_m + Y_m^T X_s Y_m) \begin{bmatrix} E'_d \\ E'_q \end{bmatrix}. \quad (39)$$

Since the network is passive, it is to be expected from (39) that $(Y_m + Y_m^T X_s Y_m) > 0$, using the notation " > 0 " to mean that the matrix is positive definite. This can be shown as follows:

$$(Y_m + Y_m^T X_s Y_m) = Y_m^T [(Y_m^T)^{-1} + X_s] Y_m. \quad (40)$$

Substituting (35) into this expression and noting the symmetry of the various matrices (in particular, note $X_a^T = -X_a$) yields

$$\begin{aligned} Y_m^T ((Y_m^T)^{-1} + X_s) Y_m &= \\ Y_m^T (T^{-1} [(Y_r^{-1})^T + X_a - T X_s T^{-1}] T + X_s) Y_m &= \\ Y_m^T (T^{-1} ((Y_r^{-1})^T + X_a) T) Y_m. \end{aligned} \quad (41)$$

The definiteness of this last expression depends on the portion in parentheses. Since a matrix is positive definite iff its inverse is positive definite and X_a is antisymmetric, the last expression is clearly positive definite iff the reduced admittance matrix Y_r is.

Now, when saliency is ignored, the total power can be expressed in the simple form:

$$\begin{aligned} P_{tot} &= \begin{bmatrix} E'_d & E'_q \end{bmatrix} Y_m \begin{bmatrix} E'_d \\ E'_q \end{bmatrix} \\ &= \begin{bmatrix} E'_d & E'_q \end{bmatrix} T^{-1} \begin{bmatrix} \bar{G}_r & -\bar{B}_r \\ \bar{B}_r & \bar{G}_r \end{bmatrix} T \begin{bmatrix} E'_d \\ E'_q \end{bmatrix}. \end{aligned} \quad (42)$$

Clearly, in this case the total power can be expressed in the network frame of reference as

$$P_{tot} = \begin{bmatrix} E'_D & E'_Q \end{bmatrix} \begin{bmatrix} \bar{G}_r & -\bar{B}_r \\ \bar{B}_r & \bar{G}_r \end{bmatrix} \begin{bmatrix} E'_D \\ E'_Q \end{bmatrix}, \quad (43)$$

where E_D and E_Q represent the projection of the machine voltage onto the network frame of reference:

$$\begin{bmatrix} E'_D \\ E'_Q \end{bmatrix} = T \begin{bmatrix} E'_d \\ E'_q \end{bmatrix}. \quad (44)$$

Since (43) is a quadratic form, the antisymmetric part of the matrix drops out and we have

$$P_{tot} = \begin{bmatrix} E'_D & E'_Q \end{bmatrix} \begin{bmatrix} \bar{G}_r & 0 \\ 0 & \bar{G}_r \end{bmatrix} \begin{bmatrix} E'_D \\ E'_Q \end{bmatrix}. \quad (45)$$

Equation (45) reflects the fact that when the output power is calculated for each individual machine, the reactive elements are multiplied by $\sin(\delta_i - \delta_j)$, which, because the sine is an odd function, cancels out when P_{e_i} is added to P_{e_j} .

5 Total Network Reactive Power

A similar expression to that presented in Section 4 can be written for the total reactive power, including the generator reactance:

$$\begin{aligned} Q_{tot} &= \begin{bmatrix} E'_d & E'_q \end{bmatrix} \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} Y_m \begin{bmatrix} E'_d \\ E'_q \end{bmatrix} \\ &= \begin{bmatrix} E'_d & E'_q \end{bmatrix} P Y_m \begin{bmatrix} E'_d \\ E'_q \end{bmatrix}. \end{aligned} \quad (46)$$

Note that $PP^T = I$, and the matrix P is of the type that commutes with T : $PT = TP$. Substituting (15) into (46), and with a little algebraic manipulation, we have:

$$Q_{tot} = \begin{bmatrix} E'_d & E'_q \end{bmatrix} (T^{-1} Y_r^{-1} P^T T - X P^T)^{-1} \begin{bmatrix} E'_d \\ E'_q \end{bmatrix}. \quad (47)$$

Observing that

$$T^{-1} Y_r^{-1} P^T T = \left\{ T^{-1} \begin{bmatrix} -B_r & -G_r \\ G_r & -B_r \end{bmatrix} T \right\}^{-1} \quad (48)$$

and

$$-X P^T = \begin{bmatrix} X_q & 0 \\ 0 & X_d \end{bmatrix}, \quad (49)$$

it is clear that one sufficient condition for positive definiteness of (46) is that $-B_r > 0$. This is true as long as line charging and shunt capacitance is not too high [7]. However, because $-X P^T > 0$, this condition is conservative.

A less conservative but still state-independent condition may be found by working with the inverse of the matrix in (47):

$$\begin{aligned} (T^{-1} Y_r^{-1} P^T T - X P^T) &= \\ T^{-1} (Y_r^{-1} P^T - T X P^T T^{-1}) T &= \end{aligned} \quad (50)$$

Observe that

$$\begin{aligned} Y_r^{-1} P^T - T X P^T T^{-1} &= \\ Y_r^{-1} P^T + \begin{bmatrix} X_d & 0 \\ 0 & X_d \end{bmatrix} &+ T \begin{bmatrix} (X_q - X_d) & 0 \\ 0 & 0 \end{bmatrix} T^{-1}. \end{aligned} \quad (51)$$

Now, since $x'_g > x'_d$, the last expression in (51) is positive semidefinite, so that a sufficient condition for positive definiteness is that

$$Y_r^{-1}P^T + \begin{bmatrix} \chi_d & 0 \\ 0 & \chi_d \end{bmatrix} > 0. \quad (52)$$

As in Section 4, when saliency is ignored, the total reactive power can be written in network coordinates as

$$\begin{aligned} Q_{tot} &= [\underline{E}_D^T \quad \underline{E}_Q^T] T^{-1} P \bar{Y}_r T \begin{bmatrix} \underline{E}_D' \\ \underline{E}_Q' \end{bmatrix} \\ &= [\underline{E}_D^T \quad \underline{E}_Q^T] \begin{bmatrix} -\bar{B}_r & -\bar{G}_r \\ \bar{G}_r & -\bar{B}_r \end{bmatrix} \begin{bmatrix} \underline{E}_D' \\ \underline{E}_Q' \end{bmatrix} \\ &= [\underline{E}_D^T \quad \underline{E}_Q^T] \begin{bmatrix} -\bar{B}_r & 0 \\ 0 & -\bar{B}_r \end{bmatrix} \begin{bmatrix} \underline{E}_D' \\ \underline{E}_Q' \end{bmatrix}. \end{aligned} \quad (53)$$

In this case, the real part of the admittance matrix is antisymmetric and drops out of the quadratic form.

6 Potential Applications

One area in which this type of model may find application is in defining a voltage-dependent system energy function. If the power system is viewed as analogous to a spring-mass system, then, assuming the damping coefficients are zero, one defines the system energy as the sum of kinetic and potential energy, after subtracting the base angular velocity ω_o :

$$\mathcal{V} = \frac{1}{2}(\underline{\omega} - \omega_o)^T M(\underline{\omega} - \omega_o) - \sum_{i=1}^m \int_{\delta_i^*}^{\delta_i(t)} \tau_i d\delta. \quad (54)$$

From (99), τ_i is identified as $-(P_{M_i} - P_{E_i})$ (note that $m_i \dot{\omega}_i$ has units of torque). This gives

$$\begin{aligned} \mathcal{V} &= \frac{1}{2}(\underline{\omega} - \omega_o)^T M(\underline{\omega} - \omega_o) \\ &\quad - \sum_{i=1}^m \int_{\delta_i^*}^{\delta_i(t)} (P_{M_i} - P_{E_i}) d\delta \end{aligned} \quad (55)$$

$$\begin{aligned} &= \frac{1}{2}(\underline{\omega} - \omega_o)^T M(\underline{\omega} - \omega_o) \\ &\quad - \sum_{i=1}^m \int_{\delta_i^*}^{\delta_i(t)} (P_{M_i} - P_{E_i}) \frac{\omega_i - \omega_o}{\omega_i - \omega_o} d\delta. \end{aligned} \quad (56)$$

$$(57)$$

Identifying $d\delta/dt$ from (98), we have

$$\begin{aligned} \mathcal{V} &= \frac{1}{2}(\underline{\omega} - \omega_o)^T M(\underline{\omega} - \omega_o) \\ &\quad - \sum_{i=1}^m \int_{t_o}^t (P_{M_i} - P_{E_i})(\omega_i - \omega_o) dt \end{aligned} \quad (58)$$

$$\begin{aligned} &= \frac{1}{2}(\underline{\omega} - \omega_o)^T M(\underline{\omega} - \omega_o) \\ &\quad - \int_{t_o}^t \sum_{i=1}^m m_i (\omega_i - \omega_o) \frac{d(\omega_i - \omega_o)}{dt} dt. \end{aligned} \quad (59)$$

With another substitution of variables, the integral becomes

$$\sum_{i=1}^m \int_{\xi(t_o)}^{\xi(t)} m_i \xi_i d\xi_i \quad (60)$$

Now, carrying out the integral, we have

$$\begin{aligned} \mathcal{V}(t) &= \frac{1}{2}(\underline{\omega}(t) - \omega_o)^T M(\underline{\omega}(t) - \omega_o) \\ &\quad - \frac{1}{2} [\underline{\xi}^T M \underline{\xi}]_{\xi(t_o)}^{\xi(t)}. \end{aligned} \quad (61)$$

Evaluating, with $\xi(t) = (\underline{\omega}(t) - \omega_o)$,

$$\mathcal{V}(t) = \frac{1}{2}(\underline{\omega}(t) - \omega_o)^T M(\underline{\omega}(t) - \omega_o) \quad (62)$$

$$- \frac{1}{2}(\underline{\omega}(t) - \omega_o)^T M(\underline{\omega}(t) - \omega_o) \quad (63)$$

$$+ \frac{1}{2}(\underline{\omega}(t_o) - \omega_o)^T M(\underline{\omega}(t_o) - \omega_o). \quad (64)$$

That is,

$$\mathcal{V}(t) = \mathcal{V}(t_o) = \frac{1}{2}(\underline{\omega}(t_o) - \omega_o)^T M(\underline{\omega}(t_o) - \omega_o) \quad (65)$$

As one would expect from the analogy to an undamped spring-mass system, the total "energy" is constant. Note that nothing has been assumed about the generator model, the load model or the network. Now, since the restoring force $-(P_{M_i} - P_{E_i})$ does not become unbounded as δ_i does, the fact that $\mathcal{V}(t)$ remains bounded does not imply that the state remains bounded, hence the need for identifying the energy of the controlling unstable equilibrium point in order to determine the stability of the system. In addition, $\mathcal{V}(t)$ clearly does not capture all of the energy exchange that occurs during a disturbance. The energy dissipated by the linear damping term $D(\omega - \omega_o)$ is one example of this, whose inclusion allows a net decay in the system energy, but which unfortunately introduces a path dependency in $\mathcal{V}$.

Another possible source of energy exchange is the total network reactive power, Q_{tot} . Although for a given value of Q_{tot} at steady state, no net energy exchange takes place, energy must be added or subtracted in order for the value of Q_{tot} to change. Therefore, one might consider an energy function of the form

$$\begin{aligned} \mathcal{V}_r &= \frac{1}{2}(\underline{\omega} - \omega_o)^T M(\underline{\omega} - \omega_o) \\ &\quad - \int_{t_o}^t \sum_{i=1}^m m_i \frac{d(\omega_i - \omega_o)}{dt} (\omega_i - \omega_o) dt + Q_{tot}. \end{aligned} \quad (66)$$

Since, in quasi-steady state, the reactive power can be thought of as an average stored energy, the units of Q_{tot} are consistent with an energy function, and its inclusion provides a term in the energy function that directly accounts for the voltage/power coupling of the system. In addition, the generator excitation voltages E_{fd} appear directly in the time derivative, allowing the effect of excitation control for stabilizing oscillations to be seen in the system energy.

It is apparent that Q_{tot} is positive as long as $\bar{B}_r > 0$. It is not clear, however, that the time derivative of $\mathcal{V}_r$ is nonpositive. Since the only part of $\mathcal{V}_r$ that is time-varying is Q_{tot} , $\dot{\mathcal{V}}_r = \dot{Q}_{tot}$. In order to evaluate this derivative, it is most convenient to use the form (53). First, it will be necessary to calculate the time derivatives of E_D and E_Q .

Only the dynamic equation for E_D will be derived here, since the equation for E_Q can be derived in almost identical fashion. We start with the definition:

$$E_D = E'_d \sin \delta + E'_q \cos(\delta), \quad (67)$$

from which the time derivative is taken:

$$\begin{aligned} \dot{E}_D = & \omega(E'_d \cos(\delta) - E'_q \sin(\delta)) \\ & + \dot{E}'_d \sin(\delta) + \dot{E}'_q \cos(\delta). \end{aligned} \quad (68)$$

Identifying E_Q in the first term and expanding the derivatives on the RHS:

$$\begin{aligned} \dot{E}_D = & -\omega E_Q - \frac{1}{T'_{q0}} [E'_d - (x_q - x'_q)i_q] \sin(\delta) \\ & - \frac{1}{T'_{d0}} [E'_q + (x_d - x'_d)i_d] \cos(\delta) \\ & + \frac{1}{T'_{d0}} \cos(\delta) E_{fd}. \end{aligned} \quad (69)$$

Observe that

$$\begin{aligned} \frac{1}{T'_{q0}} E'_d \sin(\delta) + \frac{1}{T'_{d0}} E'_q \cos(\delta) = & \\ \frac{1}{2} \left(\frac{1}{T'_{d0}} + \frac{1}{T'_{q0}} \right) (E'_d \sin(\delta) + E'_q \cos(\delta)) & \\ \frac{1}{2} \left(\frac{1}{T'_{d0}} - \frac{1}{T'_{q0}} \right) (-E'_d \sin(\delta) + E'_q \cos(\delta)). & \end{aligned} \quad (70)$$

Now, making the definitions

$$\begin{aligned} T_1 = & \frac{1}{2} \left(\frac{1}{T'_{d0}} + \frac{1}{T'_{q0}} \right) \\ T_2 = & \frac{1}{2} \left(\frac{1}{T'_{d0}} - \frac{1}{T'_{q0}} \right), \end{aligned} \quad (71)$$

identifying E_D in the first expression on the RHS of (70) and noting that

$$-E'_d \sin(\delta) + E'_q \cos(\delta) = E_D \cos(2\delta) + E_Q \sin(2\delta), \quad (72)$$

(70) becomes

$$\begin{aligned} \frac{1}{T'_{q0}} E'_d \sin(\delta) + \frac{1}{T'_{d0}} E'_q \cos(\delta) = & \\ T_1 E_D + T_2 (E_D \cos(2\delta) + E_Q \sin(2\delta)) & \end{aligned} \quad (73)$$

Similarly, if we define

$$\bar{x}_d = \frac{(x_d - x'_d)}{T'_{d0}} \quad (74)$$

$$\bar{x}_q = \frac{(x_q - x'_q)}{T'_{q0}} \quad (75)$$

$$x_1 = \frac{\bar{x}_d + \bar{x}_q}{2} \quad (76)$$

$$x_2 = \frac{\bar{x}_d - \bar{x}_q}{2}, \quad (77)$$

then we can write

$$\begin{aligned} \bar{x}_d i_d \cos(\delta) - \bar{x}_q i_q \sin(\delta) = & \\ x_1 (i_d \cos(\delta) - i_q \sin(\delta)) & \\ + x_2 (i_d \cos(\delta) + i_q \sin(\delta)), & \end{aligned} \quad (78)$$

which leads to

$$\begin{aligned} \bar{x}_d i_d \cos(\delta) - \bar{x}_q i_q \sin(\delta) = & \\ -x_1 i_Q + x_2 (i_D \sin(2\delta) - i_Q \cos(2\delta)). & \end{aligned} \quad (79)$$

At this point it is only necessary to collect terms from (73) and (79) and substitute into (68):

$$\begin{aligned} \dot{E}_D = & -T_1 E_D - T_2 (E_D \cos(2\delta) + E_Q \sin(2\delta)) \\ & + x_1 i_Q - x_2 (i_D \sin(2\delta) - i_Q \cos(2\delta)) \\ & + T_{d0}^{-1} \cos(\delta) E_{fd} \end{aligned} \quad (80)$$

By a similar procedure one can arrive at the state equation for E_Q , given here without derivation:

$$\begin{aligned} \dot{E}_Q = & -T_1 E_Q - T_2 (E_D \sin(2\delta) - E_Q \cos(2\delta)) \\ & - x_1 i_D + x_2 (i_D \cos(2\delta) + i_Q \sin(2\delta)) \\ & + T_{d0}^{-1} \sin(\delta) E_{fd}. \end{aligned} \quad (81)$$

At this point, if a constant load model is assumed, the relationship

$$\begin{bmatrix} i_D \\ i_Q \end{bmatrix} = \begin{bmatrix} \bar{G}_r & -\bar{B}_r \\ \bar{B}_r & \bar{G}_r \end{bmatrix} \begin{bmatrix} E_D \\ E_Q \end{bmatrix} \quad (82)$$

may be used to solve for the currents. By grouping the scalar equations (80) and (81) for each generator into vector form and expressing the currents as in (82), one arrives at the matrix equation (83), which appears at the top of page 7.

$$\begin{aligned}
\begin{bmatrix} \dot{E}_D \\ \dot{E}_Q \end{bmatrix} &= - \left\{ \begin{bmatrix} \mathcal{T}_1 & 0 \\ 0 & \mathcal{T}_1 \end{bmatrix} + \begin{bmatrix} \mathcal{T}_2 & 0 \\ 0 & \mathcal{T}_2 \end{bmatrix} \begin{bmatrix} C(2\delta) & S(2\delta) \\ S(2\delta) & -C(2\delta) \end{bmatrix} \right\} \begin{bmatrix} E_D \\ E_Q \end{bmatrix} \\
&- \left\{ \begin{bmatrix} 0 & -\Omega \\ \Omega & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{X}_1 & 0 \\ 0 & \mathcal{X}_1 \end{bmatrix} \begin{bmatrix} -\bar{B}_r & -\bar{G}^r \\ \bar{G}^r & -\bar{B}_r \end{bmatrix} + \begin{bmatrix} \mathcal{X}_2 & 0 \\ 0 & \mathcal{X}_2 \end{bmatrix} \begin{bmatrix} C(2\delta) & S(2\delta) \\ S(2\delta) & -C(2\delta) \end{bmatrix} \begin{bmatrix} -\bar{B}_r & -\bar{G}^r \\ \bar{G}^r & -\bar{B}_r \end{bmatrix} \right\} \begin{bmatrix} E_D \\ E_Q \end{bmatrix} \\
&+ \begin{bmatrix} C(\delta) \\ S(\delta) \end{bmatrix} \mathcal{T}_{do}^{-1} E_{fd}
\end{aligned} \tag{83}$$

The brackets in (83) group the matrices respectively into quantities that are local, primarily machine-based parameters, and quantities that depend on the external system and the transmission network. Now, we have

$$\frac{1}{2} \dot{Q}_{tot} = \begin{bmatrix} E_D^T & E_Q^T \end{bmatrix} \begin{bmatrix} -\bar{B}_r & 0 \\ 0 & -\bar{B}_r \end{bmatrix} \begin{bmatrix} \dot{E}_D \\ \dot{E}_Q \end{bmatrix}, \tag{84}$$

which can be written as

$$\begin{aligned}
\frac{1}{2} \dot{Q}_{tot} &= - \begin{bmatrix} E_D^T & E_Q^T \end{bmatrix} \{A_g(\delta) + A_n(\delta)\} \begin{bmatrix} E_D \\ E_Q \end{bmatrix} \\
&+ K(\delta) E_{fd}
\end{aligned} \tag{85}$$

where $A_g(\delta)$ and $A_n(\delta)$ are taken from the grouping in (83). These are time-varying matrices which may or may not be positive definite, but some observations can be made, based on the structure of the various component matrices. Let

$$\mathbf{B} = \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix} \tag{86}$$

$$\mathbf{X}_i = \begin{bmatrix} \mathcal{X}_i & 0 \\ 0 & \mathcal{X}_i \end{bmatrix}, \quad i = 1, 2 \tag{87}$$

$$\mathbf{T}_i = \begin{bmatrix} \mathcal{T}_i & 0 \\ 0 & \mathcal{T}_i \end{bmatrix}, \quad i = 1, 2 \tag{88}$$

$$\mathbf{R}(2\delta) = \begin{bmatrix} C(2\delta) & S(2\delta) \\ S(2\delta) & -C(2\delta) \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 0 & -\Omega \\ \Omega & 0 \end{bmatrix}. \tag{89}$$

$\mathbf{T}_1$ and $\mathbf{T}_2$ are positive diagonal matrices and it can be seen that

$$\mathbf{T}_2 \mathbf{R}(2\delta) = \mathbf{T}_2^{1/2} \mathbf{R}(2\delta) \mathbf{T}_2^{1/2}. \tag{90}$$

Moreover, $\mathbf{R}(2\delta)$ is orthogonal (hence $\|\mathbf{R}(2\delta)\|_2 = 1$) and $T_1 > T_2$ elementwise, so that

$$|x^T \mathbf{T}_2^{1/2} \mathbf{R}(2\delta) \mathbf{T}_2^{1/2} x| \leq |x^T \mathbf{T}_2 x| \leq x^T \mathbf{T}_1 x, \tag{91}$$

which means that

$$\mathbf{T}_1 + \mathbf{T}_2 \mathbf{R}(2\delta) > 0, \quad \forall \delta. \tag{92}$$

Similarly, since $\mathbf{X}_1 > \mathbf{X}_2$ elementwise and $\mathbf{W}$ is anti-symmetric,

$$\mathbf{X}_1 + \mathbf{X}_2 \mathbf{R}(2\delta) + \mathbf{W} > 0 \quad \forall \delta. \tag{93}$$

Looking first at

$$A_g(\delta) = \mathbf{B}(\mathbf{T}_1 + \mathbf{T}_2 \mathbf{R}(2\delta)), \tag{94}$$

the fact that the factors in this expression are positive definite does not mean that the product is. The matrix can easily be checked for definiteness in a neighborhood of any operating point, but the determination of the points at which definiteness is lost is a more difficult task. A similar situation holds with A_n , which can be separated into two components:

$$\begin{aligned}
A_n &= \mathbf{B}(\mathbf{X}_1 + \mathbf{X}_2 \mathbf{R}(2\delta) + \mathbf{W}) \mathbf{B}^T \\
&+ \mathbf{B}(\mathbf{X}_1 + \mathbf{X}_2 \mathbf{R}(2\delta) + \mathbf{W}) \begin{bmatrix} 0 & -\bar{G}_r \\ \bar{G}_r & 0 \end{bmatrix}.
\end{aligned} \tag{95}$$

The first component is positive definite, but the second is not. Again, the overall definiteness of the matrix can be checked at any operating point, but the critical points are more difficult to find.

7 Conclusions

A new approach to modeling the power system using the flux-decay generator model and a constant impedance loads has been proposed, which leads to a matrix form of the system dynamic equations. This formulation may be useful as a means of capturing the effect of voltage/power coupling for nonlinear system analysis and provides a reasonably simple structural insight into this aspect of the system dynamics. An addition to the simple energy function that is based on the generator rotor angle and speed has been proposed that may be useful in this regard. Although it is not presently clear over what region of the state space the proposed function maintains a nonpositive time derivative, the inclusion of a voltage-based energy term that captures the effect of voltage control is necessary if the full benefit of this type of control for short-term stability is to be realized.

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Acknowledgements

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Appendices

A The Synchronous Generator Model

Each generator is modeled by a fourth-order set of coupled nonlinear differential equations. All variables are expressed in the synchronously-rotating Parks-transformed coordinates. The coupling variables i_d and i_q depend on the machine voltages E'_d and E'_q and the rotor angle δ . The machine equations are:

$$\dot{E}'_d = \frac{1}{T'_{d0}} [-E'_d + (x_q - x'_q)i_q] \quad (96)$$

$$\dot{E}'_q = \frac{1}{T'_{d0}} [-E'_q - (x_d - x'_d)i_d + E_{fd}] \quad (97)$$

$$\dot{\delta} = \omega - \omega_o \quad (98)$$

$$\dot{\omega} = \frac{1}{m_i} \left[P_m - P_e - \frac{D}{\omega_o} (\omega - \omega_o) \right] \quad (99)$$

$$= \frac{\omega_o}{2H} \left[P_m - E'_q i_q - E'_d i_d - (x'_q - x'_d)i_d i_q - \frac{D}{\omega_o} (\omega - \omega_o) \right] \quad (100)$$

Where the notation is as follows:

D	Machine damping coefficient
δ	Rotor angle w.r.t. the synchronously rotating reference
E'_d	Direct-axis transient voltage
E_{fd}	Generator field voltage
E'_q	Quadrature-axis transient voltage
H	Normalized Generator inertia constant
i_d	Direct-axis armature current
i_q	Quadrature-axis armature current
m	Per-unit generator moment of inertia
ω	Rotor speed (electrical)
ω_o	Nominal electrical frequency
P_m	Mechanical input power
T'_{d0}	Direct-axis open-circuit transient time constant
T'_{q0}	Quadrature-axis open-circuit transient time constant
x_d	Direct-axis synchronous reactance
x'_d	Direct-axis transient reactance
x_q	Quadrature-axis synchronous reactance
x'_q	Quadrature-axis transient reactance

Considerations on a Practical Neural Network for Voltage Stability Assessment

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Abstract:

As the power system is large and complex, voltage stability has become one of the great concerns among utilities, and some indices to indicate the voltage stability have been proposed. However, most methods have to repeat power flow calculations, and therefore they require a huge amount of computing time for large-scale power systems. Recently, the artificial neural network theory has been actively researched in power system areas. In particular, the back propagation method has been frequently applied to various subjects. This method has ability of classifying nonlinear and complex patterns. However, there are problems to be considered in using the back propagation method, that is improvement of learning and generalizing abilities. Therefore, these problems have to be investigated when we apply the back propagation method to power system problems. For that reason, an application of a neural network which considers power system configuration was proposed for the voltage stability problems [1]. This paper extends the original work of [1], and tries to solve out various practical problems by applying some improvement techniques.

Keywords: Power System, Voltage Stability, Neural Network, Back Propagation, Improvement Techniques

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1. INTRODUCTION

As the power systems are becoming larger and more complex, voltage instability has become one of the great concerns among utilities, and some indices to indicate the voltage stability have been proposed. However, most conventional methods have to repeat power flow calculations, therefore they require a huge amount of computing time for large-scale power systems. Thus, a neural network approach has been applied to voltage stability assessment taking into account power system configuration [1].

Recently, artificial neural networks have been actively researched in power system areas. In particular, the back propagation method has been frequently applied to various subjects. This method arranges neurons into a number of layers, so that the network may have the ability to classify nonlinear and complex patterns which can not be achieved with the conventional 2-layered perceptron. However there are some problems in applying the back propagation method to power system problems. In this paper, we take a voltage stability assessment problem, and try to solve out various problems of the neural networks by applying some improvement techniques.

There are problems to be solved in using the back propagation method, that is, improvement of learning and generalizing abilities. In this paper, improvement of learning ability, or improvement of convergence is investigated by adding penalty terms and preprocessing data used. On the other hand, generalizing ability is an ability whether a trained neural network would create proper output data against untrained input data. Over-fitting problems can often

observed in relation to the generalizing ability. We apply the V-fold cross-validation method to avoid the over-fitting problem. An incomplete data problem is also in relation to the generalizing ability. It is on the assumption that a part of input data could be missed, and thus filling up ability of the back propagation method should be examined when the information is not complete. We evaluate the filling up ability of the back propagation method through an incomplete data problem for the neural network considering power system configuration.

2. VOLTAGE STABILITY INDEX

As power flow equations are nonlinear, they can have a number of solutions. A Power system operating condition which corresponds to closely-located stable solution x^0 and unstable solution x^* is suspected to be in an unstable situation [2]. The idea was originated by Y. Tamura.

Some indices to indicate the dangerous condition have been proposed. Among them, in this paper, we use the solid angle α between the two solutions in the voltage space. It is an extension of the solid angle VIPI(Voltage Instability Proximity Index) in the bus specification space proposed by Y. Tamura's group [3]. The angle α is defined as

$$\alpha = \cos^{-1} \frac{x^{0T} \cdot x^*}{\|x^0\| \|x^*\|}$$

With this index α , sensitivity analysis can be much simplified.

3. NEURAL NETWORK CONSIDERING POWER SYSTEM CONFIGURATION

For voltage stability evaluation we deal with in this paper, a size of neural network depends on a size of the power system, and the power system often is of a huge size, therefore the required memory becomes a problem in applying the back propagation method to power systems. Therefore a neural network considering power system configuration was proposed [1].

In this neural network, the number of the hidden units is equal to the total number of buses. And if there are no physical connections between the buses, the corresponding synapse weights from the input layer to the hidden layer are set to 0. For example, in Fig. 1, for bus 5, the synapse weights exist only between the input units corresponding to buses 2, 5, 7, and 8 (see Fig. 2), where α in the figure is the angle explained in section 2.

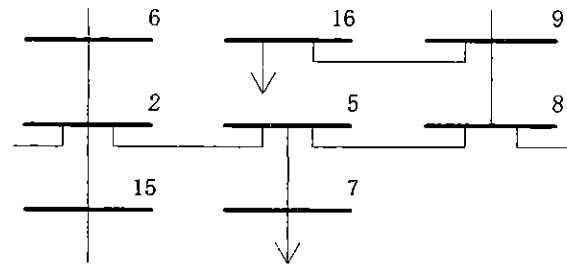


Fig. 1. A part of 43-bus system

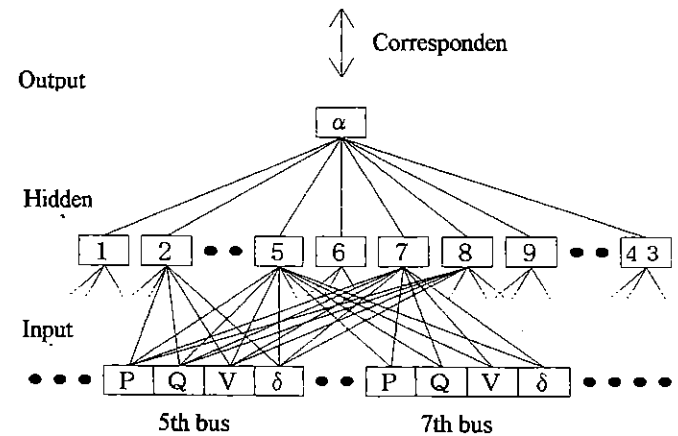


Fig. 2. A neural network model which considers power system configuration(applied to voltage stability assessment)

4. IMPROVEMENT OF LEARNING ABILITY

4.1 Data Preprocessing

The back propagation method generally uses sigmoidal functions such as in Fig. 3 as input-output functions. But under this circumstance, output data should not be so close to 0 or 1. It is because that output data against untrained input data might reach saturation values(0 or 1), and might not give

good results. We focus our attention on this data preprocessing, and observe the convergence characteristics by changing the way of data preprocessing.

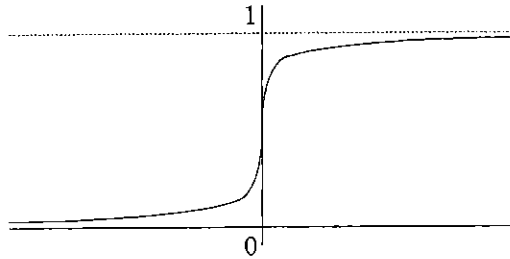


Fig. 3. Sigmoidal Function

4.2 Penalty Term Addition

In the case of using the back propagation, if the training makes weights renewal too fast, the network might reach so called local minima so that performance of the network becomes rather bad. Therefore we add a term to the back propagation cost function, which prevent the network from going to local minima. We have investigated the following terms.

(a) Weight Decay[4]

$$\text{Penalty term} = \sum_{ij} w_{ij}^2$$

(b) Weight Elimination[5]

$$\text{Penalty term} = \sum_{ij} \frac{\left(\frac{w_{ij}}{w_o}\right)^2}{1 + \left(\frac{w_{ij}}{w_o}\right)^2}$$

(c) Entropy Term[6]

$$\text{Penalty term} = - \sum_i \left\{ y_i \log_e \frac{1}{y_i} + (1 - y_i) \log_e \frac{1}{1 - y_i} \right\}$$

Although (a) and (b) are originally proposed as techniques for simplifying networks, we introduce them as techniques for improving convergence characteristics. In this paper, we compare convergence characteristics when above penalty terms are introduced.

5. IMPROVEMENT OF GENERALIZING ABILITY

5.1 Over Fitting Problem

In the case of using the back propagation method, an over-fitting problem emerges generally as a pending problem. It is a phenomenon that prediction errors increase and network's generalizing ability becomes rather inferior as repeating training process goes on too much even if the cost function for training data decreases (see Fig. 4). In order to solve this problem, various techniques has been proposed. Among them, we apply the V-fold cross-validation method to the voltage stability assessment.

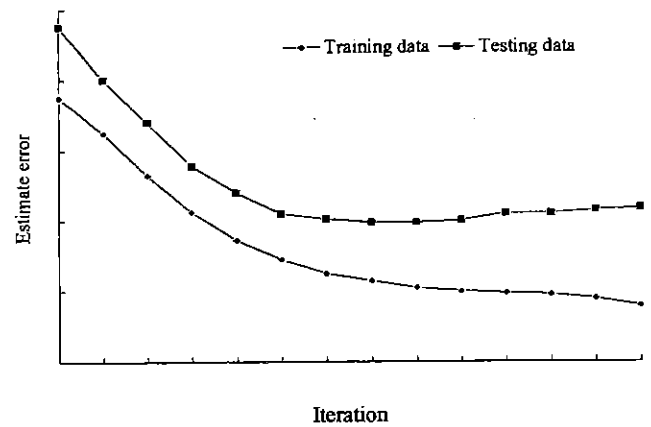


Fig. 4. Over fitting phenomenon

5.2 V-Fold Cross-Validation [7]

Although we cannot directly calculate the prediction errors in general, we can estimate them from available data. We usually separate training data and testing data from available data, build a neural network by using the training data, and estimate prediction errors from the testing data (Test-set validation). In such a case, it is effective to use the cross-validation, which is a sample re-use method for estimating prediction errors; it makes maximally efficient use of the available data.

The V-fold cross-validation is explained as below. We have M-patterns of available data set as in <1> of Fig. 5. Since the data set is not so large, we want to use all of them as training data. Namely let the M-patterns of the data be divided into v randomly selected disjoint subsets P of roughly

equal size as in <2> of Fig. 5. Compute error estimates trained on all data except for one of the P. Then repeat this calculation V times as shown in <3> of Fig. 5. After completing the above process, we can estimate prediction errors at checking before implementation. Then we can understand the over-fitting phenomenon of the neural network and fix the training iteration count so as not to increase the prediction errors. We train all M-pattern data until proper iteration counts.

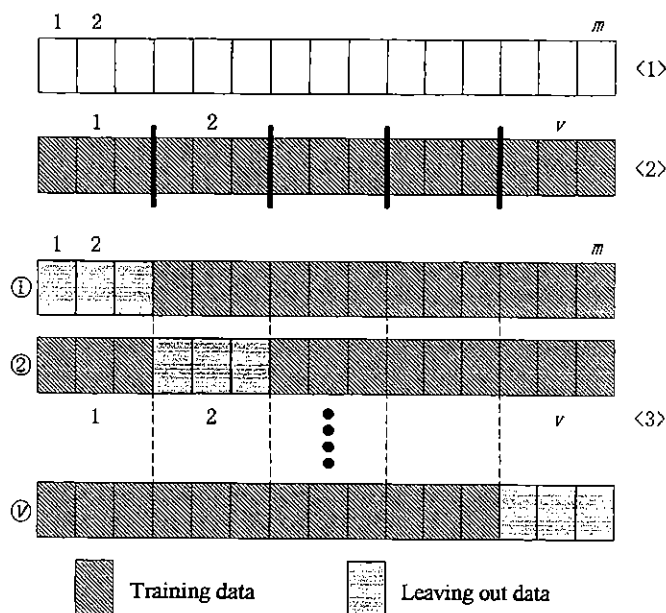


Fig. 5. V-fold CV method

5.3 Incomplete Data Problem

In applying the back propagation method to power systems, input data could be missed. It is because a part of bus information may possibly missed by reasons of many kinds. Even if there are incomplete data, it is expected that neural networks have sufficiently accurate results with filling up ability of the back propagation method. But the neural network considering network configuration reduces the number of synapse weights so much and thus might weaken the filling up ability of the back propagation method. Therefore prediction errors may be larger than the original

neural network. We investigate this viewpoint

6. SIMULATION

We train the neural network considering network configuration for a 43 bus system which has closely-located multiple solutions. Input variables are the system conditions (bus voltages $|V|$, θ , net active and reactive power injections P, Q) at each bus. The output unit is the voltage stability index α . The values of the index are obtained by repeating load flow calculations with changing load ratio of load buses. In this paper, we set load ratio of the base case at 100 percent, and change load ratios from 100% to 50% randomly.

6.1 Data Preprocessing

The differences in convergence characteristics by changing the way to preprocess data are shown in Table 1. It shows that computing time reductions are obtained by narrowing the range of preprocessing data in this simulation. It is thought that input data (bus voltages for example) have comparatively linear relations with voltage stability index α , the output data. But when one pays attention to the error for the testing data, the result which was obtained by preprocessing data (0.3-0.7) is rather bad. Namely one cannot always have the best result when one seek good convergence in the learning phase. In this example, it was good to preprocess data to (0.25-0.75).

6.2 Penalty Terms Addition

The results of adding penalty terms are shown in Table 2. Every method achieves improvement of convergence by adding penalty terms. The Weight Decay method especially shows a good result. Furthermore the Weight Decay method gives the smallest error for the testing data. But one should be careful about to fixing a parameter for the Weight Decay method because if one fixes it a little bit larger, the learning is seriously deteriorated. However the improvement of convergence characteristics shown in Table 2 is a big

advantage, and because one can know the deterioration in the initial stage of learning process one can restart learning by changing the parameter easily. Thus the Weight Decay method is still very useful.

6.3 V-Fold Cross-Validation

The relationships between learning iterations and error function values are shown in Fig. 6. Table 3 shows estimate errors by stopping the learning in each technique; V-fold cross-validation method, setting iteration count, and proper iteration stopping when the error function value becomes the smallest for the test data set.

Fig. 6 shows that the values of error function for training data continues decreasing, while the error function for over-fitting checking data changes from decreasing to increasing at iteration 3000-4000. On the other hand, the error function for testing data also changes from decreasing to increasing at iteration 2000-3000, which means so-called over-fitting phenomena have occurred. We usually fix the

learning iteration count from experience. In this case, it was fixed to 10000 times, and over-fitting phenomena have appeared. Estimate errors by 10000 iterations are twice larger than estimate errors by using the proper iteration stopping with test data set method (see Table 3). However when you stop the learning by using the V-fold cross-validation method, the estimate errors get close to estimate errors by the proper iteration stopping method. Therefore it can be said that the over fitting problem has been solved.

6.4 Incomplete Data Problem

Table 4 shows estimate errors when missing testing data are replaced by 0.5, which is the center of input data. One can see that incomplete data make errors worse, but estimate errors by the neural network considering network configuration which eliminates many synapse weights produces as small estimate errors as the original neural network. In reality, pseudo-data will be actually used instead of 0.5 when there are incomplete data.

Table 1. Iterations count for data preprocessing

Range of preprocessing	0.05-0.95	0.1-0.9	0.15-0.85	0.2-0.8	0.25-0.75	0.3-0.7
Learning iteration	3553	2458	1392	803	407	104
Estimate error ($\times 10^{-4}$)	1.045	1.361	1.250	0.968	0.946	1.950

Convergence criterion ; 1.0×10^{-4}

Table 2. Iterations count for penalty terms addition

Penalty terms	No penalty	Weight Decay	Weight Elimination	Entropy term
Parameter	—	1	0.1	0.005
Learning iteration	803	564	755	711
Estimate error ($\times 10^{-4}$)	0.968	0.504	0.881	0.866

Convergence criterion ; 1.0×10^{-4}

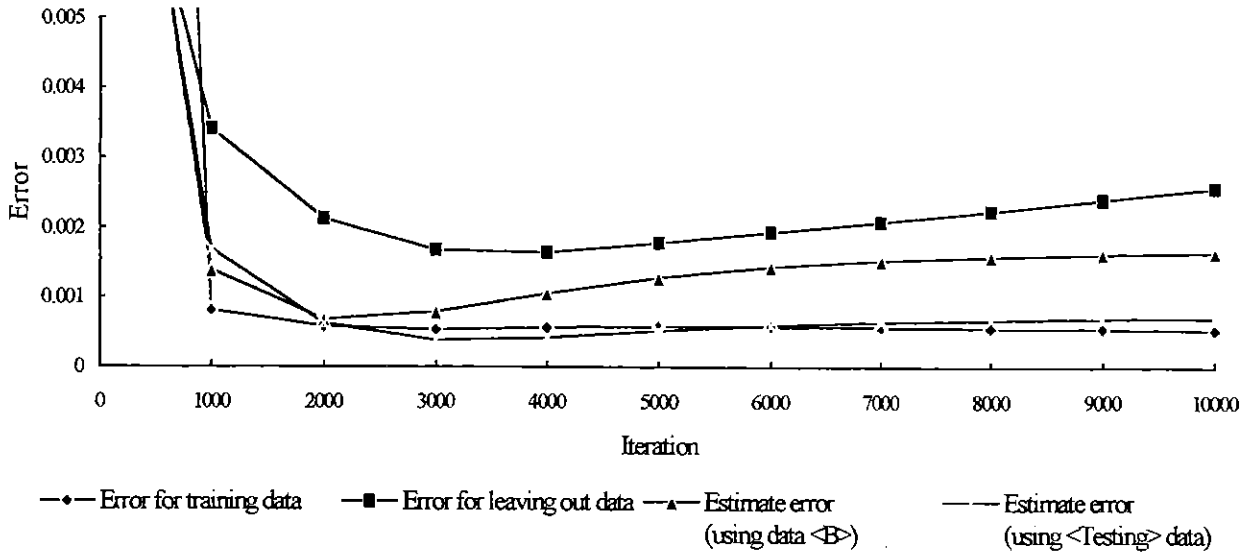


Table 3. Estimate errors in each iteration count

Training data	Data <A> ; 11 patterns					
Testing data	<Testing> data ; 10 patterns			Data ; 11 patterns		
How to stop learning	V-fold CV method	Setting iteration	Proper iteration stopping	V-fold CV method	Setting iteration	Proper iteration
Learning iteration	3600	10000	3200	3600	10000	2200
Estimate error ($\times 10^{-5}$)	3.85	7.08	3.74	9.39	16.42	6.69

Table 4. Estimate errors when data are not complete

Incomplete data	Original Neural Network	NN Considering network configuration
Nonexistent	0.547	0.779
Existent	0.850	0.901

Convergence criterion ; 1.0×10^{-4}

7. CONCLUSIONS

In this paper, for fast voltage stability assessment an neural network application has been investigated. And

various problems encountered are described. In order to solve the problems the countermeasures are proposed. For improvement of learning ability, it has been found that computing time can be reduced by narrowing the range of

data preprocessing for the voltage stability assessment. Penalty terms addition, especially the Weight Decay method made a significant effect in convergence characteristics. For improvement of generalizing ability, we have studied the over-fitting problem, and proved the effectiveness of the V-fold cross-validation method. Furthermore we have examined the neural network considering network configuration which eliminates many synapse weights, and shown that there are no significant problem in using the network even when missing data existed.

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Assessment of Transient Voltage Stability Based on Critical Operating Time of Emergency Control Using Neural Networks

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Abstract

Since unstable phenomena that load bus voltages collapse rapidly after a large disturbance occurred in a bulk power system, it is becoming necessary to develop a fast and precise evaluation method of what is called transient voltage stability. Though many indices have so far been proposed for static voltage stability assessment, there are few evaluation methods but digital simulation analysis for the transient voltage stability assessment considering dynamic characteristics of generators, loads etc.

This paper presents the transient voltage stability evaluation system which consists of two artificial neural networks (NNs). The first NN judges whether the system is stable or not under given operating condition, load composition and fault location from a viewpoint of the transient voltage stability. If it is judged to be unstable, the second NN estimates critical operating time of emergency control action as a severity index of the transient voltage instability. Here, closing of a bus tie which is switched off in the normal state is adopted as the emergency control for the voltage stabilization. Numerical examples for 26-bus model system are shown in order to check the effectiveness of the proposed evaluation system.

1. Introduction

Presently the voltage stability is an important issue for stable operation of power systems. Because of the difficulties in siting, power sources are located away from load centers. Trunk transmission lines tend to be longer and be heavily loaded. Moreover, loads are becoming of constant power type and of poor power factor with the rapid increase of cooling demand such as inverter-driven air conditioners. These factors, in combination, caused voltage collapse phenomena on some power systems in the world.[1],[2]

Voltage instability phenomena are roughly divided into two groups; slow phenomena and rapid phenomena.[2],[3] The former ones are such slow phenomena that a bus voltage collapses in one to ten minutes due to a rapid increase in demand and corresponding control actions on transformer taps and shunt capacitors. The latter ones are such rapid phenomena that the voltage collapses in several seconds after a large disturbance such as a system fault. For the assessment of voltage stability of slowly developing phenomena, some techniques have been developed. They use voltage stability indices based on analytical methods, such as single or multiple-load flow solution analysis considering actions of various voltage control equipment. These techniques have been of use for planning of a reactive power compensator and for on-line monitoring of the voltage instability.[4]-[6],[9]

It, however, is difficult for these voltage stability assessment techniques to assess the rapid voltage collapse phenomena occurred in power systems in recent years[2], the so-called transient voltage stability. The reason is that these techniques can not consider the dynamic characteristics

of generators, excitation control system and loads within a transient time region. Hence, up to the present, we have had no effective techniques for assessing the transient voltage stability, except performing dynamic simulations under various sets of conditions. Therefore, development is strongly needed of a new technique for easily and quickly assessing the transient voltage stability, from viewpoints of the system operation planning and the on-line monitoring.

Artificial neural networks, which are mathematically modelled on the brain neural networks of organisms, have many excellent features such as parallel processing of information, learning mechanism and self-organizing capability. The multilayer neural networks using the backpropagation learning algorithm excel in pattern recognition. These networks can reproduce nonlinear and discontinuous input/output relations precisely through learning. Hence active research efforts are in progress to apply them in the field of power systems.

This paper presents an application of neural networks to assessment of the transient voltage stability. The authors propose a transient voltage stability assessment system which consists of two multilayer neural networks (NNs). The NN of the first stage judges, from the viewpoint of transient voltage stability, whether the system is stable or unstable under given conditions, including the operating conditions, load composition and fault location(s). If the result of the assessment is "instability," the NN of the second stage will estimate the critical operating time for emergency control action as an index of severity of the voltage instability. In the proposed system, a control action for closing a bus tie which is normally open is adopted for the emerging control. To verify the effectiveness of the proposed transient voltage stability assessment system, numerical computations were carried out on a 26-bus model system.

2. Emergency Control against Voltage Collapse

2.1 Basic Characteristics of Model System

Since rapid collapse of a load bus voltage after a large disturbance in a power system was reported in Japan,[2] the so-called transient voltage stability problem is now attracting much attention. In this paper, a model looped system of Fig. 1 is used. To examine the transient voltage stability problem in some practical aspects, the looped system is designed to include a pumped-storage power station and a load bus of which power consumption is great at night (industrial loads having private power generation plants).

If a severe fault occurs on a transmission line in a looped network configuration, the faulted transmission line will be switched off, in the worst case, and the network configuration will change from a looped system to a radial system. Seen from a load bus near a terminal end of the radial system, the system impedance will change significantly with a rapid increase. Under the above

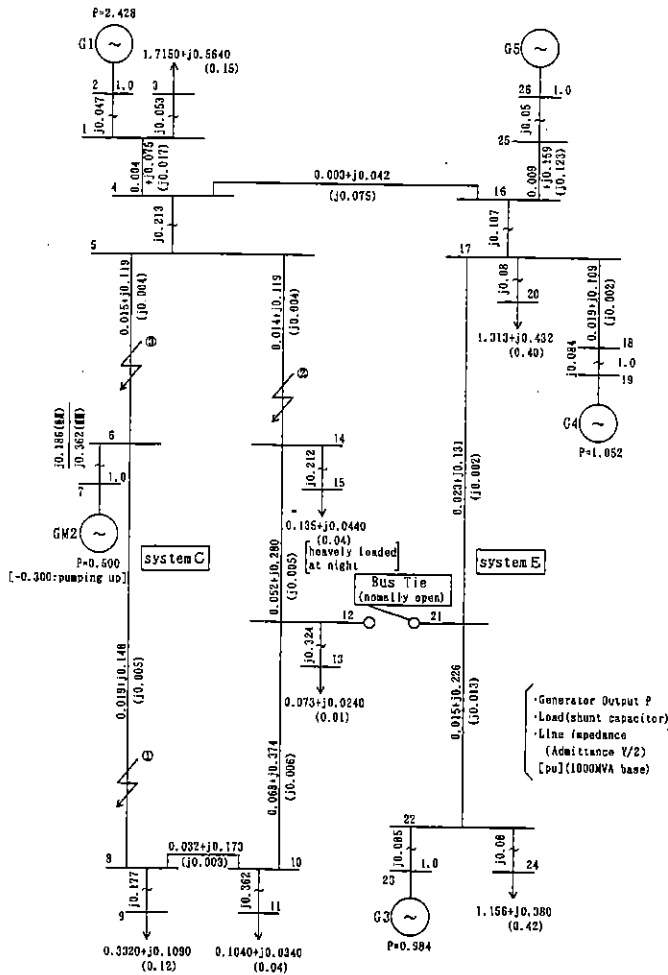


Fig. 1. Model System

conditions, a transient voltage instability phenomenon may occur at a load bus near the terminal end of the radial system.

In comparison with the slowly developing instability phenomena, the transient voltage collapse phenomena greatly depend on the dynamic characteristics of generators, excitation control systems and loads.[2],[4],[11] Hence dynamic digital simulation techniques are normally used to consider these dynamic characteristics in analyzing the transient voltage stability. The conditions and results of the dynamic simulations on the model system will be described later in this paper. The basic characteristics of the model system are summarized here from the viewpoint of transient voltage stability:

- (1) Under the heavily loaded condition during the daytime, the load of No. 9 bus is large. If a fault (2cct-3LGO) occurs at point ①, the voltage stability of buses near No. 9 bus will be threatened.
- (2) At night, if a fault occurs at point ②, the voltage stability of the buses near No. 15 bus with a large demand will be threatened.
- (3) At night, if a fault occurs at the point ③, and if No. 2 generator is under operation of pumping up, the voltage stability of the buses near No. 7 and No. 9 buses will be threatened. The generator/motor for pumped-up storage will experience decelerating phenomenon or so-called synchronous instability at the same time.

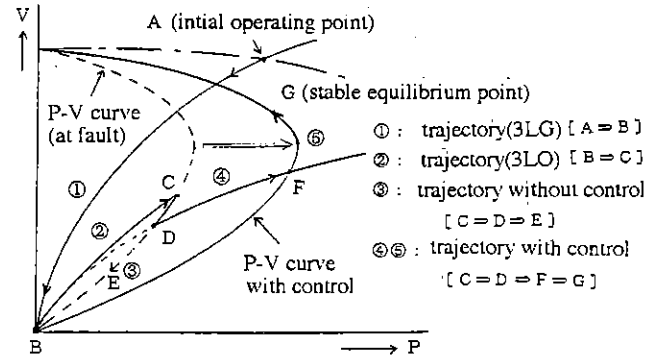


Fig. 2. Concept of Voltage Stabilizing Control.

2.2 Emergency Control against Voltage Collapse

The selected emergency control against voltage collapse for the model system of Fig. 1 is the closure of a interconnecting point between the looped system (system C) and its adjacent system (system E) after the occurrence of a system fault. The bus tie is normally open. This control interconnects two systems which are normally separated from each other. The rapid change of the network configuration decreases the impedance of the circuit. This increases the transmission capacity and stabilizes the voltage. The concept of transient voltage stabilization by this emergency control is shown in Fig. 2 with P-V curves at the receiving end. A voltage instability phenomenon is developing after clearing a system fault. When this emergency control is taken, the P-V curve will rapidly swell; the unstable operating point will shift to a stable equilibrium point and the stability will be assured.

This control is a voltage stabilizing measure from a view point of network configuration. It makes a contrast with voltage stabilizing measures from the load bus side, such as rapid switching of shunt capacitors. As the model system used here has a basic characteristic that faults at points ① through ③ cause voltage collapse on different buses, this control is very effective in stabilizing the voltages of different buses by closing only one bus tie.[10] This control also realizes an economical way of emergency control.

3. Transient Voltage Stability Assessment Technique with Neural Networks

Various indices for assessing voltage stability have been proposed to the present. These indices, however, are static or quasi-dynamic ones. Simulation analysis is, at present, the only one available method for assessing the transient voltage stability depending on the dynamic characteristics of generators and loads as mentioned above. It is necessary to develop quick and accurate assessment methods.

Neural networks, which are modelled on the brain neural networks, have a capability of learning nonlinearities of input/output relations. The multilayer neural networks are excellent in pattern recognition and can reproduce nonlinear, discontinuous relations through learning. Many efforts are under way to apply them in the field of power systems.[7]

In this section, multilayer neural networks and a transient voltage stability assessment technique using such networks are described.

3.1 Multilayer Neural Network and Learning Algorithm

A multilayer network is schematically shown in Fig. 3. This network has a total of three layers; one input layer, one middle layer, and one output layer. The backpropagation learning algorithm[12] is used for this multilayer network. The learning algorithm is outlined in the following:

[Learning Algorithm]

(i) Input data set of pattern p is given in the input layer. The desired output result " t_p " for pattern p is indicated in the output layer as the teacher signal.

(ii) The input "net" to the middle layer is calculated by equation (1) using the output from the input layer and the connection weight w between the input layer and the middle layer. Then the output from the middle layer is determined by equation (2) using net _{p} and the characteristic function. Here the characteristic function is sigmoid function of equation (3).

$$\text{net}_{pj} = \sum_i w_{ji} O_{pi} \quad (1)$$

$$O_{pj} = f(\text{net}_{pj}) \quad (2)$$

$$f(x) = (1 + e^{-x})^{-1} \quad (3)$$

(iii) The input to the output layer is calculated from the relationship between the middle layer and the output layer as in (ii). Then the output from the output layer is determined by the characteristic function.

(iv) The squared error E , which is defined by the following equation, is calculated using the actual output O_{pj} of the network and the teacher signal t_{pj} :

$$E = 1/2 \sum_j (t_{pj} - O_{pj})^2 \quad (4)$$

(v) To minimize the squared error E , the connection weight w of the network is modified by equation (5).

$$w_{ji} = w_{ji} - \eta \partial E / \partial w_{ji} \quad (5)$$

(vi) The steps (i) through (v) are repeated with different patterns of data till the squared error E is reduced sufficiently.

3.2 Transient Voltage Stability Assessment Technique Based on Multilayer Neural Networks

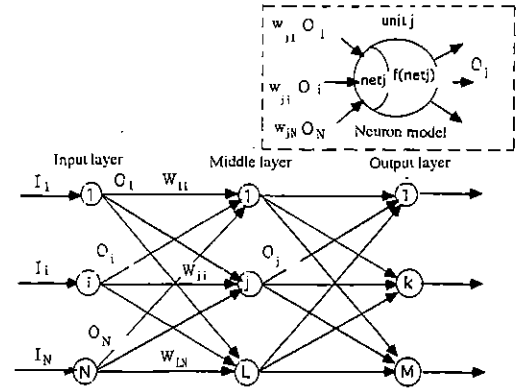


Fig. 3. Multilayer Neural Network.

An assessment system consisting of two multilayer neural networks (NNs) as shown in Fig. 4 is proposed to assess the transient voltage stability. The NN of the first stage (NN1) judges, from the viewpoint of transient voltage stability, whether the system is stable or unstable for given operating conditions, load compositions and fault location(s). If it is judged to be unstable, the NN of the second stage (NN2) will estimate the critical operating time of emergency control action as an index of severity of the voltage instability. Here, closing of a bus tie which is open in the normal state is adopted as the emergency control for the voltage stabilization as mentioned in 2.2.

For the purpose of the voltage stability monitoring, the assessment index should be one that can be easily understood by the system operators. From this viewpoint, the critical operating time of the emergency control action for the voltage stabilization, that is, urgency of stabilization control was selected as the assessment index. In a stable case, this assessment index will be infinite and have no meaning. Hence a two-stage assessment system is needed where NN1 judges whether the system is stable or unstable, and NN2 estimates the assessment index.

The features of the proposed system are summarized as follows:

- (i) The system state variable and contingency conditions provide the sufficient input information.
- (ii) The transient voltage stability can be judged quickly.
- (iii) The information concerning the severity of the transient voltage stability is provided by the "urgency of control action," which is an easy index for system operators to understand.

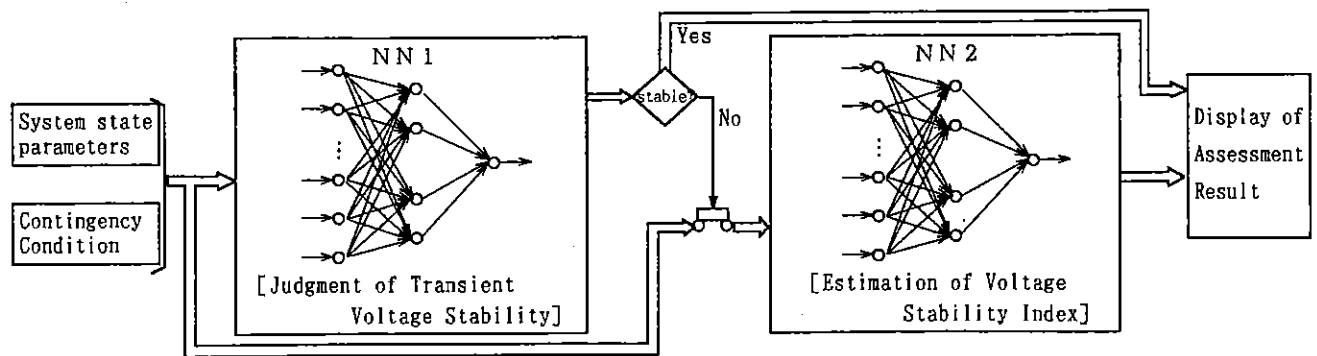


Fig. 4. Outline of Assessment of transient voltage stability.

4. Application to Model System

In this section, the effectiveness of the proposed technique of assessing transient voltage stability is examined with the use of the 26-bus model system shown in Fig. 1. NN1 and NN2 are made to assess the voltage stability under various combinations of the system operating conditions (demand, operating conditions of generators, network configuration), load composition, and fault location(s). The results are compared with those of dynamic digital simulations to demonstrate the effectiveness of the proposed technique.

4.1 Structures of the NNs and Training Data.

(1) *Structures of NN1 and NN2* NN1 comprises of three layers as shown in Fig. 5. The input layer has 48 units, the middle layer 15 units, and the output layer one unit. As the input for NN2 is identical to that for NN1, NN2 has the same structure as NN1. The number of input units was determined to cover the system state variables at generator buses and load buses and the fault locations as shown in Fig. 5. The number of units of the middle layer was determined on a trial and error basis.

The symbols used in Fig. 5 are defined as follows:
 V, δ, P_g : Magnitude and phase angle of terminal voltage and active power output from generator, respectively.
 δ, PL, QL, QC : Phase angle of voltage, active and reactive power demand, phase regulating capacity, of load bus, respectively.
 IC : Load composition index (ratio of induction motor load).
 SSC : Network configuration (with, without change).
 FK, FM, FH : fault locations ①, ②, and ③, respectively.

The magnitude of the voltage of each load bus is not used as input. It is because the number of inputs for an NN should be as small as possible for smaller memory requirement and higher computation speed. In this case, it is possible to eliminate some other parameters from input variables instead of the magnitude of the voltage. Since active and reactive power PQ of each load bus are specified in many cases of load-flow computation, however the active power, reactive power and phase regulating capacity of each load bus are selected as the inputs for the NNs. The phase angles of bus voltages are considered to be input data which indirectly give the relative relationship between buses to the NNs, where the phase angle δ of the bus voltage on the primary side of the interconnection transformer is used as a reference.

(2) *Data Set for Learning and for Verification* Data sets for learning and for verification which consist of the input data and output data for both NN1 and NN2 are prepared. The results of the dynamic simulations are used as the output data. The output for NN1 is stability or instability. The output for NN2 is the critical operating time (within 2.5 seconds) of emergency control action for the case which was judged to be unstable.

The data sets for learning and for verification are prepared as follows:

The load demand shown in Fig.1 is set to be 100% demand. Input data to NNs are obtained from demand levels from 110% to 60% in the daytime and from 60% to 40% in the nighttime, and the system operating conditions such as the operating conditions of generators and changes of the network configuration, where power consumption of each load bus is proportional to the demand in percentage given above. Large demands at industrial loads of extra high voltage are taken into account at No. 15 bus at night.

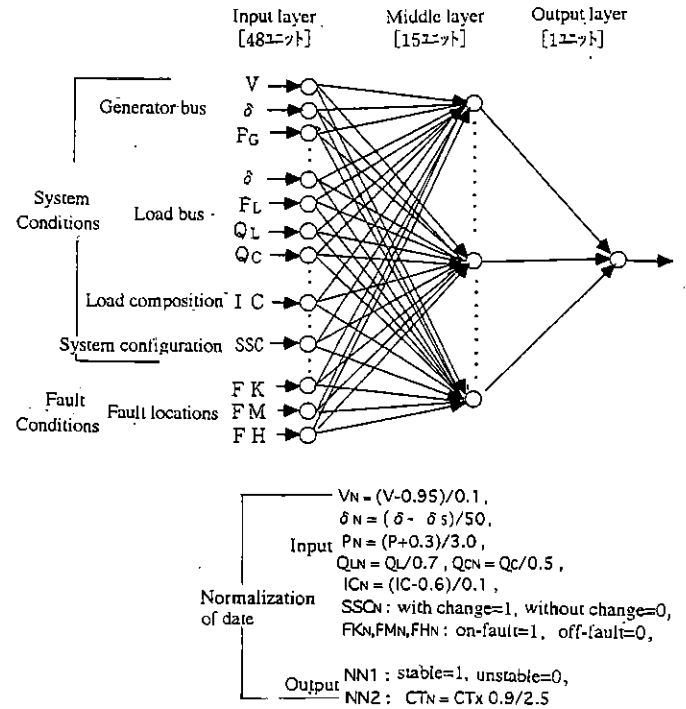


Fig. 5. Structures of NN1 and NN2.

Three kinds of the operating conditions of generators, namely, generation, shutdown and pumping-up are considered for G2 in Fig. 1. The terminal voltages of the generators (G2, G3, G4) are assumed to be 1.01 p.u. or 1.03 p.u. The active power outputs from the above three generators (for G2, only under the generating condition) are adjusted according to the load demand level. The terminal voltage and active power output of G1 are assumed to be constant. G4 is a swing generator.

Changes of the network configuration are defined as impedance changes of transmission line between No. 1 bus and No. 4 bus corresponding to the reduced system adjacent to this study system. The transmission line impedance between No. 1 and No. 4 buses is increased by 45% of the impedance due to the configuration change.

Load compositions and fault locations are also taken into account as the input data. The loads are represented as mixtures of induction machine loads and constant impedance loads. Two mixing ratios of the induction machine loads (IM) are assumed to be 60% and 70%. Three fault locations are given as points ①, ② and ③ in Fig. 1.

As a result, 192 patterns of input data for learning and 132 patterns for verification were prepared for NN1. Interpolated points of demands in the learning data are used in the verification data. For NN2, 79 patterns of input data for learning and 46 patterns for verification were prepared by extracting data in unstable cases from the input data for NN1. It is noted that the input and output data are normalized as shown in Fig. 5, where the symbol "CT" denotes the assessment index, and a subscript "N" is added to the left side member to indicate normalization of input and output data of NN.

4.2 Dynamic Digital Simulation Analysis

As the transient voltage stability after occurrence of a fault depends significantly on the dynamic characteristics of generators and loads, the dynamic digital simulation is conducted to prepare output teacher data for the input data mentioned above. We use the analytical techniques described in literature [13].

The generator model is based on Park's model with some modification and expansion. The excitation control systems are shown in Fig. 6. Load models are represented as compositions of induction machine model and constant impedance model. Table 1 shows the constants of induction machines. A two circuit three-phase grounded fault, which is cleared by opening the transmission line, is assumed as the most severe fault in this simulation analysis. The duration of fault was set at 0.07 second.

4.3 Evaluation of the Proposed System

(1) *Learning and Voltage Stability Judgement of NN1* 192 patterns of data were learned by NN1 using the backpropagation method with learning constant η of 2.0 and the maximum number of iterations of 2000 times. The mean squared error after finishing learning was 4.8470E-6.

Trained NN1 was made to judge the voltage stability for 132 patterns of data not yet learnt. In this study, the NN1 output of 0.0 to 0.2 corresponds to "stability", and the output of 0.8 to 1.0 "instability". Table 2(a) and Table 2(b) show the examples of judgement results of NN1 in comparison with the results of simulation analysis. The rate of correct answer is 99% and fairly well. It should be noted that, NN1 can make correct judgement of voltage stability even in case of network configuration change as shown in Table 2(b).

(2) *Learning and Voltage Stability Assessment Index of NN2* 79 patterns of data were learned by NN2 with learning constant η of 1.5 and the maximum number of iterations of 2,000 times. The mean squared error after finishing learning was 3.5664E-5.

Trained NN2 was made to estimate the assessment index of transient voltage stability, i.e. critical operating time of

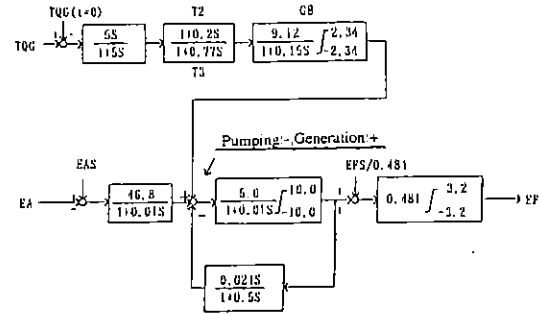


Fig. 6. Block Diagram of Excitation Control.

Table.1. Constants of Induction Motor.

	Self-Capacity base
Primary resistance (Secondary : R_2)	0.05 [PU]
Primary reactance (Secondary : X_2)	0.10 [PU]
Excitation reactance X_m	1.9 [PU]
Inertia constant M	1.0 [S]

emergency control for 46 patterns not yet learnt. Some examples of the results are shown in Table 3(a) and Table 3(b). In comparison with the simulation results, the mean error was 0.029 second, and the maximum error was 0.096 second show a fairly high precision.

Fig. 7 shows the input/output relations of NN2 in a three-dimensional space, where the demand level and the output of the pumping-up motor, both being the inputs into NN2, are taken on X-axis and Y-axis, respectively.

Table.2. Test Results of Voltage Stability Using NN1.

(a) without Network Configuration Change

[O ; Stable, X ; Unstable]

(b) with Network Configuration Change

[O ; Stable, X ; Unstable]

System Condition					Contingency Condition	Judgement of Voltage Stability	
Load Condition			Generator Operating Condition			Fault Location	NN1
Day or Night	Demand (%)	Load Composition Index [%]	P.S.G [MW]	Terminal Voltage [PU]			
Day	105	60	G (550)	1.01	①	x	x
Day	105	70	G (550)	1.01	②	o	o
Day	95	60	G (430)	1.03	①	x	x
Day	95	70	G (430)	1.01	③	o	o
Day	95	60	G (430)	1.03	②	o	o
Day	85	70	G (280)	1.01	①	x	x
Day	75	60	G (150)	1.01	③	o	o
Day	65	70	G (50)	1.01	②	o	o
Night	55	60	S	1.01	②	x	x
Night	55	70	S	1.01	①	x	x
Night	55	60	P (-300)	1.01	①	o	o
Night	45	60	S	1.03	③	x	x
Night	45	70	P (-300)	1.01	②	x	x
Night	45	70	P (-300)	1.03	①	o	o
G : Generation ; S : Shutdown ; P : Pumping					P.S.G : Pumped-Storage Generator		

G : Generation ; S : Shutdown ; P : Pumping P.S.G : Pumped-Storage Generator

System Condition					Contingency Condition	Judgement of Voltage Stability	
Load Condition			Generator Operating Condition			Fault Location	NN1
Day or Night	Demand (%)	Load Composition Index [%]	P.S.G [MW]	Terminal Voltage [PU]			
Day	105	60	G (550)	1.01	②	○	○
Day	105	70	G (550)	1.01	①	×	×
Day	95	60	G (430)	1.01	③	○	○
Day	95	60	G (430)	1.03	②	○	○
Day	95	70	G (430)	1.03	①	×	×
Day	95	70	G (280)	1.01	①	×	×
Day	75	60	G (150)	1.01	③	○	○
Day	65	70	G (50)	1.01	②	○	○
Night	55	70	S	1.01	①	×	×
Night	55	70	S	1.01	②	×	×
Night	55	60	P (-300)	1.01	②	×	×
Night	45	60	S	1.01	③	○	○
Night	45	60	P (-300)	1.03	③	×	×
Night	45	70	P (-300)	1.03	①	○	○

G : Generation S : Shutdown P : Pumping P.S.G : Pumped-Storage Generator

G : Generation ; S : Shutdown ; P : Pumping P.S.G : Pumped-Storage Generator

Table.3. Assessment Indices of Voltage Stability Using NN2.

(c) without Network Configuration Change

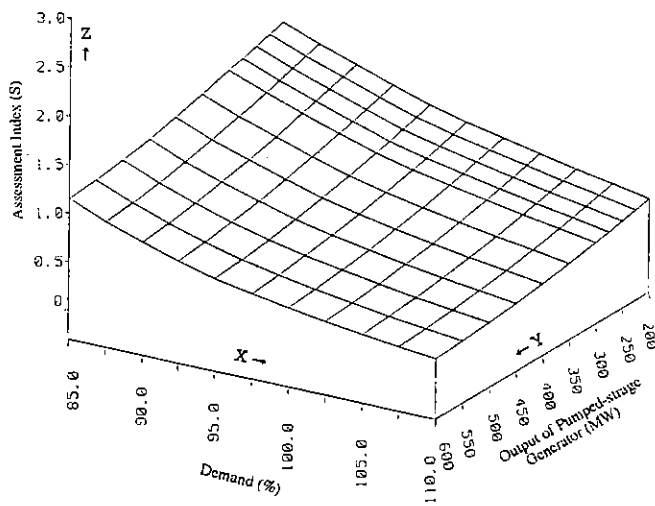
System Condition					Contingency Condition	Assessment Index	
Load Condition			Generator Operating Condition			NN2	Simulation
Day or Night	Demand (%)	Load Composition Index [%]	P.S.G [MW]	Terminal Voltage [PU]	Fault Location		
Day	105	60	G (550)	1.01	①	0.417	0.430
Day	105	70	G (550)	1.01	①	0.245	0.240
Day	95	60	G (430)	1.03	①	0.970	0.970
Day	95	70	G (430)	1.01	①	0.425	0.480
Day	95	70	G (430)	1.03	①	0.510	0.570
Day	85	70	G (280)	1.01	①	0.897	0.870
Night	55	70	S	1.01	②	0.693	0.690
Night	55	60	P (-300)	1.01	③	0.212	0.240
Night	45	70	S	1.01	②	0.738	0.760
Night	45	60	P (-300)	1.03	③	1.274	1.200
Night	45	70	P (-300)	1.01	②	0.805	0.790
Night	45	70	P (-300)	1.03	③	0.265	0.290

G : Generation ; S : Shutdown ; P : Pumping P.S.G : Pumped-Storage Generator

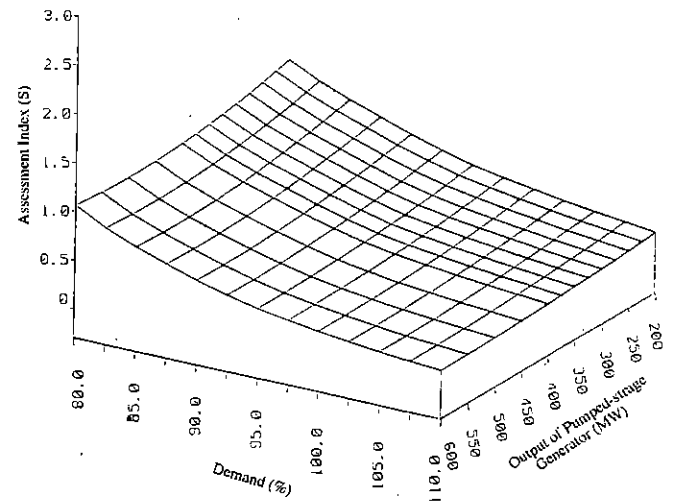
(d) with Network Configuration Change

System Condition					Contingency Condition	Assessment Index	
Load Condition			Generator Operating Condition			NN2	Simulation
Day or Night	Demand (%)	Load Composition Index [%]	P.S.G (MW)	Terminal Voltage [PU]	Fault Location		
Day	105	60	G (550)	1.01	①	0.396	0.410
Day	105	70	G (550)	1.01	①	0.244	0.230
Day	95	60	G (430)	1.01	①	0.721	0.750
Day	95	60	G (430)	1.03	①	0.922	0.930
Day	95	70	G (430)	1.01	①	0.414	0.460
Day	85	70	G (280)	1.01	①	0.829	0.830
Night	55	70	S	1.01	②	0.636	0.650
Night	55	70	P (-300)	1.01	②	0.630	0.680
Night	45	70	S	1.01	②	0.684	0.700
Night	45	60	P (-300)	1.01	③	0.270	0.260
Night	45	70	P (-300)	1.03	②	0.885	0.820
Night	45	70	P (-300)	1.03	③	0.246	0.260

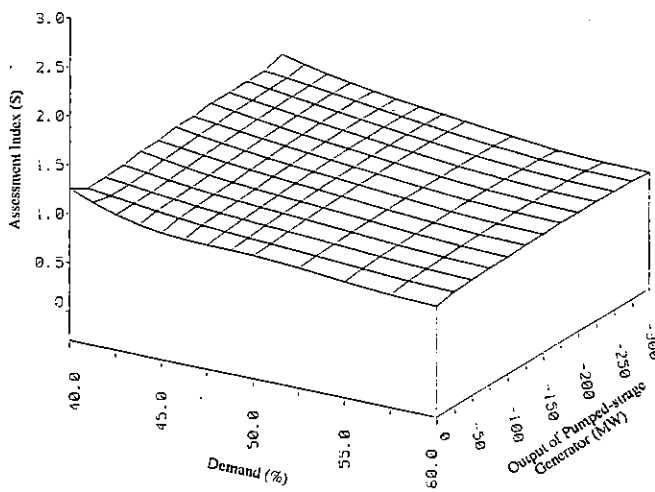
G : Generation ; S : Shutdown ; P : Pumping P.S.G : Pumped-Storage Generator



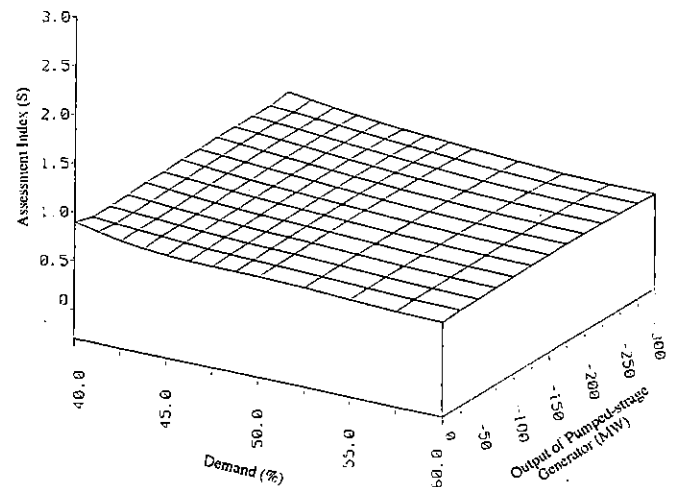
(a) Fault Location ①, Daytime,
Load Composition (Ratio of IM = 60%)



(b) Fault Location ①, Daytime,
Load Composition (Ratio of IM = 70%)



(c) Fault Location ②, Nighttime,
Load Composition (Ratio of IM = 60%)



(d) Fault Location ②, Nighttime,
Load Composition (Ratio of IM = 70%)

Fig. 7. Input / Output Relations of NN2.

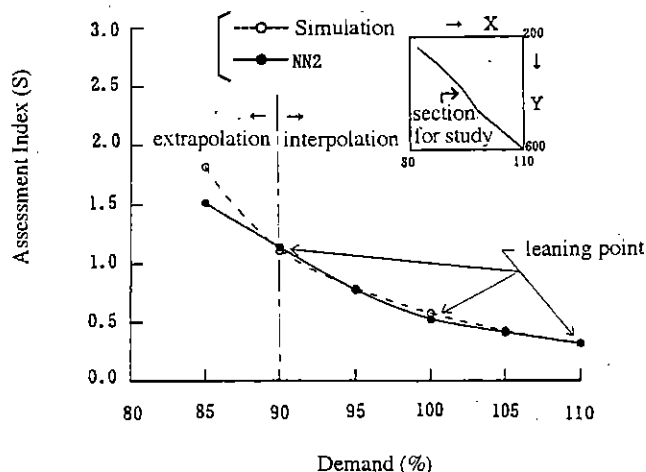


Fig. 8. Relation Between Demand and Assessment Index.

The assessment index being the output from NN2 is taken on Z-axis. Fig. 7(a) clearly shows that for fault point ① having severe impact on voltage stability during the heavily loaded daytime, the relations between the NN2 inputs of demand and pumping-up motor output, and the NN2 output of assessment index are nonlinear. In particular, the relation between the demand and the index shows a stronger nonlinearity. Fig. 7(b) has a relatively lower curved surface than Fig. 7(c) because the ratio of IM in the load composition of (b) is higher than (a). The shape of the curved surface of (b), however, is similar to that of (a). For fault point ② having severe impact on voltage stability at night, Fig. 7(c) and (d) show that the input/output relation of NN2 is also nonlinear although the changes are smaller. From these observations, we can see that NN2 has mastered the nonlinear relations between the inputs and the output. The above-mentioned satisfactory results indicate that the NNs are very effective when applied to assessment of transient voltage stability.

In the learning of NN2, no pattern of data for learning was given to NN2, in which the critical operating time of emergency control, being the output teacher data, was near its upper limit (2.5 seconds). In Fig. 8, which shows the relation between the demand and the assessment index for a certain section of Fig. 7(a), the critical operating time at 90% demand is the largest. Fig. 8 shows that when the demand level in 90% or over is interpolated from the learning data by NN2, the error is very small. When the demand level less than 90% is extrapolated, the error tends to be greater. For a practical application, it, therefore, will be necessary to make the NNs learn data in which the critical operating times of emergency control are near the upper limit so that any patterns of data not yet learnt should be interpolated.

5. Concluding Remarks

In this paper, as a basic study on the development of techniques for assessing transient voltage stability, we proposed a fast and accurate assessment system using two multilayer neural networks (NNs).

In the proposed system, the NN of the first stage judges the transient voltage stability, and the NN of the second stage makes an assessment using the critical operating time of emergency control as an index of transient voltage instability. Some examples of numerical computations for a model system with 26 buses were presented to

show the effectiveness of the proposed technique.

We hereby express our deep gratitude to Mr. Kunio Takei and Mr. Kozo Morita, System Operation Department, Shikoku Electric Power, Co. and all the people concerned for their precious suggestions. Our sincere gratitude is also due to Mr. Fumio Sangawa and Mr. Jiro Kashiba, Development Department, Yonden Technical Consulting, Inc. for their generous cooperation in analytical treatment of our study.

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Summary of Discussion

by

Chen-Ching Liu
University of Washington

1. Marija Ilic's Paper

Les Fink

The definition you started with is simplistic. This is inherently a multi-criteria optimization problem. If you consider the fact that you are possibly impairing system security by the approach you are taking, then you are impairing reliability and, over the long run, you may be delivering less power than if you have taken security into account. You also ignore economics if you ignore economic distribution of the generation. From a multi-criteria standpoint, the definition of the optimal voltage profile should give an overall sense of optimality.

Jose Fernandez

I agree with Les Fink in the sense that I find it somewhat strange to maximize the capability of a system to absorb the load variations. It sounds like maximization of the reactive power output from a generator. It is necessary to provide enough reactive power, but it would not be necessary to maximize it. The maximal reactive output may imply higher losses.

Constantine Vournas

I have a follow-up question. I think the sense of optimality as defined in this paper is actually the condition for voltage collapse. This is a point to avoid. To obtain an optimal setting of the system, one may take the active power losses as a percentage of the load.

George Knight

I would like to raise two points. The first point is the configuration change. Security has to be maintained with respect to configuration changes. The other point is that system operation should satisfy, but *not oversatisfy*, the constraints. I suggest that the amount of changes to be met should be specified but should not be maximized. In other words, it may be an outage

of a generator or two, but it is a credible contingency.

Marija Ilic

I appreciate your comments. Les Fink's comment is general to the problem. This is the best we could think of from the viewpoint that there are energy demand and resources and one tries to meet, as much as possible, the demand with the available resources. If you recall, my load model is not constant. As the loading changes, there are limits on the real power generations and the voltages on the generation side. The question is 'what is that generator good for to the system' as we look into the rest of the system. The basic task in a power system is to deliver the real power as the operating condition changes.

Now, there is a peculiar issue here about the security margins. In the past, we keep the voltage levels at the most secure operating point. We have also moved into the n-1 criterion. Even in the n-1 criterion, we do not push the voltages close to the operating constraints for fear that something may go wrong, whether static or dynamic. I believe, however, security is not neglected. In case a line contingency occurs, the impedance of a parallel line will double. No matter what happens, I want to reset either automatically or via an OPF so that as the parameters change I target the optimal voltage so that I still deliver the most real power I can. This is the best we are able to propose. There is some risk in proposing a unified optimal voltage profile.

There seems to be some confusion in Jose Fernandez's question. The task is to minimize the reactive power out of a generator so that you automatically minimize the transmission losses according to the proposed formulation. Constantine Vournas's comment about voltage collapse is true. That is a matter of philosophy; how far are we willing to push the system as we apply new control techniques or bifurcation points? If we are willing to allow the system to go there and control them, the system is likely already out of the steady-state security constraints, which may be conservative. I apologize I can not give better answers; this is a very hard topic.

2. Thierry Van Cutsem's Paper

Nelson Martins

Congratulations on your paper, except for a few points. I read some of your previous work. There is a sentence in your paper saying, 'the critical point identification does not require an explicit eigenvalue computation of the dense state matrix A; rather, sensitivities computed from the sparse Jacobian can be used.' Then, you suggest Carpentier's approximate formulation. Actually, we are doing the sparse Q-V sensitivity eigensolution first done by Andersson and Lof. It is also using an 'augmented formulation' idea on electromechanical stability, which is from my published work based on an unpublished thesis by Brian's student in 1974. Carpentier's sensitivities consider global reactive power generation. My interpretation based on the formula is that it is very close to an eigenvalue which reflects the global area voltage behavior. What is an eigenanalysis of a Q-V or P-V matrix? It is nothing more than an organized, orderly and effective way to use sensitivities.

Yakout Mansour

Your work is timely and addressing a real challenge. After completing the development of our on-line angle stability assessment tool, we initiated the on-line voltage stability tool development about 6 months ago. We were able to assess the angle stability problem at almost real-time speed, while the voltage stability is about two orders of magnitude as the real-time speed using more or less the same tools. We also found that the most time-consuming step is the Jacobian updating every time one of the hundreds of tap changers move. We also tried to minimize the number of updates of the Jacobian by looking at the snapshots. The dQ_g/dQ_l indicator is a good indicator for corrective actions when the generators are close to the load. For cases where generators are far from the load, the indicator is not the best. Did you think of other indicators to guide you in corrective actions?

Julian Barquin

You are checking the Jacobian matrix along the trajectory. Then, when you get eigenvalues, you

are in a saddle node bifurcation. However, that is only true if it is also an equilibrium point. Just after Eq. (21), you are checking the saddle node bifurcation in the fast dynamics. Suppose one looks at the system $dX/dt = -X^2 - (P-P_0)$, where P is a parameter and X is the state variable. Now, if a change of parameter occurs, the system moves from point A to B and the system suffers from a collapse. Now, the system has a collapse; however, all around the collapse X is negative or zero. Therefore, the sign of the Jacobian ($-2X$) remains constant and you do not have a zero eigenvalue.

Thomas Overbye

I like your formulation of a voltage collapse caused by the loss of the equilibrium point in the load parameter space. When the unsolvable power flow occurs, it seems that you have two options for control application. If a contingency condition becomes unsolvable, you can do a pre-disturbance or a post-disturbance control move. Looking at the pre-disturbance case, to calculate the control sensitivities to the pre-disturbance control move, I say that, rather than calculating the point Sc that you mentioned, I would calculate the closest boundary point on sigma to your original point zero. That is probably the best point to calculate and to use in your sensitivity calculation even if you choose to take the post-contingency control moves. That will save yourself having to do dynamic simulation to calculate Sc . So my question is, 'How do you calculate Sc ? Is it through time simulation?' My suggestion is to use the closest boundary point instead.

Claudio Canizares

Congratulations on your paper. My question is similar to Julian Barquin's. On the issue of ignoring the X dot term, particularly under a large disturbance, the system equation does not necessarily generate a proper system trajectory. Under a large disturbance, the system may oscillate and even move out of the stability region. So, that concerns me a bit. I do small-signal studies myself, so I would agree that for small disturbances the system follows the equilibrium very closely. For your system, if the change is a tap change or that type, I can see that happens. The other question is about the use of a linear indicator with a trajectory

generated by nonlinear dynamics. There is a problem in that it may not be caused by bifurcations.

Christopher DeMarco

I hope I can synthesize the previous questions. The question relates stability analysis along the trajectory with linearization. Let's take the ideal case where you do not approximate by eliminating the fast dynamics. The error dynamics relative to your trajectory can be obtained. If you look at the state equation for the error and do a Taylor's expansion on the nonlinear part, you linearize the nonlinear term along the trajectory and it is a linear time-varying system. If the system moves slowly enough and the eigenvalues are far enough on the left half plane you can argue that the system is stable. But, in general, if you allow switching actions etc., the left half plane criterion does not guarantee stability.

Thierry Van Cutsem

Nelson Martins's question: My point was not to suggest that I propose the sparse formulation and others do a dense one. My point is to show that sensitivity calculation exploits the full Jacobian matrix. In fact, we are tracking the zero determinant of the dense A matrix. From the system point of view, we are tracking the singularity of A, which is dense, and we are using a linear indicator which is based on J, a sparse matrix. I do not claim that all else use the dense formulation.

Yakout Mansour's question: Your suggested that dQ_g/dQ_l is not a good indicator if generators are far away from the load centers. The sensitivity here is just the inverse of the Jacobian. If you have a zero eigenvalue close to the point where you switch from small negative to small positive, you normally feel the change in sign everywhere in the system. After some time, the other modes come into picture and you may not be able to draw the same conclusion. I am just tracking the change in sign. The Jacobian update is very crucial. In our method, the Jacobian is not updated after each tap change; it is kept constant as much as possible. That is why it is efficient. When there is a generator limiting, the Jacobian is updated. Finally, I do not use other indicators.

Julian Barquin's question: I think you are saying that there are cases where I could get singularity and my midterm model fails and that I can not ignore fast dynamics. I can show you an example with a constant power and constant admittance demand. This is prone to the singularity problem because in the course of tap-changer movements you can be in a situation where it does not cross any longer. My answer to this is that the sensitivity/eigenvalue technique does not deal with the problem at S; it deals with the problem at C. S is singularity because my load characteristic does not intersect with the characteristic which is equivalent to $\dot{x}$ equal to zero. Since it does not intersect, the system has lost its fast dynamics equilibrium and this can not be detected by sensitivities. I do not worry about S because I am dealing with C from a practical point of view. That is what the sensitivities are tracking.

Thomas Overbye's question: I have to think about it carefully. I have the contingencies so I hit the hypersurface. You seem to propose a way to deal with the cases where I would not hit it but still I can get some information on corrective control.

Claudio Canizares's question: After a severe contingency, of course, you can simulate a short circuit, eliminate it and keep the line off to see whether the voltage will collapse after 2 minutes. Obviously, the short circuit could trigger the angle stability before, so I would recommend some kind of mixed time scales or you can at least check the first period with full integration and then switch to this technique after the fast dynamics settle down but before the tap changers act. But from this point up to the subsequent midterm evolution, $\dot{x}$ equal zero is reasonable, except when you lose the transient equilibrium. Very frankly, this occurs in situations where you already met the midterm stability before so that you are sure it is covered by the sensitivity criteria.

The linear indicator is an important issue. I prefer to give you a physical explanation. Let's take a system. Suppose we are operating at a pre-contingency condition. A disturbance causes one to switch from the pre-disturbance characteristic to another. In this case the tap changer is moving the operating condition and then the system goes down. There is no midterm

equilibrium because the midterm equilibrium characteristic after the tap changer restores the load is a constant power load. Indeed, we do not have an equilibrium point. Rather, we pick a point along the curve and use it as if it were an equilibrium point. This could be dangerous if there is a big angle swing. This is not a rigorous proof, but it gives a feel of the problem.

3. Marija Ilic's Paper

Eyad Abed

I would like to congratulate Marija on an excellent paper; people who do control design for power systems and voltage stability are in need of this type of work. My question is about the control design. Since you assume impedance loads and then you do a reduced admittance matrix, if there are controls other than the exciters, for instance at the load, then it may be difficult, especially when you are designing a nonlinear control, to design that control within your set up. Do you have any comment ?

Constantine Vournas

I am using a similar formulation. My experience is that if you keep the load as a constant admittance or impedance and you do not model the load restoration then you would not have a voltage stability problem.

Robert Thomas

We did a lot of work under DOE contracts during 1978-79. The issue is that if the system meets certain matching conditions you can always guarantee that your linear feedback control works. The problem is that the control tends to be very large facing uncertainties so the control is overdesigned. It may be much better to deal with the nonlinear form and only add uncertainties in places when it is absolutely necessary.

Marija Ilic

I will start with Bob's question. I think what you are saying is true. If you are taking a brute force approach to go from the differential algebraic form to a set of ordinary differential equations, it is a question of control design for nonlinear systems in the state space form, which is simpler

than the one we started with. The second problem is the one of going from the nonlinear to the uncertain linear formulation. This is what Jeff Chapman is doing, which is interesting in that the formulation and bounds are not conservative. One goes from a reference plane to the other with a rotational matrix. You can not do better or worse; this is a natural bound. The hope is that the bound is natural so maybe we can do fairly nonconservative control design. We did not look at the DOE report; I think that is an early piece of work on robustness and now it is becoming very relevant.

Eyad Abed's question: Our specific subproblem is to go from the transient stability range (first swing) to control design of the order of 10 seconds. You have a variety of control candidates. You have controls on the generator side and add to it the static var compensators. For that type of problem, one should not revisit the old constant gain control that we have on power system stabilizers; rather one should do something more effective for a larger range of stability, which naturally lends to that problem. So, in many ways, this formulation is good to that subgroup of controls and the emphasis is that we are going from the first swing stabilization problem to one that is longer and relevant to inter-area oscillations where voltage and angle dynamics interact. This model captures that fairly well.

Constantine Vournas's question: The tap-changer model proposed by Thierry van Cutsem has an intentional delay, which is typically in the order of minutes. I am designing controls which are supposed to act in 10 seconds and then settle, so the impedance load model is justifiable.

Eyad Abed's question: One would have to think carefully about the applicability to FACTS devices. These devices involve changes in parameters of the transmission system, which is different. The proposed method does not apply to that.

Les Fink's question: This is actually a general comment to the whole problem. Regarding the multi-criteria issue, we are talking about a simple network - one generator delivering power to a static nonlinear load via a linear transmission line. From Circuit Theory, if the

load is a constant impedance, the minimal transmission loss will occur at the same location as the minimal generation that one needs to deliver to this constant impedance load, and, at this same point, the minimal Q_g is generated. So this is a nice starting point which comes from the load matching condition. Now that result does not apply. You have implicitly a multi-criteria formulation because delivering the maximal power to the load does not work within the generation constraint. That was the starting point. Unfortunately we do not operate at that point because the voltage is very high. Now, instead of a constant impedance load, we have a static nonlinear load. How does the optimality change? Maybe the definition is a bit misleading in that it does not spell out exactly what happens with the loss. As we went through the derivations, however, it became clear how the minima are related. For example, to deliver a maximal P_L with a region you may have high losses. That does not happen when you have a simple network. As you minimize P_G , the optimum I chose in the entire range also minimizes the losses. For the specified load that you have to deliver, if you do that with the minimal generation, you simultaneously minimize the transmission loss. So it may appear to be over-simplified but it is a multi-criteria formulation.

Jacques Deuse

I want to underline that minimal loss for the objective function is equivalent to minimal *total* generation but not for any individual generating unit.

4. Yasuhiro Hayashi's Paper

Robert Thomas

How many inputs do you use in training ?

Shinichi Iwamoto

The total number of inputs is 132; there is only one output. There are 11 training sets.

Robert Thomas

Based on our experience in training feedforward neural nets, the number of inputs you can have and the amount of data needed to train the

neural nets to the specified accuracy generally for functions of the sinusoidal form limits the number of inputs very severely or it limits the density of the data you can use. Otherwise, the back propagation goes on forever.

Shinichi Iwamoto

This is a very good point. How many weights in the neural net do you need for the input set ? If you have only one neural net for all the purposes, then you have many training sets and you have a limited number of weights. Then, you won't get good results. My advice is that you have a certain number of weights first, then you determine a certain number of sets. Some would think it will not work if you have only one neural net.

Miroslav Begovic

This may be an excellent application area to reduce the number of inputs prior to doing anything with neural nets. Some work on voltage coherency or hierarchical voltage control that has been done before may be the prerequisites before we start using neural nets.

Hans Glavitsch

It is not clear from the presentation if one neural net is able to capture contingency cases or you have to have several networks. The analysis should also show contingencies.

Shinichi Iwamoto

We did not look into that, but I think you need a number of neural works. A single one will not be enough.

Hans Glavitsch

I have a follow-up question for the audience. You can do a simple calculation about the number of weights for a larger size system and compare the number of weights to the number of connections in the sparse system matrix. Since the way back propagation goes through the network is similar to the forward-backward substitutions in sparsity techniques, you can work out the on-line efficiency relative to the number of load flow runs. It seems to me you

will reach a crossover point where neural nets are not efficient any more.

Shinichi Iwamoto

The neural network has two stages. The off-line computation is time consuming, but the on-line computation is almost instantaneous.

Hans Glavitsch

What I said may contradict this because you are getting more weights above a certain number of nodes than the number of connections in the sparse matrix.

Shinichi Iwamoto

The number of weights in the proposed method is very small. What we said in the paper is that, if you do the general back propagation, the number of weights will increase exponentially. We are taking advantage of the power system configuration; we have the sparse matrix structure for the neural network. Neural nets sometimes do not give good results if training is not properly done. My advice is to use the neural nets as a screening tool if the system is large.

Daniel Cheng

My first question is about the input layer structure. Suppose that you train the neural net for a particular configuration and then put it on line. Now the busbar splits and the busbar now becomes busbar A and busbar B. How do you couple the dynamic structure with the input layer ? The second question is about training. I noticed that training is done through extensive off-line studies. If the neural net is working well for a while and then the power system adds a new line, do you re-train them ? How often ? Do you use the old training set or the new training set gained from the on-line experience ? Do you mix the sets or discard the old one ?

Shinichi Iwamoto

We have not dealt with the configuration changes. We did remove some lines and the proposed method works well. Re-training is necessary for on-line but we have not looked into the re-training issue.

Daniel Cheng

One solution is, instead of having each neuron correspond to a bus, you have each neuron correspond to each bus section. But, in this case, you will have a lot of neurons.

5. Tatsumi Maeda's Paper

Chen-Ching Liu

What is the stability index you are using ?

Akihiko Yokoyama

The index is the critical delay time of the emergency action to close the bus tie between the two subsystems.

The session chairman closed the session.

Session 4

Survey III

"Industry Practice in Voltage Stability Analysis of Power Systems"
Yakout Mansour: BC Hydro

Analytical Methods

"Bifurcation Flow: A Tool to Study Both Static and Dynamic Aspects of Voltage Stability"
Byongjun Lee, Venkataramana Ajjarapu
Iowa State University

"Probabilistic Approach to Steady State Voltage Stability of Bulk Power Systems"
Marian Sobierajski: The Technical University of Wroclaw

"A Continuation Method Approach to Finding the Closest Saddle Node Bifurcation Point"
Yuri V. Makarov, Ian A. Hiskens
University of Newcastle

"Improving Continuation Methods for Tracing Bifurcation Diagrams in Power Systems"
Claudio A. Cañizares, Antonio Z. de Sousa, Victor H. Quintana
University of Waterloo

**"Determination of Dynamic Voltage Stable Equilibrium Point
Based on Nonlinear Optimization Method"**
Yicheng Zhou, Keiichiro Yasuda, Ryuichi Yokoyama
Tokyo Metropolitan University

"Estimating Multiple Load Flow Solutions with the Householder Transform"
Hiroyuki Mori: Cornell University

Industry Practice in Voltage Stability Analysis of Power Systems

Yakout Mansour
B.C. Hydro, CANADA

Abstract*

In the last decade, many parts of the utility industry were caught short of necessary tools to properly assess and mitigate voltage stability limitations in their networks at a time when the phenomena began to spread its routes in the planning and operating environments. Because of the slow nature of voltage instability and collapse, load flow programs have been used to calculate the limits using power and reactive power - voltage sensitivities calculated at a bus or group of buses in the network. While this approach served the urgency to some extent, it was not perceived by many as a satisfactory solution to the problem and considerable research efforts were directed towards streamlining and enhancing P-V and Q-V computations.

Fortunately, some good efforts produced new tools and concepts that helped utilities considerably in understanding and resolving the problems. In the process, the following have to be determined.

- i) Assessment of System Voltage Stability (stable/unstable)
- ii) Transfer limit
- iii) Factors affecting voltage stability limit
- iv) Preventive/corrective actions

This presentation summarizes the techniques evolved in the last few years to the application phase in many utilities to answer the above questions. The techniques selected for discussion and demonstration, using a large network model, are step-by-step load flow, continuation load flow, modal analysis, and long-term dynamic simulation. The results will demonstrate the merits and limitations of each and how to use the combination of these techniques to answer the key questions.

Note:

- * Refer to IEEE Publication #93TH0620-5PWR "Suggested Techniques for Voltage Stability Analysis," copies to be distributed.

Bifurcation Flow: A Tool to Study Both Static and Dynamic Aspects of Voltage Stability

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Abstract

This paper attempts to develop a unified framework that can be used to study voltage stability in the structure-preserving power system model. The significance of this framework is that the methodology unifies various major contributions on voltage stability in such a way that these contributions become a subset of this framework. Salient features of this approach are (1) Both dynamic and static models that are relevant to voltage stability are included. (2) Realistic load increases and a generation sharing scheme with relevant limits are included. (3) For a given load increase and generation sharing scenario, all possible critical points that may lead to voltage instability are systematically traced and identified. (4) Systematic sensitivity analysis near all the critical points is included. A new invariant subspace parametric sensitivity (ISPS) method is developed. It gives the contribution of any parameter to a particular critical point. Results obtained from applying this framework to the IEEE New England 39-bus test system show that the ISPS method successfully identifies sources of instabilities by determining which parameters have the most influence on voltage instability. This framework systematically identifies the areas, components, and parameters that are responsible for a possible voltage collapse.

I. Introduction

Recent major changes in power system operating practices have resulted in highly interconnected and heavily loaded power system networks. These changes have produced increased problems associated with maintaining acceptable voltage profiles in the network. In electric power utilities, the system dispatcher is responsible for maintaining an acceptable system voltage profile through the use of a variety of controls. Since there is a lack of tools for adequate voltage security assessment, the system dispatcher must work with only limited information about the system state. Controls applied with such limited knowledge often have a counterpro-

ductive effect during a disturbance. Persistence of the problem is due in part to a lack of adequate understanding of voltage dynamics.

According to the industry experiences of voltage instability phenomena [1, 2, 3], voltage collapse dynamics span a range in time from a fraction of a second to tens of minutes. Time frame charts have been used to describe dynamic phenomena which show time responses for equipment that may affect voltage stability [4]. Two time frames are considered for voltage stability: transient and long term. If the system operates close to singularity-induced bifurcation, fast transient dynamics need to be considered. (Details are given in [5, 6, 7].)

Then the time frames to be considered become: very fast transient, transient, and long term. The main characteristics of the three time frames are as follows:

- A very fast transient voltage collapse involves network RLC components having very fast responses. The EMTP (Electro Magnetic Transient Program) contains a dynamic model related to network RLC components. The time is micro- to milli-seconds.
- A transient voltage collapse involves a large disturbance and loads having a rapid response. Motor dynamics following a fault are often the main concern. The time frame is one to several seconds.
- A Long-term voltage collapse usually involves a load increase or a power transfer increase. In this time frame, a voltage collapse involves slow load restoration by tap-changer and generator current limiting. Manual actions by system operators may be important. The time frame is usually 0.5 - 30 minutes.

Figure 1 shows models involved in the analysis of voltage stability in different time frames, where X_f is a state vector representing transient dynamics, X_s is a

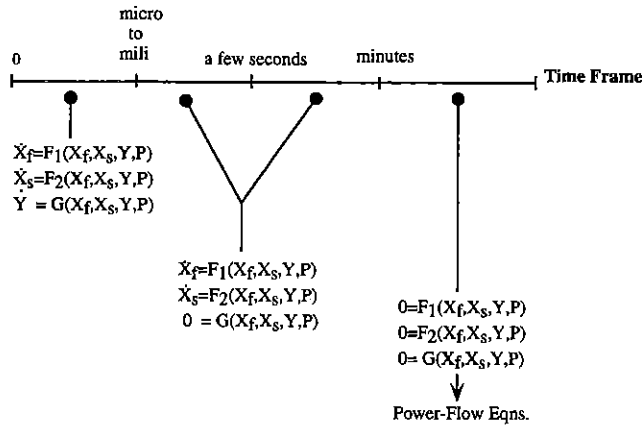


Figure 1: Time frame and relevant model in voltage stability analysis

state vector representing long-term dynamics, Y is a state vector representing very fast transient dynamics related to network components, and P is a system parameter vector. Voltage instability is affected by various system components in a wide time range. In order to tackle this problem, one must consider proper model and analysis methods.

Static (power-flow) Analysis

In its early stages, the voltage collapse studies were mainly concerned with steady-state voltage behavior. The voltage collapse is often described as a problem which results when a transfer limit is exceeded. The transfer limit of an electrical power network is the maximal real or reactive power that the system can deliver from the generation sources to the load area. Specifically, the transfer limit is the maximal amount of power that corresponds to at least one power-flow solution. From the well known PV or QV curves, one can observe that the voltage gradually decreases as the power transfer amount is increased. Beyond the maximum power transfer limit, the power-flow solution does not exist, which implies that the system has lost its steady-state equilibrium point.

From an analytical point of view, Venikov [8] proposed the first criteria for detecting the steady-state stability limit as the point where the Jacobian of power-flow equations become singular. This concept has been equivalently analyzed in terms of the minimum singular value of the Jacobian [9, 10]. However, if the system's condition is close to a critical loading, conventional power-flow calculations based on the Newton-Raphson method create numerical troubles. Therefore, additional special techniques should be implemented

in power-flow programs. The continuation method [11, 12, 13, 14] and direct method [15, 16] have been developed to overcome numerical troubles.

The static analysis using power-flow equations has the following disadvantages.

- First, if a steady-state equilibrium point of an electrical power system does not exist, the static result simply indicates that the system is unstable. However, if we consider dynamics, the system behavior may be affected by periodic or other nonlinear behavior.
- Second, according to the conventional criteria, usually in the region below the maximum power transfer point, the system is considered unstable, but it is not always true. If a proper load model is considered [17, 18], the voltage may be stable even below the maximum power transfer point.
- Third, the dynamics of various components, including load, may change the stability results. The results in [5, 19, 20, 21, 22, 23, 24, 25, 26, 27] show that the system voltage or other dynamic mechanism may become unstable before the maximum power transfer point. In general, the singularity of the power-flow Jacobian matrix does not coincide with the singularity of the system matrix at which system becomes unstable.

These observations indicate that the criteria based on power-flow equations may not be accurate, or even too optimistic. Thus, a proper dynamic model of the key devices needs to be included in the analysis of voltage stability.

Dynamic Analysis

Traditionally, power system stability is often associated with the rotor dynamics. In the detailed generator model, various flux linkage components are modeled, while in simplified swing equations the voltages are simply assumed constant. The instability phenomena caused by swing dynamics may be power-flow oscillations or voltage fluctuations. In the swing equation model the voltage variation is considered as a dependent variable which follows the angle swings. In other words, the angle swing induces the voltage fluctuation. However, in a voltage collapse study, we cannot assume that voltage is constant. In that case, the question of what mechanism can cause the voltage to collapse arises. In order to address this question, various dynamic mechanisms of a power system need to be analyzed.

The choice of analytical methods used to study dynamic voltage stability depends on the size of the disturbance and the range of the time frame involved. This, in turn, leads to transient analysis with large disturbances and long-term analysis with small disturbances.

Transient and Long-term Analysis: Although classical voltage instability evolves over several minutes, the possibility of transient voltage instability also exists because of the characteristics of fast-response system components. When a large disturbance occurs in the systems, the phenomena are usually analyzed by transient stability programs which include fast dynamic models such as induction motors.

Voltage instability often occurs due to system dynamics of the order of tens of seconds to minutes. Dynamics include increase of load, recovery of load after disturbances, and power coordination following a loss of generation. Several approaches which are based on long-term time simulation have been developed. The time simulation is valuable in determining and demonstrating the time sequence of control and protection actions with any size of disturbance.

The most general method for transient and long-term analysis involves unified solution of fast (transient) and slow (long-term) dynamics [28, 29, 30, 31, 32, 33]. These programs use various time steps to simulate the system behavior in different time frames.

However, time domain simulations have a number of shortcomings. First, they do not readily provide sensitivity information regarding the degree of system voltage stability. Second, they generally require extensive output analysis to uncover the causes of and the factors contributing to voltage instability. Time domain simulations look at the overall system stability and do not focus on voltage stability. Third, although numerical integration techniques have become more efficient, time domain dynamic simulations for long periods of time remain computationally very intensive.

Small-Disturbance Analysis: The aim of small-disturbance voltage stability analysis is to determine whether a suggested operating point of a power system will be voltage stable with respect to small disturbance which is usually gradual increasing in load. To achieve this goal, the original nonlinear dynamic equations are linearized around a specified operating point, and the system matrix is calculated. The eigenvalues of the system matrix determine the dynamic response of the system for small disturbances around the operating point. The corresponding right and left eigenvectors de-

fine the shape of the corresponding modes of response. This analysis provides an analytical tool for both the proximity to and the mechanism of voltage instability.

The papers related to this analysis are appeared in general in [1, 34, 35] and in particular with classification in [5, 20, 36, 37]. They commonly concluded that the generic bifurcations that may be a source of voltage problems are: saddle-node bifurcation, representing a collapse type of instability; Hopf bifurcation, representing oscillatory instability; and singularity-induced bifurcation, representing a fast collapse type of instability.

Scope Of The Work

Most literature reviewed has focussed on individual mechanisms based on particular devices or system components. However, most incidents of voltage instability involve a combination of individual mechanisms, and it is often difficult to isolate any one action as responsible for the voltage instability. Even though the influence of the key devices can be demonstrated using simulations which serve to isolate the individual effects of certain system characteristics, it is still unknown how individual mechanisms interact. Each mechanism can cause a different type of voltage problem, and the mechanisms can interact to create a voltage collapse.

This paper concentrates on a small-disturbance stability analysis to the structure-preserving power system model. The reason a small-disturbance stability analysis is used is as follows: Voltage stability is particularly insidious because it can occur in an apparently normal system without a significant disturbance. It is now understood that the underlying cause is typically a bifurcation associated with the nonlinear power system dynamics and can be initiated by a small change in system parameters. The aim of voltage stability analysis in this work is to determine whether a suggested operating point of a power system will be voltage stable. To achieve this goal, the problem is formulated as a small-disturbance voltage stability. The commonly classified voltage instabilities described in the preceding section are used as criteria to evaluate a small-disturbance stability.

In Section II, the paper introduces a unified framework that integrates all important mechanisms, including the swing, flux, excitation, generic load dynamics, with generator field current and armature current limits. The model under consideration includes all the key components of the power system recommended by CIGRE Task Force 38.02.10 [34] except the HVDC model. The mechanisms are combined step by step so that

system dynamics can be more thoroughly understood. Previous work concentrated only on individual components and their effect on voltage stability. This section also explain the importance of dynamic analysis.

Further insights into voltage problems can be obtained by properly identifying the areas and the parameters in these areas that are responsible for the particular type of system behavior. This paper derives parametric sensitivity in a structure-preserving power system model. In the analysis of a power system it is important to have a basic knowledge of the variations of system voltage with respect to the parameters involved. Previous approaches to voltage-stability sensitivity concentrated exclusively either on static part of the system [38, 39, 40, 10] or, to a limited extent on the dynamic part of the system [33, 41]. In Section III, an attempt is made to derive parametric sensitivities for system voltages by combining both the static and the dynamic aspects in a coherent fashion. It also introduces a promising new method, termed invariant subspace parametric sensitivity (ISPS). Parametric sensitivity gives the sensitivity of equilibrium point with respect to any parameter, whereas ISPS gives voltage stability sensitivity. Section IV describes the application of ISPS to identify the areas, components, and parameters that are responsible for a possible voltage instability.

II. Small-Disturbance Analysis Of Structure-Preserving Power System Model

Mathematical equations representing the power system model under consideration are described by a number of algebraic and differential equations, which in general are nonlinear,

$$\dot{X} = F(X, Y, P) \quad (1)$$

$$0 = G(X, Y, P) \quad (2)$$

The function F describes the dynamics of the generator, the exciter, and the load. Function G represents the stator, internal, and external network model. The state variables X , the algebraic variables Y and the parameters P are given as follows:

$$X = (\delta, \omega, E'_d, E'_q, E_{fd}, V_R, R_f, V \text{ and } \theta \text{ at load buses})$$

$$Y = (I_d, I_q, V \text{ and } \theta \text{ at other than load buses})$$

$$P = (P_L \text{ or } Q_L)$$

Equations 1 and 2 are same as the Equations corresponding to "a few seconds" time frame in Figure 1 if (X_f, X_s) is replaced with X . This is the structure-preserving model[42] or differential algebraic equations (DAEs). This section concentrates on the small-disturbance stability of this power system model.

To evaluate the small-disturbance stability, Eqs. 1 and 2 first have to be linearized around a specified operating point. While we are free to arbitrarily specify the initial conditions for all dynamic states and control inputs, most dynamic studies begin from steady-state operation. In power systems, the most widely used program for finding the steady-state equilibrium point is the power-flow program. In this paper the continuation power flow (CPF) program [11] provides robust steady-state equilibrium point.

Linearization

In Eqs. 1 and 2 if the parameter is varied, the corresponding state vector X and the eigenvalues of the system matrix evaluated on this path change accordingly. Near an equilibrium point the left-hand side term $\dot{X}$ becomes zero:

$$0 = F(X, Y, P) \quad (3)$$

Eqs. 2 and 3 specify the position of the equilibrium point X as a function of P . Let's linearize Eqs. 1 and 2 at a certain equilibrium point $(X(P_o), Y(P_o))$ as follows,

$$\begin{bmatrix} \Delta \dot{X} \\ 0 \end{bmatrix} = \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} \quad (4)$$

Matrices K_1 , K_2 , K_3 , and K_4 contain first order derivatives of F and G , evaluated at $(X(P_o), Y(P_o))$, and their elements are derived as:

$$K_{1i,j} = \left. \frac{\partial F_i}{\partial X_j} \right|_{(X(P_o), Y(P_o))}$$

$$K_{2i,j} = \left. \frac{\partial F_i}{\partial Y_j} \right|_{(X(P_o), Y(P_o))}$$

$$K_{3i,j} = \left. \frac{\partial G_i}{\partial X_j} \right|_{(X(P_o), Y(P_o))}$$

$$K_{4i,j} = \left. \frac{\partial G_i}{\partial Y_j} \right|_{(X(P_o), Y(P_o))}$$

Matrix K_4 is an algebraic Jacobian matrix that contains the power flow Jacobian matrix. The general structure of the entire Jacobian matrix of a structure-preserving model is shown in Figure 2.

In Eq. 4, if $\det(K_4)$ is not zero, ΔY can be expressed in terms of ΔX as

$$\Delta Y = -K_4^{-1} K_3 \Delta X \quad (5)$$

Replacing ΔY in Eq. 4 with Eq. 5 results in

$$\begin{aligned} \Delta \dot{X} &= A \Delta X \\ A &= [K_1 - K_2 K_4^{-1} K_3] \Delta X \end{aligned} \quad (6)$$

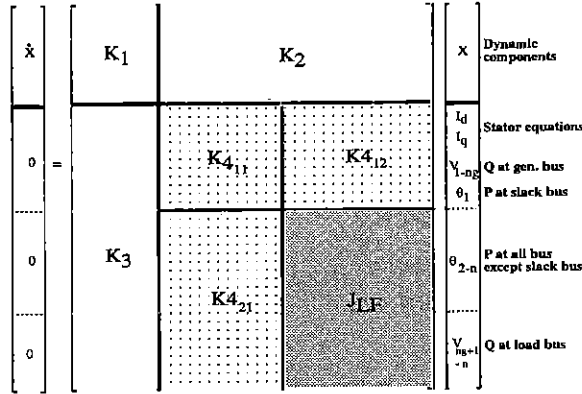


Figure 2: Jacobian matrix of structure preserving power system model

The essential small-disturbance dynamic characteristics of a structure-preserving model are expressed in terms of eigenproperties of the reduced system matrix A .

To find K_1, K_2, K_3 , and K_4 , we need initial conditions that are defined by $\delta, \omega, E'_q, E'_d, E'_{fd}, V_R, R_f, I_d, I_q$, and V, θ . For a given load increase and generation sharing scenario, the CPF systematically traces a path of steady-state equilibrium points and provides V and θ at all buses. These values can be used to calculate remaining variables [43, 44].

Determining Critical Points

The power system state on the surface defined by Eqs. 1 and 2 is asymptotically stable if the eigenvalues of the system matrix A have negative real parts. As described in the preceding section, the system can reach a critical state if an equilibrium point becomes nonhyperbolic, that is, one or a pair of eigenvalues have a zero real part at that point. The former case is a saddle-node type bifurcation, and the latter case represents a Hopf bifurcation. In the DAE formulation, however, another critical state may occur when the algebraic Jacobian matrix becomes singular, the so-called singularity-induced bifurcation [5]. In this research, in addition to those critical points, two more are defined as the node-focus point and 1:1 resonance point at which eigenvalues with multiplicity of two occur.

Saddle-Node Bifurcation (SN): At a saddle-node bifurcation point, the system has a nonhyperbolic equilibrium with a simple zero eigenvalue when transversality conditions corresponding to this point are satisfied [45]. Since the transversality conditions are generically satis-

fied [46], they are not tested in practical investigation. At this point, the eigenvector corresponding to the zero eigenvalue points in the direction of system trajectories. Once the system moves in the direction of the eigenvector corresponding to the zero eigenvalues, the system trajectories are attracted by the center manifold. The system behavior on the center manifold around this bifurcation point is monotonic, and the system may experience a monotonic type of instability, i.e., a collapse type of instability [47].

Hopf Bifurcation (H): A Hopf bifurcation occurs at a point where the system has a nonhyperbolic equilibrium with a pair of purely imaginary eigenvalues (with nonzero imaginary parts). Similar to the SN, the Hopf bifurcation should also satisfy the transversality condition [48]. In the neighborhood of this point, periodic solutions always exist. The steady-state behavior becomes unstable and the system stability is determined by the stability of periodic solutions [21, 22, 23].

Singularity-Induced Bifurcation (SI): A singularity-induced bifurcation occurs when the algebraic Jacobian matrix (K_4) becomes singular. At this point one real eigenvalue of the system matrix (A) becomes infinity, and the rest are bounded. Numerical simulation of DAE can not be performed near this point because the interrelation between the state space and the algebraic space is broken. Singularity-induced bifurcation was named by Venkatasubramanian et al. [5].

Node-Focus (NF) and 1:1 Resonance: At a node-focus (NF) point a pair of complex eigenvalues become two equal real eigenvalues. At a 1:1 resonance point two pairs of complex eigenvalues become two equal complex eigenvalues [49, 50, 51]. The aforementioned SN, H, and SI bifurcation points are critical points at which steady-state stability changes. Although the stability does not change, NF and 1:1 resonance points are important because system behavior may experience qualitative change. Around an NF point the system is overdamped or underdamped. At a 1:1 resonance point, the system may experience nonlinear oscillatory behavior due to order one resonance. It is noted that at these points participation factors, eigenvectors, and residues can be misleading because in such cases the eigenvectors are not unique and lose their physical meaning.

III. Static vs Dynamic Voltage stability Analysis

In this section, numerical simulations are conducted to see the effect of modeling on voltage stability. First, the conventional static voltage stability analysis is performed using CPF. Comparisons are made to see the effect of modeling by increasing the complexity of dy-

namic model.

Case 1 Continuation power-flow [11] including generator current limit [52] (to simulate steady-state operation)

Case 2 Classical machine model and networks [44]

Case 3 Two-axis machine model with exciter and networks [44]

Case 4 Same as case 3 plus generic load model [53]. Typical load parameters are $\alpha=1.50$, $\beta = 3.70$, $a = b = 0$ and time constant is 25 seconds.

Numerical investigations are conducted in two different test systems; the 3-machine system and the New England 39-bus system. These two test systems are well known in voltage stability analysis.

Constant power load is assumed in CPF. System loading is increased as follows:

$$P_{L_i}(\lambda) = P_{L_{i0}} + \lambda [K_{L_i} S_{\Delta_{BASE}} \cos(\psi_i)] \quad (7)$$

$$Q_{L_i}(\lambda) = Q_{L_{i0}} + \lambda [K_{L_i} S_{\Delta_{BASE}} \sin(\psi_i)] \quad (8)$$

where

$P_{L_{i0}}, Q_{L_{i0}}$ = original load at bus i , active and reactive, respectively;

K_{L_i} = multiplier designating the rate of load change at bus i as λ changes;

ψ_i = power factor angle of load change at bus i ; and

$S_{\Delta_{BASE}}$ = apparent power, which is chosen to provide appropriate scaling of λ .

The load increment is shared by the generator as follows:

$$P_{G_i}(\lambda) = P_{G_{i0}}(1 + \lambda K_{G_i}) \quad (9)$$

where

$P_{G_{i0}}$ = active generation at bus i in the base case and

K_{G_i} = constant specifying the rate of change in generation as λ varies.

Once K_{L_i} , K_{G_i} , and ψ_i are fixed, these values are maintained while CPF traces the entire PV curve. In all the case studies, the generator reactive power limit is determined by setting the maximum armature current and field current for 200% of the base case.

Numerical Investigation Procedure

The numerical investigations contain the following steps.

Step 1 Solve the power-flow equations using continuation power-flow program.

Step 2 Calculate the equilibrium points for the state variables.

Step 3 Check the SI bifurcation condition.

Step 4 Build the system matrix.

Step 5 Calculate the eigenvalues and corresponding right, and left eigenvectors.

Step 6 Check conditions for SN and H bifurcations, and for the NF and 1:1 resonance points. If SN or H bifurcation is determined, calculate participation factors [54] corresponding to critical eigenvalues.

Step 7 Go to step 1 until tracing the entire PV or QV curves.

3-machine, 9-bus System Results

In this section, the 3-machine, 9-bus system [55] shown in Figure 3, is tested. This system consists of three generator buses (1,2,3), three load buses (4,5,7), and three connecting buses (6,8,9). The same system was tested in [20, 37]. The load is increased in steps at all PQ buses (buses 4, 5, and 7) that have initial load. The load increment is supported by generators at buses 1, 2, and 3. The stability results are shown in Figures 4 – 7, in which the vertical axis is the voltage at bus 5 and the horizontal axis the real power at the same bus.

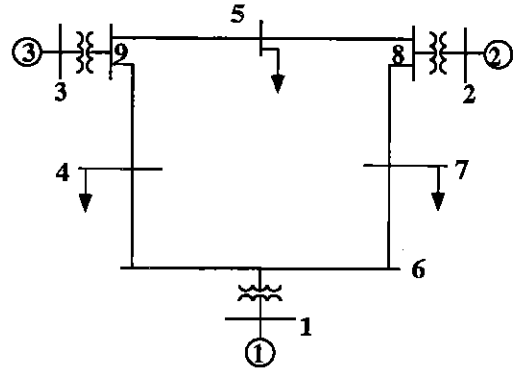


Figure 3: 3-machine, 9-bus system

Case 1 This case corresponds to a steady-state analysis. Dynamics corresponding to all the components

are neglected. No eigenvalues are involved. The load is assumed to be a constant power load. The continuation power flow program calculated the maximum power transfer point at a total load increment (ΔP_t) of 2.44 pu (P_5 of 1.01 pu). At this point generators 2 and 3 hit the reactive power limit. In Figure 4 the nose point of the PV curve is the maximum power transfer point at which J_{LF} becomes singular. The power-flow based voltage stability study assumes this maximum power transfer point as the instability point.

Case 2 The stability of generator swing dynamics is investigated. Along the steady-state trajectory obtained in the case 1, corresponding initial points are calculated. Eigenvalues calculated at the initial point evaluate the small-disturbance stability of swing dynamics near the initial point. At the starting point (base load, ΔP_t of 0 pu), all the eigenvalues are complex except for an eigenvalue corresponding to reference. When the load is gradually increased, a complex pair of eigenvalues split into two real eigenvalues through the NF point around ΔP_t of 2.28 pu (P_5 of 0.97 pu). One of the splitting real eigenvalues becomes positive immediately through SN bifurcation point. Further variation of load along the bottom of the PV curve leads to a SI bifurcation around ΔP_t of 2.08 pu (P_5 of 0.92 pu). The unstable region existing between the SN and SI bifurcations is shown in Figure 5. This instability is angle instability.

Case 3: Flux decay and excitation system dynamics are added to case 2. At ΔP_t of 2.44 pu (P_5 of 1.01 pu), the real part of a pair of complex eigenvalues becomes positive through a Hopf bifurcation. This mode is dominated by the exciter dynamics of Gen. 1. With further variation of load, the swing dynamics of Gen. 2 begin to share this mode with exciter dynamics. At ΔP_t of 2.33 pu (P_5 of 0.98 pu), these eigenvalues split through a NF point into two real eigenvalues, which move in the opposite directions along the real axis. The eigenvalue moving to the right side is dominated by the swing dynamics of Gen. 2 and the one moving to the left-hand side is dominated by exciter dynamics of Gen. 1. The eigenvalue moving to the right-hand side becomes very large around SI at ΔP_t 2.15 pu (P_5 of 0.94 pu), and finally comes back to the negative side through SI. The eigenvalue moving to the left-hand side becomes negative through a SN bifurcation around ΔP_t 1.05 pu (P_5 of 0.66 pu).

The system is unstable in the region between the Hopf bifurcation point ΔP_t of 2.44 pu (P_5 of 1.01 pu) and the SN bifurcation point ΔP_t of 1.05 pu (P_5 of 0.66 pu) as shown in Figure 6. This case was already simulated in [20]. The movement of eigenvalues observed in

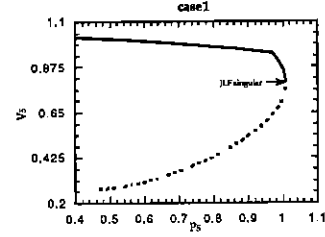


Figure 4: PV curve including stability result of case 1, 3-machine system

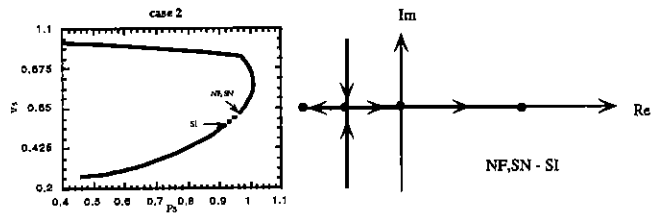


Figure 5: PV curve including stability result of case 2, 3-machine system

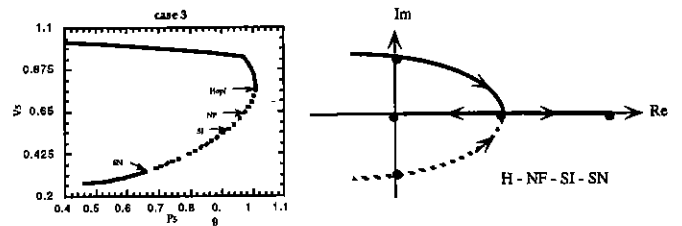


Figure 6: PV curve including stability result of case 3, 3-machine system

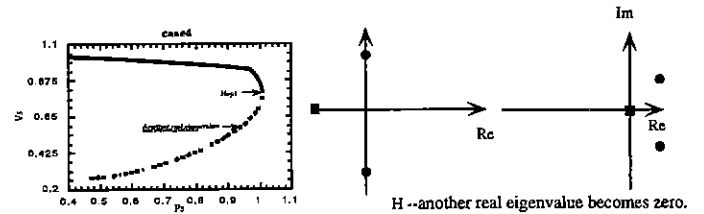


Figure 7: PV curve including stability result of case 4, 3-machine system

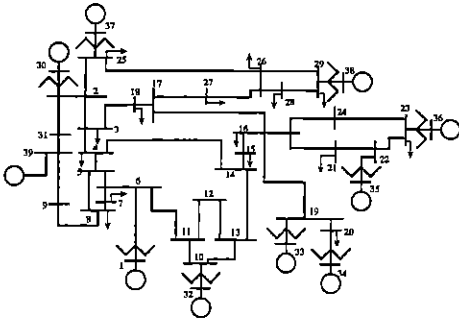


Figure 8: New England 10-machine, 39-bus system

this paper is same as shown in [20]. However, here the unstable region includes the bottom half of PV curve, whereas in [20] the unstable region exists in the upper region of PV curve. The difference may come from using different generator reactive power limit and load increase scenario. In [20], generator current limit was not considered and load was increased at a particular load bus, whereas this study includes a generator current limit and increasing loads at all load buses. The instabilities observed in this case study correspond to a control loop of the excitation system.

Case 4: Load dynamics are added to case 3. The generic load model [53] is included at all load buses (buses 4, 5, and 7) in addition to the dynamic components considered in case 3. The real part of a pair of complex conjugate eigenvalues become positive through the Hopf bifurcation point at ΔP_t of 2.44 pu (P_5 of 1.01 pu). This mode is dominated by exciter dynamics of Gen. 1. As the load varies further, a real eigenvalue approaches the origin and this real eigenvalue becomes unstable at ΔP_t of 2.15 pu (P_5 of 0.94 pu) in the bottom of PV curve. This mode is dominated by the voltage angle at bus 5 and the voltage magnitude at bus 7. These two unstable modes remain in the right half-plane until the entire curve is traced. The stability result is shown in Figure 7.

New England 10-machine, 39-bus System Results

In this section, the New England 10-machine, 39-bus system is tested. The system shown in Figure 8 consists of 10 generators (1,30, and 32–38), 15 load buses (3,4,7,8,15,16,18,20,21,23, and 25–29), and the rest connecting buses. The load is increased in steps at all load buses that have initial load. The load increment is supported by generators at all generator buses. The stability results are shown in Figures 9–12, in which the vertical axis is the voltage at bus 29 and the horizontal axis the real power at the same bus.

Results for cases 1, 2 and 3 are similar to that of 3-machine system. In case 4, inclusion of generic load

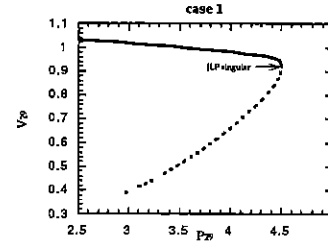


Figure 9: PV curve including stability result of case 1, 10-machine system

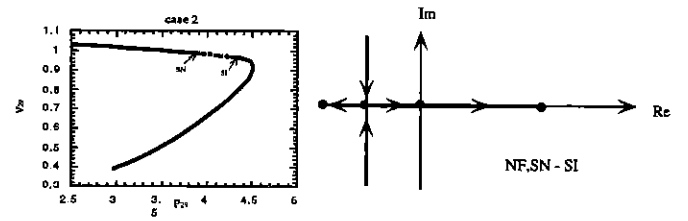


Figure 10: PV curve including stability result of case 2, 10-machine system

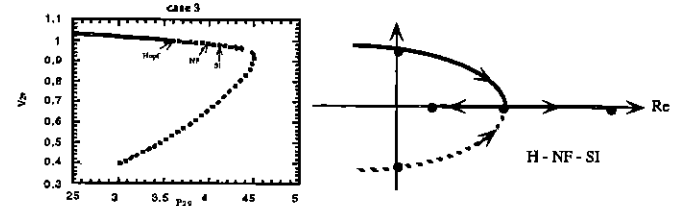


Figure 11: PV curve including stability result of case 3, 10-machine system

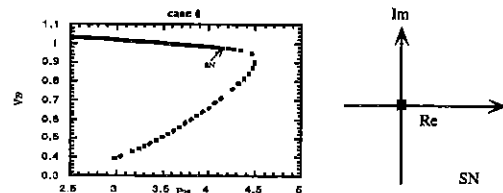


Figure 12: PV curve including stability result of case 4, 10-machine system

model replaces the bifurcations observed in case 3 with SN bifurcation.

Observations From Numerical Simulations

The following general observations can be made from the two test cases.

Determining the stability based on the J_{LF} singularity is too optimistic. In the case of constant power loads, the PV or QV curve is interpreted as a bifurcation diagram because P or Q becomes an independent system parameter. When the maximum power transfer point is approached, two equilibria coalesce and disappear as P or Q is further increased. In all the above cases, singularity of J_{LF} occurs at the nose of the PV curve. In steady-state analysis, the maximum power transfer point is assumed to be a saddle-node bifurcation point. In case 4 of the New England system, a saddle-node bifurcation occurs in the upper branch of the PV curve and is not coinciding with J_{LF} singularity. This SN bifurcation is an actual instability point. In general, the J_{LF} singular point does not coincide with one of the critical points that determine stability.

The results from the 10-machine system certainly indicate that the maximum power transfer limited by stability depends on the model considered. Table 1 shows the maximum power transfer determined by the J_{LF} singularity (case 1) is optimistic.

Table 1: Stability limited power transfer, 10-machine system

	10 machine			
	case 1	case 2	case 3	case 4
$\Delta P_{t, max}$	38.76	29.56	24.33	33.94

Including the reactive power limit associated with generator current limits did change the stability results. For the 3-machine system, the generator current limit was not included in [20, 37]. The resulting movement of the critical eigenvalues in case 3 of 3-machine system appears similar, in which the unstable region appears on the upper part of PV curve. The results of simulating New England system with different current limits are shown in Figure 13. It is revealed that the total amount of load increase is reduced with decreased reactive power limit. However, although the movement of the critical eigenvalues does not seem to change qualitatively, the unstable region moves to the lower part of PV curve. This means that critical points appear at low voltage.

In the case studies, it appears that the Hopf bifurca-

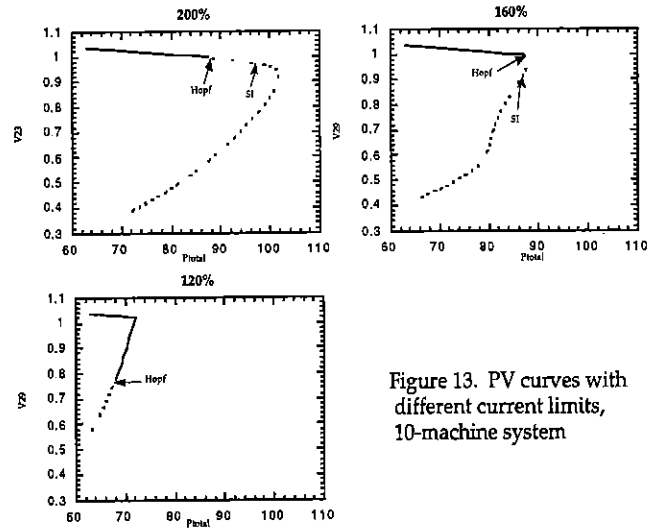


Figure 13. PV curves with different current limits, 10-machine system

tion depends on the amount of reactive power supplied by the generators and SI bifurcation on the amount of network transactions. This implies that the Hopf bifurcation may correspond to the limit of the excitation system and SI bifurcation to the limit of network structure.

Load dynamics play an important role as predicted above. In cases 1, 2, and 3, the movements of critical eigenvalues have a similar pattern for the 3-machine and 10-machine systems considered. In cases 1, 2, and 3, the entire network is modeled by a set of algebraic equations. The network in case 4 is modeled by DAEs. Including dynamics in the network model eventually alters the structure of the algebraic part of the DAE formulation. This may be the reason that case 4 is quite different from other cases.

Participation factors [54] identify the critical states associated with system instabilities. In most cases, except for the NF point or the 1:1 resonance point, participation factors are useful for finding the critical states.

So far, analysis is mainly focussed on the classification of system instabilities and dynamic behavior near or after bifurcation occurs. Further insights into the voltage problems can be obtained by properly identifying the areas, components, and parameters that are responsible for particular types of system behavior.

Invariant subspace parametric sensitivity (ISPS) derived in the following section identifies the areas, components, and parameters that are participating in voltage instabilities.

IV. Parametric Sensitivity Of Structure-Preserving Power System Model

In power system analysis, it is not enough merely to obtain the stability information. It is also important to know the areas, components, and parameters that contribute to particular instability.

In this section, a systematic sensitivity analysis method is developed that satisfies the requirements mentioned in the above paragraph. An explicit derivation of parametric sensitivity of state variables is obtained in terms of partial derivatives of known quantities. This takes into account both static and dynamic parts. Further insights into the parametric sensitivity can be obtained by properly projecting this sensitivity onto subspaces corresponding to each eigenvalue. The projected sensitivities are called invariant subspace parametric sensitivities (ISPSs). ISPS indirectly gives the relation between the eigenvalue and the parameters under consideration.

Parametric Sensitivity

Near a given solution $(X(P_o), Y(P_o))$ of a structure-preserving power system model (Eqs. 1 and 2), the derivatives $\partial X/\partial P$ and $\partial Y/\partial P$ of (X, Y) at P_o give a natural measure of the sensitivity of the solution $(X(P_o), Y(P_o))$. Here, P is a parameter vector which include all the parameters that explicitly appear in the static and dynamic parts.

At an equilibrium point $(X(P_o), Y(P_o))$, if we take partial differentiations of Eqs. 1 and 2 with respect to P and evaluate matrices at this equilibrium point, then

$$0 = \frac{\partial F}{\partial X} \frac{\partial X}{\partial P} + \frac{\partial F}{\partial Y} \frac{\partial Y}{\partial P} + \frac{\partial F}{\partial P} \quad (10)$$

$$0 = \frac{\partial G}{\partial X} \frac{\partial X}{\partial P} + \frac{\partial G}{\partial Y} \frac{\partial Y}{\partial P} + \frac{\partial G}{\partial P} \quad (11)$$

From Eq. 11

$$\frac{\partial Y}{\partial P} = - \left[\frac{\partial G}{\partial Y} \right]^{-1} \left[\frac{\partial G}{\partial X} \frac{\partial X}{\partial P} + \frac{\partial G}{\partial P} \right] \quad (12)$$

Substitute Eq. 12 in Eq. 10

$$\begin{aligned} 0 &= \frac{\partial F}{\partial X} \frac{\partial X}{\partial P} - \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \left[\frac{\partial G}{\partial X} \frac{\partial X}{\partial P} + \frac{\partial G}{\partial P} \right] + \frac{\partial F}{\partial P} \\ &= \left[\frac{\partial F}{\partial X} - \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial X} \right] \frac{\partial X}{\partial P} \\ &\quad - \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial P} + \frac{\partial F}{\partial P} \end{aligned}$$

After rearranging,

$$\underbrace{\left[\frac{\partial F}{\partial X} - \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial X} \right]}_A \frac{\partial X}{\partial P} = \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial P} - \frac{\partial F}{\partial P} \quad (13)$$

From Eq. 13, the parametric sensitivity of state variables are obtained as

$$\frac{\partial X}{\partial P} = A^{-1} \left[\frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial P} - \frac{\partial F}{\partial P} \right] \quad (14)$$

The parametric sensitivity of algebraic variables can be obtained by substituting $\partial X/\partial P$ from Eq. 14 in Eq. 12. The parametric sensitivity derived in this section provides the sensitivities of system components with respect to any parameters.

In the previous section, stability information is obtained by determining the critical points of structure-preserving power system model. Combining the stability and sensitivity information results in an understanding of the factors that contribute to instability. In this paper, the invariance property [56] of the eigenbasis is exploited for linking the stability and sensitivity aspects of the system. In a small-disturbance stability analysis, each eigenvalue belongs to a particular eigenbasis. If a critical eigenbasis refers to a critical eigenvalue, then according to the invariance property of the eigenbasis the system instabilities are governed by the dynamics on the critical eigenbasis. This is achieved by collapsing the entire state space onto the critical eigenbasis. In the next section, the parametric sensitivity is projected onto the subspace (which is the span of the eigenbasis) corresponding to each eigenvalue. The link between the eigenvalues and the parameters is achieved through this projected parametric sensitivity (or ISPS).

Background Material

It is well known that if A has all distinct eigenvalues λ_i ($i = 1, \dots, m$), then it will also have m corresponding linearly independent $m \times 1$ eigenvectors u_i ($i = 1, \dots, m$) which are related to the equations

$$A u_i = \lambda_i u_i, \quad (i = 1, \dots, m) \quad (15)$$

The set of vectors $\{u_1, \dots, u_m\}$ accordingly constitutes a basis for a m -dimensional vector space. Here, u_i is called the right eigenvector associated with λ_i . There also exists a vector v_i^t (t denotes transpose) which satisfies

$$v_j^t A = \lambda_j v_j^t, \quad (j = 1, \dots, m) \quad (16)$$

This vector is the left eigenvector. The left eigenvector set $\{v_1^t, \dots, v_m^t\}$ also constitutes a basis for an m -dimensional vector space.

Connection Between Two Eigenvectors [57]: If Eq. 16 is postmultiplied by u_i , then

$$v_j^t A u_i = \lambda_j v_j^t u_i, \quad (i \neq j, i, j = 1, \dots, m) \quad (17)$$

Similarly, if Eq. 15 is premultiplied by v_j^t :

$$v_j^t A u_i = \lambda_i v_j^t u_i, \quad (i \neq j, i, j = 1, \dots, m) \quad (18)$$

Since left-hand sides of Eqs. 17 and 18 are identical, subtraction of these two equations yields

$$(\lambda_i - \lambda_j) v_j^t u_i = 0, \quad (i \neq j, i, j = 1, \dots, m) \quad (19)$$

which in turn results

$$v_j^t u_i = 0, \quad (i \neq j, i, j = 1, \dots, m) \quad (20)$$

This implies that u_i and v_j ($i \neq j$) are orthogonal vectors. However, in the case of eigenvectors corresponding to the same eigenvalue, it is evident that

$$v_i^t u_i = \xi_i, \quad (i = 1, \dots, m) \quad (21)$$

where ξ_i is a nonzero constant.

Reciprocal Basis: In Eq. 21, since the eigenvectors are determined only to within a scalar multiplier, it is convenient to normalize these vectors as

$$v_i^t u_i = 1 \quad (i = 1, \dots, m) \quad (22)$$

Then the set of vectors $\{v_1^t, \dots, v_m^t\}$ is termed as the reciprocal basis of $\{u_1, \dots, u_m\}$. The main use of the reciprocal basis is to get a convenient expression for the components of any vector x relative to the bases $\{u_1, \dots, u_m\}$. The vector x can be represented as a linear combination of bases, i.e.,

$$x = \sum_{i=1}^m \xi_i u_i \quad (23)$$

On the reciprocal basis it can be represented as

$$x = \sum_{i=1}^m \langle v_i^t, x \rangle u_i \quad (24)$$

where ξ_i is the inner product of the left eigenvector v_i^t and a vector x , i.e., $\xi_i = \langle v_i^t, x \rangle$.

Next, we define dyad M [58]. A dyad is a linear transformation which transforms the vector x onto the vector $\langle v_i^t, x \rangle u_i$. In other words, it maps all vectors onto the invariant subspace associated with λ_i . A dyad has a one-dimensional range which is spanned by u_i that is represented by the matrix $M_i = u_i v_i^t$. Eq. 24 is represented by the dyad expression

$$x = \sum_{i=1}^m u_i v_i^t x \quad (25)$$

$$= \sum_{i=1}^m M_i x \quad (26)$$

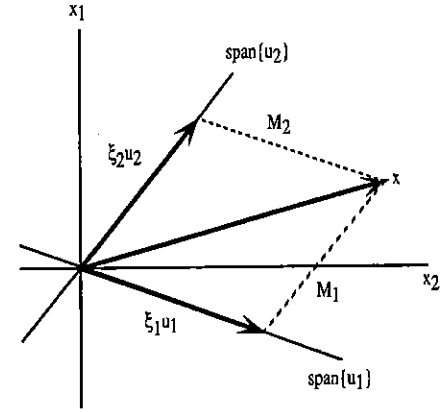


Figure 13: Projection of vector x onto invariant subspaces

The column space of M_i is the i -th right eigenvector u_i , and the row space of M_i is the i -th left eigenvector v_i^t . This implies that the null space of M_i is the total sum of the eigenspace associated with all eigenvalues of matrix A other than λ_i . For a two-dimensional linear vector field, Figure 13 shows the projection of a vector x onto each eigensubspace through the dyad M_i .

Matrix Decomposition: On the reciprocal eigenbasis, the reduced system matrix A can be written as

$$A = U \Lambda V \quad (27)$$

$$= \sum_{i=1}^m \lambda_i M_i \quad (28)$$

where

$$M_i = u_i v_i^t = \begin{bmatrix} u_{i1} v_{i1} & \dots & u_{i1} v_{im} \\ \vdots & \ddots & \vdots \\ u_{im} v_{i1} & \dots & u_{im} v_{im} \end{bmatrix}$$

$$\begin{aligned} U &= [u_1 \dots u_m] = V^{-1} \\ u_i &= (u_{i1} \ u_{i2} \ \dots \ u_{im}) \\ V &= [v_1^t \dots v_m^t]^t = U^{-1} \\ v_i &= (v_{i1} \ v_{i2} \ \dots \ v_{im}) \\ \Lambda &= \text{diag}(\lambda_1 \dots \lambda_m) \end{aligned}$$

To visualize the relation between the matrix A and a dyad M , we postmultiply Eq. 28 by a vector x as shown below.

$$Ax = \left(\sum_{i=1}^m \lambda_i M_i \right) x \quad (29)$$

For a two-dimensional system, $m = 2$ and $x = (x_1, x_2)$. Then

$$Ax = \lambda_1 M_1 x + \lambda_2 M_2 x$$

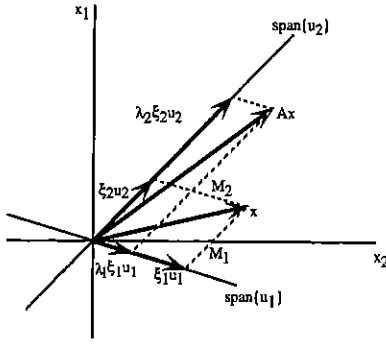


Figure 14: Relation between A and M

$$= \lambda_1 \begin{bmatrix} M_{1,1} & M_{1,2} \\ M_{2,1} & M_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \lambda_2 \begin{bmatrix} M_{2,1} & M_{2,2} \\ M_{1,1} & M_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

This relation is explained in Figure 14. The action of A on a vector x is a linear transformation. Thus the left term Ax is a new vector transformed through A . The meaning of the right term is as follows: if we project a vector x onto each eigensubspace, then it creates new vectors $\xi_i u_i$ ($i = 1, 2$). Multiplying $\xi_i u_i$ by the eigenvalue corresponding to each eigensubspace results in $\lambda_i \xi_i u_i$ ($i = 1, 2$). The sum of all these vectors is Ax .

The inverse of a matrix A can also be decomposed in the same way:

$$\begin{aligned} A^{-1} &= [U\Lambda V]^{-1} \\ &= V^{-1}\Lambda^{-1}U^{-1} \\ &= U\Lambda^{-1}V \\ &= \sum_{i=1}^m \frac{M_i}{\lambda_i} \end{aligned} \quad (30)$$

Equation 30 is visualized in Figure 15. The same discussion in Figure 14 is applicable to Figure 15 except that $\xi_i u_i$ is now divided by λ_i .

Invariant Subspace Parametric Sensitivity (ISPS)

In this section, the matrix decomposition technique is applied to define the ISPS.

Substitution of Eq. 30 in Eq. 14 results in

$$\begin{aligned} \frac{\partial X}{\partial P} &= \left[\sum_{i=1}^m \frac{M_i}{\lambda_i} \right] \left[\frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial P} - \frac{\partial F}{\partial P} \right] \\ &= \left[\sum_{i=1}^m \frac{M_i}{\lambda_i} \right] L \end{aligned} \quad (31)$$

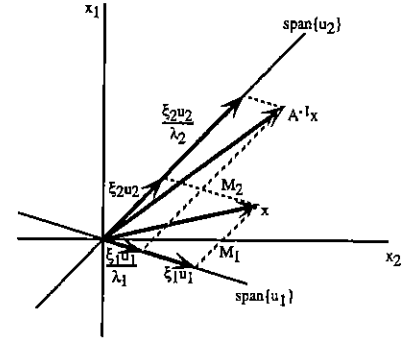


Figure 15: Relation between A^{-1} and M

where

$$L = \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial P} - \frac{\partial F}{\partial P} \quad (32)$$

L contains the sensitivities of functions F and G with respect to the parameters P . The matrix A^{-1} operating on L results in parametric sensitivity.

For a multiple parameter vector $P = (p_j, j = 1, \dots, n)$, L becomes a matrix with $l_1, l_2, \dots, l_n$ as column vectors.

Premultiply both sides of Eq. 31 with M_i

$$\begin{aligned} M_i \frac{\partial X}{\partial P} &= M_i \left(\sum_{i=1}^m \frac{M_i}{\lambda_i} \right) L \\ &= \frac{M_i}{\lambda_i} L \\ &= \frac{1}{\lambda_i} \begin{bmatrix} u_{i1} v_{i1} & \cdots & u_{i1} v_{im} \\ \vdots & \ddots & \vdots \\ u_{im} v_{i1} & \cdots & u_{im} v_{im} \end{bmatrix} \begin{bmatrix} l_{11} & \cdots & l_{n1} \\ \vdots & \ddots & \vdots \\ l_{1m} & \cdots & l_{nm} \end{bmatrix} \end{aligned} \quad (34)$$

In the above equation (Eq. 34), the vector l_j is the column of the sensitivity matrix L . Each term $M_i l_j / \lambda_i$ represents the projection of parametric sensitivity onto i -th eigensubspace. This projected parametric sensitivity is defined as the ISPS. Then, the most sensitive parameter on a particular subspace can be obtained by taking the maximum of $M_i l_j / \lambda_i$, $j = 1, \dots, n$.

Measure of ISPS

In order to select the most sensitive parameter, we do not need to calculate the entire product of $M_i l_j / \lambda_i$, $j = 1, \dots, n$. Simply taking the maximum of the inner product between v_i^T and l_j gives the same result. The explanation is given below.

If we define the new symbol $\sigma_j = \sum_{k=1}^m v_{i,k} l_{j,k}$ and rewrite Eq. 34 using the new symbol, then

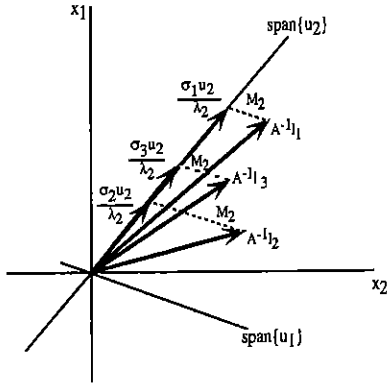


Figure 16: Projection of sensitivity vectors l_1, l_2, l_3 on the selected eigen-subspace

$$\begin{aligned} \frac{M_i L}{\lambda_i} &= \frac{1}{\lambda_i} \begin{bmatrix} u_{i1} \sigma_1 & \cdots & u_{i1} \sigma_n \\ \vdots & & \vdots \\ u_{im} \sigma_1 & \cdots & u_{im} \sigma_n \end{bmatrix} \\ &= \frac{1}{\lambda_i} \begin{bmatrix} u_{i1} \\ \vdots \\ u_{im} \end{bmatrix} \begin{bmatrix} \sigma_1 & \cdots & \sigma_n \end{bmatrix} \end{aligned} \quad (35)$$

In σ_j , the left eigenvector u_i measures the weight of contribution of sensitivity vector l_j to each λ_i^{-1} . In Eq. 35, the ISPS of a particular parameter, p_j , becomes

$$\frac{M_i l_j}{\lambda_i} = \frac{\sigma_j u_i}{\lambda_i} \quad (36)$$

Consequently,

$$\text{Max} \left\| \frac{M_i l_j}{\lambda_i} \right\|_2 = \text{Max} \left\| \frac{\sigma_j u_i}{\lambda_i} \right\|_2, \quad (j = 1, \dots, n) \quad (37)$$

In the above equation, the largest sensitive parameter is obtained from $\text{Max} \sigma_j, j = 1, \dots, n$, since u_i/λ_i is common to all the parameters. Thus, σ_j is a good measure for selecting the most influential parameters. For example, Figure 16 describes the vector l_j corresponding to parameter p_j on the selected eigensubspace in a two-dimensional vector field with three parameters. On the subspace corresponding to λ_2 , parameter p_1 is the most influential parameter in this particular example.

It should be noted that the parametric sensitivity is sum of the ISPS as represented in Eq. 38.

$$\frac{\partial X}{\partial P} = \frac{M_1 L}{\lambda_1} + \frac{M_2 L}{\lambda_2} + \cdots + \frac{M_m L}{\lambda_m} \quad (38)$$

In the close neighborhood of a saddle-node bifurcation point, i.e., if the i -th eigenvalue is almost-zero, parametric sensitivity $\partial X/\partial P$ is dominated by the almost

zero eigenvalue and is approximated by the ISPS corresponding to the i -th eigenvalue.

$$\begin{aligned} \frac{\partial X}{\partial P} &= \frac{M_1 L}{\lambda_1} + \frac{M_2 L}{\lambda_2} + \cdots + \underbrace{\frac{M_i L}{\lambda_i}}_{\infty} + \cdots + \frac{M_m L}{\lambda_m} \quad (39) \\ &\approx \frac{M_i L}{\lambda_i} \end{aligned}$$

ISPS is claimed to provide indirectly the relationship between eigenvalues and the parameters under consideration. Eigenvalue sensitivity expressions that provide the exact eigenvalue sensitivity is compared with ISPS and the result is given in the appendix.

Concluding Remarks

An explicit derivation of the parametric sensitivity of state variables is obtained in terms of partial derivatives of known quantities. Then the parametric sensitivity of algebraic variables can be derived in terms of the parametric sensitivity of the state variables.

The ISPS method developed is a byproduct of parametric sensitivity analysis. The motivation for deriving the ISPS starts from an attempt to combine the stability results in Section II and the sensitivity information obtained from the parametric sensitivity analysis. The parametric sensitivity only provides the ratio of incremental changes in the variables to changes in the parameters under consideration, i.e., sensitivity of known equilibrium due to changes in parameters. However, voltage stability is determined by the dynamic characteristics at an equilibrium point rather than the sensitivity of an equilibrium point. At a critical point, system behavior is governed by the dynamic characteristics (which are invariant) on the eigensubspace corresponding to the critical eigenvalue. Therefore, to study voltage-stability sensitivity, it is necessary to calculate the parametric sensitivity corresponding to a particular eigensubspace. As mentioned in the preceding section, this sensitivity is obtained by projecting the parametric sensitivity onto a particular subspace; This projected sensitivity is the ISPS. At a particular eigensubspace, ISPSs of all the parameters under consideration are represented by a right eigenvector multiplied by a scalar σ . Thus the parameter which has the most influence on this subspace can be determined by selecting largest value of σ .

As we mentioned before, ISPS is a byproduct of parametric sensitivity, and again as a byproduct ISPS gives indirect relation between the eigenvalues and the parameters under consideration. This relation is verified with a well known eigenvalue sensitivity. σ is also giv-

ing a cluster of eigenvalues that are most sensitive to particular parameter.

The formulation of the ISPS requires only the calculation of the desired number of eigenvalues and eigenvectors of the reduced system matrix. Since these are required to determine system stability and participation factors, no additional major calculations are involved.

In the next section, the ISPS method is applied to a New England test system.

V. Application Of The ISPS Method

In the previous section, the validity of the invariant subspace parametric sensitivity (ISPS) method for providing parametric sensitivities of eigenvalues in the structure-preserving model is given. In this section, application of the ISPS method to the IEEE New England 39-bus system is described. The CPF traces the entire PV curve to provide steady state equilibrium points that are needed to linearize the DAEs. The critical eigenvalue is traced along the entire PV curve. At each continuation step, the ISPS corresponding to the critical eigenvalue is calculated. From these results, sources of voltage instability, the parameters that mostly influence the voltage instability, are identified.

ISPS at Different Loading Conditions

This section concentrates on the general trend of the ISPS of various parameters for a gradual change in load. Case 3 of New England 39-bus system from Section II is used as a test scenario. We have total 45 parameters. The parameters are $K_{A_i}, K_{E_i}, K_{F_i}$ ($i = 1, \dots, 10$), which correspond to exciters of 10 generators, and K_{L_i} ($i = 1, \dots, 15$), which correspond to the load.

In this scenario, a particular eigenvalue is responsible for both Hopf and SI bifurcations. The ISPS corresponding to this particular eigenvalue is traced through the entire PV curve. Steps involved in calculating the ISPS curves for each parameter are given below.

Step 1 Find the equilibrium point at each load level.

Step 2 Calculate eigenvector corresponding to this particular eigenvalue.

Step 3.a Calculate the sensitivity vector for a particular parameter p_j .

$$l_j = \frac{\partial F}{\partial Y} \left[\frac{\partial G}{\partial Y} \right]^{-1} \frac{\partial G}{\partial p_j} - \frac{\partial F}{\partial p_j}$$

Step 3.b Calculate the ISPS measure.

$$\sigma_j = \sum_{k=1}^m v_{jk} l_{jk} \quad (40)$$

Go to step 3.a until all the parameters are considered.

Step 4 Repeat steps 1, 2, and 3.

Figures 17–20 show the ISPS of each parameter in the entire PV curve. The vertical axis is σ (the measure of the ISPS) $\times 10^{-2}$. The horizontal axis is the number of continuation steps representing the different system loading conditions.

Identification of The Generators, Loads, and Branches

In the previous section, the general trend of the ISPSs of all the parameters under consideration is described. Observation of the trend results in the selection of σ corresponding to two parameter groups K_F and K_L for detailed analysis. In this section, the ISPS data obtained in the preceding section are investigated at the Hopf bifurcation and at the SI bifurcation. The results provide the exciter and load that are contributing to these points.

This section also includes calculation of the ISPSs of branches in addition to exciter and load sensitivities. The ISPSs of branches are obtained as follows;

Let us consider a branch, ij . Let $V_i \angle \delta_i$ and $V_j \angle \delta_j$ be the voltages at buses i and j , respectively, and let $y_{ij} \angle \theta_{ij}$ be the line admittance. Then the losses in the line ij , neglecting the shunt charging capacitance, can be derived as (see ref[40]):

$$\begin{aligned} S_{loss} &= P_{loss} + jQ_{loss} \\ &= [V_i^2 + V_j^2 \\ &\quad - 2V_i V_j \cos(\delta_i - \delta_j)] y_{ij} \angle -\theta_{ij} \end{aligned} \quad (41)$$

The sensitivity of losses are expressed as follows:

$$\begin{aligned} \frac{dS_{loss}}{dP} &= [(2V_i - 2V_j \cos(\delta_i - \delta_j)) \frac{dV_i}{dP} \\ &\quad + (2V_j - 2V_i \cos(\delta_i - \delta_j)) \frac{dV_j}{dP} \\ &\quad + (2V_i V_j \sin(\delta_i - \delta_j)) \frac{d\delta_i}{dP} \\ &\quad - (2V_i V_j \sin(\delta_i - \delta_j)) \frac{d\delta_j}{dP}] y_{ij} \angle -\theta_{ij} \end{aligned} \quad (42)$$

The sensitivities of voltage magnitudes and angles are obtained from Eq. 12 by substituting $\partial x / \partial p$ with the

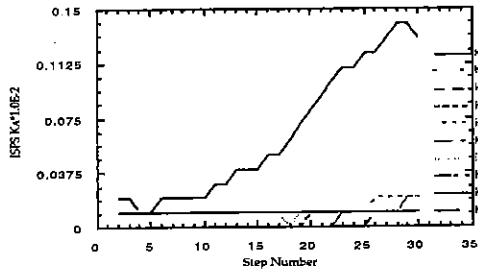


Figure 17: ISPS of K_A

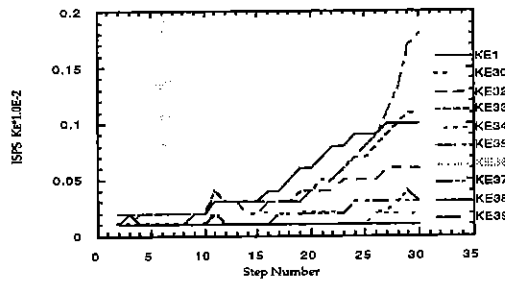


Figure 18: ISPS of K_E

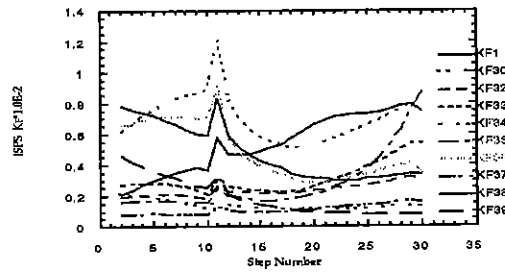


Figure 19: ISPS of K_F

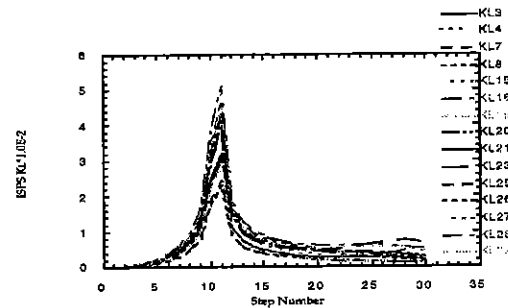


Figure 20: ISPS of K_L

corresponding ISPS. The real part of Eq. 42 gives dP_{loss}/dP and the imaginary part gives dQ_{loss}/dP . Substituting the ISPSs of voltage magnitudes and voltage angles in Eq. 42, one can obtain the corresponding ISPS of loss in each branch.

At The Hopf Bifurcation Point: At ΔP_t 24.33 pu (step number 8), a pair of complex conjugate eigenvalues becomes $0.303 \pm 5.379j$, which is already past the Hopf bifurcation point. As explained in the previous section, the magnitude of the ISPSs of K_F and K_L corresponding to the Hopf bifurcation point at step number 8 are represented in a bar graph of Figure 21.

The first three sensitive exciters and loads are selected:

Exciters: $K_{F_{34}}$ (0.86), $K_{F_{36}}$ (0.71), $K_{F_{38}}$ (0.68)

Loads: $K_{L_{16}}$ (0.97), $K_{L_{18}}$ (0.86), K_{L_3} (0.83)

The other load parameters are also significantly participated at this point. The first three sensitive branches are

Branches: from 29 to 38, from 6 to 7, from 20 to 34

The participation factors corresponding to the critical eigenvalues are calculated, and the three dominant states in this mode are

Part. factors: $E'_{q_{36}}$ (0.10), $E'_{q_{38}}$ (0.09), $E_{fd_{38}}$ (0.08)

The dominant states selected by the participation factors are variables of exciters selected by ISPS method.

In Figure 22, the one-line diagram of the 39-bus system shows the location of the critical generators, critical loads, and critical branches identified above.

At The SI Bifurcation Point: At ΔP_t 33.94 pu (step number 12), one of the real eigenvalues returns to the left half-plane through the SI bifurcation point. The magnitude of the ISPSs of K_F and K_L corresponding to the SI point at step number 12 are represented in a bar graph in Figure 23.

The first three sensitive exciters and loads are selected:

Exciters: $K_{F_{34}}$ (0.82), $K_{F_{38}}$ (0.59), $K_{F_{36}}$ (0.58)

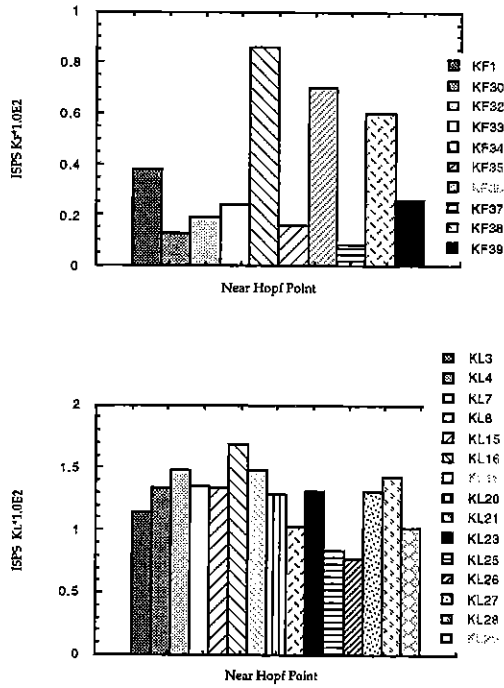


Figure 21: ISPS of K_F and K_L near Hopf point

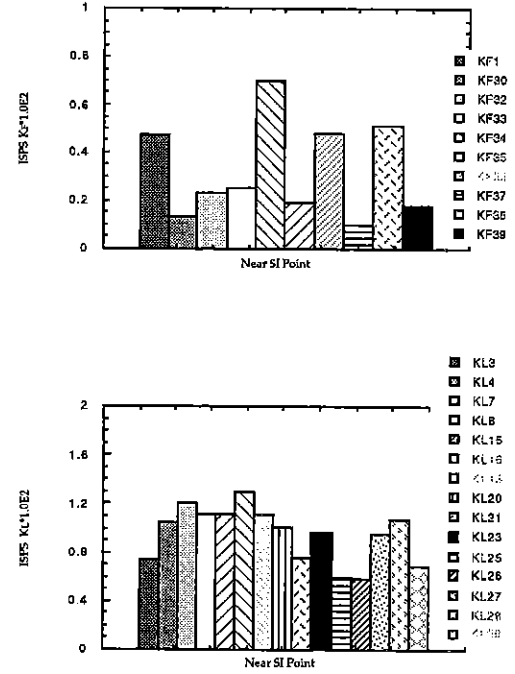


Figure 23: ISPS of K_F and K_L near SI point

Loads: $K_{L_{16}}$ (1.97), K_{L_7} (1.74), K_{L_8} (1.61)

The first three sensitive branches are

Branches: from 5 to 6, from 6 to 7, from 6 to 1

The participation factors corresponding to the critical eigenvalue are calculated, and the three dominant states in this mode are

Part. factors: $E'_{q_{36}}$ (0.09), $E'_{q_{38}}$ (0.09), $E'_{q_{34}}$ (0.09)

The dominant states selected by the participation factors are variables of exciters selected by the ISPS method.

In Figure 24, the one-line diagram of 39-bus system shows the location of the critical generators, critical loads, and critical branches identified above.

Concluding Remarks

One of the main goals of voltage stability analysis for power system operation is the identification of the sources of voltage instability and of the parameters that

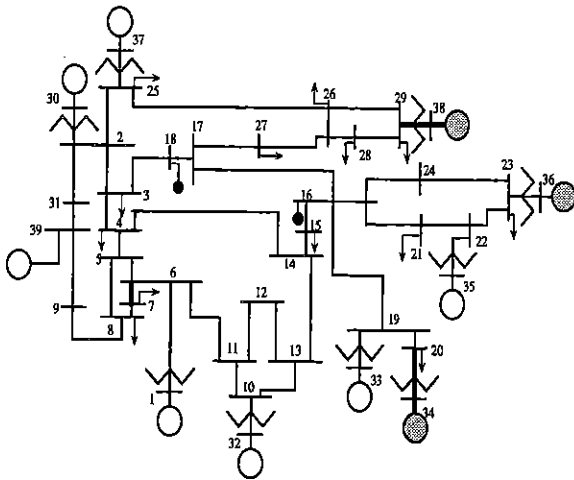


Figure 22: System configuration near Hopf bifurcation point

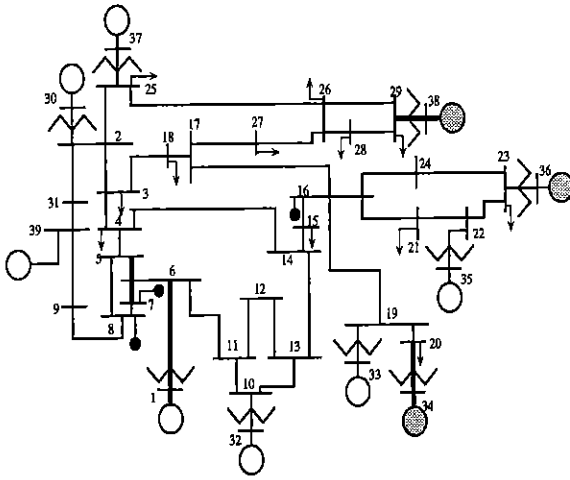


Figure 24: System configuration near SI bifurcation point

heavily influence the voltage instability. However, different modeling assumptions have often made it difficult to effectively compare various aspects of the voltage problems. In this section, results of applying the ISPS method to voltage stability analysis show that the ISPS method successfully identifies the sources of instability by determining which parameters mostly influence the instability.

In the current literature, general voltage-stability sensitivity is studied exclusively either by eigenvalue sensitivity or by parametric sensitivity. For eigenvalue sensitivity, the power system is modeled either by a set of differential equations or by a set of DAEs. In the case of the DAE form, the sensitivity analysis is done only for the parameters that correspond to dynamic aspects of the system. Parametric sensitivity analysis is done exclusively for the power-flow type of formulation. In the power-flow type of formulation, the voltage instability is associated with an operating point at which the ratio of incremental change in bus voltages to changes in reactive power demand approaches infinity. This point corresponds to the singularity of the power-flow Jacobian (J_{LF}). In the DAE formulation, infinite sensitivity of variables due to changes in parameters occurs at a saddle-node bifurcation point.

However, the saddle-node bifurcation point, in general, does not coincide with the singularity of J_{LF} . A saddle-node bifurcation that corresponds to a DAE formulation is an actual critical point, whereas the voltage instability determined in a power-flow type formulation is a maximum power transfer point. The parametric

sensitivity or ISPS derived in this section enables us to study voltage-stability sensitivity at the saddle-node bifurcation point for the parameters corresponding to both the static and the dynamic parts of the system. In Eq. 39, parametric sensitivity is represented by the sum of ISPSs. Near a saddle-node bifurcation, the ISPS corresponding to critical eigenvalue approaches infinity. So the contribution to parametric sensitivity by ISPSs corresponding to other eigenvalues can be neglected. Consequently, total parametric sensitivity is equivalent to the ISPS corresponding to the critical eigenvalue. It should be noted that the total parametric sensitivity can not select the most influential parameter at a saddle-node bifurcation point, whereas the ISPS method does.

At other critical points (Hopf and singularity-induced bifurcation points) the parametric sensitivity is not useful for studying voltage-stability sensitivity. As discussed in the previous section, voltage-stability sensitivity should be calculated on the eigensubspace corresponding to the critical eigenvalue. At these points, ISPS and total parametric sensitivity are completely different.

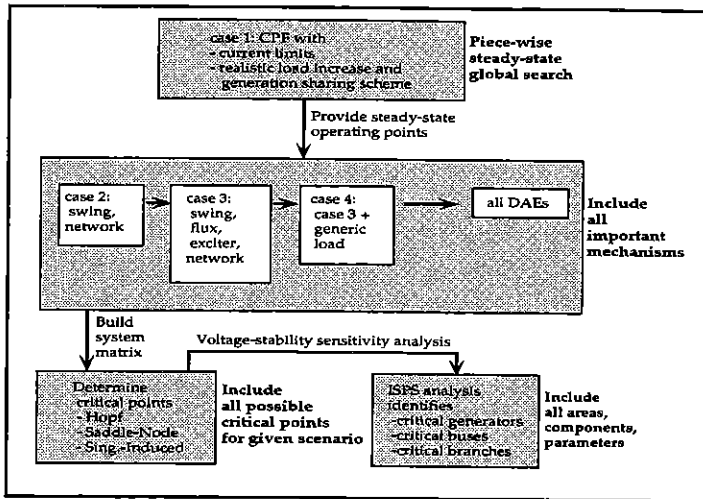
The main purpose of studying voltage-stability sensitivity is to determine which areas, components, and parameters contribute to particular instability. In the existing literature, methods that simultaneously provide the above information have not been found. For example, the eigenvalue sensitivity or parametric sensitivity alone can not provide the branch sensitivity. However, ISPS successfully identifies the most sensitive branches that are contributing to a particular instability.

In conclusion, the ISPS method is useful for studying voltage-stability sensitivity. It determines the components and the parameters that are participating in a particular instability. Consequently, the areas involving these components and parameters can be determined. The flexibility of the ISPS method gives us the freedom to perform various sensitivity analyses at all critical points.

VI. General Conclusions

In this paper, a unified framework based on small-disturbance analysis is developed. It is unified because it addresses all the relevant issues related to general small-disturbance voltage stability. Any previous work related to small-disturbance voltage stability becomes a subset of this framework. As summarized in Figure 25, salient features of this framework are as follows:

- Includes relevant dynamic and static models in the



Bifurcation Flow

Figure 25: Summary of the method

structure-preserving formulation. The model integrates all important mechanisms, including the rotor, flux, excitation system, and load dynamics with generator field current and armature current limits. A realistic load increase and a generator sharing scheme with relevant limits (in particular, it replaces the constant generator reactive power limit with generator field and armature current limits) are also included. The DAE formulation is developed in such a way that any other model that is not included here can be accommodated.

- Utilizes the continuation power flow in conjunction with the DAE formulation. If we call this combination bifurcation flow, then for a given load increase and generation sharing scenario, the method systematically traces and identifies all possible critical points where the system experiences qualitative changes in behavior. This is like a piece-wise global analysis.
- Conducts a systematic sensitivity analysis near all the critical points identified by the bifurcation flow. These sensitivities are divided into two types, parametric sensitivity and ISPS. Parametric sensitivity gives the gradient type of information, whereas the ISPS gives the parameters that are contributing to a particular subspace. The ISPS can determine the parameters and components participating in a particular instability.

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PROBABILISTIC APPROACH TO STEADY STATE VOLTAGE STABILITY OF BULK POWER SYSTEMS

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Abstract

In this paper a probabilistic approach to finding the weakest buses is proposed. From the point of view of planning it is very important to model the active and reactive loads as random variables. Pessimistically active and reactive bus loads may be treated as uniformly distributed. This aspect may be taken into account in voltage instability analysis. Some voltage stability indexes called as s-factors are estimated for all load buses. Considerations are illustrated using Ward-Hale power system.

Introduction

There is extensive literature on voltage stability. Some authors prefer the dynamic approach to the voltage stability and many other the steady state one [1,2]. Voltage stability is indeed a dynamic phenomenon and may be studied using the set of differential and algebraic equations. The eigenvalue analysis of dynamic state matrix is used to determine the power system state as stable or unstable. However the approach based only on steady state analysis is more attractive because it is quick and usually requires only small modifications of a standard load flow program.

Effective voltage instability evaluation methods are very important for transmission planners who are working with a wide variety of transfer and loading conditions. There is a need for methods that can provide information on the location and severity of var deficiency causing the voltage instability conditions.

In this paper a probabilistic approach to finding the weakest buses by means of s-factors is proposed. From the point of view of planning it is very important to model active and reactive loads as random variables. Bus loads are really not known precisely by power system planners!

Depending on the statistical and theoretical analysis the active and reactive bus loads may be pessimistically treated as uniformly distributed [3]. This aspect may be used for voltage instability analysis.

Four s-factors are proposed as the statistical indexes for voltage collapse: partial derivatives $s_1 = \partial V / \partial P$ and $s_2 = \partial V / \partial Q$, bus unbalance $s_3 = dP$ and $s_4 = dQ$. Using bus unbalances the probability of voltage collapse at each load bus may be estimated.

The probabilistic voltage instability analysis is divided into two parts:

1. Simulation: a) simulation of random active and reactive bus loads as independent random variables, b) solution of load flow equations for random loads using non-divergent load flow algorithm, c) evaluation of s-factors at each bus.
2. Statistical analysis: a) calculation of mean value vector and covariance matrix of s - factors, b) calculation of probability of voltage collapse at each load bus.

Power system model

The detailed power system dynamic and static model may be prepared under assumption that the generator at slack bus has infinite inertia. It enables to get a simple transition from the dynamic to static approach. General form of power system model is as follows:

$$\begin{aligned} \dot{x} &= g(x, z, v) & - \text{differential equations} \\ y &= f(x, z, v) & - \text{algebraic equations} \end{aligned} \quad (1)$$

where

- x - the state variables,
- z - the generator internal voltage vector,
- v - the network voltage vector,
- y - the vector of the power balance of bus injection and receiving.

For good conditioned power systems we have the zero bus unbalance vector, i.e. $y=0$.

Dynamic Voltage Stability

The dynamic stability of multiple machine systems is usually evaluated by computing the eigenvalues of the system dynamic state matrix. This matrix is obtained by linearization with all algebraic equations eliminated. All equations (1) are linearized around a point found using standard load flow with $y=0$.

From the linearization we obtain the set of matrix equations

$$A_{xx} \Delta x + A_{xz} \Delta z + A_{xv} \Delta v = \Delta \dot{x} \quad (2a)$$

$$A_{zx} \Delta x + A_{zz} \Delta z + A_{zv} \Delta v = 0 \quad (2b)$$

$$A_{vx} \Delta x + A_{vz} \Delta z + J_{LF} \Delta v = \Delta y \quad (2c)$$

From the dynamic analysis point of view we may assume that there are no bus unbalances $\Delta y = 0$ and load flow equations have solution.

The Jacobian J_{LF} is related to the load buses. The matrix A_{zz} is quasi diagonal and always non-singular, hence after elimination of Δz and Δv we obtain the following linear matrix equation

$$(A' - A_{xv}' J_{LF}'^{-1} A_{vx}') \Delta x = \Delta \dot{x} \quad (3)$$

where

$$A_{xx}' = A_{xx} - A_{xz} A_{zz}^{-1} A_{zx}$$

$$A_{xv}' = A_{xv} - A_{xz} A_{zz}^{-1} A_{zv}$$

$$A_{vx}' = A_{vx} - A_{vz} A_{zz}^{-1} A_{zx}$$

$$J_{LF}' = J_{LF} - A_{vz} A_{zz}^{-1} A_{zv}$$

Hence, the system matrix is following

$$A = A_{xx}' - A_{xv}' J_{LF}'^{-1} A_{vx}' \quad (4)$$

The dynamic instability is associated with the crossing of a real part of eigenvalue from the negative value to the positive value. It is clearly seen from formula (4) that eigenvalues must be dependent on load flow solution because the state matrix A is dependent on the inversion of the load flow Jacobian $J_{LF}'^{-1}$ related to load buses.

Steady State Voltage Stability

The proper model of the voltage stability should use the dynamics model for every power system component, but the approach based on steady state analysis is more attractive. The question to ask is whether the steady state equations resulting from the dynamic model detect voltage instability at the same points as the linearized dynamic equations.

It is considered that a power system may be treated as voltage stable at a given operating condition if for every load bus, bus voltage increases as reactive power injection at this same bus is increased [4]. A power system is voltage unstable, if for at least one bus in the system, bus voltage magnitude decreases as the reactive power injection at the same bus is increased.

This happens usually when the operating point has been found as the second close multiple load flow solution instead of the first one or when the analysed power system is very close to the tip of Q-V nose curve. In both cases it means that the analysed power system works in the vicinity of voltage collapse point. As a consequence of this fact we may say that voltage stability is connected with the sign of dV/dQ derivative at each bus. If the dV/dQ has the negative sign at least at one bus we may treat an analysed power system as voltage unstable [1,2].

For steady state analysis we put $\Delta \dot{x} = 0$ and then make elimination of the variables Δx and Δz connected with generator internal buses.

However we do not put $\Delta y = 0$, because some power unbalances may happen in the vicinity of voltage collapse point. At each load bus we have the power injection and power receiving. Load bus injection depends on system generation and network impedances. Load bus receiving depends on load voltage characteristics. In the case of ill conditioned power system the network bus voltages can have such small values that the specific bus receiving is greater than the relevant bus injection. In this case the power unbalance really happens at such a node.

Let us rewrite the matrix equation (2) in the following form

$$B \Delta u + C \Delta v = 0 \quad (5a)$$

$$D \Delta u + J_{FL} \Delta v = \Delta y \quad (5b)$$

where

B - the matrix associated with internal buses of generators,

$\Delta u = [\Delta x \quad \Delta z]^T$ - the vector of the deviation of state variables and internal voltages,

$\Delta v = [\Delta V \quad \Delta \delta]^T$ - the vector of the deviation of load voltages,

C, D - sub-matrices which depict the relation between generator internal buses and network,

J_{LF} - the load flow Jacobian for load buses,

$\Delta y = [dP \quad dQ]^T$ - the vector of the power unbalances at load buses.

After elimination of Δu we obtain the following relation between load bus unbalances and load bus voltage deviations

$$J_{mLF} \Delta v = \Delta y \quad (6a)$$

where

$$J_{mLF} = J_{LF} - D B^{-1} C \quad (6b)$$

is the modified load flow Jacobian.

The calculation of the modified Jacobian J_{mLF} is simple because it needs only some modification of the standard load flow Jacobian J_{sLF}

After inversion of the modified Jacobian J_{mLF}^{-1} we obtain the matrix which relates the load bus voltage deviations and bus power unbalances

Now, the estimating of the partial derivatives at load bus

$$s_1 = \partial V / \partial P \text{ and } s_2 = \partial V / \partial Q \quad (7)$$

is very simple for they equal the relevant elements of the modified Jacobian inversion.

The bus power unbalances $s_3 = dP$ and $s_4 = dQ$ may be calculated using non-diverged load flow algorithm. For this purpose load flow equations are solved with two close multiple solutions Iwamoto and Tamura method [4]

$$v_{it+1} = v_{it} + \mu_{it} v_{it} \quad (8)$$

where

it - the iteration number,

μ - the optimised parameter.

The optimized parameter μ is going to zero when bus load unbalances appear and this means that there is no real load flow solution in fact.

The bus unbalances are calculated as follows

$$s_3 = dP = \Delta P = \Delta P_G - \Delta P_L \quad (9a)$$

$$s_4 = dQ = \Delta Q = \Delta Q_G - \Delta Q_L \quad (9b)$$

where G and L means injection and receiving at load bus.

Additionally, the load flow solution for planned bus and branch input data may be characterised by the base determinant value of Jacobian of standard load flow. The load flow solutions for changed input data may be characterised by the similar values as well. To describe the difference between the planned and changed state we can use the both values and compute the k_d coefficient

$$k_d = \frac{\text{sign}(\det J_{sLF})_{\text{new}} \ln |\det J_{sLF}|_{\text{new}}}{\text{sign}(\det J_{sLF})_{\text{base}} \ln |\det J_{sLF}|_{\text{base}}} \quad (10)$$

The natural logarithm is used to avoid computer overflow because the value of Jacobian determinant for real sized power system can be very large.

When k_d decreases then the power system condition becomes worse. When k_d changes its sign then we may suppose that the state is close to the voltage collapse point.

Statistical analysis of voltage collapse

The probabilistic voltage instability analysis is divided into load flow simulation and statistical analysis of s-factors.

The active and reactive bus loads are given as voltage characteristics

$$P_L = pV^\alpha \text{ and } Q_L = qV^\beta \quad (11)$$

It is assumed that active and reactive loads are independent random variables with rectangular probability distributions. The expected values are equal the planned values of loads and the standard deviations are given as ε -percentage of the expected values. So we have

$$p_{\min} = \bar{p} - \sqrt{3}\sigma_p \text{ and } q_{\min} = \bar{q} - \sqrt{3}\sigma_q \quad (12a)$$

$$p_{\max} = \bar{p} + \sqrt{3}\sigma_p \text{ and } q_{\max} = \bar{q} + \sqrt{3}\sigma_q \quad (12b)$$

where

$\bar{p}, \bar{q}$ - expected values,

$\sigma_p = 0.01\varepsilon_p \bar{p}$ - the standard deviation of p,

$\sigma_q = 0.01\varepsilon_q \bar{q}$ - the standard deviation of q.

For each simulated load flow s-factors at each load bus are calculated. The sample mean value vector and covariance matrix are then calculated

- mean value

$$\bar{s}_i = \frac{1}{n} \sum_{k=1, n} s_{ik} \quad (13a)$$

- covariance

$$m_{ij} = \frac{1}{n-1} \sum_{k=1, n} (s_{ik} - \bar{s}_i)(s_{jk} - \bar{s}_j) \quad (13b)$$

- standard deviation

$$\sigma_i = \sqrt{m_{ii}} \quad (13c)$$

- correlation coefficient

$$\rho_{ij} = \frac{m_{ij}}{\sigma_i \sigma_j} \quad (13d)$$

The correlation coefficient is a measure of the degree of linear relationship between s-factors at various load buses.

The probability of voltage collapse p_{vc} at the i-th bus can be calculated as the relative frequency of bus unbalance

$$p_{vc} = \frac{m}{n} \quad (13e)$$

where m is the number of bus unbalances and n - the number of load flow runs.

The bus which has the greatest probability p_{vc} is the weakest one from the point of view of voltage collapse.

At last the weakest buses should be very carefully studied using dynamic approach to voltage collapse.

Analysis of generator-load system

The proposed steady state model has been used for the analysis of s-factors. First of all, a simple 2-bus system has been analysed. The power system has one generator bus and one load bus, fig.1.

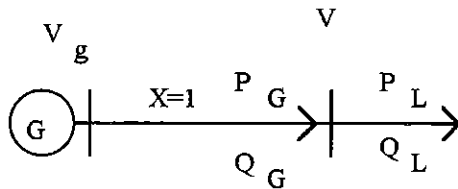


Fig.1 Generator-load example system.

The load flow equations are as follows

$$y_P = -P_L(V) + P_G(V) \quad (14a)$$

$$y_Q = -Q_L(V) + Q_G(V) \quad (14b)$$

where L means receiving at load bus and G - injection from system to load bus.

The receiving-end voltage depends on the active P_L and reactive Q_L load and the both loads depend on the bus voltage

$$P_L(V) = p\sqrt{V} \quad \text{and} \quad Q_L(V) = q\sqrt{V} \quad (15)$$

The generator terminal voltage V_g is constant and the power injections P_G and Q_G are dependent on voltage magnitudes and angles.

The injection are the following functions

$$P_G(V) = V_g V \sin \delta \quad (16a)$$

$$Q_G(V) = -V_g V \cos \delta + V^2 \quad (16b)$$

where δ means the voltage angle.

After linearization we obtain

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} -P'_{LV} + P'_{GV} & P'_{Gs} \\ -Q'_{LV} + Q'_{GV} & Q'_{Gs} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} \quad (17)$$

and finally after matrix inversion

$$\frac{\partial V}{\partial P} = \frac{\sin \delta}{V_g - (2V - Q'_{LV}) \cos \delta - P'_{LV} \sin \delta} \quad (18a)$$

$$\frac{\partial V}{\partial Q} = \frac{-\cos \delta}{V_g - (2V - Q'_{LV}) \cos \delta - P'_{LV} \sin \delta} \quad (18b)$$

From (18a,b) we can see that $\partial V / \partial P$ and $\partial V / \partial Q$ are very non-linear function of bus load voltage. To simplify this presentation let us assume that the active load is zero and only the reactive load has the exponential voltage characteristic, so we have

$$\begin{aligned} s_1 &= \frac{\partial V}{\partial P} = 0 \\ s_2 &= \frac{\partial V}{\partial Q} = \frac{-1}{V_g - 2V + 0.5Q/\sqrt{V}} \end{aligned} \quad (19)$$

In table 1 there are given results for reactive loading. It is seen that below a certain voltage the reactive power consumed increases with decrease in voltage until $\partial V / \partial Q = \infty$ when the voltage collapses. Further increase of the reactive load

gives no analytical real solution but only converged solution with bus unbalances.

Table 1 Load flow solutions of 2-bus EPS and $\partial V/\partial P$, $\partial V/\partial Q$ criteria

q	V 1-st sol.	V 2-nd sol.	s ₂ 1-st sol.	s ₂ 2-nd sol.
0.1	1.4160	0.0045	0.73	-1.34
0.2	1.3263	0.0182	0.81	-1.38
0.3	1.2294	0.0424	0.91	-1.46
0.4	1.1224	0.0793	1.07	-1.58
0.5	1	0.1340	1.33	-1.82
0.6	0.8487	0.2196	1.60	-2.10
0.7	0.5840	0.4204	7.94	-8.38
0.7071	0.5025	0.4974	262	-261
0.7072	0.5207	0.4939	153	-472
0.7073	0.0000	0.5033	-0.0006	-15.3

Note that the partial derivative change its sign between $Q=0.7072\sqrt{V}$ and $Q=0.7073\sqrt{V}$. At the point of $Q=0.7073\sqrt{V}$ load flow equations have no real solution.

Analysis of 6-bus Ward-Hale system

The Ward-Hale power system has one generator bus, three load buses, one bus with zero bus powers and infinite bus. Input data of the this power system are the same as in the paper [4] but they have been extended by generator data given in table 2.

The values of p and q are random variables uniformly distributed with the expected values given in table 3 and with standard deviations equal 50 % of expected values.

Table 2 Generator input data

S _n	U _n	X _q	X _d	X _{d'}	T _{do'} s	H s
4.25	1	2.4	2.6	0.3 5	8.17	4.25
T _A s	T _E s	T _F s	K _A	K _E	K _F	
0.55	0.3	1 s	30	1.0	0.03	

T_A, T_E, T_F mean time constants of the voltage regulator, the exciter and the stabilising circuit, K_A, K_E, K_F mean gains of the voltage regulator, the exciter and the stabilising circuit. The

generator power S_n is given in pu related to 100 MVA.

It is assumed that all bus loads have exponential voltage characteristics

$$P_L(V) = p\sqrt{V} \quad \text{and} \quad Q_L(V) = q\sqrt{V} P_L = p\sqrt{V}$$

The values of p and q are given in table 3.

Table 3 Bus loads of W&H system

	p pu	q pu	α	β
bus-3	0.55	0.13	0.5	0.5
bus-5	0.30	0.18	0.5	0.5
bus-6	0.50	1.00	0.5	0.5

Deterministic approach to voltage stability

The results of reactive loading at bus 6 are shown in table 4,5 and 6.

Load flow equations are non-linear and they have multiple solutions. Two close solutions merged into one in the vicinity of voltage collapse point. The eigenvalue criterion says that power system is unstable from the value of $q=1.438$, table 4.

It is very clearly seen from table 5 that the partial derivatives criteria make possible to estimate voltage collapse and distinguish very well the first and the second solution of load flow equations. It is important that the partial derivatives change their signs at the point where two close solutions are close in fact.

We can observe that in the vicinity of voltage collapse point partial derivatives become negative not only at the bus with heavy reactive loading but at other buses as well, table 7.

Table 4 Reactive loading at bus 6 and eigenvalues criterion

q	V 1-st LF sol.	V 2-nd LF sol.	max Re(λ) 1-st	max Re(λ) 2-nd
0	0.936	0.005	-0.037	+0.121
0.50	0.810	0.043	-0.036	-0.884
1.00	0.651	0.138	-0.036	-0.102
1.40	0.434	0.278	-0.027	-0.063
1.42	0.409	0.299	-0.003	-0.062
1.438	0.365	0.340	+0.462	-0.174
1.439	0.352	0.351	+10.39	+19.61

Table 5 Reactive loading at bus 6 and partial derivatives criteria

q	s ₁ 1-st LF sol.	s ₂ 1-st LF sol.	s ₁ 2-nd LF sol.	s ₂ 2-nd LF sol.
0	0.10	0.24	0.14	-0.04
0.50	0.12	0.31	24.4	181.0
1.00	0.16	0.48	-0.20	-0.45
1.40	0.55	1.75	-0.52	-1.67
1.42	0.81	2.58	-0.72	-2.33
1.438	4.59	14.9	-2.55	-8.30
1.439	-9.43	29.7	-8.33	-27.0

It was observed that the k_d coefficient decreased its value during reactive loading, table 7.

Table 7 The values of s-factors at bus-3 and bus-5 during reactive loading at bus-6

q	s ₁ bus-3	s ₂ bus-5	s ₁ bus-3	s ₂ bus-5	k _d
0	0.12	0.32	0.09	0.36	1.1
0.5	0.13	0.34	0.11	0.41	1.1
1	0.16	0.38	0.16	0.52	1
1.4	0.27	0.53	0.55	1.12	0.8
1.42	0.33	0.61	0.74	1.46	0.7
1.438	1.17	1.70	4.10	6.43	0.6
1.439	-2.05	-2.52	-8.92	-12.7	0.4

From table 8 we can notice when load flow equations loose their real solutions then the bus unbalances appear. Also it is seen that the determinant of Jacobian has sometimes positive and sometimes negative sign. This means that the load flow converges to minimal transmission losses but the obtained solution is accidental.

Table 8 Bus unbalances during reactive loading at bus-6

q	s ₃ bus-5	s ₄ bus-5	s ₃ bus-6	s ₄ bus-6	sign det J _{sLF}
1.43	0.00	0.00	0.00	0.00	-1
1.44	0.00	0.00	0.02	0.05	+1
1.46	0.09	0.11	0.35	1.1	-1
1.48	0.18	0.31	0.66	2.0	+1
1.5	0.26	0.42	0.99	3.0	-1

Hence, the numerical results obtained for Ward-Hale system show that the partial derivatives and bus unbalances criteria can be used to predict

system performance in the area close to voltage collapse point.

Probabilistic approach to voltage stability

The standard deviations of p and q at all load buses are the same for all load buses and equal 50% of the specific expected value. There have been simulated 500 runs of load flow. Statistical results are given in table 9. Symbol E means expected value, σ - standard deviation, P_{vc} - voltage collapse probability.

Table 9 Mean values, standard deviations and voltage collapse probabilities

	bus-3	bus-5	bus-6
E(s ₁)	0.08	-0.01	-0.11
σ_1	1.13	2.43	2.66
E(s ₂)	0.30	0.28	-0.19
σ_2	1.06	3.24	7.51
E(s ₃)	0.10	0.028	1.04
σ_3	0.27	0.66	2.24
P _{vc3}	0.20	0.23	0.26
E(s ₄)	0.30	0.60	3.26
σ_4	0.86	1.47	7.24
P _{vc4}	0.22	0.25	0.26

From table 9 it is seen that the probability of voltage collapse is different for various load buses. The probability of voltage collapse at bus-6 is the greatest one so it means that bus-6 is the weakest bus from the point of view of voltage collapse.

Additionally we can observe that the voltage collapse probabilities calculated from dP and dQ criteria are similar. The reason of such fact is coming from functional relationship between various s-factors, see table 10,11,12 and 13,14,15.

Table 10 Correlation between partial derivatives $\partial V / \partial P$

s ₁ and s ₁	bus-3	bus-5	bus-6
bus-3	1	0.89	0.89
bus-5	0.89	1	0.97
bus-6	0.89	0.97	1

Table 11 Correlation between partial derivatives $\partial V/\partial Q$

s_2 and s_2	bus-3	bus-5	bus-6
bus-3	1	0.86	0.91
bus-5	0.86	1	0.98
bus-6	0.91	0.98	1

Table 12 Correlation between partial derivatives $\partial V/\partial P$ and $\partial V/\partial Q$

s_1 and s_2	bus-3	bus-5	bus-6
bus-3	0.98	0.82	0.88
bus-5	0.82	0.96	0.95
bus-6	0.88	0.95	0.96

From table 10,11 and 12 it is seen that the partial derivatives are almost linearly dependent on each other.

Table 13 Correlation between bus unbalances dP

s_3 and s_3	bus-3	bus-5	bus-6
bus-3	1	0.72	0.76
bus-5	0.72	1	0.86
bus-6	0.76	0.86	1

Table 14 Correlation between bus unbalances dQ

s_4 and s_4	bus-3	bus-5	bus-6
bus-3	1	0.73	0.76
bus-5	0.73	1	0.85
bus-6	0.76	0.85	1

Table 15 Correlation between bus unbalances dP and dQ

s_3 and s_4	bus-3	bus-5	bus-6
bus-3	0.77	0.74	0.75
bus-5	0.74	0.78	0.75
bus-6	0.75	0.75	0.93

From table 12,13 and 15 it is seen that the power bus unbalances are non-linear functions on each other. The reason of this is coming from the fact

that the bus unbalances are the non-linear function of the same bus voltages, see for example formula (14a,b).

Conclusions

In this paper the probabilistic approach to voltage collapse analysis is presented. Starting from the set of differential and algebraic equations the modified load flow Jacobian has been obtained. The inverted modified Jacobian matrix enables us to estimate partial derivatives

$\partial V/\partial P$ and $\partial V/\partial Q$ at each load bus. When the sign of the partial derivative is negative then voltage collapse at this load bus may happen. In the area of voltage collapse point bus unbalances can appear as a result of load flow solution. These bus unbalances may be used for the estimation of the voltage collapse probability at each bus. Many improvements should be added in the future to the proposed method, especially the impact of various probability distributions on the probabilities of voltage collapse.

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A CONTINUATION METHOD APPROACH TO FINDING THE CLOSEST SADDLE NODE BIFURCATION POINT

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Abstract

The paper considers the problem of finding saddle node bifurcation points which are closest (in a local sense) to the power system operating point. This optimization problem leads to a set of equations which describe such *critical points*. Not all solutions of this set of equations are critical points. The paper therefore explores the nature and characteristics of solutions. A two stage algorithm is proposed for solving the critical point problem. The first stage is simply to find a point on the singular surface, i.e., the surface of saddle node bifurcation points, which lies in a specified direction. The second stage uses a continuation method to move from that initial point to the desired critical point. Singularity of the critical point problem can have a significant influence on the robustness of the continuation method. The paper investigates singularity conditions. The proposed algorithm is tested on an eight bus power system example.

Keywords: stability margin calculation, power flow singularities, continuation methods

1 Introduction

The trend in modern power system operation is toward greater utilisation of generation and transmission assets. This necessarily means that systems must operate much closer to stability limits. Therefore there is a need to be able to determine those limits more accurately and reliably.

The form of the desired stability margin information varies, depending on the type of power system investigation being undertaken. Investigations of large disturbance stability may make use of a stability margin based on Lyapunov ideas, see [1] for example. In an operating environment however, it is more common to use security margins based on quasi-static properties of the system. The power flow problem is central to this form of security margin. The margin is generally based on some measure of distance from the operating point to a point where the power flow problem becomes unsolvable. This paper focusses on this latter type of security margin.

A power flow problem can be described by the set of n nonlinear algebraic equations

$$y^0 + f(x) = 0 \quad (1)$$

where $y^0 \in \mathbb{R}^n$ is the vector of specified independent parameters such as active and reactive powers of loads and generators or fixed voltages, $x \in \mathbb{R}^n$ is the state, consisting of nodal voltages. The vector function $f(x)$ defines the sum of power flows or currents into each bus from the rest of the network. If nodal voltages x are expressed in rectangular coordinates then $f(x)$ is a quadratic function of x .

The power flow equations $f(x)$ generally define a mapping from state space to a subset of parameter space, i.e., $-f: \mathbb{R}^n \rightarrow \mathcal{L}$ where $\mathcal{L} \subset \mathbb{R}^n$. The region $\mathcal{L}$ defines the set of parameters for which power flow solutions exist. Therefore the inverse mapping, from parameter space to state space is only defined inside $\mathcal{L}$. This inverse mapping is in general not unique, with a number of solutions corresponding to a given value of parameters $y^0 \in \mathcal{L}$. In fact, parameter space may be divided into regions, with each region having a different number of solutions for a given value of y .

As parameters y vary, the solutions of (1) will also move in state space. Parameters y may move to a point where two solutions coalesce, with further variation of y resulting in the disappearance of that solution. Behaviour of that form is referred to as a saddle node bifurcation. It follows from the Implicit Function Theorem that at such bifurcation points

$$\det D_x f = \det J(x) = 0 \quad (2)$$

i.e., $J(x)$ is the Jacobian matrix of $f(x)$. Further, we see that regions defined on the basis of the number of solutions for a given y must be bounded by surfaces of points satisfying (2). We shall define the boundary as

$$\Sigma = \{(x, y) : x, y \in \mathbb{R}^n, y + f(x) = 0, \det J|_{(x,y)} = 0\} \quad (3)$$

The projection of Σ onto state space and parameter space will be referred to as Σ_x, Σ_y respectively. More detailed analysis of the structure of $\mathcal{L}$ was undertaken in [11, 12].

The condition (2) means that at least one real eigenvalue η_i of J must be zero. Under certain modelling assumptions, a zero eigenvalue of J corresponds to loss of small disturbance stability of the system [13, 14, 15]. Therefore the 'distance' from an operating point to points where J is singular, i.e., points in Σ , provides a useful measure of the security of a system. As an operating point moves nearer to Σ , the stability region around that point reduces [37].

An important power system control problem therefore is to prevent an operating point from moving too close to Σ . That is, operating points should always be (at least) some specified distance from Σ . As parameters y correspond to physical quantities that can be measured and controlled, it is useful to consider this distance in terms of parameter space, i.e.,

$$d(y) = \|y - y^0\| \quad (4)$$

where y^0 is the operating point, and $y \in \Sigma_y$, i.e., y is a point on the solution boundary [16, 17, 18, 19]. The shortest (or critical) distance $\min_{y \in \Sigma_y} d(y)$ gives a measure of power system security in the most dangerous direction of

loading [19, 21, 22, 23, 24, 26, 27]. In addition, the critical vector $(y^0 - y)$ defines the optimal way of controlling the power system to maximise security. Its largest components indicate parameters which contribute most to the security conditions [17, 20, 22, 26].

In this paper we address the issue of robustly finding the minimum distance to Σ . This question has been investigated before [19, 21, 22, 23, 24, 26, 27]. We are proposing a continuation approach to finding the points on Σ which are closest (in a local sense) to the operating point. We call these points *critical points*. Singularities of the problem which affect such methods are investigated.

The paper is organised as follows. Section 2 establishes the mathematical description of critical points. Properties of the solutions of the critical point problem are discussed in Section 3. Singularities of that problem are considered in Section 4. Section 5 proposes an algorithm for finding the critical points. An eight bus example is considered in Section 6. An numerical technique which is useful for the critical point algorithm of Section 5 is outlined in Appendix B.

2 Formulation of the problem

2.1 Parameter weighting factors

When considering the minimum distance from an operating point to Σ , and optimal control strategies for increasing that distance, it is necessary to take account of the fact that parameters may have different dimensions, e.g., bus powers and nodal voltages. It may also be necessary to weight parameters of the same type differently. For example a small but critical load may need to be weighted differently to a large, but not so critical load. Further, some parameters are fixed. An example in this case would be bus powers at nodes which have no generators or load connected.

Therefore, to make the parameters compatible, ‘normalising’ coefficients can be used [19, 22, for example]. The distance function would then be redefined as

$$d(y) = \|\mu[y - y^0]\| \quad (5)$$

where μ is a diagonal matrix of weight coefficients, with diagonal elements

$$\mu_i = 1/y_i^b \quad (6)$$

Each y_i^b is a ‘normalising’ factor for the i -th parameter. The distance (5) can be used as an aperiodic stability (or security) index [16, 19]. By comparing the distance $d(y)$ with a specified safe value of the stability index I_s , it is possible to decide whether the current operating state is dangerous or not.

It should be noted that the y^b in (6) should be constants, and not dependent on the operating point values y^0 . It was shown in [20] that difficulties arise when $y^b = y^0$, due to the resulting nonlinear dependence of $d(y)$ on y^0 .

2.2 Optimization formulation

One of the first things to consider in the formulation of the optimization problem

$$\min_{y \in \Sigma_v} d(y) \quad (7)$$

is that not all parameters are free to vary. Some parameters, such as power injected at buses which have no

load or generation, must always be fixed. Parameters can be effectively held constant by assigning very large values of weight coefficients μ in (5). However this leads to an ill-conditioned problem, and associated numerical difficulties. We therefore adopt the following power flow formulation.

Let m equations in (1) contain fixed values of parameters $y_2 = y_2^0 = \text{const}$. The other $(n - m)$ parameters y_1 are free to vary. Then the system (1) can be rewritten as

$$y_1 + f_1(x) = 0 \quad (8)$$

$$y_2^0 + f_2(x) = 0 \quad (9)$$

Using (8),(9), we see that the square of the distance $d(y)$ defined at (5) can be written¹

$$d(y)^2 = \|y - y^0\|^2 \quad (10)$$

$$= \|y_1 - y_1^0\|^2 \quad (11)$$

$$= \|y_1^0 + f_1(x)\|^2 \quad (12)$$

Consider the optimization problem

$$\text{ext}_x \|y_1^0 + f_1(x)\|^2 \quad (13)$$

$$y_2^0 + f_2(x) = 0 \quad (14)$$

where ‘ext’ denotes extrema of the cost function (13). The optimization is subject to nonlinear constraints (14). From (10)-(12) it can be seen that the cost function (13) defines the square of the distance between the points y_1 and y_1^0 , with both points belonging to the constraint hyperplane $y_2 = y_2^0 = \text{const}$, see Figure 1.

If we define the Lagrange function

$$l(x, \lambda) = \|y_1^0 + f_1(x)\|^2 + 2[y_2^0 + f_2(x)]^t \lambda \quad (15)$$

then the constrained optimization problem (13),(14) can be formulated as an unconstrained problem

$$\text{ext}_{x, \lambda} l(x, \lambda) \quad (16)$$

Solutions of (16) satisfy the nonlinear system

$$J_1^t(x)[y_1^0 + f_1(x)] + J_2^t(x)\lambda = 0 \quad (17)$$

$$y_2^0 + f_2(x) = 0 \quad (18)$$

where $J_1 = \frac{\partial f_1}{\partial x}$ and $J_2 = \frac{\partial f_2}{\partial x}$. This system of equations can be written in more general form as

$$\phi(x, \lambda) = 0 \quad (19)$$

Using the substitution

$$s = y_1^0 + f_1(x) \quad (20)$$

in (17) allows us to represent the system (17),(18) as

$$\begin{aligned} -s + y_1^0 + f_1(x) &= 0 \\ y_2^0 + f_2(x) &= 0 \\ J_1^t(x)s + J_2^t(x)\lambda &= 0 \end{aligned} \quad (21)$$

¹To simplify expressions we take $\mu = I$, where I is the identity matrix. The case when $\mu \neq I$ requires trivial transformations of all the following equations.

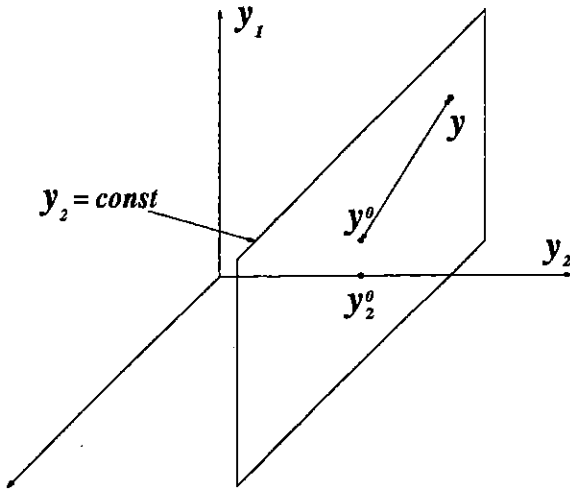


Figure 1: The constraint hyperplane (14).

It is clear that the system (21) has the same solutions as the system (17),(18), so

$$\phi(x, \lambda) = 0 \Leftrightarrow \underline{\phi}(x, s, \lambda) = 0 \quad (22)$$

where $\underline{\phi}(x, s, \lambda) = 0$ represents the system (21). The last equation in (21) can be rewritten as

$$J^t(x)s_\lambda = 0 \quad (23)$$

where

$$J^t = [J_1^t \ J_2^t] \quad (24)$$

$$s_\lambda = [s^t \ \lambda^t]^t \quad (25)$$

If $s_\lambda \neq 0$, the Jacobian matrix $J(x)$ is singular, and the vector s_λ is a left eigenvector corresponding to a zero eigenvalue. Therefore, considering the original optimization problem (13),(14), and the condition (23) for $s_\lambda \neq 0$, we can conclude that critical points, i.e., points on Σ that are minimal distance (locally) from the operating point y^0 , satisfy the system (21).

3 Solutions of the critical point problem

The system (21) can be considered as an extended power flow problem where the usual power flow equations (the first two equations in (21)) are supplemented by the singularity condition (23). State space, i.e., the space of unknown variables, is now extended to include the additional variables s_λ . So the variables are $(x, s_\lambda) \in \mathbb{R}^{2n}$.

3.1 Trivial and nontrivial solutions

There are two kinds of solutions to the critical point problem (21):

- Trivial solutions corresponding to the condition $s_\lambda = 0$. Those solutions are actually the solutions of the usual power flow problem (1). They are global minima (zeros) of the distance function (4). All trivial solutions coincide in parameter space.
- Nontrivial solutions conforming to the condition $s_\lambda \neq 0$. Those solutions belong to the singular margin Σ given by (3).

Solution of (21) can result in either trivial or nontrivial solutions, depending on initial estimates of the variables and the numerical solution technique used for solving the problem. A technique which produces nontrivial solutions is proposed in Section 5.

3.2 Distance and the left eigenvector

Let us analyze the nontrivial solutions of the system (21). It is known, that the vector s_λ is a normal vector to the singular hypersurface Σ_y . However, we see from the first equation of (21), and (8) that at critical points, i.e., solutions of (21),

$$y_1 - y_1^0 = y_1 - s + f_1(x) = -s \quad (26)$$

So, because $y_2 = y_2^0$, the component $-s$ of the vector $-s_\lambda$ is the distance vector $y - y^0$.

Since the distance vector is an orthogonal vector to the singular hypersurface Σ_y , nontrivial solutions correspond to local minima or maxima of the distance from the point y^0 to the singular margin. The minimum of the distances associated with the nontrivial solution points characterizes the "level" of power system security.

3.3 A graphical illustration

The graphical illustration of a nontrivial solution point of (21) is given by Figure 2. The solution point must lie somewhere on the intersection of the singular margin Σ_y and constraint hyperplane $y_2 = y_2^0 = \text{const}$. The left eigenvector s_λ at the nontrivial solution point y is perpendicular to the singular boundary, and its component s coincides with the vector from the singular point y to the operating point y^0 . The component λ (the Lagrange multiplier vector) of s_λ is orthogonal to the constraint hyperplane.

3.4 Constraint impact upon the critical distance

The relative length of the vector λ ,

$$l_\lambda = \frac{\|\lambda\|}{\|s_\lambda\|} \quad (27)$$

indicates the significance of the constraint set (14) on the optimization (13). Consider the following cases,

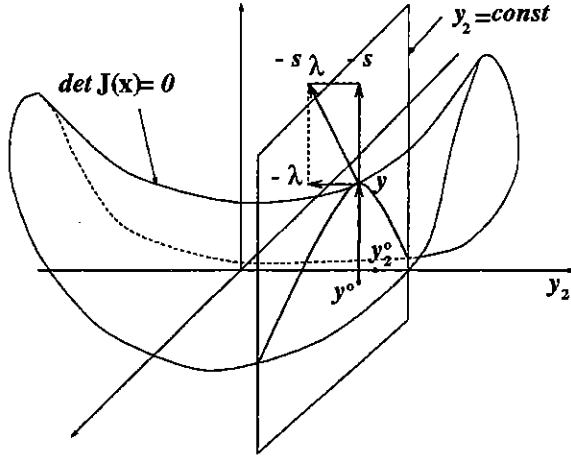


Figure 2: Graphical illustration of a nontrivial solution point.

1. $l_\lambda = 0$, ($s \neq 0, \lambda = 0$). The vector s_λ lies on the constraint hyperplane $y_2 = y_2^0 = \text{const}$, and the critical point corresponds to a solution of the unrestricted optimization problem

$$\text{ext}_x \|y^0 + f(x)\|^2 \quad (28)$$

Hence the constraints have no influence on the solution.

2. $l_\lambda = 1$, ($s = 0, \lambda \neq 0$). The vector s_λ is orthogonal to the constraint hyperplane. This means that the constraint hyperplane is tangent to the singular margin Σ_y at the critical point. It will be shown in Section 4.2 that the corresponding point is a singular point of (21). Traditional numerical techniques encounter difficulties at such points in the same way that power flow techniques exhibit poor convergence near singular points. The constraints are particularly significant in this case.
3. $0 < l_\lambda < 1$, ($s \neq 0, \lambda \neq 0$). The corresponding non-trivial solution is affected by the constraints (14). The value of l_λ indicates the extent to which the constraints influence the minimum distance from the operating point to the singular margin. A small value of l_λ indicates modest influence of constraints.

3.5 Minima, maxima and saddle points

The Jacobian matrix J_ϕ of (21),

$$J_\phi = \begin{bmatrix} J_1(x) & -I & 0 \\ J_2(x) & 0 & 0 \\ D(s_\lambda) & J_1^t(x) & J_2^t(x) \end{bmatrix} \quad (29)$$

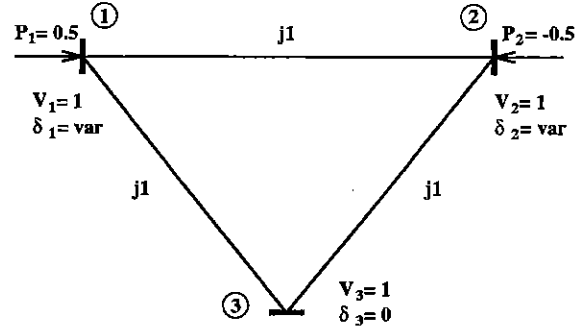


Figure 3: Simple 3 bus power system.

with

$$D(s_\lambda) = \frac{\partial}{\partial x} [J^t(x) s_\lambda] \quad (30)$$

is effectively a Hessian matrix of the Lagrangian cost function (15).

It is known [28, for example] that the solution point is a minimum when the matrix J_ϕ is positive definite, and a maximum when it is negative definite. If the Hessian matrix is indefinite, the corresponding solution is a saddle. All those cases can be encountered in this optimization problem. To illustrate this, we consider the following simple 3-bus power system example.

Example (3 bus)

Power balance equations for the system shown in Figure 3 can be written as

$$\begin{aligned} P_1 + \sin(\delta_2 - \delta_1) - \sin \delta_1 &= 0 \\ P_2 + \sin(\delta_1 - \delta_2) - \sin \delta_2 &= 0 \end{aligned} \quad (31)$$

The values P_1, P_2 are considered as free parameters y_1 in (8). The voltage magnitudes $V_1 = 1\text{pu}$ and $V_2 = 1\text{pu}$ are taken as fixed parameters y_2^0 in (9). The optimization problem (13) therefore transforms to

$$\text{ext}_{\delta_1, \delta_2} [(P_1 - 0.5)^2 + (P_2 + 0.5)^2] \quad (32)$$

Figure 4 shows the plane of state variables δ_1, δ_2 , and Figure 5 the plane of free parameters P_1, P_2 , with parameters V_1, V_2 fixed. Singular margins Σ_x for the system (31) (dashed lines) and contours of the cost function (32) are plotted on the plane δ_1, δ_2 in Figure 4. Solutions of the optimization problem (32) are also shown. Points A1, A2 are minima. They correspond to trivial solutions of the critical point problem (21). A1 is the 'normal' operating point. Points marked B and C are non-trivial solutions of (21), with B1...B5 being saddle points of (32), and C1...C3 maxima. All non-trivial solutions of (21) lie on the singular margin Σ_x .

Note that the section of Σ_x which surrounds the operating point A1, i.e., the oval that contains points B1, B2, C1, C2, is of primary interest. If parameters were varied continuously from their operating point values, then

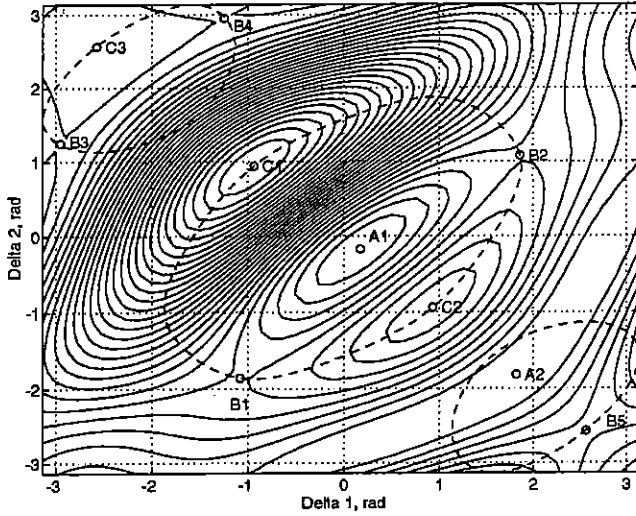


Figure 4: The δ_1, δ_2 plane for the 3 bus example.

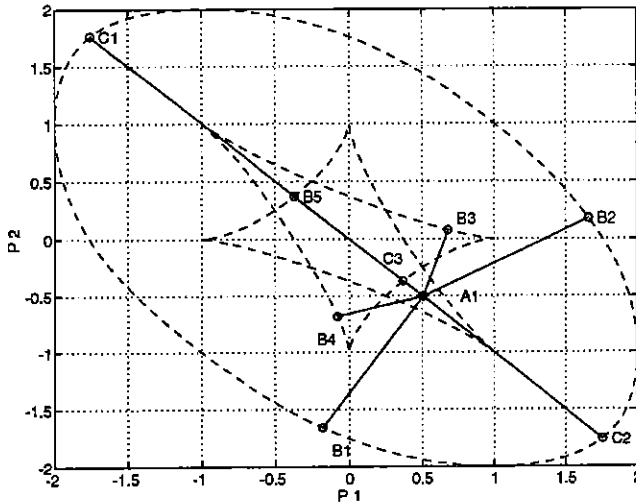


Figure 5: The P_1, P_2 plane for the 3 bus example.

a path, beginning from A1, would be mapped out on Figure 4. Such a path would first intersect Σ_x at a point on that oval. At points along the path up to that singular point, all real eigenvalues of the power flow Jacobian would be negative. At the singular point, an eigenvalue would become zero. If the path crossed the oval transversally, then at points immediately outside the oval, one real eigenvalue would be positive. Note that real eigenvalues can only change sign at points on Σ . The region contained inside this primary section of Σ_x shall be called the *security region*. The primary section of Σ_x is therefore the boundary of that region.

Figure 5 shows the sections of the singular margin Σ_y plotted in the plane of free parameters P_1, P_2 . The solution space $\mathcal{L}$ has three 'layers', with each layer restricted by a section of the singular margin. These layers reflect the non-uniqueness of solutions of the power flow equations. Each layer effectively generates a pair of power flow solutions. For example, there are two layers at the point A1. Consequently, there are four distinct solutions of the power flow problem (31). Only two of them, A1 and A2, are shown in Figure 4.

All solutions of (32) except A2 are shown in Figure 5. (A2 coincides with A1, so is not marked.) This figure also shows vectors from the operating point A1 to the solutions of (21). Each of these vectors is normal to the singular margin Σ_y .² The vectors A1-B1, A1-B2, A1-C1, and A1-C2 are particularly significant as they show the distances and direction from A1 to the critical points on the boundary of the security region. Figure 4 shows clearly that B1, B2 are saddle points of the optimization problem (32). From Figure 5, we see that they satisfy (7) locally. On the other hand, C1, C2 are local maxima of (32). They are also points that locally satisfy $\max_{y \in \Sigma_y} d(y)$. $\square$

Depending on initial guesses of variables, and the solution technique, any of the points identified in Figures 4 or 5 could be obtained as a solution of (21). As seen in the example though, only some of those points are of interest to us. Thus, the problem is to find saddle points on the boundary of the security region. This problem is quite different to usual optimization problems where minima or maxima are desired. Appropriate techniques are discussed in Section 5.

Comment

It is interesting to note that because J_ϕ is effectively the Hessian matrix of (15), it can be represented as a symmetric matrix by transpositions of its rows and columns. Such transpositions give

$$\tilde{J}_\phi = \begin{bmatrix} -I & J_1(x) & 0 \\ J_1^t(x) & D(s_\lambda) & J_2^t(x) \\ 0 & J_2(x) & 0 \end{bmatrix} \quad (33)$$

The matrix $D(s_\lambda)$ in (33) is symmetric as its elements d_{ij} can be expressed like

$$d_{ij} = \frac{\partial}{\partial x_j} \left[\sum_{k=1}^n \frac{\partial f_k}{\partial x_i} s_{\lambda_k} \right] = \frac{\partial}{\partial x_i} \left[\sum_{k=1}^n \frac{\partial f_k}{\partial x_j} s_{\lambda_k} \right] = d_{ji}$$

Therefore $\tilde{J}_\phi$ is symmetric.

²The apparent absence of orthogonality of the vectors with respect to the singular margin in Figure 5 is caused by a difference in the horizontal and vertical scales of the figure.

4 Singularity of the critical point problem

In this paper we are interested in numerical techniques for finding solutions of the critical point problem (21). Singular points of (21) can have a significant influence on the convergence characteristics of such numerical techniques. (The influence is similar to that of a singular power flow Jacobian on convergence of power flow algorithms.) Therefore we shall explore general conditions of singularity, then focus on some particular cases which are of interest. We then undertake an analysis of singular points which is aimed at providing a better understanding of the nature and significance of singularity.

4.1 General singularity conditions

In later sections, we investigate continuation techniques which are based on following paths along the intersection of the solution space boundary Σ and the hyperplane $y_2 = y_2^0$. Therefore we are interested in singularity of the critical point Jacobian J_ϕ , given by (29), at arbitrary points on that intersection. Such arbitrary points (x, y) are given by

$$y_1 + f_1(x) = 0 \quad (34)$$

$$y_2^0 + f_2(x) = 0 \quad (35)$$

$$J_1^t(x)s + J_2^t(x)\lambda = J^t(x)s_\lambda = 0 \quad (36)$$

where $s_\lambda \neq 0$. Linearizing gives

$$dy_1 + J_1(x)dx = 0 \quad (37)$$

$$J_2(x)dx = 0 \quad (38)$$

$$D(s_\lambda)dx + J_1^t(x)ds + J_2^t(x)d\lambda = 0 \quad (39)$$

So, for small changes dx in x along the intersection of Σ_x and the hyperplane $y_2 = y_2^0$, (37)-(39) provide the corresponding changes dy_1 and ds_λ in y_1 and s_λ respectively.

Consider now a point where J_ϕ is singular, i.e., a point at which

$$J_\phi(x, s, \lambda)r = 0 \quad (40)$$

where $r \neq 0$ is a right eigenvector corresponding to a zero eigenvalue. Let $r = [dx^t ds^t d\lambda^t]^t$. Then from (29) we find that at a singular point

$$J_1(x)dx - ds = 0 \quad (41)$$

$$J_2(x)dx = 0 \quad (42)$$

$$D(s_\lambda)dx + J_1^t(x)ds + J_2^t(x)d\lambda = 0 \quad (43)$$

From (38),(39), it can be seen that (42),(43) are satisfied at all points on the intersection of Σ and the hyperplane $y_2 = y_2^0$.

So, at a point where J_ϕ is singular, a small change dx in x will cause a change $ds = J_1(x)dx$ in s . But (37) indicates that the increment dx would cause y_1 to change by $dy_1 = -J_1(x)dx$. Therefore, at a singular point,

$$ds + dy_1 = 0 \quad (44)$$

We can rewrite (34) as

$$\phi_1(x, s) = -s + y_1^0 + f_1(x) = C \quad (45)$$

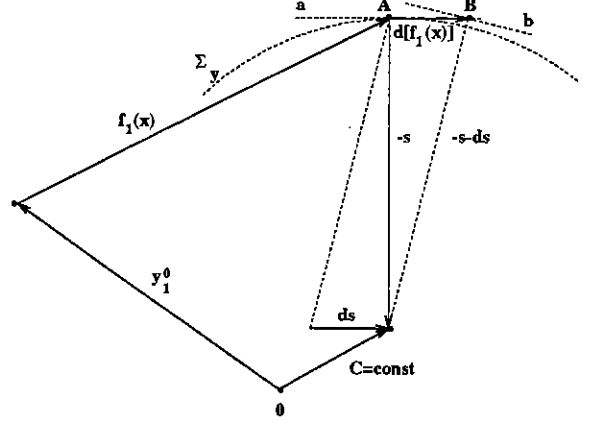


Figure 6: Geometrical interpretation of singularity of J_ϕ on Σ_y .

where $\phi_1(x, s)$ is the first equation of (21), and

$$C = -s + y_1^0 - y_1 \quad (46)$$

is the mismatch in ϕ_1 at the current arbitrary point. (At a solution point of (21), $\phi_1 = 0$.) From (44),(46), we see that at a singular point, the change dC in the mismatch C in response to a change dx is

$$dC = -ds - dy_1 = 0 \quad (47)$$

So, at a point where J_ϕ is singular, $\phi_1(x, s) = C$ remains constant for small changes in x .

Figure 6 provides a geometrical interpretation of behaviour at a singular point. The figure represents the constraint hyperplane $y_2 = y_2^0$, with the intersection of Σ_y plotted. The sum of the vectors y_1^0 and $f_1(x)$ gives the point A on Σ_y . By adding $-s$, the mismatch vector C is obtained. The tangent plane a taken at the point A is an orthogonal plane with respect to s . Variation of x by dx causes a small increment $d[f_1(x)]$ of $f_1(x)$ along Σ_y . This gives the point B, a new eigenvector $-s-ds$, and tangent plane b . The point A on Σ_y is a singular point of J_ϕ if the mismatch vector C is constant, and the vector $-s-ds$ is directed from the new point B to the same point C. It is easy to demonstrate that points of Σ_y form a singular continuum if Σ_y is a part of the hypersphere whose centre is at the point C. The equation of the hypersphere is

$$\|y_1^0 + f_1(x) - C\|^2 - \|s\|^2 = 0 \quad (48)$$

Linearizing (48) gives

$$2s^t(dy_1 + ds) = 0 \quad (49)$$

This equation satisfies the singularity conditions (44).

4.2 Particular cases of singularity

In this section we consider three interesting cases of singularity of J_ϕ .

1. Consider the second case of Section 3.4, where the point of interest is a point of tangency of the constraint hyperplane $y_2 = y_2^0$ and the singular margin Σ_y . The vector s_λ is orthogonal to the hyperplane, so $J^t(x)s_\lambda = J_\phi^t(x)\lambda = 0$. Therefore, if $r = [0 \ 0 \ \lambda^t]^t$, then $J_\phi r = 0$, i.e., r is a right eigenvector of J_ϕ corresponding to a zero eigenvalue. Hence J_ϕ is singular.
2. It is shown in Appendix A that if the power balance functions $f(x)$ are formulated using the rectangular form of voltages (rather than the polar form), J_ϕ is singular at the midpoint of a line in state space connecting any pair of distinct solutions $z_1 = (x_1, s_1, \lambda_1)$ and $z_2 = (x_2, s_2, \lambda_2)$ of (21). Because solutions of the power flow problem (1) are trivial solutions of (21), it follows that J_ϕ is singular at the midpoint of a straight line in state space joining any distinct power flow solutions.
3. Any solution point x of the power flow problem (1) which lies on the singular margin Σ_x corresponds to a singular point of (21). In this case, (21) is satisfied by vectors $x \neq 0$, $s_\lambda = 0$. Therefore, in the Jacobian matrix J_ϕ given at (29), the submatrix $D(s_\lambda)$ is a zero matrix. But because $x \in \Sigma_x$, $\det J(x) = 0$. So

$$\det(J_\phi) = \det^2 J(x) = 0$$

4.3 Analysis of singular points

Once a point of singularity of J_ϕ has been found, it is possible to obtain information about the cause of the singularity. The following approaches are helpful.

The left eigenvector u corresponding to a zero eigenvalue of J_ϕ satisfies

$$J_\phi^t(x, s, \lambda)u = 0 \quad (50)$$

Because $D(s_\lambda)$ is symmetric, it follows that if $r = [dx^t \ ds^t \ d\lambda^t]^t$ satisfies (40), then $u = [ds^t \ d\lambda^t \ dx^t]^t$ satisfies (50). So the left eigenvector u can be obtained by transpositions of elements of the right eigenvector r .

The role of the left eigenvector u in (50) is similar to that of s_λ in (23). Therefore u can provide information about singularity of the critical point problem (21) in the same way that s_λ provides information about singularity of the power flow problem (1). Let $\phi(x, s, \lambda) = q$, i.e., q is the mismatch in the critical point equations ϕ at the point (x, s, λ) . (Of course $q = 0$ at a critical point.) Then u is normal to the surface of singularity of (21) in the space of mismatches q . Therefore normalised values $\tilde{u}_i = u_i/\|u\|$ of the elements of u provide qualitative information about the factors influencing singularity. For example, if some values of $\tilde{u}$ corresponding to ds are large, then the corresponding parameters y_1 have a significant influence on singularity. Large values of $\tilde{u}$ corresponding to $d\lambda$ indicate that parameters y_2 are significant. Likewise, if the values of $\tilde{u}$ corresponding to dx are large, then the power

flow singularity condition (23) has a significant influence on singularity of J_ϕ .

This information can be particularly helpful with path following techniques, such as those used in the algorithm of Section 5. If a singular point is encountered as the path is traversed, the elements of $\tilde{u}$ provide a guide as to the reason why the path cannot be continued.

In fact, in the vicinity of a singular point, an estimate of u can be obtained directly from increments of the numerical technique outlined in Appendix B. To see this, consider the general nonlinear equation

$$g(z, \beta) = 0 \quad (51)$$

If, during variation of the scalar parameter β , a point of singularity is encountered, i.e., a point where

$$u^t \frac{\partial g}{\partial z} = 0 \quad u \neq 0 \quad (52)$$

then the curve $z(\beta)$ tends to the right eigenvector r of $\frac{\partial g}{\partial z}$ [39].

This can be simply proved, as follows. Suppose that the values of z, β at the singular point are z_*, β_* . Denote

$$\left. \frac{\partial g}{\partial z} \right|_{z=z_*, \beta=\beta_*} = J_g^* ; \quad \left. \frac{dg}{d\beta} \right|_{z=z_*, \beta=\beta_*} = \Delta g^* \quad (53)$$

From (51),

$$J_g^* dz + \Delta g^* d\beta = 0 \quad (54)$$

Multiplying (54) by u^t gives

$$u^t J_g^* dz + u^t \Delta g^* d\beta = u^t \Delta g^* d\beta = 0 \quad (55)$$

In general $u^t \Delta g^* \neq 0$. Therefore, at the singular point $d\beta = 0$. From (54),

$$J_g^* dz = 0 \quad (56)$$

The increment dz of the curve $z(\beta)$ at the singular point therefore aligns with the right eigenvector r .

The numerical method given in the Appendix B closely follows the trajectory $z(\beta)$. So, the last increments of variables $\Delta x, \Delta s, \Delta \lambda$ obtained as the singular point is approached can be used to estimate the left eigenvector u ,

$$u \approx [\Delta s^t \ \Delta \lambda^t \ \Delta x^t]^t \quad (57)$$

4.4 The 3 bus example (continued)

A graphical illustration of singularity of J_ϕ for the 3 bus example is given in Figures 7 and 8. Figure 7 shows curves of singularity of the Hessian matrix of (32) (effectively J_ϕ), and of the Jacobian of (31). The latter are shown as dashed lines. Intersections of these singularity curves, i.e., points of singularity of J_ϕ on Σ , are marked as T1...T10. Figure 8 shows the solution space boundary Σ_y in parameter space. The critical points are marked, along with the singular points T1...T10.

It appears that the curves of singularity of the Hessian of (32) restrict the regions of convergence (of 'traditional' numerical techniques) that surround solutions of (32). Notice in Figure 8 that along Σ_y , a point of singularity of J_ϕ lies between each solution of the critical point problem. Further exploration of these ideas is required.

5 An algorithm for finding critical points

The equations (21) that describe critical points also have solutions that are not of interest, e.g., trivial solutions where $s_\lambda = 0$. Therefore, an algorithm for finding critical points must consist of two parts, (1) a way of obtaining a good estimate of the unknown state variables x, s, λ in the vicinity of the critical point, and (2) a numerical technique that will converge reliably from that initial estimate to the critical point. Such an algorithm is described in this section.

5.1 Stage 1: Obtaining a good initial estimate

The first stage of the critical point algorithm must produce a good estimate of state variables x, s, λ in the vicinity of the critical point. To achieve this, it is necessary to have some idea of the direction in parameter space from the operating point to the desired critical point. In practice this requirement does not restrict the usefulness of the method, as power system operators and planners will usually have a good idea of the way in which parameters of their system, such as loads, vary. Let the estimated loading direction be Δy_1 . Recall $y_2 = y_2^0$.

The initial estimate of the critical point can be taken as the point on the solution boundary Σ in the direction Δy_1 from the operating point. That point is given by

$$\beta \Delta y_1 + y_1^0 + f_1(x) = 0 \quad (58)$$

$$y_2^0 + f_2(x) = 0 \quad (59)$$

$$J^t(x) s_\lambda = 0 \quad (60)$$

$$s_\lambda^t s_\lambda = 1 \quad (61)$$

where β is the loading parameter in the specified direction Δy_1 , and $\|\Delta y_1\| = 1$. An alternative formulation of (60),(61) uses the right eigenvector to achieve the singularity condition, rather than the left eigenvector s_λ . Many techniques have been proposed for solving this problem, for example [2, 3, 4, 5, 6, 7]. In some cases direct methods have been used, whilst others have applied continuation methods [8, 9] to obtain the solution.

The numerical solution technique outlined in Appendix B can be used to obtain the desired point on Σ . Using that technique, the equations (58),(59) are solved for β varying from zero to some large value β_i , where β_i is chosen to ensure that no solutions exists for the parameter value $\beta_i \Delta y_1 + y_1^0$. The characteristics of the method ensure that it follows the line $\beta \Delta y_1$, approaching (but not quite reaching) the point where that line intersects Σ . The values of x, β given by this loading technique are then used as initial estimates of variables for solving (58)-(61). An estimate of s_λ is also required. It is given by $-\Delta y_1$. This procedure gives fast and reliable convergence.

5.2 Stage 2: Motion along the singular margin

Stage one of the algorithm provided us with a point on Σ in the vicinity of the desired critical point. Let that point be $x^*, \beta^*, s_\lambda^* = [s^{*t} \lambda^{*t}]^t$. We now wish to move from that point to the critical point. Consider the equations

$$\alpha(\beta^* \Delta y_1 + s^*) - s + y_1^0 + f_1(x) = 0 \quad (62)$$

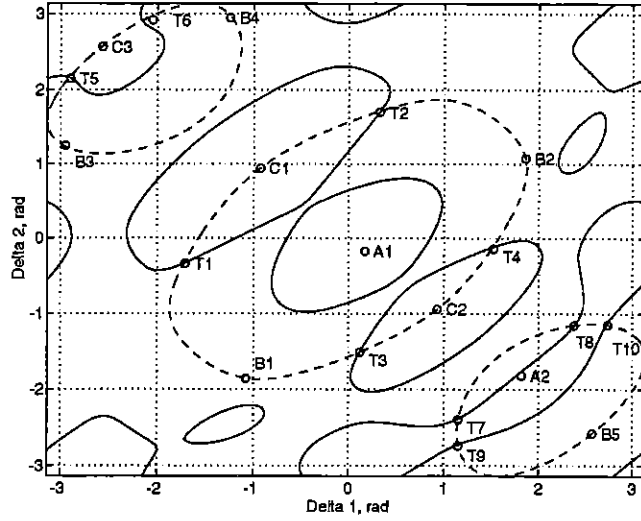


Figure 7: Power flow and critical point singularity curves (state space).

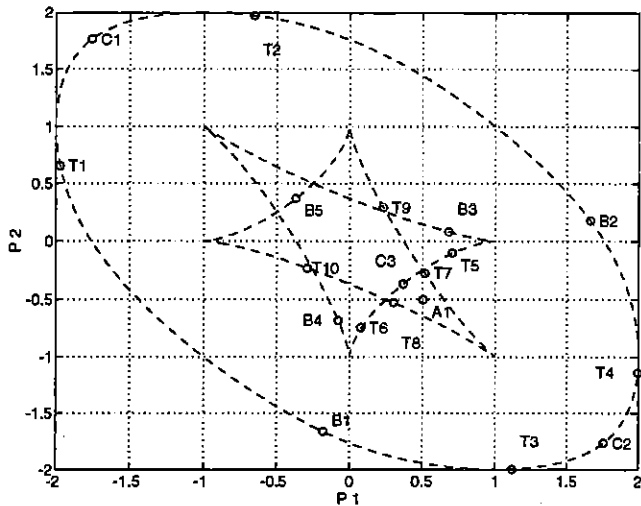


Figure 8: Power flow and critical point singularities (parameter space).

$$y_2^0 + f_2(x) = 0 \quad (63)$$

$$J'(x)s_\lambda = 0 \quad (64)$$

The initial point $x^*, \beta^*, s_\lambda^*$ is a solution of (62)-(64) when $\alpha = 1$. But when $\alpha = 0$, the problem is exactly that of (21). So the critical point is a solution when $\alpha = 0$. Therefore, as α is varied from 1 to 0, the solution of (62)-(64) is distorted from the initial point given by stage one, to the critical point. Notice that because of (64), all points along that path lie on Σ . Also, (63) ensures that the path lies on the $y_2 = y_2^0$ hyperplane. In solving this continuation problem, it is helpful to scale s_λ so that $\|s\| = \beta^*$ at the initial point $p = 0$. This scaled s_λ will still satisfy (64).

This problem has a form which is naturally suited to the numerical solution technique outlined in Appendix B, see (75). That technique has the following features:

- For an appropriate choice of the maximum deviation ϵ_g defined in (88), and the corresponding step sizes α_i , the technique will follow the linear path through parameter space given by $\alpha(\beta^* \Delta y_1 + s^*)$.
- The method always gives a solution if there are no points of singularity along the line $\alpha(\beta^* \Delta y_1 + s^*)$. If a singular point does occur on that line, successive iterations will approach, but never quite reach, that point. The step size α_i decreases to a very small value. The method does not diverge, but is blocked from proceeding further along the path. It is possible that in some cases the singular point could be stepped over, and the solution process continued. Further exploration of these ideas is required.
- Computational time of this method is of the same order as the Newton-Raphson method. The increase in cost of each iteration is offset by the reduction in the number of iterations [32].

This solution method was used for the example given in Section 6.

An alternative approach to moving from the point given by stage one to the critical point makes use of the equations

$$\beta^* \Delta y_1(1 - p) - ps + y_1^0 + f_1(x) = 0 \quad (65)$$

$$y_2^0 + f_2(x) = 0 \quad (66)$$

$$J'(x)s_\lambda = 0 \quad (67)$$

When $p = 0$, the initial point $x^*, \beta^*, s_\lambda^*$ satisfies (65)-(67). But $p = 1$ corresponds to the critical point problem. So in this case, by varying p from 0 to 1, the solution of (65)-(67) is distorted from the stage one point to the desired critical point. Because of (66), (67), this path again traverses the intersection of the $y_2 = y_2^0$ hyperplane and Σ . As with the previous case, it is helpful to scale s_λ such that $\|s\| = \beta^*$ at the initial point $p = 0$. Many numerical techniques exist for solving this continuation problem [7, 8, 9]. Further investigations of the characteristics of this approach are required.

5.3 Locally optimal direction of distance minimization

Stage one of the critical point algorithm provides a point on the intersection of Σ and the $y_2 = y_2^0$ hyperplane. Stage two seeks to minimize the distance $\|y_1 - y_1^0\|$, where $y_1 \in$

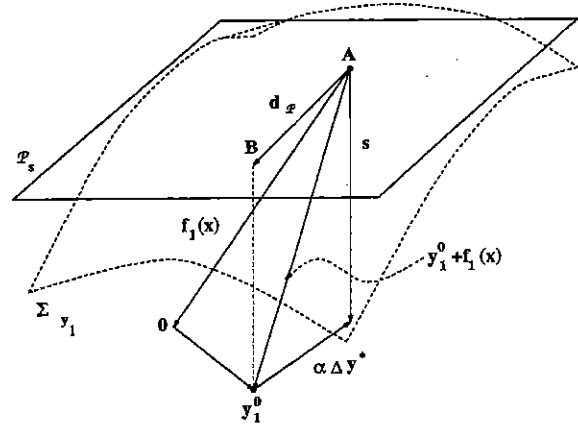


Figure 9: Locally optimal direction of distance minimization.

Σ_{y_1} , by moving from that initial point. It is interesting to consider which direction of movement would (locally) give the greatest reduction in $\|y_1 - y_1^0\|$, whilst still maintaining $y_2 = y_2^0$.

At a point on the power flow solution space boundary Σ_{y_1} , the surface Σ_{y_1} can be approximated (locally) by its tangent hyperplane $\mathcal{P}$. Of interest are the intersections of Σ_{y_1} and $\mathcal{P}$ with the $y_2 = y_2^0$ hyperplane. These intersections shall be referred to as Σ_{y_1} and $\mathcal{P}_s$, respectively. Figure 9 shows a point $y_1 = -f_1(x)$ (marked A) on Σ_{y_1} , and the hyperplane $\mathcal{P}_s$ which is tangent to Σ_{y_1} at that point. It can be seen from Figure 9 that at point B, i.e., the point on $\mathcal{P}_s$ which is closest to the operating point y_1^0 , the vector from y_1^0 to B is orthogonal to $\mathcal{P}_s$. Therefore, if Σ_{y_1} was the plane $\mathcal{P}_s$, the distance $\|y_1 - y_1^0\|$ would be minimized by moving from A to B, i.e., along the vector $d_p \in \mathcal{P}_s$. So d_p gives the locally optimal direction in which to move to minimize $\|y_1 - y_1^0\|$.

We can use these ideas to investigate solution motion for the first solution approach considered for stage two in Section 5.2. The governing equations are (62)-(64). Consider (62), with (63) satisfied at all points. Figure 9 shows that the projection of $y_1^0 + f_1(x)$ onto $\mathcal{P}_s$ coincides with d_p . From Figures 2 and 9, it is clear that s is orthogonal to $\mathcal{P}_s$, so its projection onto $\mathcal{P}_s$ is a point. Therefore, (62) provides a connection between the projections onto $\mathcal{P}_s$ of $\alpha(\beta^* \Delta y_1 + s^*) = \alpha \Delta y^*$ and $y_1^0 + f_1(x)$, viz.,

$$(\alpha \Delta y^*)_{\mathcal{P}} + (y_1^0 + f_1(x))_{\mathcal{P}} = 0 \quad (68)$$

So,

$$-(\alpha \Delta y^*)_{\mathcal{P}} = (y_1^0 + f_1(x))_{\mathcal{P}} = d_p \quad (69)$$

During the solution process, α is varied from 1 to 0. We see from (69) that the component of motion in the $\mathcal{P}_s$ plane is in the (locally) optimal direction. (Note though that there may be a component of motion normal to $\mathcal{P}_s$.)

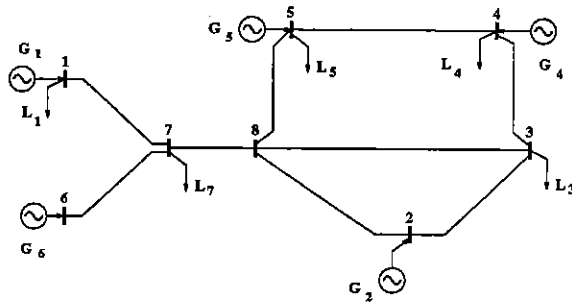


Figure 10: Eight bus test power system.

TABLE 1
BUS PARAMETERS FOR THE 8-BUS SYSTEM

Bus no. p	Generation, voltage		Load	
	Active power MW	Fixed voltage kV	Active power MW	Reactive power Mvar
1	-	-	16.0	10.0
2	32.0	220.0	-	-
3	-	-	64.0	40.0
4	600.0	220.0	-	-
5	-	-	256.0	160.0
6	-	220.0	-	-
7	-	-	1020.0	640.0
8	-	-	-	-

Further, as α reduces from 1 to 0, the vector $(\alpha \Delta y^*)_p$ will reduce to zero. From (68), the vector $(y_1^0 + f_1(x))_p$ will also reduce to zero. It follows that $\|y_1^0 + f_1(x)\| = \|y_1 - y_1^0\|$ must reduce along the continuation path. Therefore this approach will never converge to solutions of (21) that are maxima.

6 An 8 bus example

The algorithm of Section 5 was tested on the eight bus example shown in Figure 10. Operating point values of generation and load parameters are given in Table 1. The system contains three generators with fixed terminal voltage, and four nonzero loads. Bus 6 is the slack bus. The nominal voltage of all buses is 220kV. The electrical network consists of 9 lines. Their parameters are given in Table 2.

The voltage profile at the operating point is given in

TABLE 2
IMPEDANCES FOR THE 8-BUS SYSTEM

Buses p-q	Impedance Z_{pq} , Ohm	Buses p-q	Impedance Z_{pq} , Ohm
1-7	5.41+j20.8	2-3	15.85+j61
2-8	13.9+j53.4	3-4	6.09+j23.4
3-8	18.4+j70.8	4-5	3.38+j21.6
5-8	10.9+j69.9	6-7	2.03+j10.0
7-8	4.35+j27.0		

TABLE 3
COMPLEX BUS VOLTAGES AT THE OPERATING POINT

Node p	Voltage kV	Node p	Voltage kV
1	167.10-j30.71	2	218.13+j28.63
3	202.51+j48.54	4	206.19+j76.72
5	188.92+j37.88	6	220.00+j0.00
7	169.10-j29.41	8	187.75+j8.60

TABLE 4
THE CLOSEST SINGULAR POINTS (CRITICAL POINTS)

Bus p	I-st critical point			II-nd critical point		
	Voltage kV	Vector $y - y^0$ MW Mvar or kV ²	Eigen- vector MW Mvar or kV ²	Voltage kV	Vector $y - y^0$ MW Mvar or kV ²	Eigen- vector MW Mvar or kV ²
	Re/Im			Re/Im		
1	126.7 -27.9	-48.5 0.0	48.5 94.1	81.6 -74.4	-297.8 0.0	297.8 232.0
2	123.3 182.2	0.0 0.0	-238.5 -1.3	219.1 20.4	0.0 0.0	-23.9 -0.5
3	50.3 181.7	0.0 0.0	-217.7 8.2	199.3 45.5	0.0 0.0	-23.5 9.1
4	2.9 219.9	199.8 0.0	-199.8 -0.8	206.0 77.2	25.3 0.0	-25.3 -0.5
5	24.4 173.7	210.4 0.0	-210.4 11.8	184.5 37.9	22.5 0.0	-22.5 12.5
6	220.0 0.0	0.0 0.0	-1263.0 0.0	220.0 0.0	0.0 0.0	-1826.3 0.0
7	133.0 -19.2	0.0 0.0	32.3 90.3	133.9 -42.7	0.0 0.0	64.6 115.8
8	95.3 91.2	0.0 0.0	-188.0 109.6	171.0 1.0	0.0 0.0	-3.6 53.1

Table 3. Due to the lack of reactive power in the system, voltage magnitudes at buses 1, 7, and 8 are very low (0.77pu, 0.78pu, and 0.85pu respectively).

The aim of the example was to determine the closest points on the power flow solution boundary Σ (the critical points), if the real power injections at buses 1, 4, and 5 were free parameters, i.e., allowed to vary from their operating point values. Two critical points were obtained using the algorithm of Section 5. Details of these points are given in Table 4. The length of the vector $y - y^0$ is 294.1MW for the first critical point, and 299.7MW for the second critical point. In both cases the angle between $y - y^0$ (columns 3, 6 in Table 4) and s , which is formed from the elements of the left eigenvector s_λ (columns 4, 7 in Table 4) that correspond to free parameters, is equal to 180deg.

In obtaining these solutions, a number of different loading directions Δy_1 were used in the first stage of the critical point algorithm. These loading directions are given in Table 5 (columns 2 to 4). The loading directions gave different points on the solution boundary Σ . Each of these points was used as the starting point for the second stage of the critical point algorithm. Table 5 (columns 5, 6) shows convergence results for the second stage when the solution technique of Appendix B was used. Column 6 indicates which critical point was converged to, or whether a singular point (s.p.) was encountered. Figure 11 shows trajectories of the second stage solution process. Each trajectory starts from the point on Σ obtained from stage one for the different loading directions. The labels of these starting points correspond to the loading directions given in Table 5.

The following observations were made about the stage two solution process:

TABLE 5
INITIAL DIRECTIONS AND CONVERGENCE OF THE METHOD

Experiment	Initial loading directions			Convergence results	
	ΔP_1 MW	ΔP_4 MW	ΔP_5 MW	Number of iterations	Solution
1	300.0	0.0	0.0	24	I
2	0.0	300.0	0.0	4	I
3	0.0	0.0	300.0	4	I
4	-300.0	0.0	0.0	4	II
5	0.0	-300.0	0.0	-	s.p.
6	0.0	0.0	-300.0	-	s.p.
7	300.0	300.0	0.0	7	I
8	300.0	0.0	300.0	6	I
9	0.0	300.0	300.0	4	I
10	300.0	-300.0	0.0	-	s.p.
11	300.0	0.0	-300.0	-	s.p.
12	0.0	300.0	-300.0	-	s.p.
13	-300.0	300.0	0.0	7	II
14	-300.0	0.0	300.0	8	II
15	0.0	-300.0	300.0	-	s.p.
16	300.0	300.0	300.0	6	I
17	-300.0	300.0	300.0	4	I
18	300.0	-300.0	300.0	22	I
19	300.0	300.0	-300.0	-	s.p.
20	300.0	-300.0	-300.0	-	s.p.
21	-300.0	300.0	-300.0	5	II
22	-300.0	-300.0	300.0	4	II
23	-300.0	-300.0	-300.0	10	II
24	-48.5	199.8	210.4	0	I
25	-297.8	25.3	22.5	0	II

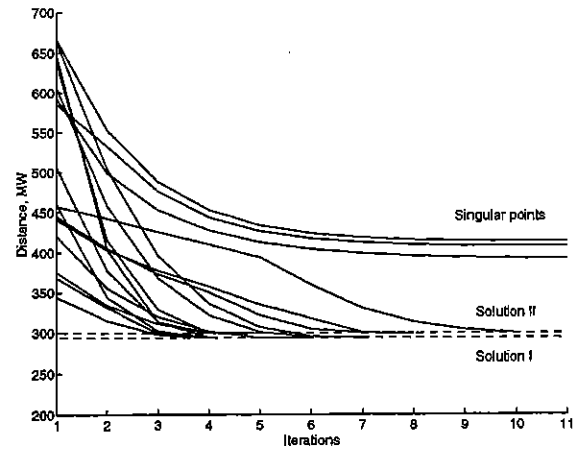


Figure 12: Distance changes during the stage two iterative process.

- The iterative solution process always moved along the solution space boundary Σ , i.e., one eigenvalue of the power flow Jacobian $J(x)$ was always zero.
- The distance $d(y) = \|y - y^0\|$ steadily decreased as the iterative process moved from the initial point on Σ given by stage one, to the final point (either of the critical points, or a singular point). This behaviour is shown in Figure 12.
- In most cases solutions were obtained after 4-8 iterations. More iterations were required (10-24), and singular points were encountered, when inappropriate initial loading directions Δy_1 were chosen.
- Convergence to trivial solutions or maxima never occurred.

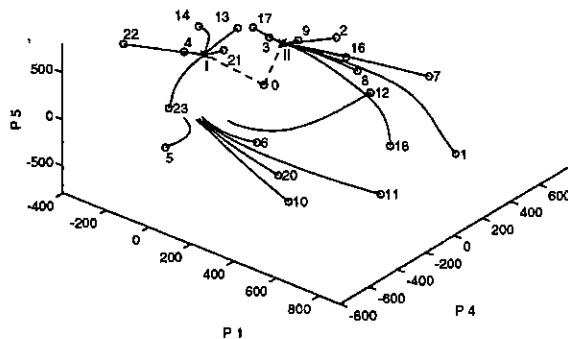


Figure 11: Trajectories of the stage two iterative process in the space of free parameters.

7 Conclusions

The minimum distance from an operating point to the power flow solution space boundary gives a measure of the security of a power system. Points which (locally) provide this minimum distance satisfy a constrained optimization problem. The optimization problem leads to a set of equations which describe such critical points. Not all solutions of this set of equations are critical points however. Trivial solutions correspond to solutions of the usual power flow problem. Other nontrivial solutions describe extrema of the optimization problem that are not of interest. Care must therefore be taken to ensure that algorithms for finding critical points do in fact find the correct type of points.

A two stage algorithm can be used to find critical points. Because there may be many critical points, it is necessary to provide an estimate of the direction in parameter space of the desired critical point. The first stage of the algorithm finds a point on the solution space boundary which lies in that specified direction. The second stage uses a

continuation method to move from that initial point to the desired critical point. Two ways of formulating the continuation problem are given in the paper. A numerical technique that can be used to solve the continuation problem is outlined.

The proposed algorithm converges reliably to desired critical points under normal conditions. However, if a very bad estimate of the direction of the critical point is used, singularity of the Jacobian of the critical point equations may occur. Information contained in the left eigenvector of the Jacobian can be used to determine the nature and cause of the singularity.

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A Midpoint singularity of J_{ϕ}

If voltages are expressed in rectangular form, then $f_1(x)$ and $f_2(x)$ are quadratic functions of x . The Jacobians $J_1(x)$ and $J_2(x)$ are then linear functions of x . It follows that the critical point function $\phi(x, s, \lambda)$ described by (21) is a quadratic function of x, s, λ .

Let $z_1 = (x_1, s_1, \lambda_1)$ and $z_2 = (x_2, s_2, \lambda_2)$ be solutions of (21), with $z_1 \neq z_2$. Then the Taylor series expansion of ϕ yields

$$\begin{aligned}\phi(z_2) &= \phi(z_1) + J_{\phi}(z_1)(z_2 - z_1) + \frac{1}{2}W_{\phi}(z_2 - z_1) \\ \phi(z_1) &= \phi(z_2) + J_{\phi}(z_2)(z_1 - z_2) + \frac{1}{2}W_{\phi}(z_1 - z_2)\end{aligned}\quad (70)$$

where $\phi(z_1) = \phi(z_2) = 0$, and W_{ϕ} is the quadratic term of the expansion. The vector W_{ϕ} has dimension $2n$. Its elements are given by

$$w_i(z_2 - z_1) = \sum_{k=1}^{2n} \sum_{l=1}^{2n} \frac{\partial^2 \phi_i}{\partial z_k \partial z_l} (z_{2,k} - z_{1,k})(z_{2,l} - z_{1,l}) \quad (71)$$

As the elements of the Hessian matrices in (71) are constants, it is clear that

$$W_{\phi}(z_2 - z_1) = W_{\phi}(z_1 - z_2)$$

So, from (70), we get

$$[J_{\phi}(z_1) + J_{\phi}(z_2)](z_1 - z_2) = 0 \quad (72)$$

Because all elements of $J_{\phi}(z)$ are linear functions of z , (72) becomes

$$2J_{\phi}\left(\frac{z_1 + z_2}{2}\right)(z_1 - z_2) = 0 \quad (73)$$

Now $z_1 \neq z_2$, so the matrix J_{ϕ} calculated at the point $(z_1 + z_2)/2$ is singular.

For a pair of distinct trivial solutions of (21), i.e., power flow solutions, we obtain

$$J_{\phi}\left(\frac{x_1 + x_2}{2}, 0, 0\right) \begin{bmatrix} x_1 - x_2 \\ 0 \\ 0 \end{bmatrix} = 0 \quad (74)$$

indicating singularity of J_{ϕ} at the point $(x_1 + x_2)/2$. It should be noted that the Jacobian of the quadratic power flow problem (1) is singular at the same point. It can be proved in the same way [38].

B A numerical solution technique

A numerical technique which can be applied to both stages of the critical point algorithm of Section 5 is outlined in this appendix. The method has a number of characteristics which are very useful for the algorithm. It was originally developed in [29, 30, 31]. Generalizations and improvements were presented in [32].

B.1 Solution motion and its Taylor series expansion

Let us consider a general set of smooth nonlinear equations

$$g(z, \beta) = \beta \Delta g + \underline{g}(z) = 0 \quad (75)$$

where z is a vector of dependent variables, β is a scalar parameter, and Δg is a vector of increments.

If $\frac{\partial \underline{g}}{\partial z}$ is nonsingular, then the function $g(z, \beta)$ can be considered as an implicit function which defines the dependence $z(\beta)$. Differentiation of (75) yields

$$\frac{\partial g}{\partial z} \frac{dz}{d\beta} + \frac{dg}{d\beta} = 0 \quad (76)$$

or

$$\frac{\partial g}{\partial z} \frac{dz}{d\beta} + \Delta g = 0 \quad (77)$$

If the Jacobian matrix $\frac{\partial g}{\partial z}$ is nonsingular, we get the differential equation

$$\frac{dz}{d\beta} = - \left(\frac{\partial g}{\partial z} \right)^{-1} \Delta g \quad (78)$$

The equation (78) defines motion of a solution of (75) as the parameter β varies.

A solution of (78) can be represented as the Taylor series expansion [33]

$$z(\beta) = z^0 + \sum_{k=1}^{\infty} \frac{1}{k!} \left(\frac{d^k z}{d\beta^k} \Big|_{\substack{\beta=\beta^0 \\ z=z^0}} \right) (\beta - \beta^0)^k \quad (79)$$

where $\beta = \beta^0$, $z = z^0$ is a solution of (75). Substituting $\alpha = (\beta^0 - \beta)$ gives

$$z(\alpha) = z^0 + \sum_{k=1}^{\infty} \frac{\alpha^k}{k!} \Delta z_k \quad (80)$$

where

$$\Delta z_k = \frac{d^k z}{d\alpha^k} \Big|_{\substack{\alpha=0 \\ z=z^0}} \quad (81)$$

The expansion (80) represents the solution function $z(\alpha)$ as a polynomial of the scalar parameter α . It can be used in two ways.

1. If $\beta^0 = 1$, and Δg is thought of as a mismatch vector of (75) at the point $z = z^0$, then if the series expansion (80) converges for $\alpha = 1$, it will give a solution of the problem $g(z) = 0$. Using this idea, (80) can be used for solving the power flow problem (1), or for solving the critical point problem (21).
2. For z^0 corresponding to $\beta = \beta^0 = 0$, and any given Δg , (75) can be considered as a loading procedure, with $z(\alpha)$ in (80) giving the loading trajectory in the space of dependent variables z . For example, (80) could be used in the first stage of the proposed critical point algorithm to obtain a point on the power flow singular margin.

Note that the trajectory $z(\alpha)$ given by (80) corresponds to motion along the direct line $(\beta^0 - \beta)\Delta g$ in the space of parameters g .

Due to the possibility of poor convergence of the series (80), and the impracticality of computing an infinite number of Δz_k , the summation (80) must be restricted to a finite number of terms K . Accordingly, (80) becomes an iterative procedure

$$z_{i+1} = z_i + \sum_{k=1}^K \frac{\alpha_i^k}{k!} \Delta z_{k,i} \quad (82)$$

where i is the iteration number and $\Delta z_{k,i}$ is the k -th correction vector. The *correction coefficient* α_i influences convergence reliability. This is discussed further in Section B.3. It can be easily shown that for $K = 1$, (82) corresponds to the Newton-Raphson method with an optimal multiplier. If $K > 1$, then (82) becomes a generalization of the Newton-Raphson method which takes into account nonlinear terms of the Taylor series expansion. The linear approximation of $g(z)$ that is used in the Newton-Raphson method is replaced by an approximation that is nonlinear.

B.2 Computation of the correction vectors Δz_k

Expressions for correction vectors Δz_k can be obtained by successive differentiation of (77) with respect to β . Following the ideas of [32], we set $\alpha = (\beta^0 - \beta) = 1$ in (75) and express $g(z)$ as a Taylor series, so giving

$$(1 - \beta^0)\Delta g = g(z) \\ = g(z^0) + J_g(z^0)\Delta z + \sum_{l=2}^{\infty} \frac{1}{l!} W_l(\underbrace{\Delta z, \dots, \Delta z}_l) \quad (83)$$

where $J_g(\cdot)$ is the Jacobian matrix, and $W_l(\cdot)$ is the l -th order term of the Taylor series. It is shown in [40] that by substituting

$$\Delta z = \sum_{k=1}^K \frac{1}{k!} \Delta z_k \quad (84)$$

into (83), the following expressions can be obtained

$$\begin{aligned} \Delta z_1 &= -J_g^{-1}(z^0)[(\beta^0 - 1)\Delta g + g(z^0)] = J_g^{-1}(z^0)\Delta g \\ \Delta z_2 &= -J_g^{-1}(z^0)[W_2(\Delta z_1, \Delta z_1)] \\ \Delta z_3 &= -J_g^{-1}(z^0)[3W_2(\Delta z_1, \Delta z_2) \\ &\quad + W_3(\Delta z_1, \Delta z_1, \Delta z_1)] \\ &\vdots \end{aligned} \quad (85)$$

$$\Delta z_i = -i! J_g^{-1}(z^0) \left(\sum_{l=2}^i \sum_{\substack{s_1, s_2, \dots, s_K = 0, \dots, i \\ s_1 + s_2 + \dots + s_K = l \\ s_1 + 2s_2 + \dots + Ks_K = i}} W_l(\underbrace{\Delta z_1, \dots, \Delta z_1}_{s_1}, \dots, \underbrace{\Delta z_K, \dots, \Delta z_K}_{s_K}) \right) \\ \frac{1}{(1!)^{s_1} (2!)^{s_2} \dots (K!)^{s_K} s_1! s_2! \dots s_K!}$$

The high order terms $W_l(\cdot)$ in (85) can be expressed through values of the function $g(z)$ [32]. For example, if (75) was a set of quadratic equations, then for $K = 5$ we obtain the following recurrent equalities [32]

$$\begin{aligned} \Delta z_1 &= -J_g^{-1}(z^0)[(\beta^0 - 1)\Delta g + g(z^0)] = J_g^{-1}(z^0)\Delta g \\ \Delta z_2 &= -J_g^{-1}(z^0)[W_2(\Delta z_1, \Delta z_1)] \\ \Delta z_3 &= -J_g^{-1}(z^0)[3W_2(\Delta z_1, \Delta z_2)] \end{aligned} \quad (86)$$

$$\begin{aligned}\Delta z_4 &= -J_g^{-1}(z^0)[3W_2(\Delta z_2, \Delta z_2) + 4W_2(\Delta z_1, \Delta z_3)] \\ \Delta z_5 &= -J_g^{-1}(z^0)[5W_2(\Delta z_1, \Delta z_4) + 10W_2(\Delta z_2, \Delta z_3)]\end{aligned}$$

where

$$W_2(\Delta z_i, \Delta z_j) = \underline{g}(\Delta z_i + \Delta z_j) - \underline{g}(\Delta z_i) - \underline{g}(\Delta z_j) + \underline{g}(0) \quad (87)$$

The expressions (86), (87) are used at each iteration (82) of the method.

B.3 Correction coefficients

To provide reliable convergence of the method, it is necessary to use appropriate values of the correction coefficients α_i in (82). The correct choice of α_i gives direct motion in the space of parameters g . It was shown in [32] that the deviation of the method from the direct line $(\beta^0 - \beta)\Delta g$ can be evaluated by the norm

$$\left\| (\beta^0 - \alpha)\Delta g + \underline{g} \left(z^0 + \sum_{k=1}^K \frac{\alpha^k}{k!} \Delta z_k \right) \right\| \leq \epsilon_g \quad (88)$$

It is clear that for $\alpha = 0$ the norm (88) is equal to zero. Increasing α results in the method taking larger steps, but the deviation (88) can also increase. However, having calculated the correction vectors $z_{k,i}$ at the i -th iteration, and knowing the specified maximum deviation ϵ_g , it is not difficult to obtain the corresponding value of α_i which keeps the deviation (88) within the desired accuracy ϵ_g . If the value of ϵ_g is small enough, the method will converge up to a singular point of (75) [32].

