

IMPROVING CONTINUATION METHODS FOR TRACING BIFURCATION DIAGRAMS IN POWER SYSTEMS

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Abstract: This paper proposes several techniques for reducing the computational burden of tracing bifurcation diagrams in power systems with continuation methods. The first technique described in the paper is an implementation of the fast-decoupled method in the predictor-corrector continuation algorithm. The other proposed techniques consist on reducing the system size. One reduction method is based on the tangent vector calculated during the predictor step, whereas the second one is based on network partition and detection of voltage-weak areas. The computational issues related to the actual implementation of these methods are also discussed. The system reduction techniques are tested in the 300 bus IEEE system, so that these methods can be analyzed in a realistic environment.

Keywords: continuation methods, generic saddle-node bifurcations, system reduction, fast-decoupled methods.

1. Introduction

Voltage stability problems in power systems arise from a variety of reasons and operating conditions, from voltage control problems (taps, automatic voltage regulators, HVDC controls [2]–[6]) to system bifurcations (saddle-nodes, transcriticals, Hopfs [2, 6]–[18]). Local bifurcations are characterized by one or two system eigenvalues lying on the imaginary axis of the complex plane for certain system parameters values, i.e., a real eigenvalue (saddle-node, transcritical, pitchfork bifurcations) or the real part of a complex conjugate pair (Hopf bifurcation) become zero for given values of these parameters. Thus, the power system moves from one stable equilibrium point (s.e.p.) to another as the parameters (load and generation powers, taps, control settings, etc.) change slowly, until certain parameters values are reached and the system “bifurcates,” i.e., the eigenvalues reach the imaginary axis. This phenomenon can be classified as a steady-state stability problem [2]; however the final result is a system-wide dynamic instability [7]–[10].

Bifurcations of one-parameter dynamic systems have been thoroughly studied in differential equation models [19, 20]. Several authors have extended this theory to the analysis of similar phenomena in power system models [8, 9, 10, 11, 15, 16, 17, 18], which are typically represented by a set of differential equations and algebraic constraints. Particularly, in [16, 17] the author thoroughly discusses the conditions and methods used for detecting saddle-node bifurcations in these kinds of models.

Of all local bifurcations, only saddle-nodes and Hopfs are generic, i.e., one expects to encounter these types of bifurcations unless there are some special assumptions regarding the system modeling [19, 20]; this can be demonstrated in power system models for certain system parameters [18]. Therefore,

this paper also concentrates on these two kinds of bifurcations in power networks.

Bifurcations points can be located using direct and/or continuation methods [20]. Direct methods are designed to find the bifurcation from a known operating condition in one solution attempt, whereas continuation methods trace a succession the equilibrium points by automatically changing the parameter value until the bifurcation is encountered, and beyond. The manifold resulting from joining the system equilibria generated by the continuation method is known as the bifurcation diagram (“PV”, “QV”, or “nose” curves are particular cases of these diagrams). For power systems models, direct and continuation methods have been proven to be numerically well-behaved [16, 17]. Direct methods have been shown to be an efficient way of locating saddle-nodes when a system is close to these bifurcations points [6]; however, when the system is far from bifurcation, and especially when all system control and limits are considered, this method tends to fail [6]. Locating Hopf bifurcations with these methods is completely impractical as demonstrated in [18]. On the other hand, continuation methods do not present these difficulties and have the additional advantage of yielding more information than direct methods, since complete bifurcation diagrams for any type of bifurcation can be generated with this technique; however, this method is time consuming, particularly when dealing with large networks [6].

This paper concentrates on discussing several issues associated to the detection of generic saddle-node bifurcations with the continuation method. Locating Hopf bifurcations does not represent a particularly difficult numerical problem when continuation methods are used, since these bifurcations do not produce a singularity of the system equations Jacobian or a change in the number of system equilibrium points; one can detect these bifurcations by monitoring the eigenvalues of system equilibria on the bifurcation diagram using typical eigenvalue computational techniques.

Several authors have proposed different implementations of the continuation method to detect saddle-node bifurcations in power systems [6, 21]–[26]. These programs can also be used to detect other types of bifurcations as shown in [18], since they are slight variations, based on some typical characteristics of power system models, of the techniques described in [20]. In [22], a tangent vector predictor together with a perpendicular intersection corrector are used to trace part of the bifurcation diagram, whereas in [23, 25] a parameterization technique with a fixed parameter corrector are proposed to avoid singularities of the continuation method Jacobians at saddle-node bifurcation points, so that complete bifurcation diagrams can be traced. The authors in [6, 21] use similar predictor and corrector techniques as in [22], but introduce a step length control technique to avoid the need of parameter-

ization when tracing whole bifurcation diagrams, and to efficiently handle system limits and control mode changes, particularly when high voltage direct current (HVDC) systems are included. In [24] an optimal step length control is proposed to deal effectively with Q-limits in generators. Finally, the authors in [26] propose the use of a secant predictor and an arc-length parameterization. Section 2 discusses with detail all of these techniques, concentrating on the advantages and disadvantages of each one of them.

The problem of large CPU times required by the continuation method when tracing bifurcation diagrams was discussed in [6]; little has been reported in the literature, however, on trying to overcome this difficulty. This paper addresses this issue by proposing different techniques to reduce CPU time, so that continuation methods can be efficiently used in a control center environment for “real time” applications. Two distinct techniques are proposed and studied, namely fast-decoupled methods and system reductions. The introduction of fast-decoupled methods into the continuation method is analyzed in detail in section 3, concentrating on the computational issues related to the implementation of these techniques in existing programs. In section 4, two different methods to reduce the number of system buses to trace bifurcation diagrams are proposed and discussed. The system reduction techniques are finally tested in the 300 bus IEEE system, so that several computational issues can be addressed in a realistic environment.

2. Continuation Method

A power system can be modeled using a set of nonlinear ordinary differential equations and algebraic constraints, such as

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{y}, \lambda) \\ \mathbf{0} &= \mathbf{g}(\mathbf{x}, \mathbf{y}, \lambda)\end{aligned}\quad (1)$$

where the vector $\mathbf{x} \in \mathbb{R}^n$ represents the system state variables, the vector $\mathbf{y} \in \mathbb{R}^m$ corresponds to the system variables associated to the nonlinear algebraic constraints, and $\lambda \in \mathbb{R}^+$ is a slow changing parameter that drives the reduced system

$$\dot{\mathbf{x}} = \mathbf{s}(\mathbf{x}, \lambda) = \mathbf{f}(\mathbf{x}, \mathbf{h}(\mathbf{x}), \lambda) \quad (2)$$

to a local generic saddle-node or Hopf bifurcation, if along system trajectories of interest one has a nonsingular Jacobian of the algebraic constraints $D_y \mathbf{g}(\cdot)$. Furthermore, as demonstrated in [16, 17] for this particular assumption, a saddle-node bifurcation point of (2) corresponds to a system equilibrium point where the Jacobian $D_z \mathbf{F}(\mathbf{z}_0, \lambda_0)$ is singular ($\mathbf{F}(\cdot) \triangleq [\mathbf{f}^T(\cdot) \mathbf{g}^T(\cdot)]^T$, $\mathbf{z} \triangleq [\mathbf{x}^T \mathbf{y}^T]^T$, $\mathbf{F}(\mathbf{z}_0, \lambda_0) = \mathbf{0}$). Thus, a bifurcation manifold of differential equations (2) can be traced by monitoring the equilibria of (1).

Voltage profiles of power systems have been typically obtained by calculating a series of equilibrium points of equations (1) with successive power flow solutions [2, 27]. These profiles correspond to parts of the bifurcation diagram. Hence, by solving $\mathbf{F}(\mathbf{z}, \lambda) = \mathbf{0}$ for increasing values of λ , one can easily trace relevant parts of the bifurcation diagram; however, since the system Jacobian becomes singular at a saddle-node bifurcation ($\mathbf{z}_0, \lambda_0$), and no equilibrium points or solutions exist for values of $\lambda > \lambda_0$, one cannot accurately compute λ_0 using this simple technique, because divergence of the Newton-

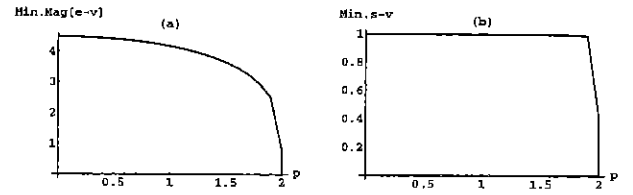


Fig. 1: (a) Minimum magnitude of the eigenvalues and (b) singular values of the s.e.p.s of a simple generator-line-load power system, as a function of the system load P .

Raphson based numerical methods does not guarantee nonexistence of solutions of the corresponding equations. Nevertheless, as shown in [2, 10], this method can be used to obtain solutions rather close to the saddle-node bifurcation point due to the highly nonlinear behavior of the eigenvalues of the system Jacobian. Figure 1 depicts the minimum magnitudes of the eigenvalues and singular values of the steady-state Jacobian for a simple power system constituted of a generator, a transmission line, and a constant power factor PQ load, where the active load power P is chosen as the slow varying parameter that drives the system to bifurcation. Notice the sharp change in the magnitude of the eigenvalues and singular values when the system is close to the bifurcation point. Similar behavior has been also observed by several researchers in a variety of power systems [7, 13, 28]. The Voltage Stability Analysis (VSTAB) program distributed by the Electric Power Research Institute (EPRI), uses this technique to successfully trace the stable equilibrium region of the bifurcation diagram for a variety of power system models [27].

The continuation method overcomes the difficulties of the successive power flow solutions method, allowing the user to trace the complete bifurcation diagram by automatically changing the value of λ . The strategy used in this method is illustrated in Fig. 2, where a known equilibrium point ($\mathbf{z}_1, \lambda_1$) of the bifurcation manifold is used to compute the direction vector $\Delta \mathbf{z}_1$ and a change $\Delta \lambda_1$ of the system parameter. This first step is known as the predictor, since it generates an initial guess ($\mathbf{z}_1 + \Delta \mathbf{z}_1, \lambda_1 + \Delta \lambda_1$), which is then used in the corrector step to compute a new equilibrium point ($\mathbf{z}_2, \lambda_2$) on the bifurcation diagram. Since the Jacobian $D_z \mathbf{F}|_0$ is singular at the bifurcation point, a parameterization is sometimes needed in the predictor and/or corrector steps, depending on the techniques used, to guarantee a well behaved numerical solution of the related equations. A detailed description of these techniques follows.

2.1. Predictor and Parameterization

One way of calculating the direction vector $\Delta \mathbf{z}_1$ at an equilibrium point ($\mathbf{z}_1, \lambda_1$) on the bifurcation manifold, is to compute the tangent vector to the manifold at that point. Hence, since $\mathbf{F}(\mathbf{z}_1, \lambda_1) = \mathbf{0}$, then

$$\begin{aligned}\frac{d\mathbf{F}}{d\lambda}(\mathbf{z}_1, \lambda_1) &= D_z \mathbf{F}(\mathbf{z}_1, \lambda_1) \left. \frac{d\mathbf{z}}{d\lambda} \right|_1 + \left. \frac{\partial \mathbf{F}}{\partial \lambda} \right|_1 = \mathbf{0} \\ \Rightarrow D_z \mathbf{F}|_1 \left. \frac{d\mathbf{z}}{d\lambda} \right|_1 &= - \left. \frac{\partial \mathbf{F}}{\partial \lambda} \right|_1\end{aligned}\quad (3)$$

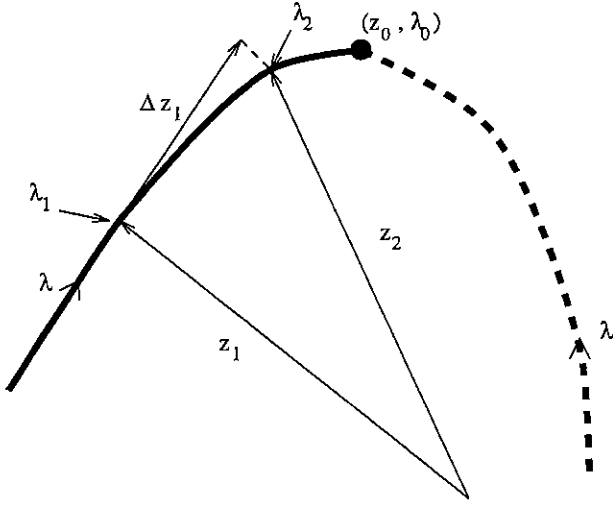


Fig. 2: Bifurcation diagram in state and parameter space depicting the s.e.p.s with a continuous bold line, the u.e.p.s with a bold dashed line, and the bifurcation point at (z_0, λ_0) . A continuation methodology to move from one equilibrium point (z_1, λ_1) to another (z_2, λ_2) is also shown.

Thus, the direction vector and the parameter step come from the normalization of the tangent vector, i.e.,

$$\begin{aligned} \Delta \lambda_1 &= \frac{k}{\|dz/d\lambda|_1\|} \\ \Delta z_1 &= \Delta \lambda_1 \frac{dz}{d\lambda}|_1 \end{aligned} \quad (4)$$

where $k \in \mathbb{R}^+$ is a constant that controls the size of the predictor step. The normalization in (4) results on the reduction of the step size as the system approaches the saddle-node bifurcation point, since the magnitude of the tangent vector increases as the system gets closer to this point. If the step is too large, then the initial guess $(z_1 + \Delta z_1, \lambda_1 + \Delta \lambda_1)$ yield convergence problems in the corrector, whereas if the step is too small, the method takes too many steps to trace the bifurcation manifold. A technique to determine the optimal value of k was proposed in [24], considering the reactive power limits of the generators. Good results were reported in [6] for $k = 1$ in various system sizes, by using step cutting when limits or convergence problems are encountered.

This predictor technique has the particular advantage of generating an approximation to the zero right-eigenvector at the bifurcation point, since the tangent vector smoothly converges to this eigenvector [16, 17]. The zero right-eigenvector yields important information regarding voltage sensitive areas in the system [12, 29]

Computing the tangent vector in (3) does not represent a significant computational cost, since one can use the last factored Jacobian matrix $D_z F(z_1, \lambda_1)$. However, this method has difficulties when the equilibrium point is close to the bifurcation point, since the system Jacobian becomes ill-conditioned. To avoid this problem parameterization techniques can be used [20]. A relatively simple technique successfully applied in [6, 21, 23, 24] is local parameterization, which consists on interchanging close to the bifurcation the parameter λ with the system variable $z_i \in z$ that has the largest normalized entry in the tangent vector, so that λ becomes part of the equations

variables, whereas z_i becomes the new parameter p , i.e., for $N \triangleq n + m$

$$p \leftarrow \max \left\{ \left| \frac{\Delta z_1}{z_1} \right|, \left| \frac{\Delta z_2}{z_2} \right|, \dots, \left| \frac{\Delta z_N}{z_N} \right|, \left| \frac{\Delta \lambda}{\lambda} \right| \right\} \quad (5)$$

Nevertheless, when step cutting and perpendicular intersection correctors are used, this local parameterization is not needed in practice [6, 21], due to the highly nonlinear behavior of the Jacobian eigenvalues; one must be rather close to the saddle-node bifurcation point in order to have an ill-conditioned Jacobian matrix.

Another type of predictor with parameterization used to take the system around the singularity of the bifurcation point is the arclength method [20, 26]. This technique is based on the idea that the system variables and parameter at the equilibrium points can be represented as a function of the arclength s of the bifurcation manifold, i.e., for $F(z_1(s), \lambda_1(s)) = 0$,

$$D_z F|_1 \frac{dz}{ds}|_1 + \frac{\partial F}{\partial \lambda}|_1 \frac{d\lambda}{ds}|_1 = 0 \quad (6)$$

where the arclength s must satisfy the condition

$$\frac{dz}{ds}|_1^T \frac{dz}{ds}|_1 + \frac{d\lambda}{ds}|_1^2 = 1 \quad (7)$$

Therefore, by approximating $\Delta z \approx dz$, $\Delta \lambda \approx d\lambda$, and $k = \Delta s \approx ds$ ($k \in \mathbb{R}^+$), equations (6) and (7) become

$$\begin{aligned} D_z F|_1 \Delta z_1 + \frac{\partial F}{\partial \lambda}|_1 \Delta \lambda_1 &= 0 \\ \Delta z_1^T \Delta z_1 + \Delta \lambda_1^2 &= k \end{aligned} \quad (8)$$

where k is a scalar constant that defines the length of the arc, and consequently the size of the predictor step. Equations (8) can be used to calculate the predictor step instead of equations (3) and (4), with a guaranteed nonsingular Jacobian at the bifurcation point.

Finally, a simpler predictor method that does not require of parameterization is the secant method, which was used in power systems bifurcation analysis in [26]. This technique consists on approximating the tangent vector $dz/d\lambda$ using two or more previously determined equilibria on the bifurcation manifold. Thus, given two equilibrium points (z_1^a, λ_1^a) and (z_1^b, λ_1^b) on the bifurcation diagram, such that $\lambda_1^b > \lambda_1^a$,

$$\frac{dz}{d\lambda}|_1 \approx z_1^b - z_1^a$$

Equations (4) can then be used to calculate the direction vector and the parameter step. Notice that the closer these two points are, within reasonable numerical tolerances, the better the approximation of the tangent vector; however, more equilibrium points have to be computed, taking longer to trace the desired bifurcation diagram. On the other hand, points too far apart generate inadequate approximations of $dz/d\lambda$, yielding initial guesses that lead to convergence difficulties during the corrector step. Using more points on the bifurcation manifold to better predict its curvature, requires of greater computational resources, but can be used as an alternative procedure to approximate the tangent vector when the bifurcation diagram changes direction rapidly, particularly when

control limits are encountered. Hence, depending on the curvature of the bifurcation diagram, the secant method presents advantages and disadvantages with respect to the two predictors previously described. A mixed approach is proposed in [20, 26], using the secant predictor when the tangent vector changes slowly on the “flat” part of the bifurcation diagram, to then switch to tangent vector or arclength predictors when the manifold presents larger curvatures.

2.2. Corrector

Once an initial guess $(z_1 + \Delta z_1, \lambda_1 + \Delta \lambda_1)$ is determined in the predictor step, with or without parameterization, the actual equilibrium point (z_2, λ_2) on the bifurcation manifold must be calculated by solving the following set of equations for z and λ [20]:

$$\begin{aligned} F(z, \lambda) &= 0 \\ \rho(z, \lambda) &= 0 \end{aligned} \quad (9)$$

The first vector equation in (9) corresponds to the steady-state system equations, which have a singular Jacobian $D_z F|_0$ at the saddle-node bifurcation point (z_0, λ_0) . The second scalar equation represents a phase condition $\rho : \mathbb{R}^N \times \mathbb{R} \mapsto \mathbb{R}$ that guarantees non singularity of the corrector equations Jacobian

$$\begin{bmatrix} D_z F & \frac{\partial F}{\partial \lambda} \\ D_z \rho & \frac{\partial \rho}{\partial \lambda} \end{bmatrix}_{(N+1) \times (N+1)}$$

at all equilibria on the bifurcation manifold.

Two different phase conditions $\rho(\cdot)$ have been successfully used in bifurcation studies of power systems. The first condition consists on defining a perpendicular vector to Δz_1 , which starts at $(z_1 + \Delta z_1, \lambda_1 + \Delta \lambda_1)$ and intersects the bifurcation manifold at (z, λ) ; thus,

$$\rho(z, \lambda) = \Delta z_1^T (z - z_1 - \Delta z_1) + \Delta \lambda_1 (\lambda - \lambda_1 - \Delta \lambda_1) \quad (10)$$

This was introduced in [22] and successfully applied to various systems in [6, 21]. This condition does not require of any kind of parameterization to guarantee non singularity of equations (9) for all system equilibria [16, 17].

A simpler phase condition was used in [23, 24, 25, 26], based on the local parameterization of the system around the bifurcation point. In this case, a local parameter p (λ or $z_i \in z$), is set to a constant value, i.e.,

$$\rho(z, \lambda) = p - p_1 - \Delta p_1$$

The parameter p is chosen based on parameterization (5), guaranteeing a nonsingular Jacobian of equations (9) [20].

Of this two corrector techniques, the perpendicular intersection has the advantage of not requiring of parameterization. However, it introduces an almost full row in the Jacobian matrix, which must be taken into consideration during the factorization process to avoid sparsity degradation.

3. Fast-Decoupled Techniques

For certain ac system models, saddle-node bifurcations can be directly detected using load flow equations

$$\begin{aligned} \Delta P(\delta, V, \lambda) &= 0 \\ \Delta Q(\delta, V, \lambda) &= 0 \end{aligned} \quad (11)$$

since these equations have the same saddle-node transversality conditions as differential equations (2) [16, 17]. Hence, continuation methods can be applied to these equations to obtain the desired bifurcation diagram. $\Delta P(\cdot)$ and $\Delta Q(\cdot)$ are a subset of $F(\cdot)$, and represent the active and reactive power mismatches at the system buses; the vectors V and δ stand for the bus phasor voltage magnitudes and angles, respectively.

Equations (11) can be efficiently solved for a given parameter value λ^* using fast-decoupled load flow techniques [30, 31, 32]. Thus, during a Newton-Raphson iteration i one has to solve matrix equation

$$\begin{bmatrix} D_\delta \Delta P|_i & D_V \Delta P|_i \\ D_\delta \Delta Q|_i & D_V \Delta Q|_i \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta V_i \end{bmatrix} = \begin{bmatrix} H_i & N_i \\ M_i & L_i \end{bmatrix} \begin{bmatrix} \Delta \delta_i \\ \Delta V_i \end{bmatrix} = \begin{bmatrix} \Delta P_{i-1} \\ \Delta Q_{i-1} \end{bmatrix} \quad (12)$$

for vector $[\Delta \delta_i^T \Delta V_i^T]^T$, where $\Delta P_{i-1} \triangleq \Delta P(\delta_{i-1}, V_{i-1}, \lambda^*)$, and $\Delta Q_{i-1} \triangleq \Delta Q(\delta_{i-1}, V_{i-1}, \lambda^*)$. Fast-decoupled methods are based first on the assumption that $\Delta V_i \approx 0$ when solving (12), yielding equation

$$B' \Delta \delta_i = \Delta P(\delta_{i-1}, V_{i-1}, \lambda^*) \quad (13)$$

Assuming then that $\Delta P_i \approx 0$, leads to

$$B'' \Delta V_i = \Delta Q(\delta_{i-1} + \Delta \delta_i, V_{i-1}, \lambda^*) \quad (14)$$

Equations (13) and (14) are solved sequentially (BX version). B' (B matrix) and B'' (X^{-1} matrix) are constant matrices defined as [32]:

$$\begin{aligned} B' &\triangleq H|_{\delta=0, V=1} \\ B'' &\triangleq L|_{\delta=0, V=1, R=0} \end{aligned}$$

where $V \angle \delta$ stand for all bus phasor voltages, and R represents all system resistances.

Similar assumptions can be used to solve the following corrector equations using the fast-decoupled method:

$$\begin{aligned} \Delta P(\delta, V, \lambda) &= 0 \\ \Delta Q(\delta, V, \lambda) &= 0 \\ \rho(\delta, V, \lambda) &= 0 \end{aligned}$$

Thus, for the perpendicular intersection equation (10), the matrix equation of the i^{th} Newton-Raphson iteration becomes

$$\begin{bmatrix} H_i & N_i & \frac{\partial \Delta P}{\partial \lambda} \\ M_i & L_i & \frac{\partial \Delta Q}{\partial \lambda} \\ \Delta \delta_1^T & \Delta V_1^T & \Delta \lambda_1 \end{bmatrix}_i \begin{bmatrix} \Delta \delta_i \\ \Delta V_i \\ \Delta \lambda_i \end{bmatrix} = \begin{bmatrix} \Delta P_{i-1} \\ \Delta Q_{i-1} \\ \Delta \rho_{i-1} \end{bmatrix}$$

where $\Delta \delta_1$, ΔV_1 , and $\Delta \lambda_1$ are generated by the predictor step. Then, by assuming that $\Delta \delta_i \approx 0$, $\Delta V_i \approx 0$, $\Delta \rho_i \approx 0$, and $\Delta P_i \approx 0$, respectively, the following fast-decoupled corrector equations can be obtained:

$$\begin{aligned} \Delta \lambda_1 \Delta \lambda_i &= \rho(\delta_{i-1}, V_{i-1}, \lambda_{i-1}) \\ B' \Delta \delta_i &= \Delta P(\delta_{i-1}, V_{i-1}, \lambda_{i-1} + \Delta \lambda_i) \\ B'' \Delta V_i &= \Delta Q(\delta_{i-1} + \Delta \delta_i, V_{i-1}, \lambda_{i-1} + \Delta \lambda_i) \end{aligned} \quad (15)$$

These equations can be solved sequentially to obtain the desired equilibrium point on the bifurcation manifold. For small

changes $\Delta\delta_1$ and ΔV_1 , i.e., small variations in the system variables z , equations (15) can be shown to have similar convergence characteristics as fast-decoupled load flow equations (13) and (14). The latter condition can be met by reducing the predictor's parameter step $\Delta\lambda_1$ as the system approaches a saddle-node bifurcation.

HVDC systems can be included in these fast-decoupled techniques by treating the ac/dc converters at each iteration as constant PQ loads, while updating the system ac bus voltages, and then solving the DC system equations with fixed ac converter voltages to calculate new values of the corresponding ac active and reactive powers; this process is repeated until achieving convergence.

Although the proposed fast-decoupled corrector has not been implemented and tested, the previous analysis justifies incorporating this technique into an existing continuation power flow program. Work is currently underway at the University of Waterloo to add these methods to the PFLOW program originally developed at the University of Wisconsin-Madison [6].

A secant predictor together with the fast-decoupled corrector described above were proposed for inclusion in future versions of EPRI's program VSTAB [27]. The current version of this program does not have a true implementation of a predictor-corrector continuation algorithm; however, the program successfully uses fast-decoupled load flows to determine s.e.p.s on the bifurcation manifold for fixed values of λ , which could be viewed as a fast-decoupled implementation of the fixed parameter corrector described above, without local parameterizations. When the fast-decoupled method fails to converge, the program switches to a full Newton-Raphson iteration.

4. System Reduction

Another approach to reduce the computational burden of tracing bifurcation diagrams and locating bifurcation points, is to reduce the system size while maintaining acceptable levels of accuracy in the calculations. A method to reduce the number of system variables for these types of studies was proposed in [28]; in this case, the system is reduced based on coherency considerations. This paper proposes two different methods to reduce the system size, based on certain local characteristics of saddle-node bifurcations. A discussion of these techniques follows.

4.1. Tangent Vector Technique

Saddle-node bifurcations in power systems can be associated to angle and/or voltage magnitude problems at specific system buses through the zero right-eigenvector [6, 12, 16, 29]. This can be easily justified based on the fact that the tangent vector $dz/d\lambda$ converges to this eigenvector at the bifurcation point [16, 17]; hence, the largest entries on the zero right-eigenvector pinpoint the critical areas and buses affected the most by the bifurcation. On the other hand, small entries on this vector indicate that the bifurcation has little steady-state effect on the corresponding system variables or buses. Based on this observations, the system variables that suffer little changes during the calculation of the bifurcation diagram, can be safely eliminated from the computational process without affecting the final results, i.e., a variable $z_i \in z$ can be kept

constant at its last equilibrium value if

$$|\Delta z_{i1}| \leq u$$

where $\Delta z_{i1} \in \Delta z_1$ in (4), and u is a user defined tolerance. Notice that vector Δz_1 has been already normalized, which avoids errors of elimination of important system variables due to small values of all entries in $dz/d\lambda|_1$, especially for "flat" bifurcation profiles far from the bifurcation. This technique reduces the number of variables and equations used during the continuation process, decreasing consequently the computational burden of tracing bifurcation diagrams.

A problem with this method is the possible elimination of variables associated to system controls with limits, particularly bus voltages with large reactive power support. These voltages change slowly or do not change at all, until voltage control is lost when minimum or maximum limits are reached. Furthermore, depending on the system variations produced by the parameter λ , an area that initially is not considered critical due to small changes on the associated system variables, could become critical when close to bifurcation. Hence, variables that are eliminated in early stages of the continuation process, might become later a source for significant computational errors. To minimize this problem one can allow for the inclusion of these variables as they start changing rapidly; however, this procedure implies having to calculate the tangent vector for the full system every few steps, eliminating some of the computational benefits of the proposed reduction technique. A simpler but effective approach is to carry out a system reduction every U steps of the continuation method.

This reduction technique was implemented into the PFLOW program [6], and tested in the 300 bus IEEE system. The results of these tests and some additional computational issues are presented and discussed in section 5.

4.2. Network Partition

Several network partition techniques have been proposed to reduce the size of the power system for steady-state voltage stability analysis [33, 34, 35]. These methods also yield the critical areas and buses in the system, so that control actions can be taken to steer the system away from the bifurcation point. A new method based on these techniques is proposed here to partition the system and reduce the computational burden of calculating the bifurcation manifold.

The idea is to first detect the critical bus for a given operating point, i.e., determine the load bus with the largest voltage variations when the system parameter λ is changed. This can be easily done using the tangent vector generated by the predictor step. The largest entry on that vector determines the critical system variable and its associated load bus, as discussed above. Once the critical bus is located, an initial critical area is formed with the first neighbors of this bus (level 1). New levels of neighboring buses can be added to this basic critical area to increase its size. The buses that do not belong to the critical area but connect this area to the rest of the system (border buses), are considered as part of the reduced system and are treated as constant voltage buses, i.e., PV buses, based on the assumption that their steady-state voltage magnitudes are not significantly affected by the parameter changes in the system. Hence, to determine a connection level that defines an adequate system partition, one of the following criteria can be used:

- Determine the determinant of the "critical matrix" $\mathbf{D}'$ for the full and partitioned systems, as defined in [34], i.e.,

$$\begin{aligned} \mathbf{D}' &\triangleq \mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B} \\ \Rightarrow \det(\mathbf{D}_z\mathbf{F}|_1) &= \det(\mathbf{D}')\det(\mathbf{A}) \end{aligned} \quad (16)$$

where matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are part of the system Jacobian matrix $\mathbf{D}_z\mathbf{F}|_1$ evaluated at the equilibrium point $(\mathbf{z}_1, \lambda_1)$,

$$\mathbf{D}_z\mathbf{F}|_1 = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

$\mathbf{D}$ is a 2×2 matrix partition corresponding to equations and variables of the critical bus l , thus

$$\mathbf{D} \triangleq \begin{bmatrix} \left. \frac{\partial \Delta P_l}{\partial \delta_l} \right|_1 & \left. \frac{\partial \Delta P_l}{\partial V_l} \right|_1 \\ \left. \frac{\partial \Delta Q_l}{\partial \delta_l} \right|_1 & \left. \frac{\partial \Delta Q_l}{\partial V_l} \right|_1 \end{bmatrix}$$

where $[\Delta P_l(\cdot) \Delta Q_l(\cdot)]^T \in \mathbf{F}(\cdot)$, and $[\delta_l V_l]^T \in \mathbf{z}$. Notice that $\mathbf{D}'$ for the complete and reduced systems in (16), can be obtained by factorizing the corresponding Jacobian matrix $\mathbf{D}_z\mathbf{F}|_1$. Furthermore, since $\mathbf{D}_z\mathbf{F}|_1$ is nonsingular for all equilibrium points other than the saddle-node bifurcation $(\mathbf{z}_0, \lambda_0)$, $\det(\mathbf{D}') \neq 0$ and $\det(\mathbf{A}) \neq 0$ for those equilibria. (At the bifurcation point $\det(\mathbf{D}') = 0$ and $\det(\mathbf{A}) \neq 0$, since $\det(\mathbf{D}_z\mathbf{F}|_0) = 0$ and matrix $\mathbf{A}$ is nonsingular, due to the elimination from the singular Jacobian $\mathbf{D}_z\mathbf{F}|_0$ of the rows and columns with the largest entries in the zero right-eigenvector.) Hence, by monitoring the difference between the determinants of $\mathbf{D}'_F$ for the full system and $\mathbf{D}'_R$ for the reduced system, an adequate partition is assumed to be obtained when

$$|\det(\mathbf{D}'_R) - \det(\mathbf{D}'_F)| \leq \epsilon \quad (17)$$

where ϵ is a positive scalar that defines the desired criterion tolerance.

- Compare the changes in the tangent vector $d\mathbf{z}/d\lambda|_1$ between the full and the reduced system. Thus, when

$$\left\| \left. \frac{d\mathbf{z}_R}{d\lambda} \right|_1 - \left. \frac{d\mathbf{z}_F}{d\lambda} \right|_1 \right\| \leq \epsilon \quad (18)$$

the desired system reduction level is attained. When the tangent vector predictor is used, the computational cost of applying this criterion is lower than calculating matrices $\mathbf{D}'$, since one only needs to solve equation (3) for each reduction level until convergence condition (18) is met.

Of these two criteria, the tangent vector appears to be more sensible to changes in system size than the determinant of $\mathbf{D}'$; however, the norm of $d\mathbf{z}/d\lambda$ seems to have a highly nonlinear behavior with respect to changes in λ , similar to the previously depicted behavior of the Jacobian eigenvalues, whereas the determinant of the reduced matrix associated to the critical bus seems to have an almost linear behavior as shown in [36]. More tests have to be carried out before a definite conclusion can be reached.

This reduction process can be repeated every few steps of the continuation method, and as with the tangent vector reduction method, one expects to obtain a smaller critical area partition as the system approaches bifurcation.

Since the parameter λ is usually chosen to simulate varying load levels throughout the system, the corresponding generator powers must also be allowed to change to supply the desired system load for each value of λ . For the full system this does not represent a problem; all system generators are assigned a participation factor of the total available power, depending on its size and initial active and reactive powers, so that each generator can change its base power to supply the desired load demand. For the reduced system, where some, or all, border buses have been converted from load (PQ) to generator (PV) buses, one must first determine the level of power that is being delivered to or absorbed from the critical area through these buses, so that an appropriate generation participation factor can be defined. Hence, these factors can be calculated by comparing the power change on these buses for two different equilibria on the bifurcation manifold, i.e., for two distinct values of λ the power differences on the border buses yield the desired participation factors.

One problem that still has to be resolved in order to make the proposed method a practical tool, is the handling of voltage control limits, particularly generator Q-limits, which make a significant difference in the shape of the bifurcation manifolds and the location of the corresponding bifurcation points, as illustrated in section 5 and discussed in [4, 6, 21]. Studies are underway to find an appropriate answer to this question, but so far it does not appear to have a simple solution.

The partition technique presents the same problem as the tangent vector reduction method regarding the possible changes in location of the critical area with changes on the parameter λ . Hence, if the critical area at the bifurcation is not part of the reduced system, the final results can have significant errors as demonstrated for the 300 bus IEEE test system below. This is especially true for the partition method, which, contrary to the tangent vector technique, is based on a local area analysis of the system; therefore, this method seems to be better suited to handle problems where changes in λ only affect a particular area in the system.

Due to the unresolved difficulties in the partition method, it was not directly implemented in an existing continuation program. The preliminary results shown in the next section were obtained using the PFLOW program [6] and MATLAB [37].

5. Computational Results

The 300 bus IEEE system was used to test the system reduction techniques described above. This is an ac system divided in 3 areas, with 69 generators and 411 transmission lines and transformers, including 51 under-load tap changer (ULTC) transformers. The tests were carried without enforcing power flow control between areas and treating the ULTCs as regular transformers with fixed taps, to simplify the analysis of the results. The system loads were modeled as constant PQ loads, and the parameter λ was used to simulate active and reactive power load increases throughout the system, i.e., load powers at bus j were simulated as

$$\begin{aligned} P_j &= P_j^0(1 + \lambda) \\ Q_j &= Q_j^0(1 + \lambda) \end{aligned}$$

where P_j^0 and Q_j^0 represent the base load power. For this particular load model, λ represents the loading factor in p.u. or total MVA. The generators were assigned a participation fac-

tor of the load demand based on the initial generated power, without enforcing maximum active power limits. Several tests were run with and without generator Q-limits, as shown below.

The tangent vector reduction technique was incorporated into the PFLOW program, to obtain the results shown in Tables 1 through 4 for several values of the tolerance u and number of steps U in (18). Figures 3-6 depict the bifurcation diagrams corresponding to each of the test cases in Tables 1-4; the tip of these curves correspond to a saddle-node bifurcation. No distinction is made between stable (s.e.p.) and unstable (u.e.p.) equilibrium points on these manifolds, since the eigenvalues were only monitored to detect a singularity or saddle-node bifurcation point; Hopf bifurcations were not tracked on these cases.

Tables 2 and 1, and their corresponding Figs. 4 and 3, show the results obtained for increasing load throughout the whole system, with and without enforcing generator Q-limits, respectively. On the other hand, Tables 4 and 3 with their respective Figs. 6 and 5, depict the results obtained with and without Q-limits when the load is changed in area 3 only. From these results one can conclude, in general, that the smaller the values of u and U , the smaller the number of equations and, therefore, the larger the errors. Also, as the criterion tolerance u and the number of steps U decreases, the CPU time per step of the continuation method tends to decrease, as expected, particularly for the cases where the load is only changing in a small system area. Notice that in some cases the CPU time tends to increase even though the number of equations decreases, this is due to the need of having to refactorize the Jacobian matrix when the equations change; thus, the smaller the value of U , the more often this has to be done along the bifurcation manifold, and consequently the larger the CPU time. In some cases cycling and back-tracking problems were detected, especially when the values of u and U are small, increasing the overall CPU time required to trace the bifurcation manifolds.

Figure 6 depicts the problem of immediate Q-limit instability [4, 5, 6], i.e., the test system becomes immediately unstable when voltage control is lost at generator bus 191.

Table 5 and Fig. 7 depict the results and bifurcation manifold, respectively, obtained with the network partition method when using convergence criterion (17) with $\epsilon = 10^{-4}$. The load powers are assumed to increase in all system load buses without generator Q-limit enforcement. The saddle-node bifurcation λ_{max} values and critical buses shown in Table 5 were obtained using a direct method. CPU times were not determined in this case since the method was not incorporated into a continuation program; the test shown here was run only to depict some of the problems with the partition method. Notice that by using the determinant convergence criterion the system is reduced from 300 buses to 110 (level 9) at the initial operating point, i.e., for $\lambda = 0$. This reduced system does not contain the critical bus at the saddle-node bifurcation point (bus 191), consequently yielding a significant error in the bifurcation manifold depicted in Fig. 5, and in the location of the corresponding bifurcation point. More tests must be run to determine whether tangent vector criterion (18) yields better results, and to resolve the problem of how to handle voltage control limits.

Case No.	Reduction Data	Eqns.	Time [sec.]	Time/step [sec.]	λ_{max} [p.u.]	λ Error [%]
1	$u = 0$ $U = \infty$	600	43.2	1.4897	0.36908	-0.07
2	$u = 10^{-6}$ $U = 2$	596	46.7	1.6103	0.36908	-0.07
3	$u = 10^{-4}$ $U = 2$	460	46.2	1.5931	0.36908	-0.07
4	$u = 10^{-3}$ $U = 10$	407	46.5	1.6034	0.37337	+1.09
5	$u = 10^{-3}$ $U = 5$	395	51.6	1.7793	0.37598	+1.80
6†	$u = 10^{-3}$ $U = 2$	64	67.1	1.7205	0.37968	+2.80

†Cycling problems in the u.e.p. region

Table 1: Results of the tangent vector reduction technique for the 300 bus IEEE system with no limits considered, when the active and reactive load powers are increased throughout the system. The direct method applied to the complete system yields: $\lambda_{max} = 0.36934p.u.$, and bus 191 as the critical bus.

Case No.	Reduction Data	Eqns.	Time [sec.]	Time/step [sec.]	λ_{max} [p.u.]	λ Error [%]
7	$u = 0$ $U = \infty$	600	75.6	1.3263	0.04575	-0.03
8	$u = 10^{-6}$ $U = 2$	598	76.9	1.3491	0.04575	-0.03
9	$u = 10^{-4}$ $U = 2$	582	82.0	1.4386	0.04563	-0.29
10	$u = 10^{-3}$ $U = 10$	513	87.1	0.9678	0.04927	+7.67
11†	$u = 10^{-3}$ $U = 5$	446	114.6	1.6141	0.04943	+8.01
12†	$u = 10^{-3}$ $U = 2$	450	104.4	1.5353	0.04953	+8.23

†Back-tracking

Table 2: Results of the tangent vector reduction technique for the 300 bus IEEE system with limits considered, when the active and reactive load powers are increased throughout the system. The direct method applied to the complete system yields: $\lambda_{max} = 0.04576p.u.$, and bus 576 as the critical bus.

Case No.	Reduction Data	Eqns.	Time [sec.]	Time/step [sec.]	λ_{max} [p.u.]	λ Error [%]
13	$u = 0$ $U = \infty$	600	48.9	1.4382	1.0002	-0.08
14	$u = 10^{-6}$ $U = 2$	451	76.2	2.2412	1.0082	+0.71
15†	$u = 10^{-4}$ $U = 2$	263	47.7	1.1634	1.0466	+4.55
16	$u = 10^{-3}$ $U = 10$	161	49.3	1.1465	1.3928	+39.13
17	$u = 10^{-3}$ $U = 5$	163	36.0	1.0000	1.1085	+10.73
18†	$u = 10^{-3}$ $U = 2$	136	54.8	0.8839	1.9209	+91.89

†Back-tracking

Table 3: Results of the tangent vector reduction technique for the 300 bus IEEE system with no limits considered, when the active and reactive load powers are increased in a particular area of the system only. The direct method applied to the complete system yields: $\lambda_{max} = 1.00105p.u.$, and bus 191 as the critical bus.

Case No.	Reduction Data	Eqns.	Time [sec.]	Time/step [sec.]	λ_{max} [p.u.]	λ Error [%]
19	$u = 0$ $U = \infty$	600	77.8	1.2754	0.12021	0
20	$u = 10^{-6}$ $U = 2$	535	81.0	1.5577	0.12021	0
21	$u = 10^{-4}$ $U = 2$	324	46.5	1.2237	0.12050	+0.24
22	$u = 10^{-3}$ $U = 10$	211	43.2	1.1077	0.12110	+0.74
23	$u = 10^{-3}$ $U = 5$	210	41.5	1.0921	0.12192	+1.42
24	$u = 10^{-3}$ $U = 2$	210	40.0	1.1111	0.12231	+1.75

Table 4: Results of the tangent vector reduction technique for the 300 bus IEEE system with limits considered, when the active and reactive load powers are increased in a particular area of the system only. The system becomes immediately unstable before the saddle-node bifurcation when a maximum Q-limit is reached at bus 191 when $\lambda_{Qlim} = 0.12021p.u.$

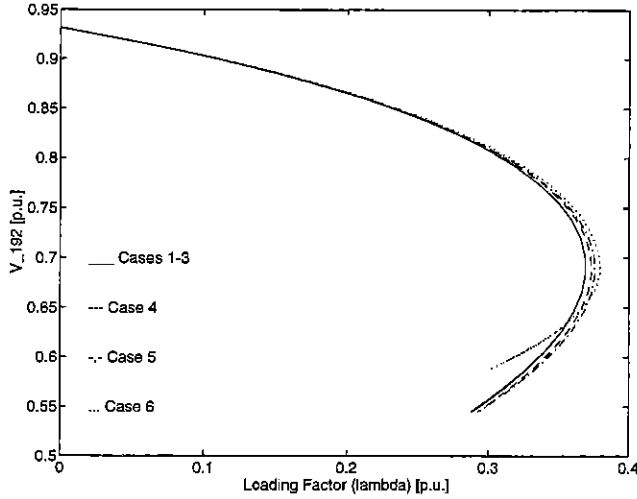


Fig. 3: Bifurcation diagrams (PV or nose curves) of a system bus voltage for the original and reduced systems when no limits are considered. The active and reactive powers increase in all system load buses. A saddle-node bifurcation takes place at the maximum system loading $\lambda_{max} = 0.36934$ p.u.

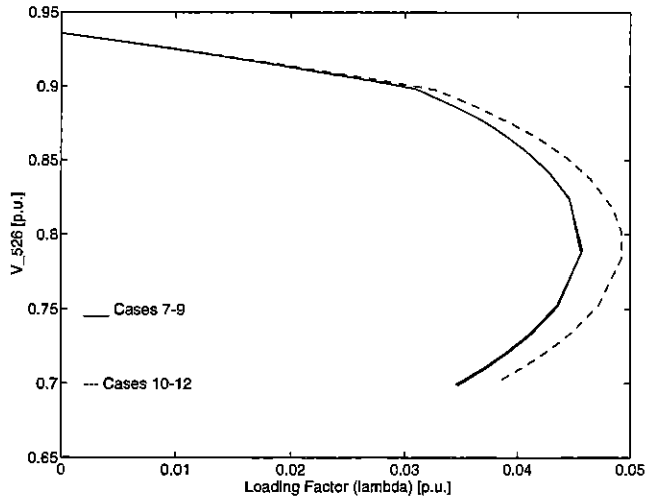


Fig. 4: Bifurcation diagrams (PV or nose curves) of a system bus voltage for the original and reduced systems when limits are considered. The active and reactive powers increase in all system load buses. A saddle-node bifurcation takes place at the maximum system loading $\lambda_{max} = 0.04576$ p.u.

Case No.	Buses	Gens.	Critical Bus	$\det(D')$	λ_{max} [p.u.]	λ Error [%]
1	300	69	191	0.0211	0.36934	0
25	110	36	9025	0.0210	0.59566	+61.28

Table 5: Results of the network partition reduction technique for the 300 bus IEEE system with no generator Q -limits and load increasing throughout the system.

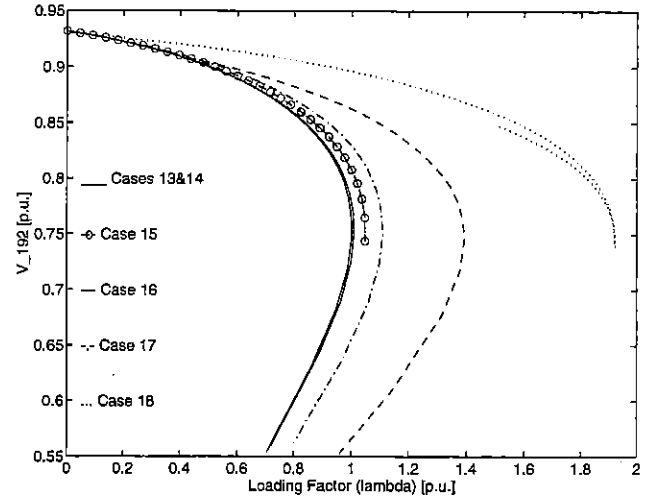


Fig. 5: Bifurcation diagrams (PV or nose curves) of a system bus voltage for the original and reduced systems when no limits are considered. The active and reactive load powers increase in a local area only. A saddle-node bifurcation takes place at the maximum system loading $\lambda_{max} = 1.00105$ p.u.

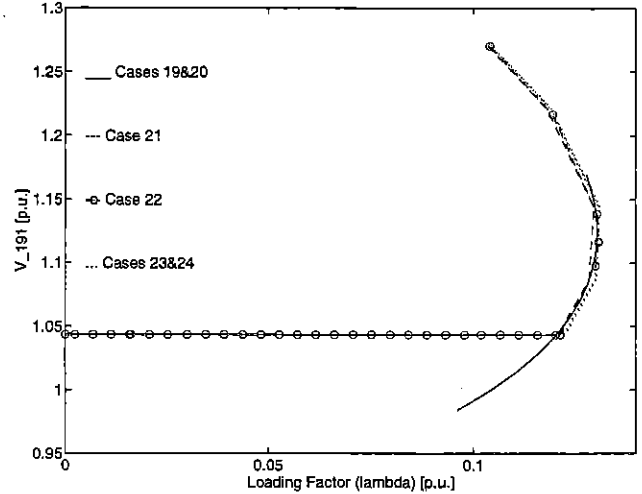


Fig. 6: Bifurcation diagrams (PV or nose curves) of a system bus voltage for the original and reduced systems when limits are considered. The active and reactive load powers increase in a local area only. A Q -limit immediate instability occurs at $\lambda_{Qlim} = 0.12021$ p.u., before the saddle-node bifurcation point.

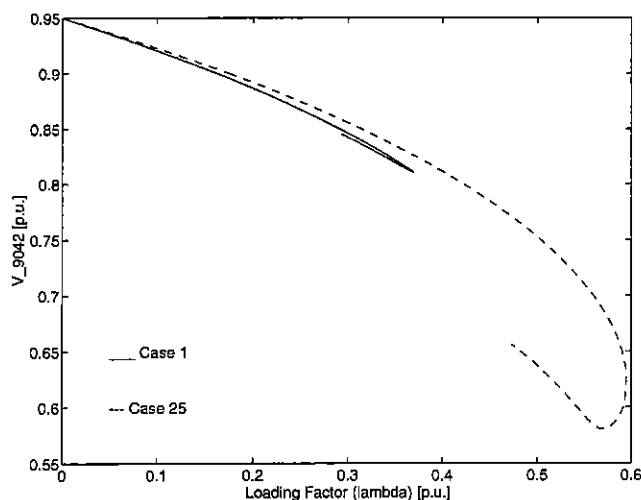


Fig. 7: Bifurcation diagrams (PV or nose curves) of a system bus voltage for the original and partitioned systems when no limits are considered and the active and reactive load powers increase throughout the system. A saddle-node bifurcation takes place at the maximum system loading $\lambda_{max} = 0.36934$ p.u.

6. Conclusions

A comprehensive review of the current literature and state of the art of continuation methods in power systems bifurcation analysis is presented. The paper then proposes three different methods to accelerate the computation of saddle-node bifurcation points and their associated manifolds with the continuation method, namely, fast-decoupled correctors, tangent vector system reduction, and network partition. Of these methods, only the tangent vector was incorporated and tested in a the production-grade continuation program PFLOW; however, additional tests on larger systems are required.

Work is currently underway to include and test the proposed fast-decoupled corrector in PFLOW, based on favorable previous experience with successive fast-decoupled power flows in VSTAB. In the case of system reduction through network partition, several issues, particularly the handling of voltage control limits, have to be resolved before this method can be programmed and tested in realistic conditions. Once all these methods are implemented in a similar computational environment, a thorough comparison of their performance will be carried out to determine their applicability in control centers.

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Determination of Dynamic Voltage Stable Equilibrium Point Based on Nonlinear Optimization Method

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This paper presents a practical method for computing a dynamic voltage stable equilibrium point, which corresponds to an asymptotic stable point, on the basis of the nonlinear optimization method. The problem for finding the dynamic voltage stable equilibrium point is formulated as a nonlinear optimization problem so as to meet the necessary condition for the dynamic voltage stable equilibrium point, and solved by using the successive quadratic programming method which is one of the most powerful methods for solving constrained nonlinear optimization problems. Since the proposed method is based on the nonlinear optimization method, nonlinearities of voltage dynamics in electric power systems can be easily considered. The effectiveness and feasibility of the proposed method are verified by means of simulations using simple models that take dynamics of OLTC and load into consideration.

Keywords: *dynamic voltage stability, stable equilibrium point, nonlinear optimization method, differential equations*

I. INTRODUCTION

With increasing scale and complexity of the power system, the importance of the voltage stability problem in the power system is further being enhanced. Studies hitherto undertaken in connection with voltage stability can be broadly classified into the static analysis and the dynamic analysis. The former consists of studies largely based on the multiple load flow solutions, while the latter consists of studies based on the differential equation stability with the dynamics of generators, loads and the controllers taken into consideration.[1]-[5]

According to the above classification, the present paper comes under the study based on dynamic analysis, and while it provides the definition of the stable equilibrium point having pertinent meaning that takes into account the engineering characteristic and significance of voltage control in the power system, the derivation of the sufficient condition and the necessary condition relative to the sufficient condition, it also proposes the method of calculating the stability equilibrium point on the basis of the nonlinear optimization technique.

Now, as has already been stated, dynamic voltage phenomena in the power system can be described by means of the differential equation determined by the dynamics of the system components. Accordingly, the dynamic voltage stability would be determined by the stability of the differential equation describing the dynamics. However, since

the differential equation that describes the dynamic phenomena of the power system voltage not only includes non-linearity factors in the load flow equation and the load characteristics but it also come to have an important meaning under situations where the voltage stability itself assumes conspicuous non-linearity in the system with heavier load, realization of more realistic analysis and evaluation of dynamic voltage stability should require analysis of the nonlinear differential equation itself, which conserves the essential non-linearity without allowing linearelazation of dynamic phenomena of the voltage.

Incidentally, the concept of stability of the nonlinear dynamic system is, in many cases, based ordinarily on the stability theory of Liapunov, in which the equilibrium point is classified into the three types consisting of stable, asymptotically stable and unstable. However, when the fact that voltage control in the power system fundamentally is the regulation for maintaining the control value constant is taken into account, the equilibrium point that is most important in the analysis and evaluation of voltage stability requires more than the stable equilibrium point of Liapunov that remains in a certain fixed zone against small disturbances; it should rather be called the asymptotically stable equilibrium point of Liapunov that gradually approaches the equilibrium point of itself against small disturbances. In the present paper this asymptotically stable equilibrium point of Liapunov will be defined as the dynamic voltage stable equilibrium point of the power system and the sufficient condition and the necessary condition relative to sufficient condition are given.

On the other hand, obtaining of the asymptotic stable equilibrium point of the nonlinear system means giving extremely important data that form the foundation for structural stability discriminating and stable region determination for the system. The stable equilibrium point has hitherto been calculated by solving, in coalition, the differential equations describing the dynamic phenomena of the system, but under this method the equilibrium point obtained was not necessarily the stable equilibrium point; it could often be the unstable equilibrium point or the saddle point. One other defect of the method was that the volume of calculations rapidly increased along with the scale enlargement of the system, making the application of the method to actual systems extremely difficult.

In the present paper an algorithm based on the nonlinear optimization technique capable of calculating the dynamic voltage stable equilibrium point has been proposed. The proposed technique features these points. It derives the necessary condition relative to the sufficient condition to satisfy the dynamic voltage stable equilibrium point, and composing the equivalent nonlinear optimization problem

that satisfies the necessary condition, it solves the optimization problem, thus making it possible to calculate efficiently the dynamic voltage stable equilibrium point. It is further capable of easily taking into account the non-linearity of the differential equation since the nonlinear programming is used.

If efficient calculation of the dynamic voltage stable equilibrium point is possible, the technique will be highly useful to quantitative analysis and evaluation of voltage stability, to planning of preventive control and emergency control measures for voltage stabilization, to planning of reactive power distributing schedule and further to construction of the linear model around the asymptotic stable equilibrium point.

Subsequently, while defining the dynamic voltage stable equilibrium point, sufficient condition relative to the dynamic voltage stable equilibrium point will be derived and the necessary condition relative to the sufficient condition will additionally be derived in Chapter 2. And continuing into Chapter 3, the method of calculating the dynamic voltage stable equilibrium point will be shown on the basis of the nonlinear optimization programming. Finally, in Chapter 4, the proposed technique will be applied to the typical system that takes the OLTC or the dynamics of the load into consideration to clarify the effectiveness of the proposed technique.

2. DYNAMIC VOLTAGE STABLE EQUILIBRIUM POINT AND ITS EVALUATION

2.1 Dynamic Voltage Stable Equilibrium Point

As has already been stated, the differential equations describing dynamic voltage phenomena of the power system include the non-linearity. In addition, as the voltage stability itself assumed an important meaning under the situation where the non-linearity of the system laboring under heavier load appears conspicuously, it becomes necessary to analyze the nonlinear differential equation itself, which retains the essential non-linearity relative to the dynamic character of the voltage, in order to materialize more realistic analysis and evaluation of the dynamic voltage stability.

Meanwhile, when the fact that voltage control is basically doing the work of a regulator is taken into account, the stable equilibrium point may be taken as Liapunov's asymptotic stable equilibrium point under which the behavior of the system towards the initial point near the equilibrium point undergoes successive approximation to the equilibrium point. In the present paper the Liapunov's asymptotic stable equilibrium point will be defined as the dynamic voltage equilibrium point, the details of which will be given below.

Now, the n -dimension space R^n composed of the state variable $x(t)$, which indicates the dynamic phenomena of the voltage, is considered, and the locus of solution of the state variable

$$x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \quad \dots\dots\dots(1)$$

is taken as the successively differentiable curve c . However, when the fact that the differential equation indicating the voltage dynamics is nonlinear is taken into account, it is clear that examination of the stability of point a in the space R^n must be limited to the vicinity of point a . The vicinity of point a is defined as hereunder.

Definition 1. a is a point in R^n , ϵ is taken as a positive number. ϵ -vicinity $N_\epsilon(a)$ in the point a of R^n means the set formed by the whole of point x whose distance from a is short of ϵ . In other words,

$$N_\epsilon(a) = \{x \in R^n \mid d(x, a) < \epsilon\} \quad \dots\dots\dots(2)$$

provided

$$d(x, a) = \sqrt{\sum_{i=1}^n (x_i - a_i)^2}$$

where a is the point where stability should be decided while ϵ is the range of consideration.

Next, the definition of the dynamic voltage stable equilibrium point will be composed on the basis of the voltage locus of the ϵ -vicinity of point a .

Definition 2. The fact that point a is the dynamic voltage stable equilibrium point in the ϵ -vicinity related to the point sequence of R^n means that the solution $\{x_n\}$ of R^n formed using an optional point in this ϵ -vicinity as the initial point converges with point a as time passes.

The geometric image of the dynamic voltage stable equilibrium point thus defined is shown in Fig.1. As is evident from the figure, the dynamic voltage stable equilibrium point is one of the stabilizing points of the system. Moreover, the locus of the system behavior, when the initial point is taken in the ϵ -vicinity of this point, is the Liapunov's asymptotic stable equilibrium point which approaches this point in all cases. If the system state is that of this stable equilibrium point, it will return to the original equilibrium point without fail even when small disturbances are sustained in any direction.

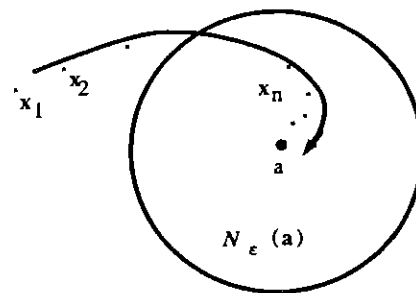


Fig.1. Geometric Representation Of Dynamic Voltage Stable Equilibrium Point.

2.2 Sufficient Condition for Dynamic Voltage Stable Equilibrium Point

In this section the sufficient condition related to the dynamic voltage stable equilibrium point defined in the preceding section will be derived.

Now, the locus of the solution of the state variable

$$x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \quad \dots\dots\dots(3)$$

showing the behavior of the system voltage that indicates the voltage behaviors of the system is considered as the continual and differentiable curve c in the real number space R^n of n dimension. The time t is the parameter.

At this instance, the speed vector of curve c at time t is

$$v(t) = \frac{dx(t)}{dt} = \left(\frac{dx_1}{dt}, \frac{dx_2}{dt}, \dots, \frac{dx_n}{dt} \right)^T = (f_1, f_2, \dots, f_n)^T \quad \dots\dots\dots(4)$$

and the distance function $F(v(t))$ of $v(t)$ is defined as hereunder.

$$F(v(t)) = \sqrt{\sum_{i=1}^n (dx_i(t)/dt)^2} = \sqrt{\sum_{i=1}^n f_i^2} \quad \dots\dots\dots(5)$$

Next, the straight line h that recedes from point $a = x_0$ at the speed of $v(t) = (f_1, f_2, \dots, f_n)^T$ on the constant semi-straight line $\theta = (\theta_1, \theta_2, \dots, \theta_n)^T$ is considered. the h can be expressed as

$$x_i(r) = x_{i0} + r \cos \theta_i \quad \dots\dots\dots(6)$$

using $r(>0)$ as parameter.

Furthermore, the unit vector $r(t)$ is taken as

$$r(t) = \frac{x(t) - x_0}{r} = (\cos \theta_1, \cos \theta_2, \dots, \cos \theta_n)^T \quad \dots\dots\dots(7)$$

However, if the distance function r is defined as

$$r(t) = d(x(t), x_0) = \sqrt{\sum_{i=1}^n (x_i(t) - x_{i0})^2}$$

the variation rate $dv(t)/dr$ of the unit speed vector $v(t)$ against r can be given with the equation that follows.

$$v_r(t) = \frac{dv(t)}{dr} = \left(\frac{df_1}{dr}, \frac{df_2}{dr}, \dots, \frac{df_n}{dr} \right)^T \quad \dots\dots\dots(8)$$

provided df_i/dr is the differential of f_i against r .

$$\frac{df_i}{dr} = \sum_{j=1}^n \cos \theta_j \left(\frac{\partial f_i}{\partial x_j} \right) \quad \dots\dots\dots(9)$$

Also, the inner products of vector $v_r(t)$ and vector $r(t)$ become as given in the equation below.

$$\Pi = \frac{dv(t)}{dr} \cdot r(t) = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial f_i}{\partial x_j} \cos \theta_i \cos \theta_j \quad \dots\dots\dots(10)$$

The relations among these various vectors are shown in Fig.2. As is evident from direct view, if the inner product Π of $v_r(t)$ and $r(t)$ at point b is positive, the locus of the solution of state variable $x(t)$ departs from point b and goes off in the direction opposite from point a . if Π is negative, the locus of $x(t)$ moves towards point a , and if it is zero, the locus of $x(t)$ shifts while intersecting with the vector $r(t)$. As, has already been defined, since dynamic voltage stable equilibrium point means that the locus of system behavior towards all initial points in the $\mathcal{E}$ -vicinity makes asymptotic approach to the equilibrium point, the sufficient condition for being the dynamic voltage stable equilibrium point in the $\mathcal{E}$ -vicinity is as given below.

(1) The speed vector $v(t)$ at point x_0 is zero vector. in short,

$$v(t) \text{ at } x_0 = 0 \quad \dots\dots\dots(11)$$

(2) Against all the angles θ at point x_0 , the inner product Π is non-positive. In other words,

$$\sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial f_i}{\partial x_j} \cos \theta_i \cos \theta_j \right) \text{ at } x_0 \leq 0 \quad \dots\dots\dots(12)$$

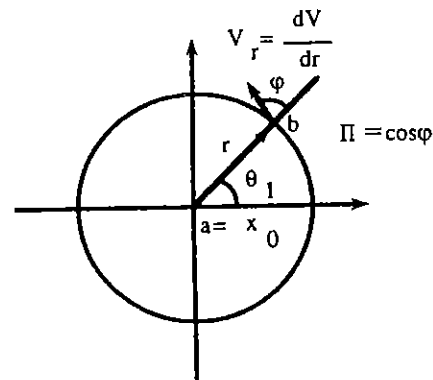


Fig.2. Relation of the Vectors V_r and r .

Provided

$$\sum_{i=1}^n \cos^2 \theta_i = 1$$

If the above two conditions are satisfied, it is self-evident that, the locus of voltage behavior about the initial point in the $\mathcal{E}$ -vicinity will make asymptotic approach to the stable equilibrium point in accordance with $t \rightarrow \infty$. If the initial point is taken in the $\mathcal{E}$ -vicinity of the equilibrium point that satisfied the sufficient condition, the locus of the system will make uniform asymptotic approach without receding from the equilibrium point. accordingly, the equilibrium point that satisfies this sufficient condition will correspond to the local stability nodal point and the stability center point in the topological analysis.

These equations can be used as means of discriminating the asymptotic stability, but since the condition is the sufficient condition to the end, caution is necessary regarding the possible existence of asymptotic stable equilibrium points that do not satisfy the condition. When the existing state of power system operation and control requiring stern reliability is taken into account, it may well be said that it should be more desirable to look for the stable equilibrium point that will satisfy more moderate and more strict conditions.

Incidentally, verification of the soundness of the sufficient condition cited above in connection with the typical example of analytically solving the differential equations that indicate the dynamic characteristics of the system is given in appendix 1.

2.3 Necessary Condition Relative to the Sufficient Condition of the Dynamic Voltage Stable Equilibrium Point

The sufficient condition relative to the dynamic voltage stable equilibrium point in the $\mathcal{E}$ -vicinity that was brought up in the preceding section is the condition for the locus indicating the behavior of the system voltage to make asymptotic approach without at least receding from the equilibrium point along with passage of time, and a moderate but stern condition relative to the asymptotic stable equilibrium point of Liapunov is given. However, as the condition is in the form of complex function taking the inner product for every direction into account, direct incorporation of this condition as the constraint condition of the optimization problem in order to calculate the dynamic voltage stable equilibrium point has become highly difficult from the aspect of calculation volume.

In this paper, thereupon, the sufficient condition derived previously is simplified by projecting the sufficient condition on the n number of intersecting axes of the space of n diminution that correspond the state variable. This simplification can be realized by making only the angle that corresponds to the specified axes to be projected zero and making all the other angles $\pi/2$. In other words, if, in equation (12),

$$\theta_i = 0, \quad \theta_j = \pi/2 \quad j=1,2,\dots,n \quad j \neq i \\ i=1,2,\dots,n$$

The equation cited below is obtained.

$$\frac{\partial f_i}{\partial x_i} \leq 0 \quad i=1,2,\dots,n \quad \dots\dots\dots(13)$$

The above equation requires the locus of the system behavior that has been projected on the orthogonal axes corresponding to the various variables of the space of n dimension to make asymptotic approach to the equilibrium point in uniform manner.

In an intuitive view this means the direction of speed against the displacement of the specific state variable in the vicinity of stable equilibrium point is turned toward this equilibrium point. If, by means of the simplification cited above, the dynamic characteristics of the system are projected on the specific state variable axes, the relationship between the state variable x and its time differential dx/dt (speed) shown in Fig.3 can be obtained. And if point x^* is assumed to be the stable equilibrium point at this juncture, equation(13), which is the simplified condition, requires the speed $\dot{x}$ to be negative if x is larger than x^* and to be positive if x is smaller than x^* .

When attention is directed towards specific state variable only, it means that the system behavior is always making asymptotic approach to the equilibrium point, if the initial point found, when the state variable only has been displaced to the vicinity of the equilibrium point, is selected, but caution is needed regarding the fact that no consideration is given to simultaneous displacement of all the variables. In other words, by projecting on the intersecting state variable axes of the space of n dimension, equation(13), which constitutes the condition related to the simplified dynamic voltage stable equilibrium point, gives the necessary condition relative to the sufficient condition given in equation(12).

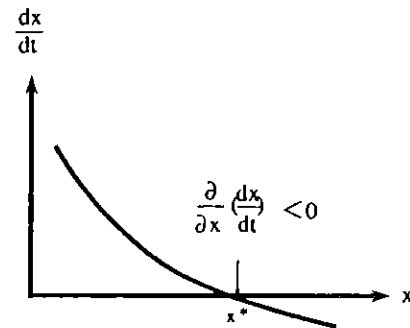


Fig.3. Relation Between x and $\dot{x}$ for specified axis.

With such conditions taken into account, the present paper proposals not a direct use of equation(12), which is the sufficient condition for the dynamic voltage stable equilibrium point, but obtaining of the dynamic voltage stable equilibrium point by the use of equation(13), which is equivalent to this condition as far as possible and is a simple necessary condition. Such simplification of conditions makes possible a substantial reduction in the volume of calculations.

3. METHOD OF CALCULATING THE DYNAMIC VOLTAGE STABLE EQUILIBRIUM POINT BASED ON NONLINEAR OPTIMIZATION TECHNIQUE

3.1. Dynamic Model of Voltage Stability

The general formula cited below are used to indicate the dynamic characteristics of voltages in the power system.

$$\dot{x} = f_1(x, u, \lambda) \quad \text{.....(14)}$$

$$f_2(x, u, \lambda) = 0 \quad \text{.....(15)}$$

Provided,

$$f_1 = (f_{11}, f_{12}, \dots, f_{1n})^T$$

$$f_2 = (f_{21}, f_{22}, \dots, f_{2m})^T$$

$$x = (x_1, x_2, \dots, x_n)^T$$

$$u = (u_1, u_2, \dots, u_k)^T$$

$$\lambda = (\lambda_1, \lambda_2, \dots, \lambda_l)^T$$

Here x is the state vector, u is the control vector and λ is the time changing parameter vector. Moreover, equation(14) is the differential equation showing the dynamics of the system, while equation(15) is the static constraint equation(load flow equation, for instance) of the system. Moreover, equation(14), equation(15) and the selection of the various vectors differ according to the purpose of study and the model.

3.2. Formulations as Nonlinear Optimization Problems

The dynamic characteristics of voltages can be described by means of the behavior of the solution of the first order simultaneous differential equation of equation(14). The dynamic voltage stable equilibrium point defined in the present paper must at least simultaneously satisfy the three conditions cited below.

1) Static constraint equation.

$$f_2(x, u, \lambda) = 0 \quad \text{.....(16)}$$

2) Zero right member of the differential equation(14).

$$f_1(x, u, \lambda) = 0 \quad \text{.....(17)}$$

3) necessary condition of the stable equilibrium point.

$$\frac{\partial f_{1i}}{\partial x_i} \leq 0 \quad \text{.....(18)}$$

One of the method of obtaining the stable equilibrium point that satisfies these three conditions is to obtain the equilibrium point by allying equations(16)and(17) and by examining consecutively as to whether or not the equilibrium point obtained satisfied the stability condition given in equation(18). However, with enlargement of system scales the number of equilibrium points that satisfy equation(16) and (17) rapidly increases. On the other hand, it is difficult for efficiently calculating all the roots (equilibrium points) of nonlinear simultaneous equation, such a method is not necessarily practical.

Meanwhile, since differential equations that express the behaviors of the voltage of the power system contain nonlinear factors in the load flow equations and load dynamics, and, in addition, since the voltage stability itself assumes an important meaning under the situation where the non-linearity of the system appears conspicuously as in the case of heavy load, the nonlinear differential equations that retain essential non-linearity should be analyzed without linearizing the voltage dynamics, in order to materialize more practical analysis and evaluation of dynamic voltage stability.

In the present paper that stands on such a position the equilibrium point that satisfies the three conditions, that is to say, an equivalent nonlinear optimization problem that makes possible efficient calculation of the dynamic voltage stable equilibrium point, is constituted, and by solving this problem, the algorithm capable of efficiently calculating the dynamic voltage stable equilibrium point is proposed. As the proposed algorithm is based on the nonlinear optimization programming, it easily allows consideration of the non-linearity of the differential equation describing the voltage dynamics.

To be concrete, a nonlinear optimization problem with constraints, which takes the square of the left side member of equation(17)as the objective function, equation(16)as the equation constraint and equation(18) as the non-equation constraint, is formulated as given below.

$$\min \sum_{i=1}^n f_{1i}^2(x, u, \lambda) \quad \text{.....(19)}$$

$$\text{subj.to } f_2(x, u, \lambda) = 0 \quad i = 1, 2, \dots, m \quad \text{.....(20)}$$

$$\frac{\partial f_{1j}}{\partial x_j} \leq 0 \quad j = 1, 2, \dots, n \quad \text{.....(21)}$$

In the present paper the nonlinear optimization problem with constraints thus formulated is solved by using the successive quadratic programming method. At each repetition of the original problem in the successive quadratic programming method, the objective function is taken as secondary and the constraint condition is approximated to the linear and the so-called quadratic programming problem; the

method thus consists of successively solving this quadratic programming problem. The successive quadratic programming method is an extension of the quasi-Newton method against the non-constraint optimization method, and it is taken as the most potent method among the nonlinear optimization methods with constraints. An outline of the algorithm of the successive quadratic programming method is given in appendix 2.

4. SIMULATION RESULTS AND EXAMINATION

4.1. Examples of Calculating the 2-Bus System with OLTC Dynamics Taken into Consideration

In this section the calculation method of stable equilibrium point based on the proposed nonlinear optimization techniques will be applied to the simple 2-bus system with two units of OLTC shown in Fig.4, and the possibility of calculating the dynamic voltage stable equilibrium point and the relationship with the unstable equilibrium point will be clarified. Incidentally, the values of the various constants are as show in Table 1, and the meanings of the various symbols are as shows in Table 2.

For simplifying the calculation, all impedance are assumed to be inductive as in the cases of literatures(6) an (7). Namely,

$$Z_a = jZ_a, Z_b = jZ_b, Z_c = jZ_c, Z_1 = jZ_1, Z_2 = jZ_2,$$

Then when the formula of the secondary voltage of OLTC is obtained from the circuit equation and is arranged in order, the equations given below are obtained[7].

$$\frac{dn_1}{dt} = f_1(n_1, n_2) = \frac{g_1(n_1, n_2)}{T_1 \cdot g_3(n_1, n_2)} \quad \dots\dots\dots(22)$$

$$\frac{dn_2}{dt} = f_2(n_1, n_2) = \frac{g_2(n_1, n_2)}{T_2 \cdot g_3(n_1, n_2)} \quad \dots\dots\dots(23)$$

Provided, however,

$$\begin{aligned} g_1 &= (Y_2 + n_2^2/Z_2)V_{10}n_1^2/Z_1 - (H + n_2^2/Z_2Z_a)En_1 \\ &\quad + (H + Y_1n_2^2/Z_2)V_{10} \\ g_2 &= (Y_1 + n_1^2/Z_1)V_{20}n_2^2/Z_2 - (H + n_1^2/Z_1Z_b)En_2 \\ &\quad + (H + Y_2n_1^2/Z_1)V_{20} \\ g_3 &= n_1^2n_2^2/(Z_1Z_2) + Y_1n_2^2/Z_2 + Y_2n_1^2/Z_1 + H \end{aligned}$$

Table.1. Parameter of 2-bus system with OLTC[p.u.]

Za	0.3	V01	0.8
Zb	0.2	V02	1.0
Zc	0.25	E	1.312
Z1	0.2603	Z2	-1.0

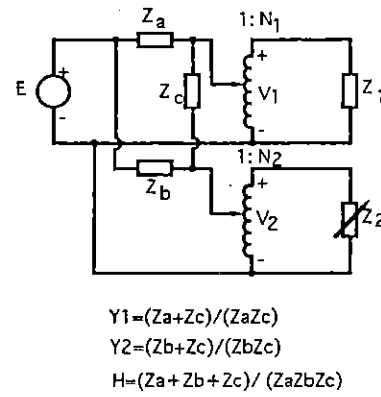


Fig.4. 2-bus test System With OLTC.

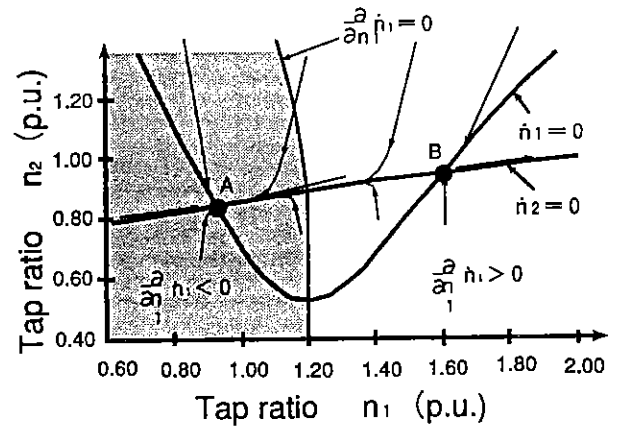


Fig.5. Simulation Result of 2-bus system with OLTC

Before obtaining the dynamic voltage stable equilibrium point of the 2-bus system, a detailed investigation into the types of equilibrium points that are likely to exist by means of the topological analysis using n_1 and n_2 as the state variable is undertaken. The result is shown in Fig.5. The two curves in Fig.5 respectively express $\dot{n}_1=0$ and $\dot{n}_2=0$, and the intersection points of these two curves forms the equilibrium points of this dynamic system. Incidentally, detailed examination of the condition given in equation(13) in relation to this 2-bus system is shown in appendix 3.

Table 2. List of symbols (2-bus system with OLTC).

$N1, N2$	: Tap position of OLTC.
$V10, V20$	: Preset voltage.
$V1, V2$	: Secondary voltage of OLTC.
$T1, T2$	: Time constant of OLTC.
E	: Generator terminal voltage.

In this 2-bus system, equation(13), which is the condition equation related to n_2 , satisfies this condition equation in relation to all n_1, n_2 batten than the equation(sup.3-2) given in appendix 3. For this reason, the "speed" of n_2 in all the regions is turned towards the $n_2=0$ curve. In other words, it means that in this system the asymptotic stability related to the state valuable n_2 in all the regions is satisfied. Equation(13), which is the condition equation related to n_1 , is equation(sub.3-1) of appendix 3, and the curve that divides Fig.5 into two regions is the curve that allows formation of

the equal sign $\partial \dot{n}_1 / \partial n_1 = 0$ of the condition equation(13) relative to n_1 , and it gives a symptotic stability related only to the sate variable n_1 .

Accordingly, the asymptotic stability region n_1 is determined by the asymptotic stability condition related to n_1 and the left half of Fig.5 that is divided by $\partial \dot{n}_1 / \partial n_1 = 0$ asymptotic stable region of this system, And point A forms the dynamic voltage stable equilibrium point defined in this paper.

Now, to obtain the dynamic stable equilibrium point of the 2-bus system described above, it would be sufficient to compose the nonlinear optimization problem stated in the preceding chapter as below and to solve it.

$$\min \quad g_1^2(n_1, n_2) + g_2^2(n_1, n_2) \quad \dots\dots\dots(24)$$

$$\text{subj.to} \quad g_3 \frac{\partial g_1}{\partial n_1} g_1 \frac{\partial g_3}{\partial n_1} \leq 0 \quad \dots\dots\dots(25)$$

$$g_3 \frac{\partial g_2}{\partial n_2} g_1 \frac{\partial g_3}{\partial n_2} \leq 0 \quad \dots\dots\dots(26)$$

And as a result of solving this nonlinear optimization problem from a number of initial points, it has been possible to obtain, in all cases, the point A, of Fig.5, i.e., the dynamic stable equilibrium point.

4.2. Examples of Calculating the 6-bus System

In the preceding section, the proposed technique was applied to the simple 2-bus system having two state variables in order to grasp the intuitive image of the dynamic voltage stable equilibrium point, but in this section application of the proposed technique to the r_1 load bus, $r-r_1$ substation bus, $(n-r-1)$ generator bus and to the n bus system formed with 1 slack bus will be considered.

The group of equations representing the static and dynamic phenomena of power systems giving consideration to load dynamics is as given below.

(1) Load flow equations of load bus

$$-P_{Li} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad \dots\dots\dots(27)$$

$$-Q_{Li} - V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \quad \dots\dots\dots(28)$$

$$i=1, 2, \dots, r_1$$

(2) Load flow equations of bus without load and generator

$$V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad \dots\dots\dots(29)$$

$$V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \quad \dots\dots\dots(30)$$

$$i=r_1+1, r_1+2, \dots, r$$

(3) Load flow equations of generator bus

$$P_{Gi} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad \dots\dots\dots(31)$$

$$V_i = V_{i0} \quad i=r+1, r+2, \dots, n-1 \quad \dots\dots\dots(32)$$

(4) Slack bus

$$V_n \angle \theta_n = V_{n0} \angle 0 \quad \dots\dots\dots(33)$$

(5) Dynamics of load

$$P_{Li}(V_i) = P_{si}(V_i) \quad \dots\dots\dots(34)$$

$$Q_{Li}(V_i, \dot{V}_i) = Q_{si}(V_i) + K_{Qi} \dot{V}_i \quad \dots\dots\dots(35)$$

Provided,

$$P_{si}(V_i) = \alpha_{Pi} V_i^2 + \beta_{Pi} V_i + \gamma_{Pi} \quad \dots\dots\dots(36)$$

$$Q_{si}(V_i) = \alpha_{Qi} V_i^2 + \beta_{Qi} V_i + \gamma_{Qi} \quad i=1, 2, \dots, r_1 \quad \dots\dots\dots(37)$$

If dynamic equations concerning voltages should be derived from equations(28)and(35), the equations given below would be obtained.

$$\dot{V}_i = \frac{1}{K_{Qi}} \{ -V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) - (\alpha_{Qi} V_i^2 + \beta_{Qi} V_i + \gamma_{Qi}) \} \quad i=1, 2, \dots, r_1 \quad \dots\dots\dots(38)$$

From the foregoing, nonlinear optimization problems for obtaining dynamic voltage stable equilibrium point of n -bus system that takes load dynamics into consideration can be composed as below.

$$\min \sum_{i=1}^{r_1} \left\{ V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) + (\alpha_{Qi} V_i^2 + \beta_{Qi} V_i + \gamma_{Qi}) \right\}^2 \quad \text{....(39)}$$

$$\text{subj.to } -\frac{\partial}{\partial V_i} \left(\frac{dV_i}{dt} \right) \geq 0 \quad i=1, 2, \dots, r_1 \quad \text{.....(40)}$$

$$-P_{Li} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad i=1, 2, \dots, r_1 \quad \text{.....(41)}$$

$$V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad \text{.....(42)}$$

$$V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \quad i=r_1+1, r_1+2, \dots, r \quad \text{.....(43)}$$

$$P_{Gi} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad \text{.....(44)}$$

$$V_i = V_{i0} \quad i=r+1, r+2, \dots, n-1 \quad \text{.....(45)}$$

$$V_n \angle \theta = V_{n0} \angle 0 \quad \text{.....(46)}$$

Provided,

$$\frac{\partial}{\partial V_i} \left(\frac{dV_i}{dt} \right) = \left\{ - \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) + V_i B_{ii} - (2\alpha_{Qi} V_i + \beta_{Qi}) \right\} / K_{Qi} \quad i=1, 2, \dots, r_1 \quad \text{.....(47)}$$

The result of seeking the dynamic voltage stable equilibrium point, that takes load dynamics into consideration in the 6-bus system shown in Fig.6 as $T_{kq1} = T_{kq2} = T_{kq3} = 1.2$ on the basis of such formulation, is as show in table 4. table 4 also shows the unstable equilibrium point obtained by solving the nonlinear optimization problem, excluding the asymptotic stability condition, that have been formulated in the above.

Meanwhile, Fig.7 and Fig.8 show variations in every bus voltage and phase angles of buses based on slack bus at the dynamic voltage stable equilibrium point obtained by this technique, when the parameter γ_{Qi} of equation(37) that gives the load its static characteristics (parameters T_{kq1}, T_{kq2} and T_{kq3} in Fig.6 are varied simultaneously and uniformly) is varied. Moreover, variation in the parameter γ_{Qi} corresponds to the variation in the static part of the reactive load.

Table 3. Definitions of the variables(p.u.)

V_i	:	Voltage of the i-th bus.
θ_i	:	Phase angle of the i-th bus voltage
G_{ij}, B_{ij}	:	The real part and the imaginary part of the admittance matrix
P_L, Q_L	:	Active and reactive power supplied to the each load
P_{Gi}	:	Active power of the i-th generator bus.
K_{Qi}	:	Time constant of dynamics of reactive load
$\alpha_{Pi}, \beta_{Pi}, \gamma_{Pi}$	:	Constants of static characteristics of active load

As can be seen from Figs.7 and 8, the bus voltage that corresponds to the dynamic voltage stable equilibrium point gradually declines as the static part of the reactive load increases. Also, the phase angle (based on the slack bus) of the bus corresponding to the dynamic voltage stable equilibrium point gradually increases along with increase in the static part of the reactive load. This trend qualitatively coincides with the characteristics of the so-called high solution which is the stable solution of the nose curve in the static analysis that does not take the dynamic characteristics of the load into consideration.

Incidentally, the time required for calculations for obtaining the dynamic stable equilibrium point in 6-bus systems was about 0.5 second at VAX 9210VP (about 80 MFLOPS)

5. CONCLUSION

In the present paper, while defining the dynamic voltage stable equilibrium point that is important in the dynamic voltage stability analysis and evaluation, its sufficient condition and necessary condition relative to the sufficient condition have been derived. In addition, nonlinear optimization problems that are likely to satisfy the necessary condition are composed, and by applying the successive quadratic programming method, algorithm capable of calculating, at high speed and high efficiency, the dynamic voltage stable equilibrium point has been composed. Furthermore, while showing the effectiveness of the technique by means of simulations using a simple system, brief examination of the dynamic voltage stable equilibrium point obtained has been made.

The features of the proposed technique and of the dynamic voltage stable equilibrium point obtained are as given below.

(1) As nonlinear optimization problem has been formulated so as to satisfy the necessary condition of the sufficient condition relative to the dynamic voltage stable equilibrium point, dynamic voltage stable equilibrium point can be calculated.

(2) As formation of the nonlinear optimization problem has been completed, the non-linearity of the system arising from OLTC and the dynamics of load that has essential meaning in the dynamic voltage stability analysis, for instance, can easily be considered.

Table 4. Simulation result of 6-bus system

Variab	Stable equilibrium point	Unstable equilibrium point
V_1	0.95735	0.54094
V_2	0.87360	0.47177.
V_3	0.87178	0.38380
V_4	0.89037	0.49624
θ_1	-0.31320	-0.98591
θ_2	-0.32734	-1.38695
θ_3	-0.30097	-1.00736
θ_4	-0.23976	-0.65466
θ_5	-0.17927	-1.43389

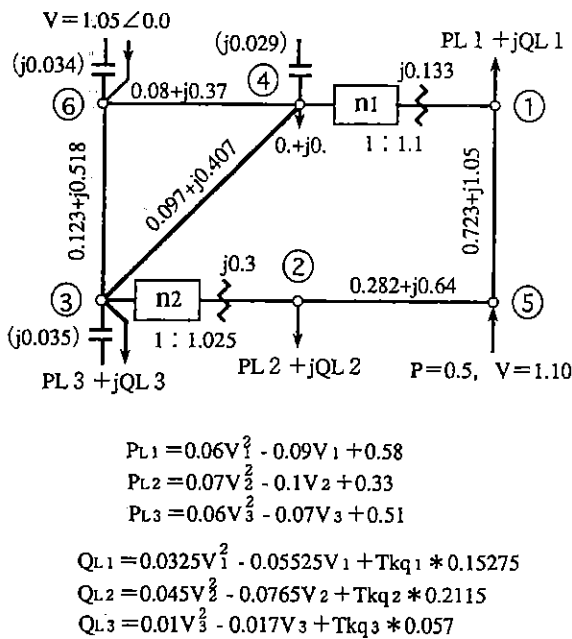


Fig.6. Simulation Result of 6-bus system.

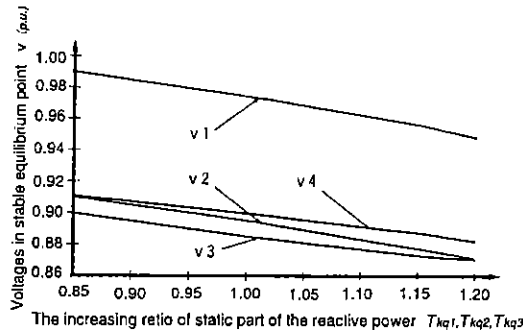


Fig.7. Relation Between Bus Voltage Magnitude And Increase Of Static Part Of Reactive Load.

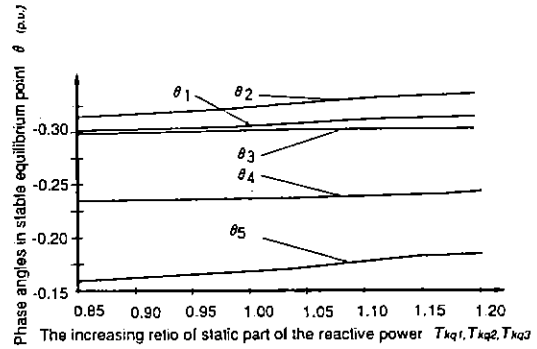


Fig.8. Relation Between Bus Voltage Angle And Increase Of Static Part Of Reactive Load.

(3) As the successive quadratic programming method, which is one of the most potent optimization techniques with constraint, is applied, the dynamic voltage stable equilibrium point can be calculated at high speed and efficiency. For example, the calculation time required for obtaining the dynamic voltage stable equilibrium point in the 6-bus system is VAX 9210VP (about 80 MFLOPS) or about 0.5 second.

(4) The tendency of the voltage and the phase angle of the dynamic voltage stable equilibrium point to change along with increase in the reactive power qualitatively agrees with that of high voltage solution in the static analysis.

This study, moreover, is the basic study of the calculation method of the dynamic voltage stability analysis, and in order to make the technique more practical, all kinds of investigations are needed in the future. Some of the more important themes are as given below.

(1) Consideration of dynamics other than load characteristics and OLTC, and application to large-scale systems.

(2) Possibility of composing the Liapunov function and determination of stable region.

(3) Applications to preventive control, emergency control and reactive power distribution plans.

In conclusion we wish to express our deep gratitude to Prof. Yasuo Tamura of Waseda University for his invaluable guidance in all aspects of the study and for permitting the use of his Test system during prosecution of this study.

Appendix 1: Verification of the Sufficient Condition of the Dynamic Stable Equilibrium Point

The result of discriminating the dynamic stable equilibrium point derived in this study by means of the sufficient condition in connection with the 4 simple examples by which the system behavior can analytically be obtained is as given below.

Example 1. Unstable System

$$\dot{x} = x \quad (\text{general solution} \quad x(t) = x_0 e^t)$$

Discriminating equation; More unstable than

$$\frac{\partial f}{\partial x} = \frac{dx}{dx} = 1 > 0$$

Example 2. Asymptotically Stable System

$$\dot{x} = -\frac{x}{1+t} \quad (\text{general solution } x(t) = \frac{1+t_0}{1+t} x_0)$$

Discriminating equation:

$$\frac{\partial f}{\partial x} = \frac{dx}{d} \left(-\frac{x}{1+t} \right) = -\frac{1}{1+t} < 0$$

More asymptotically stable.

Example 3. Asymptotically Stable System

$$\dot{x} = -x \quad \dot{y} = -y \quad (\text{general solution } x(t) = A e^{-t}, y(t) = B e^{-t})$$

$$\frac{\partial f_1}{\partial x} = -1 \quad \frac{\partial f_2}{\partial y} = -1$$

$$\frac{\partial f_2}{\partial x} = 0 \quad \frac{\partial f_1}{\partial y} = 0$$

Discriminating equation:

$$\Pi = -(\cos^2 \theta + \sin^2 \theta) = -1 < 0$$

More asymptotically stable.

Example 4. Unstable System

$$\dot{x} = x \quad \dot{y} = y \quad (\text{general solution } x(t) = A e^t, y(t) = B e^t) \quad \text{Provided}$$

$$\frac{\partial f_1}{\partial x} = 1 \quad \frac{\partial f_2}{\partial y} = 1$$

$$\frac{\partial f_2}{\partial x} = 0 \quad \frac{\partial f_1}{\partial y} = 0$$

Discriminating equation:

$$\Pi = (\cos^2 \theta + \sin^2 \theta) = 1 > 0$$

More unstable.

Appendix 2: Successive Quadratic Programming Method

Nonlinear optimization problem with equation and non-equation constraint is shown below..

$$\begin{aligned} \min & f(x) \\ \text{subj.to} & g(x) \leq 0 \\ & h(x) = 0 \end{aligned}$$

The successive quadratic programming method's algorithm against the above is as given below.

Step 0: The Lagrange function would be

$$L(x, \lambda, \phi) = f(x) + \lambda^T g(x) + \phi^T h(x)$$

Step 1: Selecting the initial point x^1 and the positive definiteness value symmetric matrix B^1 , $k=1$ is given.

Step 2: Supporting quadratic programming against x^k consisting of

$$\begin{aligned} \min & \frac{1}{2} s^T B^k s + f(x^k) + \nabla f(x^k)^T s \\ \text{subj.to} & \nabla g(x^k)^T s + g(x^k) \leq 0 \\ & \nabla h(x^k)^T s + h(x^k) = 0 \end{aligned}$$

is solved, and obtaining the Lagrange multipliers u^k, v^k corresponding to the solution s^k is obtained. The later are taken as λ^k and ϕ^k respectively.

Step 3: If $s^k < \epsilon$, calculations are concluded, but if not, calculations are continued to step 4.

Step 4: Against suitable step $\alpha^k > 0$, $x^{k+1} = x^k + \alpha^k s^k$ is set up and B^k is given as

$$B^{k+1} = B^k + \frac{q^k q^{kT}}{q^{kT} p^k} - \frac{B^k p^k p^{kT} B^k}{p^{kT} B^k p^k}$$

$$p^k = x^{k+1} - x^k$$

$$q^k = \nabla_x L(x^{k+1}, \lambda^k, \phi^k)^T - \nabla_x L(x^k, \lambda^k, \phi^k)^T$$

The renewal is made using and making $k=k+1$, calculations are reverted to step2.

Appendix 3: Condition of Equation(13) in Relation to n_1 and n_2

The condition of equation(13) in relation to n_1 is

$$T_1 \frac{\partial}{\partial n_1} \left(\frac{dn_1}{dt} \right) = \frac{\partial}{\partial n_1} \left(\frac{g_1}{g_3} \right) = \left(g_3 \frac{\partial g_1}{\partial n_1} - g_1 \frac{\partial g_3}{\partial n_1} \right) / g_3^2$$

With the condition of equation(13), only the symbols of the above equation are the issue, but since $T_1 g_3^2 > 0$, the condition of equation(13) can be judged by means of the symbols of the following equations:

$$\begin{aligned} g_3 \frac{\partial g_1}{\partial n_1} - g_1 \frac{\partial g_3}{\partial n_1} = & (HE + En_1^2 / (Z_2 Z_a)) * ((Y_2 / Z_1 \\ & + n_1^2 / (Z_1 Z_2)) n_1^2 - H - Y_1 n_1^2 / Z_2 \\ & \dots\dots\dots(\text{sub.3-1}) \end{aligned}$$

The condition of equation(13) in relation to n_2 would be

$$T_2 \frac{\partial}{\partial n_2} \left(\frac{dn_2}{dt} \right) = \frac{\partial}{\partial n_2} \left(\frac{g_2}{g_3} \right) = \left(g_3 \frac{\partial g_2}{\partial n_2} - g_2 \frac{\partial g_3}{\partial n_2} \right) / g_3^2$$

and if the fact $T_2 g_3^2 > 0$ is taken into account, distinction can be made by means of the symbols of the following equation as in the case of

$$\begin{aligned} g_3 \frac{\partial g_2}{\partial n_2} - g_2 \frac{\partial g_3}{\partial n_2} = & (HE + En_1^2 / (Z_1 Z_b)) * ((Y_1 / Z_2 \\ & + n_1^2 / (Z_2 Z_1)) n_2^2 - H - Y_2 n_2^2 / Z_1 \\ & \dots\dots\dots(\text{sub.3-2}) \end{aligned}$$

Against constants given in Table 1,

$$\begin{aligned} (HE + En_1^2 / (Z_1 Z_b)) & > 0 \\ (Y_1 / Z_2 + n_1^2 / (Z_2 Z_1)) n_2^2 & < 0 \\ H + Y_2 n_2^2 / Z_1 & > 0 \end{aligned}$$

are given. Therefore, equation(sub.3-2) would be negative in all the regions under the conditions given in Table 1. In other words, it is evident that the condition of equation(13) in relation to n_2 is always satisfied.

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Estimating Multiple Load Flow Solutions with the Householder Transform

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Abstract -This paper proposes a new method for estimating closely-located multiple load flow solutions. The load flow equations have multiple solutions due to the nonlinearity. As power systems become more heavily-loaded, the number of the solutions is inclined to two and eventually the power system approaches a critical point in a static voltage instability sense. Multiple load solutions allow us to evaluate the voltage instability index. However, the calculation of the solutions is required to be more efficient in practice. The proposed method is based on the Householder transform that makes use of "mirror images". The operating solution is mapped into another solution symmetrical with a plane with the origin to estimate the counterpart. Although the transform is linear, this paper focuses on the fact that the center of closely-located multiple load flow solutions is a singular point. The proposed method is tested to demonstrate the effectiveness in sample systems.

Keywords: multiple load solutions, Householder transform, static voltage instability, voltage instability index

I. INTRODUCTION

Voltage instability is one of important issues in power system operation and planning. The voltage instability problem discusses the limitation of reactive power supply. It is known that voltage instability is inclined to occur under heavily-loaded conditions. So far, several approaches are studied for qualitative analysis of the load flow equation[2-3], voltage instability

indices[8-12], [23], [25] evaluation of bifurcation points[13-15], [20], [21], [24], dynamic models[16-17], etc.

This paper focuses on evaluation of the voltage instability indices with multiple load flow solutions. In particular, estimation of the closely-located multiple load flow solutions is discussed to reduce computational effort. It is known that the load flow equation has multiple load flow solutions and is influenced by the initial guess[1-7]. The problem is referred to as the multiple load flow problem. The number of multiple load flow solutions is gradually reduced to a pair of solutions as power system loads become more heavy[6]. Eventually, the load flow equation has the critical point. In fact, relationship between the feasibility of load flow solutions static voltage instability has been investigated. The conventional methods for finding closely-located multiple load flow solutions may be classified as follows:

- 1) Method with combination of initial conditions [4], [19]
- 2) Homotopy continuous method[22]
- 3) Optimal multiplier μ_3 [18]
- 4) Parameterized method[26]

Methods 1), 2) and 4) are time-consuming in evaluating the counterpart. Method 3) is more practical and attractive method in a sense that a scalar multiplier is introduced to obtain a closely-located multiple load flow solution.

In this paper, an alternative method is proposed to estimate the counterpart with the Householder transform. It allows us to map the operating

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solution into the counterpart. The method makes use of the fact that the center of a pair of closely-located solutions is a singular point. It is expected that a good approximate solution is available without calculation of the multiple load flow solutions. The proposed method is tested in sample systems.

II. Multiple Load Flow Solutions

This section outlines an index for static voltage instability with multiple load flow solutions. The load flow equation has a set of solutions due to the nonlinearity. Now, consider the load flow equation for a power system with n -nodes.

$$f(x) = \lambda \quad (1)$$

where

x : nodal voltage (state space) vector($m \times 1$)

f : nonlinear function($m \times 1$)

λ : specified value (parameter space) vector($m \times 1$)
 $m = 2(n-1)$

Fig. 1 shows non-uniqueness of load flow solutions for a given specified value. As power systems become more heavily-loaded, the number of solutions decreases. It is known that the load flow equation has a pair of solutions before approaching the critical point.

Expressing vector x of Eqn. (1) in rectangular coordinates gives the following quadratic equation[2-3], [5]:

$$\lambda = \frac{1}{2} x^T H x \quad (2)$$

where

H : cubic matrix($m \times m \times m$)

Applying the Taylor expansion to Eqn. (2) around $x = x_0$, we have the exact equation due to the quadratic equation of Eqn. (2).

$$\begin{aligned} \lambda &= f(x) = f(x_0 + \Delta x) \\ &= f(x_0) + x_0^T H \Delta x + f(\Delta x) \end{aligned} \quad (3)$$

or

$$\lambda = f(x_0) + J(x_0) \Delta x + f(\Delta x) \quad (4)$$

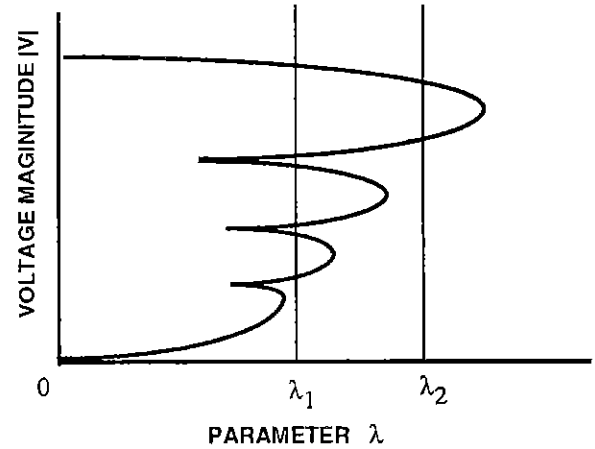


Fig. 1 Relationship between Parameter λ and Voltage Magnitude $|V|$

where

J : Jacobian matrix ($m \times m$) such as

$$J(x_0) = x_0^T H \quad (5)$$

Assuming that Eqn. (2) has a pair of multiple solutions under heavily-loaded conditions, we can express them as follows[5-6]:

$$x_1 = a + b \quad (6)$$

$$x_2 = a - b \quad (7)$$

where

a : singular vector in x -space

Substituting Eqns. (6) and (7) into the Eqn. (4), respectively, we have

$$\lambda = f(a) + J(b)a + f(b) \quad (8)$$

$$\lambda = f(a) - J(b)a + f(b) \quad (9)$$

Adding Eqns. (8) and (9) gives

$$\lambda = f(a) + f(b) \quad (10)$$

It should be noted that vector $f(a)$ corresponds to singular vector in λ -space.

Fig. 2 shows the concept of multiple load flow solutions in state and parameter space,

respectively. In Fig. 2 (a), a pair of multiple solutions, x_1 and x_2 , approaches singular vector a as power system conditions become more heavy-loaded. Similarly, operational power system conditions moves to singular vector $f(a)$ in Fig. 2 (b).

From Eqns.(6) and (7), we have

$$a = \frac{1}{2}(x_1 + x_2) \quad (11)$$

Thus, it can be seen that singular vector corresponds to the center of multiple solutions x_1 and x_2 . Using the fact, this paper estimates x_2 with the Householder transform of x_1 .

A pair of multiple load flow solutions allows us to evaluate global or local voltage instability indices (see Appendix A1). The global indices show the closeness of the power system to the critical point under current system conditions rather than near-future ones. In other words, they indicate a measure of voltage stability at Point A in Fig. 3. As shown in Fig. 3, there is a variety of cases where the power system approach one of singular points, say, $B_1, B_2, B_3, \dots, B_n$ in the figure. This paper aims at identifying weaker nodes at current power system conditions (Point A). The "weaker" nodes in terms of static voltage stability are identified by the local index. For example, Fig. 4 shows the local index at each node. Preventive control is taken to improve voltage stability at the weakest node.

III. HOUSEHOLDER TRANSFORM

This section deals with the Householder Transform that maps a point into another point symmetrical with respect to a plane involving the origin. Making use of the fact that the center of a pair of multiple solutions corresponds to singular point a , the Householder Transform of x_1 allows us to obtain the approximate solution of counterpart x_2 .

Let us consider two vectors u ($m \times 1$) and v ($m \times 1$), and make a assumption that they satisfy the following equation:

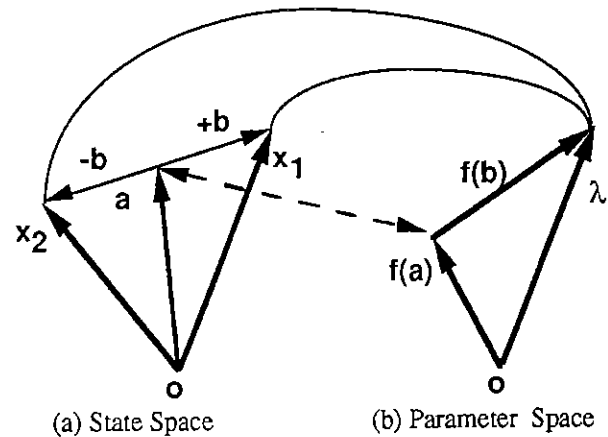


Fig. 2 Concept of Multiple Load Flow Solutions

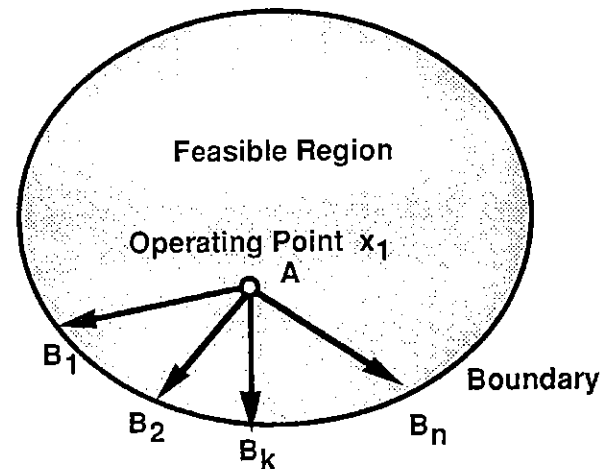


Fig. 3 Operating Point and a Set of Singular Points

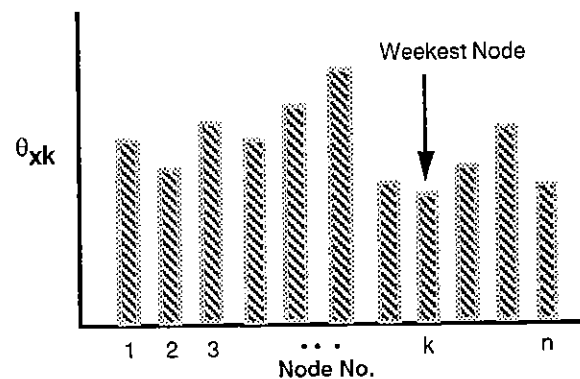


Fig. 4 Local Index at Each Node

$$\|u\| = \|v\| \quad (12)$$

where

$\|\cdot\|$: magnitude of vector •

Under the conditions, we have a linear transformation from u to v .

$$P u = v \quad (13)$$

or

$$P v = u \quad (14)$$

where

P : transform matrix ($m \times m$) defined by

$$P = I - 2 w w^T \quad (15)$$

I : unit matrix ($m \times m$)

w : normalized vector ($m \times 1$) of difference between vectors u and v

$$w = (u - v) / \|u - v\| \quad (16)$$

Fig. 5 show the concept of the Householder Transform. It can be seen that this linear transformation allows point C to map to point D symmetrical with origin O . Point M is the middle point of line CD .

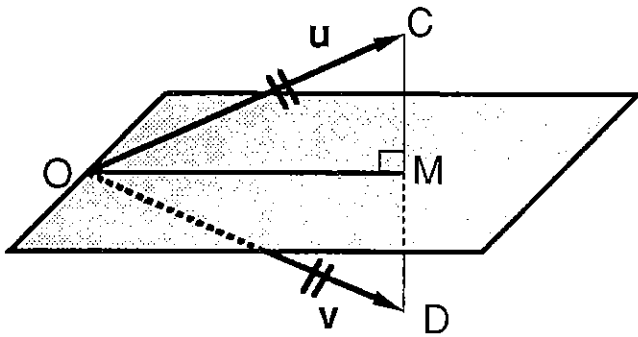


Fig. 5 Householder Transform

IV. PROPOSED METHOD

This section explains the proposed method for estimating counterpart x_2 . Using the Householder transform of load flow solution x_1 , the counterpart is calculated to obtain approximate solution. A scalar is introduced to employ the transform since the magnitude of x_1 is difference from that of x_2 .

Mathematically, the counterpart may be approximated by

$$x_2 \approx P x_1 \quad (17)$$

As shown in Fig. 6, the magnitude of solution x_1 is smaller than that of x_2 . Defining x_2' by

$$x_2' = \alpha x_2 \quad (18)$$

$$\|x_1\| = \|x_2'\| \quad (19)$$

where, α : scalar ($\alpha \geq 1$), we have

$$\alpha = \frac{\|x_1\|}{\|x_2\|} \quad (20)$$

$$x_2' = P x_1 \quad (21)$$

Eqns. (17) and (18) give

$$x_2 = \frac{1}{\alpha} P x_1 \quad (22)$$

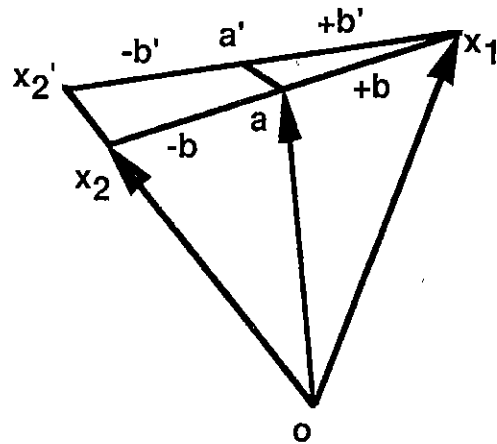


Fig. 6 Geometrical Meaning of Proposed Method

Now, let us define iteration count as k in successively evaluating load flow solution x_1 each time specified value λ is changed. The initial guess of counterpart x_2 may be written as

$$x_2^{(k)} = \frac{1}{\alpha^{(k)}} (I - 2w^{(k)}w^{(k)T})x_1^{(k)} \quad (23)$$

where

$x_2^{(k)}$: initial guess of counterpart x_2 for k -th multiple load flow calculation

$\alpha^{(k)}$: modification coefficient of counterpart x_2 for k -th multiple load flow calculation

$$\alpha^{(k)} = \frac{\|2a^{(k-1)} - x_1^{(k)}\|}{\|x_1^{(k)}\|} \quad (24)$$

$a^{(k-1)}$: known singular vector in state space, say

$$a^{(k-1)} = \frac{1}{2}(x_1^{(k-1)} + x_2^{(k-1)}) \quad (25)$$

$w^{(k)}$: normalized vector for k -th multiple load flow calculation

$$w^{(k)} = \frac{x_1^{(k)} - \alpha^{(k)}(2a^{(k-1)} - x_1^{(k)})}{\|x_1^{(k)} - \alpha^{(k)}(2a^{(k-1)} - x_1^{(k)})\|} \quad (26)$$

Eqn. (23) provides good initial guess of solution x_2 without carrying out the multiple load flow calculation. Using the results, the singular vector is obtained by Eqn. (11).

V. SIMULATION

The proposed method was tested to demonstrate the effectiveness in sample systems. The following simulation conditions were used:

1) The proposed method was applied to the following sample systems:

Stagg & El-Abiad 5 node system

Table 1 Average Error in Sample Systems

No. of Nodes	Average Error[%]
5	2.167×10^{-3}
14	5.267×10^{-4}
30	1.1383×10^{-3}
57	1.114×10^{-4}

IEEE 14 node system

IEEE 30 node system

IEEE 57 node system

The proposed method was compared with the conventional method of the multiple load flow solution for evaluating x_2 [?]. The accuracy of the proposed method was examined by

$$\varepsilon = \frac{1}{m} \sum_i \frac{|x_i^t - x_i^e|}{|x_i^t|} \times 100[\%] \quad (27)$$

where

ε : average absolute error

x_i^t : true value obtained by the conventional method[18]

x_i^e : estimate value obtained by the proposed method

2) The mismatch of the load flow calculation was

$$\max_i \|\Delta\lambda_i\| \leq 0.001 \quad (28)$$

where

$\max\|\cdot\|$: maximum value of $\cdot$

$\Delta\lambda_i$: i -th element of power mismatch vector

$$\Delta\lambda_i (= \lambda - f(x))$$

3) The calculation was performed on Fujitsu FACOM VP2200.

Table 1 shows simulation results in sample systems. The errors are less than 0.003[%] in individual case. In other words, the proposed method is acceptable as an approximation method for estimating multiple load flow solution.

VI. CONCLUSIONS

This paper has proposed a method for estimating multiple load flow solutions. Focusing on the fact that the center of a pair of load flow solutions corresponds to a singular point in state space, this paper applied the Householder Transform to the estimation of the counterpart. The simulation results has shown that the solutions of the proposed method have a good agreement with those of the conventional multiple load flow calculation.

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Appendix

A1. Local Index for Voltage Instability

A pair of multiple load flow solutions is used to provide indices. With the inner product of vectors, they may be expressed as follows:

$$\theta_x = \cos^{-1} \frac{x_1^T x_2}{\|x_1\| \|x_2\|} \quad (A1)$$

$$\theta_\lambda = \cos^{-1} \frac{\lambda^T f(a)}{\|\lambda\| \|f(a)\|} \quad (A2)$$

where,

θ_x : index for voltage instability in state space

x_1, x_2 : multiple load flow solutions for a given specified value λ

θ_λ : index for voltage instability in parameter space

$f(a)$: singular vector in parameter space

The index of Eqn. (A2) is referred to as VIPI[27].

The idea of Eqn. (A1) allows us to define a local index as follows:

$$\theta_{xk} = \cos^{-1} \frac{x_{1k}^T x_{2k}}{\|x_{1k}\| \|x_{2k}\|} \quad (A3)$$

where

θ_{xk} : index at node k for voltage instability in state space

x_{1k}, x_{2k} : multiple load flow solutions at node k

BIOGRAPHIES

Hiroyuki Mori (S'82, M'85) was born in Tokyo, Japan on November 10, 1954. He received the B.S., M.S. and Ph.D. degrees all in electrical engineering from Waseda University, Tokyo, Japan in 1979, 1981, and 1985, respectively. From 1984 to 1985 he was a Research Assistant at Waseda University. In 1985 he joined the faculty in Electrical Engineering at Meiji University, Kawasaki, Japan, where he is currently an Associate Professor. He has been a Visiting Associate Professor at Cornell University, Ithaca, New York, U.S.A since 1994. His research interests include harmonics, voltage instability, load flows, state estimation, system identification and neural network applications. Dr. Mori is a member of ACM, AIAA, INNS, and IEE of Japan.

DISCUSSION - SESSION FOUR

Edited by Eyad H. Abed, University of Maryland

Peter W. Sauer (University of Illinois):

Before continuing with the regular talks in the Analytical Methods session, we'll take some questions for Yakout Mansour's survey talk. Yakout doesn't need to answer them now but can note them and respond to them later during the discussion period.

Christopher L. DeMarco (University of Wisconsin):

This question has to do with a detail, having spent some time with ETMSP, which is your tool for the simulations that you showed. In that tool, the default is a switch in load models when voltage drops below a certain threshold, and in fact the default is about 0.5 pu. This is a switch from nonlinear load models to pure impedance load models. Did you disable that for the simulations that you showed? Did you keep the nonlinear load models all the way through the range of voltage?

Yakout Mansour (B.C. Hydro):

The simulations that we used were point-by-point, changing the load in the load flow.

Christopher L. DeMarco (University of Wisconsin):

So you were changing it on the fly.

Nelson Martins (CEPEL - Centro de Pesquisas de Energia Eletrica, Brazil):

Yakout, congratulations. You showed a ranking list in histogram form for SVC location, and I saw many buses with high participation. That's my experience just the same. And this doesn't happen, for example, for TCSC, series compensation thyristor-controlled devices. Only a few lines come high. We have a scaling problem here which I haven't solved completely yet but that's the point. Thanks for your priorities for extra developments with which I agree, and I find very nice coming from industry. I took notes of all of them. I disagree a little regarding tools for finding points of collapse. I find them very elegant and automatic. They are still very slow. I tried Fernando's (Alvarado) and Claudio's (Cañizares) point of collapse software tool, and it's very slow for large-order systems. But, we are thinking of ways of making it faster, and that's very nice to integrate these automatic planning tools so as to get maximum loadability points and do planning studies based on that. Thank you.

Peter W. Sauer (University of Illinois):

What was the question?

Nelson Martins (CEPEL - Centro de Pesquisas de Energia Eletrica, Brazil):

Well, his comments on my comments.

Robert A. Shlueter (Michigan State University):

Yakout (Mansour) and I would probably disagree on this, but I'd be interested in his answers to it. My experience is that several eigenvalues approach collapse, and each one of them is a group of areas. Our experience is that QV curves computed in a particular area of the system drive one of those eigenvalues to zero. So a QV curve gives a particular picture of why a particular eigenvalue goes to zero, and it goes to zero because a particular set of reactive resources, generators or whatever, hit limits. So, I would say that looking at QV curves is not a loss, but a tremendous gain. It also helps you understand why particular transfers or loading levels will cause very dramatic changes in why the collapse occurs and what you need to do to prevent each and every one of the collapses. Is that consistent with your experience?

Hans Glavitsch (ETH, Zurich):

Yakout (Mansour), if one sees these numbers one wonders if it's necessary to take a model of 5000 buses. I could rephrase the question: How many buses are really affected by a serious voltage instability? Or, in another fashion, how many buses would you have to cut out in order to save the system? Because it's not probable that if you have a serious situation like this that the whole system would drop.

Fernando Alvarado (University of Wisconsin):

Yakout (Mansour), I saw that in your studies you don't just do them, you actually classify them by area in presenting the results, which is very useful; you see patterns. Have you looked into the issue of classification by voltage level as well? In some of our studies that's a very important thing to distinguish. What are you stressing, and do you think a distinction by voltage level would be useful or important?

Nelson Martins (CEPEL - Centro de Pesquisas de Energia Eletrica, Brazil):

That's the scaling I was talking about.

Fernando Alvarado (University of Wisconsin):

Sorry, scaling.

Robert J. Thomas (Cornell University):

I probably heard this wrong, but I would just like to clarify. What I saw in one of your examples is that

if you would have used the minimum singular value as the indicator for the mode of collapse you would have gotten the wrong answer. I think this is probably the wrong interpretation for the minimum singular value. If you try to associate that value with a particular mode of collapse, you certainly can associate them with eigenvalues, but that's a wrong interpretation of the minimum singular value. It's sort of a matrix norm that tells you something about distance, but it doesn't tell you anything about what individual eigenvalues and how they will move in response to parameter perturbations. That's asking way too much. So, I wondered if I misinterpreted what you were saying in terms of the interpretation and use of the minimum singular value.

Peter W. Sauer (University of Illinois):

We'll now have questions on the papers that were presented by Drs. Ajjarapu and Sobierajski.

Fernando Alvarado (University of Wisconsin):

The question/comment is for Dr. Sobierajski regarding the probability load flows. We've done related work a few years ago. Just to make it very simple, we've all seen demand parameter space characterizations in terms of P_1 - P_2 or S_1 - S_2 figures, whatever you like, and a given operating point. (Showed a sketch like the one in Figure 1. The curve represents the limit of existence of operating points. The point with closed curves encircling it illustrates the operating point and the closed curves show the distance of the operating point from other conditions.)

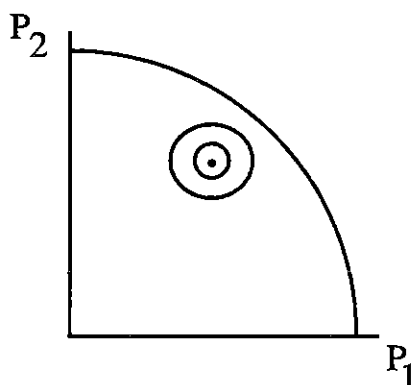


Figure 1.

I believe a lot of the problem boils down to a characterization of how likely it is for that point to exist in that position or to move around, and also whether contingencies are going to move you in or out of that space. We had an IEEE paper about three or four years ago that essentially describes the whole problem. We have an EPRI report coming out that describes the technique. To make it short, the problem is very tractable if you take a correlation assumption. It becomes very simple to determine how likely it is to lie outside vs. inside the region of stability. Also,

if you use the minimum distance concept, you cover yourself because you find essentially the probability that you'll exit the region of stability and how likely it is for the worst possible case to occur. It's a little different from what he presented, but I think it's very related to what he presented. This is more a comment than a question. Any comments on this?

Peter W. Sauer (University of Illinois):

Mario (Sobierajski), would you like to comment on that?

M. Sobierajski (Technical University of Wroclaw):

Yes. People who study the solution of load flow equations know that this is the curve for the load flow Jacobian to be zero. This is standard. Using Thevenin's Theorem, we could obtain such characteristics for each bus. When we consider rectangular distributions, we can be in this area (showed sketch as in Figure 2). We can be here (shaded area in Figure 2), and this is a power imbalance. I think that when we consider random loading of buses, we should take into consideration this area. There's no load flow solution there. We didn't consider contingencies like branch or transformer outage, only changes of loads and buses. There are papers on this subject, but this is a deterministic approach to this subject. I tried to use imbalances as indices for finding the weakest bus.

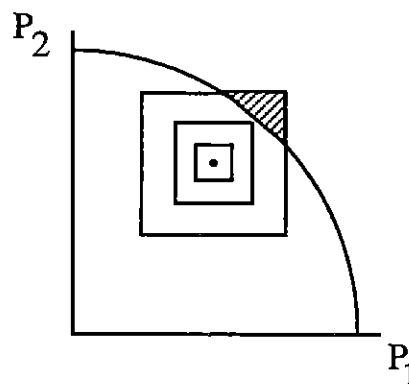


Figure 2.

Ian Dobson (University of Wisconsin):

I have a question for Venkata (Ajjarapu) about his sensitivity calculation. I have a suggestion for an interpretation, and I wanted to check whether it is correct or useful. My understanding is that the paper looks at the sensitivity of the equilibrium near the bifurcation point with respect to parameters, so that if you wiggle the parameters a bit you get the change in the state variables. You could also look at the unstable equilibrium point, and since it approaches the stable equilibrium point, you'll find the same sensitivity for that with the sign reversed. The two equilibria approach each other along the direction of the right eigenvector, and that's reflected in your index. Now,

if you were to look at the difference between those two equilibria in the state space, that is look at the distance between the two of them, if you subtract the two of them, the sensitivity of that quantity will be just double your ISPS number. So what I'm trying to suggest here is that the ISPS is proportional to the sensitivity of an index of voltage collapse which is the difference between the stable equilibrium position and the unstable equilibrium position. I wanted to hear your opinion of whether you thought this is a correct or useful interpretation, because it would interpret it as the sensitivity of an index of voltage collapse with respect to a parameter.

Venkata Ramana Ajjarapu (Iowa State University):

I haven't thought about that. We compared this with the actual eigenvalue sensitivity for a DAE model using a reduced matrix and checking how the parameters of K_1 affect the eigenvalues. This is actually not eigenvalue sensitivity, even though it picks up the same parameters as picked up by eigenvalue sensitivity. Your suggestion is very good and I will consider it.

Xin Jiang (Washington University):

My question relates to Venkata's (Ajjarapu) paper. First, I quite like your discussion on the relationship and differences between static analysis and dynamic analysis of voltage stability. Second, it's interesting to me that you say in your paper that, besides the saddle-node, Hopf and singularity-induced bifurcations, the node-focus points and 1:1 resonance points are also important because at these points, participation factors, eigenvectors and residues can be misleading. This is because at these points the eigenvectors are not unique and lose their physical meaning. Could you give some numerical results showing how the system behaves near these points? Also, could you show how these points can be characterized mathematically?

Venkata Ramana Ajjarapu (Iowa State University):

We didn't do any time simulations because we wanted to use time domain simulations to solve DAE systems. We wanted to test exactly what happens at those points, so we didn't check that. But these two points are well established in the mathematics literature. One remark with respect to node-focus bifurcations is that as shown in the paper there is a short peak at the node-focus with respect to the ISPS because the eigenvalue is very sensitive to small changes in parameter. So the comment here is that it may be difficult to calculate the eigenvalue near that point. With respect to the mathematical point of view, it's already there in the mathematics literature; there's nothing new. However with respect to physical interpretation, we are still struggling to determine what happens at 1:1 resonance. That's why

we didn't go into detail; because we also don't know what happens at 1:1 resonance.

Nelson Martins (CEPEL - Centro de Pesquisas de Energia Eletrica, Brazil):

This is a transparency that I wasn't able to show in my talk yesterday. (Showed transparency entitled "New Horizons in Voltage Stability Analysis.") The transparency shows new things to be done. Voltage stability tools added to generation-transmission reliability software: it's right there. At CEPEL we have this NH2 software and we're starting there a working task force to include voltage stability functions. The question then is directed to Dr. Sobierajski. Since you are doing Monte Carlo simulation you could have probably also considered line outages in your approach, instead of just PQ load uncertainties. That's what we are doing with our generation-transmission-reliability software, which is still restricted to adequacy studies. This would be finding not just eigenvalues, because you need to assess margins for every scenario—voltage stability margins. Doing that is a nonlinear issue, you have to initially use continuation software or simple power flow. However, later, and this is my objective in the future, using point of collapse software for that. Also my colleagues in CEPEL—and this is a provocative question—say that probabilistic load flow has been highly superseded by generation-transmission-reliability tools. It's very important to consider line outages and equipment outages and their failure rates and all that. I also have a question for Dr. Ajjarapu and some comments on Ian Dobson's point.

Peter W. Sauer (University of Illinois):

If there's time, you can ask questions later. David (Hill), did you want to say something? We have time but we have to let Yakout (Mansour) answer his questions.

David Hill (University of Sydney):

My question is to Lee and Ajjarapu and relates to the static vs. dynamic stability issue. I commend the authors for doing sensitivity analysis in the way it should be done, i.e., relating eigenvalues of dynamic A matrix to parameters. The usual voltage stability modal analysis just studies static sensitivities which may not be closely related to dynamic behaviour.

Let me refer to some results I will present tomorrow. These are based on a more focused model of the load system, i.e., without generator dynamics. The generic load models have three important features: static load characteristic, transient load characteristic and recovery time. The static sensitivities are obtained from the usual power flow Jacobian J_s based on static loads; the dynamic sensitivities (as studied in this paper) are dependent on J_s and J_t , a transient

power flow Jacobian based on the transient loads. Typically, without generator dynamics intruding, a statically stable—we call it regular—operating point is also transiently stable; however, irregular points can be stable. The gap between static and dynamic properties may involve more exotic behavior.

My questions are:

1. Have you explored this gap between static and dynamic load end behavior?
2. In particular, have you found any of your Hopf bifurcations to be caused by loads?

Anyway, it seems that you are in a good position to make such investigations.

Peter W. Sauer (University of Illinois):

You can just say “yes” and “yes.”

Venkataramana Ajjarapu (Iowa State University):

We didn’t include the transient characteristics of the load. That’s why we calculated the ISPS for Case 3, where we didn’t include the load dynamics. Basically, we wanted to demonstrate the concept of ISPS. Assuming that is valid, then we wanted to extend it to other characteristics. With respect to the static sensitivity vs. dynamic sensitivity, maybe saddle node bifurcation can be guessed from the static analysis, but static analysis is no longer useful at Hopf bifurcation.

Peter W. Sauer (University of Illinois):

Thirty seconds (for a general comment)? Go ahead.

M. Sobierajski (Technical University of Wroclaw):

There are some interesting methods of probabilistic load flow calculations. I think that they are very general: they use reliability of generation, probabilistic character of line and transformer outages, and so on. But, when we complicate this task, so some new decisions must be made, for example when we consider reliability of generation we should take into account the dispatcher function. There is no good way to determine the dispatcher function. I think that the stochastic load flows, probabilistic load flows, wait for the future. Now we have expert systems. Maybe the dispatcher function should be determined from experts. This is one point. As for branch outages, some outages can change the state of a power system so far that they can be considered probabilistically. My opinion is that the probabilistic approach, from the point of view of voltage stability, is a good one to show the deficiency of reactive power. So, don’t leave active power in peace. Study the loading of reactive power, and consider some outages for some scenarios. I think that the results, the probabilities showing which buses are weakest when we consider a large power system, can be very useful for practice.

Peter W. Sauer (University of Illinois):

Yakout (Mansour), would you read back the questions the way you understand them, so we can get them properly recorded?

Yakout Mansour (B.C. Hydro):

The first set of questions was from Nelson Martins, and I think his questions basically related to bus participation vs. line participation, and weighing the benefits of shunt vs. series compensation. This point is well taken. Both are looked at. However, from a formulation point of view, it was reasonable, straightforward and easy to get good information from the bus participation. Line participation, as described in the document you have associated with modal analysis, is in part judgemental. Our experience so far is that if you use that element we have not been getting 100% consistent results. In most cases, yes, they do identify lines, but there are some odd cases where they do not really identify the right thing. So, if you want to take it from the point of view of bus vs. line participation, I could say bus participation is a robust index, but I can’t yet say the same about line participation. This applies if you want to use modal analysis to justify either of the two. However, regardless of modal analysis, the relative merits of series vs. shunt compensation is an issue that will have to be dealt with. You rightly stated it; I just didn’t show an example that shows you that difference. Does that address the first point? Okay. The second point is related to the point of collapse methods, and you agreed that they are slow but you’re working on ways to make them faster. As I said, they’re an elegant way of formulating the problem. If you want high precision in getting the nose point and you want the opposite side of the curve, by all means they are beneficial. So far I have seen that you seldom need to do this. In the odd cases, when you need it, then maybe the state-of-the-art today is good enough with respect to the speed issue. Yes, there is room for speeding up, but I wouldn’t put this as a top priority among my needs today. In a forum like this, we are trying to focus on the list of top priorities. So, by all means there is room and I hope that we can see it at some point in time, but yet I if suggest issues to focus on for my colleagues in the research community including ourselves, it would not be at the top of my list. Does that answer both questions for you? Thank you.

Bob (Shlueter), I don’t know if you misunderstood what I said. You said you disagree with me about the use of QV curves. I didn’t say that QV curves aren’t useful. What I’m saying is that blind use of QV curves could be very misleading. What I’ve seen so far of how people use QV curves is they pick a bus and say: “We’ll pick this bus because it’s in the

load center," or "because it's a big bus." You put in a synchronous condenser and you fix the voltage or vary the voltage and you see the contribution of the condensers and you plot QV. Then you build all kinds of things around this. What I showed in the examples is that, really, if you do this, you could be changing the mode to a point where the results you're getting are not good. They do not relate to the actual mode of collapse. I have shown these two cases (referred to a transparency entitled "BCH 230 kV busses and areas"), and actually these two buses are only 20 kilometers apart, connected by two 500 kV cables, two 130 kV lines, and a 700 MW HVDC line. So, anyone looking at this would think that since they are both in the load center, and both very close, either one could be used. Here I am showing that they give you two different modes, if you use QV. These modes are calculated just based on Q-stressing, just the way you would do QV. Interestingly enough, if I now take this system (reference to transparency showing graphs of mode shape vs. "S-stress"), with two lines that are very close, in fact common in most of the right of way but going to two different buses, you get two different mode shapes depending on which of the lines you take out. Two completely different mode shapes. Interestingly enough, you'll find that one of these is very close to one of the plots I did with the QV analysis, on a certain bus, and the other one is the other bus. (Showed a transparency with a bar graph showing participation levels in the Vancouver Island Area.) So all I'm saying is that, with this knowledge about the modes, you can select your proper QV, that would actually be a bus that would represent this particular mode. I could put this in mathematical terms that would show the composition of a modal QV, and how it relates to actual buses. If you do that step and you pick the proper bus for a QV then you're correct; otherwise you're wrong. Do you agree? Thank you.

I think the next question was from Hans (Glavitsch). I think your question, Hans, was related to the size of system I would use, right?

Hans Glavitsch (ETH, Zurich):

In principle, that was the first question. The next question is: if you know a little bit about the system, can you reduce it? The third is: if you have to do load shedding, how many buses must really be affected to save the system?

Yakout Mansour (B.C. Hydro):

Right. That's a loaded question that we go through, and if I can add another point to the list I put for research priorities, it would be system modeling or system reduction for voltage stability studies. That would be a good topic for research. We start with a highly interconnected large power pool, say the WSCC or whatever. They're highly intercon-

nected and they're related in many ways so that in the beginning I don't know what affects what. So the first step is to go through the steps of identifying the clusters. Once I identify the clusters then I'm educated in how to reduce it. Any time I study one such cluster then that area would have to be detailed down basically to the load level. Once you do that in such a system, typically you find that a study system is around 2000 buses. Voltage collapse in that cluster doesn't mean that you collapse the whole system. However, you can't ignore the interconnections with the rest of the system. If you have three or four systems surrounding that study area, your system would get to around 3000 buses or so. That is typical. So these are the steps we go through.

Regarding Fernando's (Alvarado) question of whether or not we can cluster by voltage level, really the clustering I reported just came out of the modal analysis. I didn't specify a desired kind of clustering or grouping. It just came as a result of the participation; it grouped itself this way. (Showed a transparency with a bar graph of bus participation levels.) If I look at the top ten buses and knowing what they are, they are actually of different voltage levels. So if I cluster by voltage level, I don't know if I'm going to get any picture that would identify something.

Fernando Alvarado (University of Wisconsin):

Stressing by different voltage levels gives you very different results.

Yakout Mansour (B.C. Hydro):

Well, in these cases I just take contingencies, line outages basically. So these are the cases I was stressing.

Now for the last question, which was asked by Bob (Thomas). I think I might have said something wrong about the use of singular values. What I meant is that there were trials to take the proximity of the eigenvalues to zero as an indication of how far you are. Now, in this case I'm saying that you really can't do that, because it shows that the smallest one isn't the one that controls stability. (Showed a transparency with a table of eigenvalues and participation levels.) As a matter of fact, there are cases where the one that controls stability doesn't even appear in the base case. It just comes later on. If I say it that way, would you agree then?

Robert J. Thomas (Cornell University):

You said minimum singular value, but you meant eigenvalue. Other than that, I would agree with you.

Yakout Mansour (B.C. Hydro):

Sorry.

Peter W. Sauer (University of Illinois):

We have about forty minutes for discussion of the last four papers of the Analytical Methods session. Why don't we devote approximately ten minutes to each of the four papers. If there's any time left after that, then we'll let Yakout talk some more. First, the paper presented by Ian Hiskens.

Claudio A. Cañizares (University of Waterloo):

I just had a couple questions, Ian (Hiskens). I didn't read the whole paper but I was quite interested. If I understand correctly you get on the manifold first, and then you follow the manifold to what you define as the critical point or the closest saddle node bifurcation, right? Now, from my experience the first step is already quite a bit of a problem, especially in large systems, it can take long depending on what methods you use. But that's not my question. The question really is if you can elaborate a bit on the shape of that Σ surface that you define. Obviously, as you discuss in the paper, that shape affects the performance of the algorithm. Right—the directions of movement on Σ ? You talk about singularities on Σ , which I find quite interesting. I'm just curious whether you've seen how system limits, like limits on generators, taps, etc., affect the shape of the Σ surface.

Ian Hiskens (University of Newcastle, Australia):

Regarding the shape of the boundary surface, we don't have a lot of experience about these things, but in this traditional three bus case that is the shape of it. (Showed transparency.) That's the most trivial and the cleanest example. In this particular example, there's symmetry in the line admittances, which gives the nice symmetric appearance of this thing. You can play with the symmetry of the example; you can include line resistances which aren't there. This starts to distort this middle bit, which ultimately coalesces with the outside, and it becomes a rather complicated shape. In the eight bus example (another transparency shown), all of these trajectories that you see here, which is in the space of the three free powers are all sitting on Σ . So you can see that it gets quite complicated. We've been doing some initial work to get a picture of that surface and it's a little difficult to get hold of—there's a few dimensions to play with—but we're getting there. It is kind of a complicated surface, but it's kind of elongated as well. Apart from that, I haven't the foggiest what it's like. In the paper there's another figure which I don't have on an overhead, which shows the singular points of the critical point problem. They actually lie between each of the critical points, which you imagine from symmetry properties, and likewise you get the same kind of thing happening in the middle. That's in the paper.

Peter W. Sauer (University of Illinois):

The second paper was presented by Claudio (Cañizares) and Antonio (de Souza). Any questions for the authors?

Venkataramana Ajjarapu (Iowa State University):

I wanted to know whether the authors played with different step lengths, and also I wanted to know if during the corrector iterations whether they kept the Jacobian constant. What we observed is that as long as the generators are not hitting their limits if you keep the Jacobian constant during the corrector iterations, we didn't get any convergence problems. I wanted to know your experience.

Claudio A. Cañizares (University of Waterloo):

I suppose you're referring to the length of the step here, right? Our experience has been that towards the flat end of the curve, large steps are okay. Once you get toward the tip of the curve, especially when you have sharp corners, if you take too long of a step, then you end up missing the point and having problems. You can always cut back and go back. So that hasn't been a major issue. It becomes an issue when you use these tangent vectors to reduce the system. (Showed a transparency entitled "Continuation Methods," illustrating tangent vectors to a QV curve at several loading levels.) We're using a part of the tangent vector to eliminate the variables that don't change much, and then we continue with the rest. That way we reduce the space or the domain of our analysis. As you can see, we made this the size of the step we are taking. (Showed a transparency entitled "Tangent Vector," which showed the upper bound on the step size used in the method.) Obviously if we take too long of a step, then maybe we can eliminate more variables or less variables depending on the situation of the system.

Here it does become an issue. So, this concept or property we are using here, or definition, if you will, to eliminate variables definitely depends on the step size. Regarding the way we handle the Jacobian, going back to this figure, we've had exactly the same experience you had. If we don't hit limits, we keep the Jacobian pretty much constant. We keep it (constant) for quite a long period of time. Particularly when this is flat; that means it's not changing much. But as you said, you hit a limit and this tends to change significantly the direction in which you're moving. Then you end up factorizing the matrix. That's an issue that shows up here, when tracing the continuation, and that's what makes continuation methods slow for large systems. When you start hitting lots of limits, and that happens especially when that bifurcation point is far away, you end up re-factorizing the matrix very often. That basically makes it slow.

Nelson Martins (CEPEL - Centro de Pesquisas de Energia Eletrica, Brazil):

I was very glad to use your and Fernando's (Alvarado) software, but I think step length is quite an issue, as you mentioned when you get near the nose tip. So, the idea is, and I don't know if this is a new question in this field, why not use VSVO, that's Variable Step Variable Order integration methods, in your continuation power flow. This is used in EU-ROSTAG (spelling) which is a dynamic simulation program, and it has been used in a similar problem, which is finding the unstable equilibrium point (UEP) of interest in transient stability analysis. This is in a paper by Eresary et al. (spelling) published in PICA. They've used it for a similar problem, that of finding this unstable equilibrium point. So I think that's a good way to go for step length control. Leave it to the algorithm to decide it on the basis of accuracy.

Claudio A. Cañizares (University of Waterloo):

The algorithm does decide the step size by itself. It depends on the length of the tangent vector. In the very flat parts of the curve, the norm of this tangent vector tends to be small. Now you can play with the size. The user can increase or reduce the step size. We've shown that with $K = 1$, you basically don't interfere and the algorithm does fairly well. It goes fast in the flat parts and slow towards the end of the shape. (Showed a transparency with approximate title "Predictor and Parametrization Tangent Vector.") So, that's included in the algorithm.

Nelson Martins (CEPEL - Centro de Pesquisas de Energia Eletrica, Brazil):

I know that. The question regards VSVO methods, and probably that's a better solution than having your own strategy for step length control. I think it should be pursued.

Claudio A. Cañizares (University of Waterloo):

Well, I could look into that. I'm not familiar with the terms you're using. We could look into that, but the algorithm already includes the step size. It's not just left to the user to decide. Now, you have to be careful how you take the steps, especially when you have limits. So that's another issue that you have to consider.

T. van Cutsem (University of Liege):

I have two remarks, maybe not directly addressed to a particular speaker, preferably to all of them. My first remark has some relationship with Nelson Martins' remark. I would like here to propose my own continuation method. (Showed a transparency.) This one is obviously based on the fast mid-term simulation approach which I have described yesterday. Physically, you simulate a ramp increase in the load demand, and in this simulation, which is very fast, you just record P as a function of time, V as a function of time, and then you just eliminate time and

you get your PV curve, if you are interested in getting such a curve in addition to your dynamic simulation. In some sense, you use the system's physical behavior as a continuation parameter, and as the system goes through the saddle node bifurcation it goes smoothly, so there is no problem physically. Advantages and drawbacks: Well, I have heard that continuation methods may be slow. I think that the simplifying simulation can be fast. Of course it requires a little more modeling, but this may be significant in studying the system. In addition, I'd like to come back to some ideas of yesterday evening that you have a single tool to perform dynamic simulation and draw PV curves, if you want to draw PV curves.

I have another small remark. Maybe it's not well placed now, but I would like to suggest setting up a good realistic test case to allow researchers to investigate the real implications of Hopf bifurcation. This is related to reading Venkata's (Ajjarapu) paper. You didn't speak about it a lot in your presentation, but it is in your paper. I would really like to get a realistic example of Hopf bifurcation, in order to evaluate the risk of meeting this bifurcation before the traditional saddle node bifurcation, and also to see if this doesn't come from modeling simplifications, like neglecting the power system stabilizer or something like that, or also maybe using a constant power load assumption, which is, according to my experience, a source of artificial oscillations. Sorry for the badly placed second remark.

Ian Hiskens (University of Newcastle, Australia):

This is just a comment regarding continuation methods in general. There's a lot of talk about using continuation methods for finding nose curves, but there's a lot more information that you could obtain out of those curves. This is a picture taken from a paper published ten years ago, which shows the response or the behavior of a section of the Queensland power system. (Showed a transparency with an excerpt from a 1984 paper by G.B. Price, showing curves produced by a continuation method.) The curves in this were produced using a continuation method which is an Euler homotopy method. The paper of ten years ago describes the tangent method for getting started, and the secant method for obtaining subsequent points. I'd just like to say that one of the things that really annoys me is that it's never referenced, while it's about six years ahead of any other work on continuation methods (in power systems). It's really quite a valuable paper.

Peter W. Sauer (University of Illinois):

You can even go back to Bob Thomas' Ph.D. thesis. It's also in there.

Claudio A. Cañizares (University of Waterloo):

Answering the question you were suggesting to me regarding the continuation method you were proposing, I think VSTAB is already doing that. Now, the idea of continuation methods is obviously how to move intelligently along the way. Obviously once you move around the bifurcation point, you may end up in trouble using just power flow techniques, let's say. But, you can get really close to it. So, why bother? Right? But, then again, the continuation method not only gives you how to move along that path, which in this case in VSTAB you decide how to do it, but the steps to take, which is the issue I was discussing with Nelson (Martins). The program does it automatically for you. But the interesting thing about it is that as a by-product of the continuation method, you get a tangent vector. That tangent vector happens to converge to the eigenvector of the system at the bifurcation point. Basically that's what has been called modal or participation factors, and that's a by-product of the method. You don't have to do anything else; it just comes out of it. So, that's another advantage of continuation methods.

Peter W. Sauer (University of Illinois):

Let's continue on from these continuation methods to multiple load flow solutions, the last two papers. First, Professor Shinichi Iwamoto will assist in answering questions (by translation) on the paper "Determination of Dynamic Voltage Stable Equilibrium Point Based on Nonlinear Optimization Method" (the paper presented by Yicheng Zhou). Any questions or comments on that paper?

Daniel Karlsson (Sydkraft AB, Sweden):

I am Daniel Karlsson from Sydkraft. It's a power utility company in the south of Sweden, and I've been working a bit with load models, and I have a question about the structure of the load model on page seven. What's the reason for the chosen structure, and what's the background of the parameter values? I want to know what load dynamic behavior you want to capture with your model. Is it induction machines, heating, air conditioning units, or what? Finally, what about the sensitivity of the results? If some of the parameter values are given with an accuracy of about five figures, if you change that a little, what about the stability result?

Shinichi Iwamoto (Waseda University):

I'm sorry—I didn't follow the last question.

Daniel Karlsson (Sydkraft AB, Sweden):

In Figure 6, the parameter values are given with great accuracy—about five figures. If you change them by about 10%, what happens to the results of the simulation? Thank you.

Shinichi Iwamoto (Waseda University), interpreting for Yicheng Zhou (Tokyo Metropolitan University):

I don't think there's any specific reason for the load model, they have just chosen Equations (34) and (35). The values of the parameters are taken from some references. They are not really talking about an induction motor load or something like that. The load model is just equations (34) and (35). About the sensitivity, if you increase the parameter values by 10%, he said that in Figure 7 the voltage magnitude is going to decrease, and then in Figure 8 the angle is going to increase. That is his answer. You can talk to him later. I will be the interpreter.

Chen-Ching Liu (University of Washington):

I have a question about the simple system that you're using—that's Figure 4 and Figure 5. The question is, if you suitably change the parameters, then there would not be any stable equilibrium points. In that case, how does your algorithm behave?

Peter W. Sauer (University of Illinois):

Can you repeat that, Chen-Ching?

Chen-Ching Liu (University of Washington):

Yes. The question is: when there is no stable equilibrium point, and that can happen in the two-bus system of Figure 4, how does the algorithm behave?—because the algorithm is trying to find a stable equilibrium point, and when that disappears how does it behave?

Shinichi Iwamoto (Waseda University), interpreting for Yicheng Zhou (Tokyo Metropolitan University):

I don't know if he understood my interpretation, but he said that if there is no stable equilibrium point then voltage collapse is always going to occur.

Chen-Ching Liu (University of Washington):

The algorithm is supposed to find the stable equilibrium point, but when there is no stable equilibrium point, what does the algorithm do? Does it keep searching? Or does it just say "infeasible" and stop?

Shinichi Iwamoto (Waseda University), interpreting for Yicheng Zhou (Tokyo Metropolitan University):

Okay. If the solution doesn't converge, then the algorithm says "stop."

Peter W. Sauer (University of Illinois):

Okay, and so we'll go to the last paper—Hiroyuki Mori's paper.

Thomas J. Overbye (University of Illinois):

I've got two questions. I've played around a little bit with solving for low voltage solutions myself, and I've found that, up until very close to the point of singularity you'll actually have a number of these low voltage solutions. In fact, closer than you'd ever want to get is where you finally get that final just two solutions, or the operable solution and a single

type-1 solution. The method you're proposing just finds that last solution, so presumably it won't work real well until you get close to the point of maximum loadability. What I found is that when you're far from that point the other multiple solutions can actually be used to develop multiple measures for voltage stability of the system—measures for different areas in the system to tell you the voltage stability. So, I'd ask for your comments on whether you've run into this situation where you've got a number of low voltage solutions that you'd like to find. The second question I have is that you mentioned the VIPI method which I know is very popular in Japan for assessment of voltage stability. This method uses the angle between the two solutions to come up with a norm of voltage stability. This is what energy methods do, too: they come up with a norm of voltage stability by measuring the distance between the two solutions. Just comparing energy methods with the VIPI method, I see that once you have the low voltage solution they both require negligible computation. You can compute them both very fast. Just one comment on the energy methods is that I see that they have an advantage in the sense that, based on some research we've done at Wisconsin and Illinois, you can interpret the energy measure in terms of an MVA, an MW or an MVAR margin that could then be used for the operator. So I was just wondering if you've taken your results, which are concerned with finding the low voltage solution, and looked into using energy methods as well as VIPI.

Peter W. Sauer (University of Illinois):

You may answer by saying "yes" and "yes."

Hiroyuki Mori (Cornell University):

Regarding the first question, actually the method is not general. I mean first we have to use another method, for example, Dr. Iba's method. If we capture the structure of the effect of the load flow equation in the transform matrix P then we can get a counterpart under different power system conditions. I am not saying that this method is not the same as Dr. Iba's method, in the sense that we can directly evaluate its counterpart under any power system condition. But if we capture the power flow condition, for example, the Jacobian matrix, then we can directly evaluate its counterpart very quickly: the method is based on linear transformation. Talking about the second question, I know your and Chris' (DeMarco) work on energy function methods, but what we have done is the so-called static voltage instability. In a sense, I think the energy method includes the dynamics of the voltage instability problem, so it is difficult to make a comparison of the two methods.

Thomas J. Overbye (University of Illinois):

They're both static measures.

Peter W. Sauer (University of Illinois):

The answers are "yes" and "no," not "yes" and "yes." We have time for one more question.

Fernando Alvarado (University of Wisconsin):

I have two comments and a question. One comment is regarding your Figures 3 and 4. You talk here about this local index that you use to determine the degree of stability. Many of us actually think these local indices really are very dangerous—that you really don't want to do them under operating point conditions. So, that's just a comment. Another comment is on Figure 1. I hope that this is a conceptual figure. It would have been, I think, much nicer to have actual figures of a true example of these multiple curves, because the real ones don't look quite like your Figure 1, although they have some resemblance. The third, which is really a question, is: in trying to study this, going to my room and trying to program this and seeing if it works, I find that there is not enough information here. I'm not clear whether a is computed first or computed last; where does the vector u come from? I think, at least from what I was trying to do, either I missed a point in the algorithm or it's missing something.

Peter W. Sauer (University of Illinois):

He'll check that out and let you know after lunch. That's the end of our session. We are adjourned.

**Comments on Bifurcation flow: A tool to study both static
and dynamic aspects of voltage stability by Byongjun Lee and Venkataramana Ajjarapu**

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We make comments and pose questions about the sensitivity computations of the paper as applied to saddle node, Hopf and singularity-induced bifurcations. We would appreciate the authors' comments and replies.

Saddle node bifurcation: First some notation to augment that of the paper: The dynamics of the differential-algebraic model (1),(2) when the algebraic equations are soluble are given by

$$\dot{X} = f(X, P) = F(X, h(X), P)$$

where h satisfies $G(X, h(X), P) = 0$. We write $|_*$ for evaluation at the bifurcation.

Let the i th mode be the mode with zero eigenvalue at the saddle-node bifurcation. Then

$$(\sigma_1, \dots, \sigma_n)|_* = v_i^t \frac{\partial f}{\partial P}|_* = N$$

where N is the normal vector to the set Σ of parameters yielding saddle-node bifurcation. Let Δ be the loading margin from a nominal loading assuming a given direction of load increase. Then reference [A] proves that $\frac{\partial \Delta}{\partial P_j}$ is proportional to $N_j = \sigma_j$. That is, the authors' measure σ_j of sensitivity is proportional to the sensitivity of the loading margin in [A]. An advantage of the interpretation of σ as a sensitivity of the loading margin is that the changes in loading margin due to a given change in power system parameters or controls can be estimated to first order.

The ISPS can also be interpreted as the sensitivity of a voltage collapse index. Define the voltage collapse index $|X - X_*|$, where X is the stable equilibrium before bifurcation and X_* is the bifurcating equilibrium. Then

$$\frac{\partial |X - X_*|}{\partial P_j} = \frac{(X - X_*)^t}{|X - X_*|} \frac{\partial (X - X_*)}{\partial P_j} = \frac{(X - X_*)^t}{|X - X_*|} \frac{\partial X}{\partial P_j}$$

Near the bifurcation,

$$\frac{\partial |X - X_*|}{\partial P_j} \approx \frac{u_i^t}{|u_i|} \frac{u_i}{\lambda} v_i^t \frac{\partial f}{\partial P_j} = \frac{|u_i|}{\lambda} v_i^t \frac{\partial f}{\partial P_j} = |\text{ISPS}|$$

(We suspect that a similar result holds for the voltage collapse index $|X - X'|/2$, where X' is the unstable equilibrium involved in the bifurcation.) We favor the use of σ rather than ISPS for a saddle-node bifurcation because σ specifies the loading margin sensitivity and is finite at the bifurcation.

Hopf bifurcation: ISPS and the measure σ_j are derived from the sensitivity of the equilibrium position X with respect to a parameter changes P_j . Thus if the equilibrium does not move when P_j is varied then ISPS and σ_j will be zero. However, the dynamics of the power system equations can change with parameters in such a way that the location of a Hopf bifurcation changes but the equilibrium position X does not change. For example, if P_j is a time constant multiplying the left hand side of one of

the differential equations, then changing P_j does not affect the equilibrium position X , but will generally change the eigenvalues and hence change the the location of a Hopf bifurcation. Therefore we question whether ISPS or σ_j capture the part of the sensitivity of the Hopf bifurcation due to changes in eigenvalues that do not affect X . Reference [41] of the paper derives the formula for the sensitivity of the loading margin to Hopf bifurcation which captures both the part of the sensitivity due to changes in X and the part of the sensitivity due to eigenvalue changes alone.

Singularity induced bifurcation: One way to study the sensitivity of a singularity induced bifurcation is to regard the singularity induced bifurcation as a fold bifurcation of the algebraic equations and study the sensitivity of this fold bifurcation. That is, let Δ^{*i} be the loading margin to the singularity induced bifurcation and v_i^{*i} be the left eigenvector corresponding to the zero eigenvalue of the Jacobian $\frac{\partial G}{\partial X}$. Then following the approach of [A] yields

$$\frac{\partial \Delta^{*i}}{\partial P_j} \text{ is proportional to } v_i^{*i} \frac{\partial G}{\partial P_j}$$

(The constants of proportionality are easily obtained.) We would appreciate the authors' comments on this suggestion.

[A] I. Dobson and L. Lu, Computing an optimum direction in control space to avoid saddle node bifurcation and voltage collapse in electric power systems, *IEEE Transactions on Automatic Control*, vol 37, no. 10, October 1992, pp. 1616-1620.

Closure

Bifurcation Flow: A tool to study both static and dynamic aspects of voltage stability

B. Lee & V. Ajjarapu (Iowa State University) : We thank Dr. Dobson for going into details of our paper. He made valuable comments, suggestions, and questions regarding this paper. In the following paragraph we clarify some of the discussor's interpretation of σ and ISPS with respect to saddle-node and singularity-induced bifurcations.

(i) Saddle-Node bifurcation: σ_j is similar to the j -th elements of normal vector (N_j) defined in the discussor's paper. However, the approach to come up with ISPS and σ is quite different from the discussor's method.

In this paper, the motivation of deriving the ISPS starts from the parametric sensitivity of DAE (Eqs. 12 and 14). Near saddle-node bifurcation the parametric sensitivities are always infinite with respect to any parameter p_j that satisfy $l_j \neq 0$. Thus, the parametric sensitivities are enough to identify the saddle-node bifurcation point. However, because the parametric sensitivities are always infinite, the most critical parameters cannot be obtained. Our aim to study the sensitivity analysis is to find the most critical parameters near saddle-node bifurcation point and it was achieved by projecting the parametric sensitivity onto eigensubspaces. In Eq. 38 the parametric sensitivity is represented by the sum of ISPSs.

$$\frac{\partial X}{\partial P} = \frac{M_1 L}{\lambda_1} + \frac{M_2 L}{\lambda_2} + \dots + \frac{M_m L}{\lambda_m}$$

In Eq. 39 the parametric sensitivity near the saddle-node bifurcation point is approximated by the ISPS corresponding to the critical eigenvalue λ_i , which is as follows,

$$\begin{aligned} \frac{\partial X}{\partial P} &\approx \frac{M_i L}{\lambda_i} \\ &= \frac{u_i}{\lambda_i} [\sigma_1 \dots \sigma_j \dots \sigma_n] \end{aligned}$$

This equations indicate that the parametric sensitivity is equivalent to the ISPS corresponding to the almost zero eigenvalue (λ_i) and ISPSs of all the parameters can be represented by $\frac{u_i}{\lambda_i}$ multiplied by a scalar σ_j , $j = 1, \dots, n$. Here, n is the number of parameters.

The discussor gave an important interpretation of ISPS where ISPS is the sensitivity of voltage collapse index. We agree with the discussor as long as the operating point X is close to the saddle-node bifurcation point

In state space, ISPSs are vectors and the length of vectors representing the ISPSs is proportional to σ . ISPSs on the critical eigensubspace u_2 are vectors that have different length and same direction as shown in Figure 16. Obviously, the parameter that has the most influence on this eigensubspace will correspond to the longest vector. Consequently, the largest σ_j will select the parameter p_j that has the most influence on the critical eigensubspace.

The geometrical meaning of σ is the length of ISPS vector with proper scale, which is defined in the state space, whereas the elements of the normal vector N is defined in the parameter space.

We want to give a general comment regarding the σ and the normal vector relation given by the discussor (i.e., $v_i^T \frac{\partial F}{\partial P} \big|_*$). Here, care must be taken to include explicit representation of parameters corresponding to the algebraic part. Otherwise, $\frac{\partial F}{\partial P}$ will be zero. σ defined in this paper takes into account all the parameters of interest.

(ii) Singularity-Induced bifurcation : SI bifurcation occurs if algebraic Jacobian matrix K_A becomes singular.

As far as identification of SI bifurcation is concerned, the discussor's suggestion that regards the SI bifurcation as a fold bifurcation of the algebraic equations is valid. However, for the sensitivity of SI bifurcation, two cases arise depending on the derivation of function G .

1. G corresponds to algebraic part only,
2. G satisfies $G(Y, \Psi(Y), P) = 0$ where Ψ satisfies $F(\Psi(Y), Y, P) = 0$.

In case 1, G may not contain all the parameters that are related to dynamic part (such as excitation system parameters $K_{A_i}, K_{E_i}, K_{F_i}$). In case 2, the parameters correspond to dynamic part don't appear explicitly in G . For these cases, the sensitivity $\frac{\partial G}{\partial P}$ becomes zero.

However, the fold bifurcation point of the algebraic equations is sensitive to the parameters related to the dynamic part. For example, near the SI bifurcation point,

$$\det \left[\frac{\partial G(X, Y, P)}{\partial Y} \right] \bigg|_{(X(P^*), Y(P^*), P^*)} \approx 0$$

where $(X(P^*), Y(P^*), P^*)$ is a fold bifurcation point. If the parameters P are varied slightly as $P = P^* + \Delta P$, then the equilibrium point $(X(P^*), Y(P^*))$ is also varied to $(X(P^* + \Delta P), Y(P^* + \Delta P))$. Obviously, the

algebraic Jacobian matrix $\frac{\partial G}{\partial Y}$ is also varied and this new equilibrium may not be a fold bifurcation point,

$$\det \left[\frac{\partial G(X, Y, P)}{\partial Y} \right] \bigg|_{(X(P^* + \Delta P), Y(P^* + \Delta P), P^* + \Delta P)} \neq 0$$

Figure 23 shows that the ISPSs of the exciter parameters K_{F_i} are nonzero.

Based on these two cases, the discussor's suggestion that studies the sensitivity of a fold bifurcation of the algebraic equations is valid as long as the effect of dynamic parameters are taken into account.

(iii) Hopf bifurcation: We agree with the discussor's comments that the ISPS proposed in this paper is not valid for the parameters that are coupled to $\dot{x}$ terms such as time constants. But for the other parameters (such as the exciter parameters K_A, K_E, K_F), the ISPS method works well.

Again we thank Dr. Dobson for his interest in our paper.

Session 5

Security Analysis - II

"Determination of Voltage Collapse Areas Through Generalized Singular Value Analysis"

Julián Barquín, Tomás Gómez
Universidad Pontificia Comillas

**"Assessment of Static Voltage Stability Limit
Based on the Complex-valued NR Load Flow in Power Systems"**

Hiroyuki Kita, Eiichi Tanaka, Jun Hasegawa
Hokkaido University
Ken-ichi Nishiya: Hokkaido Institute of Technology
Tomoyuki Fukushima: Hokkaido Electric Power Company

"Control of Bifurcations Associated with Voltage Instability"

Eyad H. Abed: University of Maryland

"Voltage Collapse Proximity Indicators for Very Short Term Security Assessment"

Paolo Marannino: ENEL Centro Ricerca di Automatica
Paolo Bresesti: CESI Spa
M.Delfanti, G.P.Granelli, M.Montagna
Università di Pavia

"Circuit-Based Perspectives on Voltage Stability"

Christopher L. DeMarco: University of Wisconsin

"PV-Curve in Unbalanced Transmission Lines"

Kunio Matsuzawa, Mamoru Suzuki, Masanobu Kaminaga
The Tokyo Electric Power Company
Kenji Iba: Mitsubishi Electric Corporation

Determination of voltage collapse areas through generalized singular value analysis

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1 Introduction

Voltage collapse phenomena are considered to be, very often, geographically local. On the other hand, they have been related to the nullification of the minimum singular value of the load-flow jacobian matrix [1]. In this paper, a new algorithm to divide the power system in coherent areas is proposed to perform voltage collapse analysis.

Two buses are considered to belong to the same area when their response to active and reactive power demand perturbations are similar. The response is measured by the increment of the voltage magnitudes and angles. It is shown that most of the response is determined by the right singular vectors of the load-flow jacobian matrix associated to the smallest singular values.

The proposed algorithm has two phases:

1. An efficient algorithm for the computation of the n smallest singular values and singular vectors of the load-flow jacobian is proposed. This algorithm is a generalization of the one proposed in [2] for the computation of the smallest singular value. The algorithm is also similar to the one proposed in [3] for the computation of the n smallest jacobian eigenvalues. The computations are performed without loss of sparsity of the jacobian matrix preserving the computational efficiency.

In addition, the proposed algorithm allows to focus the singular analysis on any subsystem. A subsystem is defined as any subset of the load-flow equations, such as the active or reactive power flows, or the flows in a given geographical zone. It is worth to emphasize that the proposed algorithm takes also into account the equations out of the selected subsystem.

2. A standard data-grouping algorithm (the mediod algorithm [4]) is used to divide the system in coherent areas. This algorithm has the singular values and singular vectors computed in the previous step as input.

The algorithm can be used to compute the power system weak boundaries. High power flow across these boundaries can cause a voltage collapse. It can be also used to assess the effects of possible disturbances and corrective actions.

The sequel of this paper is organized as follows: in section 2 the power system model is described and the meaning of the singular values and vectors of the load flow jacobian addressed. In section 3 the generalized singular value analysis is introduced and its utility to focus on different power system areas explained. Section 4 deals with the application of a grouping algorithm to the clustering power system areas problem. Results corresponding to the New England 30 buses and IEEE 118 buses networks are provided in section 5. Finally, some additional material related to these topics is to be found in the appendixes.

2 Singular analysis and area partition

The steady-state behaviour of the power system is assumed to be modelled through a set of load flow equations. In short, this equations relate the active and reactive power bus injections (jointly denoted by $\mathbf{p}$) with the buses voltages magnitudes and angles (jointly denoted by $\mathbf{x}$):

$$\mathbf{p} = \mathbf{f}(\mathbf{x}) \quad (1)$$

Linearisation around a solution point $(\mathbf{x}_0, \mathbf{p}_0)$ yields the system

$$\Delta \mathbf{p} = \mathbf{f}_\mathbf{x} \Delta \mathbf{x} \quad (2)$$

where $\Delta \mathbf{p}$ and $\Delta \mathbf{x}$ are the increments $\mathbf{p} - \mathbf{p}_0$ and $\mathbf{x} - \mathbf{x}_0$, respectively. $\mathbf{f}_\mathbf{x}$ is the jacobian matrix evaluated at $(\mathbf{x}_0, \mathbf{p}_0)$.

We can invert (2) to obtain

$$\Delta \mathbf{x} = \mathbf{f}_\mathbf{x}^{-1} \Delta \mathbf{p} \quad (3)$$

so long as $\mathbf{f}_\mathbf{x}$ is non-singular.

On the other hand, it is possible to write $\mathbf{f}_\mathbf{x}$ using the singular value decomposition as:

$$\mathbf{f}_\mathbf{x} = \sum_i \sigma_i \mathbf{w}_i \mathbf{v}_i^T \quad (4)$$

where the singular values σ_i and singular vectors $\mathbf{v}_i$ and $\mathbf{w}_i$ are reals quantities which fulfill:

$$\mathbf{f}_\mathbf{x} \mathbf{v}_i = \sigma_i \mathbf{w}_i \quad (5)$$

$$\mathbf{f}_\mathbf{x}^T \mathbf{w}_i = \sigma_i \mathbf{v}_i \quad (6)$$

$$\mathbf{v}_i^T \mathbf{v}_j = \delta_{ij} \quad (7)$$

$$\mathbf{w}_i^T \mathbf{w}_j = \delta_{ij} \quad (8)$$

$$\sigma_i > 0 \quad (9)$$

It is easy to check that

$$\mathbf{f}_\mathbf{x}^{-1} = \sum_i \frac{1}{\sigma_i} \mathbf{v}_i \mathbf{w}_i^T \quad (10)$$

Therefore, equation (3) can be written as:

$$\Delta \mathbf{x} = \sum_i \frac{1}{\sigma_i} \mathbf{v}_i (\mathbf{w}_i^T \Delta \mathbf{p}) \quad (11)$$

Therefore, as the system response along each direction $\mathbf{v}_i$ is weighted by $\frac{1}{\sigma_i}$, it should be expected that the response is dominated by the smallest singular values. Moreover, when

some singular value gets close to zero, the state variations Δx for a given increment Δp grow unbounded. This is considered to be a hallmark of the voltage collapse phenomena. Deeper treatments of this subject are in [1, 5].

Close to the voltage collapse point, when a singular value is almost zero, the system response is almost entirely determined by the minimum singular value σ_1 and its singular vectors v_1 and w_1 . It is possible to approximate:

$$\Delta x \approx \frac{1}{\sigma_1} v_1 (w_1^T \Delta p) \quad (12)$$

Therefore, the different components of w_1 can be considered as the weights of each component of p in the total increment.

Power system areas can be defined according two criteria:

1. Two buses are in the same area when their response to power injections perturbations are similar. That means that the components of the vector v_1 related to the voltage magnitudes or angles in both buses should be similar.
2. Two buses are in the same area when the effect of a given perturbation of the injected power in either bus is similar. That means that the components of the vector w_1 related to the power injection in both buses are similar

On the other hand, it can be shown that the right and left singular vectors of a symmetric matrix are equals. f_x is not a symmetric matrix, but it is not very far from being one. Then, the right and left singular vectors are similar. This fact, which has been checked in [1], should be also expected on physical basis. Indeed, a big value of the component $w_1(i)$ in some bus i means that a given increment of the power injection Δp in that bus causes a big increment in the voltages magnitudes and angles across the system. In particular, the increment in bus j is given by equation (12):

$$\Delta x(j) \approx \frac{1}{\sigma_1} (w_1(i) \Delta p) v_1(j) \quad (13)$$

Note that $\Delta x(j)$ is proportional to $v_1(j)$. As it is expected that this change is particularly remarkable at the bus i , $v_1(i)$ is expected to be "big".

Therefore, both ways of obtaining power system areas (by using the left or the right singular vectors) should yield essentially the same result.

Consider now a situation where the power injection is increased in bus i and decreased by the same amount in bus j , that is, a power transfer between buses i and j . Then, the change in the system variables is given by:

$$\Delta x \approx \frac{1}{\sigma_1} v_1 (w_1(i) - w_1(j)) \Delta p \quad (14)$$

If buses i and j are in the same area, $w_1(i) - w_1(j)$ is small and the variables increment Δx is small too. But if buses i and j are in different areas $w_1(i) - w_1(j)$ is "big" and Δx is "big" too. This power transfer only causes great effects when buses i and j are in different areas. In other words, the power transfer only causes great effects when it crosses the areas boundaries. As a consequence, the areas boundaries are the system weak boundaries.

Summarizing, if the static behaviour of the power system is dominated by a singular value σ_1 and a weak boundary can be identified, then

1. In each one of the power system areas the right v_1 or left w_1 singular vectors, which are similar, have similar components for the variables in the area.

2. The system can be divided in areas which boundaries are the system weak boundaries.

Now, it is easy to generalize this theory to the case when the static behaviour is dominated by more than one singular value. Let us assume that:

$$\Delta x \approx \sum_{i=1}^N \frac{1}{\sigma_i} v_i w_i^T \Delta p \quad (15)$$

where N is the number of dominant singular values, which is assumed to be small. Then, the power system can be divided in several areas in which the components of w_i and v_i should remain similar. Once that the N smallest singular values and its associated singular vectors are computed, this task can be performed with the grouping algorithm proposed in section 4 and detailed in Appendix B.

3 Focusing on subsystems: Generalized singular analysis

The complete power-flow equations set has, very often, much more equations and variables than those which are of direct interest for the system partitioning. There are two main reasons:

1. It is a well-known fact that voltage collapse phenomena often arise as reactive power transfer problems. In this case only the reactive power injections and buses voltages magnitudes are of interest.
2. In most power systems the interconnections with neighbouring systems have non-negligible effects. One way to account them is to include an equivalent of the neighbouring systems. However, the internal behaviour of the equivalents is devoid of interest.

Both problems can be solved in a similar way. For this purpose, divide the whole set of power-flow variables x in two sets: one set of relevant variables x_R and another one of non-relevant variables x_N . As every variable is naturally paired with an equation (the buses angles with the active power injection in the same bus, and the buses voltages magnitudes with the reactive power injection), it can be written:

$$f^R(x_R, x_N) = p_R \quad (16)$$

$$f^N(x_R, x_N) = p_N \quad (17)$$

Linearization of this equation yields

$$f_{x_R}^R \Delta x_R + f_{x_N}^R \Delta x_N = \Delta p_R \quad (18)$$

$$f_{x_R}^N \Delta x_R + f_{x_N}^N \Delta x_N = \Delta p_N \quad (19)$$

In principle, Δx_N could be solved from (19) and inserted in (18). The result is

$$(f_{x_R}^R - f_{x_N}^R (f_{x_N}^N)^{-1} f_{x_R}^N) \Delta x_R = \Delta p_R - (f_{x_N}^N)^{-1} \Delta p_N \quad (20)$$

Therefore, in this case the singular values and vectors of interest are those of the Kron complement $f_{x_R}^R - f_{x_N}^R (f_{x_N}^N)^{-1} f_{x_R}^N$. This matrix is not usually sparse. In [2] an algorithm (the Smed algorithm) was proposed to compute the minimum singular value and vectors of the Kron complement of a sparse matrix without loss of sparsity. This algorithm

has been applied in [1], where, in our terminology, the relevant variables are the voltage magnitudes in all of the system buses. The Smed algorithm is essentially a generalization of the inverse power algorithm for the eigenvalue computation. The algorithm is explained in appendix A.

In a more formal way, the generalized singular value analysis could be defined as:

$$f_x v_i = \sigma_i E E^T w_i \quad (21)$$

$$f_x^T w_i = \sigma_i E^T E v_i \quad (22)$$

$$v_i^T E^T E v_j = \delta_{ij} \quad (23)$$

$$w_i^T E E^T w_j = \delta_{ij} \quad (24)$$

$$\sigma_i > 0 \quad (25)$$

where E is a possible singular matrix. Note that in the case that E is the identity, these equations are those that define the singular analysis of f_x (5-9). It can be proven that the σ_i are positive reals and that there are so many as the rank of E . Furthermore, let us assume that E is a 0-1 diagonal matrix such that the diagonal elements which correspond to the relevant variables are 1's, and the elements corresponding to the non-relevant variables are 0's. Then, the singular values and singular vectors of the generalized problem are the same than those obtained by the singular value of the Kron complement of f_x respect to the relevant variables. Summarizing, the 0-1 E matrix is an flexible way to specify the relevant variables in order to focus the analysis on the problem of interest.

The Smed algorithm can be generalized to compute the n -smallest singular values. The modified algorithm is alike to the simultaneous iteration inverse power algorithm for the eigenvalues computation. The n -dimensional subspace spanned by n -vectors is iterated instead of a single vector, as in the inverse power algorithm or in the Smed algorithm. This algorithm is well suited to the analysis of large power systems. The algorithm is properly described in appendix A.

4 Grouping the areas: the mediod algorithm

In section 2 was stated that the static behaviour of the power system is mainly determined by the smallest singular values and their singular vectors. Furthermore, as the right and left vectors are almost equals, the relevant information must be contained in any of both. In the sequel the right vectors v_i shall be used.

The algorithm used in this paper cluster all the buses in a predetermined number of areas. It uses the information contained in the vectors v_i of a given number of singular values. It also yields a measure of the "goodness" of the partition. Comparing the value of this measure for a given number of singular values and different numbers of areas it can be detected an "optimal" number of areas, i. e., the number of areas which is "meaningful" to identify. As the number of considered singular values increases, the number of identified areas increases too.

The behaviour of each bus voltage magnitude and angle is determined by the two corresponding components of each selected singular vector v_i . In the sequel it shall be assumed that the voltage magnitudes are the relevant variables. Generalizations involve no difficulty.

With this assumption, the component of the singular vector v_i corresponding to the bus j voltage magnitude is denoted by $v_i[j]$.

For a predetermined number of selected singular values, two buses j and k belong to the same area when all the components $v_i[j]$ and $v_i[k]$ are similar. Then, it is defined the distance between buses j and k as:

$$D(j, k) = \sum_i \|v_i[j] - v_i[k]\| \quad (26)$$

The sumatory index runs over all the selected singular values and vectors. This quantity is small when both sets of components are similar. In this case, any clustering algorithm should cluster both buses in the same area.

Let us assume that it is desired to divide the power system in N areas. The mediod algorithm used by the authors chooses a representative bus from each area, known as the mediod. This bus is the one which distance to all the remaining buses in the cluster is minimal. Formally, the mediod bus m of the area A_l is

$$m = \text{sol}_k \min \sum_{j \in A_l} D(j, k) \quad (27)$$

And the index

$$O_l = \sum_{j \in A_l} D(j, m) \quad (28)$$

is a measure of the "goodness" of the area. Then, it can be considered that the global index

$$O = \sum_{l=1}^N O_l \quad (29)$$

is a measure of the "goodness" of the partition. A smaller O implies a better partition.

So, to find the better partition the mediod algorithm minimize the index O . See Appendix B for further information.

Note that the optimal O_{opt} is a decreasing function of the number of areas. Furthermore, if there are so many areas as buses, every area "goodness" O_l is zero, and the system "goodness" O is also zero.

Then, there is a function $O(N)$ which decreases from its maximum value when $N = 1$ to zero for N equal to the number of buses. It is observed that this decrement is not uniform. The function decreases faster when N is low than when N is high. The transition between the two behaviours is neatly detected in the cases studied. It is proposed that the value of N in which the transition happens should be considered as the number of areas meaningful for the set of singular vectors analyzed.

5 Results

Two systems were studied with the proposed algorithms. The first one is the New England 30 buses system (see figure 1). Data on the network parameters and operating point can be found on [6].

The set of selected relevant variables are the voltage magnitudes in all the PQ buses. Table 1 shows the complement to 1 of the correlation factors between the singular left and right vectors w and v . Only the components corresponding to the reactive power injections and voltage magnitudes in each bus were correlated. The data show that the vectors are almost equals.

The figures 1, 2, 3 and 4 show the different clustering areas obtained by the mediod algorithm, using 1, 2, 3, 4 or 5 singular values and vectors.

The load was homothetically increased by 50%. The new singular values and the correlations between their singular vectors are shown in table 2. The areas obtained by the clustering algorithm are shown in figures 5 and 6. Note the high similarity with the areas previously obtained.

The correlation factors between the right singular vectors of both cases (base case and 50% load increment case) are shown in table 3. The figures in the border are the correlations of the right vector associated with the singular value at

Singular value	$1 - \frac{w^T v}{\ w\ \ v\ }$
28.3528	5.3716×10^{-6}
40.9919	1.1112×10^{-6}
57.9957	4.9868×10^{-6}
59.6554	3.9445×10^{-6}
66.5749	2.4539×10^{-8}

Table 1: Singular values and correlation factors between singular vectors. Base case of the New England System.

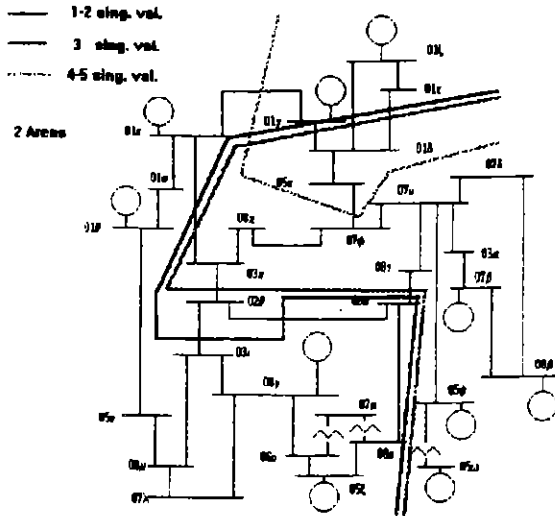


Figure 1: 2 Areas Clustering. Base case of the New England System

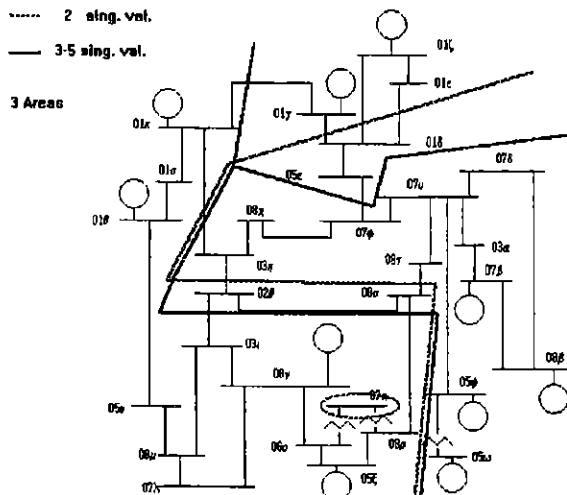


Figure 2: 3 Areas Clustering. Base case of the New England System

Singular value	$1 - \frac{w^T v}{\ w\ \ v\ }$
21.3147	1.9574×10^{-5}
30.8061	3.5646×10^{-5}
40.5594	4.9604×10^{-6}
56.8821	1.5759×10^{-5}
58.5176	1.8775×10^{-5}

Table 2: Singular values and correlation factors between singular vectors. 50% load increment case of the New England System.

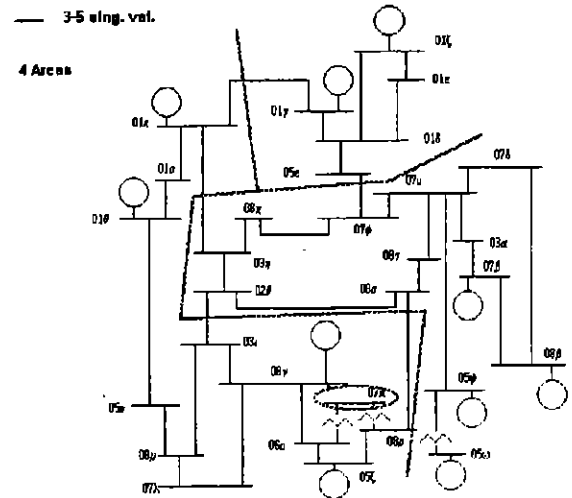


Figure 3: 4 Areas Clustering. Base case of the New England System

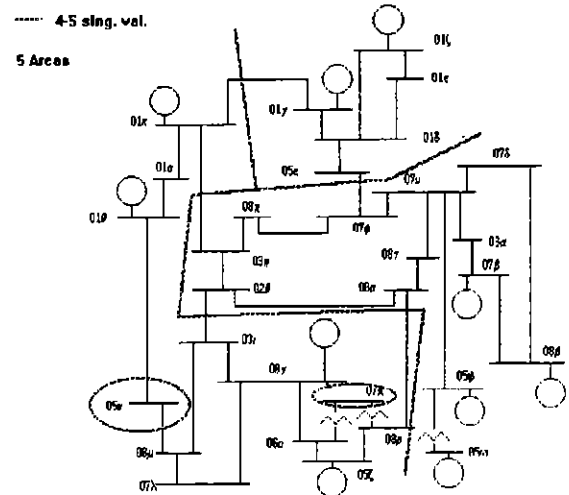


Figure 4: 5 Areas Clustering. Base case of the New England System

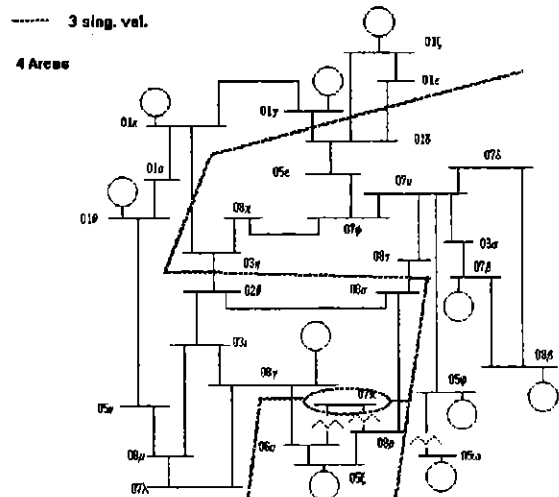


Figure 5: 4 Areas Clustering. 50% load increment case of the New England System

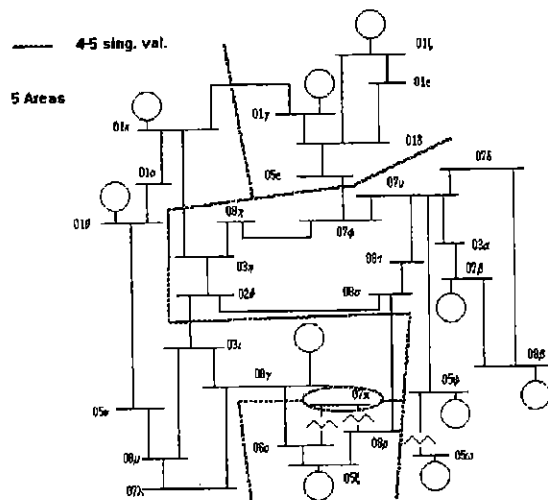


Figure 6: 5 Areas Clustering. 50% load increment case of the New England System

Singular value	$1 - \frac{w^T v}{\ w\ \ v\ }$
4.1270	1.8577×10^{-4}
22.5960	1.4541×10^{-4}
36.4403	3.2231×10^{-5}
52.3505	3.5462×10^{-5}
56.4522	3.0640×10^{-5}

Table 4: Singular values and correlation factors between singular vectors. 25% additional reactive load increment case of the New England System.

the beginning of the row or column with the subspace spanned by all the right singular vectors of the other case. The high figures mean that the subspace spanned by the right vectors corresponding to the first five singular values is almost invariant. These figures show that the singular vectors and, specially, the subspace that span, remain fairly stable when the operation point changes.

The reactive load in all buses was increased and additional 25%. The singular values and correlation factors for the new operating point are given in table 4. Note that for this increment, a singular value is far smaller than the rest. For a reactive load slightly greater, it becomes zero and the power system suffers a voltage collapse. The height of the bars in figure 7 is proportional to the magnitude of the component of the right singular vector associated to the smallest singular value. The voltage collapse area is identified by the clustering algorithm. Note the similarity between this area and the West area determined at the initial operating point (see figure 1 as an example).

The same algorithms were run on the IEEE 118 buses network. Again, the set of relevant variables are the voltage magnitudes in all the PQ buses. The system parameters data and the operating point can be found in [7]. The correlation factors for the left and right singular vectors v and w are shown in table 5. Again, only the reactive power injections and voltage magnitudes components were correlated. Note again the almost perfect parallelism between these vectors.

Figure 8 shows the number of components for the 3 first right singular vectors v . It can be seen that most of the components are very small. The components with greater magnitude should define an area.

The clustering algorithm was run for different number of singular vectors. The behaviour of the objective function for several number of singular values as the number of clustering

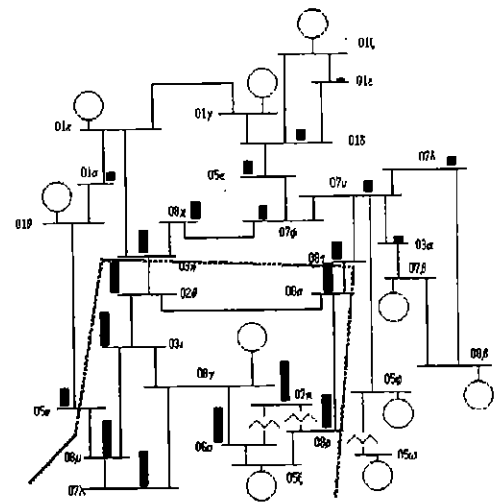


Figure 7: Singular vector components. 25% additional reactive load increment case of the New England System.

Singular value	$1 - \frac{w^T v}{\ w\ \ v\ }$
3.6666	3.1670×10^{-6}
3.9281	3.8255×10^{-6}
5.0243	6.7866×10^{-6}
5.4126	1.2345×10^{-4}
5.9908	8.9652×10^{-6}
8.2888	7.6403×10^{-5}
8.7421	1.1024×10^{-6}
8.9336	3.4794×10^{-5}
10.6359	1.5573×10^{-4}
11.0741	1.4349×10^{-4}

Table 5: Singular values and correlation factors between singular vectors. IEEE-118 System.

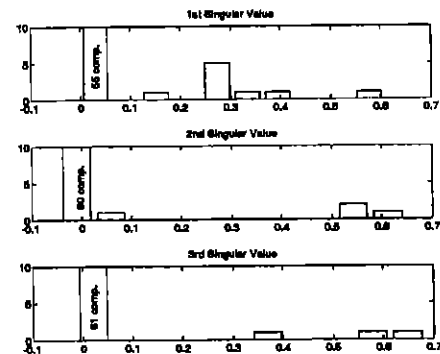


Figure 8: Number of components versus magnitude of the right singular vectors. IEEE-118 System.

	21.3147	30.8061	40.5594	56.8821	58.5176	
28.3528	0.7362	0.1494	-0.3600	-0.3018	-0.0825	0.8899
40.9919	0.6659	-0.2748	0.3988	0.3702	0.0084	0.9028
57.9957	-0.0748	-0.9461	-0.2110	-0.1883	-0.0419	0.9912
59.6554	0.0406	-0.0077	0.5948	-0.7278	0.3247	0.9954
66.5749	0.0427	-0.0354	-0.1942	0.2701	0.9183	0.9782
	0.9972	0.9972	0.8513	0.9308	0.9784	

Table 3: Correlation factors between right singular vectors, for the base case and the 50% load increment case of the New England System.

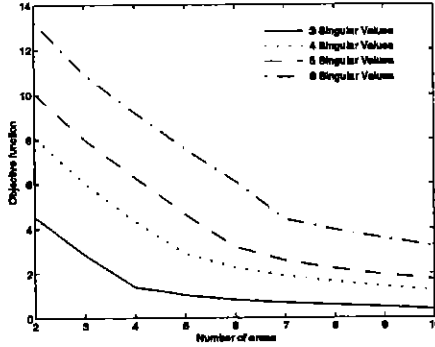


Figure 9: Objective function evolution. IEEE-118 System.

areas is increased is shown in figure 9. It can be seen that there is a discontinuity in the function decreasing rate when the number of areas is equal to the number of singular values plus one. This is considered to be the meaningful value of the number of areas.

The clustering areas for each one of these optimal selections is shown in figure 10. In addition to the bordered areas there is a background area which includes all the buses which do not belong to these areas. Each singular value adds a new area:

- With three singular values there are four areas: the areas 1, 2 and 3, and the background area.
- With four singular values, the area 4 is added.
- With five singular values, the area 5 is added.
- With six singular values, the area 4 is split in areas 4a and 4b.

This system was also analyzed focusing on the south zone of the system. The border of this zone is the dashed curve in figure 11. That means that the set of selected relevant variables are the voltage magnitudes of the PQ buses in this zone. The two smallest singular values and the correlation between singular vectors are shown in table 6. It is also shown the difference with the most similar singular value computed when all the PQ buses voltage magnitudes were relevant variables, and the complement to 1 of the correlation between the corresponding right vectors.

Again, the clustering algorithm is run with this singular vectors. The results are shown in figure 11:

- With 1 singular value, it is obtained the area 1 and the background area.
- With 2 singular values, the area 1 is split in areas 1a and 1b, which are identical to areas 4a and 4b of the previous case.

Sing. val.	$1 - \frac{\mathbf{w}^T \mathbf{v}}{\ \mathbf{w}\ \ \mathbf{v}\ }$	$\sigma_{old} - \sigma_{new}$	$1 - \frac{\mathbf{v}_1^T \mathbf{v}_2}{\ \mathbf{v}_1\ \ \mathbf{v}_2\ }$
5.4126	$1.2366 * 10^{-4}$	$7.7068 * 10^{-7}$	$1.5446 * 10^{-6}$
8.2888	$7.6383 * 10^{-5}$	$5.4084 * 10^{-9}$	$4.8637 * 10^{-9}$

Table 6: Singular values and correlation factors between singular vectors. IEEE-118 System. South zone.

Therefore, the area clustering of the South zone can be accomplished more efficiently by focusing the algorithm on this zone, taking the rest of the system as non relevant variables.

6 Conclusions

This paper address the power system division in voltage collapse areas through the generalized singular values analysis. It has been shown that the system static behaviour is dominated by the smallest singular values. The system is divided in areas by using the information contained in the right and left singular vectors associated with these singular values. Two buses belong to the same area when the two corresponding components of the right and left singular vectors are similar. As the right and left vectors are alike, this condition can be checked in either one.

The power system variables can be divided in two sets of relevant and non-relevant variables. This fact allows to focus the analysis on a zone or a type of variables of the power system (for instance, just the voltage magnitudes).

Two algorithms have been developed. The first one computes the smallest singular values and their singular vectors corresponding to the subset of relevant variables. The second one is an optimization algorithm to perform the areas clustering of the power system. The number of areas which can be meaningfully computed has been found to be equal to the number of singular values plus one.

These algorithms have been applied to two systems widely studied in the literature. It is found that the singular vectors and the areas determined by the clustering algorithm remain fairly stable when the operation point changes.

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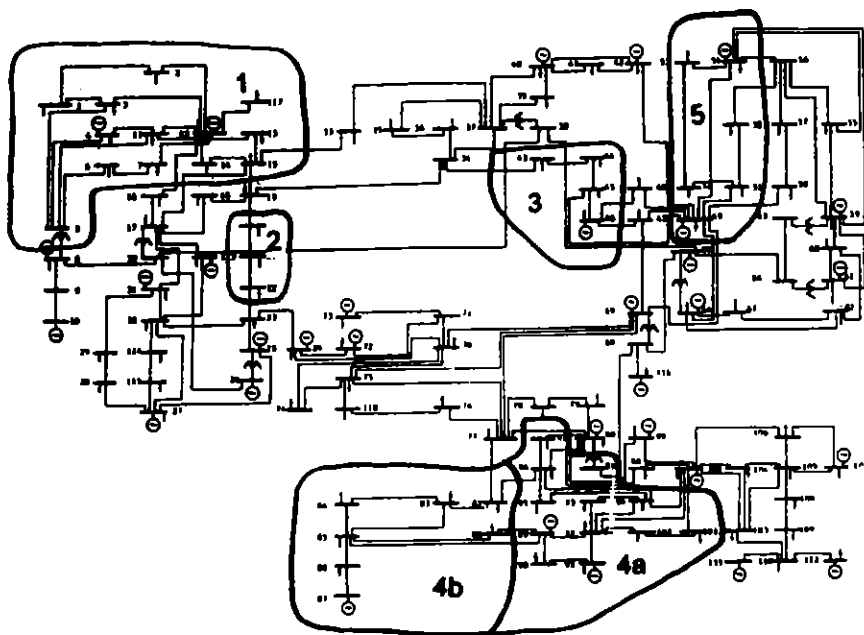


Figure 10: Area clustering. IEEE-118 System.

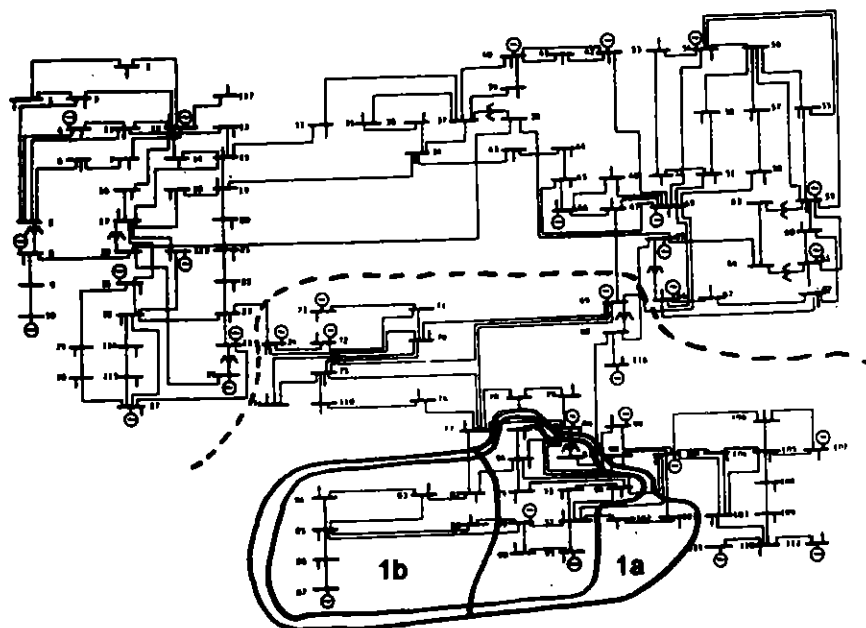


Figure 11: Area clustering. South zone of the IEEE-118 System.

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A The generalized singular value decomposition algorithm

The proposed algorithms are based upon the Smed algorithm for the computation of the generalized smallest singular value. This algorithm is alike to the inverse power algorithm for eigenvalues computations [8]. Using the section 3 notation the Smed algorithm is in pseudo-code:

```

set tolerance
set  $w_\sigma^0$ 
 $w_\sigma^0 \leftarrow [w_\sigma^{0T} E E^T w_\sigma^0]^{-1} w_\sigma^0$ 
while 1
   $v_\sigma^{k+1} \leftarrow f_x^{-1}[E E^T w_\sigma^k]$ 
   $\sigma_\sigma^{k+1} \leftarrow [v_\sigma^{k+1,T} E^T E v_\sigma^{k+1}]^{-1}$ 
   $v_\sigma^{k+1} \leftarrow \sigma_\sigma^{k+1} v_\sigma^{k+1}$ 
   $w_\sigma^{k+1} \leftarrow f_x^{-T}[E^T E v_\sigma^{k+1}]$ 
   $\sigma_\sigma^{k+1} \leftarrow [w_\sigma^{k+1,T} E E^T w_\sigma^{k+1}]^{-1}$ 
   $w_\sigma^{k+1} \leftarrow \sigma_\sigma^{k+1} w_\sigma^{k+1}$ 
  if  $\|\sigma_\sigma^{k+1} - \sigma_\sigma^k\| < \text{tolerance}$ 
    break
  else end
end

```

The basic structure of the generalized singular value computation algorithm is very similar to that of the simultaneous iteration algorithm for the simultaneous computation of several eigenvalues. In the inverse power or the Smed algorithm a vector, which can be considered a 1 times m matrix, is iterated. m is the dimension of the jacobian matrix. The result is a single eigenvalue or singular value, and its corresponding right and left vectors. In the simultaneous iteration algorithm or the generalized algorithm a n times m matrix is iterated. The result are n eigenvalues or singular values and their corresponding vectors. The iterated matrix must be orthogonalized in each iteration.

For the description of the proposed algorithm the function svd is useful:

$$[U, \Sigma, V] = \text{svd}(J)$$

which returns the singular value decomposition of J :

$$J = U^T \Sigma V$$

being Σ a diagonal matrix with the singular values in increasing order, and U and V matrices which columns are the corresponding singular left and right vectors. Note that, in the proposed algorithm, this function is only applied on reduced dimension matrices.

Then, the pseudo-code of the generalized algorithm is:

```

set tolerance
set  $W^0$ 
while 1
   $V_a \leftarrow f_x^{-1}[E E^T W^k]$ 
   $[U_1, \Sigma_1, V_1] \leftarrow \text{svd}(V_a^T E^T E V_a)$ 
   $V^{k+1} \leftarrow V_a V_1^{-1} \Sigma_1^{-\frac{1}{2}}$ 
   $W_a \leftarrow f_x^{-T}[E^T E V^{k+1}]$ 
   $[U_2, \Sigma_2, V_2] \leftarrow \text{svd}(W_a^T E E^T W_a)$ 
   $W^{k+1} \leftarrow W_a U_2^{-1} \Sigma_2^{-\frac{1}{2}}$ 
  if  $\|\Sigma_1 - \Sigma_2\| < \text{tolerance}$ 
    break
  else end
end

```

Of course, operations of the form $f_x^{-1}b$ are done using the LU factorization of f_x .

B The medioid algorithm

This appendix contains an explanation of the medioid algorithm used to obtain the area clustering from the singular vectors. The algorithm is throughly described in [4].

Let us recall that the purpose of the algorithm is to obtain the areas which minimize the sum of the distances of each bus to the "central" bus of their cluster (the medioid). The distance between buses i and j is given by:

$$D(i, j) = \sum_k \|\mathbf{v}_k[i] - \mathbf{v}_k[j]\|$$

The sumatory index runs over all the selected singular values and vectors. The medioid is the bus in each area which has minimal mean distance to the rest of the area buses.

For small and medium size systems (up to about 100 or 200 buses) the simple version of the algorithm can be used. This version has two phases:

- Phase I. The purpose of phase I is to obtain an initial clustering of all the buses. This is done through the following sequential steps:

1. Consider a non-selected bus i .
2. Consider a non-selected bus j and compute its distance D_j to the closest previously selected bus and its distance $D(i, j)$ to bus i .
3. Compute

$$C_{ji} = \max(D_j - D(i, j), 0)$$

4. Compute the total gain obtained by choosing bus i :

$$\sum_j C_{ij}$$

5. Choose the non-selected bus i which maximizes $\sum_j C_{ij}$.

The process is iterated until k buses have been selected, being k the number of desired areas. Then, the k areas are made by associating to each selected bus the buses which are closer to it than to any other selected bus.

- Phase II. The purpose of this phase is to improve the initial clustering obtained in the previous phase. This is done by swapping a medioid (selected) bus for a non-medioid (non-selected) bus. More precisely, the algorithm is:

1. Consider a non-selected bus j and calculate its contribution C_{ijh} to the swap:

- (a) If j is more distant from both i and h than from the other selected buses, C_{ijh} is zero.
- (b) If j is not further from any other selected bus ($D(j,i) = D_j$), two situations must be considered:
 - i. j is closer to h than to the second closest representative bus

$$D(j,h) < E_j$$

where E_j is the distance between j and the second most closest bus. In this case, the contribution C_{ijh} to the swap between buses j and h is:

$$C_{ijh} = D(j,h) - D(j,i)$$

- ii. j is at least as distant from h than from the second closest selected bus

$$D(j,h) \geq E_j$$

In this case

$$C_{ijh} = E_j - D_j$$

- (c) j is more distant from bus i than from at least one of the selected buses but closer to h than to any selected bus. In this case

$$C_{ijh} = D(j,h) - D_j$$

2. Calculate the total result of a swap by adding the contributions

$$T_{ih} = \sum_j C_{ijh}$$

3. Select the pair (i,h) which minimizes T_{ih} .
4. If the minimum T_{ih} is negative, the swap is carried out and the phase II is iterated again. If not, the selected clustering is optimum and the algorithm stops.

It is worth to remark that in the reported cases the selection made by the phase I was almost optimal, requiring only 3 iterations of the phase II.

In the case of having a large number of buses, the same algorithm than before can be used, but applied to a smaller set of buses randomly chosen. The non-chosen buses are added after finishing the algorithm to the area of the closest mediod. This procedure is iterated 5 or 6 times, and the clustering with smallest objective function is finally selected.

Assessment of Static Voltage Stability Limit Based on the Complex-valued NR Load Flow in Power Systems

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Abstract

This paper presents a methodology for determining a 'real' voltage stability limit (RVSL) of power systems considering operating constraints and voltage stabilizing effects by the installation of VAR sources. The necessary and sufficient conditions for a system state to be on the RVSL are derived, and the marginal power demand and the marginal SC installation are determined based on these conditions. To search for the RVSL mathematically, a preventive control method previously developed by the authors for enhancing the voltage stability is applied, and two types of algorithm which differs in search direction fundamentally are presented. In one method, we can obtain the RVSL where marginal power is larger relatively. In the other method, the RVSL which is closest to a current operating condition can be obtained. The basic causes for its difference are discussed in detail and illustrated by numerical examples.

Introduction

In recent power systems, the increased loading on long-distance transmission lines and insufficient local reactive power capacity have been observed; therefore, problems related to voltage instability in power systems are today, in many countries, a major concern [1]. To maintain the voltage stability, it is necessary to monitor and control power system states continuously. Further, even in power system planning such as the installation of VAR sources and changes of voltage classes of transmission lines, it has been indispensable to consider the voltage stability [2].

Voltage stability has been studied extensively using both static and dynamic approaches [3]. In the former approach, the voltage instability is analyzed by algebraic equations such as load flow equations, and in the latter approach, differential equations for apparatus such as induction motor loads, tap changing transformers are considered. For power system planning it is sufficient to use the former approach. More specifically, it is necessary to know the static voltage stability limit (VSL) which shows to what extent a system configuration can withstand load increase. The VSL corresponds mathematically to the tip of the power-voltage (P-V) curve [4], and naturally, voltage collapse results when the load demand exceeds this point. However, in actual operation consideration of this point alone is not adequate because the system state may violate operating constraints before the load demand reaches this point [5]-[8]. Further, since operators attempt to correct the voltage profile by introducing existing VAR sources in accordance with the load increase, it is undesirable to determine the VSL of the particular P-V curve for the VAR sources being fixed.

The authors define the 'real' VSL (RVSL) as follows: the point representing the largest load increase without violating operating constraints and without causing the P-V curve's lower voltage portion for each respective bus to emerge inside the constraint range. More specifically, if constraint violation does not occur, it corresponds to the point where the lowest

voltage magnitude in each bus equals the lower limit at the tip of its P-V curve. Since the lower voltage portions are considered, an operator can monitor the voltage instability by observing whether or not the bus voltage magnitude violates the lower limit.

This paper presents a new method for calculating the above defined RVSL. This method uses the complex-valued Newton-Raphson method for the load flow; therefore, it has the advantage of being able to obtain a load flow solution without diverging near the critical point.

Basic Idea of Voltage Stability Limit

As is well known, in order to maintain a load bus voltage magnitude at a desirable value, it is necessary to adequately adjust voltage control apparatus such as static condensers (SC) as power demand increases. Although the successive adjustment of SC contributes to an increase of the marginal power demand, the voltage magnitude at the tip of the P-V curve comes close to its lower limit as shown in Fig.1. If this adjustment is further continued, the voltage magnitude increases over the lower limit. More specifically, the lower voltage portions of the PV curve may lie within the usual operating voltage range. In this case, an operator can not assess the voltage stability by determining whether or not voltage magnitudes violate their limits; therefore, it is thought that introducing SC to this point is undesirable in the power system operation. Thus, when we consider voltage adjustment by SC, the 'real' voltage stability limit (RVSL) becomes the point where a target bus voltage magnitude at the tip of the PV curve equals its lower limit. In this paper, we call the amount of SC installed to this point the marginal SC installation. Here, the bus with the lowest voltage magnitude in a power system should be selected as the target bus.

The sensitivity of the total power demand with respect to load bus voltage magnitude, dP_d/dV_d has a positive value at the usual higher voltage portions, and conversely it has a negative value at the lower voltage portions. Further, as the load demand gets close to the tip of the PV curve, the absolute value of the above sensitivity becomes smaller (it becomes zero at the tip of the PV curve). Thus, for an operating state to be at the RVSL, the conditions necessary have to conform with

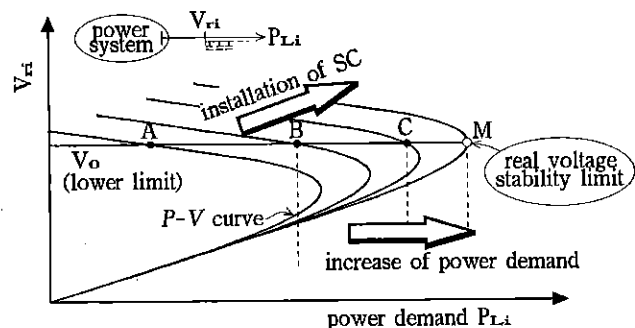


Fig.1 Changes of the P-V curve by the introduction of SC.

the following equations:

$$\begin{aligned} (a) dP_d/dV_d &= 0 & (1) \\ (b) V_{di} &= V_{dimin} & (2) \end{aligned}$$

Here, the subscript i is the target bus number. Note that a solution which satisfies the above conditions is not unique, however, a number of solutions are possible depending on the SC rating at each bus. More specifically, even if the total installation of SC is the same, a different RVSL can be obtained depending on its distribution rate. In power system planning, it is necessary to obtain the RVSL where the marginal power demand becomes a maximum. Conversely, in system operation, it is necessary to calculate the closest RVSL to a current operating point.

Determination of Real Voltage Stability Limit

Calculation of RVSL Index

As described above, it is necessary to assess the value of dP_d/dV_d when calculating the RVSL. In this section, we explain how this index can be calculated for a particular PV curve for the value of SC being fixed.

First, we express the load flow equation as follows:

$$\underline{f}(\underline{h}^s, \underline{x}, \underline{y}_{sc}) = \underline{h}^s - \underline{h}(\underline{x}, \underline{y}_{sc}) = \underline{0} \quad (3)$$

where,

$\underline{h}^s = [\underline{\alpha}^T P_d, \underline{y}_{sc}^T]^T$ (vector composed of specific variables)

P_d : total power demand

$\underline{\alpha}$: vector for projecting an assumed total load demand on real and reactive powers injected in individual bus (it provides the so-called load increase scenario)

$\underline{y}_{sc}$: vector composed of specified generator voltage magnitudes

$\underline{x}$: vector composed of bus voltage phasor, $\underline{x}_i$ expressed in the rectangular form

$\underline{y}_{sc}$: vector composed of the susceptance of SC

The total differential of $\underline{f}$ in eq.(3) can be expressed as follows:

$$\begin{aligned} d\underline{f} &= \left[\frac{\partial \underline{f}}{\partial \underline{h}^s} \right] d\underline{h}^s + \left[\frac{\partial \underline{f}}{\partial \underline{x}} \right] d\underline{x} + \left[\frac{\partial \underline{f}}{\partial \underline{y}_{sc}} \right] d\underline{y}_{sc} \\ &= \left[\frac{\partial \underline{f}}{\partial \underline{h}^s} \right] \left[\frac{\underline{\alpha} dP_d}{d\underline{y}_{sc}} \right] + \left[\frac{\partial \underline{f}}{\partial \underline{x}} \right] d\underline{x} + \left[\frac{\partial \underline{f}}{\partial \underline{y}_{sc}} \right] d\underline{y}_{sc} \quad (4) \end{aligned}$$

Since $d\underline{f} = \underline{0}$ and $d\underline{y}_{sc} = \underline{0}$, and also $[\partial \underline{f} / \partial \underline{h}^s]$ is a unit matrix, the following equations are derived.

$$d\underline{x} = - \left[\frac{\partial \underline{f}}{\partial \underline{x}} \right]^{-1} \left[\frac{\underline{\alpha} dP_d}{d\underline{y}_{sc}} \right] \quad (5)$$

However, the following equation has to be satisfied for target bus i .

$$dV_{di} = [\partial V_{di} / \partial \underline{x}_i] d\underline{x}_i \quad (6)$$

Thus, by instituting the above equation into eq.(5), the following equations can be obtained under conditions that $d\underline{y}_{sc} = \underline{0}$.

$$dV_{di} = - \left[\frac{\partial V_{di}}{\partial \underline{x}_i} \right] \left[\frac{\partial \underline{x}_i}{\partial \underline{f}} \right] \left[\frac{\underline{\alpha} P_d}{0} \right] \quad (7)$$

Thus, by assessing the above sensitivity matrix for a steady state, the value of dP_d/dV_{di} in the state can be obtained. In the following section, we explain the specific algorithms for calculating the RVSL.

Solution Algorithm Using dP_d/dV_d (Method 1)

The algorithm proposed in this section updates the to-

tal demand according to the following equation, and thereafter adjusts the values of SC so that every constraint condition is satisfied.

$$\Delta P_d = -k(dP_d/dV_{di}) \quad (8)$$

$$P_d^{new} = P_d^{old} + \Delta P_d \quad (9)$$

where, k : an appropriate positive multiplier

Since the absolute value of the sensitivity, dP_d/dV_{di} , becomes smaller as the power demand becomes close to the marginal power demand, the updated P_d are large during the early conditions, however fine searches are implemented near the RVSL.

Here, if P_d^{new} exceeds the marginal power demand, there is no load flow solution, that is, a stable operating state does not exist. In this case, we have to adjust the value of SC so that a load flow solution exists. In this paper, we call this adjustment Control V. The authors presented a complex-valued NR load flow method in which real and imaginary parts of individual bus voltage phasor in rectangular form are extended to complex numbers [9]. Outlines of this method are given in Appendix. When the assumed power demand exceeds the marginal value, a load flow solution by this method converges to a complex number. Thus, to shift the power system state into a region where a stable solution exists, it is desirable to adjust the value of SC so that the extended imaginary parts become non-existent.

On the other hand, even if a load flow solution exists, it is undesirable for a state obtained by Control V to violate constraints such as upper and lower limits of load bus voltage magnitude and generator reactive power. In this case, it is necessary to further adjust the value of SC. In this paper, we call this adjustment Control S. Naturally, at the target bus the condition that the voltage magnitude equals the lower limit is constrained. The amount of SC to be adjusted in Control V and Control S can be calculated using the preventive control method proposed by the authors [10]. In the Appendix, objective functions for each control are given.

The specific algorithm for calculating the RVSL is summarized as follows:

[Step 1]: Assume the amount of load increase according to eqs.(8) and (9).

[Step 2]: The complex-valued NR load flow is implemented for the assumed load increase. If a solution converges to a real number, go to Step 3. Otherwise, the optimization based on the preventive control algorithm in the reference [10] is implemented so that the extended imaginary parts become non-existent (Control V).

[Step 3]: Assess constraint violations of the load bus voltages and generator reactive power outputs. If the violations do not exist, go to step 4. Otherwise, the optimizing calculations for eliminating the violations are implemented (Control S).

[Step 4]: Assess the sensitivity index, dP_d/dV_{di} . If the value is zero, exist. Otherwise, update P_d according to eqs.(8) and (9), and go back to Step 2.

Here, when the assumed power demand exceeds the RVSL, a solution which satisfies $V_{di} = V_{dimin}$ in the Step 3 does not exist in the higher voltage portions. In this case, the algorithm searches for values of P_d and y_{sc} by the dichotomous method so as to satisfy this constraint condition, and goes to Step 2. More specifically, the value of dP_d/dV_{di} is not assessed at the lower voltage portions.

Note that this method is not an exact optimizing method, because multiplier k is fixed. Thus, even by this method a solution can not always be obtained.

Solution Algorithm Based on Non-linear Optimization (Method 2)

The algorithm proposed in this section is based on an exact non-linear optimization: the steepest descent method. The objective function is defined as $F=(dP_d/dV_d)|_{V_d=V_{dmin}}$, and the amount of SC for minimizing this function is determined by a successive one-dimensional search. The optimal step length of Y_{sc} , β which minimizes the objective function along the given descent direction is calculated by the quadratic interpolation method. Using this β , Y_{sc} is updated according to the following equations.

$$\Delta Y_{sc} = -\beta(dF/dY_{sc}) \quad (10)$$

$$Y_{sc}^{new} = Y_{sc}^{old} + \Delta Y_{sc} \quad (11)$$

These operations are continued until the objective function becomes zero. Here, note that the one-dimensional search has to be implemented considering constraints on the voltage magnitude of the target bus, because the objective function is defined as dP_d/dV_d when this equality constraint is satisfied. Thus, to calculate the objective function for the given Y_{sc} , optimization with respect to the total power demand has to be performed, so that this equality constraint is satisfied. The desirable power demand is calculated in the same manner for the Control S of Method 1. In this case, the complex valued NR load flow is applied in every search point, because the given power demand may exceed its marginal value.

The complete algorithm is summarized as follows:

[Step 1]: Calculate the steepest descent vector, dF/dY_{sc} and determine the optimal step length of Y_{sc} by the one-dimensional search method. Here, at every search point, P_d is also updated so that $V_d = V_{dmin}$ is satisfied.

[Step 2]: Assess dP_d/dV_d for the optimal Y_{sc} and P_d determined in Step 1. If this value is zero, exit. Otherwise, go to Step 1.

If there is no level of power demand which satisfies constraints at a search point of Step 1, the algorithm updates the value of Y_{sc} by the dichotomous method in order to meet the constraints.

Calculation of dF/dY_{sc}

In this section, a method for calculating the sensitivity, dF/dY_{sc} , in eq.(10) is described. First, in eq.(4), $d\mathbf{f}$ and $d\mathbf{y}^s$ are zero, and $[\partial \mathbf{f} / \partial \mathbf{h}^s]$ is the unit matrix; therefore, this equation is expressed as follows:

$$\begin{bmatrix} \alpha dP_d \\ 0 \end{bmatrix} + \begin{bmatrix} \partial \mathbf{f} \\ \partial \mathbf{x} \end{bmatrix} d\mathbf{x} + \begin{bmatrix} \partial \mathbf{f} \\ \partial \mathbf{y}_{sc} \end{bmatrix} d\mathbf{y}_{sc} = 0 \quad (12)$$

Here, since $dV_{di}=0$ and $\mathbf{x}_i = [e_i, f_i]^T$, $df_i = [\partial f_i / \partial e_i] de_i = -(f_i/e_i) de_i$. Thus, the above equation can be expressed as follows:

$$\begin{bmatrix} \alpha \\ 0 \end{bmatrix} \begin{bmatrix} \partial \mathbf{f} \\ \partial \mathbf{x} \end{bmatrix} \begin{bmatrix} dP_d \\ d\mathbf{x}' \end{bmatrix} + \begin{bmatrix} \partial \mathbf{f} \\ \partial \mathbf{y}_{sc} \end{bmatrix} d\mathbf{y}_{sc} = 0 \quad (13)$$

Here, $\mathbf{x}'$ is a voltage vector in which f_i is removed from vector $\mathbf{x}$. In addition, the total differential of F is calculated as follows:

$$dF = \begin{bmatrix} \partial F \\ \partial \mathbf{h}^s \end{bmatrix} d\mathbf{h}^s + \begin{bmatrix} \partial F \\ \partial \mathbf{x} \end{bmatrix} d\mathbf{x} + \begin{bmatrix} \partial F \\ \partial \mathbf{y}_{sc} \end{bmatrix} d\mathbf{y}_{sc} \quad (14)$$

Therefore, since $[\partial F / \partial \mathbf{h}^s] = 0$ and $dV_{di}=0$, the following equations are obtained using eq.(13).

$$dF = \begin{bmatrix} 0 \\ \partial \mathbf{f} \\ \partial \mathbf{x}' \end{bmatrix} \begin{bmatrix} dP_d \\ d\mathbf{x}' \end{bmatrix} + \begin{bmatrix} \partial F \\ \partial \mathbf{y}_{sc} \end{bmatrix} d\mathbf{y}_{sc} = - \left[\begin{bmatrix} 0 \\ \partial \mathbf{f} \\ \partial \mathbf{x}' \end{bmatrix} \begin{bmatrix} \alpha \\ 0 \end{bmatrix} \begin{bmatrix} \partial \mathbf{f} \\ \partial \mathbf{y}_{sc} \end{bmatrix}^{-1} \begin{bmatrix} \partial \mathbf{f} \\ \partial \mathbf{y}_{sc} \end{bmatrix} + \begin{bmatrix} \partial F \\ \partial \mathbf{y}_{sc} \end{bmatrix} \right] d\mathbf{y}_{sc} \quad (15)$$

Numerical Examples

Simulation Results

The proposed methods were applied to a 22-bus model system as shown in Fig.2 and Table 1. The assumed scenario is the following: (1) the power demand increases uniformly under a constant power factor, (2) this increase is distributed among all the generators at a constant rate. In this simulation, however, upper and lower limits on generator real and reactive power outputs are not considered. The upper and lower limits of every load bus voltage magnitude are set as 1.1 [pu] and 0.9 [pu] respectively. Further, when the sensitivity index, dP_d/dV_d , becomes smaller than 1.0×10^{-6} , we judge that the power system state is at the RVSL. Here, in the initial state, the voltage magnitude of bus 22 is a minimum.

Table 2 shows the marginal SC installation, marginal power demand and computation time, and Fig.3 shows the PV curve for this case. Here, multiplier k in Method 1 was set at 0.4. As is evident from these results, the marginal power by Method 2 is smaller than that by Method 1. More exactly, since the steepest descent method is applied in Method 2, under the given scenario is obtained the closest RVSL to the current operating state is obtained.

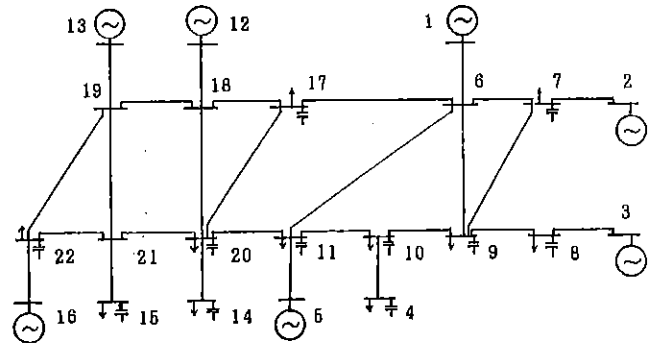


Fig.2 22-bus model system.

Table 1. The specified values to 22-bus system.

Bus No.	The specified real power	The specified reactive power	Bus No.	The specified real power	The specified reactive power
1		1.04*	12	4.5520	1.001*
2	5.4151	1.051*	13	3.6448	1.002*
3	4.5188	1.051*	14	-1.8143	-0.5982
4	-0.6772	-0.2344	15	-1.3661	-0.4599
5	0.2254	0.0790	16	0.2272	0.0763
6	0.0	0.0	17	-2.2535	-0.7832
7	-2.2508	-0.7886	18	0.0	0.0
8	-1.8000	-0.6251	19	0.0	0.0
9	-2.2508	-0.7877	20	-2.2571	-0.7760
10	-1.5781	-0.5353	21	0.0	0.0
11	-1.8044	-0.6117	22	-5.4062	-1.4524

*: The specified voltage magnitude

Table 2. Marginal SC installations and marginal power demands by each method.

Bus No.	Initial value	ΔY_{sc} (pu)	
		Method 1	Method 2
4	0.33	0.4764	0.1713
7	1.49	0.3265	0.1086
8	1.14	0.1327	0.0500
9	1.27	0.3302	0.1126
10	0.77	0.3956	0.1596
11	0.62	0.3821	0.1996
14	0.79	0.4292	0.2592
15	0.55	0.3916	0.4056
17	1.74	0.3072	0.1747
20	0.55	0.3474	0.2322
22	1.27	0.3046	0.6109
Total installation (pu)		3.8246	2.4840
Marginal power demand (pu)		27.188	26.156
Load increase (%)		15.9	11.5
Comp. time (sec)		107	150

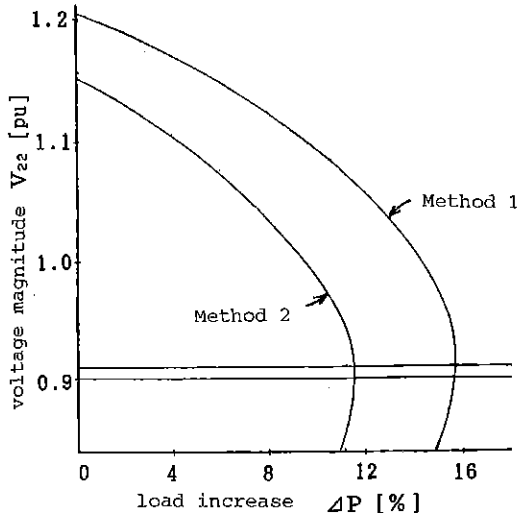


Fig.3 P-V curve by each method.

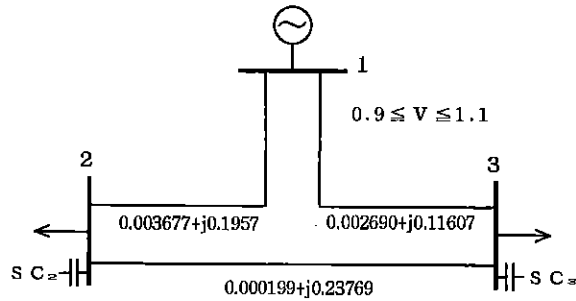


Fig.4 3-bus model system.

Investigation on Characteristics of Both Methods

As shown in the simulation results, the proposed twotypes of methods differ in the amounts of power demand at the RVSL. In this chapter, we shall clarify the difference in the basic characteristics by investigating the voltage profiles and installation patterns of SC.

First, using a simple model power system as shown in Fig.4, we calculated the RVSL to the variable introduction rates of SC_2 and SC_3 . This model system, in the

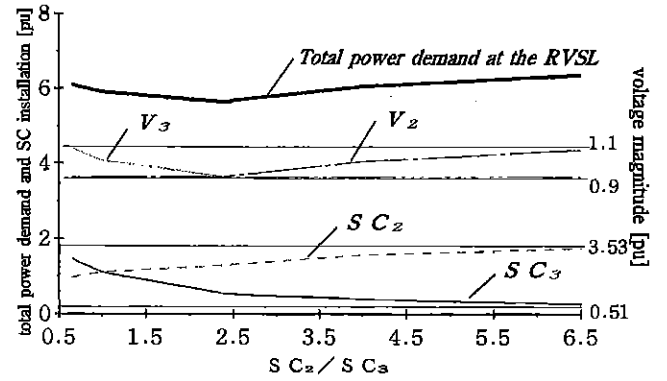


Fig.5 Simulation Results to 3-bus system.

initial state before installing SC, has the characteristics that the voltage magnitude at bus 2 is smaller than that of the other buses. Fig.5 shows power demands and bus voltage magnitudes at each RVSL. As is evident from this figure, as the amount of SC_2 increases, the bus where the voltage magnitude becomes a minimum changes from bus 2 to bus 3. Further, it is also shown that there are two maximum values in power demands at the RVSL, corresponding to each voltage profile.

Fig.6 shows changes of voltage magnitudes until a system state reaches the RVSL for the following three cases.

[Case 1]: $SC_2/SC_3=1.0$

Since the installation of SC_2 is the same amount as SC_3 , the power system state reaches the RVSL, maintaining the initial voltage profile. More specifically, the bus with the minimum voltage magnitude does not change. In this case, it can be understood that the marginal power demand is relatively large as shown in Fig.6.

[Case 2]: $SC_2/SC_3=2.5$

Since a larger SC is installed at bus 2 where voltage magnitude is a minimum in the initial state, the increase in V_2 is larger than that of V_1 . Also, the power system state reaches the RVSL when V_2 almost equals V_3 . In this case, the marginal power demand is relatively small.

[Case 3]: $SC_2/SC_3=4.0$

Since the installation of SC_2 becomes much larger than Case 2, the value of V_2 increases drastically; therefore, the bus where the voltage magnitude becomes a minimum changes from bus 2 to bus 3. Also in this case, the marginal power demand is large.

Thus, when voltage magnitudes become flat as a whole by introducing SC, the marginal power demand tends to become smaller. Since the proposed Method 1 searches for the RVSL successively so that the extended imaginary parts of complex-valued NR load flow become non-existent for the assumed power demand, the static condensers are introduced evenly. Thus, the RVSL where voltage profiles in an initial state are maintained is obtained. Thus, it is considered that the obtained marginal power demand becomes relatively large. On the other hand, since in Method 2 the RVSL close to a current operating state is obtained, it is considered that the voltage profiles at the RVSL tend to become flat.

Next, we investigated the voltage profiles for the RVSL obtained by the 22-bus system. Fig.7 shows the results obtained. As mentioned before, the voltage magnitude at bus 22 is a minimum in the initial state. In Method 1, the RVSL such that the initial profiles are maintained is obtained. This corresponds to Case 1. On the

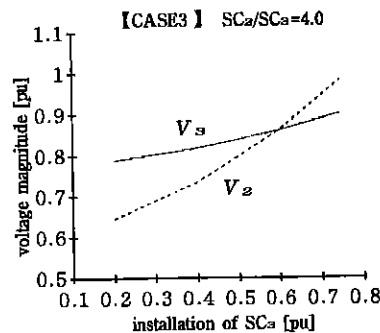
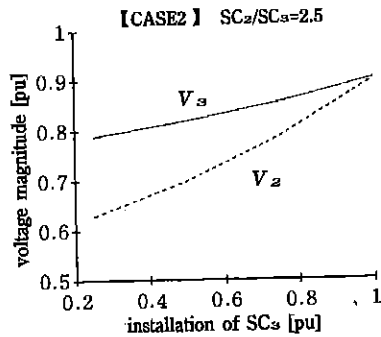
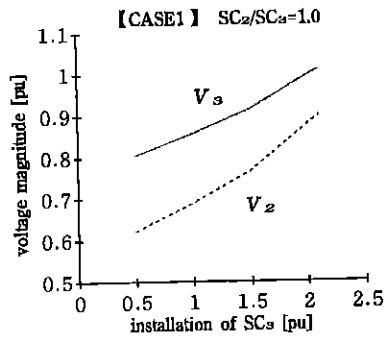


Fig.6 Voltage magnitudes versus introduction rates of SC_2 and SC_3 .

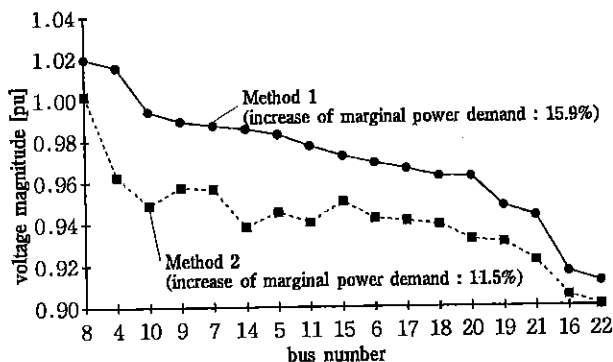


Fig.7 Voltage profiles to 22-bus system.

other hand, in Method 2, the fluctuations of voltage magnitudes are small; therefore, this corresponds to Case 2 and it is considered that the RVSL is the point where the marginal power demand is a minimum.

Next, to confirm the existence of the RVSL corresponding to Case 3, a simulation was implemented under the following condition:

- (1) Initial value of SC at bus 14 is set to half value.
- (2) SC is introduced only at bus 22.

Table 3. Simulation results corresponding to Case 3 (22-bus system).

Bus No.	Initial value(pu)	ΔY_{sc} (pu)	Voltage magnitude(pu)
14	0.40	0.0000	0.900506
22	1.27	3.1623	1.043032
Total installation(pu)		3.1623	
load increase(%)		14.9	

Table 3 shows the results of this case. The bus with a minimum voltage magnitude changed from bus 22 to bus 14. Further, the increase rate of the marginal power demand is 14.9%; it is larger than that of Method 2. Thus, we conclude that marginal power demands have several maximum values, corresponding to changes of voltage profiles.

Conclusions

This paper presented two methods for determining the real voltage stability limit (RVSL) considering a voltage compensation by static condensers (SC) and operating constraints. The effectiveness of the proposed methods was ascertained through numerical examples. At the RVSL, the lowest voltage magnitude in a power system coincides with its lower limit and the sensitivity of power demand with respect to voltage magnitude, dP_d/dV_d becomes zero. Thus, the RVSL is determined by updating the values of the total power demand and SC so that the above conditions are satisfied. Both methods differ in the determination of the search direction; Method 1 uses dP_d/dV_d directly and Method 2 uses the sensitivity of dP_d/dV_d with respect to SC. In particular, we consider that the RVSL by Method 1 should be used in planning aspects, because the marginal power demand is larger. In addition, in both methods the preventive control algorithm based on the complex-valued load flow is applied; therefore, they have the advantage of being able to obtain a load flow solution without diverging near the critical point.

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Appendix

Outlines of Complex-valued NR Load Flow

The k-th bus voltage in rectangular form is expressed by the following equation.

$$V_k = e_k + j f_k \quad (A-1)$$

Although the real part, e_k , and the imaginary part, f_k , are essentially real numbers, they are extended to complex numbers respectively by introducing another complex number plane (i-plane) as follows:

$$e_k = e_{kR} + i e_{kI} \quad (A-2)$$

$$f_k = f_{kR} + i f_{kI} \quad (A-3)$$

Similar to the above equations, active and reactive power injections, P_k and Q_k , and squared voltage magnitudes, V_k^2 , are also extended to a complex number plane.

$$P_k = P_{kR} + i P_{kI} \quad (A-4)$$

$$Q_k = Q_{kR} + i Q_{kI} \quad (A-5)$$

$$V_k^2 = V_{kR}^2 + i V_{kI}^2 \quad (A-6)$$

Here, note that this i-plane is different from j-plane and thus, j cross i is not equal to -1. Representing the (k,m) element of the admittance matrix as $Y_{km} = G_{km} + jB_{km}$, the load flow equations to be solved are expressed as follows:

$$\begin{aligned} P_{kR} &= e_{kR}a_{kR} - e_{kI}a_{kI} + f_{kR}b_{kR} - f_{kI}b_{kI} \\ P_{kI} &= e_{kR}a_{kI} + e_{kI}a_{kR} + f_{kR}b_{kI} + f_{kI}b_{kR} \end{aligned} \quad (A-7)$$

$$\begin{aligned} Q_{kR} &= -e_{kR}b_{kR} + e_{kI}b_{kI} + f_{kR}a_{kR} - f_{kI}a_{kI} \\ Q_{kI} &= -e_{kR}b_{kI} - e_{kI}b_{kR} + f_{kR}a_{kI} + f_{kI}a_{kR} \end{aligned} \quad (A-8)$$

or

$$\begin{aligned} V_{kR}^2 &= e_{kR}^2 - e_{kI}^2 + f_{kR}^2 - f_{kI}^2 \\ V_{kI}^2 &= 2e_{kR}e_{kI} + 2f_{kR}f_{kI} \end{aligned} \quad (A-9)$$

where,

$$\begin{aligned} a_{kR} &= \sum_{n=1}^N (G_{kn}e_{nR} - B_{kn}f_{nR}), & a_{kI} &= \sum_{n=1}^N (G_{kn}e_{nI} - B_{kn}f_{nI}) \\ b_{kR} &= \sum_{n=1}^N (B_{kn}e_{nR} + G_{kn}f_{nR}), & b_{kI} &= \sum_{n=1}^N (B_{kn}e_{nI} + G_{kn}f_{nI}) \end{aligned}$$

The equations at each node consist of eqs.(A-7) and (A-8) or eqs.(A-7) and (A-9) and the total number of equations is twice as many as that in the conventional load-flow analysis, that is, there exists $4(N-1)$ simultaneous equations for a N-bus system. These equations are summarized by

$$h(x) = h_s \quad (A-10)$$

In the above equation, x is a $4(N-1)$ -dimensional vector of bus voltages, e_{kR}, e_{kI}, f_{kR} and f_{kI} , and h represents a non-linear function vector which relates the vector, x , to specified values for each bus.

The extended imaginary parts of the specified variables, P_{kI}, Q_{kI} and V_{kI}^2 are set to zero and the Newton-Raphson method is applied to eq.(A-10) by selecting appropriate non-zero values as initial estimates of e_{kI} and f_{kI} . This method has the advantages that a solution can be always obtained independent on the existence of feasible load-flow solutions. Further, if the assumed power demand exceeds the voltage stability limit, the solution converges to a complex number, otherwise to a real number. Thus, for an assumed pattern of load increase, based on the extended

imaginary parts of solutions it is easily judged whether or not the static voltage stability can be maintained.

Objective Functions for Control V and Control S

The authors proposed the following index as an objective function for Control V,

$$VI = \sum_{k=1}^N e_{kI}^2 + \sum_{k=1}^N f_{kI}^2 \quad (A-11)$$

This index expresses a squared Euclidean norm of the extended imaginary parts of bus voltages. Since feasible solutions become non-existent when the load demand continues to increase over the voltage stability limit, the bus voltages converge on the complex number plane. This means that the value of VI is non-zero only if the forecasted power demand exceeds the voltage stability limit. Thus, this objective function can be regarded as a security index for assessing the degree of insecurity of the system condition after the load demand has increased.

On the other hand, the following index defined by the sum of the squared violations is used for Control S.

$$SI = \sum_i^m w_d (V_d - V_{d \max}^{(min)})^2 + \sum_i^{N-m} w_g (Q_g - Q_{g \max}^{(min)})^2 \quad (A-12)$$

where

- m : total number of generator-buses
- Q_g : reactive power output of generators
- V_d : magnitude of load-bus voltages
- w : weighting factor
- min, max : lower and upper limits

Here, the value of w is set to zero if the corresponding violation does not occur. Further, the equality constraint to a target bus is considered approximately by setting the upper and lower limits to its voltage magnitude as 0.91 and 0.90 respectively.

Discussion of papers presented by Barquín and by Kita

Edited by Thierry Van Cutsem, University of Liege

Marija Ilic (M.I.T.):

I would like to congratulate Julián [Barquín] for the effort put into this algorithm. I think it is a very important technique which acknowledges the fact that, when you are looking at the voltage collapse point, one variable actually contributes to the problem that is inherently not seen in the classical PV curve. So, there is something which is qualitatively new, independently from the algorithm that one uses.

Now, just a suggestion: I believe that it would be very interesting to include also the effects of relevant controls, because as the collapse takes place, there are different resettings - things hit limits and so forth - so if that is included I think the technique would be much more powerful.

My question is not related to the controls. Maybe I sensed this wrong but, are you attempting to use this technique only for $\sigma_1 > 0$ or for $\sigma_1 < 0$ also? Because if you were to get the participation of state variables to σ_1 negative, then the basic assumption of localized response that you have made in the paper fails. There are rigorous proofs on that, so that you should be very careful; it is a very tricky business when you have a non positive definite Jacobian.

Julián Barquín (Universidad Pontificia Comillas)

With respect to your question: singular values cannot be negative, they are not eigenvalues. It is one advantage of the method not to worry about negative or complex eigenvalues. From the numerical point of view, it is very convenient.

Ilic: When the Jacobian is negative definite, how is that related to the singular values?

Barquín: I really do not know about that situation.

Göran Andersson (Royal Institute of Technology):

Marija mentioned the effect of controls. I'd like just to comment that Thomas Smed in his thesis actually investigated the stability of different controls for HVDC by using this method. So, some of that work has already been done, but still much more is to be done. Second is a comment to Marija about the sign of the minimum singular value. As you said, by definition, it is always positive or could be zero. What we found corresponds to eigenvalues. When you get a negative eigenvalue and take the scalar product between the v and w vectors, if that is negative then you are on the unstable side. We couldn't prove that mathematically, but we found by inspection that it was correct.

Barquín: I could prove that to you. In fact, near a singular point, the singular values and the eigenvalues are proportional. The constant of proportionality is related to a normalization constant, and one can prove in this way what you said.

Andersson: So, you could in fact use that scalar product as I suggest?

Barquín: Yes. The problem is that I do not find it very useful, because if you compute the eigenvalues of the Jacobian - there is no dynamic meaning for that.

Andersson: No, all right, you know where you are anyhow. Now, I come to a question to you and perhaps also to Yakout and others here. What we found was that the difference between the eigenvalues and the minimum singular values was very small. They were almost the same all the time; what you call the v and w vectors correspond to the right and left

eigenvectors, and we could hardly distinguish them. The interpretation is different because the minimum singular value is norm related, whereas the eigenvalue is not. Then my question is: shouldn't we just stick to one of those, because it is the same information?

Barquín: I agree that most of the time it is the same thing. However, I feel more comfortable for numerical algorithms to know that, whatever happens, it is going to converge well - it is not going to give me complex or strange things, etc. Otherwise, I agree with you.

Claudio Cañizares (University of Waterloo):

This is actually not a question but a comment on why eigenvalues are very close to singular values. That just happens to be because of the matrix that they have been using, the $\Delta Q/\Delta V$ matrix, after reduction. If you look at the full Jacobian matrix, this does not happen; singular values are significantly distinct from eigenvalues. When you reduce the matrix to $\Delta Q/\Delta V$, assuming ΔP is zero, it happens that the matrix becomes very much symmetric. We had the same problem you mentioned, that the eigenvalues were really close to the singular values. For the full Jacobian that was not true, but when you reduce to $\Delta Q/\Delta V$ case, it certainly was, which is a very interesting idea and a very useful one, by the way.

Nelson Martins (CEPEL):

[Addressing Kita] Complex-valued Newton Raphson power flow has been used in publications by Japanese colleagues for probably about 8 or 10 years - I am not so sure about how long - but I haven't seen a full paper dedicated to it. I'm curious about it. I thought for a while about going this way; I think is a nice way. You go, and you don't have a solution in the real domain, you go to the complex domain, find a solution and then probably find your eigenvalues there and move them back to the real domain through control actions. That was nice, until I went to a different problem: multiple pole location - the control problem. I allowed my problem to be complex and I had an enormous amount of solutions that wouldn't happen (they are all complex solutions). So, when you go to the complex domain, you have a lot of solutions: how do you deal with them? I think that this idea should be carefully explained in a full paper dedicated just to it. That's my suggestion.

Chair (Hans Glavitsch, E.T.H. Zurich):

I have a more practical question. If I understood this right, the idea is to place static condensers on a system. I'm a little bit worried about static condensers just like this. If you have load shedding, you have a lot of VAr's on your system. Voltage goes up, as your curves have shown. So you get overvoltages and people would be afraid of such overvoltages. Do you provide for disconnecting your static condensers?

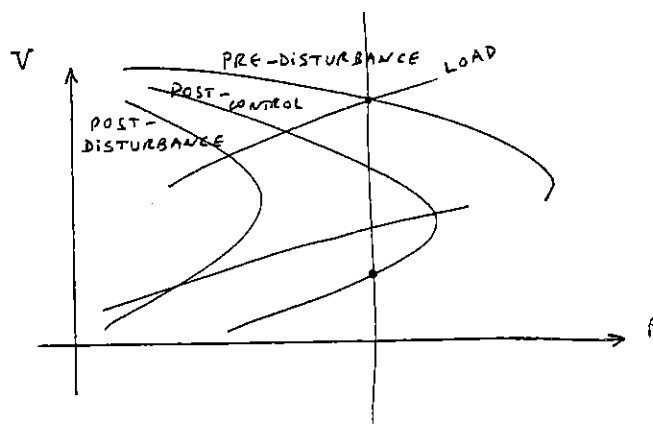
Hiroyuki Kita (Hokkaido Univ.):

In this paper, contingencies such as a forced outage are not considered. Naturally, I also think that at least N-1 security has to be considered. In this case the marginal power demand considering contingencies becomes a minimum, corresponding to each contingency.

Chair: It's a practical problem you have to take care of. It has nothing to do with a solution method, but it is a practical problem that one should think of.

Yakout Mansour (B.C. Hydro):

A thing I mentioned in the morning, and was mentioned before - I think Thierry addressed it briefly in his presentation, and also Carson addressed it in his presentations a number of times. Really, when you talk about PV curves near the tip, and then add something that would change the power factor to extend the point: these are very simplistic ways of looking at voltage stability. Many of you might have heard this before, but I am repeating it because it is really important to look carefully at this point of view. Take a system with a pre-disturbance characteristic as shown (Figure), and then a disturbance that would get you to another point as shown. If your load does not change characteristics and is constant MVA - which is not very realistic at all - then if the tip is below the MVA, this is where you get instability. But the fact of the matter is that the load characteristic changes transiently to something else [as shown], so you always have a solution. Now, control is very important. Once you talk about synchronous or static condensers, then you are talking about control. Control moves you to something in between the post-disturbance and the pre-disturbance. When does it change, as related to load recovery, is very important. So, if you change the characteristics of the load, you get a point on the post-disturbance curve, you always have a solution and you are always stable for the short period of time after disturbance, as it moves on that curve. But at that time, as the load characteristic changes this way, your control has to go from one point to the other point before the intersection of the load recovery characteristics with the new characteristics-with-control. If you wait long enough below this point, you collapse. If you are fast enough before this point, you recover to the stable point. So, tip of the curve, and power factor, and all these really are simplistic ways to look at things.



CONTROL OF BIFURCATIONS ASSOCIATED WITH VOLTAGE INSTABILITY

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Abstract

Control of power system voltage instability is considered, with emphasis on relevant bifurcation control problems. Among the issues discussed are ramifications of model uncertainty for the design of bifurcation control laws, stabilization of bifurcations by feedback, and integration of the design of control laws with on-line prognosis and diagnosis of impending instability.

1. Introduction

In this paper we discuss the role of bifurcations in the analysis, control and diagnosis of voltage instability (and voltage collapse) in electric power systems. As motivation for our discussions we begin with two observations. First, it is now widely accepted that bifurcations are closely connected to voltage instability. Indeed, voltage security measures taking bifurcation phenomena into account are being developed [14],[15],[13],[18],[39]. Second, advanced nonlinear control techniques can now be seriously considered for future power system control, in light of high power electronic devices now being developed and the emergence of Flexible AC Transmission Systems (FACTS) technology [28],[30],[29],[24],[19],[34],[52],[31].

An important conclusion of studies of voltage collapse undertaken thus far is the general consensus that bifurcations in underlying mathematical models of a power system are closely linked with voltage collapse. Several different types of bifurcation phenomena have indeed been detected in power system models exhibiting voltage collapse. These include local static and dynamic bifurcations, as well as global bifurcations and routes to chaos [49],[50],[35],[16],[4],[2],[61],[8],[12],[51],[58].

It is therefore essential that bifurcations be accounted for in the design of control systems and diagnostic tools for stressed electric power systems. To facilitate further discussion and elaboration of this observation, we recall some basic results from bifur-

cation analysis in Section 2. In Section 3, we discuss problems in the control and diagnosis of nonlinear systems exhibiting bifurcation. The problems discussed are chosen for their relevance to control of voltage instability. Conclusions are collected in Section 4.

2. Bifurcations

In this section we briefly discuss several local bifurcations that have been observed in analytical and numerical studies of voltage collapse. This discussion is an abridged version of [1, Section 2], which also addresses degenerate and global bifurcations.

Since an electric power system normally functions at a stable operating *point*, it is this condition (an equilibrium) which, upon slow variation of a system parameter, yields, after one or many bifurcations, the decisive bifurcation at which collapse occurs. The first bifurcation experienced coincides with loss of stability of the nominal operating point. The bifurcation at which collapse occurs is distinguished by the 'local' vanishing of any stable limit set of the system model (1).

Consider a one-parameter family of autonomous systems of ordinary differential equations

$$\dot{x}(t) = f_{\mu}(x(t)). \quad (1)$$

In (1), the state $x \in \mathbb{R}^n$, time t is continuous, and $\mu \in \mathbb{R}$ is the bifurcation parameter. The mappings f_{μ} is assumed sufficiently smooth in x and μ .

Suppose that (1) possesses an equilibrium point $x_0(\mu)$ for a range of parameter values of interest. We assume that this is an asymptotically stable equilibrium for a large portion of this range. Thus, the equilibrium can be viewed as a possible operating condition for the power system modeled by (1). When the power system operates in a highly stressed environment, it is possible for the equilibrium $x_0(\mu)$ to lose stability for some parameter value μ_c . At such a loss of stability, the nonlinear system (1) typically undergoes a local bifurcation. *Local bifurcations* for Eq.

(1) are bifurcations from an equilibrium point of (1). Among the possible local bifurcations for (1), only two are *generic*, i.e., preserved under small perturbations of the right side of (1). These are the saddle node bifurcation and the Andronov-Hopf bifurcation. The former bifurcation involves the coalescence and disappearance of two neighboring equilibrium points as a parameter is varied, while the latter involves the emergence of a small-amplitude periodic orbit from an equilibrium point as a parameter is varied.

Before proceeding to state the basic theorems on saddle node bifurcation and Andronov-Hopf bifurcation, we note that limit sets emerging from these bifurcations often undergo further bifurcations. The first bifurcation, which occurs along the nominal solution branch, is termed a primary bifurcation. A bifurcation from the solution emanating from the primary bifurcation is termed a secondary bifurcation. Successive bifurcations can become rather complex, and can lead to chaotic (or turbulent) behavior. Chaos is an irregular, seemingly random dynamic behavior displaying extreme sensitivity to initial conditions. Nearby initial conditions result, at least initially, in trajectories that diverge exponentially fast.

2.1. Saddle Node Bifurcation

Figure 1 depicts the *bifurcation diagram* for a saddle node (or fold) bifurcation of Eq. (1), as well as the dynamics associated with this bifurcation. The amplitudes of two equilibrium solutions are plotted in the diagram against the bifurcation parameter μ . A dark curve indicates an asymptotically stable equilibrium point (node), while a thin curve indicates an unstable equilibrium point (saddle). The arrows indicate the dynamics of the system for various parameter values.

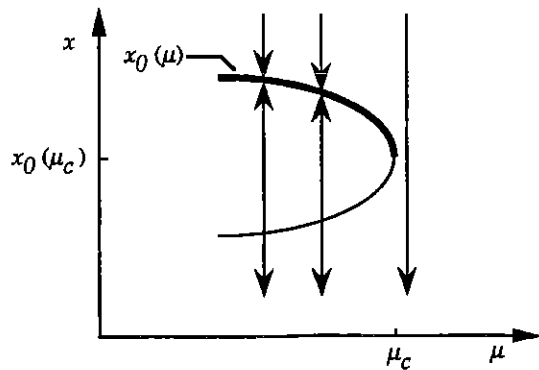


Figure 1: Saddle node bifurcation

To state a theorem on saddle node bifurcation, we consider the system (1) where f is sufficiently smooth and $f_0(0) = 0$. Express $f_\mu(x)$ in a Taylor series about

$x = 0, \mu = 0$ as follows:

$$f_\mu(x) = Ax + g\mu + Q(x, x) + \dots \quad (2)$$

Here, $A = D_x f_0(0)$ is simply the Jacobian matrix of f_μ at the origin for $\mu = 0$, $g = D_\mu f_0(0)$, $Q(x, x)$ includes all quadratic terms in x in the Taylor expansion of $f_0(x)$ near $x = 0$, and the dots denote terms of still higher order. The quadratic form $Q(x, x)$ is written such that it is generated by a symmetric bilinear form $Q(x, y)$. The following two hypotheses will be invoked.

(SN1) The Jacobian A is of rank $n - 1$. Thus, A possesses a zero eigenvalue to which there correspond unique right and left eigenvectors.

With this hypothesis in force, denote by r (resp. l) the right column (resp. left row) eigenvector of the critical Jacobian A corresponding to $\lambda(0) = 0$. Normalize r and l by setting the first component of r to 1 and then choosing l so that $lr = 1$. (This may require interchanging the first state variable with another state variable.)

(SN2) $lg \neq 0$ and $lQ(r, r) \neq 0$.

Under these hypotheses, Eq. (1) undergoes a saddle node bifurcation from the origin at $\mu = 0$. The precise statement is given in the following theorem. Note that usually this result is stated for a one-dimensional reduced system model, whereas we give a statement which applies directly to a general n -dimensional system [32].

Theorem 1: (Saddle Node Bifurcation Theorem) With the notation above, if (SN1) and (SN2) hold, then there is an $\epsilon_0 > 0$ and a function

$$\mu(\epsilon) = \mu_2 \epsilon^2 + O(\epsilon^3) \quad (3)$$

such that $\mu_2 \neq 0$ and for each $\epsilon \in [-\epsilon_0, \epsilon_0]$, (1) possesses an equilibrium $x_0(\epsilon)$ near 0 for $\mu = \mu(\epsilon)$. These are the only equilibrium points of (1) near the origin for μ near 0. Thus, (1) undergoes a saddle node (or fold) bifurcation from the origin at $\mu = 0$. Specifically, for each μ sufficiently small and positive (if $\mu_2 > 0$), or negative (if $\mu_2 < 0$), there are exactly two equilibrium points near the origin, one occurring for some $\epsilon > 0$ and another occurring for some $\epsilon < 0$. Moreover, the origin is unstable at the critical parameter value $\mu = 0$.

The fold bifurcation has been linked to voltage collapse in [50],[35],[16], [8]. An important feature of the saddle node bifurcation is the *disappearance, locally, of any stable bounded solution of the system* (1). In the next subsection, we discuss the Andronov-Hopf bifurcation, which can give rise to this same feature.

2.2. Andronov-Hopf Bifurcation

Suppose that the instability of the nominal equilibrium $x_0(\mu)$ is the result of a pair of eigenvalues of the system linearization crossing the imaginary axis transversely in the complex plane. Then, as is well known, generically it follows that a small amplitude periodic orbit of (1) emerges from the equilibrium $x_0(\mu)$. More accurately, the following hypotheses are invoked in the Andronov-Hopf Bifurcation Theorem, detailed statements of which can be found in many references (e.g., [26],[5]). The essence of this bifurcation theorem was originally stated by Poincaré and used in his study of lunar orbital dynamics [45, Secs. 51-52].

(AH1) f of system (1) is sufficiently smooth in x, μ , and $f_0(0) = 0$. The Jacobian $D_x f_\mu(0)$ possesses a complex-conjugate pair of (algebraically) simple eigenvalues $\lambda(\mu) = \alpha(\mu) + i\omega(\mu)$, $\bar{\lambda}(\mu)$, such that $\alpha(0) = 0$, $\alpha'(0) \neq 0$ and $\omega_c := \omega(0) > 0$.

(AH2) $\pm i\omega_c$ are the only pure imaginary eigenvalues of the critical Jacobian $D_x f_0(0)$.

The Andronov-Hopf Bifurcation Theorem asserts that, under (AH1) and (AH2), a small-amplitude nonconstant periodic orbit of Eq. (1) emerges from the origin at $\mu = \mu_c$. Moreover, locally it is generically true that the bifurcated periodic orbit is either orbitally asymptotically stable or is unstable. In the former case the bifurcation is said to be supercritical, while in the latter case it is termed subcritical.

In Figure 2 bifurcation diagrams are given for both situations. In this figure, a solid circle indicates a stable periodic orbit, while an open circle indicates an unstable periodic orbit. As in Figure 1, a dark line represents a stable equilibrium branch, and a thin line represents an unstable equilibrium branch. Note the general property that supercritical solutions are stable while subcritical solutions are unstable.

Stability calculations for Andronov-Hopf bifurcation have been developed by several authors (e.g., [26]). For instance, one can calculate the coefficient of the lowest order term in a Taylor expansion of the Floquet exponent determining stability of the periodic orbits. This 'bifurcation stability coefficient' is

sometimes denoted β_2 [26]. If $\beta_2 < 0$ then the bifurcation is supercritical, while if $\beta_2 > 0$ the bifurcation is subcritical.

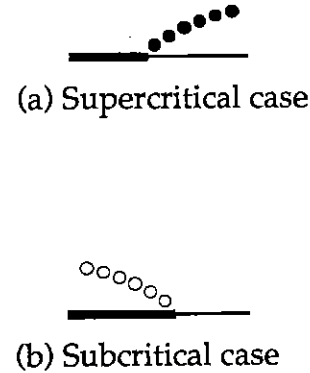


Figure 2: Andronov-Hopf bifurcation

3. Control and Diagnostics

In this section we discuss the role of bifurcation analysis in the control and diagnosis of voltage instability. Theoretical work on bifurcation control has addressed use of feedback to modify the bifurcation characteristics of a given system (see [6],[1] for a review). Feedback controls have been designed for stabilization and/or delay of a given bifurcation, reduction of the amplitude of bifurcated solutions, introduction of bifurcations at desired parameter values, optimization of a performance index near bifurcation, re-shaping of a bifurcation diagram, or a combination of these. Control systems for alleviation of power system voltage instability must be designed with attention to the inherent uncertainty and nonlinearity of large electric power systems [27],[28]. Next we discuss basic problems of control of bifurcations for enhancing voltage stability. This is followed by a discussion of the issue of control design to facilitate diagnosis of instabilities. An overall control design should take into account both objectives: stability and diagnosis.

3.1. Control

Consider a one-parameter family of nonlinear control systems

$$\dot{x} = f_\mu(x, u), \quad (4)$$

where $x \in \mathbb{R}^n$ is the state vector, u is the scalar control, $\mu \in \mathbb{R}$ is the bifurcation parameter, and the vector function f is smooth in x, u and μ .

Denote by $x_0(\mu)$ the nominal operating point (equilibrium point) of the open-loop version of Eq. (4). Thus, $x_0(\mu)$ solves the algebraic equation

$$f_\mu(x, 0) = 0 \quad (5)$$

for a range of values of the parameter μ , say $\mu \leq \mu_c$.

Suppose that $x_0(\mu)$ loses stability before it undergoes a hypothesized saddle node bifurcation. At $\mu = \mu_c$, the equilibrium $x_0(\mu)$ merges with another equilibrium in a saddle node bifurcation, and no longer appears for $\mu > \mu_c$. Now suppose that the operating point $x_0(\mu)$ loses stability at $\mu = \mu_1 < \mu_c$. The situation is sketched in Figure 3. This figure is reminiscent of a power-voltage curve, with x being the voltage magnitude and μ the power (real or reactive) delivered to a load. Power system models exhibiting behavior as in Fig. 3 have been studied in several papers, including [8], [49], [61], [12], [51], [58]. Although this particular scenario need not occur in voltage collapse, it is assumed here to facilitate illustration of control objectives.

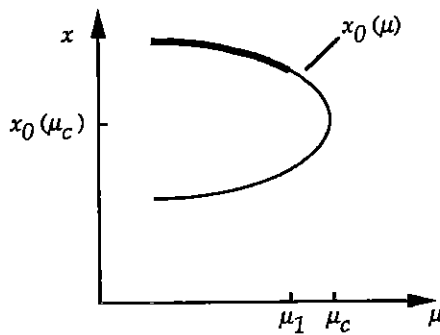


Figure 3: Loss of stability before fold bifurcation

Given a power-voltage curve such as that depicted in Figure 3 the following major control problems arise in sequence:

(a) *Feedforward control for voltage regulation.* In this problem, some type of open-loop control, possibly depending on the parameter μ , is applied. The purpose of this control law is to raise the upper branch of the power-voltage curve so that the voltage is maintained at a desired steady state value for as large a parameter range as possible. This can be achieved, for instance, using static var compensators (SVCs), as discussed in [41].

(b) *Linear feedback control.* The control law designed in Step (a) does not necessarily provide the local stability properties of the achieved operating point that may be desired. In this second step, feedback control is employed to achieve stabilization of this equilibrium path over a range of parameter values.

(c) *Nonlinear feedback control.* The system resulting from application of the control law of Step (a) will generally display a bifurcation diagram similar to that of Fig. 3, except that μ_c will have been increased and the voltage will tend to be higher and nearly constant for a larger parameter range. Suppose that the nominal equilibrium for this system still loses stability at some critical parameter value $\mu = \mu_1$, and that this loss of stability is through an Andronov-Hopf bifurcation. Then nonlinear feedback control may be used to ensure what Tamura [49] has called “quasi-stable” behavior in the parameter range $\mu_1 < \mu < \mu_c$. This means that although the nominal equilibrium is unstable, it is surrounded by a stable limit cycle or some other small-amplitude stable motion.

Let us elaborate on the control design procedure outlined above. Step (a) concerns allocation of real and reactive power resources to meet changing demand. Included in these resources are the reactive power injections available from static VAR compensators (SVCs). Thus from Step (a) there results a baseline control design with less severe limits on deliverable reactive power. At the conclusion of this step, however, it is possible that the stability properties of the (new) nominal equilibrium point are unsatisfactory. Then Step (b) of the procedure above is applied, and a possibly gain-scheduled feedback controller is designed to result in improved stability of the nominal equilibrium over a range of parameter values. Finally, the loss of stability scenario depicted in Fig. 3 may apply to the closed-loop system resulting after Steps (a) and (b). Step (c) addresses the problem of ensuring that the system does not undergo sudden collapse at the parameter value μ_1 where loss of stability occurs. This can often be achieved by a nonlinear feedback law. An example of this procedure is given in [60]. It is helpful to consider more explicitly the reasoning in Step (c).

Suppose that either an Andronov-Hopf bifurcation from the nominal equilibrium point $x_0(\mu)$ occurs at the critical parameter value $\mu = \mu_1$. For definiteness, let the conjugate pair of eigenvalues crossing the imaginary axis at $\mu = \mu_c$ traverse the axis from the left half of the complex plane to the right half as μ increases. Suppose the Andronov-Hopf bifurcation is subcritical (as in Fig. 2(b)). Then the Andronov-Hopf bifurcation gives rise to an unstable periodic solution of small amplitude in the vicinity of the equilibrium point for values of the parameter μ less than μ_1 . This means that for μ slightly greater than μ_1 , system trajectories beginning near the (unstable) nominal equilibrium point will tend to diverge away from this point. If the control u can be designed so as to render the bifurcation supercritical, then this situation will be remedied to a certain extent. That is, the same system trajectories will now converge to

a small amplitude periodic solution. Since this periodic solution is *near the nominal equilibrium point*, divergence of the system to a distant (and unacceptable) operating mode is thus prevented. This is an instance of the "quasi-stable" behavior discussed by Tamura [49]. This aspect of the control design procedure is discussed further in the next subsection, which emphasizes diagnosis of impending collapse.

Smooth feedback control laws rendering supercritical an initially subcritical Andronov-Hopf bifurcation are derived in [3]. Such control laws are said to 'stabilize' the bifurcation. Application of local bifurcation control results to voltage dynamics in electric power systems was reported in [36] and [60]. References to applications in other areas are given in [6].

We note that several important considerations in the control design procedure discussed above have not been addressed here. For instance, the controls may be required to depend on certain readily available measurements, i.e., an output feedback controller is sought. Another consideration, which we elaborate on briefly, is that model uncertainty might motivate use of dynamic washout filter-based feedback controllers in Steps (b) and (c). Such controllers include sufficient freedom to modify the stability of equilibrium points and bifurcated solutions, but do not change the position of any equilibrium point of the system. This is useful conceptually, since the equilibrium structure is to be specified in Step (a) of the procedure, and is not to be modified in later steps.

Since the use of washout filters in nonlinear control design is not commonly discussed in the literature, we discuss this concept a bit further. Suppose that Step (a) has been completed. Use of a static state feedback control law $u = u(x)$ in later steps of the design has potential disadvantages, especially for systems with multiple equilibria. In general, a static state feedback

$$u = u(x - x_0(\mu)), \quad (6)$$

designed with reference to the nominal equilibrium path $x_0(\mu)$ of (4) will affect not only the stability of this equilibrium but also the location and stability of other equilibria. Indeed, even a linear feedback control law can introduce extraneous equilibrium points in the system. Now suppose that (4) is only an approximate model for the physical system of interest. Then the nominal equilibrium branch will also be altered by the feedback. A main disadvantage of such an effect is the wasted control energy that is associated with the forced alteration of the system equilibrium structure. Another disadvantage is that system performance is often degraded by operating at an equilibrium which differs from the one at which the system is designed to operate.

For these reasons, we have developed [38] bifurcation control laws for systems (4) which are *dynamic state feedback* control laws of a special form. Specifically, we have incorporated high pass filters known as washout filters into the structure of the allowed controllers. In this way, we guarantee preservation of all system equilibria even under model uncertainty. Note that washout filters are common in power system stabilizers [9],[34] and aircraft control systems [48].

A washout filter is a stable high pass filter with transfer function

$$G(s) = \frac{y(s)}{x(s)} = \frac{s}{(s+d)}. \quad (7)$$

In the following, washout filters are incorporated into bifurcation control laws for (4). Specifically, in (4), for each system state variable x_i , $i = 1, \dots, n$, introduce a washout filter governed by the dynamic equation

$$\dot{z}_i = x_i - d_i z_i \quad (8)$$

along with output equation

$$y_i = x_i - d_i z_i. \quad (9)$$

Here, the d_i are positive parameters (this corresponds to using stable washout filters). Finally, we require that the control u depend only on the measured variables y , and that $u(y)$ satisfy $u(0) = 0$.

In this formulation, n washout filters, one for each system state, are present. In fact, the actual number of washout filters needed, and hence also the resulting increase in system order, can usually be taken less than n .

The advantages of using washout filters stem from the resulting properties of equilibrium preservation and automatic equilibrium (operating point) following. Indeed, since $u(0) = 0$, it is clear that y vanishes at steady state. Hence the x subvector of a closed loop equilibrium point (x, z) agrees exactly with the open loop equilibrium value of x . Also, since

$$y_i = x_i - d_i z_i = (x_i - x_{0i}(\mu)) - d_i(z_i - z_{0i}(\mu)), \quad (10)$$

the control function $u = u(y)$ is guaranteed to center at the correct operating point.

3.2. Diagnostics

The main message of this subsection is that control design to increase the region of operability of a stressed power system should also have another goal, which is *prognosis and diagnosis of impending collapse*. Because of the inherent uncertainty of large electric power system models, the prediction of stable behavior over a parameter range need not be accurate. Loss of stability of an operating point, or loss

of the operating point itself, may occur in the predicted 'safe' parameter range. Because of this, what appears to be an excellent control design might turn out to be less than ideal. This is because the system may behave so well up to the point of collapse that no warning of the impending collapse is given. We pursue this further next, and then briefly discuss some relevant work on diagnostics and integrated control and diagnostics.

In the scenario depicted in Fig. 1, the operating point is stable up to the saddle node bifurcation point. The system displays no discernible change in its dynamic behavior as the parameter μ approaches the critical value μ_c . It may then be desirable not to have such stable operation in an uncertain system.

An alternative involves stabilizing the nominal equilibrium up to a value of μ close to, but less than, μ_c . Then we obtain Figure 3 with $\mu_c - \mu_1$ small. Generically, it will then be the case that an Andronov-Hopf bifurcation will occur at the nominal equilibrium point for $\mu = \mu_1$. There are generically two possibilities for this bifurcation: it can be subcritical or supercritical. These possibilities are illustrated in Figure 4 and Figure 5, respectively.

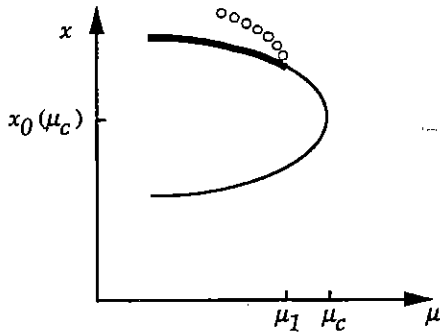


Figure 4: Subcritical Andronov-Hopf bifurcation before fold bifurcation

It is now natural to ask if a design of this type can facilitate automatic recognition of the proximity to the fold even in the absence of an accurate model. If the Andronov-Hopf bifurcation occurring at $\mu = \mu_1$ is subcritical, then trajectories starting at the nominal equilibrium for μ slightly greater than μ_1 will diverge. However, if the Andronov-Hopf bifurcation is supercritical, then these same trajectories will converge to small-amplitude periodic motions near the nominal (now unstable) equilibrium. An automatic monitoring system could conceivably then detect proximity to the fold bifurcation by detecting the periodic solutions. Thus periodic solutions, which usually are considered undesirable, might provide a warning signal for impending voltage collapse. Note that if the open

loop system does not exhibit Andronov-Hopf bifurcation from the nominal equilibrium, by this discussion it appears that in some situations it may be advantageous to introduce such bifurcations. Another possibility involves introducing a chaotic attractor near $x_0(\mu_c)$ for $\mu > \mu_1$.

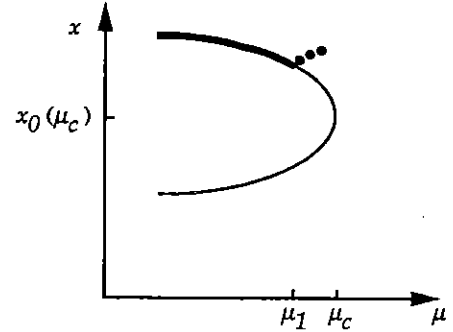


Figure 5: Supercritical Andronov-Hopf bifurcation before fold bifurcation

Regarding the related problem of designing controllers which ensure persistence of the limit cycle for values of $\mu > \mu_c$, considered in [1], we observe the following.

Theorem 2: Suppose that for $u = 0$ the control system (4) undergoes a saddle node bifurcation from the equilibrium $x_0(\mu)$ at $\mu = \mu_c$. Suppose the direction of this bifurcation is such that no equilibrium of the open-loop system exists near $x_0(\mu_c)$ for all $\mu > \mu_c$ with $\mu - \mu_c$ sufficiently small. Suppose there is a small amplitude continuous feedback $u(x, \mu)$ such that a sufficiently small open set in state space, located in a neighborhood of the point $x_0(\mu_c)$, is locally attracting for all $\mu > \mu_c$ with $\mu - \mu_c$ sufficiently small. Then there also exists a small amplitude continuous state feedback depending on μ which introduces a new equilibrium path near $x_0(\mu_c)$ for $\mu > \mu_c$.

Proof. Brockett's necessary condition for continuous stabilizability of equilibria [11] applies also to small open sets. It implies that under the assumptions of the theorem, the function $f_\mu(x, u)$ viewed as a mapping from $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}$ is onto a neighborhood of $x_0(\mu_c)$ for μ near μ_c , x near $x_0(\mu_c)$, and u near 0. Thus for any $\mu > \mu_c$ with $\mu - \mu_c$ small, there is a constant value of u such that x takes an equilibrium value near $x_0(\mu_c)$.

Note that this result says nothing about the use of discontinuous feedback control laws. It may be that such control laws can yield superior performance over a wider operating range than is achievable with continuous feedback.

The foregoing discussion is purely motivational. It points to the possible advantages of small-amplitude oscillatory behavior from the point of view that this may provide a warning signal of impending stall. In some engineering systems, such as rotating machinery, vibrations always appear prior to failure [63]. Diagnostics for these systems are based on extrapolation from experience with the vibration "warning signals" produced by machinery under different conditions of stress. In other engineering systems, identifying possible warning signals is in itself a major research problem. For example, the study of stall in axial compressors is crucial for safe high performance operation of jet engines. The search for "stall precursors" in axial compressors has attracted considerable attention [23]. In voltage collapse of electric power systems, there is as yet no clear precursor signal. Simulations for the same model might or might not display oscillatory motion prior to collapse, depending on initial conditions [2],[61],[51].

Thus we may pose the following loosely stated problem. In the design of a control system for mitigation of voltage instability, arrange that stable oscillatory solutions of small amplitude occur prior to reaching reactive power limits. These oscillatory solutions must be of a nature so as to be adequately reflected in measurable system outputs. This goal should be achieved in the face of model uncertainty. Basically, we seek a controller which facilitates both stable operation and prognosis and diagnosis of impending instability. The control and diagnostics functions should ideally be designed in an integrated fashion. Note that due to the size and network structure of large power systems, modal analysis techniques [59],[55] will likely be important in addressing this design problem. Similar problems have been considered in linear control theory, starting with the paper by Nett [44] (see also [54]). There, diagnostics of sensor and actuator failures is considered alongside control system design. Design of diagnostic systems has also been considered as a separate problem in many references. See, e.g., the survey papers [62],[42], as well as the references [10],[33],[47]. Of course, monitoring of power systems is a subject in itself (see, e.g., [21, Section 3]), but we feel that special techniques will be needed for control and diagnostics in the context of voltage collapse.

4. Conclusions

Mechanisms of loss of stability are inherently connected to bifurcations. Voltage collapse is a power system instability which may arise in different ways for different systems. A variety of bifurcation phenomena, including routes to chaos, have been de-

tected for power system models exhibiting voltage collapse. Bifurcations are likely to play an important role in the integrated design of controllers which simultaneously address stabilization and diagnostics. The difficulty of this problem area stems to a large extent from the need to account for significant model uncertainty in the design.

Acknowledgments

The author is grateful to Prof. A.M.A. Hamdan for useful discussions. This research has been supported in part by the Electric Power Research Institute under contract RP8050-5, and in part by the NSF Engineering Research Centers Program: NSFD CDR-88-03012.

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VOLTAGE COLLAPSE PROXIMITY INDICATORS FOR VERY SHORT TERM SECURITY ASSESSMENT

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Abstract: Accurate time domain simulations, which take into account the complex models of network components and of regulating devices, are suited to off-line planning studies of the voltage collapse phenomenon. In the very short term operation, many voltage collapse proximity indicators have been proposed using steady-state sensitivities or examining the occurrence of singularities in the load flow (LF) Jacobian. In the present paper some different steady-state indicators are proposed and compared with other ones available in the technical literature. Beside nodal sensitivities, area-wise or system-wise indicators are shown to provide valuable and easy-to-use insights of the voltage collapse phenomenon. Their compatibility with the eigenvalue analysis of particular matrices is investigated. Test cases using both the EHV transmission network and a subtransmission area of the ENEL system are presented. The hierarchy in voltage control and the action of the LTC transformers are simulated.

Keywords - Voltage collapse, sensitivity indices, eigenvalue analysis, area-wise indicators, very short term applications.

1. INTRODUCTION

In the planning and in the operation of large electric systems power engineers are more frequently involved in the solution of voltage/reactive security problems due to the heavy power transfers from the generation plants to the load areas.

In particular, an unacceptable degradation of the network voltage profile, that leads to voltage instability, may occur either because severe contingencies stress the system or because of the lack of reactive generation margins during load pick-up periods. A comprehensive survey of the voltage instability phenomenon is presented in [1,2,3] together with the current practices adopted by power utilities, to operate the electrical system as far as possible from a voltage collapse point. Among the different methods proposed to assess the voltage collapse, the problem is analysed in [4,5] from the point of view of the maximum loadability of the power systems. Besides, the critical role of certain control devices (such as those of LTC transformers) which turn unstable in proximity of the voltage collapse point is highlighted in [5].

The relationships between voltage instability and the presence of multiple load flow solutions are pointed out in [6]. The determination of voltage collapse proximity indicators based on a steady-state analysis of the power system (LF equations) has been undertaken by many researchers. In [7] some indices are proposed and some examples of calculations are given. Global indicators, given by the use of an optimal power flow program, are defined in [8,9].

Sensitivity methods which provide a measure of the distance from the collapse point have been proposed in [10,11,12]. In [13] the important roles played by the Secondary Voltage Regulators, by the LTC transformers and by the system non linearity are envisaged. A lot of work has been done to detect the Jacobian matrix singularity conditions; among many others, [14,15] make use of continuation methods to detect the point of voltage collapse. The loss of the steady state stability in voltage collapse is seen as a static bifurcation in [16]. An iterative and direct method to compute the load increase pattern for which the bifurcation occurs is given in [17], while a method for computing an optimal direction in the control space for avoiding the collapse is outlined in [18]. Suggestions for reducing the risk of voltage instability and for localising the most critical nodes are given by [19,20]. A method which encompasses different existing models, like multiple LF solutions, static bifurcation, singular Jacobian, Jacobian sensitivities, minimum singular values is described in [21]. Reference [22] presents a clear partition in static and dynamic sets of the methods adopted in the studies of voltage collapse. The static set includes all the methods which investigate LF feasibility regions and steady state stability approaches. The dynamic methods adopt non linear extended transient stability models. A suitable classification of the voltage instability patterns based on the singular perturbation theory is provided in [23]. Some relationships between static and dynamic approaches are deeply examined in [24,25]. The characteristics of the equilibrium points of the dynamic systems are focused on by [26,27,28]. The impact of the load models (static or dynamic) on the voltage stability conditions is analysed in [29]. Beside the steady-state approaches, the voltage instability problem has also been addressed by using long term dynamic simulation of the electrical system and of the reactive control apparatus [24,31] with more or less sophisticated models. Time domain simulations, however, are very expensive from both the CPU time and the engineering judgement required for the result analysis. Their application appears to be practicable for planning or at most for operational planning studies or only to examine in more detail the most critical situations detected with static applications. The most accurate steady state methods seems to be based on the modal analysis of the power system Jacobian matrices [19,20,30] as confirmed by comparisons with some time domain simulations[32]. Eigenvalues and eigenvectors of reactive power flow Jacobians are shown to provide a valuable tool to analyse the network behaviour with respect to voltage and reactive power problems. The number of significant

entries of the eigenvectors allows one to discriminate between local and global modes and also to identify the size of the areas involved in the voltage collapse phenomenon [20,30].

2. VERY SHORT TERM APPROACH

In the present paper, a fast steady-state approach is presented. It should give system operators a useful tool to evaluate the distance of the actual operating point from a voltage collapse situation. Use is made of appropriate sensitivity indices obtained from exact load flow solutions which take into account all the system non-linearity, the steady-state responses of LTC transformers, the voltage dependence of real and reactive load, and the primary and secondary voltage regulator actions. Actual load ramping patterns (given by on-line load forecasts) and generation re-assignment practices are adopted in an iterative process which simulates the system behaviour under more and more stressed conditions. Beside nodal sensitivities, area or system-wise indicators are available to give effective and easy-to-use measures of the operating margins at disposal of system operators to avoid the collapse. The validity of some global indicators is strengthened by means of the results of the modal analysis. Applications of the proposed approach to the security assessment of the EHV Italian transmission system are presented with reference to critical situations typically occurring in summer working days. The impact of the steady state load models and of the LTC transformers is investigated by examining in detail a subtransmission area of the ENEL network.

3. ANALYTICAL NETWORK MODEL

3.1 Generalities

For a fast detection of the distance from voltage collapse in a very short term environment a slow variation of the system parameters (loads and generations, bus voltages and so on) is considered. Steady-state models are assumed for all the network components and for their pertinent control apparatus. Therefore, the AGC, AVR, Secondary Voltage Regulator (SVR) and LTC regulator dynamics are not taken into account. Their responses to perturbations are evaluated at the end of the transient phenomena. In the adopted scenario, the real power generations are updated in order to track the load variation pattern by considering the effect of both primary and secondary regulations. The AVR controlled units are simulated as PV buses until one of the binding reactive generation limit is reached. In this case the bus is switched to PQ-type. A detailed (polynomial in the P-V variables) representation of the capability curves is included to provide a more realistic simulation of the voltage-reactive behaviour of the synchronous generators. The SVR controlled units are modelled as PQ busses. They provide the reactive power demand variation needed by each area to maintain the pilot node voltage at the reference value assigned either by a tertiary voltage regulation [33] function or by a very short term reactive power scheduling program. The required reactive generation shifts are obtained by the use of the area sensitivity matrix [34] containing the derivatives of the pilot node

reactive injections with respect to the reactive generations of the controlled units. The pilot nodes are maintained as PV-type busses, with the assigned reference voltages, as long as the reactive level of the corresponding areas do not attain a bounding limit. If saturation of the area reactive level occurs, the pilot node is converted to PQ-type and its voltage is no longer sustained. The bus voltages on the controlled side of LTC's are kept constant by using sensitivity methods to change the transformer ratios within their allowable limits. The dependence of the real and reactive loads on the bus voltages is accounted for by using exponential laws obtained from experimental data.

3.2 Load and Generation Scenario

An iterative procedure is adopted for the simulation of the real and reactive load variation and for the re-assignment of the generations. A realistic hypothesis about the distribution of loads among the buses of each area of the system is assumed. The distribution of the global load among the areas depends on the very short term load forecast when operating in a real time framework. For operational planning studies several choices can be made according to the user perspectives. The real power generation variation is distributed among the units which still have sufficient real and reactive generation margins at the current iteration, in compliance with the assumed regulation policies. The following load and generation distribution coefficients are defined:

β_j , distribution coefficients of the global load among the N_{ar} areas, η_{hj} , distribution coefficients of the load of the area j

among its N_{Lj} load buses $(\sum_{j=1}^{N_{ar}} \beta_j = 1, \sum_{h=1}^{N_{Lj}} \eta_{hj} = 1)$ and

ρ_m , distribution coefficients of real power generation among the N_{gr} still regulating units. The coefficient ρ_m is the participation factor to ED (Economic Dispatch) of the unit m or is given by:

$$\rho_m = P_{max,m} / \sum_{i=1}^{N_{gr}} P_{max,i} \quad \text{with} \quad \sum_{m=1}^{N_{gr}} \rho_m = 1$$

where $P_{max,m}$ is the maximum real generation of unit m . At each iteration k the procedure determines the minimum value of the total real load variation ΔC_{min}^k , which leads to the attainment of the capability limits of at least one of the synchronous generators:

$$\Delta C_{min}^k = \min_{i \in N_{PV}^k} \frac{Q_{gi}^k - Q_{gi}^{max,i}}{\partial Q_{gi}^k / \partial C_{tot}}$$

where N_{PV}^k is the number of the generators with reactive

power margin at iteration k . Q_{gi}^k , $Q_{gi}^{max,i}$, $\partial Q_{gi}^k / \partial C_{tot}$ are for each unit i , the actual value, the upper bound and the sensitivity of the reactive power generation with respect to the global real load increase at iteration k . Sensitivity coefficients

$\partial Q_{gi}^k / \partial C_{tot}$ are computed by means of a linear combination of the sensitivities of the reactive power generations of PV-type nodes with respect to the real and reactive injections in

the load buses. Assuming a constant power factor for each bus during the load ramp, the following formula is obtained:

$$\frac{\partial Q_{gi}^k}{\partial C_{tot}} = \sum_{j=1}^{N_{gr}} \left[\sum_{h=1}^{N_{Lj}} \left(\frac{\partial Q_{gi}^k}{\partial C_{hj}} + \frac{\partial Q_{gi}^k}{\partial D_{hj}} \tan \Phi_{hj} \right) \eta_{hj} \right] \beta_j + \sum_{m=1}^{N_{gr}} \frac{\partial Q_{gi}^k}{\partial P_{gm}} \rho_m$$

where $\cos \Phi_{hj}$ is the power factor of load bus h located in the area j , and $\partial Q_{gi}^k / \partial C_{hj}$, $\partial Q_{gi}^k / \partial D_{hj}$, $\partial Q_{gi}^k / \partial P_{gm}$ are the sensitivities of the reactive power generation in node i with respect to the real and reactive load of node h of area j and to the active generation of the regulating unit m .

The evaluated active load increment ΔC_{min}^k is shared among the load busses according to the β_j , η_{hj} coefficients while the generated real powers are increased by the ρ_m factors. The pertinent reactive load increments are proportional to the active ones by means of the factors $\tan \Phi_{hj}$.

A new steady-state situation is computed for the system by a power flow computation where a coupled Newton-Raphson method is employed. Voltage profiles and reactive power generations of PV buses are updated. All units that have reached their reactive generation limits ask for a node type switching (from PV to PQ).

3.3 Performance Indices

Before starting a further iteration with a new ΔC_{min}^{k+1} , the following voltage collapse proximity indicators are computed:

$\partial V_{hj} / \partial C_{hj}$, $\partial V_{hj} / \partial D_{hj}$, sensitivity value of the voltage in PQ bus h of area j with respect to the corresponding active and reactive load increase.

$\partial Q_{gtot} / \partial D_{tot}$, sensitivity value of the global reactive power production of all regulating units with respect to the reactive load increase in the whole system.

$\partial V_{pilj} / \partial D_{aj}$ sensitivity value of the voltage of the pilot node of area j with respect to the area reactive load increase.

$\partial Q_{gaj} / \partial D_{aj}$ sensitivity value of the reactive power generation of area j with respect to the area reactive load increase. Sensitivity $\partial V_{hj} / \partial D_{hj}$ is usually employed to provide a local collapse proximity indication. In normal operating conditions, it assumes a moderate negative value. When increasing the bus load, it increases in magnitude until its sign changes at the point of voltage collapse. Some approaches are based on this kind of local proximity indicators for obtaining indications about the loadability margins of a single node. These procedures may be useful in the planning stage when the system load and generation expansion is examined on a long time horizon. In contrast, it may provide useless responses in the operation planning or in very short term security assessment, when realistic load trend scenarios are analysed. The performance index $\partial Q_{gtot} / \partial D_{tot}$ is given by the following relation:

$$\frac{\partial Q_{gtot}}{\partial D_{tot}} = \sum_{m=1}^{N_{gr}} \frac{\partial Q_{gi}}{\partial D_{tot}} \quad (1)$$

The terms $\partial Q_{gi} / \partial D_{tot}$ can be obtained in two different ways. In the first way only the reactive Jacobian matrix is used:

$$\frac{\partial Q_{gi}}{\partial D_{tot}} = \sum_{j=1}^{N_{gr}} \left[\sum_{h=1}^{N_{Lj}} \frac{\partial Q_{gi}}{\partial D_{hj}} \eta_{hj} \right] \beta_j \quad (2)$$

In the second one the complete Jacobian matrix is used and the increase of the reactive generation in the P,V busses is computed taking into account also the active load and generation increases. Therefore:

$$\frac{\partial Q_{gi}}{\partial D_{tot}} = \frac{\partial Q_{gi}}{\partial C_{tot}} \frac{1}{\tan \Phi_{tot}} \quad (3)$$

$$\text{with } \tan \Phi_{tot} = \frac{\Delta D_{tot}}{\Delta C_{tot}} = \sum_{j=1}^{N_{gr}} \left[\sum_{h=1}^{N_{Lj}} \eta_{hj} \tan \Phi_{hj} \right] \beta_j$$

In normal operating conditions the performance index $\partial Q_{gtot} / \partial D_{tot}$ is positive and close to 1.0. It increases with load demand, assuming very high values in proximity of the voltage collapse point, where it may change in sign. Its behaviour is consequent to the fast reactive power loss increase caused by the lack of the reactive generation margin available on the

generating units. The sensitivities $\partial V_{pilj} / \partial D_{aj}$ have a similar behaviour to that of the node wise : they are negative (with increasing magnitude) until the voltage collapse occurs.

They give a satisfactory idea of the average voltage variation in the area following a reactive load increase distributed by means of the area distribution coefficients. Instead of examining the voltage reactive behaviour of individual nodes, only one pilot node is selected to represent each area. This allows one to greatly reduce the amount of data to be considered in studying the voltage collapse problem while still retaining the main local features of the phenomenon. The

sensitivities $\partial Q_{gaj} / \partial D_{aj}$ provide indications about the operating MVAR margins of each area and are computed as follows:

$$\frac{\partial Q_{gaj}}{\partial D_{aj}} = \sum_{m_j=1}^{N_{grj}} \left[\sum_{h=1}^{N_{Lj}} \frac{\partial Q_{gmj}}{\partial D_{hj}} \right] \quad (4)$$

where N_{grj} is the number of generators belonging to the area j with reactive power margins. The area response in reactive power generation delivery due to the reactive load variation occurring in the other areas can be measured as well by the

$$\text{sensitivity: } \frac{\partial Q_{gaj}}{\partial D_{ai}} = \sum_{m_j=1}^{N_{grj}} \left[\sum_{h=1}^{N_{Lj}} \frac{\partial Q_{gmj}}{\partial D_{hi}} \right] \quad (5)$$

3.4 Simulation of the Secondary Voltage Regulation

For what concerns the introduction of the SVR actions in the analysis of the voltage collapse phenomenon, load variations, line (or transformer) trips and generator outages can be simulated. An alignment constraint is introduced for the reactive production of the N_{grj} generators controlled by the SVR in area j . Therefore the reactive power productions satisfy the constraint:

$$Q_{g1j} / Q_{g \max 1j} = Q_{g2j} / Q_{g \max 2j} = \dots = Q_{gN_{grj}j} / Q_{g \max N_{grj}j} \quad (6)$$

Denoting with:

$$Q_{A_j} = \sum_{i=1}^{N_{grj}} Q_{gi_j}, Q_{A_{\min_j}} = \sum_{i=1}^{N_{grj}} Q_{g_{\min_i j}}, Q_{A_{\max_j}} = \sum_{i=1}^{N_{grj}} Q_{g_{\max_i j}},$$

the reactive power production of area j and its lower and upper bounds, the following relation handles for each unit in the area:

$$Q_{gi_j}/Q_{g_{\max_i j}} = Q_{A_j}/Q_{A_{\max_j}} = q_{A_j} \quad i = 1, \dots, N_{grj} \quad (7)$$

The variable q_{A_j} is the area reactive level and it is the reference signal sent by the SVR to the plants in the area for preserving the required reference voltage V_{ref_j} in the pilot node of the area j . The new steady-state operation point consequent to a given perturbation is obtained with the help of an iterative procedure [34]. At the beginning, all the pilot

nodes are assumed as P,V busses with $V_{pil_j} = V_{ref_j}$, while the generator busses involved in the SVR are of the P,Q type and satisfy constraints (6), (7). After the load increase, the reference voltages of the pilot nodes can be preserved if the reactive power injections in the nodes are suitably changed. Since any kind of reactive power generation is truly available in the pilot nodes, the reactive power unbalances have to be eliminated by an adequate modification of the reactive power levels of the different areas and as a consequence by updating the reactive productions of the pertinent generators. Therefore an iterative solution scheme is adopted in order to eliminate the reactive power unbalances in the pilot nodes. At the current iteration k the new reactive productions are computed for the SVR controlled areas:

$$[Q_A^k] = [Q_A^{k-1}] + [\Delta Q_A^k] \quad (8)$$

where the modifications $[\Delta Q_A^k]$ are obtained by the linear approximation:

$$[\Delta Q_A^k] = \left[\frac{\partial Q_{pil}}{\partial Q_A} \right]_k^{-1} [\Delta Q_{pil}^k] \quad (9)$$

with:

$[\Delta Q_{pil}^k]$ reactive power unbalances in the N_{ar} pilot nodes at

iteration k , $[\Delta Q_A^k]$ modifications of the reactive power

productions in the N_{ar} areas at iteration k , $[\partial Q_{pil}/\partial Q_A]_k$ square sensitivity matrix of dimension $N_{ar} \times N_{ar}$ when all the areas are regulating. In each area k the reactive power variation is shared among the area generators following the criterion of (7). If in the iterative process the voltages of some generators controlled by the SVR reach the upper or lower limits, the bus type is changed from P,Q to P,V with V_g fixed on the binding limit; as a consequence these generators are no longer controlled by the SVR. If

$Q_{A_j}^k = Q_{A_j}^{k-1} + \Delta Q_{A_j}^k \geq Q_{A_{\max_j}}$, the reactive level of the area is fixed to 1 (maximum value) and the pilot bus type is changed from P,V to P,Q, with $\Delta Q_{pil} = 0$ (with a consequent voltage reduction).

The iterative process, which utilizes an a.c. LF program, stops when all the reactive unbalances ΔQ_{pil} fall below a given tolerance. At the end of the procedure the new pilot voltages

and the new reactive levels are given. When SVR is in operation the sensitivity matrix $\partial Q_A/\partial D_a$ is obtained by:

$$\frac{\partial Q_A}{\partial D_a} = \left[\frac{\partial Q_{pil}}{\partial Q_A} \right]^{-1} \frac{\partial Q_{pil}}{\partial D_a} \quad (10)$$

4. MODAL ANALYSIS

In the modal analysis the suggestions of [30] are followed.

In order to confirm the adequacy of the performance indices adopted for the detection of voltage collapse proximity, the dominant eigenvalue λ_i and relevant right and left eigenvectors r_i and l_i of the factorized inverse of the reactive Jacobian matrix J_R are determined. Therefore:

$$J_R^{-1} = R \Lambda L$$

where Λ is the diagonal matrix whose entries are the eigenvalues λ_i while R and L are square matrices whose columns and rows are, respectively, the right and left eigenvectors r_i and l_i .

The knowledge of the above mentioned matrices allows one to determine the following linear relation between the voltage variations in the n load busses and the reactive load increases:

$$\Delta V = R \Lambda L \Delta Q \quad \text{or} \quad \Delta V = \sum_{i=1}^n r_i \lambda_i l_i \Delta Q$$

As a consequence the sensitivity of the voltage with respect to the reactive power demand at bus h is:

$$\frac{\partial V_h}{\partial Q_h} = \sum_{i=1}^n r_{hi} l_{ih} \lambda_i = \sum_{i=1}^n P_{hi} \lambda_i,$$

with P_{hi} participation factor [30] of bus h to the mode i .

If only the eigenvalues and eigenvectors of the dominant modes are calculated, approximate values of the sensitivities are obtained; the comparison between these values and the exact ones, derived by the factorized inverse Jacobian matrix, allows us to assess the lowest dimension m of the set of dominant eigenvalues. As a consequence, the rectangular matrices $R_m (n \times m)$ and $L_m (m \times n)$ whose columns and rows are, respectively, the right and left eigenvectors of the m dominant modes, can be determined. Furthermore these matrices are normalised in order to obtain for the dominant mode i :

$$\sum_{h=1}^n r_{m_{hi}} l_{m_{ih}} = \sum_{h=1}^n P_{hi} = 1 \quad (11)$$

The analysis of the biggest participation factors relevant to the m dominant eigenvectors allows to point out the areas (and the busses) of the network closest to the voltage instability.

By assuming a positive sign for ΔQ when the reactive load increases, in a stable operation condition the eigenvalues λ_i are negative and limited in size; their absolute values increase while increasing load demands. In voltage collapse proximity the dominant λ_i assumes a very large negative value ($1/\lambda_i \rightarrow -0$ since J_R is becoming singular), changing in sign when the voltage instability occurs.

Besides the eigenvalues of the sensitivity matrix $\partial Q_{ga}/\partial D_a$ are calculated. In stable operating condition the largest eigenvalue is positive and it approximates $\partial Q_{gtot}/\partial D_{tot}$. Its value increases quickly near the voltage instability region, changing in sign as $\partial Q_{gtot}/\partial D_{tot}$ when voltage collapse occurs.

5. TESTS AND RESULTS

The automatic procedure "COLLAS" has been employed for measuring the distance from the voltage collapse point both for the ENEL EHV transmission system and for the subtransmission system of Sicily. In the first application a critical situation characterized by low voltage levels and poor reactive generation margins has been examined. These operating conditions can occur during summer working days if several thermal units are in maintenance, the hydro plant are delivering their rating active power, the load power factors are lowering for the high air conditioning load and large amount of active power is driven by the neighbourhood countries. The examined scenario corresponds to a real situation occurring on the morning pick-up of a working day of July. The data adopted in the study are drawn by the archives of the on-line state estimation program running in the ENEL National Control Center. In this framework the data relevant to the subtransmission system and to the operation of the LTC transformers feeding the MV network are not available.

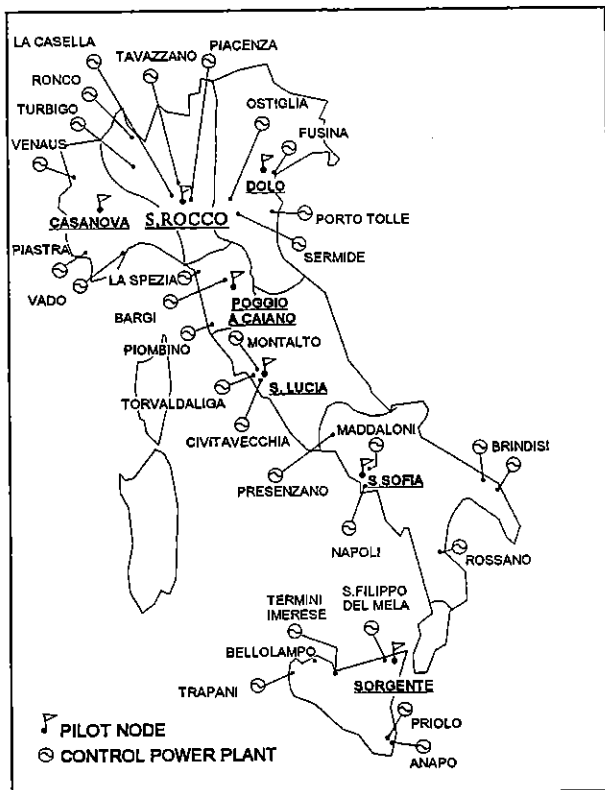


Figure 1: Secondary Voltage Regulation Areas

Therefore the action of the LTC transformers is not taken into account in the analysis. As a consequence the voltage dependence of active and reactive loads is neglected. In the simulation tests the impact of the SVR action on the security margins is examined by the comparison between the results obtained with the actual regulating devices (only local AVR

control) and the ones attainable with the adoption of a more centralized control function (SVR). In the simulation performed seven SVR areas are considered (Fig.1). For both the hypotheses the modal analysis is employed in order to strengthen the validity of the global and of the area-wise performance indices adopted and to point out the most critical area while approaching the voltage collapse point. Besides, the maximum loadability of the interconnection lines is determined by assuming a suitable equivalent network of the external system (UCPTE countries). In the second application the adopted scenario consists of the subtransmission system (represented in detail as far as the MV terminals of the HV/MV load transformers) of the Palermo area. As a consequence, the effects of the LTC transformers and of the voltage dependent loads is considered in this analysis. Exponential laws are adopted both for active and reactive loads with coefficients similar to the ones obtained by experimental tests (1.5-2.7 for active and reactive loads respectively)

5.1 Tests on the EHV ENEL System.

In the morning pick-up of the examined scenario [35] the voltage profiles in all the network are quite satisfactory until 8:00 a.m., but the reactive generation margins in Milano area are strongly reduced due to the maintenance of several thermal groups and to the massive use of air conditioning. The load increases and reaches the maximum at 10:00 a.m. In the first 15 min. of the load ramp a reduction of 6 kV occurs in the 380 kV bus of Brugherio in Milano area whose pilot node is S.Rocco (Fig.1). At 9:00 a.m. the voltage drop experienced in Brugherio reaches 25 kV (Fig.2).

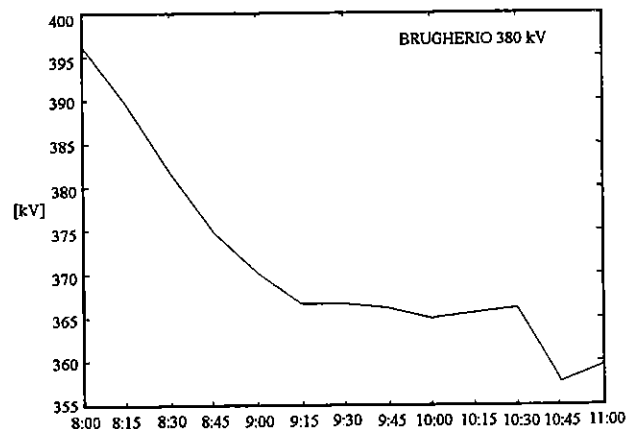


Figure 2: Brugherio voltage drop during the load ramp.

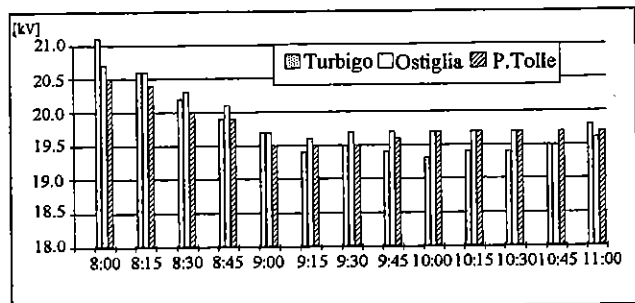


Figure 3: Terminal voltages of some northern generators during load ramp.

The voltage drop in the areas of Torino, Milano and Venezia is consequent to the attainment of the capability limits of almost all the synchronous generators in the northern Italy.

In Fig.3 the behaviour of the terminal voltages of some important generators which lose their regulation capability during the load pick-up, is shown.

In the time interval 10:30-10:45 the trip of a loaded 380 kV line near to the Brugherio bus and the outage of a thermal unit in the Turbigo plant occur. As a consequence the voltage of Brugherio slopes down to 357 kV. The voltage recovering in the next 15 min. is due to the characteristic load decrease after 10:00 a.m.. The assessment of the voltage stability is performed for the most critical point.

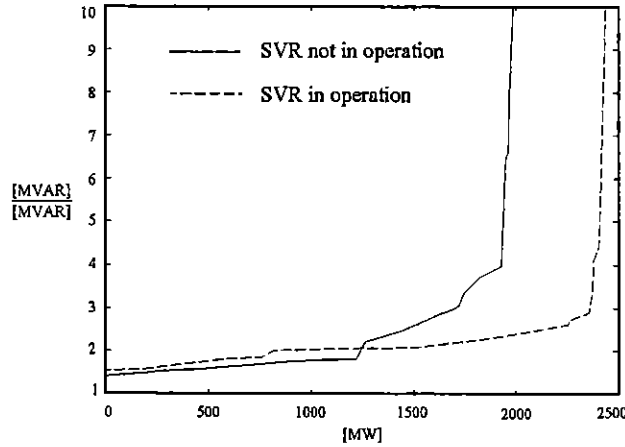


Figure 4: Performance Index $\partial Q_{tot}/\partial D_{tot}$
ENEL EHV System

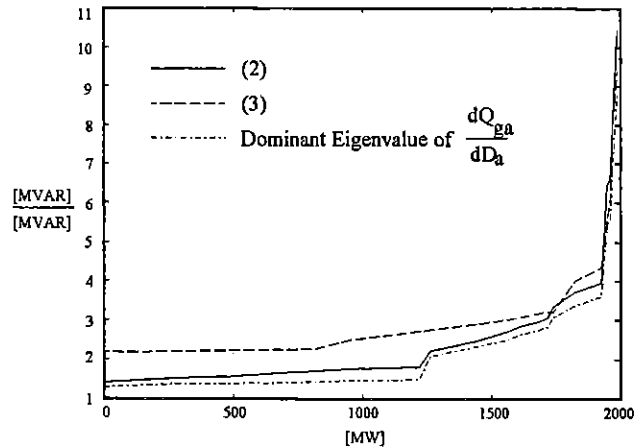


Figure 5: Performance Indices $\partial Q_{tot}/\partial D_{tot}$ and dominant eigenvalue of $\partial Q_{ga}/\partial D_a$.

In Fig.4 the behaviour of the global performance index $\partial Q_{tot}/\partial D_{tot}$ is shown while the load increases up to the voltage collapse. The action of the SVR provides an improvement of the security margin of about 400 MW.

In Fig.5 the values of the global performance index obtained according to the equations (2) and (3) are compared. They are significantly different for small value of the load increase and nearly equal close to the voltage instability point where they become negative. In the same picture the behaviour of the dominant eigenvalue of the area matrix $\partial Q_{ga}/\partial D_a$ is presented. In Fig.6 the bus voltage drop in Brugherio is shown. The voltage collapse occurs at 337 kV if the SVR^s are not in operation and at 341 kV if the SVR^s are in operation.

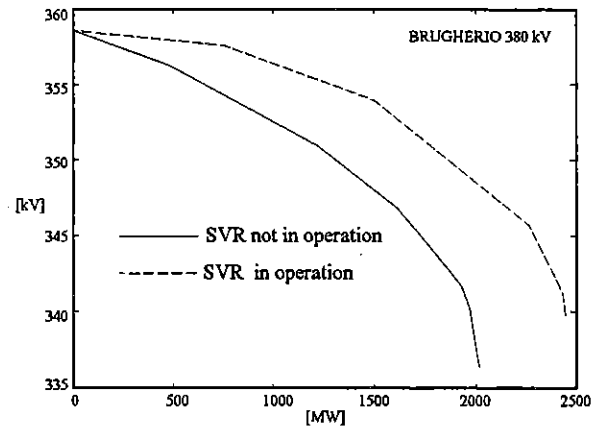


Figure 6: Voltage of the Brugherio bus.

The modal analysis, performed for both the situations under study, allows to strengthen the validity of the global and of the area-wise performance indices adopted and to point out the most critical area while approaching the voltage collapse point. The absolute value of the biggest eigenvalue of J_R^{-1} (inverse of the reactive Jacobian matrix) as a function of the active load increase which leads to voltage collapse, is shown in Fig.7.

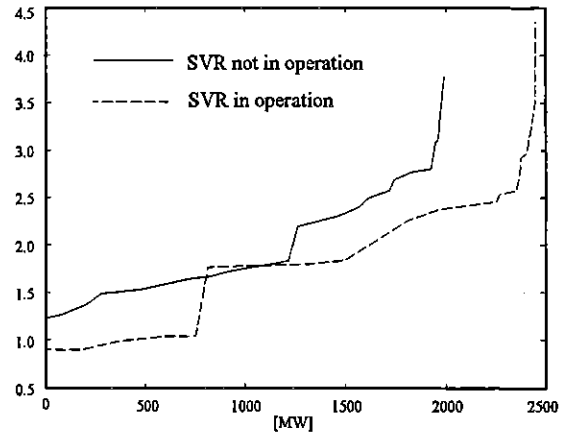


Figure 7: Absolute value of the dominant eigenvalue of J_R^{-1}

The computation of the most significant participation factors pertinent to the dominant eigenvalue is performed at each iteration of the procedure "COLLAS". For small value of the load increase there are few critical nodes characterised by significant values of the participation factor and, according to equation (11), the dominant P_{hi} are large. While approaching the collapse point, the set of the critical nodes increases and the critical zone spreads out more and more.

In Fig.8 the expansion of the critical zone when the SVR^s are not in operation is shown. For low load increases the zone consists of the neighbouring busses of Torino (darkest circle in Fig.8). When the load increase reaches 1440 MW the zone includes the Milano area and its barycentre is still Torino 220 kV bus. Between 1440 MW and 1970 MW the zone spreads up to the Firenze and Venezia areas and reaches the northern busses of the Roma area. The bus of Brugherio becomes the most critical one (barycentre). At last when voltage instability occurs (2020 MW) the critical zone reaches the first node of the Napoli area. In Fig.9 the expansion of the critical area when the SVR is in operation is shown. Near the actual load (at the operation point 10:45) the dominant mode(local type) is influenced by the busses closest to Scilla

(a longitudinal structure of the electric network is evident in the South).

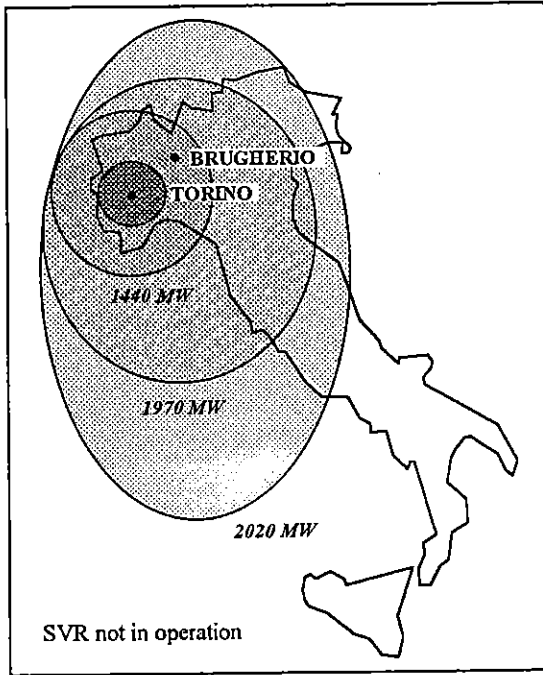


Figure 8 : Expansion of the critical zone.

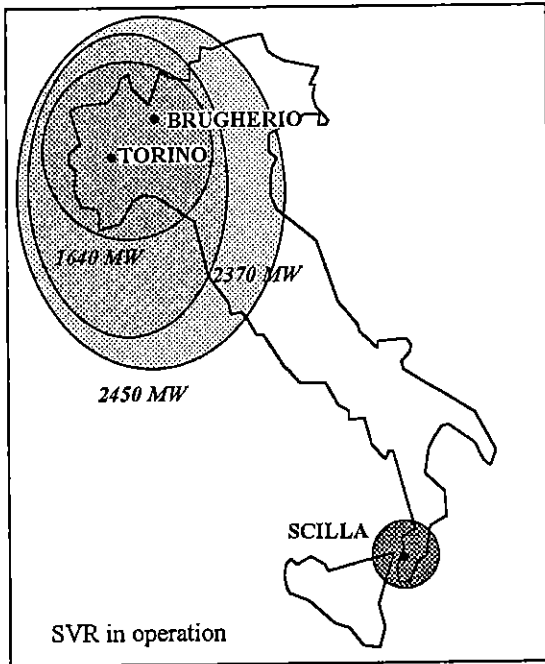


Figure 9 : Expansion of the critical zone. SVR in operation.

When the load increase reaches 1640 MW the critical zone moves toward the northern areas (before reaching Torino and after Milano). When voltage collapse occurs the bus of Brugherio becomes the most critical one and the zone interested by the voltage decline reaches at most the Firenze and Venezia areas. These results point out the positive effects of the SVR on voltage stability as also confirmed in [36]. The procedure has been adopted for determining the maximum loadability of the international interconnection of the ENEL system in this critical situation. The value of the real power importation obtained by the state estimation

records at 10:45 a.m. is 4200 MW. Two interties are out of service (the 380 kV line Redipuglia-Divaça and the 220 kV line Sondrio- Robbia). The 380 kV line Soazza-Bulciago (with second thermal in Milano area) is driving 1000 MW. A Ward equivalent of the UCPT network is connected to the Italian frontier busses. In the simulation all the additional loads are satisfied by increasing the imported real power. The point of collapse occurs when the exchanged power reaches 6130 MW and the voltage in the critical node of Brugherio falls to 319 kV. The barycentre of the critical zone is in the Venezia area (where the interconnection appears weaker).

5.2 Tests on the Sicily Network.

The operation point adopted in the study is pertinent to the peak load of a short term planning winter day. The assessment of the voltage stability is performed for four situations relevant to different operation strategies of the LTC transformers and to the happening of a severe disturbance. Test 1 is carried out assuming that all HV/MV and EHV/HV LTC transformers of Sicily control the voltages of their lower side bus. In Test 2 the HV/MV transformers have fixed ratios while the EHV/HV ones perform the voltage regulation. The exponential laws above mentioned are employed for the loads.

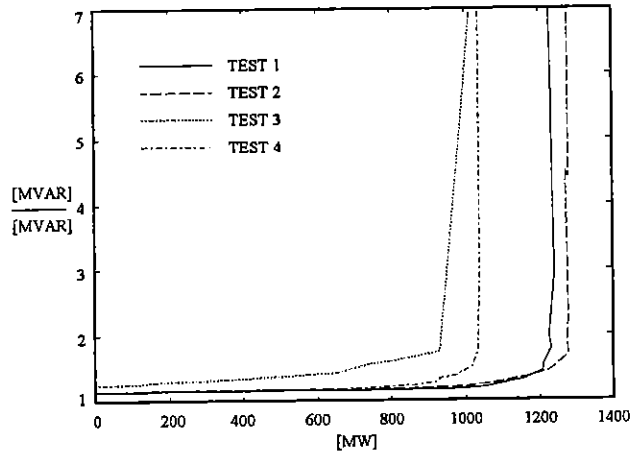


Figure 10: Performance Index $\frac{\partial Q_{\text{tot}}}{\partial D_{\text{tot}}}$

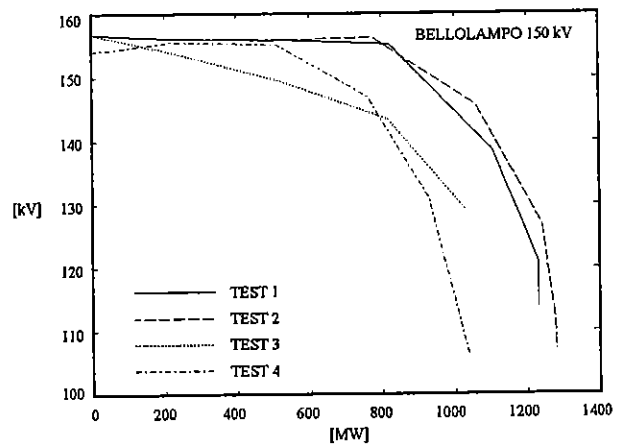


Figure 11 : Secondary voltages of the Bellolampo transformer.

In Test 3 all the transformers have fixed ratios and a P,Q constant model is adopted for the load busses. After all, Test 4 allows to assess the effect of the 220 kV line Bellolampo-Caracoli trip on the operative margins of the system when only the EHV/HV transformers perform the voltage regulation. The

performance indices shown in Fig.10 point out the weakness of the load model adopted in Test 3 (an extremely low margin is obtained). Besides the positive action of blocking the tape changers (Test 2) of the HV/MV LTC appears by the comparison between Test 1 and 2. Fig. 11 and 12 depict the behaviour of the voltages in both sides of the Bellolampo 220/150 kV transformer.

The comparison between Test 1-2 and Test 3 (all LTC with fixed ratio) outlines how the preservation of a constant voltage on the 150 kV side as long as possible advances the voltage drop on the primary terminal.

The behaviour of the voltage in the MV load bus of Sciacca is depicted in Fig.13.

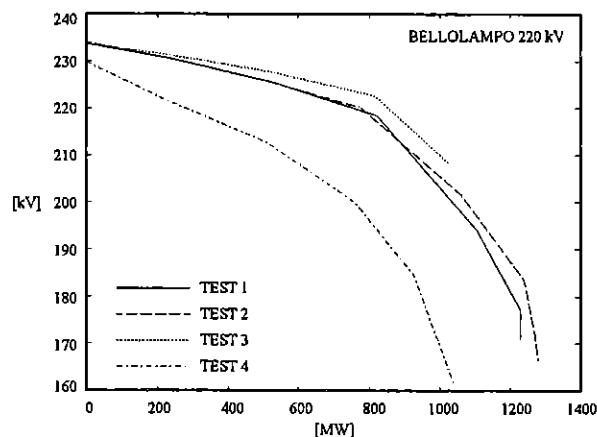


Figure 12 : Primary voltages of the Bellolampo transformer.

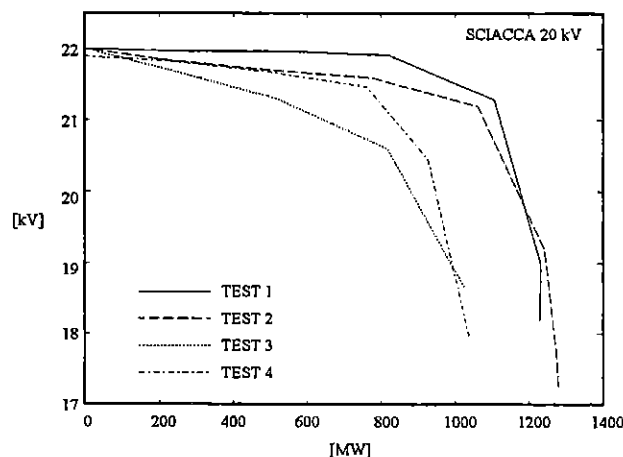


Figure 13: Secondary voltages of the Sciacca transformer.

CONCLUSIONS

In the paper a steady state approach useful for the assessment of the voltage stability in the very short term environment of an EMS is presented. The procedure "COLLAS" is based both on sensitivity (performance) indices (global and area wise) obtained by the LF and on the modal analysis performed, in an alternative way to the very large Jacobian matrices of LF equations, on a more manageable matrix containing the linear relations between reactive generations and loads in predefined areas. Some of the drawbacks presented by previous similar approaches consisting in an impracticable distribution of loads and generations, in an unsatisfactory approximation of the system non linearity (reactive

generation bounds, constant Jacobian in continuation methods, load models etc.) are eliminated. The computation times obtained (~100 sec on VAX 9000 computer in the critical situations) are encouraging for the application of the program in the ENEL National Control Center and for extending the research to the area of the voltage stability enhancement (rescheduling, load shedding, SVS placement etc.).

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Discussion of papers presented by Abed and by Marannino

Edited by Thierry Van Cutsem, University of Liege

Claudio Cañizares (Univ. of Waterloo):

My question to Eyad [Abed] is regarding computational issues. I have been playing lately with the Hopf bifurcation. I would very much appreciate his comments regarding the determination of stability of limit cycles. Once you get to a certain number of state variables, things become messy, and complicated, and very expensive. I might not be using the right technique, so I would appreciate his comments on that, because that would help me a lot.

Eyad Abed (Univ. of Maryland):

We have been looking at small systems and have been using the AUTO package. That can analyze systems up to 25 and it does a good job. Other than that, as far as the control design is concerned, it seems to me that finding the structure of controls that can give you stable limit cycles is a useful objective and then to test that on large systems would be the next step, through simulations maybe.

Ian Hiskens (University of Newcastle):

You have shown a slide that indicates there was a transition to chaos. Would you please explain in more detail what is going on. Also it does not appear in your paper, does it? Is there any reference on this?

Abed: That slide was from another paper, from the 1993 American Control Conference. That is the control paper; the analysis part is in the IEEE Transactions on Circuits and Systems, April of this year. As regards chaos, there is a rule of period doubling bifurcation to chaos. Starting from the limit cycle generated at each Hopf bifurcation point, and then those chaotic attractors that are generated disappear in a boundary crisis when they meet with the unstable equilibrium point, which is the bottom, the lower voltage equilibrium.

Nelson Martins (CEPEL):

You said linear feedback control was not used in this conference to avoid saddle-node bifurcation. It actually was used. I used it in my paper, using SVCs to increase system loadability. Yakout also did that, so we are using that. I have shown there the best place to put feedback control by critical zero's, comparing locations and then actually placing the SVC. It was a first-order function if you recall.

A second point is about having limit cycles in your system imposed through your controller or something. We have in our own system a lot of uncontrolled limit cycles, which bothers us. We do not like the idea very much.

Abed: What I was talking about is: once the SVC is placed, further linear control, for stabilization control of your equilibria, and so on - I hadn't heard about. That is what I really meant. Regarding oscillations in the system, that is a big subject, and we'll have to talk off-line.

Daniel Karlsson (Sydkraft):

I have a very practical question. You showed a number of slides showing the stable area, and then you introduced some sort of bifurcation, and extracted the stable area from the top of the nose. As I am working in a utility, this is my reality [showing a diagram of the Swedish system], and when I'll get back home to Sweden, they'll ask: what did you learn in Davos? In a way, I have to translate these curves into my system.

But my question will be a bit nicer: Is it possible to give an example of what you do in a system in order to introduce that nice bifurcation to make the system better?

Abed: There is a simple answer. When you get to Sweden, do not mention that you met me and that you heard my talk! That solves the problem!

These are just conceptual comments. Actually, this bifurcation that you picked is the most complicated one in my presentation, and it's not in the math books yet.

Karlsson: So, I have to wait?

Abed: Yes, you have to wait.

Chair (Hans Glavitsch, ETH Zurich):

I may add, as being the chairman of this session, that there is a real need - I have myself almost 20 years of industry experience and I am in the academia, and it is my intention, and always has been my objective - to bridge academia and the practical applications, and this I think, in the long run, is a real need.

Yakout Mansour (B.C. Hydro):

[Addressing Marannino] If the objective of the paper is to discuss or propose voltage collapse proximity indicators, I have a problem with just about everyone of them. When I looked at Figure 7 - and it is shown also by Figures 8 and 9 - you are plotting an absolute value of the dominant eigenvalue of different modes. You did indicate that as you increase the load the mode changes. Then you are just tracking the dominant eigenvalue as an indicator, and of course I cannot see that you tracking the same things.

That is one indicator. Then you go to the other indicator which is dQ/dD_{total} or whatever. They are very nonlinear. They are really not proximity indicators; they just indicate that you are collapsing when you actually collapse, but how close are you to collapse? - none of those indicators gives you a proximity indication.

I'd like a comment on those two objections, and the third one is actually a question: how do you select a pilot bus?

Paolo Marannino (ENEL-CRA)

The dominant eigenvalue in the case of Figure 8 is always the same. It does not change. We consider that the secondary voltage regulation is not in operation. The dominant eigenvalue is located in the Torino area in the north. Also it is the zone that expands, but the mode does not change.

Mansour: When the zone expands, to me that is a mode change.

Chair: To be more precise, is it the same eigenvalue, and does just the eigenvector change?

Marannino: Yes, the same eigenvalue.

Mansour: If the eigenvector changes, then it is not the same mode.

Marannino: Ok.

Mansour: The other indicators are really not proximity indicators. They are very nonlinear. They are flat all over the range, then suddenly tell you "too late."

Marannino: These are proximity indicators, because the question I have to ask with the program is "was the operating point of 20 July near to voltage collapse? What was the distance?" In this sense it is a proximity indicator. It is obvious that when you arrive near the voltage collapse point, for instance when you arrive near 2000 MW from 1000 MW, the procedure makes a lot of small steps of load increase. For a small increase the value of this index certainly changes a lot, but if you see this picture, these are only 50 or 70 MW. For my practice, for instance, when I am studying the system, I can say until this point I have no problem, but when the figure reaches a value of 3 or 4, I have very critical problem, so that I have to take actions. I do not have to wait for the point of collapse. In this sense, it is a practical application.

Mansour: I do not know. We would disagree on that.

Marannino: For instance, I do not have to know if the collapse is after 2000 or 2010 or 1950 MW. The problem is, when we are in the region near this. Certainly, they are non-linear, especially when we arrive there. And you have a lot of discontinuities when you have generator buses that switch from PV to PQ because they reached their capability limits.

Marija Ilic (M.I.T.):

Would you please put your slide on the power transfer that you attempted, the one where you had your 4 curves, Figure 10?

Marannino: Figure 10 is on the study of the Sicily sub-transmission system.

Ilic: I have a question regarding my definition of yesterday, the proposed definition on what we mean by good voltage support. What you drew there was the largest power that you can transfer from the north to south. Or the other way round - I do not know which way the power flow went. Your means of comparing what is better or worse was in terms of how much real power you can transfer between two points. Is that correct?

Marannino: No, Figure 10 has nothing to do with power transfer between northern Italy and south. It shows only the analysis of the impact of blocking tap changers in sub-transmission system of Sicily. But I said something about our investigations on the maximum transfer capability of the international interties, between Italy and neighbouring countries.

Ilic: That is correct. What was the objective there as you compared methods? What is better? How do you compare them?

Marannino: No, we encountered a lot of difficulties in this investigation. The first one is related to a problem of which Dr. Mansour reminded us this morning: the external system equivalent. There are big problems for making this investigation. Usually our agreement with the other companies is that for what concerns the reactive power exchange, this exchange could be zero between the different companies. So, what happens if I simulate the generators in the external system as PV buses? This sustains voltage in our network. In this case, we made two different simulations. One with external network generators as PV buses, did not provide clear results. In the other case, with external generators as PQ buses, we obtained the results reported in the paper, with an increment of 2000 MW before reaching the point of collapse.

Ilic: Well, I think I'm asking a much more basic question. Even in Figure 10 or 11, on the x-axis you always draw megawatts. Those are the ones that you worry about. So, I am going to compare two performance criteria, and the worse one, that you are saying is the more conservative, is telling me that I can transfer less, or deliver less, whatever your

megawatts stand for there. So my question is, why did you use that? That is the same definition I was proposing yesterday in my first paper?

Marannino: I think it is not the point. The comparison is made between a policy where we use LTCs both in transmission-subtransmission system and subtransmission-MVA in operation, and another where we block the secondary side of MV load tap changers. Curve number 3 is pertinent to the easiest simulation that we can do: PQ constant load and blocked tap changers.

Ilic: Maybe we'll talk later. Thank you.

DISCUSSION ON LINEAR INDICES AND EIGEN-SYSTEM DECOMPOSITION

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The authors in [1] present the successful application of well-known voltage stability analysis techniques to the ENEL system, namely, linear sensitivity indicators to determine the presence of system singularities [2, 3, 4], also known as fold or saddle-node bifurcations [5, 6], and eigen-analysis techniques to study sensitive system areas and determine mechanisms to avoid the onset of a bifurcation problem [7, 8].

This discussor believes that two important issues must be clarified in the paper, particularly since these points have been misinterpreted by several researchers, and misused by some industry engineers. Thus, the authors' comments to the following points would be appreciated:

1. The authors seem to suggest throughout the paper that linear indicators, whether these are sensitivity measures, or equivalently singular values or eigenvalues, are good measures of proximity to fold bifurcations. Although these indices are useful to denote the presence of a system singularity, it has been demonstrated that they are not adequate indicators of proximity due to their highly nonlinear behavior [3, 9], i.e., these indices give a false sense of security since they remain practically unchanged until very close to the bifurcation point, by then it is already too late to save the system. These comments are certainly supported by the results shown in the paper: when comparing Figures 4 and 10, one observes that what constitutes a good proximity value of the proposed performance index for the system in Fig. 4, e.g., $\partial Q_{gtot}/\partial D_{tot} = 3$, it is not an adequate value in all tests cases depicted in Fig. 10.
2. The decomposition of the system Jacobian J_R in its eigenvalues Λ , and right R and left L eigenvectors, i.e.,

$$J_R = R\Lambda L^T \quad (1)$$

is only valid if J_R is a "diagonalizable" or "semi-simple" matrix [10], i.e., the matrix has no repeated eigenvalues and eigenvectors; otherwise this matrix can only be decomposed in its Jordan canonical form. Although (1) is widely used in power systems, one must be careful in studying the structure of the matrix before applying it. Interestingly enough, a closer look at J_R , as defined in the paper and [8, 11], shows that it has a quasi-symmetric structure. Therefore, since a real symmetric matrix is also a Hermitian matrix, (1) can be safely applied to J_R .

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Closure of the Discussion of Linear Indices and Eigen-System Decomposition

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The reactive problems connected to the voltage collapse are clearly non linear, both for the known Q,V relations and in the most stressed situations for the discontinuity of the jacobian matrix entries consequent to the attainment of the synchronous generator capability limits in the evolution of the phenomena.

The authors of the paper referenced with [1] agree with Dr. Caffizares that sensitivities and eigenvalues show a strongly non linear behaviour in a narrow proximity to the voltage collapse. Their utilisation has not been proposed in [1] for the definition of on-line control actions when the voltage instability impends over the system. Therefore it is worthy to clarify that the aim of the automatic procedure COLLAS, in phase of development at the Automation and Computation Research Center of the ENEL, is to give to the operation planning engineers and to the National Control Center operators very useful guide lines for their activities. At the end of the project the outputs of the procedure will be:

a) the voltage collapse distance (MW, Mvar margins) of the scheduled operation points in a short term (some days or 24 hours in advance) or in a very short term (from 2-3 hours to 1/2 hour in advance) environment,

b) the preventive (corrective) control actions (blocking of the ratios of the OLTC transformers feeding the loads at the MV levels, lowering the voltage targets of the consumers, switching reactive compensation devices, shedding loads and so on) to be taken in alarm (emergency) states.

The global sensitivity $\partial Q_{gtot}/\partial D_{tot}$ and the area sensitivities

$\partial Q_{ga}/\partial D_{a_i}$ are familiar figures to our utility operators and they are useful for a simple classification of the system state. For giving an example in the ENEL transmission system the sensitivity $\partial Q_{gtot}/\partial D_{tot}$ ranges from the values 1.2-1.3 at peak loads in normal states. The value 1.6 reached in the critical situation analysed in [1] represented a severe alarm. An estimated distance of 2000 MW (the summer peak-load was about 33000 MW) from the voltage collapse point did not guarantee a secure operation with respect to the outage of one of the largest (750 MVA) thermal generator or to the trip of one of the most loaded lines. For several days the operators asked for the block of the OLTC ratios on all the network areas at the beginning of the morning pick-up period (7 a.m). The availability of an automatic procedure, like COLLAS, could give to the system operators most suitable information on the critical areas where preventive counter measures could be taken.

The aim of the tests performed on the Sicily network is to demonstrate the ability of the procedure to evidenziate the influence of the load models and of the OLTC transformer operation policy on the voltage collapse phenomenon. The MW margins are very high (1200 MW for a peak forecast of 2300 MW), the sensitivity $\partial Q_{gtot}/\partial D_{tot}$ is near to 1.1 in the actual operation point. In the performed tests the Sicily network is connected to the powerful system of the Naples department. The loads are increased only in the Sicily area. Some small hydro generators are in service in the subtransmission system (150 kV level), the largest thermal and hydro generators deliver their power in the EHV transmission system of Naples and Palermo departments. The

voltage collapse appears in part of the western coast when all the local reactive generations (in the Palermo department)

reach the capability limits. The value of $\partial Q_{gtot}/\partial D_{tot}$ suddenly grows (starting near the value 2.) to infinite for the huge overload (160 % of the rating) of some HV/MV transformers of the weak 150 kV islands and for the loss of the regulation capability of the inner synchronous generators. The reactive power available on the generators in service in the Naples area cannot be delivered for sustaining the voltage in the 150kV network of the Palermo area. Ultimately it is not possible to assign an alarm (emergency) value to

$\partial Q_{gtot}/\partial D_{tot}$ independently from the characteristics of the system. Then a comparison between Fig. 4 and Fig. 10 of [1] is misleading.

For what concerns the eigen value analysis performed for the detection of the critical areas in the Italian EHV system, the incremental active and reactive injections have been expressed as functions of $\Delta\vartheta, \Delta V$ by the relation

$$\begin{bmatrix} \Delta F \\ \Delta G \end{bmatrix} = \begin{bmatrix} F_\theta & F_V \\ G_\theta & G_V \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}. \quad \text{The eigenvalues presented in}$$

[1] are pertinent to the reactive jacobian matrix G_v . This matrix is a rougher approximation of the Q,V relations but its symmetry characteristics are undoubtedly more apparent

than the ones of the matrix $J_R = (G_v - G_\theta F_\theta^{-1} F_V)$ defined by J. Carpentier [1c] and adopted in [8], [11]. The new updated version of the procedure COLLAS allows us to obtain the eigen (singular) values (vectors) of G_v , J_R and of the complete jacobian J of the LF equations. In the tests performed for the Italian EHV system the dominant eigen values of J_R and of J change in sign at the collapse point

where the dominant eigen value of G_v is near to the value 4. (see Fig. 7 in [1]). These results are in agreement with the ones of reference [11]. The analysis of the participation factors of the 150 kV busses of Sicily network to the dominant eigenvalue of matrix J shows the important contribution given by the angle variation $\Delta\vartheta$ of some nodes. Beside the security enhancement module has been implemented. Singular value [2c] sensitivities to the load variations and singular vectors are utilized for the definition of preventive (corrective) control actions in alarm (emergency) situations.

Finally, matrix G_v has certainly no multiple eigenvalues, as required for a safe application [3c] of the formula (1) of the discussion. Several eigenvalues have been calculated for matrices J_R and J and no repeated values have been detected. As suggested by our experience, the heuristic sensibleness combined with the theoretical knowledges allow the development of practical automatic procedure for the analysis and the control of the large and complex power systems.

Then only a strong co-operation between University, Institution Researchers and Utility Engineers can be successful.

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CIRCUIT-BASED PERSPECTIVES ON VOLTAGE STABILITY

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To be presented at:

*International Seminar on Bulk Power System Voltage Phenomena - III:
Voltage Stability, Security, and Control.*

Interlaken, Switzerland

23-27 August 1994

ABSTRACT

In recent years, a number of works have illustrated the promise of "energy-like" Lyapunov functions for characterizing vulnerability to voltage stability problems in electric power systems. However, similar to the use of such functions in transient stability characterization, the underlying system model and Lyapunov function have been unable to exactly represent a number of common effects and devices, with the most important of these being resistive losses and induction motor loads. Using a circuit-based approach to system modeling, this work will offer an improvement on these methods by proposing a true energy-based Lyapunov function for the time domain behavior (*not* sinusoidal quasi-steady state) of rotating machines and network elements. This function and its associated model will handle resistive losses and induction motor loads in a straightforward fashion. Moreover, standard textbook results imply that at equilibrium, the time-domain, three-phase circuit quantities can be calculated from a per-phase sinusoidal steady state model of much reduced dimension. Therefore, characterizing voltage stability margins as the difference in energy between a stable operating point, and that at an unstable equilibrium, requires much less computation than would be suggested by the dimension of the underlying circuit-based state space model.

I. INTRODUCTION AND MOTIVATION

Given the nature of electric power grids as interconnections of electro-mechanical energy conversion devices with the circuit elements composing the transmission network, it is natural to suppose the exchange of energy between various storage elements is the key physical phenomena underlying the dynamics of the network. Indeed, several classic texts in the field of electro-mechanical energy conversion derive the dynamic equations for individual machines using the Lagrangian/Hamiltonian formalisms, in which energy storage terms play a key role (see, for example, [1]). By contrast, most studies of large scale electric power networks begin by

adopting sinusoidal quasi-steady state approximations, thereby ignoring the dynamics of network circuit elements (i.e. only the algebraic constraints of the power flow remain), and use dynamic models for synchronous machines that employ the terminal voltage phasor and complex power as the input-output pair to characterize interaction between the machine and network. In such a formulation, exact characterization of energy storage has generally been of less use. Indeed, in the extensive literature on "energy-based" methods of stability analysis, there is little effort to relate the energy-like Lyapunov function to true energy storage. Instead, when these works have motivated the candidate Lyapunov function by energy considerations, they typically examine energy storage in an analogous mechanical system that possesses the same form of dynamic equations as the sinusoidal quasi-steady state model of the power system. The reader is referred to [2], [3] for summaries of these approaches.

In recent years, several works [4], [5], [6] have proposed using "energy-like" Lyapunov functions to characterize the security of power system operating points with respect to "voltage instability" and "voltage collapse." Given that the observed phenomena in systems experiencing voltage problems typically does not involve significant deviations in synchronous machine rotational velocities, this suggests that the underlying stability phenomena does not involve significant energy transfer between mechanical kinetic energy of generators and energy stored in the electric network (primarily magnetic energy storage in the inductance of transmission lines). Indeed, if one examines the use of "energy-functions" for voltage stability analysis, it becomes apparent that one of the key terms used to define the energy margin is a difference between total reactive power absorbed by the network at a stable equilibrium, versus that at an unstable equilibrium. Recalling that reactive power absorbed by a linear RLC circuit is, in sinusoidal steady state, proportional to the difference between magnetic energy storage in inductors, and electric energy storage in capacitors, this suggests that these terms of the energy-like Lyapunov function do have a close relation to

physical energy storage. However, the use of the sinusoidal quasi-steady state approximation makes it difficult to interpret these terms under non-sinusoidal conditions.

Motivated by the above observations, this work will propose a Lyapunov function candidate based on physical energy storage in a time-domain, circuit-based model for an electric power network. The key contribution of this paper can be simply stated as follows. Given models of rotating machines (synchronous generators or induction motors) as multi-terminal inductors dependent on shaft position, and using linear, lumped-element models for the transmission network, this paper will show that the resulting state space model takes the form:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\nabla\vartheta(\mathbf{x}). \quad (1)$$

where

$$\mathbf{x} \in \mathbb{R}^n, \mathbf{A}: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}, \vartheta: \mathbb{R}^n \rightarrow \mathbb{R}.$$

The function $\vartheta(\mathbf{x})$ will be composed of a sum of terms that are obviously identifiable as energy stored in the magnetic fields of inductive elements, and in the electric fields of capacitive elements, plus a term relating to mechanical energy input from the prime mover. Moreover, the matrix $\mathbf{A}(\mathbf{x})$ will have the property that its symmetric part is constant and negative semi-definite. As will be illustrated in the development to follow, resistive and conductive terms appear as negative definite diagonal blocks of $\mathbf{A}$ in this formulation, and contribute to energy dissipation. Therefore, as suggested by physical energy considerations, the function $\vartheta(\mathbf{x})$ will be non-increasing along trajectories. Moreover, the structure of $\mathbf{A}(\mathbf{x})$ and $\nabla\vartheta(\mathbf{x})$ will be shown to have the property that trajectories away from equilibria must cross the surface where $\vartheta(\mathbf{x}) = 0$ transversally; i.e. a dynamic trajectory will only display isolated points where $\vartheta(\mathbf{x})$ is zero. This is in marked contrast to sinusoidal quasi-steady state formulations of "energy" functions, in which the addition of resistive terms can cause the function to *increase* along portions of trajectories.

The capability of accurately representing resistive losses may, on its surface, appear relatively unimportant for voltage stability studies. However, one should note that a key quantity in an energy based security measure is the difference in energy between the operating point of interest, and an unstable equilibrium having depressed voltages at certain buses. Moreover, even the stable operating point may display unusually depressed voltages under conditions associated with voltage stability problems. These depressed voltages contribute to large current flows, making accurate representation of resistive losses important. More generally, the underlying premise of this work is that the system model and candidate Lyapunov function to be proposed will be better suited to stability studies of voltage phenomena, where one expects the underlying phenomena to relate to exchange of energy between various magnetic and electric storage

mechanisms. In particular, this formulation will identify distinct terms for capacitive energy storage versus inductive energy storage, and we may therefore hope to characterize instabilities that exchange energy between such storage mechanisms. Sinusoidal quasi-steady state models invariably rely on Y-bus formulations, in which inductive and capacitive terms show up as complex admittances of opposite sign, and cancel one another. Such a Y-bus based representation offers no mechanism to isolate the energy storage associated with each type of element. This limitation may prove even more problematic as various types of thyristor controlled series elements are added to the network. These designs explicitly depend on inductive and capacitive energy exchange to achieve a steady state characteristic that approximates a variable impedance.

As the reader will note in the derivations to follow, the use of a full circuit based model for rotating machines (i.e., treatment of rotating machines as multi-terminal inductors with shaft position dependent inductance) yields a state space model of very large dimension. Indeed, this is precisely the type of machine model employed in the well-known EMTP stability package [7]. It is generally accepted that this type of model (and the EMTP package) provides very accurate predictions of physical system behavior over a much wider range of time-scales than can be accommodated by sinusoidal quasi-steady state models. However, it is also widely acknowledged that the large state dimension of such models makes their use for systems of hundreds or thousands of buses intractable. However, standard textbook results [8] ensure that for suitable formulations of the models, a per phase sinusoidal steady state model and the full three-phase circuit-based model must produce consistent predictions of equilibria in balanced networks. Therefore, in the type of application that has been advocated for voltage security characterization, in which one seeks differences in the value of energy between two equilibria under *unfaulted* conditions (hence no imbalance), calculation of the equilibrium can be carried out in the much lower dimensional per-phase sinusoidal steady state model. This perspective may also aid in interpretation of the physical meaning of the existing "energy-like" functions in the literature.

II. REVIEW OF ROTATING MACHINE AND LUMPED TRANSMISSION LINE MODELS

This section will briefly review formulation of state equations for rotating machines through a shaft position dependent inductance matrix. The notation in the development to follow will mimic that of [8]. However, unlike [8], the dynamics will be derived using the Lagrangian/Hamiltonian formalisms, as per [1], to help emphasize the tie between energy storage and the structure of the differential equations governing the machine.

Many types of rotating machines can be modeled in this general fashion, with modifications only to the nature of the inductance matrix. The key assumption necessary for the approach here is that the machine be, in the terminology of [9], "Park-Transformable." This same assumption is reflected in the modeling used in the EMTF package, where a "universal machine model" [7] is offered to represent various types of large induction motors satisfying this assumption. This paper will not attempt to explicitly illustrate the terms of inductance matrices associated with the rotating machines to be modeled, though the two machine example to follow will, through the presence of a field winding, implicitly assume a synchronous machine. Given the availability of circuit-based, Park-transformable inductance matrix representations for induction motors, the extension to these types of machines is straightforward.

To limit the dimension of the illustrative two machine example to follow in Section III, we will consider a characteristic machine as having four electrical ports: the a-b-c (to neutral) terminals, and the single port of the field winding. Damper windings will be neglected. From a circuit perspective, electrical interactions at these ports can be characterized by a voltage/current pair, though flux associated with the machine's coils will also prove a key quantity in the derivation to follow. Note that, as in [8], the passive sign convention is *not* adopted for the electrical ports; rather a generator convention is used, yielding a positive v_i product when power is delivered from the port to the "outside world." Mechanical interactions are characterized by an additional port, with applied shaft torque and shaft position used to characterize this interaction. Therefore, the interaction of a given synchronous machine with the remainder of the system may be characterized by a five-port model, as illustrated in Figure 1.

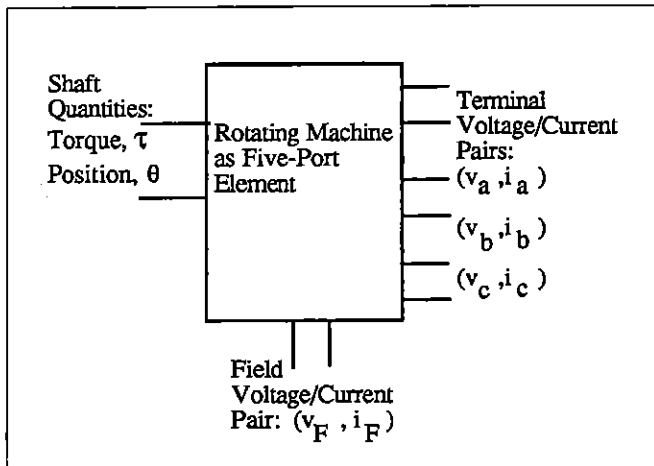


FIGURE 1: Rotating Machine as Five Port Element

Following the notation of [8], with damper windings and winding resistance neglected, let the port voltages and currents be given by:

$$\mathbf{v} = \begin{bmatrix} v_{aa'} \\ v_{bb'} \\ v_{cc'} \\ -v_F \end{bmatrix}; \quad \mathbf{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \end{bmatrix}.$$

Associated with the current in the machine coils is the flux, determined by:

$$\lambda = \mathbf{L}(\theta)\mathbf{i},$$

and charge:

$$\frac{dq}{dt} = \mathbf{i}.$$

In the Lagrangian and Hamiltonian derivations of the state equations to follow, charge will be treated as the fundamental electrical "position" variable, while port voltages will be viewed as applied forces. For the mechanical states, associated with the shaft position of the machine is its rotational velocity:

$$\frac{d\theta}{dt} = \omega.$$

Therefore, in the Lagrangian formalism, one has a composite "position" vector $\bar{\mathbf{q}} := [\theta, \mathbf{q}^T]^T$, and an associated composite "velocity" vector $\dot{\bar{\mathbf{q}}} := [\omega, \mathbf{i}^T]^T$. For the single machine, the stored energy is purely kinetic, being independently quadratic in ω and in $\mathbf{i}$, yielding a Lagrangian $\mathcal{L}$ dependent on parameters of machine inertia, J , and inductance matrix $\mathbf{L}(\theta)$:

$$\mathcal{L}(\dot{\bar{\mathbf{q}}}, \bar{\mathbf{q}}) = \frac{1}{2}J\omega^2 + \frac{1}{2}\mathbf{i}^T\mathbf{L}(\theta)\mathbf{i}.$$

Clearly, the electromechanical interactions arise due to the mechanical position dependence of the inductance matrix. Treating mechanical torque, τ , and the negative of port voltages, $-v$, as applied forces (voltage sign reversal due to the generator sign convention adopted above), the dynamics of the machine must satisfy:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}(\dot{\bar{\mathbf{q}}}, \bar{\mathbf{q}})}{\partial \dot{\bar{\mathbf{q}}}} \right) - \frac{\partial \mathcal{L}(\dot{\bar{\mathbf{q}}}, \bar{\mathbf{q}})}{\partial \bar{\mathbf{q}}} = \begin{bmatrix} \tau \\ -\mathbf{v} \end{bmatrix}.$$

Given that winding resistance and damper windings have been neglected to this point in the development, this is consistent with the state equations as derived in [8]:

$$J\dot{\omega} - \frac{1}{2}\mathbf{i}^T \frac{d\mathbf{L}}{d\theta} \mathbf{i} = \tau$$

$$\mathbf{L}(\theta)\dot{\mathbf{i}} + \omega \frac{d\mathbf{L}}{d\theta} \mathbf{i} = -\mathbf{v}.$$

To motivate a state variable formulation that will include resistive losses, it is useful to first re-express the system dynamics in a Hamiltonian form, neglecting losses and external applied forces. Clearly a system without external forces is of little practical interest, but the structure of the Hamiltonian for the unforced system motivates the choice of states and the general structure that will be used to generate a Lyapunov function. Therefore, starting from the Lagrangian, define the "generalized momenta" [10] variables as:

$$\mathbf{p} = \frac{\partial \mathcal{L}}{\partial \dot{\mathbf{q}}}.$$

For the rotating machine system, it follows that:

$$\mathbf{p} = \begin{bmatrix} J\omega \\ \mathbf{L}(\theta)\mathbf{i} \end{bmatrix} = \begin{bmatrix} p_1 \\ \lambda \end{bmatrix}.$$

The system Hamiltonian is then expressed as a function of position, $\mathbf{q}$, and generalized momenta, $\mathbf{p}$, as [10]:

$$\begin{aligned} H(\mathbf{p}, \mathbf{q}) &:= \dot{\mathbf{q}}^T \mathbf{p} - \mathcal{L}(\mathbf{q}, \dot{\mathbf{q}}) \\ &= \frac{1}{2} \mathbf{J}^{-1} (p_1)^2 + \frac{1}{2} \lambda^T \mathbf{L}^{-1}(\theta) \lambda. \end{aligned}$$

It will be useful to view the Hamiltonian as a function of composite vector of system states:

$$\mathbf{x} := [\mathbf{p}_1, \lambda^T, \theta, \mathbf{q}^T]^T.$$

The state equations for the unforced system then take the form:

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{0} & -\mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \nabla_{\mathbf{x}} H(\mathbf{x});$$

where

$$\nabla_{\mathbf{x}} H = \left[\mathbf{J}^{-1} p_1, \lambda^T \mathbf{L}^{-1}(\theta), -\frac{1}{2} \lambda^T \mathbf{L}^{-1}(\theta) \frac{d\mathbf{L}}{d\theta} \mathbf{L}^{-1}(\theta) \lambda, 0^T \right]^T.$$

To obtain the state equations for the forced, lossy system, consider a new function obtained from the Hamiltonian by the addition of terms representing energy input by the external forces; in particular, define:

$$\vartheta(\mathbf{x}) := H(\mathbf{x}) - \tau \cdot \theta + \mathbf{v}^T \mathbf{q}.$$

It follows that the state equations with losses and forcing terms can then be written in the form:

$$\dot{\mathbf{x}} = \mathbf{A} \nabla \vartheta(\mathbf{x});$$

where

$$\mathbf{A} :=$$

$$\left[\begin{array}{cc|cc} -D & \mathbf{0} & -1 & \mathbf{0} \\ \hline \mathbf{0} & -R & \mathbf{0} & -\mathbf{I} \\ \hline 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{array} \right]$$

Note that the symmetric part of $\mathbf{A}$ is negative semi-definite. It follows that the derivative of $\vartheta(\mathbf{x})$ along trajectories of the state equations, which will be given by:

$$\dot{\vartheta}(\mathbf{x}) = \frac{1}{2} \nabla \vartheta^T(\mathbf{x}) (\mathbf{A} + \mathbf{A}^T) \nabla \vartheta(\mathbf{x}),$$

must be non-positive. This same structure of state equations was previously identified in models based on the sinusoidal quasi-steady state approximation, and was termed a "quasi-gradient" form in [11]. However, for the traditional swing dynamic models, the trajectory corresponding to normal operation is a stationary equilibrium point. For the formulation of the single machine model above, normal steady state will display sinusoidal a-b-c currents, and the shaft position will be linearly increasing with time. Lyapunov analysis to establish stability about this type of trajectory requires additional machinery, and $\vartheta(\mathbf{x})$ would not directly serve as a Lyapunov function candidate. However, in the derivation to follow in the subsequent sections, the use of the Park/Blondel transformation will yield a coordinate system in which the desired steady state is an equilibrium point, and a structure very similar to the quasi-gradient form above will be obtained. The one added complication of the multi-machine case will lie in the fact that $\mathbf{A}$ becomes a function of state. However, its symmetric part will remain a constant matrix.

III. MODELING MULTIMACHINE SYSTEMS WITH A NETWORK WIDE PARK/BLONDEL TRANSFORMATION

In the "textbook" approach to coupling detailed synchronous machine models through a transmission network, one typically uses the Park/Blondel transformed internal variables for a given machine, while network quantities remain in the a-b-c phase coordinates. In this form, the internal state equations of the machine are independent of the shaft position, but this position does appear in the coupling equations that interface internal quantities to network quantities. In such an

approach, the overall dynamic equations depend on each machine's shaft angle, and that therefore each machine's shaft position is kept as a state variable. For contrast, consider the standard approach in transient stability studies using the sinusoidal quasi-steady state approximation. There one often selects a given machine as reference, deletes that angle from the set of states, and maintains in the state vector only the phase angles of the remaining machines *relative to* that reference machine. As described in [2], this choice yields a minimal dimension state space for swing dynamic models, provided the special case of uniform damping is excluded. One consequence of the minimal dimension state space is that the model displays isolated equilibrium points, rather than a manifold of equilibrium positions, as would be the case if all machine angles were maintained in the state vector. The goal of this section will be to illustrate that a similar phenomena occurs when coupling the detail circuit-based machine model with network element dynamics. In particular, this section will show that if all machine and network variables are represented in the 0-d-q frame of a reference machine, that machine's shaft position may be deleted from the set of states, and only relative shaft positions maintained.

The notation used for the Park/Blondel transformation will follow that of [8]. In particular, define:

$$P(\theta) := \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin \theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \end{bmatrix}$$

For our machine model having four windings, it will prove useful to define a 4x4 transformation matrix that preserves the field quantities, represented by:

$$B(\theta) := \begin{bmatrix} P(\theta) & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}. \quad (2)$$

A key observation for the model development to follow is that:

$$P(\theta_1)P(\theta_2)^{-1} = S_r(\theta_1 - \theta_2) = S_r^{-1}(\theta_2 - \theta_1) = S_r^T(\theta_2 - \theta_1);$$

where

$$S_r(\delta) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \delta & -\sin \delta \\ 0 & \sin \delta & \cos \delta \end{bmatrix}.$$

This equivalence is straightforward to verify algebraically, but can be made intuitively clear by the following argument. The transformation $P(\theta_1)P(\theta_2)^{-1}$ takes the 0-d-q quantities of a machine with shaft position θ_2 to the 0-d-q frame of a machine with shaft position θ_1 . The result is a rotation of the d-q quantities of generator 2 by the relative angle difference between the two coordinate frames; that is, rotation by $\theta_1 - \theta_2$, as represented by the transformation $S_r(\theta_1 - \theta_2)$.

To illustrate the form of state equations that result when all quantities are expressed in the 0-d-q frame of a single reference machine, a two machine example will be developed in detail here. Extensions to the multi-machine case are conceptually straightforward. The one line diagram for this example system is shown in Figure 2; note, however, that the equations to be formulated will represent three-phase circuit behavior, including inductive and capacitive coupling between phases. Hence the series inductance L , series resistance, R , shunt capacitances C , and shunt conductances G will all be three port elements, represented by the appropriate 3x3 matrix quantities. While the model to be developed here does not assume that balanced three phase voltages and currents are maintained along all trajectories, we *will* assume that these network matrices are balanced in the following sense. For example, for inductance matrices, all diagonal elements are assumed equal (same self inductance in each phase), and all off-diagonal elements are equal (same mutual inductance between any two phases). Shunt capacitance matrices are assumed to have the same structure. Series and shunt resistance matrices are assumed to be purely diagonal, with equal resistance in the a-b-c elements, and equivalently, in an 0-d-q frame, equal 0-d-q elements. Linear behavior is assumed throughout, and therefore, frequency dependence of line behavior is not represented. While three-phase transformers will not be explicitly considered in this initial two machine example, they can be represented as six-port inductive circuit elements (three primary port quantities, three secondary port quantities), with corresponding 6x6 inductance matrices. Partitioning these matrices according to primary and secondary, each 3x3 block would be assumed to have the structure described above.

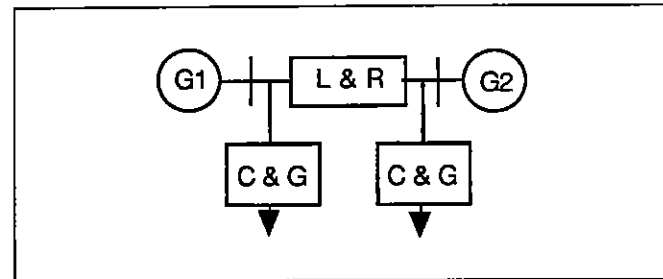


FIGURE 2: One-Line Diagram for Example System

Given the above assumptions, it is useful to illustrate the specific form that these types of balanced inductance and capacitance matrices take under the Park/Blondel transformation. Consider a representative inductance matrix.

The structure described above allows two free parameters: the self-inductance value, L_s , and the coupling inductance value, L_c . Hence, in the a-b-c frame we have:

$$\mathbf{L}_{abc} = \begin{bmatrix} L_s & L_c & L_c \\ L_c & L_s & L_c \\ L_c & L_c & L_s \end{bmatrix}$$

Straightforward algebra confirms that application of the Park/Blondel transformation yields a constant matrix of the form:

$$\mathbf{P}(\theta)\mathbf{L}_{abc}\mathbf{P}^{-1}(\theta) = \begin{bmatrix} L_s+2L_c & 0 & 0 \\ 0 & L_s-L_c & 0 \\ 0 & 0 & L_s-L_c \end{bmatrix} \quad (3)$$

Of particular importance to the derivations to follow is the fact that the (2,2) element and (3,3) element are equal. Clearly, the same structure results when an a-b-c coordinate frame capacitance matrix, satisfying the balanced network assumptions above, is transformed to an 0-d-q frame.

Machine Equations - Reference Generator

A key element of the synchronous machine dynamics relates to the machine's current vs. flux characteristics, which, in the Park's transformable machine models employed here, are given by a constant inductance matrix in the 0-d-q frame. Note that there is an 0-d-q coordinate frame associated with each machine. With these observations in mind, define:

$\mathbf{L}_{B,1}$:= Constant 4x4 inductance matrix of machine 1, in machine 1 0-d-q coordinates;

$$\mathbf{R}_{B,1} = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 \\ 0 & 0 & r_1 & 0 \\ 0 & 0 & 0 & r_F \end{bmatrix}$$

:= Constant 4x4 resistance matrix of machine 1;

J_1 := rotational inertia of machine 1;

D_1 := rotational damping of machine 1;

$\tau_{M,1}$:= mechanical torque on machine 1;

$$\Lambda_r := \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix};$$

$$\Lambda := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Standard results for synchronous machines (see, for example (10.38) in [8]) yield electrical equations of the form:

$$\dot{\lambda}_{B,1} = -v_{B,1} - R_{B,1}L_{B,1}^{-1}\lambda_{B,1} - \omega_1\Lambda\lambda_{B,1}.$$

The machine mechanical equation (as may be found in [8] (10.43)) is:

$$\dot{\omega}_1 = J_1^{-1} \{ -D_1\omega_1 + \tau_{M,1} + \lambda_{B,1}^T L_{B,1}^{-1} \Lambda \lambda_{B,1} \}.$$

For the development here, it will prove convenient to introduce a simple dynamic model for the governor/turbine action that determines the value of applied mechanical torque. To this end, let ω_0 represent the synchronous speed setpoint, and define:

$$x_{M,1} := \frac{1}{\sqrt{KG_1}} (\tau_{M,1} + \omega_0 \frac{KG_1}{RG_1})$$

with

$$\dot{x}_{M,1} = -\sqrt{KG_1} (\omega_1 - \omega_0) - RG_1 x_{M,1}.$$

Here the constant $\sqrt{KG_1}$ determines the gain of the integral control characteristic, while the ratio of the constant RG_1 to $\sqrt{KG_1}$ determines the steady state speed error, or "droop." Note that neither mechanical nor electrical equations display any dependence on shaft position θ_1 .

KCL Equations for Reference Generator Voltages

Dynamics of the 0-d-q voltages to be developed here are a consequence of the shunt capacitance matrix in the three phase π -equivalent. As an analytic convenience, we will also introduce capacitive dynamics as an approximation for a constant applied field voltage. Physically, it is clear that replacing a two-terminal dc voltage source with a large capacitance having the same initial voltage should yield qualitatively similar behavior. In particular, when the voltage source is replaced by a sufficiently large capacitance, the

solution of the new circuit will approximate that of the original circuit on a finite time interval, provided the current supplied by the voltage source remains uniformly bounded for all solutions of interest. However, while this is a convenient approximation for the dynamics, to calculate equilibria, one must return to the constant voltage model. The ultimate goal in this work will be to define the field voltage as a feedback from the terminal voltages; however, that level of detail will not be attempted here. In such a model, the difficulties associated with an external energy source (constant voltage) feeding the field terminals would disappear, as the energy driving the field circuit is actually obtained from within the closed system being modeled; i.e., the excitation power is ultimately supplied either directly from the a-b-c terminals of the machine, for a static exciter, or from a combination of terminal electrical quantities and mechanical torque, in a rotating exciter.

While field voltage capacitor approximation can be made rigorous with singular perturbation techniques, we will not attempt a formal proof here, but will rely on the reader's acceptance of circuit-based intuition. With this large fictitious capacitance attached to the field terminals, let C_1 denote resulting 4x4 capacitance matrix. The upper left 3x3 block of this capacitance matrix is assumed to satisfy the balanced assumptions outlined above, while the fourth row and column have a single non-zero entry on the diagonal (i.e., the capacitance driving the field circuit is decoupled from the shunt capacitance seen at the a-b-c terminals of the machine). Let λ_f denote the 3-vector of a-b-c frame flux in the series element of the line's π -equivalent, with reference direction leaving terminal bus 1, and v_1 denote the 4-vector of terminal and field voltages in a-b-c coordinates; i.e.

$$v_1 := B(\theta_1)^{-1} v_{B,1}.$$

In the a-b-c frame, simple application of KCL then yields:

$$\dot{v}_1 = C_1^{-1} \{ B(\theta_1)^{-1} L_{B,1}^{-1} \lambda_{B,1} - \begin{bmatrix} I \\ 0 \end{bmatrix} L_f^{-1} \lambda_f \}.$$

Using $v_{B,1} := B(\theta_1)v_1$, transforming to generator #1's 0-d-q frame yields:

$$\begin{aligned} \dot{v}_{B,1} &= \omega_1 \frac{dB}{d\theta_1} B(\theta_1)^{-1} v_{B,1} \\ &+ \tilde{C}_1^{-1} \{ L_{B,1}^{-1} \lambda_{B,1} - \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{L}_f^{-1} \lambda_f^1 \}. \end{aligned}$$

which in turn implies:

$$\dot{v}_{B,1} = -\omega_1 \Lambda v_{B,1} + \tilde{C}_1^{-1} \{ L_{B,1}^{-1} \lambda_{B,1} - \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{L}_f^{-1} \lambda_f^1 \} \quad (4)$$

where

$$\tilde{C}_1 := B(\theta_1) C_1 B(\theta_1)^{-1};$$

$$\tilde{L}_f := B(\theta_1) L_f B(\theta_1)^{-1};$$

$$\lambda_f^1 := P(\theta_1)^{-1} \lambda_f,$$

and we have used the fact that

$$\frac{dB}{d\theta_1} B^{-1}(\theta_1) = -\Lambda = \Lambda^T.$$

Note that previously stated assumptions on the structure of network capacitance and inductance matrices' structure imply that $\tilde{C}_1$ and $\tilde{L}_f$ will be constant matrices, with structure as given in (3).

KVL Equation Across Line Series Inductance

Note that starting from the network a-b-c coordinate frame, and letting λ_f denote the line flux in this coordinate system, KVL and the inductance characteristic yield:

$$\dot{\lambda}_f = \{ P(\theta_1)^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} v_{B,1} - P(\theta_2)^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} v_{B,2} \}$$

Converting to the 0-d-q coordinate frame of generator #1:

$$\begin{aligned} \dot{\lambda}_f^1 &= \omega_1 \frac{dP}{d\theta} P(\theta_1)^{-1} \lambda_f^1 \\ &+ P(\theta_1) \{ P(\theta_1)^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} v_{B,1} - P(\theta_2)^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} v_{B,2} \} \\ &= -\omega_1 \Lambda \lambda_f^1 + \{ \begin{bmatrix} I \\ 0 \end{bmatrix} v_{B,1} - \begin{bmatrix} I \\ 0 \end{bmatrix} v_{B,2}^1 \}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} v_{B,2}^1 &:= B(\theta_1) B(\theta_2)^{-1} v_{B,2} \\ &= S^{-1}(\alpha) v_{B,2} \end{aligned}$$

(gen #2 voltage vector in gen #1 0-d-q frame);

and

$$\alpha := \theta_2 - \theta_1$$

$$S(\alpha) := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The reader may also find it helpful to recall the simple relation between $P(\theta)$ and $B(\theta)$ transformations, as given in (2).

Machine Equations for Non-Reference Generator

The equations governing the internal behavior of the non-reference machine are, in principle, identical to in structure to those for the reference machine. However, our goal is to express these using voltages and fluxes in the reference machine's 0-d-q frame. We begin with the standard relation expressed in machine 2' 0-d-q frame:

$$\dot{\lambda}_{B,2} = -v_{B,2} - R_{B,2} L_{B,2}^{-1} \lambda_{B,2} - \omega_2 \Lambda \lambda_{B,2}$$

Analogous to the notation introduced above to represent generator 2's voltage vector in generator 1's 0-d-q frame, define:

$$\begin{aligned} \lambda_{B,2}^1 &:= B(\theta_1) B(\theta_2)^{-1} \lambda_{B,2} \\ &= S^{-1}(\alpha) \lambda_{B,2} \\ &\text{(gen \#2 flux vector in gen \#1 0-d-q frame);} \end{aligned}$$

and

$$\lambda_{B,2}^1 = \tilde{L}_{B,2}(\alpha) \lambda_{B,2}$$

with

$$\tilde{L}_{B,2}(\alpha) := S^{-1}(\alpha) L_{B,2} S(\alpha)$$

Then

$$\begin{aligned} \dot{\lambda}_{B,2}^1 &= \dot{\alpha} \frac{d\{S^{-1}\}}{d\alpha} S(\alpha) \lambda_{B,2}^1 \\ &\quad + \{-v_{B,2}^1 - R_{B,2} \tilde{L}_{B,2}^{-1}(\alpha) \lambda_{B,2}^1 - \omega_2 \Lambda \lambda_{B,2}^1\} \\ &= (\omega_2 - \omega_1) \Lambda \lambda_{B,2}^1 \\ &\quad + \{-v_{B,2}^1 - R_{B,2} \tilde{L}_{B,2}^{-1}(\alpha) \lambda_{B,2}^1 - \omega_2 \Lambda \lambda_{B,2}^1\} \\ &= \{-v_{B,2}^1 - R_{B,2} \tilde{L}_{B,2}^{-1}(\alpha) \lambda_{B,2}^1 - \omega_1 \Lambda \lambda_{B,2}^1\} \end{aligned} \quad (6)$$

Key observations used in arriving at this result are:

$$\frac{d\{S^{-1}\}}{d\alpha} S(\alpha) = \Lambda;$$

$$S^{-1}(\alpha) R_{B,2} S(\alpha) = R_{B,2};$$

and

$$S^{-1}(\alpha) \Lambda = \Lambda S^{-1}(\alpha).$$

The reader should note that in the machine 1 coordinate frame, the "speed voltage" term for machine 2 is proportional to the rotational speed of generator 1.

The mechanical equation keeps the same form as that of generator 1, even with the change of coordinates to generator 1's 0-d-q frame. The generator acceleration equation takes the form:

$$\dot{\omega}_2 = J_2^{-1} \{ -D_2 \omega_2 + \tau_{M,2} + (\lambda_{B,2}^1)^T \tilde{L}_{B,2}^{-1}(\alpha) \Lambda \lambda_{B,2}^1 \}.$$

The governor equations determining torque are:

$$x_{M,2} := \frac{1}{\sqrt{KG_2}} (\tau_{M,2} + \omega_0 \frac{KG_2}{RG_2})$$

with

$$\dot{x}_{M,2} := -\sqrt{KG_2} (\omega_2 - \omega_0) - RG_2 x_{M,2}.$$

KCL Equations for Non-Reference Generator Voltages

As with the reference generator, let v_2 denote the voltage vector in a-b-c coordinates; i.e.

$$v_2 := B(\theta_2)^{-1} v_{B,2}.$$

Again, KCL in this coordinate frame yields:

$$\dot{v}_2 = C_2^{-1} \{ B(\theta_2)^{-1} L_{B,2}^{-1}(\theta_2) \lambda_{B,2} + \begin{bmatrix} I \\ 0 \end{bmatrix} L_f^{-1} \lambda_f \}.$$

(note change in sign on the i_f term because the line current has reference direction entering this bus).

Transforming to generator #1's 0-d-q frame :

$$\begin{aligned} \dot{v}_{B,2}^1 &= \frac{d}{dt} \{ B(\theta_1) v_2 \} = \omega_1 \frac{B}{d\theta_1} v_2 + B(\theta_1) \dot{v}_2 \\ &= \omega_1 \frac{B}{d\theta_1} B(\theta_1)^{-1} v_{B,2}^1 + B(\theta_1) \dot{v}_2 \\ &= -\omega_1 \Lambda v_{B,2}^1 + B(\theta_1) C_2^{-1} \{ B(\theta_2)^{-1} L_{B,2}^{-1}(\theta_2) \lambda_{B,2} \\ &\quad + \begin{bmatrix} I \\ 0 \end{bmatrix} L_f^{-1} \lambda_f \} \\ &= -\omega_1 \Lambda v_{B,2}^1 + \tilde{C}_2^{-1} \{ \tilde{L}_{B,2}^{-1}(\alpha) \lambda_{B,2}^1 + \begin{bmatrix} I \\ 0 \end{bmatrix} \tilde{L}_f^{-1} \lambda_f^1 \} \end{aligned} \quad (7)$$

where

$$\tilde{C}_2 := B(\theta_1) C_2 B(\theta_1)^{-1}.$$

IV. STRUCTURE OF COMPOSITE SYSTEM STATE EQUATIONS

As illustrated in (1), the structure we seek is a right hand side composed as a product of a state dependent matrix and the gradient of scalar function of state. The scalar function will become our energy-based candidate Lyapunov function. To illustrate the specifics of this construction in a context where derivations are easily verified by the reader, this section will continue with the two machine example outlined above, indicating the general structure necessary for an m-machine system wherever possible.

For notational convenience, it will be useful to index the components of α starting from 2, so that each non-reference machine k has an associated α_k , whose index matches that of the machine.

$$x := [\alpha^T \quad \omega^T x_M^T \quad \lambda_B^T \quad v_B^T \quad \lambda_f^T]^T$$

where the currents and voltages associated with each machine are grouped together as the vectors λ_B and v_B , and, for the

multiple line case, v line currents are grouped together as a single vector i_f . Next, define a scalar function of state of the form:

$$\begin{aligned} \vartheta(x) &:= \\ &\frac{1}{2} \omega^T J \omega + \frac{1}{2} \sum_{k=1}^m \lambda_{B,k}^T L_{B,k}^{-1}(\alpha_k) \lambda_{B,k} \\ &\quad + \sum_{k=1}^v \lambda_{f,k}^T \tilde{L}_{f,k}^{-1} \lambda_{f,k} + \frac{1}{2} \sum_{k=1}^m v_{B,k}^T \tilde{C}_k v_{B,k} \\ &\quad + \sum_{k=1}^m \left(\frac{1}{2} x_{M,k}^2 - \omega_0 \frac{\sqrt{KG_k}}{RG_k} x_{M,k} \right) \end{aligned}$$

The quadratic terms relating to currents, voltages and rotational speeds are true energy storage in magnetic and electric fields, along with kinetic energy stored by the mechanical rotation of the machine, all expressed in the 0-d-q coordinate system of the reference machine. The superscript 1, previously used to emphasize this choice of coordinates, has been dropped here, where *all* quantities are expressed in machine 1's reference frame. Unlike the single machine formulation of Section II, where the Hamiltonian formalism was strictly followed, and generalized mechanical momenta was used as a state, here we have maintained rotational speed as a component of the state vector. The choice is acceptable because the mechanical inertia matrix is constant. For the electrical equations, the generalized momenta (flux) is a more convenient choice as a state, because the effective "inertia" (inductance) is dependent on position.

Given the relatively simple linear or quadratic dependence of $\vartheta(x)$ on most its arguments, most of the partial derivatives that compose its gradient are obvious. However, the form of the partial derivative with respect to α bears examination. In particular, at the non-reference machines with indices 2 through m, the structure of $L_{B,k}(\alpha_k)$ implies

$$\begin{aligned} \frac{d \left\{ \frac{1}{2} \lambda_{B,k}^T \tilde{L}_{B,k}^{-1}(\alpha) \lambda_{B,k}^1 \right\}}{d\alpha_k} &= -\lambda_{B,k}^1{}^T \Lambda^T \tilde{L}_{B,k}^{-1}(\alpha) \lambda_{B,k}^1 \\ &= -\lambda_{B,2}^1{}^T \tilde{L}_{B,2}^{-1}(\alpha) \Lambda \lambda_{B,2}^1, \end{aligned}$$

which is precisely the structure of the electrical torque term appearing in the generator acceleration equation, but with opposite sign.

With the above relations and definitions established, direct calculation will confirm that the system state equations can be re-expressed in the form (1), i.e.:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\nabla\vartheta(\mathbf{x}),$$

with specific form of $\mathbf{A}(\mathbf{x})$ for the two machine example illustrated in Appendix A.

The structure of $\mathbf{A}(\mathbf{x})$ as given in Appendix A makes it obvious that its symmetric part will be a constant diagonal matrix, with all diagonal elements negative or zero. The zero entries along the diagonal appear only in the block associated with the α state. With this structure, the following observations lead to the conclusion that trajectories not at equilibrium must cross the surface where $\vartheta(\mathbf{x}) = 0$ transversally.

Observation 1:

$\nabla\vartheta(\mathbf{x}^e) = \mathbf{0}$ implies that $\mathbf{x}^e$ is an equilibrium point

$\Leftrightarrow$

$\hat{\mathbf{x}}$ not an equilibrium point implies $\nabla\vartheta(\hat{\mathbf{x}}) \neq \mathbf{0}$.

Observation 2:

$\hat{\mathbf{x}}$ not an equilibrium point (hence $\nabla\vartheta(\hat{\mathbf{x}}) \neq \mathbf{0}$) and $\vartheta(\hat{\mathbf{x}}) = 0$ implies that:

$$\frac{\partial\vartheta}{\partial\alpha}(\hat{\mathbf{x}}) \neq 0; \omega_2 = 0; \text{ and } \frac{\partial\vartheta}{\partial x_{M,2}}(\hat{\mathbf{x}}) = 0.$$

Observation 3:

$$\frac{\partial\vartheta}{\partial\alpha}(\hat{\mathbf{x}}) \neq 0; \omega_2 = 0; \text{ and } \frac{\partial\vartheta}{\partial x_{M,2}}(\hat{\mathbf{x}}) = 0 \text{ implies that for}$$

machine 2, electrical torque is nonzero, while damping torque and mechanical torque are zero. Hence, $\dot{\omega}_2 \neq 0$, so the vector field has a non-zero projection onto the normal of the $\vartheta(\mathbf{x}) = 0$ manifold at the point $\hat{\mathbf{x}}$, i.e., the trajectory must immediately exit the $\vartheta(\mathbf{x}) = 0$ surface.

Observation 4:

Let $\mathbf{x}(t)$ be any non-equilibrium trajectory. For any $\varepsilon > 0$, let t_2, t_1 be chosen such that $t_2 > t_1 + \varepsilon$. Based on Observation 3, along a non-equilibrium trajectory $\vartheta(\mathbf{x}(t))$ can equal zero only on a set of time points having measure zero. It follows immediately that $\vartheta(\mathbf{x}(t_2)) < \vartheta(\mathbf{x}(t_1))$.

This result has another key consequence for use of $\vartheta(\mathbf{x})$ as a Lyapunov function, as shown in the proposition below.

Proposition 1:

Let $\mathbf{x}^e$ be an equilibrium point known to have a strictly stable linearization; this point is therefore locally asymptotically stable with respect to the system (1). It follows that $\vartheta(\mathbf{x})$ is locally positive definite about such a point.

Proof:

By contradiction: suppose $\vartheta(\mathbf{x})$ is not locally positive definite about $\mathbf{x}^e$. Therefore, within any neighborhood of $\mathbf{x}^e$ there must exist an $\hat{\mathbf{x}} \neq \mathbf{x}^e$ with the property that $\vartheta(\mathbf{x}^e) \geq \vartheta(\hat{\mathbf{x}})$. Moreover, for a sufficiently small neighborhood, a trajectory initiated at $\hat{\mathbf{x}}$ must asymptotically approach $\mathbf{x}^e$. Hence, along this trajectory, $\vartheta(\mathbf{x}(t))$ must either increase for a portion of the trajectory, or remain constant for all $t \geq 0$. Either conclusion results in a contradiction, based on Observation 4.

As a result of the observations above, and Proposition 1, we have established that $\vartheta(\mathbf{x}(t))$ is non-increasing along trajectories, and $\vartheta(\mathbf{x})$ is locally positive definite in the neighborhood of an equilibrium point having a strictly stable linearization. This suggests that $\vartheta(\mathbf{x})$ is well suited for estimation of subsets of the region of attraction for such a known stable equilibrium, as is typically done in applications of energy based Lyapunov functions for power system security analysis. Future investigations will seek to apply this function in benchmark voltage stability cases.

V. CONCLUSIONS

In this paper we have established an energy based Lyapunov function candidate that is non-increasing along system trajectories, and locally positive definite about asymptotically stable equilibria. The underlying model on which this Lyapunov function is based accommodates exact representation of resistive losses, as well as circuit based models for induction motor loads. This suggests that this Lyapunov function may be particularly well suited to voltage stability applications.

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Appendix A

$A(x) :=$

0	$-J_1^{-1}$	J_2^{-1}	0	0	0	0	0	0	0
J_1^{-1}	$-J_1^{-1}D_1J_1^{-1}$	0	$J_1^{-1}\sqrt{KG_1}$	0	$J_1^{-1}\lambda_{B,1}^T\Lambda^T$	$J_1^{-1}\lambda_{B,2}^T\Lambda^T$	0	0	0
$-J_2^{-1}$	0	$-J_2^{-1}D_2J_2^{-1}$	0	$J_2^{-1}\sqrt{KG_2}$	0	0	0	0	0
0	$-\sqrt{KG_1}J_1^{-1}$	0	$-RG_1$		0		0	0	0
0		$-\sqrt{KG_2}J_2^{-1}$		$-RG_2$	0	0	0	0	0
0	$-\Lambda\lambda_{B,1}J_1^{-1}$	0	0	0	$-R_{B,1}$	0	$-\tilde{C}_{B,1}^{-1}$	0	0
0	$-\Lambda\lambda_{B,2}J_1^{-1}$	0	0	0		$-R_{B,2}$	0	$-\tilde{L}_{B,2}^{-1}(\alpha)\tilde{C}_{B,2}^{-1}$	0
0	0	0	0	0	$\tilde{C}_{B,1}^{-1}$	0	$-\omega_1c_{B,1}^{-1}\Lambda$ $-\tilde{C}_{B,1}^{-1}G_{B,1}\tilde{C}_{B,1}^{-1}$	0	$-\tilde{C}_{B,1}^{-1}\begin{bmatrix} I \\ 0 \end{bmatrix}$
0	0	0	0	0	0	$\tilde{C}_{B,2}^{-1}\tilde{L}_{B,2}^{-1}(\alpha)$	0	$-\omega_1c_{B,2}^{-1}\Lambda$ $-\tilde{C}_{B,2}^{-1}G_{B,2}\tilde{C}_{B,2}^{-1}$	$\tilde{C}_{B,2}^{-1}\begin{bmatrix} I \\ 0 \end{bmatrix}$
0	0	0	0	0	0	0	$[I\ 0]\tilde{C}_{B,1}^{-1}$	$-\begin{bmatrix} I \\ 0 \end{bmatrix}\tilde{C}_{B,2}^{-1}$	$-\omega_1l_f^{-1}\Lambda_r$ $-\tilde{L}_f^{-1}R_f\tilde{L}_f^{-1}$

where

$$l_f^{-1}\Lambda_r = \tilde{L}_f^{-1}\Lambda_r,$$

with $l_f := \tilde{L}_{s,f} - \tilde{L}_{c,f}$ (self-inductance less coupling inductance, as per (3)),

and

$$c_{B,2}\Lambda_r = \tilde{C}_{B,2}\Lambda_r,$$

with $c_{B,2} := \tilde{C}_{s,B,2} - \tilde{C}_{c,B,2}$ (self-capacitance less coupling capacitance, as per (3)).

PV-Curve in Unbalanced Transmission Lines

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Abstract: Modern power system networks are comprised of long untransposed transmission lines. This unbalanced networks cause various phenomena which can not be analyzed by conventional tools based on one line diagram. For some kinds of analysis including voltage instability, it is now almost impossible to treat a system as though it were a symmetrical network. In this paper, the practical unbalanced load flow program (UNBPF) developed for large power systems and PV-Curve analysis with it are reported. PV-Curve for each three phases and critical loading condition obtained by UNBPF are unusual compare with traditional approach, if load is directly connected to untransposed lines. If zero-sequence networks of load and line are not separated by delta-winding transformers, untransposed systems will reach to a critical point faster than transposed systems.

Keywords: Unbalanced Network, Multi-Phase Load flow, Voltage Stability, PV-Curve, Critical Load

1. Introduction

In the past, unbalanced voltage/current was a problem for local networks, since local unbalanced loads and unusual transformer connections can cause negative/zero sequence current. A network itself can be treated as if it is balanced since most transmission lines are transposed. Modern EHV/UHV power system networks, however, consist of long untransposed transmission lines. It is no longer valid to treat networks as though they were completely balanced for some kinds of power system analysis. As network imbalance deteriorates, zero-sequence current may cause transformer delta windings to overheat; and negative-sequence current may cause generators to overheat. The influence to the voltage instability has not been analyzed yet. It is important to analyze these phenomena in advance.

It is also worthy of note that most transmission lines in Japan are double circuit lines, since they save space of right-of-way with large capacity and they have some advantages such as high reliability, flexible operation and easy maintenance. During a scheduled outage for maintenance, only one circuit need be stopped while the other circuit remains active. The circuit imbalance, however, become worst during this time.

The idea of multi-phase/unbalanced load flow calculation was proposed many years ago. Laughton[1][2] introduced multi-phase coordinates to replace symmetrical components and derived a representation of power system elements in phase-coordinates. Birt, et al [3]; Wasley,

etal [4] and Arrillaga, et al [5] developed a three phase load flow program for small systems. A representation of unbalanced systems was proposed by Wortman, et al [6]. Recently, Xu et al [7] proposed a multiple harmonic load flow technique. Unbalanced load flow analysis is essential for accurate harmonic analysis, since each phase have individual resonant frequency.

Although the basic methodology and models for the unbalanced load flow calculation have been established by previous research, the following important issues and difficulties still remain for practical analysis:

- Previous work have been successfully applied to some small systems. However, no experience with practical large-scale systems has been reported.
- The multi-phase analysis requires more detailed and specific data. A double circuit line needs independent 36 items of data (individual sets of three 6x6 symmetric matrices). Intensive data preparation and construction of database are essential.

The EMTP might be applied to solve the problem[13]. However, it has following disadvantages;

- EMTP's insufficient ability of load flow calculation forces users to set voltage source angle manually, using other supporting program.
- Generator's negative-sequence impedance is not considered in phasor solution. This impedance is essential for calculation of generator's negative sequence current.
- Data file setting is exhausting. No matter how excellent this program may be, however, nobody is willing to prepare enormous data file without special supporting routines.

Concerning to the voltage problems, various indices are proposed by many researchers. The attempt to detect the critical loading condition was carried by Alvalado [8], Semlyen[9], Dobson[10] and Iba[11][12]. Among many indices, PV-Curve or nose curve is preferred by the power system planners and/or operators, since it is intuitive. Although we have some tools to get PV curve and the critical loading point now, they are assuming that all transmission lines are symmetrical.

This paper reports the newly developed unbalanced load flow program (UNBPF) and reports on the results of PV-Curve analysis using this program. Section 2 provides an overview of the program and its test results applied to TEPCO's 192 bus system. Section 3 describes the each phase voltage behavior in PV-Curve observed in a simple untransposed transmission line. Section 4 summarized the conclusions of the study.

2. Unbalanced Load Flow Program

2.1 Modeling and Solution

Three phase unbalanced load flow is similar to conventional single phase load flow. Since there is no advantage in using symmetrical components, phase coordinates, i.e., actual phase quantities in per unit are used. Most nodal quantities becomes three element vectors. Each line admittance is represented by a three by three or a six by six matrix. Since modeling of network components has been studied and summarized in previous works, only key points are described here.

• Transmission Lines and Transformers

Each unbalanced transmission line was treated as a pi-equivalent circuit and each line constants were calculated by EMTP. Fig.1 shows sketch of towers and an equivalent circuit. All line constant's data on TEPCO's untransposed overhead lines were calculated by EMTP and stored to a database. Users of the unbalanced load flow program merely specify the name of transmission lines, instead of inputting all the elements contained in the admittance matrix. Any kind of transformer's connection, e.g. YY, YΔ, ΔΔ, YYΔ and YΔΔ can be used. An off-nominal tap ratio can be handled for each phase individually. An unbalanced tap scheme might be a counter-measure to improve system imbalance. The program at present does not handle ungrounded neutral nodes.

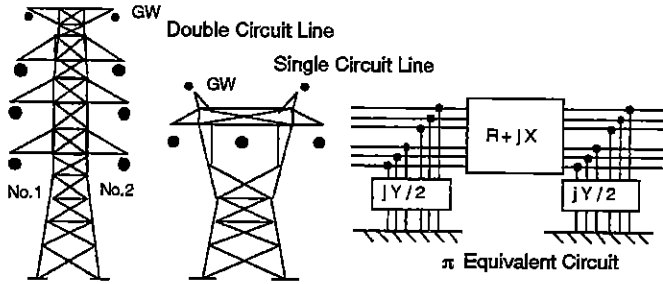


Figure 1: Tower and Equivalent Circuit

• Generators

Since generators are sensitive to unbalanced current, modeling is important. The model described here is based on Reference [3]. An internal node in which voltage is completely balanced is introduced to allow unbalanced currents and voltages at generator terminals as shown in Fig. 2. It should be noted that the impedance matrix X_g for internal reactance should be carefully selected. If the values of X_1 and X_2 are identical, the matrix X_g will be symmetric. Although it is easy to handle, the resultant negative sequence current does not match with actual behavior. For positive and negative sequence reactances X_1 and X_2 , $j\omega L_d$ and $j\omega(L_d'' + L_q'')/2$ should be used, respectively.

A generator which is represented as a PG node has eight

variables and eight equations as follows:

variable : $Et_a, Et_b, Et_c, \delta t_a, \delta t_b, \delta t_c, E_g, \delta g_a$

$$Pt_a = Qt_a = Pt_b = Qt_b = Pt_c = Qt_c = 0 \quad (1)$$

$$P_a + P_b + P_c = P_{total} \quad (2)$$

$$(Et_a + Et_b + Et_c)/3 = Et_{av} \quad (3)$$

$$X_g = T \begin{bmatrix} X_0 & & \\ & X_1 & \\ & & X_2 \end{bmatrix} T^{-1} \quad (4)$$

For slack generator, δg_a is set to zero and the equation for total power is dropped.

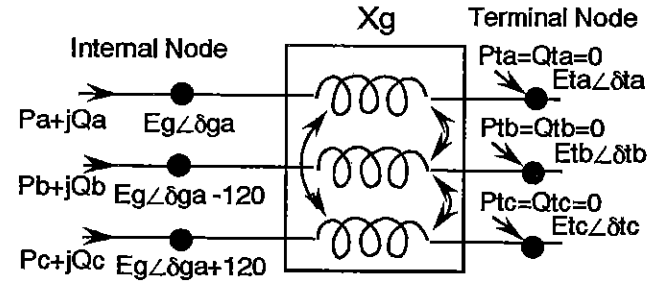


Figure 2: Generator Model

• Loads

Representation of loads is a most difficult issue, since load characteristics of actual systems is not well-defined. Loads on local systems can be identified and modeled properly. However, loads on relatively large substations consist of different kinds of load. Field tests and/or detailed measurements are indispensable to represent such mixed loads properly. The proposed program uses three types of load models. All types of load can be mixed at arbitrary rates.

• Constant Load

Constant P and Q loads are specified for each phase. This type of load is generally used as constant PQ node in conventional programs.

• Impedance Load

Two types of impedance load are prepared. A star-connected load allows zero-sequence current to flow. A delta-connected load does not allow zero-sequence current to flow at all.

• Constant Voltage Load

Constant P-load and V-magnitude are specified for each phase. Note that this type of load should be used carefully in unbalanced load flow programs. Artificially balanced voltage should not be specified; otherwise extreme imbalance may appear in current or reactive power.

• Special Considerations

It is well known that bus voltages may be undefined on the delta side of star-delta transformer connections. This

is because the relation between the ground and delta-node is not defined, if no impedance exists between them. In this situation, dummy branches should be inserted.

A mutual coupling by transmission lines might form a weak connection in networks, if no physical connections exists between them. In this case, numerical calculations of load flow become unstable. This situation will appear in maintenance condition of double circuit lines. During maintenance, only one circuit is stopped while the other circuit remains active. To overcome this problem, 6x6 matrices of double circuit lines are reduced to 3x3 matrices by eliminating floating/grounding nodes.

• Numerical Formulation

Among several methods, the conventional Newton-Raphson method in polar coordinates is used. Although the NR method of full-Jacobian type has most exact formulation, it requires more memories and computing-time, compare with other methods such as fast-decouple method or sequence-decoupled method. Such fast and compact methods, however, use some assumptions and neglects in their formulation. This fact deteriorates numerical stability and accuracy of obtained solution. Disadvantage of NR method can be easily overcome on recent EWSs.

It should be noted that the symmetrical components are not used in internal logical formulations. All inputted data in symmetrical components are converted to internal data in phase components.

• Initial Condition for Load Flow

Choosing initial conditions is important in the multi-phase load flow. At least, the phase shifting effects through star-delta transformer connections should be included. In the first stage, simple flat start (i.e., $V_a=V_b=V_c=1.0$ p.u. $\delta_a=0^\circ, \delta_b=120^\circ$ and $\delta_c=240^\circ$) was applied. Since this flat start is not stable for practical large systems, conventional single phase load flow calculations were applied to one phase (positive-sequence network) to set voltage angle and magnitude at individual bus.

Although this initial condition costs too much, this accurate starting point improves the convergent characteristic quite well and ensures fast and stable calculation.

2.2 Test Results

• Validity Check by EMTP

Program validity was checked by EMTP. Since there are limitations to EMTP, a small 43 bus system representative of TEPCO's 500kV network was used. The proposed program was executed first. Results on generators' internal voltage were used to specify EMTP's input data. All loads are modeled on the star-connected impedance load. Comparing the solutions of the proposed program and EMTP's phasor solutions, we confirmed that the two

results coincided.

The analyses revealed that, EMTP requires 1080 data lines to be input while output listing contains data 3383 lines; with the proposed program, input is only 120 lines long, and output listing contains 419 lines.

• Numerical Performance

The program was applied in practical large case studies. The models contained all of the 500kV and most of the 275kV overhead transmission lines in TEPCO. Table 1 shows the sizes of input data and required memory capacity.

Table 1: Detail of Test Case

Total Bus	192	Internal node	603
PQ-Bus	165	Ybus non zero	3726
Generator	27	Jacobian size	1205
Overhead Line	110	Jacobian non zero	28111
YΔ Transformer	27		
YYΔ Transformer	23		

It should be noted that three-phase load flow requires more dense Jacobian matrix than traditional single-phase load flow. Convergent characteristics from precise initial conditions were excellent in most cases. There is no numerical instability. Only four iterations are required to converge into a solution. The total elapsed computing time which includes single phase load flow calculations for initial sets and other I/O processes, was 6 seconds (HP-9000/735). Since the full-type Jacobian is used in the formulation, the proposed method is very stable around the neighborhood of a critical point. PV-Curve analysis in the next section proves this fact. During case studies, we find that some lightly loaded cases refused to converge. This unusual situation is caused by *balanced* specifications.

Since the transmission line is unbalanced, loads and/or voltages should be unbalanced. If we specify completely balanced loads or voltages as constraints, "no solution" must be a reasonable response.

• Voltage/Current Imbalance

The result of calculation for a practical 192 bus system is illustrated in Fig.3 and Fig.4. Although the figures show only major 500kV substations and principal power plants, other 275 kV buses have been included in the analysis. The bar charts in Fig.3 show the profile of unbalanced voltages at each bus. The black upper section of the bars indicates the rate of negative-sequence voltage. The white lower section of the bars indicate the rate of zero-sequence voltage. It should be noted that negative-sequence voltage is high in tree/end buses. On the other hand zero-sequence voltage is high in looped buses.

Fig.4 shows the profile of unbalanced currents in each line. The thickness of the line indicates the rate of negative-sequence current. The thicker the line, the

higher the rate of negative-sequence current through the line. It is interesting to note that the relationship between voltage imbalance and current imbalance is not very significant. Although not shown here, the profile of zero-sequence current is similar to that in Fig.4.

It should be noted that network reductions without careful considerations should be avoided. Fig.5 shows the results of analysis on a reduced system. This reduced system which ignores lower voltage systems is used for comparisons with EMTP. The results on this reduced 43 bus system differ from those on the precise 192 bus system. Some field test support the validity of the precise analysis. Such network reductions which lack any qualifications may lead to incorrect results. From this point of view, analytical tools that can handle large systems are indispensable.

Comparing calculated network unbalance with field tests, measured data currently available show good agreement with calculated solutions. However, further improvement of the measuring system will be essential in more advanced checking of the validity.

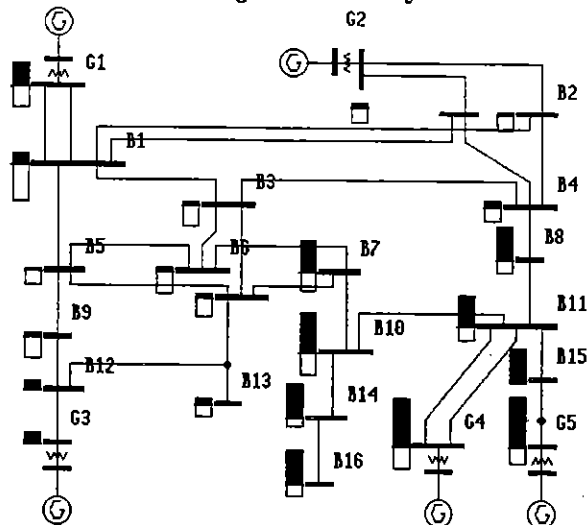


Figure 3: Voltage Imbalance in a 192 Bus System

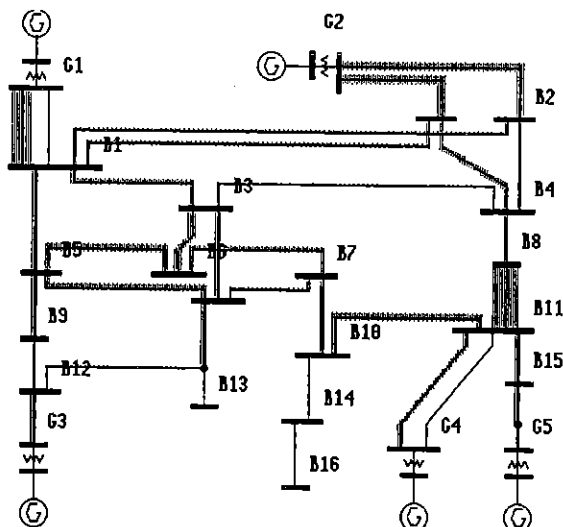


Figure 4: Current Imbalance in a 192 Bus System

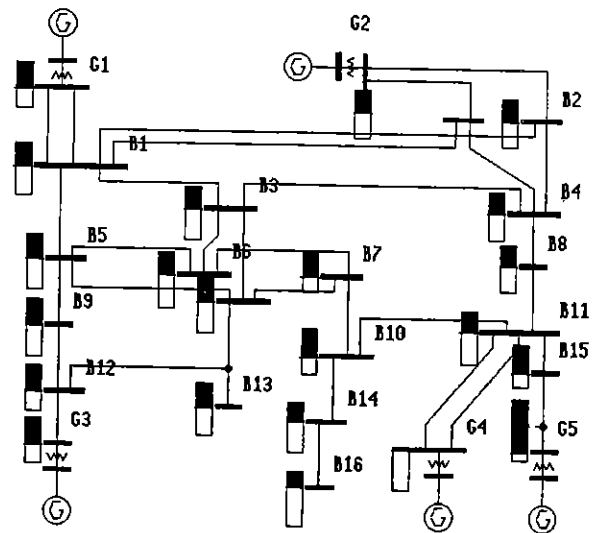


Figure 5: Voltage Imbalance in a Reduced System

2.3 Recent Developments

After development of UNBPF, voltage profile in 500kV systems and local systems which contains some unbalanced load are analyzed. New module which can calculate fault current in unbalanced systems is also developed. Any kinds of complexed faults can be analyzed using the admittance matrix in UNBPF. This program can analyze the difference between each phase faults, e.g., A-phase-to-ground and B-phase-to-ground fault are not identical in this program. Traditional programs treat them as identical one-line-to-ground (1LG) fault.

3. PV-Curve in Untransposed Lines

3.1 Model Description

UNBPF is also applied to voltage stability problems. Behavior of voltage imbalance at the critical loading condition has not been investigated so far. It is important whether voltage imbalance may or may not affect to the critical condition. Some simple models in Fig.6 are analyzed. Details of individual elements are described below;

• Transmission Line

A typical single circuit tower sketched in Fig.1 is used for unbalanced/untransposed transmission lines. The length of this line is about 105km and voltage level is 500kV. Pi equivalent circuit consists of following three matrices.

$$R = \begin{bmatrix} 9.37 & 5.59 & 5.23 \\ 5.59 & 9.07 & 5.59 \\ 5.23 & 5.59 & 9.37 \end{bmatrix} [\Omega] \quad (5)$$

$$X = \begin{bmatrix} 41.6 & 13.2 & 9.36 \\ 13.2 & 42.1 & 13.2 \\ 9.36 & 13.2 & 41.6 \end{bmatrix} [\Omega] \quad (6)$$

$$Y = \begin{bmatrix} 3.66 & -0.633 & -0.207 \\ -0.633 & 3.68 & -0.633 \\ -0.207 & -0.633 & 3.66 \end{bmatrix} \times 10^{-4} [S] \quad (7)$$

• Voltage Source

Constant voltage sources are located in the left ends of all models in Fig.6. Three phase voltages are completely balanced, but current's imbalance is allowed. Therefore, active and reactive power injection may be unbalanced. Voltage magnitude at arrow point is kept 1 pu. Generator model described in Fig.2 is utilized in Case A-3 through A-5. In this model, the mean value of terminal voltage E_{tav} in Equation(3) is kept 1 pu. Three phase voltages are balanced at internal voltage E_g , but imbalance at terminal voltage is allowed. Parameters of this generator model (correspond to two generators of 1000MW class machine) is as follows;

Generator : $X_1 = X_d = 0.8$, $X_0 = 0.05$, $X_2 = 0.1$ (pu)

• Transformer

ΔY and/or YY transformers are used. Neutral points of Y-windings are grounded. Leakage impedance is specified as follows;

Transformer : $X_{t_{gen\ side}} = X_{t_{load\ side}} = 0.1$ (pu)

• Load

Impedance load and constant load are mixed. They are all balanced. " $P_z=30\%$ " indicates 30% of load is specified as impedance load and remaining 70% of load is specified as constant load. For example, a 1800MW with $P_z=25\%$ means;

$$\begin{aligned} P_{spec} &= P_a + P_b + P_c = 1800 \text{ MW} \\ P_a &= P_b = P_c = P_z + P_{const} = 600 \text{ MW} \\ P_z &= 150 \text{ MW}, P_{const} = 450 \text{ MW} \quad (\text{each phase}) \\ G_a &= \frac{150 \text{ MW}}{(500 \text{ kV} / \sqrt{3})^2} = 0.0018 \text{ [S]} \quad (\text{conductance}) \\ &= G_b = G_c = 0.15 \text{ [pu]} \end{aligned}$$

It should be noted that P_{spec} is specification value. If obtained V_a, V_b, V_c are unbalanced and/or declined (e.g., $V_a = 0.9, V_b = 0.8, V_c = 1.1$), net load P_{net} is no longer equal to 1800MW, and each net load may be unbalanced, as listed below;

$$\begin{aligned} V_a &= 0.9, V_b = 0.8, V_c = 1.1 \text{ [pu]} \\ P_{a_{net}} &= P_{za_{net}} + P_{const_a} = G_a \cdot V_a^2 + P_{const_a} \\ &= 0.15 \cdot 0.9^2 + 0.45 = 0.5715 \\ P_{b_{net}} &= 0.15 \cdot 0.8^2 + 0.45 = 0.5460 \\ P_{c_{net}} &= 0.15 \cdot 1.1^2 + 0.45 = 0.6315 \\ P_{net} &= P_{a_{net}} + P_{b_{net}} + P_{c_{net}} = 1749 \text{ [MW]} \end{aligned}$$

PV-Curve in this paper indicates the relation between P_{net} and V_a, V_b, V_c . If the rate of constant load is high, this system will be forced to deliver well-balanced load through the unbalanced line. If voltage at a phase is lower than others, current injection to this phase should be higher than the others. It is a quite severe requirement to satisfy. If the rate of impedance load is high, low voltage at a phase means low load consumption at this phase. This condition must be easy to satisfy.

Both Y-connected and Δ -connected impedance load are examined. Although this impedance connection is a dominant factor to the zero-sequence network, the difference is not significant in many cases. All PV-Curves in Fig.7-11 are drawn using the Y-connected impedance load with grounded neutral. In this paper, power factor was supposed to be 100% for all load (i.e., $Q_{spec} = Q_{net} = 0$). Of course, it is easy to specify an arbitrary power factor in this program.

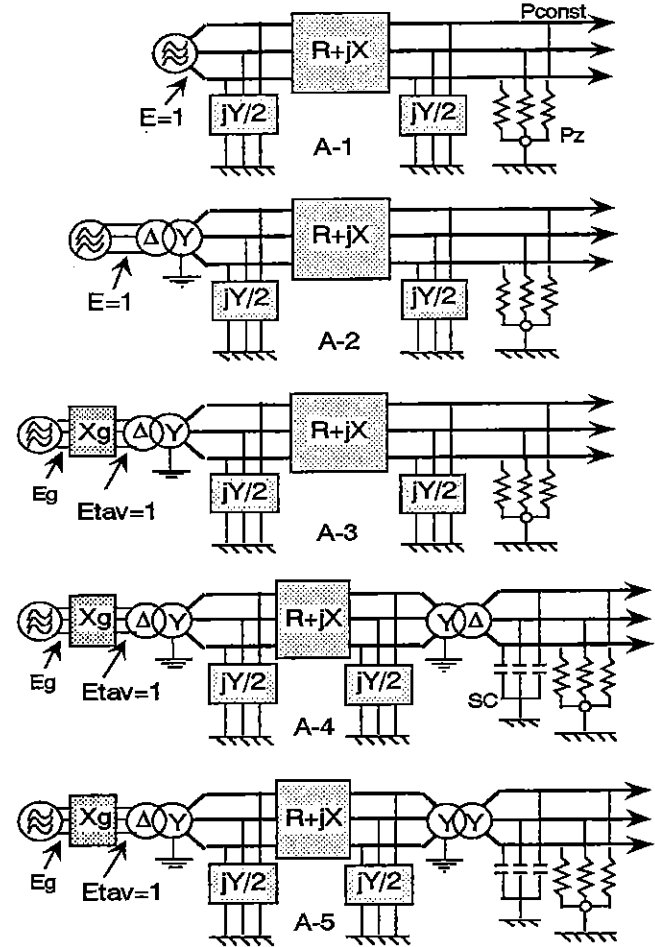


Figure 6: Model Systems

3.2 Results of Analysis

• Case A-1 (Fig. 7)

This model is most simple one. All elements in network consists of an unbalanced line except impedance load. As load P_{net} increase, voltage imbalance expand in general.

In the case of $P_z = 0\%$ (i.e., all load consists of balanced constant load and $P_{net} = P_{spec}$), voltage differences between each phases expand more than 20% at the critical loading condition. The voltage stability limits is also deteriorated. P_{max} at $P_z=0\%$ was 2850MW. This value is 76% of that at $P_z=100\%$ (3726MW). If the transmission line are completely transposed, P_{max} will be 3739MW. Although the impedance load is Y-connected in this case

study, no significant difference is observed in the case of Δ -connected load.

• Case A-2 (Fig. 8)

Since a step-up transformer is added, P_{max} becomes smaller in this case. Voltage behavior at each phases is quite similar to the Case A-1. The inference of Δ -connected winding at step-up transformer is not significant.

• Case A-3 (Fig. 9)

Generator model is used in this case. Since average of terminal voltage is kept to 1 pu, P_{max} is almost same as Case A-2 at $P_z=100\%$. In the case of constant load $P_z=0\%$ and $P_z=25\%$, P_{max} is smaller than that in Case A-2. They are 1765MW and 1967MW, respectively. It should be noted that X_2 must be negative-sequence impedance. If we ignore X_2 and replace it by $X_1=X_d$, calculated P_{max} at $P_z=0\%$ and $P_z=25\%$ become 1234MW and 1525MW, respectively.

• Case A-4 (Fig. 10)

Load is connected through a ΔY transformer. Neutral point of Y-connected winding at line side is grounded. In order to prevent the numerical trouble caused by singularity, shunt capacitors are inserted between load terminal and ground (40MVA/Phase) in this Case A-4 and A-5.

The constant balanced load ($P_z=0\%$) can be successfully delivered up to the same level of P_{max} at $P_z=100\%$. All P_{max} obtained from any percent of P_z is almost same as that obtained from an ideally-transposed line. In this case, the inference of Δ -connected winding at load the transformer is significant.

• Case A-5 (Fig. 11)

Load is connected through a YY transformer. Neutral point of Y-connected winding at both sides are grounded. In the case of constant load $P_z=0\%$ and $P_z=25\%$, P_{max} is smaller than that in Case A-4. The constant balanced load ($P_z=0\%$) can not be successfully delivered up to the same level of P_{max} at $P_z=100\%$.

Table 2: Critical Loading Condition P_{max}

Case	$L^\dagger$	Impedance Load Rate P_z [%]					$S^\ddagger$
		0	25	50	75	100	
A-1	Y	2851	3225	3581	3715	3727	3739
	Δ	2851	3128	3441	3702	3735	3739
A-2	Y	1886	2047	2147	2169	2171	2177
	Δ	1886	2011	2126	2168	2173	2177
A-3	Y	1765	1967	2135	2169	2171	2177
	Δ	1765	1918	2109	2169	2172	2177
A-4	Y	1588	1588	1588	1588	1588	1591
	Δ	1588	1589	1589	1590	1588	1591
A-5	Y	1376	1501	1577	1588	1588	1591
	Δ	1376	1477	1573	1587	1587	1591

$L^\dagger$: Type of Impedance Connection

$S^\ddagger$: Ideally-Transposed Line

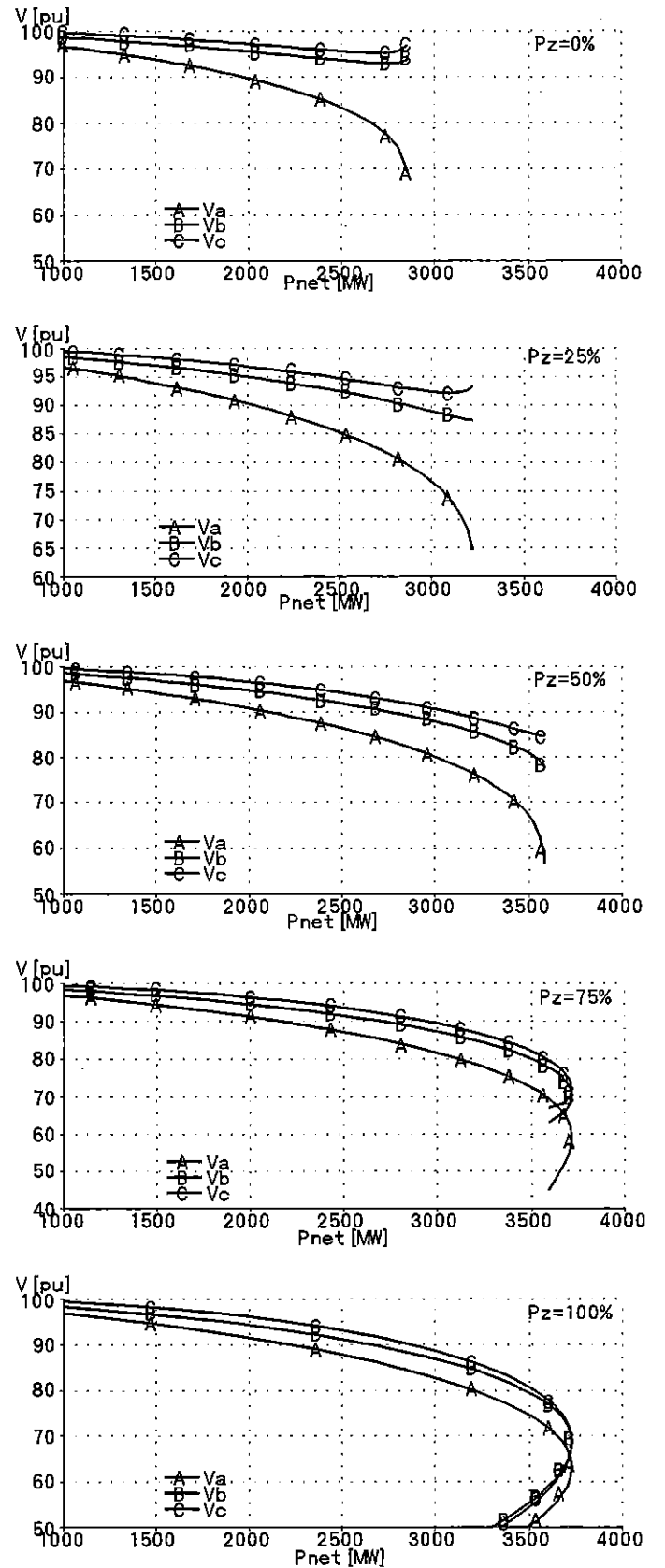


Figure 7: PV-Curve in Case A-1

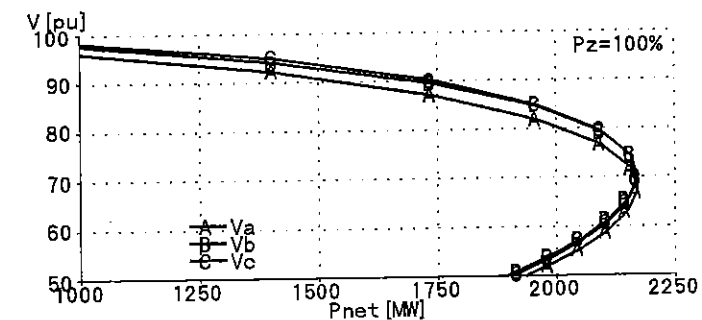
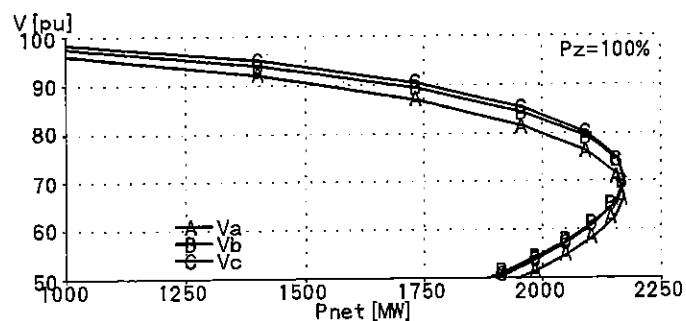
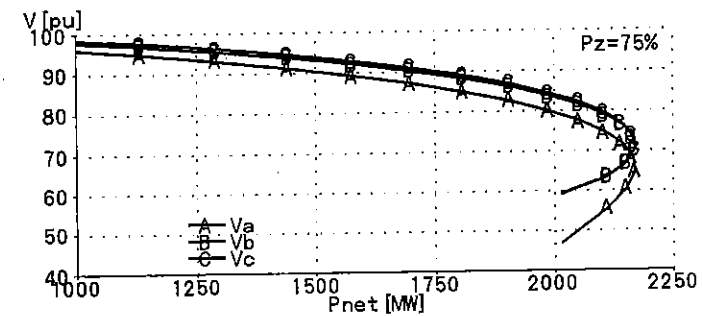
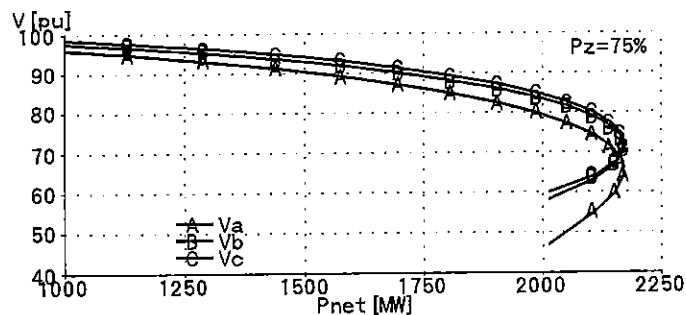
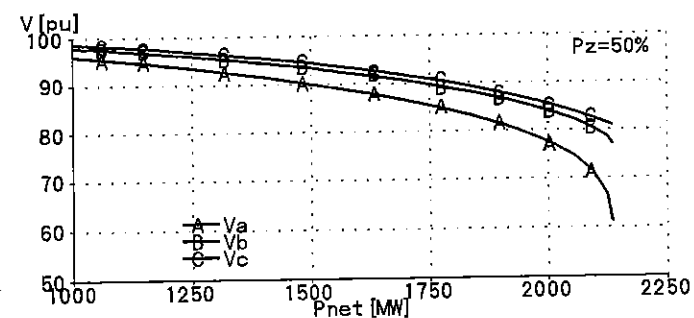
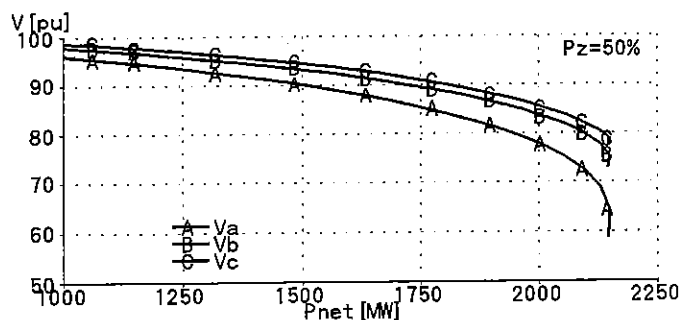
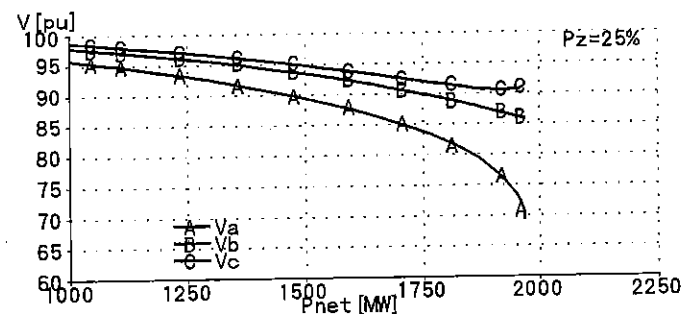
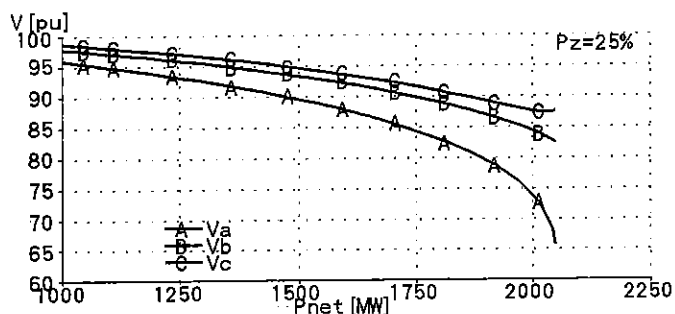
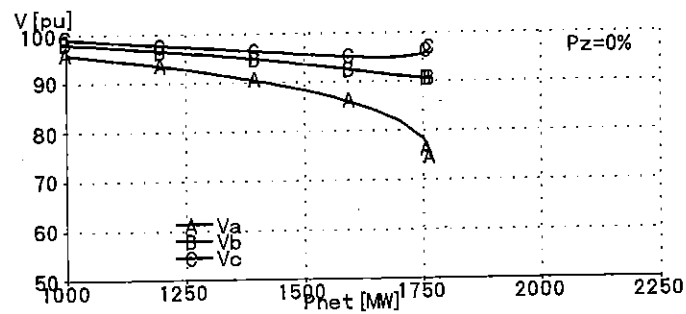
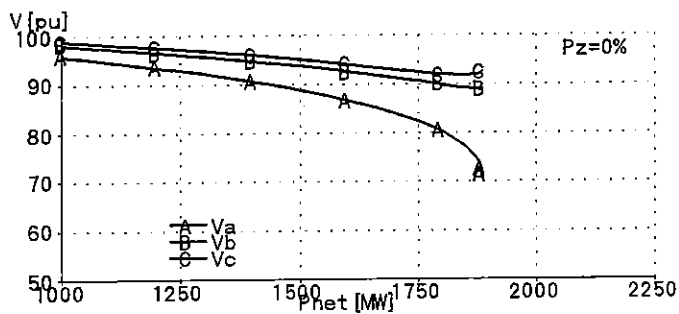


Figure 8: PV-Curve in Case A-2

Figure 9: PV-Curve in Case A-3

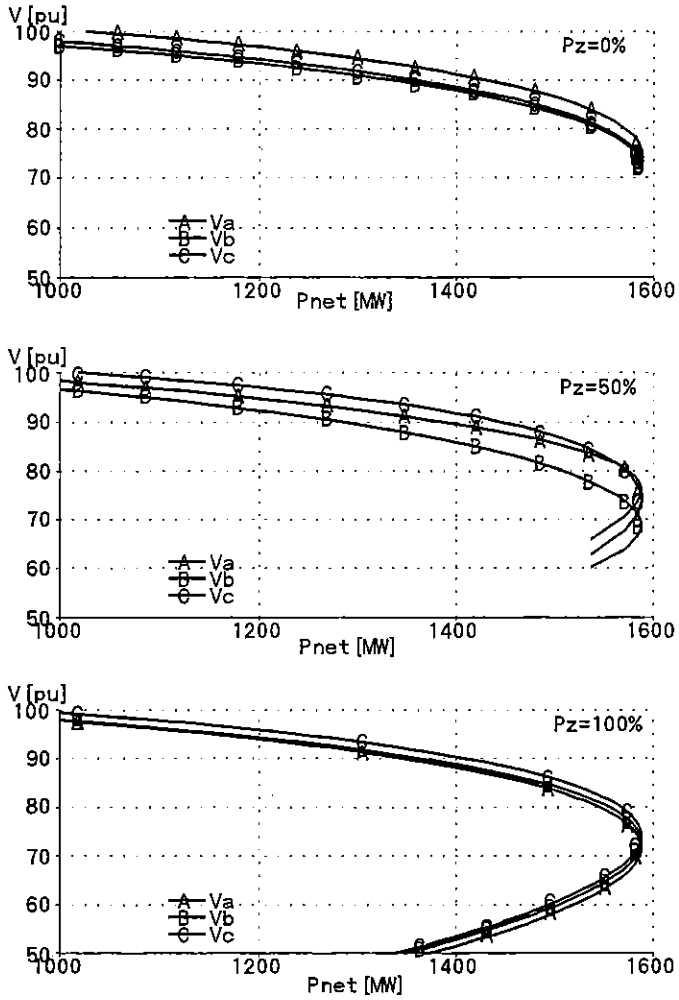


Figure 10: PV-Curve in Case A-4

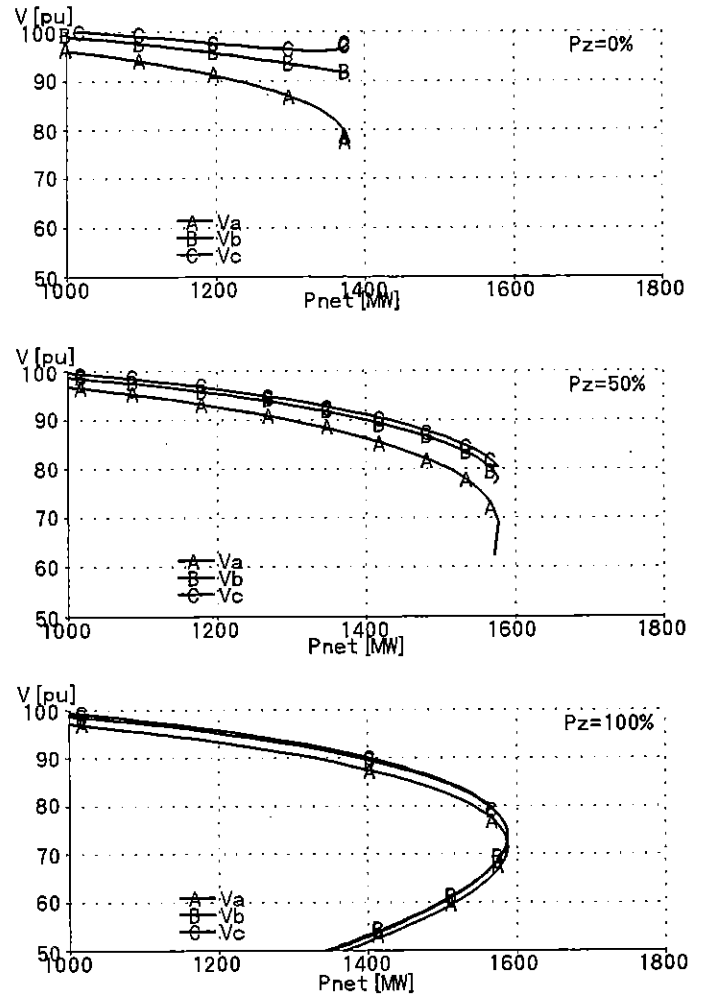


Figure 11: PV-Curve in Case A-5

3.3 Considerations

P_{max} for all cases are summarized in Table 2. If an ideally transposed (i.e., symmetrical) transmission line is used, P_{max} will be identical to that obtained by a traditional single phase load flow program. Since there is no imbalance in transposed system, the percentage of P_z does not affect to the PV-Curve at all. Suppose $P_{net}/3 = P_{a_{net}} = P_{b_{net}} = P_{c_{net}}$ and $V_a = V_b = V_c$ are given, we can divide $P_{a_{net}}$ into $P_{za_{net}}$ and P_{const_a} , arbitrarily.

Fig.12 shows the positive, negative and zero-sequence network for the Case A-4,A-5. Although line charge is not appeared in this figure, it is included with mutual coupling. Since the zero-sequence circuit is divided by Δ -winding of transformers, the zero-sequence current to load (I_{L0}) in Case A-4 is not affected by the untransposed line. On the contrary, if the zero-sequence circuit is connected through Y-winding of transformers, the zero-sequence current I_{L0} in Case A-5 is strongly affected by the untransposed line.

After some investigation, it became clear that no singularity exists in the system. The main reason of insufficient

P_{max} in Case A-5 ($P_z=0\%$) should be in the constraints of *balanced* load. Since the requirement to a *balanced* load is severe, no more *balanced* load can not be sent at the critical condition. It seems to be possible to send more unbalanced P_{max} , but its hard to satisfy the equal-constraints $P_a=P_b=P_c$.

Since untransposed transmission lines are usually connected to loads through Δ -windings of transformers in practical power systems, we do not have to worry about the results of this analysis.

Fig.13 shows some examples of other interesting analysis.

- Negative Sequence Current to Generators

Some large generators have severe limitations on the negative-sequence current. Since negative-sequence current tends to increase as load comes close to a critical point, it is important to analyze whether it exceeds the operating limit before reaching to the critical point.

- Unbalanced Load

Although impedance load allows larger P_{max} than constant load, imbalance in load and current will be large. It is worthwhile to analyze such load conditions.

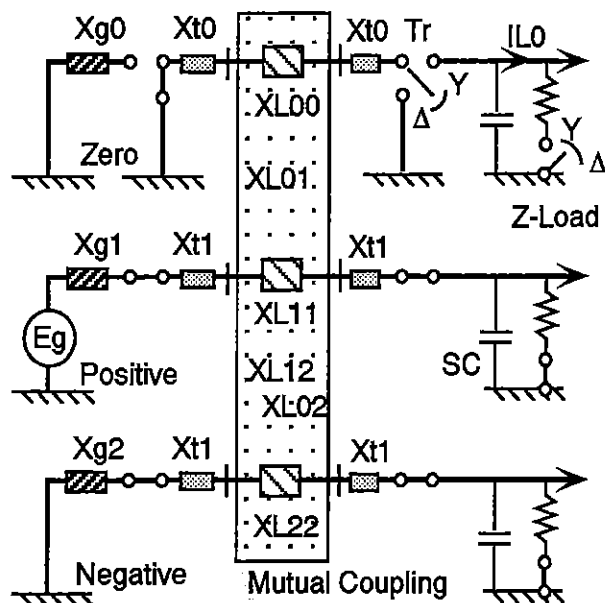


Figure 12: Positive,Negative,Zero-Sequence Circuit

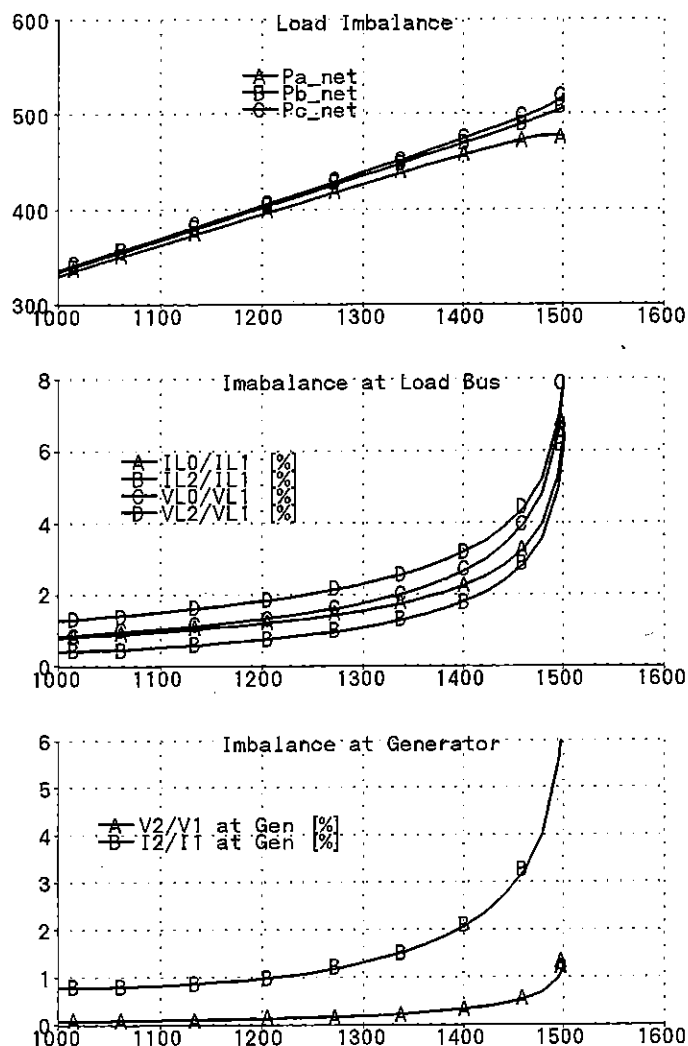


Figure 13: Imbalance in Current and Load

4. Conclusion

A multi-phase load flow program is developed. It can analyze unbalanced power systems without EMTP's phasor solution. This program is successfully applied to a practical 192-bus(576 node) system. Although the mathematical method used in this program is not new at all, the performance, i.e., computing speed and convergent characteristic is excellent. This program is already implemented TEPCO's EWSs and used for engineering and planning.

This program (UNBPF) is applied to the analysis of voltage instability. PV-Curves and critical loading conditions are calculated in a simple system which consists of an unbalanced transmission line. Among load models, the constant and balanced load requirement is very severe to satisfy in general. Low voltage at a phase means high current injection to this phase. Under this condition, PV-Curve and its critical load may not reach to the point obtained by conventional single phase load flow program. This critical condition is caused by network imbalance. If the balanced load is connected to unbalanced transmission lines through ΔY transformer, this severe situation is completely improved. Since the ΔY transformer divides zero-sequence circuit, zero-sequence current to the load (I_{L0}) become free from line's imbalance. In most of existing power system, load is connected to lines through transformers' Δ -windings. We do not have to worry about the reduced critical loading caused by lines' imbalance from a practical view point.

Voltage stability problems have been extensively studied from various view points in TEPCO. Consideration of imbalance of transmission lines is one of them. The results of this study support the validity of our previous studies based on single-phase load flow program. The detailed network model with constant load model gives most severe case. Although the imbalance of lines does not yield particular difficulties in normal operating conditions, it should be essential to analyze unbalanced systems carefully using sophisticated tools like UNBPF.

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Discussion of papers presented by DeMarco and by Iba

Edited by Thierry Van Cutsem, University of Liege

Thomas Overbye (University of Illinois)

Just a brief comment first, on the last [Iba] paper, then I'm going to talk about Christopher's [DeMarco] paper. Actually Chris DeMarco and I looked at fractals back in 1989 in a CDC paper, but Jim Thorp has done a lot at Cornell with fractals. He has got them on T-shirts, coffee tables, etc. He has got a video tape too. So this is well-known. As far I know, he hasn't really got them eliminated except through a very computationally intensive method. So, if you are interested, talk to Jim Thorp.

Concerning Chris DeMarco's paper: have you formulated an energy function for this yet? We know that an energy function gives sufficient conditions for stability that tell you whether you are stable or not. But if the energy function says you are not stable, it does not mean you are not stable. It does not give necessary conditions. So it could be very conservative! I was just wondering if you have any results on the conservativeness of the system. Also, the results of the second paper are interesting. You are doing 3-phase analysis, and I presume that 3-phase unbalances could be solved by the method that you are doing. Mike Adibi also has worked on this unbalanced system, and he has a paper that was among the preprints, but since he couldn't be here, it will not be presented.

Christopher DeMarco (Univ. of Wisconsin):

As a first comment, I'll say yes, for very preliminary results. We could formulate what we call the energy function in this framework, and not surprisingly the key elements - let's say we are focusing on rotating machines - remember I said I would choose a single Park's frame for the whole network. So I no longer get constant inductance matrices for every machine, because I am not in that machine's Park's reference frame - I mean some global frame. So I still have inductance matrices that are functions of the relative phase angle of that machine to the reference frame. So, all in all, the kind of sine-cosine functions that show up in sinusoidal quasi-steady-state energy functions do play some role here, and I create

the quadratic form $\frac{1}{2}i^T L(\Delta\theta)i$, and that is one of the key elements. And the reason we can get multiple equilibria has to do with the nonlinear dependence of this inductance matrix on the shaft angles relative to the coordinate frame. If we had just a quadratic form with positive definite L , nothing very interesting would happen.

But let me say that one of the things we are looking at is, whether it would not make more sense to also put the currents in a frame relative to the equilibrium angles. That seems to have some advantages, but I am not there yet. In terms of the fact that the end result is no longer a Hamiltonian system, it has losses, what I am doing is sort of adapting the lossless Hamiltonian to become a Liapunov function, so the shortcomings of the Liapunov stability analysis remain. The best I can hope to do is to get sufficient conditions, get subsets of regions of attraction. But I will point out that I believe the loss mechanisms in a more accurate model are fewer and in a sense smaller than the way we represent loss in a more approximate model. And the less losses there are, the stability boundary in a rough sense is better approximated by a constant contour of this function.

Yakout Mansour (BC Hydro):

Chris, I have always fun with your ideas, especially when you agree with me! I really do not want anything of what you said to be taken lightly, because this is a real good idea. I have been trying to convince my colleagues - including myself I guess - to take time to try to formulate the energy-

based dynamic representation of systems in Lagrange-like formulation instead of Liapunov-like, for the following reasons. First of all, it is energy, so it can be an index. Second, as you indicated, you could derive the state equations very nicely. Third, because you have separate functions for energy, element by element, you can also separate the states related to - say - the flux dynamics from the angle dynamics, which so far we cannot do with the general dynamic formulation, when we want to subtract the voltage stability related aspects of it. Those are three reasons on which I agree with you. A fourth one - which I do not agree with, and you are understating: you are indicating that the Lagrange formulation does not have room for losses. The original Lagrange formulation, yes. But if you include the Rayleigh (Dissipation) function in the generalized Lagrangian, it does include losses. Now, it does include actually more than losses. If you want to look at energy of damped systems, damping could be a Rayleigh function, and if you look at the voltage stability related issues, then even reactive losses could be included in that function. This is where you are understating things, as if Lagrange does not have room for that.

Now, you referred to the physicist's book, but are you aware of Jerry Meisel's book "Principles of electromechanical energy conversion"? This is the best book if you want to adopt one for your textbook for machines. This is the one. Chapter 1 puts all the bases for the Lagrange formulation, including all items we talked about. After that, every chapter deals with one machine based on that. I urge you to look at it.

DeMarco: I'll definitely take a look. Thank you.

Peter Sauer (University of Illinois):

This idea of using the a, b, c variables and going back to the time domain always runs into a big snag when you hit the load models. Now if you want to model any load as an induction motor, then that works, but what would you be proposing to do as an alternative to that in any type of load other than an induction motor? Aggregate loads for example.

DeMarco: There is certainly a whole range of things to worry about. I would say the first step that I am interested in worrying about is elements that have switching behaviour in them. In these days of FACTS being so interesting, let us say we want to bring in things that involve thyristor bridges. Let me point out that I do not necessary want to keep all this detail: I would again argue that more detail actually becomes easier for a while. Go back to the circuit model for thyristor. It's a dissipative multiterminal resistive n-port, with a couple of linear capacitors to take into account the memory that you need for gate turn-on and turn-off. So I have got 1st and 3rd quadrant dissipative nonlinear resistors, but still dissipative - easy to deal with - and simple linear capacitors. So I know something about that sort of model, and if you tell me that you give me a big circuit - and I am sidestepping your question a little bit - but let me say I want to look at the thyristor controlled bridge exciter - I would look at three terminals coming in that have the three phase voltages and currents as their variables. In currently used transfer function type models for an exciter you tell me how the measured terminal voltage - which of course would be the square root of the sum of squares of those - relates to v_f . You rarely tell me how the currents relate, and if I want a circuit model I want to know not only how voltages relate: I want enough constraints to fully define the n-port. If I know that that device is reciprocal and passive, then I can do it. I can fill in the rest of those constraints. So, as a rough and immediately vague answer - because this work is still pretty preliminary - what I want to do so is - say - tell me a few things about the

kinds of elements that make this out. Are they reciprocal? For example, thyristors: you might want to go all the way by approximating them as lossless switches. You tell me a few of those characteristics, and I will give you an n-port circuit model, and if in particular the device is reciprocal - and I claim that a lot of things that we are interested in are - then I still have something that fits the Lagrangian-Hamiltonian formulation very well. If it truly becomes antireciprocal, I am in trouble.

Fernando Alvarado (University of Wisconsin)

First of all, I want to say that Chris's [DeMarco] approach is the first approach I have really liked of all the energy function approaches, because it is fairly comprehensive. Now, as long as books are being mentioned, I believe it was Woodson and Melcher that was also based on the energy concept, along the same general lines, a variational approach. So there is a lot of precedent, but in the past these efforts have not - I would say - caught the fancy of a greater population. Part of it is - by the way I am not prepared to throw away my induction motor model - that it was not totally clear - and maybe this was obvious but I missed it - what happens to the notion of frequency? That is a very valuable concept that we have been working with. It gives you an intuitive feel for the system harmonics and Fourier analysis and all those things. I do not know whether it fits with your formulation. Maybe it does.

DeMarco: Let me now back-up and put some more honest qualification on this. If you read some of the things I say in the paper at this point, if I want to go a global system-wide Park's transformation, I want to know the steady-state frequency that that network goes to. Now I claim that if you want to go so far as to think about AGC control, ultimately you do know what frequency is the system-wide set-point. It may take you a long time to get there. If you want to approximate a little bit, then give me a frequency that is to good approximation the steady-state frequency. Even with that, if I want to do a Liapunov-type stability analysis, my life is going to be a lot easier if I have an equilibrium point. If I just took EMTP-type coordinates, I would have gotten some variables that are sinusoidal, which keeps a, b, c elements in the network, I would have got some variables that go to constants, I would have got shaft angle increasing linearly with time. That would be a pain. Part of the goal in this is to adopt a network-wide transformation such that when I am in steady-state, the steady-state is a constant equilibrium point. *Caveat*: That does not work if I have got significant harmonics, and I think the next key step in this analysis is not just the Lagrangian-Hamiltonian stuff, but some sort of harmonic expansion where I keep the state variables, not just fundamental component Park's transform variables, but some representation of higher harmonics as state variables, so that all those things go to constants when I reach steady-state. Can I do that? I am not really sure. That is one of the things I am going to work on.

Chair (Hans Glavitsch, ETH, Zurich):

The question probably is: are you really doing something different from what Aziz [Fouad] and Vittal are doing? I mean, if you do a comprehensive analysis we have to follow the conventional Liapunov-type analysis for some way, then we have to include some additional features to account for voltage stability. Isn't it more complex, what you have done? Or are there some easier features if you just look at the voltage stability? Maybe we are not able to answer at this point but ... I'm just throwing in a few points of caution.

DeMarco: I'll make a quick comment. The original abstract that Les got - I think - was considerably more ambitious than what I was able to accomplish up to this point. But I do believe that one should be able to start from this more detailed formulation, and step back and recover what is done in the classical energy analysis that Vijay [Vittal] and Aziz [Fouad] and others have done a lot of work on. You see a

few hints of that, that I think many people recognize, but let me just quickly throw something up. Anybody who's looked at the direct methods would recognize a term that shows up all the time: a double sum of line susceptances times some cosine of the angle difference across a line ($\sum \sum b_{ij} \cos(\theta_{ij})$). If you work through this, this is the sinusoidal steady-state approximation to $\frac{1}{2} Li^2$ summed over all the lines. So, there are certainly connections. The one caution I would throw out: People in that field, and myself as well, have wrestled for a long time with problems of loss mechanisms like resistance, and I think we may have been barking up the wrong tree. The loss resistances introduce a kind of loss of symmetry that makes the problem harder. I am increasingly coming to believe that that only happens because of the sinusoidal quasi steady-state approximation. If we took into account the true time rate of change of voltage and current variables, there would be extra terms in our power balance equations, and we would keep symmetry even with resistances. So there is a small perhaps addition that this point of view can make.

Chair: Well, there are a few simple models of lines and, if you think of the stored energy in a three-phase inductance, it is constant and we should maybe build on that.

DeMarco: Yes, exactly.

Mansour: I would like to make just a quick comment on the difference between this formulation and the energy formulation we have been using for transient stability. The mechanical (kinetic energy) side is ok, but the electrical side in the energy function formulation that we have now is very short of a lot of things. And this is actually why we cannot use that formulation beyond first swing. And Aziz [Fouad] could stretch it maybe beyond first swing, as long as you know that the effect is purely inertial. That is again indicating that the electrical representation is very short, mainly from the flux side of machines.

Daniel Cheng (National Grid Company):

[Addressing Iba] The three-phase load flow analysis for PV curves is very interesting. I would like to ask you whether you used point-by-point power flow analysis or have you tried continuation methods to speed up the process. The second point is a related one. Is there any mileage in going for 3-phase eigenvalue analysis? Basically, what I have in mind is a sort of two-level analysis. On a first level, balanced analysis, maybe we have several modes pretty much similar. Then you do a second level, 3-phase one, and maybe you realize there are differences.

Kenji Iba (Mitsubishi Electric Corporation):

Concerning the first question, in order to draw this PV curve we made it in just a successive trial-and-error way. So I did not apply a special continuation method, just successive load flow calculations.

Cheng: Did you meet convergence problems when you met the nose point?

Iba: Not so many, because the recent engineering workstation is very fast so that the trial-and-error is not so bad to draw one curve. So we did not need any special continuation method. Concerning your second question, we have not yet done eigenvalue analysis, but we have a very good Jacobian matrix so it may be interesting. One more point: a difficulty is, if we apply three-phase models, we have to solve using a Newton-Raphson method. It is hard to apply this mathematical approach - at least we have to get a solution. So the result of our paper is just applied to solve for the solution but from the mathematical viewpoint we could do nothing better.

Nicholas Miller (General Electric):

I want to compliment Mr. Iba on wrestling with this 3-phase problem, and I'm thinking the utility people in the room are shuddering over the prospects of gathering three phase data. I want to add just a little anecdote. We have seen similar results in our system studies for US utilities that depend very heavily on a 500 kV interconnection. We went and modelled just the major tie-lines involved in this fashion and used essentially balanced representation of the rest of the world. We found that the voltage stability limit was reduced by almost 200 MW over what was calculated in a balanced load flow which was a bit of a shock. So this is most assuredly not an academic exercise, and it does not necessarily require that huge amounts of 3-phase data be brought into the picture in order to increase the resolution of your decision making from a utility point of view.

Chair: Apparently, if you capture the major unsymmetry, you get most of the answer, a qualitative answer at least.

Nelson Martins (CEPEL):

Was it 200 out of 1000 MW?

Miller: It was 200 MW out of 4000 MW.

Fink: As far as I know, most utilities do not use 3-phase metering. They are only metering one phase on an unbalanced system. This leads to some further inaccuracies in calculations and conclusions.

Chair: I think that if they are interested in MW they will meter three phases. Here in Europe they do.

Session 6

Survey IV

"Review of Modeling Requirements for Voltage Stability Analysis"

Carson W. Taylor
Bonneville Power Administration

Load Modeling

"Voltage Stability Including Detailed Load Models"

Fernando L. Alvarado: The University of Wisconsin

"Load Structure: Identification and Impact on Voltage Stability"

Miroslav M. Begovic, Roger Q Mills III
Georgia Institute of Technology

"Asynchronous Motor Models for Voltage Stability Analysis"

Ettore Bompard, Enrico Carpaneto, Gianfranco Chicco, Roberto Napoli
Politecnico di Torino

"Modelling of Dynamic Load Behaviour"

Ian A. Hiskens, The University of Newcastle
David J. Hill: University of Sydney

"The Irrelevance of Load Dynamics for the Loading Margin to Voltage Collapse and its Sensitivities"

Ian Dobson: University of Wisconsin

Review of Modeling Requirements for Voltage Stability Analysis

Carson W. Taylor
Bonneville Power Administration
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Abstract

Requirements for modeling in power flow and dynamic simulation of longer-term voltage stability are reviewed. We identify the state-of-the-art, indicating where further research is needed. We describe gaps between modeling capabilities and routine utility practice. Needs include standardization of models and data for generator field current limiters, and improved distribution network equivalents. Bonneville Power Administration field test results showing long term load response are described.

Keywords voltage stability, voltage collapse, load modeling, power flow program, long term dynamics

Introduction

In the past few years, industry understanding of voltage stability has matured, and many new computer programs for analysis have been developed. Very large power systems can be efficiently analyzed by these new tools. Essential to these new tools are adequate models. Equally essential is good data for the models. The models and data should be validated to the extent possible by field tests.

I've recently had the privilege of leading several IEEE and CIGRÉ activities concerned with modeling for voltage stability and other power system dynamics [1-3]. This review paper is based, in part, on the resulting reports, particularly the CIGRÉ Task Force 38.02.10 report *Modelling of Voltage Collapse Including Dynamic Phenomena*.

References 1 and 4 recommend definitions and terminology for voltage stability. The terms *transient voltage stability* and *longer-term voltage stability* are recommended. Longer-term voltage stability involving time frames of tens of seconds or minutes will be assumed in this paper unless otherwise noted.

Longer-term voltage stability may be studied by either power flow or dynamic simulation. Since the slow, often discontinuous, dynamics often dominate

voltage stability, power flow snapshots of the dynamic phenomena are often adequate for at least initial voltages stability analysis.

The scope of this review is limited to generator voltage and excitation control modeling, and load modeling.

Voltage Instability Mechanisms

Voltage stability involves three aspects [4]:

1. the load characteristics as seen from the bulk power network;
2. the available means for voltage control at generators and in the network; and
3. the ability of the network to transfer power, particularly reactive power, from the point of production to the point of consumption.

Regarding the ability of the network to transfer power (aspect 3), transmission network modeling in power flow and dynamic (stability) programs is well understood and usually requires only static models. In some cases special controls must be modeled for reactive power compensation switching or LTC autotransformer switching.

Dynamic or time-coordinated models may be required for aspects 1 and 2. In particular, the main instability mechanisms are the load restoration dynamics following a disturbance and the generator current limiting dynamics.

Modeling of Generation

Power flow program modeling

In power flow simulation, the traditional model of PQ/PV generator bus switching for generator reactive power output limits may not be adequate [1]. The generator reactive power output is a function of active power output, terminal voltage, and machine design. For important generators, there is little reason not to use a more detailed model based on field (and armature) current ratings.

Generator current limiting

Following considerable industry effort, detailed generator and excitation system models are fairly well standardized [5,6]. Of greatest interest for voltage stability, however, are models for field and armature current limiters which are not included in the standard models.

Field and armature current limiters allow time-overload consistent with generator capability as given by, for example, ANSI Standard C50.13-1977. Based on initial operation at rated conditions, and depending on amount of overload, the standards allow overload from tens of seconds to about two minutes.

Armature current limiters are not widely used in most countries, but reference 1 provides a model for the equipment of one manufacturer.

Reference 1 describes several types of field current limiters used on different types of excitation equipment. These controls are also termed maximum excitation limiters or overexcitation limiters. There are many variations of the limiters based on the equipment manufacturer and the equipment vintage. References 4, 7, and 8 provide other examples of field current limiter models. Figure 1 shows a generic model that may often be adequate [1,4].

During normal conditions with field current less than the setpoint (typically 105% of rated field current), paths 1 and 2 both force the integrator to its lower limit (-A). The input to the voltage regulator summing junction is zero. If field current is above the setpoint, path 2 drives the integrator towards a positive value; once the integrator output becomes positive, the voltage regulator will start to

reduce field current. For a step increase in field current above the setpoint, the time to the start of current limiting (time for integrator output to ramp to zero) will be:

$$t = \frac{A}{K_2 \cdot K_3 \cdot (I_{fd} - 1.05 I_{fd \text{ rated}})}$$

The parameters can be adjusted so this time approximates the desired time. Once field current is reduced to the setpoint, path 1 rapidly resets the integrator to -A.

At present, limiters are often represented by user-defined models. In some applications, special strategies can be evaluated such as reduction of active power [9] or reduction of terminal voltage setpoint rather than reduction of field current.

The IEEE Task Force on Excitation Limiters is currently preparing the paper "Recommended Models for Overexcitation Limiting Devices." These models are associated with the IEEE excitation system models [6], and include details such as the required cool-down time before a second overload is allowed. Two categories of limiters are defined. The first type are limiters that take over control of excitation. These can be called *hard* limiters or *auctioneering* limiters. In older equipment, these are simply inverse time relays that transfer the regulator to manual control. The second type are summed limiters. These limiters are often referred to as *soft* because the limit level can be influenced by other inputs to the voltage regulator summing junction.

The IEEE work will be an important step forward in modeling for voltage stability. Hopefully, production-

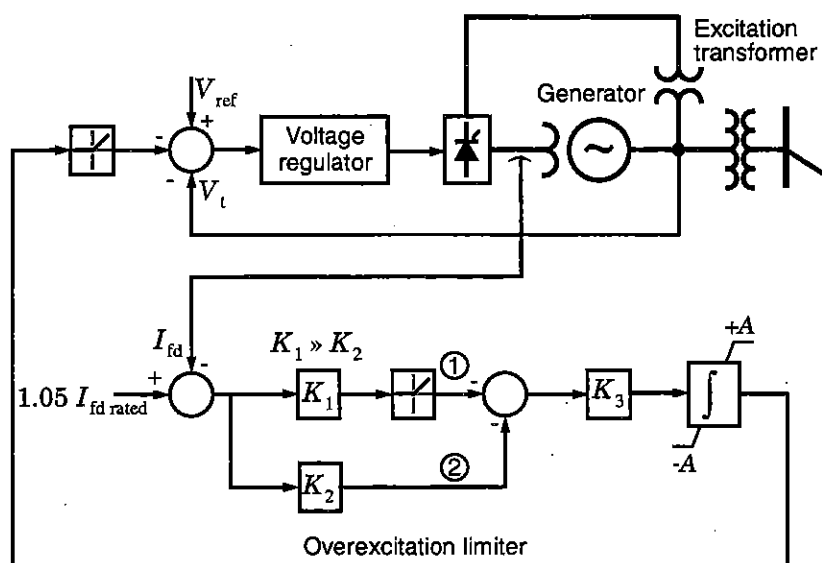


Fig. 1. Model of overexcitation limiter.

grade computer programs will rapidly incorporate these models. The challenge to utilities is to develop the required data sets. Typical data can be used if specific data is not available.

Generator voltage control

An important means to improve voltage stability is control of generator transformer high side voltage rather than terminal voltage. The most common form is transformer drop compensation of 50–80% of the transformer reactance [10,11]; models [6] are commonly available in production-grade programs. This control is simple, very low cost, and fast.

An outer voltage control loop to regulate high side voltage may be used as an effective voltage instability countermeasure. This control should be an order of magnitude slower than the primary voltage regulation (ten seconds or slower time frame). This is sometimes called secondary voltage control and may either be local or centralized. Tokyo Electric Power Company is using a microprocessor-based local control that likely represents the state-of-the-art [12]. Several European utilities are using centralized voltage control that may also control transmission network reactive power compensation. User-defined modeling is normally appropriate for these special controls.

Modeling of Loads

Voltage stability is load stability so most of this paper is devoted to load modeling.

Large-scale bulk transmission networks are simulated for interarea stability analysis. Voltage stability is different, and *we should not expect to use the same network representation*. Compared to interarea stability simulation, voltage stability analysis requires more detail of subtransmission network and bulk power delivery transformers, and possibly equivalents for distribution feeders. On the other hand, representation of remote parts of a large power system is usually not important.

Load dynamics are of two types:

1. Large load increases during a morning or afternoon pickup. In production-grade programs, provision should be made to exclude non-conforming (e.g., industrial) loads from the load increase.
2. Load restoration dynamics following a large disturbance.

The second type will be emphasized in this paper.

Load restoration is generally by three mechanisms:

1. Induction motor slip (inertial) dynamics. This occurs within a few seconds and discussion is

outside the scope of this paper.

2. Load tap changing (LTC) bulk power transformers or distribution voltage regulators that regulate voltage near the loads, and thus restore static (voltage-sensitive) loads.
3. Thermostatic or manual control of constant energy loads. This results in a loss of load diversity. Restoration starts after voltage at the loads is no longer controlled by tap changing.

Suggested load structure

With today's computer capabilities, there is no reason not to represent loads in some detail for voltage stability analysis. Voltage stability is load stability. Figure 2 shows a typical load equivalent as seen from the bulk transmission network.

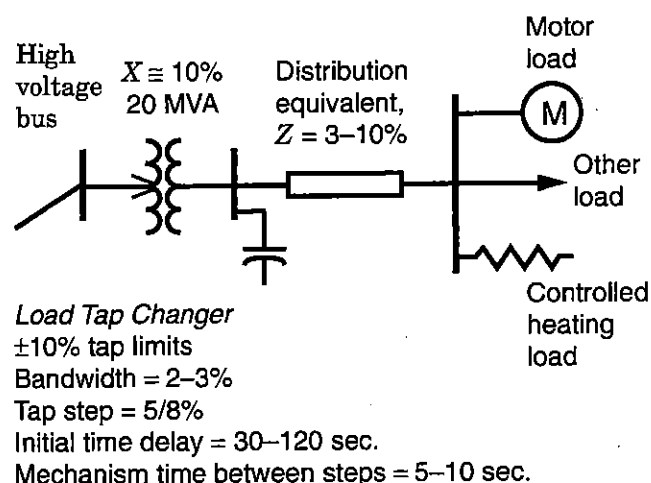


Fig. 2. Typical bulk power delivery LTC transformer and distribution equivalent (load model) for voltage stability simulation. LTC data reflects North American practice.

In Figure 2, the aggregated load components are induction motor equivalents, heating loads such as thermostatically-controlled electric space heating, and other loads such as lighting and transformer saturation. Except for the final stages of collapse, dynamic induction motor loads may not be needed for longer-term voltage stability analysis. The IEEE provides recommendations for modeling of the load components [3]. Reference 4 summarizes load component characteristics. Programs such as EPRI's LOADSYN [13] can be used as an aid in determining the load composition and data.

Note that Figure 2 represents all three mechanisms for load restoration.

The impedance of a bulk power delivery transformer is typically at least 10% on its own base, corre-

sponding to around 10% on the load base for heavy load conditions. Distribution transformer impedance is typically around 2–3%. Thus we can say with certainty that at least 10% impedance is present even if distribution feeder impedance is ignored. It's desirable to at least represent the impedance of the bulk power delivery transformer. A simplified version of Figure 2 would have only one distribution voltage bus, with the distribution feeder impedance ignored.

Tap changing effects are usually basic to voltage stability. The effect depends on the type of load, the impedance between the tap changer location and the load, and the shunt compensation. In some cases tap changing is beneficial to voltage stability. These effects can be captured in the Figure 2 model.

Compared to the usual practice of representing load at the high voltage busses, the Figure 2 model requires, for a large load area, some hundreds of additional busses. This is not a problem today.

In some special cases, even more detail may be appropriate such as distribution voltage regulators on long feeders, or shunt compensation at other locations.

The specific characteristics of individual bulk power delivery transformers can be incorporated into data sets relatively easily.

The literature shows the importance of the impedance between the high voltage bus and the load components [4,14].

Loads dynamics are discontinuous

The load restoration dynamics are discontinuous and nonlinear. The controlled heating load have limits that may be critical to voltage stability. Voltage decay under cold weather, heavy load conditions may lead to "all heaters on" resulting in stable, resistive load characteristic.

For the LTC transformers (or equivalent distribution voltage regulators), the tap limits are obviously important. During the heavy load conditions where voltage stability is a concern, taps may be near their boost limit; indeed, a voltage instability countermeasure is to adjust no-load taps so that only a few tap steps are available during heavy load.

The bandwidth (deadband), tap step size, and time delays indicated on Figure 2 may also be important. The time delay of at least thirty seconds allows stable operation initially; sensitive automatic controls may be applied during this time to restore conditions so that voltage instability mechanisms do not even start. In Chapter 7 of my book [4], I show how this may be done by reactive power compensa-

tion switching based on large generator reactive power output changes and small voltage drops. A voltage instability countermeasure is to simply lengthen the initial time delays to allow more time for either automatic or manual corrective actions.

The deadbands of tap changers may act to slow voltage decay. Particularly if loads are highly voltage sensitive, the initial voltage drops following a large disturbance will be quite small. Only some voltages will be outside of tap changer deadbands. After the time delay, one or two tap operations may return regulated side voltage to within the deadband with timer resetting. The resulting load restoration will, however, drive voltages outside deadbands of other tap changers. After timer operation, these tap changers will restore voltage and reset timers. This process continues until enough load is restored to stress the system so that tapping does not restore voltage within deadbands. The voltage decay is then more or less progressive at all locations until limits are reached, other stabilizing actions are taken, or collapse occurs.

Phenomena involving tap changer resetting and different time delay settings is believed to have slowed voltage decay and prevented a major voltage collapse in England on May 20, 1986 [1,15]. After five minutes of voltage decay, corrective actions (gas turbine response and circuit restoration) were effective in restoring voltage.

The IEEE provides recommendations for modeling of the tap changing transformers in dynamic simulation [3]. The recommended model is incorporated into a production-grade program and used in large scale studies [16].

With the notable exception of Van Cutsem [17,18], most researchers have ignored the discontinuous nature of the most important aspects of voltage stability. The discontinuous nature of the loads can be incorporated in sequential power flow simulation of the time evolution. Essentially, difference rather than differential equations are simulated.

Many investigators have approximated load restoration by tap changing and thermostats by a single time constant continuous model. Except for very approximate analysis, and when required for linear analysis, there is little justification or need for this simplification. The pure time delays, resetting of timers, and tap changer limits are too important to ignore.

Electric space heating load

Load restoration by tap changing is generally faster than load restoration of constant energy, thermostatically-controlled loads. Assuming the Figure 2

configuration, heating load will thus be restored by tap changing — unless tap limits are reached.

For cold weather, heavy load conditions, constant energy load will often be dominated by electric space heating. In some areas, the electric space heating during cold spells will be largely heat pump supplementary resistive heating.

Referring to Figure 2, for aggregated thermostatically-controlled electric space heating load comprised of hundreds or thousands of heaters of different characteristics, a single time constant model with limits is sufficient and appropriate [4,18]. Results by Graf [19] and some very valuable tests in Sweden [20–22] indicate time constants on the order of 3–5 minutes. Figure 3 shows a model [4,19].

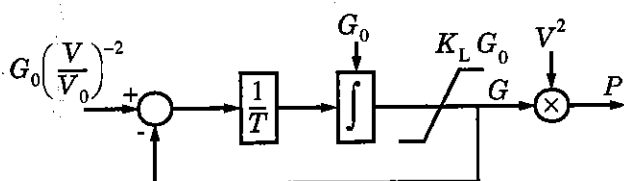


Fig. 3. Single time constant model of response of many controlled heating loads to voltage changes. G is load conductance, G_0 is initially connected load conductance. $K_L G_0$ is conductance with all loads on.

For cold weather voltage stability conditions, the amount and time constant of electric heating load restoration may be critical. Obtaining data for Figure 3 model, however, is difficult. While blocking de-stabilizing load restoration on bulk power delivery LTC transformers is relatively easy, blocking of load restoration of heating load is more difficult (but possible as part of direct load control automation).

As an example, reference 16 describes a large-scale study that assumed 20% additional conductance available during abnormally cold weather. For a very large disturbance, stable conditions with abnormally low voltages resulted after all tap changers and heating load reached limits.

BPA field tests of load restoration dynamics

Following identification of voltage stability problems in the Puget Sound (Seattle) area, BPA and other utilities conducted several field tests. Tests involving load restoration by tap changing are described in Chapters 4 and 7 of my book [4]. Results were pretty much as expected.

Similar to the tests in Sweden [20–22], other tests were made to determine response of thermostati-

cally-controlled electric space heating load. Tests were made in January of 1991, 1992, and 1993. The tests so far are largely inconclusive without the clear response observed in the Swedish tests. Because cold weather* did not occur during the test periods, the tests were mostly done during mild weather (around 40°F) and no load restoration was apparent.

The tests were done on two Snohomish County Public Utility District 12.5-kV residential feeders in the northern part of the Seattle metropolitan area. Voltage reductions of around 5% were made using distribution voltage regulators. Voltage, active power, reactive power, and temperature were monitored. One of the feeders, Meadowdale 12-1838, had older homes. The other feeder, Mukilteo 12-128, had newer homes with higher insulation levels. All tests were done during mid-morning.

Figure 4 shows the clearest indication of load restoration obtained. Figure 4 shows voltage and active power response for the Meadowdale feeder with older homes. The feeder load was nearly unity power factor. The temperature was around 26 °F, but there were some difficulties with the temperature monitoring. Temperature fluctuations of up to 7°F during the test period were recorded which, if correct, would influence the power response. The initial power changes following step voltage changes indicate close to a resistive load characteristic ($\Delta P/\Delta V$ close to 2 pu/pu).

The responses for the plus and minus voltage changes are not exponential. The time for a 63% response is around 12 minutes.

More testing is needed, particularly for colder weather conditions. Similar to the Swedish tests, tests should include evening and nighttime conditions. Analytic and "load research" work is also planned based on work completed for cold load pickup problems and energy conservation. There has been some success in predicting feeder load response as a function of temperature; see for example Figure 6 of reference 24.

R&D Areas for Voltage Stability Modeling

As a review paper for a research conference, it's appropriate to summarize areas for R&D. After around seven years of intense industry activity

* BPA forecasts peak winter loads for minimum temperature with one in two years probability of occurrence. This corresponds to -8°C (17°F) in the Seattle area. Voltage stability planning criteria also considers extremely cold weather conditions with one in twenty year probability of occurrence [4]; this corresponds to -17°C (2°F) in the Seattle area.

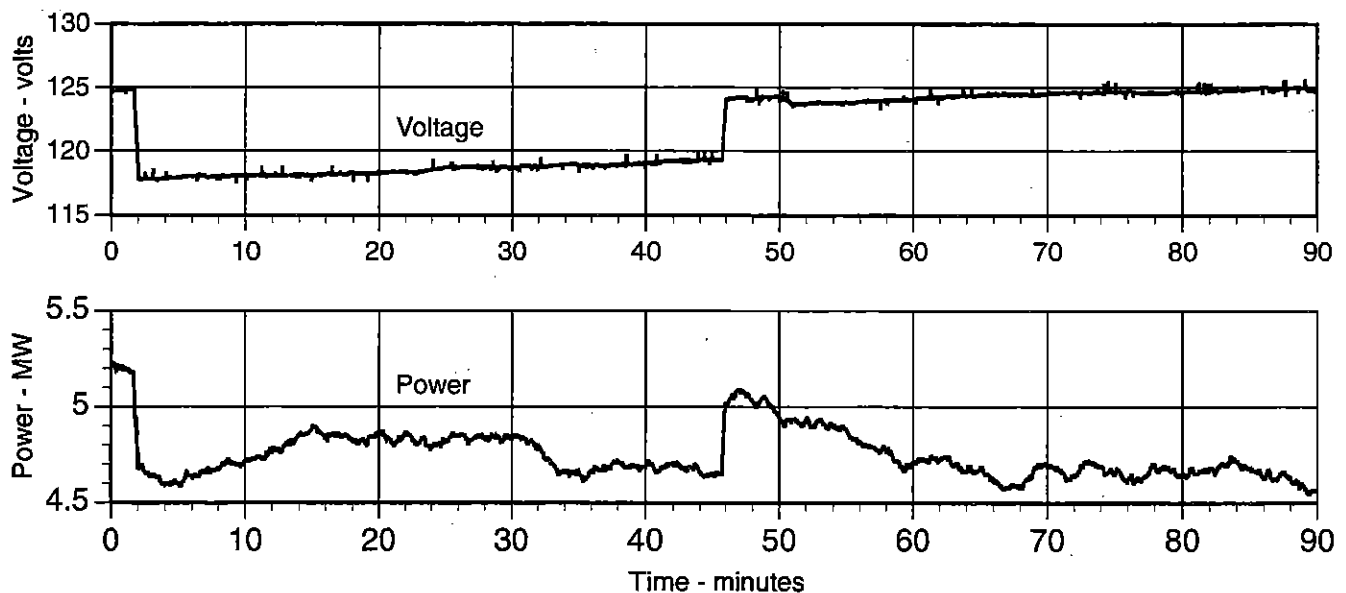


Fig. 4. Response of active power of Meadowdale feeder 12-128 for step changes in voltage (utilization voltage base). Recording taken on January 12, 1993 starting at 0939 hours. Temperature was about 26°F.

(starting, say, following the 1987 Tokyo blackout), we have a good understanding of most aspects of voltage stability phenomena and modeling. Some good simulation tools are now available—although more work needs to be done, particularly with regard to user friendliness.

Much of the remaining work is reducing the gap between our knowledge and actual power company practice.

Much of voltage stability analysis is still quite crude. The $V-Q$ curve method using constant power loads at high voltage busses is the standard method for many utilities. Can the voltage stability performance of a large load area really be judged by applying reactive power load at a single bus? In some cases, this crude analysis has resulted in over-design or overly conservative operation.

Research areas

Case studies are needed to show the sensitivity of voltage stability to power flow program generator modeling (PV/PQ bus type switching versus detailed generator model).

Although time domain simulation is available, more analysis of the discontinuous characteristics of tap changers on the time evolution of voltage stability or instability may be desirable. The May 1986 event in England mentioned above is not well understood.

Probably the most important area for modeling research relates to the characteristics of electric heating load. This is very important for wintertime

voltage stability problems. Computer programs such as LOADSYN need to be upgraded so that electric heating load data is cleanly interfaced with longer-term dynamics programs such as ETMSP [14].

Reactive power load effects are not well understood and seldom modeled very accurately. Tests usually indicate very high voltage sensitivity due to distribution transformers being operated in saturation at normal voltage. Utility tests have shown that distribution voltage reduction often leads to large, sustained reduction in reactive power demand.

Considerably more field testing of aggregated load response is warranted. With today's technology, the tests can be done at low cost from control centers. BPA power system monitoring uses Macintosh platform LabVIEW virtual instrument technology [25]. The recording computer can be located at the test site, with remote computer control and data downloading via modems. Alternatively, the remote signals can be directly telemetered over wide-area networks to desktops. SCADA may be used for capacitor bank or tap changer switching. By low-cost remote testing, a much better understanding of load characteristics can be obtained. Load characteristics could almost be determined in real-time. The main limitation is the understandable reluctance of operators to allow large voltage changes.

The nature of power system loads is changing and better understanding and modeling of power electronically-controlled loads is needed. Loads include various types of adjustable speed drives including adjustable speed air conditioners and heat pumps.

Improved practice areas

New computer program tools with improved modeling should be more widely used. Increased usage will provide justification for further improvement of the production-grade tools.

Work is required to determine models and data for field current limiters. In many cases, this will require visits to the power plants for verification of what is actually installed, and what the settings are. Database development in conjunction with the forthcoming IEEE models is required.

For large-scale simulation, network representation tailored to voltage stability analysis should be developed. This involves expansion of subtransmission networks and equivalents for distribution feeders. Available network reduction software can be used to reduce representation of remote portions of the interconnected power system.

Improved load models and data sets as described above (e.g., the Figure 2 structure), should be developed for voltage instability prone load areas. Methods such as LOADSYN should be used to determine load components as a function of season, load level (on or off peak), temperature, etc.

Although related more to transient voltage and angle stability, routine representation of dynamic induction motor equivalents has been recommended [3,26]. For the western North American interconnection, the WSCC Modeling Work Group has recommended that all loads larger than 30 MW include an equivalent motor model for a portion of the load; default data is used if specific data is not supplied. This will likely become standard practice within the Western Systems Coordinating Council.

Concluding Remarks

I've reviewed some of the more important aspects of modeling for voltage stability. My emphasis is on detailed models and dynamic simulation that should be used, at a minimum, as the final step in the planning or operating decision process. I've emphasized the discontinuous characteristics of the load restoration process. If desired, simpler models and other methods (including linear methods) may be used for preliminary assessment. Sequential, time-coordinated power flow simulation is one form of dynamic simulation.

Dynamic simulation may be time consuming, but this is a small problem with today's computing capabilities. Most workstations are not highly utilized, especially during off-hours.

Voltage stability analysis and modeling has many facets, and many aspects have not been mentioned,

but are available in references 1 and 4, and elsewhere. For example, modeling of automatic generation control, generator prime mover and energy supply system, undervoltage load shedding, and even expected operator actions is often important.

Models are never perfect and there will always be uncertainty in simulation results. Sensitivity studies and field validation are necessary. Decentralized undervoltage load shedding is a lost-cost means to protect against model shortcomings.

"Good information is valuable and much cheaper than hardware" [27]. Detailed modeling and simulation methods, along with good judgement, results in appropriate, cost-effective solutions to voltage stability challenges.

Acknowledgments. The load test results shown on Figure 4 are due to Dennis Erickson of BPA and Mike Winton of Snohomish County PUD.

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Biography

Carson W. Taylor joined Bonneville Power Administration in 1969 after earning degrees from the University of Wisconsin and Rensselaer Polytechnic Institute. His engineering at BPA includes power system control and protection, system dynamic performance, ac/dc interaction, and power system planning. His current position at BPA is Principal Engineer in System Engineering and Transmission Planning.

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Mr. Taylor is the author of the EPRI-sponsored book *Power System Voltage Stability* and has authored or co-authored many IEEE and CIGRÉ papers.

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VOLTAGE STABILITY INCLUDING DETAILED LOAD MODELS

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Abstract

This paper considers the importance of choice of bifurcation parameter in the determination of voltage stability. Using purely static models it illustrates the effect of composite load representations. When combined constant P constant Z loads are used, the effect is a "perception" that voltage bifurcations occur on the lower portion of the nose curve. The paper also illustrates the impact that various representations of speed versus mechanical torque characteristics can have on voltage collapse.

Introduction

This paper considers the importance of the choice of bifurcation parameter and the effect of load modeling in the determination of the voltage stability and maximum power loadability of a system. The paper considers both the point of maximum loadability of the system, and the saddle node bifurcation associated with increasing power demand. When the only "parameter" that is permitted to vary is power demand, both the bifurcation parameter and the maximum loadability points coincide. However, when demand depends on some other parameter, then the maximum loadability point differs from the bifurcation point.

This paper uses the accepted terminology that defines voltage collapse as a saddle node bifurcation, which corresponds to the appearance of a singular (real) eigenvalue in the Jacobian matrix for the system dynamic equations.

This paper depends crucially on the idea that, for the case of a saddle node bifurcation, a static model yields the same bifurcation point as the complete dynamic model [Dobson94]. This permits this paper to concentrate exclusively on static models.

This paper is based on numerous prior efforts, almost too numerous to consider in this paper. A paper that prompted the present work is [Lesiuetre93]. Important recent references that consider load modeling aspects of voltage collapse (and which consider the dynamics of load representation) include [Canizares94, Hill93, Hill94, Pal92, CIGRE92, EPRI87]. Refer to any of these for more extensive lists of further references. Suitable computational references for the determination of the saddle node bifurcation point include [Mansour90, Canizares92], among others.

Composite Load Models

Loads consist of the aggregation of diverse physical components. These components include impedances, such as arc furnaces and electric lighting; induction motors, as in many industrial processes and in air conditioners; electronic loads; voltage dependent loads of various kinds; and many more physical devices. An emerging class of loads are induction motors served by solid state drives. The effect of increasing use of these types of loads may have profound effects on the voltage collapse characteristics of the system. By illustrating the differences between the behavior of induction motor loads and constant power loads, this paper sheds some light on the significance of this change.

Composite loads can be treated in two different ways: first, they can be considered as "aggregate black boxes," or they can be considered as "gray boxes" that show some of their internal structure. Reference [Hill93] presented very successfully the use of the black box approach to load representation. This model is very good. However, it fails to capture some of the information available from load structure.

Time frame is important when discussing quasi-steady state load models. A load that may behave as a constant impedance load in a short time frame, may

very well behave as a constant power load in a different time frame. Two examples of this type of behavior include loads served by adjustable tap-changing transformers, and multiple thermostatically controlled heating loads. The behavior of the load as either a constant power or a constant impedance, in the former case, depends also on whether the tap changing transformer has reached a "limit" or has otherwise been "blocked."

Each physical load can vary its demand on the network. The parameter that initiates the demand variation plays an important role in the determination of voltage collapse. For example, the increase of torque within an induction motor leads to an increase in power demand. Thus, power demand should not be used as the bifurcation parameter. Torque should be the parameter. If, in turn, torque depends on motor speed, this dependence also warrants representation. Specific examples of possible bifurcation parameters include the following:

- For constant P constant Q loads, the power factor remains constant and the active power component P of the constant power portion of a load is the bifurcation parameter of interest. Bus demands can be distributed among a set of buses according to given distribution factors.
- For constant impedance loads, the admittance magnitude Y can be used as a bifurcation parameter.
- For induction motor loads there are a number of choices. It is possible to consider the torque T as a bifurcation parameter. However, in many cases it will only be necessary to consider T not as a bifurcation parameter, but rather as a quantity that depends on the speed (slip) of the motor.

One of the main objectives of this paper is to illustrate the dramatic effect that various torque-speed assumptions can have on voltage stability.

Summarizing, this paper proposes: (1) the use of static models of loads suitable for a pre-selected time frame of interest, (2) the representation of each load in some detail, and (3) the representation of composite loads explicitly.

The issues raised in this paper are best illustrated by means of a small examples. The next two sections consider two small examples. The first example considers a composite constant power constant impedance load. The second example illustrates the

case of a load consisting of an induction motor in parallel with a constant power load. In both cases, the system is represented as a Thevenin equivalent.

Constant Power Constant Impedance Load

The system studied is illustrated in Fig. 1. The upper load emulates a load that is under tap changer regulation. As a result of the tap changer, and in the time frame of interest, the load sees a constant voltage, and therefore behaves as a constant power load. The percentage of the total load that is constant impedance under base case conditions is allowed to vary.

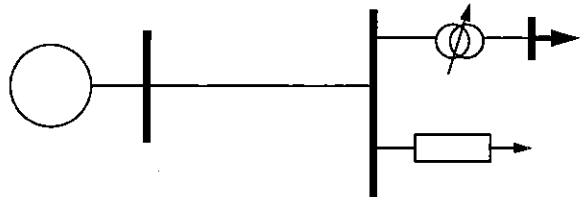


Figure 1: Constant P constant Z sample system.

The following are additional facts about this system:

- The Thevenin voltage and the Thevenin impedance are fixed and known. The voltage is 1.1 pu and the pu admittance is 5.6.
- The bifurcation parameter of interest is the active power demand in the constant power load. The power factor is assumed fixed and equal to 0.852.
- The base case power is 0.2875 pu.
- The power of interest is the total combined power delivered to both loads.

The first study using this system (illustrated in Fig. 2) assumes that there is no constant impedance load. This is just a conventional "nose curve." The next figure, Fig. 3, illustrates the same case under the assumption that 10% of the base case demand is dissipated in the constant impedance load. The constant admittance is calculated so that under base case conditions the power delivered to the constant impedance load is 10% of the power delivered to the constant P load.

The result of this is that now the point at which the constant power load parameter P bifurcates is different from the point of maximum total power loadability of the composite system. This is clearly shown in Fig. 3. Fig. 3 also illustrates that, under these conditions, there is a *perception* that the system bifurcation occurs in the

“lower” part of the bifurcation diagram. It actually occurs in the lower part of the nose curve.

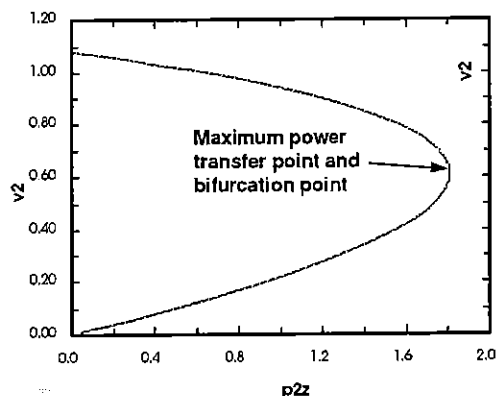


Figure 2: PV curve assuming 0% of the base case load is constant impedance load. This is just a conventional “nose curve.”

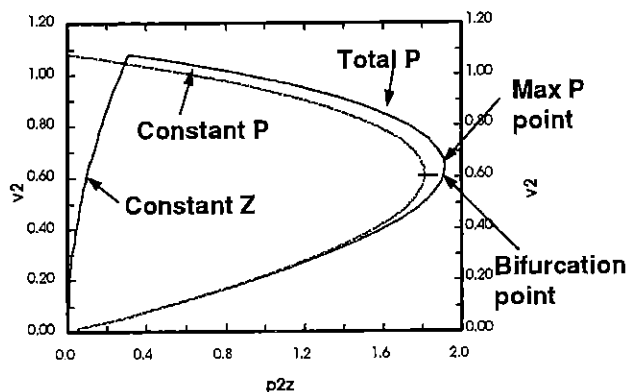


Figure 3: Total P vs. V curve assuming 10% of the base case load is constant impedance load. Maximum power point differs from bifurcation point.

Constant Power Constant Impedance Load

We now consider a composite load with a constant power component and an induction motor load component. This composite load system is illustrated in Fig. 4. As before, it is assumed that, for the time frame of interest, the tap changer holds the load power constant. The induction motor model is a conventional, with torque-speed characteristics as illustrated in Fig. 5.

For this example, the size of the induction motor relative to the size of the constant power load was varied from negligibly small to equal to the fixed load under base case conditions. The mechanical torque-speed characteristics of the mechanical load to which

the motor is connected was also varied to study its influence on the collapse. The following types of mechanical torque-speed characteristics were used in these studies:

- Torque varies as the square of speed. This emulates a “fan” type load.
- Torque that varies linearly with speed.
- Torque that varies as the square root of speed, in an attempt to emulate approximately a constant torque load that eventually diminishes.

Figure 6 illustrates the first example. This example assumes a negligibly small induction motor, with a mechanical torque that varies as the square. Observe that, for this motor, stall occurs at a voltage slightly higher than the maximum loadability point of the system.

Figure 7 illustrates the same example, but for a situation where the size of the induction motor is equal to the base case constant power load. The influence of the motor stall is very evident. The motor stalls at the “top” nose. However, the remainder of the load is capable of continuing operation at increasing levels of power (at least theoretically) until the second nose is reached. In this example, the second nose power is actually lower than the power prior to motor stall, also giving the impression that the bifurcation occurs on the lower portion of the nose curve. The line labeled “constant torque operation” indicates what happens to the nose curve if truly constant torque operation is assumed: the motor actually becomes an induction generator and starts producing power that helps support the voltage. Reference [Alvarado94] considered this assumption, but this is probably unrealistic.

Fig. 8 illustrates a similar case, but where the mechanical load is a linear function of speed. Observe that for a certain region of voltages there are actually three possible motor power solutions. Of course, in reality the system will likely “bypass” the loop and proceed directly into a stalled motor condition.

Finally, Fig. 9 illustrates the case of a motor with a mechanical load with a torque proportional to the square root of the speed, in an attempt to coarsely approximate those mechanical loads where torque is reasonably constant, at least for a while. The second nose, obtainable after the stall, can be larger than the first nose.

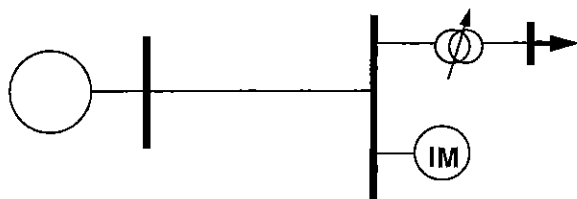


Figure 4: Sample system for composite constant P plus induction motor load.

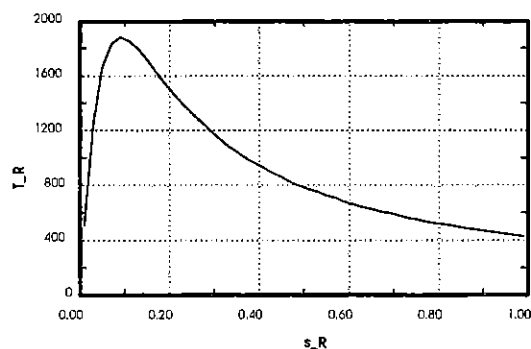


Figure 5: Torque-slip characteristics for induction motor load.

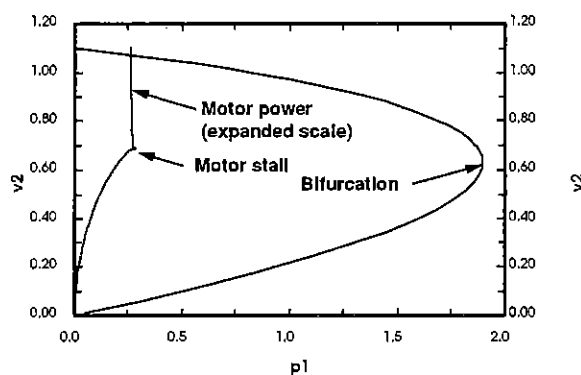


Figure 6: Nose curve assuming a negligibly small induction motor. The power versus voltage characteristics for the motor assuming a torque that varies as the square of speed is also illustrated using an expanded horizontal scale for the motor power. The stall voltage point is illustrated.

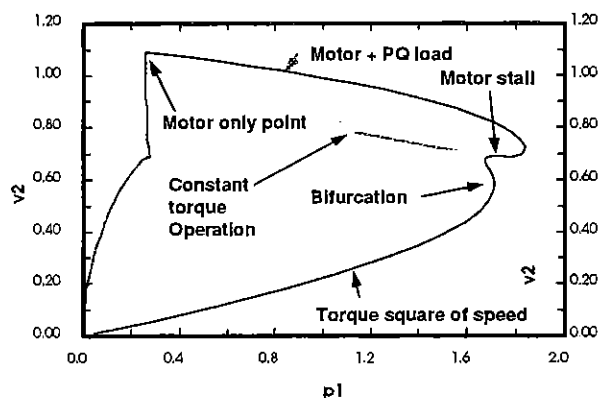


Figure 7: Nose curve for operation with a motor with a fan load (torque is proportional to the square of speed) and an induction motor size that is equal to the constant power load.

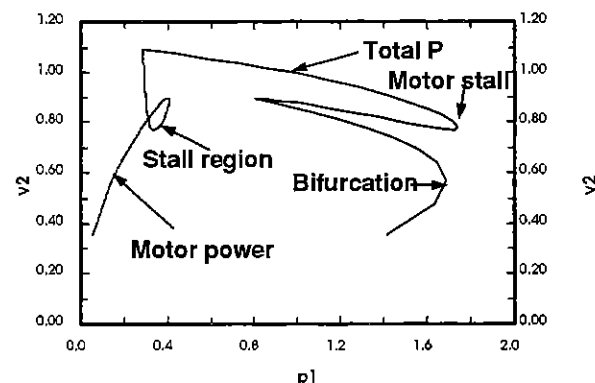


Figure 8: Nose curve when the motor mechanical torque is linearly proportional to speed.

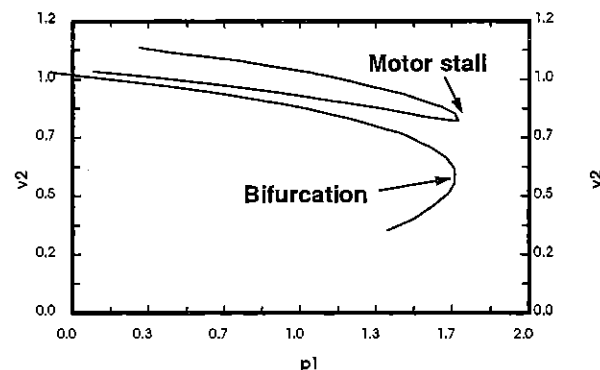


Figure 9: Nose curve when the motor mechanical torque is proportional to the square root of the speed, approximating a torque that is reasonably constant over the operable range.

Conclusions

The static characteristics of a load play a significant role in the determination of voltage collapse. A phenomenon as simple as the stalling of an induction motor (including the nature of the mechanical load connected to the motor) all play a role.

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Load Structure: Identification and Impact on Voltage Stability

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1. Introduction

The role of power system load has long been acknowledged as an influential factor in development of voltage instabilities, and recent studies [2] further clarify its relationship with structural stability of the power network. Sometimes they produce the complete system blackout following the large contingencies [5], sometimes the blackouts appear to happen regardless of seemingly appropriate network conditions [4].

Quite often, voltage problems occur in milder forms, not causing the complete system collapse, but rather creating problems in localized parts of the network. It is well known that voltage problems are intimately related to the loading level (heavy system loading always accompanies voltage problems) and that they are very sensitive to the balance of reactive powers in the network [4], [5], [10], [21], [23], and dynamics of tap-changing transformers [7]. Many authors have proposed measures of proximity to voltage instability [6], [8], [9-11], [22], which utilize state, or parameter based functions to assess the stability margins. The problem of load structure, both from the standpoint of nonlinearity with respect to voltage and time dynamics, has been studied by many researchers. Pal [12] addresses the influence of static and dynamic load components and concludes that the increased presence of the static load components may benefit the stability. Langevin and Auriol [13]

consider the power function-based load model and calculate the effects of its parameters on stability parameters such as damping and synchronizing torques in the network. Price et al. [15] describe the load model synthesis algorithm for the commercial grade computer software. Hill and Karlsson [14], [1] recognize the impact of a dynamic nonlinear voltage dependent load model and develop a model based on substantial field testing. It represents an experimental assessment of the composite load behavior under voltage perturbations, and it is characterized by relatively slow dynamics, which could be decoupled from the network short-term dynamics for the analysis purposes.

Monitoring of the structure of the composite load in the transmission network, and assessment of its impact on voltage stability are very interesting problems, at least from the practical standpoint. Extensive measurements [1] suggest that those parameters are subject to rather large variations. An approximate on-line procedure for parameter identification, based on recursive least squares technique [18-20], has recently been proposed [25] for tracking parameter variations in time. Further refinements and impact of the real-time load structure monitoring on voltage stability have been investigated in [26], where significant impact of load disturbances on voltage stability boundary was observed in simulations. In this paper, we investigate some of the effects of changing structure and extent of the loads on the stability of the power network.

2. Load Modeling and Identification

The importance of load modeling with respect to voltage stability analysis is considerable. From the system standpoint, one can ask different questions about the impact of loads on the power system:

- What is a load composition during a voltage disturbance and how does it impact the voltage stability?
- How does the difference in patterns of loading in the power network affect the stability boundary, and is the impact of load composition different for different loading patterns?

Pai, Sauer, Lesieutre, and Adapa [2] have studied the impact of load formulation on structural stability of the power system. They investigate, among other things, a load formulation

$$S(V) = P_0 \cdot V^{K_p} + jQ_0 \cdot V^{K_q} \quad (2.1)$$

and show that the bifurcation is possible only when $K_p < 1$, $K_q < 1$. Karlsson and Hill [1] investigate a dynamic load structure which can be described in steady state by a formulation similar to (2.1). They propose a model

$$\begin{aligned} T_p \frac{dP_r}{dt} + P_r &= P_0 \left(\frac{V}{V_0} \right)^{\alpha_s} - P_0 \left(\frac{V}{V_0} \right)^{\alpha_t} \\ P_d &= P_r + P_0 \left(\frac{V}{V_0} \right)^{\alpha_t} \end{aligned} \quad (2.2)$$

where P_0 , α_s , α_t , are constants, P_r is active power load recovery, while P_d is the composite load model. T_p is a time constant. The model (2.2) is a result of extensive experimentation and measurements, and it is believed that it represents accurately the dynamics of the load with respect to the step and ramp changes of the voltage. The dynamics described by (2.2) are slow, and for the purposes of voltage stability analysis, it may be

assumed that at any point of time the value of the load is $P_d(t)$ (2.2), which has a slowly changing structure having voltage characteristics in between the steady state characteristics $P_0(V/V_0)^{\alpha_s}$ and the one associated with the transient $P_0(V/V_0)^{\alpha_t}$. These nonlinear characteristics have been proposed to be decomposed into a time varying combination of constant power, constant current injection, and/or constant impedance loads [26], which can be obtained from the first three terms of the Taylor series of (2.1) at any time instant. This form is convenient for subsequent analysis and real-time parameter identification.

The authors have investigated the relative participation of various load components during the transient [26] by using a recursive load identification of an assumed combination of constant power and constant impedance load. Such identification, even if applied on a nonlinear load of the type (2.1), may produce useful information about the distribution of equivalent constant power and impedance loads, which may then be utilized to correct the estimates of the stability margins. The recursive relationship that produces this result is

$$\begin{aligned} \hat{\theta}_{k+1} &= \hat{\theta}_k + G_k [P_{k+1} - \varphi_{k+1}^T \hat{\theta}_k] \\ G_k &= R_k \varphi_{k+1} [1 + \varphi_{k+1}^T R_k \varphi_{k+1}]^{-1} \\ R_{k+1} &= [I - G_k \varphi_{k+1}^T] R_k / \beta \end{aligned} \quad (2.3)$$

where $\hat{\theta}_k$ is a vector of estimates of the unknown load parameters, while $\varphi_k = [1 V_k^2]^T$ is a vector containing voltage magnitude measurement, and P is active power measurement. Subscript ' k ' denotes the time instant, and β is a forgetting factor, which assigns lower weight to old measurements in the recursive scheme. The results obtained by simulations [25, 26] suggest that this technique could be effectively used to assess the changes in the load composition (of the loads described by (2.2)) during slow voltage

transients and apply the results for assessment of voltage stability while taking into account that the loads are changing not only quantitatively, but also qualitatively during those disturbances.

3. Critical Load in a Radial Network

Let us assume that an ideal voltage source is connected to a nonlinear load via a lossless transmission line of reactance X . The terminal voltage of the source (a generator) is maintained at V_1 .

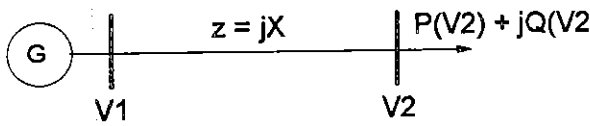


Figure 1. A simple network with a nonlinear load.

The load is a nonlinear function of voltage

$$S_L = P(V_2) + jQ(V_2) \quad (3.1)$$

which will, for the purposes of this example, be chosen to be a composite of the constant power and constant impedance load components, parametrized via a single loading parameter which we call a load factor k . By increasing, or decreasing k , we may simulate the process of system loading, or unloading.

$$\begin{aligned} P(k) &= k(P_0 + G \cdot V_2^2) \\ Q(k) &= k(Q_0 + B \cdot V_2^2) \end{aligned} \quad (3.2)$$

The relationship between active and reactive load component has to satisfy the equation

$$\left(Q(k) + \frac{V_2^2}{X} \right)^2 = \left(\frac{V_1 V_2}{X} \right)^2 + (P(k))^2 \quad (3.3)$$

When (3.2) is substituted into (3.3), the resultant expression yields

$$A \cdot V_2^4 + B \cdot V_2^2 + C = 0 \quad (3.4)$$

where

$$\begin{aligned} A(k) &= k^2 (G^2 + B^2) X^2 + 2kBX + 1 \\ B(k) &= 2k^2 (Q_0 B + P_0 G) X^2 + 2kQ_0 X - V_1^2 \\ C(k) &= k^2 Q_0^2 X^2 \end{aligned} \quad (3.5)$$

Equation (3.4) is a bi-quadratic equation whose solution yields a well-known P-V curve when the load is a varying constant power load ($G = B = 0$). Each of the coefficients $A(k)$, $B(k)$, $C(k)$ is a quadratic function of k

$$\begin{aligned} A(k) &= a_2 k^2 + a_1 k + a_0 \\ B(k) &= b_2 k^2 + b_1 k + b_0 \\ C(k) &= c_2 k^2 \end{aligned} \quad (3.6)$$

where the values of coefficients in (3.6) are obvious from (3.5). The bifurcation value of (3.4) can be calculated by solving for zero of discriminant of its solution, which produces a fourth order polynomial equation

$$\alpha_4 k^4 + \alpha_3 k^3 + \alpha_2 k^2 + \alpha_1 k + \alpha_0 = 0 \quad (3.7)$$

where

$$\begin{aligned} \alpha_4 &= b_2^2 - 4a_2 c_2 \\ \alpha_3 &= 2(b_1 b_2 - 2a_1 c_2) \\ \alpha_2 &= b_1^2 + 2b_0 b_2 - 4a_0 c_2 \\ \alpha_1 &= 2b_0 b_1 \\ \alpha_0 &= b_0^2 \end{aligned} \quad (3.8)$$

The smallest positive real root of (3.7) represents the critical value of the load factor. Figure 2 shows critical load factors for the various mixtures of the constant power and impedance loads at various power factors in a sample system having a transmission line reactance 0.2 pu and maintaining the source voltage at 1.0 pu .

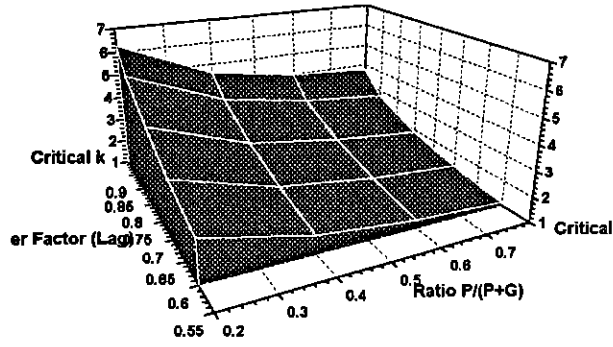


Figure 2. Critical load factor vs. power factor and ratio $P_0/(P_0+G I^2)$ for a system shown in Figure 1 having $X = 0.2$ pu and $V_1 = 1.0$ pu.

When the critical load factor is calculated for a certain combination of parameters, it is possible to estimate its perturbation with respect to the change of any of the parameters. If p_0 is the value of a changing parameter for which the critical load factor is k_0 (solution of (3.7)), we can write

$$f(k_0, p_0) = \sum_{n=0}^4 \alpha^n(p_0) k^n = 0 \quad (3.9)$$

When the parameter assumes a perturbed value p , linear expansion of (3.9) into Taylor series produces

$$\begin{aligned} f(k, p) &= f(k_0, p_0) + \\ &\left. \frac{\partial f}{\partial k} \right|_{(k, p)=(k_0, p_0)} (k - k_0) + \\ &\left. \frac{\partial f}{\partial p} \right|_{(k, p)=(k_0, p_0)} (p - p_0) + h.o.t. \end{aligned} \quad (3.10)$$

from which the new value of critical load factor may be estimated as

$$k(p) \approx k_0 - \left[\frac{\partial f}{\partial k} \right]^{-1} \left[\frac{\partial f}{\partial p} \right] (p - p_0) \quad (3.11)$$

For example, the critical load factor for a system having 80 percent constant power load at rated voltage, power factor 0.95 lagging, line reactance 0.2 pu and source voltage 1.0 pu, is $k_0 = 3.71$. If

the parameter of interest is the source voltage ($p = V_1$), the application of (3.11) indicates that a critical load factor would be reduced to 3.63 if the source voltage were reduced by 1 percent. The exact calculation would produce $k = 3.65$.

The illustration of the proposed monitoring follows in Figures 3-8. A load (2.2) is subjected to a five percent voltage reduction (Figure 3). The parameters of the load [1] are

$$\begin{aligned} \beta_s &= 1.72 & \alpha_s &= 0.54 \\ \beta_t &= 4.76 & \alpha_t &= 2.20 \\ T_q &= 2 \text{ min} & T_p &= 2 \text{ min} \\ Q_0 &= 1 \text{ pu} & P_0 &= 1 \text{ pu} \end{aligned}$$

(Parameters in the left column are the reactive load's counterpart of the active load parameters (2.2)). Figures 4 and 5 represent the actual active and reactive power of the nonlinear load during the above described slow disturbance. Figures 6 and 7 are the result of estimation of the load components using (2.3). Figure 8 is the estimate of the critical loading factor using (3.7) with time varying components of the load. It is quite clear from that Figure that a disturbance results in a significant reduction of the transient voltage stability margin under the described circumstances.

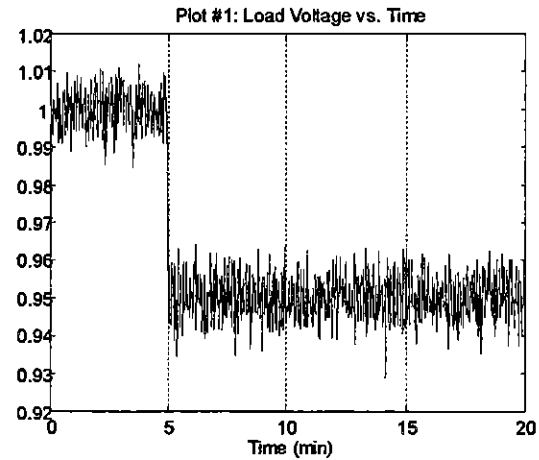


Figure 3. Five percent step change in load voltage.

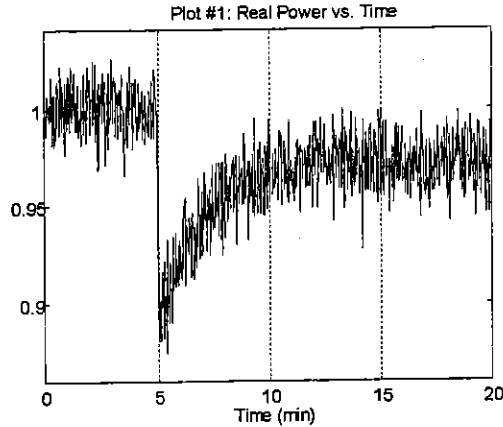


Figure 4. Real power in pu vs. time for a step change in voltage from 1.00 pu to 0.95 pu.

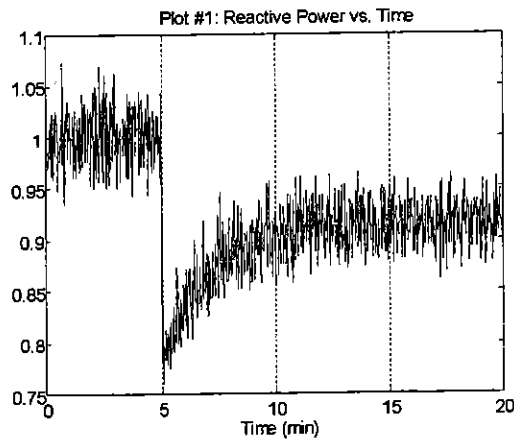


Figure 5. Reactive power in pu vs. time for a step change in voltage from 1.00 to 0.95 pu.

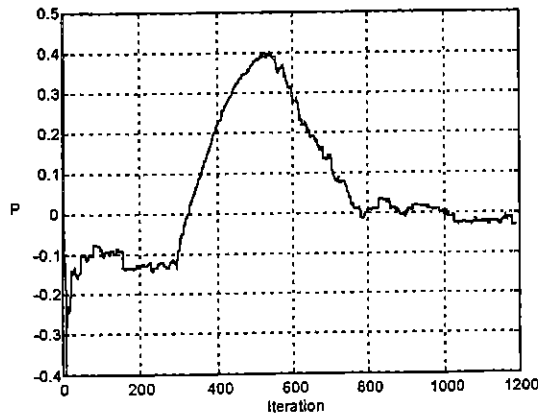


Figure 6. Estimated constant active power component of the load during the disturbance shown in Figures 3-5.

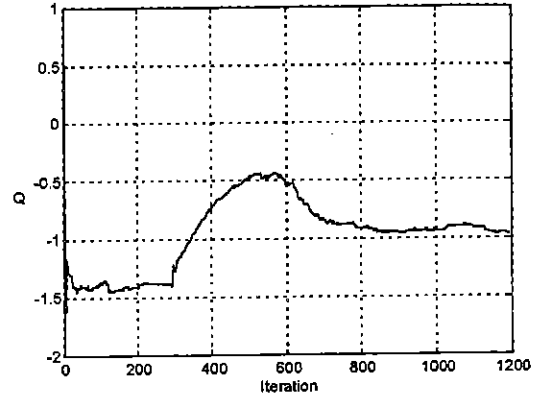


Figure 7. Estimated constant reactive power component of the load during the disturbance shown in Figures 3-5.

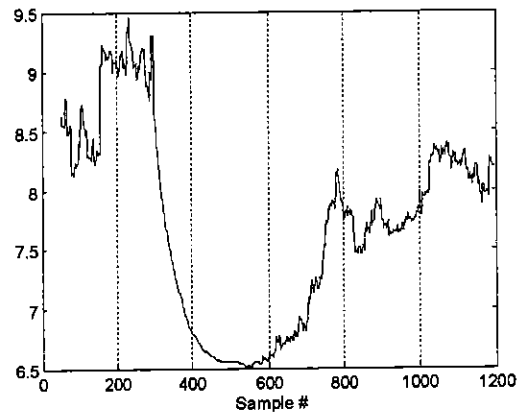


Figure 8. Variation of the estimated critical load factor during the disturbance of a nonlinear load (see text).

4. Loading Patterns and Voltage Stability

A dynamic model of a power network may be represented by a differential equation

$$\dot{x} = f(x, \lambda)$$

where $f: \mathcal{R}^n \times \mathcal{R}^p \mapsto \mathcal{R}^n$ is a nonlinear vector valued function of the state ($x \in \mathcal{R}^n$) and parameter ($\lambda \in \mathcal{R}^p$). In the steady state, the power system model can be characterized by equilibria of the above equation, which are solutions of the system of nonlinear algebraic equations

$$f(x, \lambda) = 0$$

It is well established that many cases of voltage instabilities in power networks are due to the static bifurcations of the dynamic model, caused by the slow perturbations of the parameter vector. In general, patterns of change of the parameter vector are spanning the $\mathcal{R}^p$ space, which, in the case of even small power systems, may be dimensionally very challenging. In practice, loads in large scale systems behave relatively coherently and may be parametrized via a small number of independent parameters

$$\lambda = h(\vartheta)$$

where $h: \mathcal{R}^m \mapsto \mathcal{R}^p$ is a parametrizing function and $\vartheta \in \mathcal{R}^m$ is a parameter vector ($m \ll p$). Thus, the steady state model of the power system may be written in the form

$$g(x, \vartheta) = 0$$

where $g: \mathcal{R}^n \times \mathcal{R}^p \mapsto \mathcal{R}^n$ reflects the dependence of the model on a small number of newly introduced parameters.

Some simulation results are presented below. The IEEE 39-bus system, which is shown in Figure 11a, was used to illustrate the effect of loading patterns on voltage stability. The test system was modified such that it could accommodate four different types of load: 100% constant power, 50% constant power and 50% constant impedance, 25% constant power and 75% constant impedance, and 10% constant power and 90% constant impedance. In all simulations, initial setting for load impedance was found from requirement that the modified system consumes the same power under base case scenario. The loading was then accomplished via selective multiplier

$$S_i(k) = k \left\{ (P_{io} + jG_{io} \cdot V_i^2) + j(Q_{io} + jB_{io} \cdot V_i^2) \right\}$$

which can be applied on some, or all of the buses in the system. Subscripts "io" indicate the initial values of parameters of bus i at the base case, and V_i is the voltage magnitude at bus i .

The voltage behavior of the 39-bus test system with respect to four different types of loads is demonstrated in Figures 9-11. Figures 9 and 10 show the voltage descent as the load factor k progressively increases (applied on all the system buses) for the four most critical buses in the system. Figure 11 shows the voltage responses under different types of loads for bus 12. Predictably, the most dramatic voltage drops occur in the case when the loads are constant power.

For the illustration of different loading patterns, we choose the three-parameter loading of the system

$$S_i(k) = \sum_{j=1}^3 k_j(i) [P_{io}(V_i) + jQ_{io}(V_i)]$$

The test system is divided into four regions:

- Region 1: buses 1, 2, 3, 17, 18, 25, 26, 27, 30, and 37.
- Region 2: buses 5, 6, 7, 8, 9, 11, and 12.
- Region 3: buses 19, 20, 21, 22, 23, 24, 29, 33, 34, 35, 36, and 38.
- Region 4: buses are 4, 10, 13, 14, 15, 16, 28, 31, 32, and 39.

The three-parameter loading is designed so that regions 1, 2, and 3 are independently loaded via selective multipliers k_1 , k_2 , and k_3 . Region 4, which is an interface between the first three regions, is loaded by all three multipliers: $k_4 = (k_1 + k_2 + k_3)/3$.

The three load factors were examined and a critical surface was found. Four different points on the surface were chosen to study the sensitivity of the generated reactive power with

respect to the load power requirements. They correspond to the cases of extreme loadings in any one of the directions along k_1 , k_2 , k_3 , while the others are kept at their base case loading values, and one loading direction when all three factors are uniformly applied. Figures 12-21 illustrate the different cases studied. The first point chosen for this study is $k_1 = 2.47$, $k_2 = 1.00$, and $k_3 = 1.00$. Figures 12 and 13 illustrate the sensitivity of the generated reactive powers [23-24] with respect to real and reactive load powers, respectively. The most sensitive generator buses are 31, 33, 34, 36, and 38. All of these buses except bus 31 are in region 3.

The next case is $k_1 = 1.00$, $k_2 = 1.00$, and $k_3 = 1.81$. This case is shown in Figures 14 and 15. From Figure 14, the sensitivities of generators at buses 31, 32, and 37 show the largest values. Buses 31 and 32 are located in the boundary region, while bus 37 is in region 1. From Figure 15, the sensitivities with respect to reactive load powers are much larger in magnitude than the sensitivities with respect to real powers, which is expected. Again, generators 31 and 32 are the most sensitive.

The third case corresponds to $k_1 = 1.00$, $k_2 = 2.14$, $k_3 = 1.00$. Figures 16 and 17 illustrate this case. Reactive power sensitivities of generators 31, 32, 35, and 38 are the largest in this case. Generators 35 and 38 are located in region 3 and 31 and 32 are located in the boundary region.

The last case was chosen to correspond to a uniform loading of all three, specifically $k_1 = 1.74$, $k_2 = 1.77$, $k_3 = 1.20$, which is shown in Figures 18 and 19. From Figure 18, the sensitivities of generators 31, 33, 34, 36, 37, and 38 are the largest. All of these buses are located in region 3 except 31 and 37.

Figures 20 and 21 illustrate one case for the 25% constant power and 75% constant impedance load at the point of uniform loading ($k_1 = 1.90$, $k_2 = 2.90$, $k_3 = 1.19$). The sensitivities with this type of load are very similar in magnitude and

location to the constant power case shown in Figures 17 and 18.

Conclusions

Both quality (voltage dependence) and quantity (the pattern of loading) influence strongly the voltage stability. A recursive identification procedure has been proposed for assessment of the basic components of the composite load model equivalent to the recently established dynamic nonlinear models obtained through intensive field testing. The transient growth of the constant power component of the equivalent composite load has been consistently associated with the slow transient voltage disturbances. Subsequent analysis of the critical loading factor in a radial network revealed that transients may be characterized by a substantial reduction of the transient voltage stability margin.

The analysis of voltage stability in parameter space is made difficult by many degrees of freedom that various load components have. In practice, loads behave very coherently and may be parametrized via a small number of independent parameters. By means of examples of 3-parameter loading in a 39-bus test system, it was shown how the sensitivities of generators with respect to load parameters reveal different areas of critical load and generator buses. The results of simulations also suggest that critical areas in the network depend more on the loading pattern than on the structure of the load.

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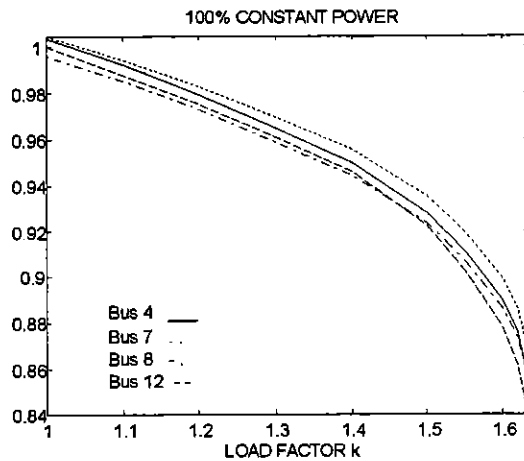


Figure 9. Voltage Magnitude in PU vs Load Factor on four buses for 100% Constant Power Load.

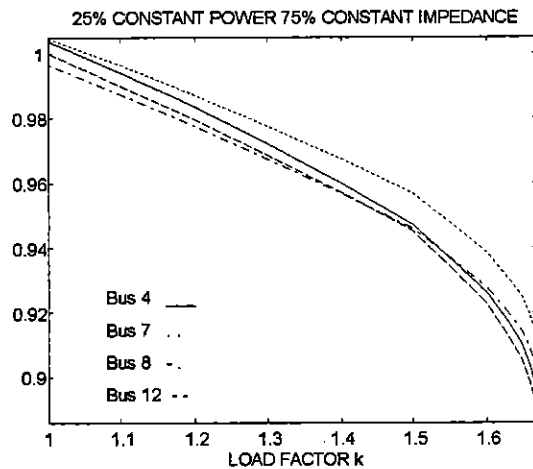


Figure 10. Magnitude in PU vs Load Factor on four Voltage buses for 25% Constant Power and 75% Constant Impedance Load.

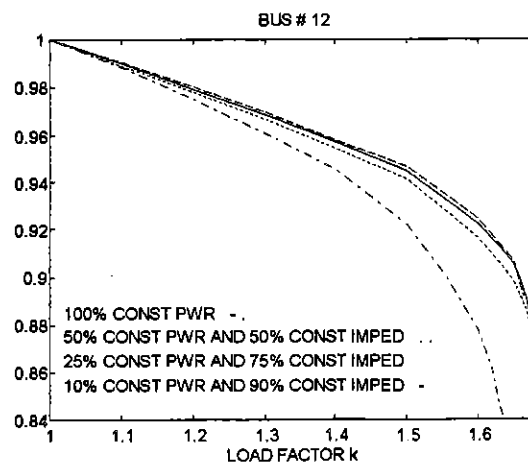


Figure 11. Voltage Magnitude in PU on Bus 12 vs. Load Factor for four different Loads.

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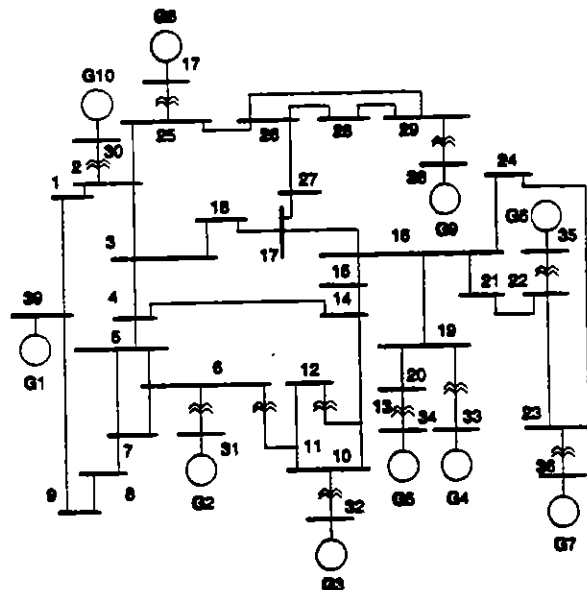


Figure 11a. A 39-bus test system used in simulations.

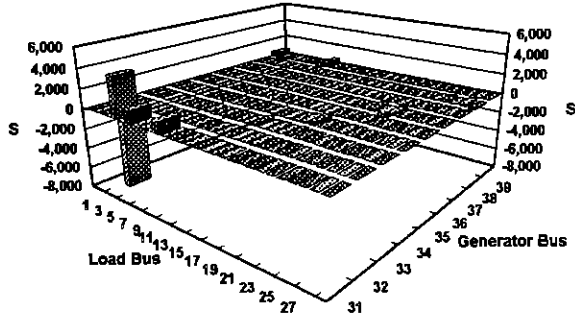


Figure 12. Sensitivity of Generators with respect to Real Power of each Load Bus for the case of 100% Constant Power and $k_1 = 2.47$, $k_2 = 1.00$, $k_3 = 1.00$

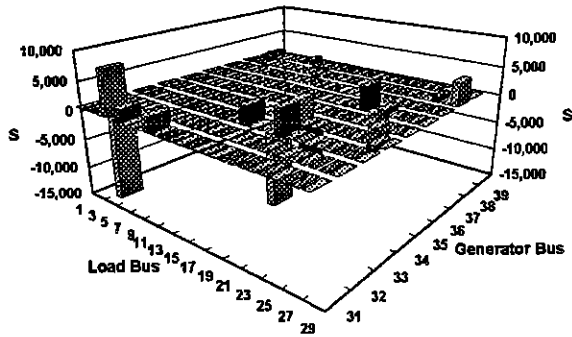


Figure 13. Sensitivity of Generators with respect to Reactive Power of each Load Bus for the case of 100% Constant Power and $k_1 = 2.47$, $k_2 = 1.00$, $k_3 = 1.00$

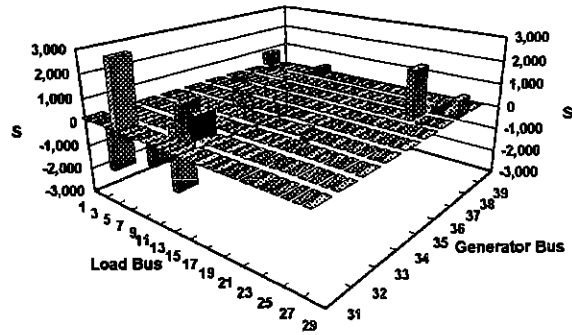


Figure 14. Sensitivity of Generators with respect to Real Power of each Load Bus for the case of 100% Constant Power and $k_1 = 1.00$, $k_2 = 1.00$, $k_3 = 1.81$

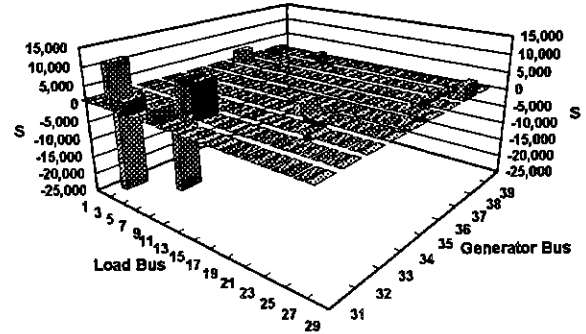


Figure 15. Sensitivity of Generators with respect to Reactive Power of each Load Bus for the case of 100% Constant Power and $k_1 = 1.00$, $k_2 = 1.00$, $k_3 = 1.81$

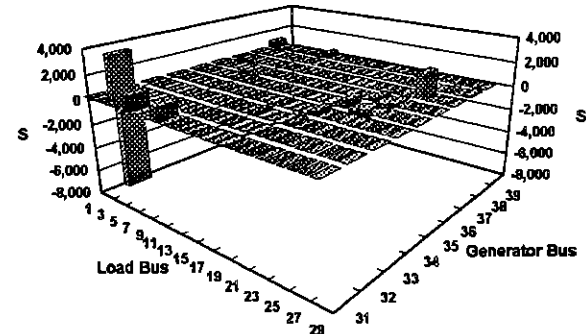


Figure 16. Sensitivity of Generators with respect to Real Power of each Load Bus for the case of 100% Constant Power and $k_1 = 1.00$, $k_2 = 2.14$, $k_3 = 1.00$

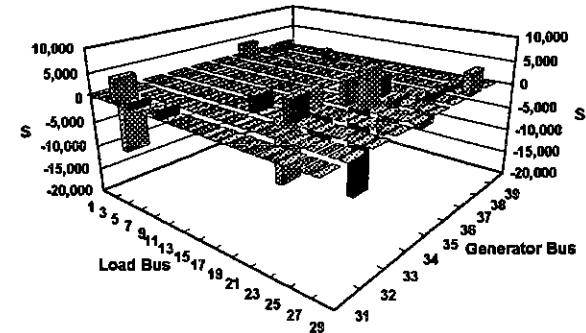


Figure 17. Sensitivity of Generators with respect to Reactive Power of each Load Bus for the case of 100% Constant Power and $k_1 = 1.00$, $k_2 = 2.14$, $k_3 = 1.00$.

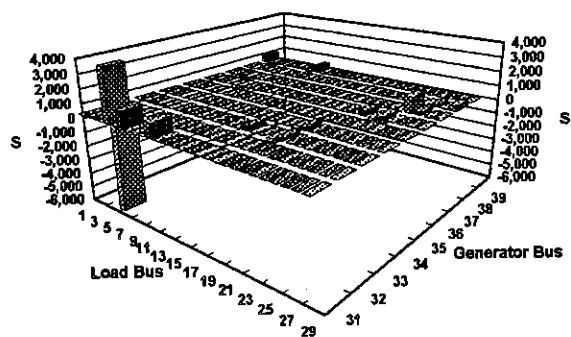


Figure 18. Sensitivity of Generators with respect to Real Power Load for 100% Constant Power and $k_1 = 1.74$, $k_2 = 1.77$, and $k_3 = 1.20$.

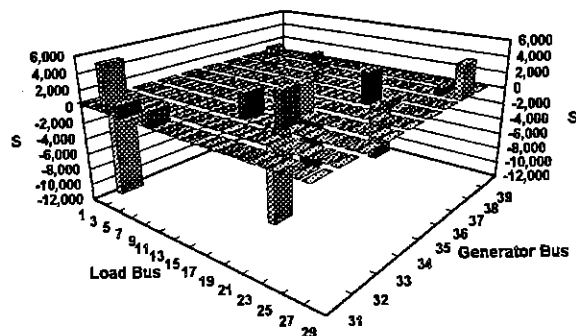


Figure 21. Sensitivity of Generators with respect to Reactive Power Load for 25% Constant Power and 75% Constant Impedance and $k_1 = 1.90$, $k_2 = 2.90$, and $k_3 = 1.19$.

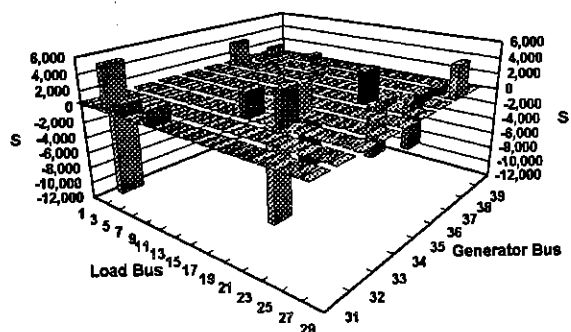


Figure 19. Sensitivity of Generators with respect to Reactive Power Load for 100% Constant Power and $k_1 = 1.74$, $k_2 = 1.77$, and $k_3 = 1.20$.

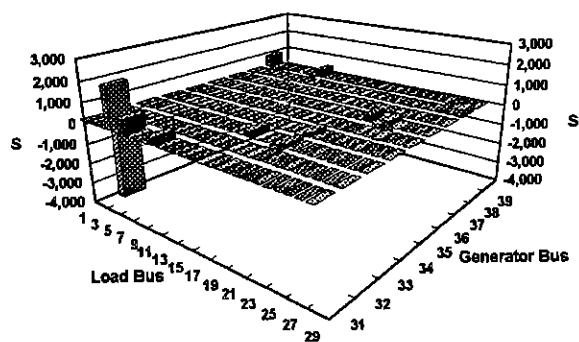


Figure 20. Sensitivity of Generators with respect to Real Power Load for 25% Constant Power and 75% Constant Impedance and $k_1 = 1.90$, $k_2 = 2.90$, and $k_3 = 1.19$.

ASYNCHRONOUS MOTOR MODELS FOR VOLTAGE STABILITY ANALYSIS

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Abstract

In voltage stability analysis load modeling is one of the most relevant aspects. Although the voltage stability phenomenon is essentially dynamic, static methods based on the load-flow equations are widely used to calculate the system operating points and the stability limits. As it may be pointed out from a small disturbance dynamic analysis (eigenvalues computation), the usual load-flow load models are not adequate when the load is totally or partially composed of induction motors. The differences between dynamic and static critical conditions are investigated in the paper and the assumptions required to obtain the same limits are discussed.

The paper presents a steady-state model for asynchronous motor loads. The proposed model adopts the same variables as the ones used in the dynamic analysis. It is fully consistent with the standard load-flow formulation if an auxiliary node is introduced for each motor load bus. The small disturbance dynamic critical conditions can be then exactly computed by the usual algorithms based on the load-flow Jacobian singularity.

The effectiveness of the proposed model is discussed and the most important aspects of the analysis are shown with numerical examples. The influence of induction motor loads with constant mechanical load is discussed, then two different types of motor load growth are examined. In the first case, the load growth is due to an increase of the mechanical load of a fixed number of motors. In the second case, an increase of the number of motors with fixed mechanical load is considered.

Introduction

The voltage instability phenomena are essentially of dynamic nature. A correct and detailed system modeling is

very important to achieve good results in voltage collapse assessment; in particular, load modeling plays a very important role. However, the use of detailed models for rotating machines implies high-dimension algebraic-differential systems with heavy computational burden. An additional complication stems from the fact that, not only at various nodes we may have loads of different type, but also at the same bus many different types of loads can be connected, requiring some kind of aggregate model. Various types of aggregated models have been proposed in literature for the dynamic analysis.

The complete analysis of the dynamic behaviour of a power system with induction motor loads requires a large disturbance dynamic analysis which can be performed only with the full time-domain simulation of the system. Large disturbance dynamic analysis is not treated in this paper.

This paper considers steady-state induction motor models for the exact assessment of the small disturbance dynamic critical conditions for the voltage collapse phenomenon. These models are used in a load-flow framework in order to readily assess the stability characteristics of the equilibrium point, and particularly to find the critical point.

For the steady-state approach validation, it is necessary to justify that the conclusions drawn via static methods and the ones obtained through conventional dynamic analysis are equivalent. For the small disturbance dynamic analysis some arguments can be provided to show the consistency between static and dynamic approaches. Under well defined hypotheses, it is possible to extend the use of static methods to assess the dynamic equilibrium point features, with evident computational advantages [2]. Clarification of these hypotheses is very important to increase the confidence in the static approach, which is still the only viable method for voltage stability assessment of real power systems.

Induction motor model

With the following assumptions:

- stator joule losses and iron core losses neglected
- linear magnetic characteristic
- motor parameters independent on rotor speed
- stator transients neglected

the induction motor dynamic is described by a third order model given by the stator equivalent circuit of Fig.1 and by the following equations (see Appendix I for symbols):

- electromagnetic differential equation:

$$\frac{d\mathbf{E}'}{dt} = -\frac{1}{T_0} [\mathbf{E}' - j(X_0 - X')\mathbf{I}] - js\omega_0 \mathbf{E}' \quad (1)$$

- mechanical differential equation:

$$\frac{ds}{dt} = \frac{1}{2H} [T_m \omega_0 - \text{Re}(\mathbf{E}' \mathbf{I}^*)] \quad (2)$$

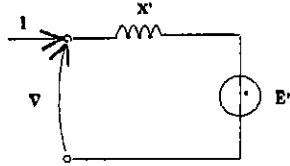


Fig.1 - Induction motor dynamic equivalent circuit

Setting to zero the time derivative in equation (1) we obtain the steady-state model shown in Fig.2, where the value of R_2 is given by:

$$R_2 = \frac{(X_0 - X')}{\omega_0 T_0} \quad (3)$$

This model can be also obtained from the classical steady-state model with three reactances (stator and rotor leakage reactances, magnetizing reactance) introducing a scaling factor (transformer ratio) in rotor current and flux in order to have a null rotor leakage reactance. The scaling factor is the same used in the definition of the transient e.m.f..

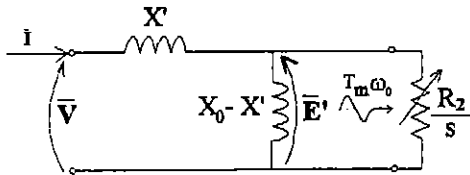


Fig.2 - Induction motor steady-state equivalent circuit

With respect to the classical model, the proposed model has the following advantages:

- it uses the same variables ($\mathbf{E}', s, T_m$) and the same parameters (X_0, X', T_0) of the third order dynamic model;
- it has a two-node structure, with one node representing the motor terminals and the other representing the transient e.m.f. and the mechanical torque ($P = T_m \omega_0$), which can be interpreted as a two-node standard load-flow problem if the mechanical torque is independent on rotor speed.

For voltage stability assessment applications, it is important to investigate the motor steady-state behaviour for a wide range of voltages. Given a mechanical torque independent on rotor speed and the terminal voltage, there are two steady-state solutions of equations (1) and (2), the one with "low" slip and the other with "high" slip (see also the following section).

Since the stator joule losses are ignored, the active input power does not depend on voltage and it is equal to the air-gap power $P = T_m \omega_0$.

Fig.3 shows the input reactive power characteristic with respect to terminal voltage: the lower branch is the low-slip solution, the upper branch is the high-slip solution; the vertical tangent point corresponds to the voltage for which the load torque is equal to the motor maximum torque (motor stall).

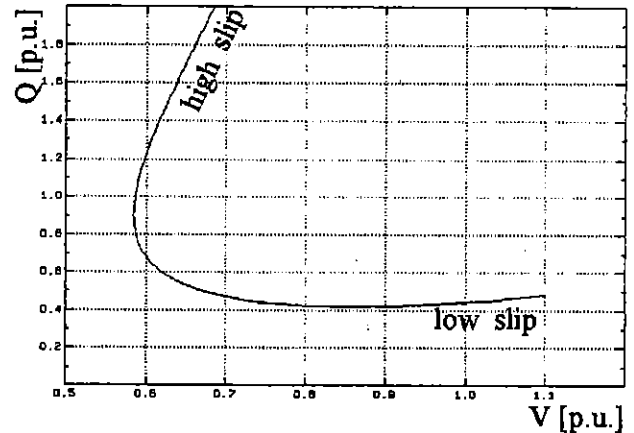


Fig.3 - Reactive power vs. voltage

Fig.4 shows the power factor with respect to the terminal voltage. In this case, the upper branch is the low-slip solution while the lower one is the high-slip solution. In the normal operating range the motor power factor is slightly influenced by voltage, but approaching the

minimum voltage it considerably decreases. This behaviour shows that the motor cannot be modeled as a constant power factor load in voltage stability studies.

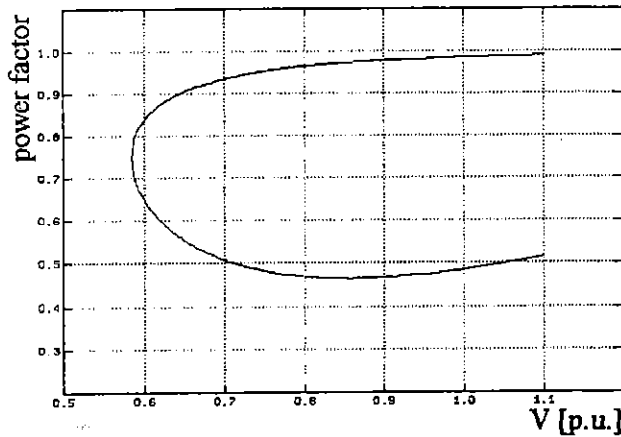


Fig. 4 - Power factor vs. voltage

Small-disturbance dynamic analysis

In order to compare the small disturbance stability limit with the static limit, let's consider the simple system of Fig. 5 composed by an induction motor fed by a constant (ideal) voltage generator through a system represented by a reactance (see Appendix II for parameter values).

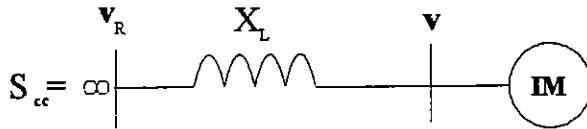


Fig. 5 - One-line diagram for a two-bus system with induction motor

The dynamic model of the system may be represented with the differential system:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \quad (4)$$

where $\mathbf{x}$ is the state variables vector:

$$\mathbf{x} = [E_d', E_q', s]^T \quad (5)$$

Including the external network the differential equations (4) may be written as:

$$\frac{dE_d'}{dt} = s\omega_0 E_q' - \frac{1}{T'} \left[E_d' - V_R \frac{(X_0 - X')}{X_0 + X_L} \right] \quad (6a)$$

$$\frac{dE_q'}{dt} = -s\omega_0 E_d' - \frac{1}{T'} E_q' \quad (6b)$$

$$\frac{ds}{dt} = \frac{1}{2H} \left[T_m \omega_0 + \frac{V_R}{X' + X_L} E_q' \right] \quad (6c)$$

From equations (6), the steady-state equilibrium points may be computed setting to zero the time derivatives. For a given mechanical torque T_m , independent on rotor speed, there are two different solutions: the one with higher values of slip corresponds to lower motor terminal voltages and viceversa.

Fig. 6 shows the motor voltage V with respect to air-gap power $P = T_m \omega_0$: the upper branch is the low-slip solution, the lower branch is the high-slip solution; the vertical tangent point corresponds to the maximum torque.

It is interesting to note that the equilibrium point calculation may be interpreted as the solution of a standard load-flow problem for a two-bus system, where the slack bus represents the infinite bus V_R and the PQ bus represents the motor rotor with voltage E' and active power $T_m \omega_0$ (independent on voltage if the mechanical torque is independent on rotor speed). In this case the usual P-V curve can be obtained (the low branch terminates in the origin).

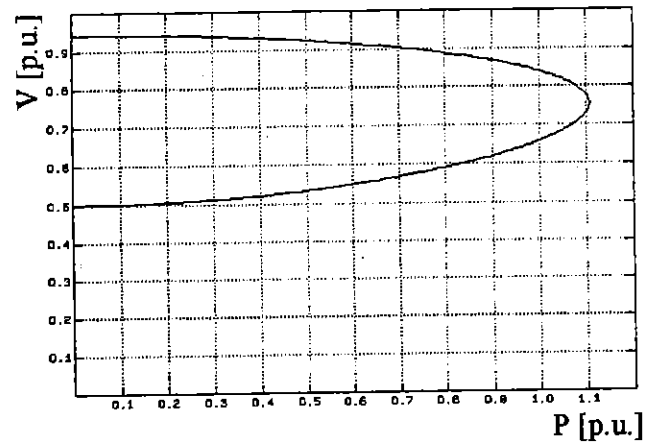


Fig. 6 - Voltage vs. active power (P-V curve)

Fig. 7 shows the power factor vs. air-gap power characteristic: the constant power factor assumption is only acceptable for mechanical torque near the rated value.

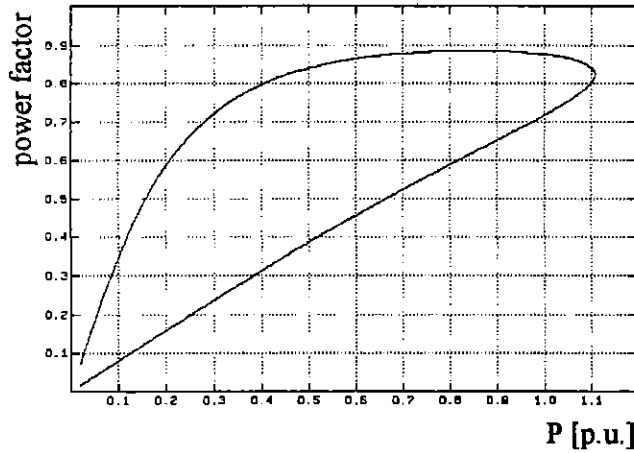


Fig. 7 - Power factor vs. active power

The small disturbance dynamic analysis is performed through the linearisation of the equation (4) around each equilibrium point:

$$\Delta \dot{\mathbf{x}} = \mathbf{J} \Delta \mathbf{x} \quad (7)$$

where $\mathbf{J}$ is the Jacobian matrix of the linearized dynamic system:

$$\mathbf{J} = \begin{bmatrix} -\frac{1}{T'} & s\omega_0 & \omega_0 E'_q \\ -s\omega_0 & -\frac{1}{T'} & -\omega_0 E'_d \\ 0 & \frac{V_R}{2H(X' + X_L)} & 0 \end{bmatrix} \quad (8)$$

Fig. 8 shows the matrix $\mathbf{J}$ eigenvalues locus corresponding to the P-V characteristic of Fig. 6: the thin line represents the equilibrium points of the upper branch ("low" slip), while the thick line represents the equilibrium points of the lower branch ("high" slip):

- two eigenvalues λ_1, λ_2 are always conjugate complex and lie in the left half-plane;
- the real eigenvalue λ_3 is negative on the upper branch, positive on the lower branch and null for the maximum mechanical torque (Fig. 9).

Therefore, all the equilibrium points on the upper branch are stable for small disturbances, while all the equilibrium

points of the lower branch are unstable. The loss of stability is of aperiodic type (voltage collapse).

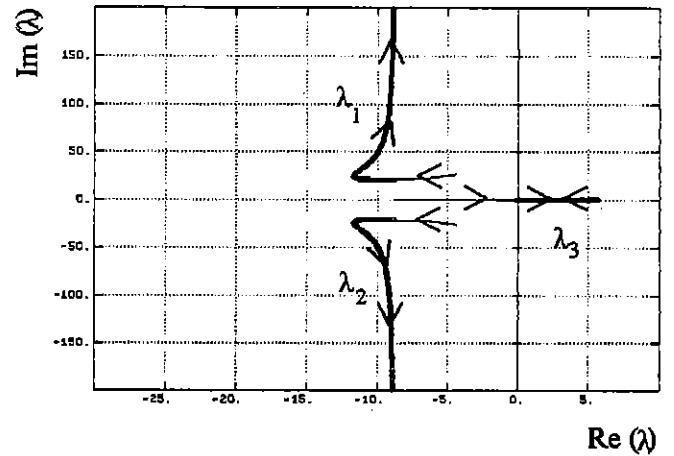


Fig. 8 - Eigenvalues locus

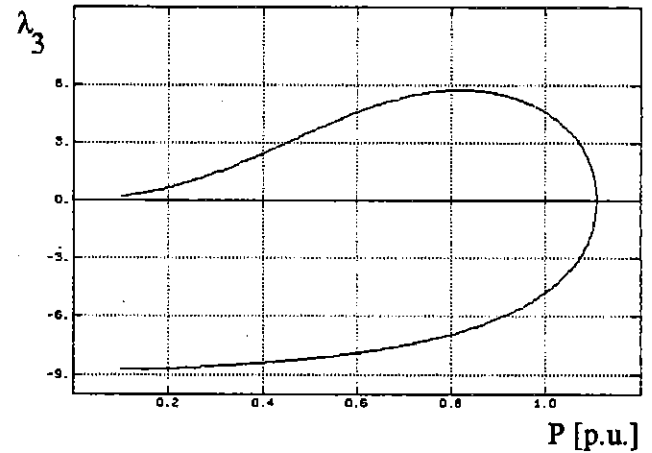


Fig. 9 - Real eigenvalue as the load varies

According to bifurcation theory terminology [1,2] there is no Hopf bifurcation point (purely imaginary eigenvalues, oscillatory instability) and there is a saddle-node bifurcation point (null eigenvalue, aperiodic instability) which corresponds to the singularity of the Jacobian matrix $\mathbf{J}$:

$$\det \mathbf{J} = 0 \quad (9)$$

For the two-bus system of Fig. 5, equation (9) can be expanded as:

$$\det \mathbf{J} = \frac{V_R \omega_0}{2H(X' + X_L)T'} (E'_d + s\omega_0 T' E'_q) = 0 \quad (10)$$

From equations (10) and (6) it can be shown that the dynamic stability limit coincides with the condition of maximum power (mechanical torque) delivered to the variable resistance R_2/s and then with the matching condition between the resistance R_2/s and the equivalent reactance X_{eq} seen from its terminals:

$$R_2/s = X_{eq} \quad (11)$$

with:

$$X_{eq} = (X_0 - X') \frac{X' + X_L}{X_0 + X_L} \quad (12)$$

Since the equilibrium point calculation is equivalent to a standard load-flow calculation, the dynamic stability limit is also equivalent to the singularity of the load-flow Jacobian matrix.

Therefore the following conditions are equivalent:

1. small disturbance stability limit (null eigenvalue);
2. singularity of the Jacobian matrix of the equilibrium point equations;
3. singularity of the Jacobian matrix of the load-flow equations corresponding to the equilibrium point calculation (usual static limit);
4. maximum torque developed by the motor (matching condition).

Induction motor model in load-flow equations

In the standard load-flow formulation loads are modeled as constant power (active and reactive power independent on voltage) or as constant impedance. Some programs allow more elaborated model with static dependence of active and reactive power on bus voltage. Two typical examples are the ZIP model (combination of constant impedance, constant current and constant power loads) and the improperly called exponential model:

$$P = P_0 \left(\frac{V}{V_0} \right)^\alpha, \quad Q = Q_0 \left(\frac{V}{V_0} \right)^\beta \quad (13)$$

The steady state analysis of the induction motor shows that, neglecting stator losses, the motor active power does not depend on voltage, but the motor reactive power strongly depends on voltage (Fig.3) in a way that cannot be represented with the usual load models near the minimum voltage.

Since the steady-state model of the induction motor (Fig.2) with mechanical torque independent on speed may be interpreted in terms of standard load-flow formulation, it is easy to incorporate it in the load-flow equations of a generic system with no modification to the equation structure. In fact, it is sufficient to add a PQ auxiliary node, as shown in Fig.10, with the following features:

- its complex voltage is equal to the transient e.m.f. E' ;
- it has a constant power load, with $P = T_m \omega_0$ and $Q=0$;
- it has a constant reactance load ($X_0 - X'$) which represents the shunt branch of the equivalent circuit.
- it is connected to the motor bus through the transient reactance X' .

The Joule stator losses may be easily modeled by adding the stator resistance in series with the transient reactance.

Using the voltage-dependent load models available in load-flow programs it is also possible to approximately represent the speed dependence of the mechanical load torque $T_m = T_m(\omega)$. For example, with reference to an operating point with torque T_{m0} , active power P_0 , slip s_0 and e.m.f. E'_0 , the load model (13) used at the auxiliary bus becomes:

$$P = P_0 \left(\frac{E'}{E'_0} \right)^\alpha \quad (14)$$

and corresponds to the following expression of the mechanical torque as a function of the motor slip:

$$T_m = T_{m0} \left(\frac{s_0}{s} \right)^\gamma \quad (15)$$

where: $\gamma = \frac{\alpha}{2 - \alpha}$. With the exponent α in the range $[0,2]$ the exponent γ varies in the range $[0, +\infty]$.

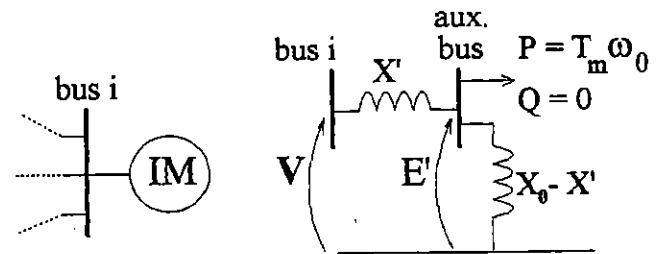


Fig.10 - Induction motor model for load-flow studies

The proposed model increases the number of the load-flow nodes but allows an exact representation of induction motor loads in any existent program with only modification in data input. In particular it is possible to include motor loads in all the programs for voltage stability assessment based on static approaches. Of course, the usual care must be taken to avoid the problems related to the ill-conditioning of the Jacobian matrix near the critical conditions [3-5].

Induction motor models and voltage collapse

This section deals with the influence of induction motor loads on static indexes for voltage collapse assessment. The static indexes are usually based on a load variation in the load-flow equations described by a scalar parameter. The following types of indexes are considered:

- a) with motor load not involved in the load variation:
 - a1. active power margin (P-V curve)
 - a2. reactive power margin (V-Q curve)
- b) with motor load involved in the load variation
 - b1. mechanical torque margin for a single motor
 - b2. maximum number of motors connected to the node

The induction motor loads are represented with the auxiliary bus model described in the preceding section (Fig.10) with mechanical torque independent on rotor speed.

The classical P-V and V-Q curves are computed with a standard load-flow program based on the full Newton-Raphson method. The tests are performed:

- for case a): on a 30-bus system derived from the standard IEEE 30-bus system (see Appendix III)
- for case b): on the two-bus system of Fig.5.

a1. Active power margin with fixed motor mechanical load

The active power margin is evaluated on P-V curves, as described in the following examples.

Example a1.1

The P-V curves are drawn for the 30-bus system varying the loads in a system area (the dashed area in Fig.A1 with nodes 12, 15, 16, 17, 18, 20, 21 and 23). Load variations are driven by a scalar factor which multiplies the base power. The power factor is constant (the same as in the base case), active power variations are compensated by the slack bus generator.

Loads at buses 14, 19 and 26 are assumed to be composed of both constant power and induction motor loads. Three types of load composition are considered (100% PQ load; 50% PQ load and 50% induction motors; 100% induction motors), with the same total active power at each bus as in the base case. The corresponding P-V curves are shown in Fig.11. It can be easily noted that an increase in the portion of induction motor load causes a reduction in the active power margin.

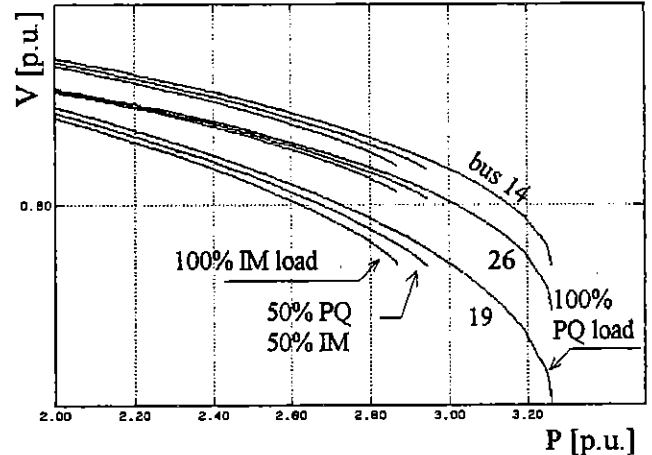


Fig.11 - P-V curves for different load composition

Example a1.2

The P-V curves are drawn varying a single load (bus 19) containing both PQ loads and induction motors with fixed mechanical torque. Assuming the same initial active power and allowing changes in PQ load power only (with constant power factor), the cases of all-PQ load and mixed loads (50% PQ load, 50% induction motors) are compared. Also in this case, as shown in Fig.12, the active power margin is reduced by the presence of induction motors.

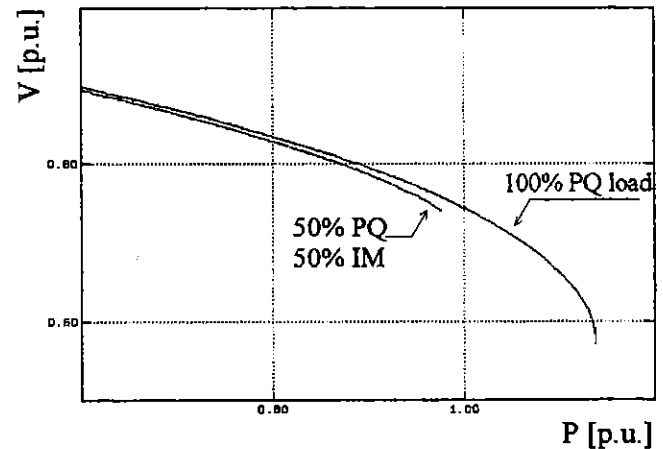


Fig.12 - P-V curves at bus 19

Example a1.3

The active power margin depends on the mechanical load level of the motors. In Fig.13 the cases of induction motors operating with rated mechanical torque (load level 100%) and with reduced mechanical torque (load level 50%) are compared. The P-V curves are obtained assuming the same load increase in a network area as in the previous example a1.1. The results confirm that the reduction in the mechanical load level increases the active power margin.

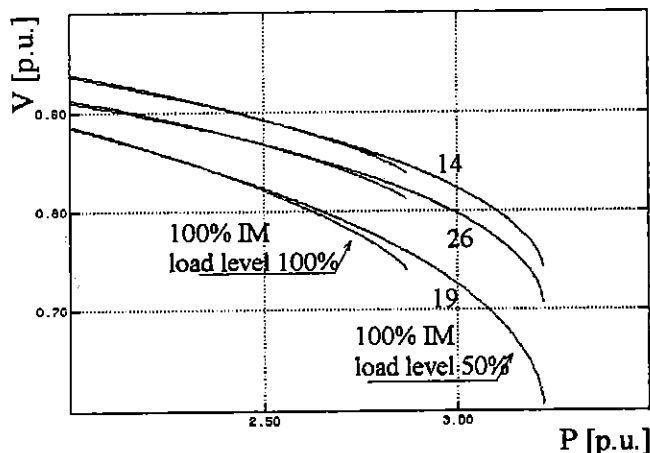


Fig.13 - P-V curves for different mechanical load levels

a2. Reactive power margin with fixed motor mechanical load

The dependence of the reactive power of the induction motor on voltage has already been described (Fig.3). In usual operating conditions (voltage near rated value), if voltage decreases the motor reactive power also decreases, but there is a voltage value above which a decrease in voltage corresponds to an increase in reactive power. The evaluation of this conditions is performed drawing the V-Q curve which allows to define the reactive power margin.

The V-Q curves are drawn for any selected PQ node by a succession of load-flows, in which the node is transformed from PQ to PV type and the voltage is assigned as independent variable. With PQ loads, the V-Q curve can be traced for a wide range of voltages. In presence of induction motor load at the bus, the voltage assignment is limited by the minimum voltage of the motor V-Q characteristic, that is, by the voltage which causes the motor stall.

Fig.14 shows the V-Q curves at bus 19 in three different loading conditions:

- PQ load only (base case of the IEEE 30-bus network);

- mixed load (50% PQ load, 50% induction motors) with total active power equal to the base case and PQ load with constant power factor;
- induction motors only, with active power as in the base case.

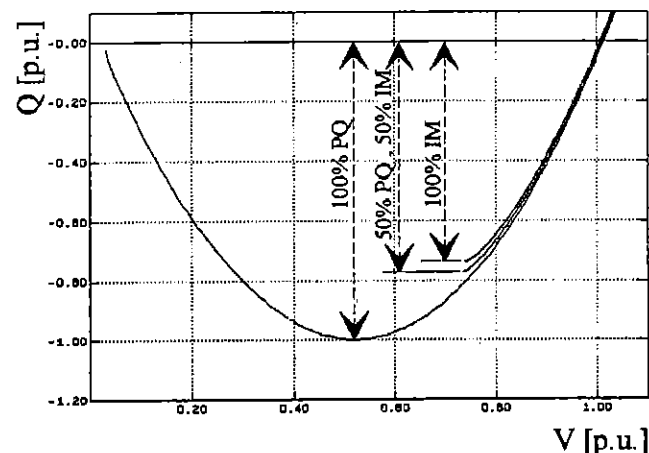


Fig.14 - V-Q curves and reactive margins at bus 19 for different load composition

In Fig.14 the reactive power margin in the various cases is also shown. This margin is reduced by the presence of induction motors. The stalling voltage does not vary with different amount of induction motor load at the node. A zoom of the critical portion of the V-Q curves is shown in Fig.15: near the minimum voltage the V-Q curves tend to have vertical tangent (see Fig.3), and there are load-flow convergence problems.

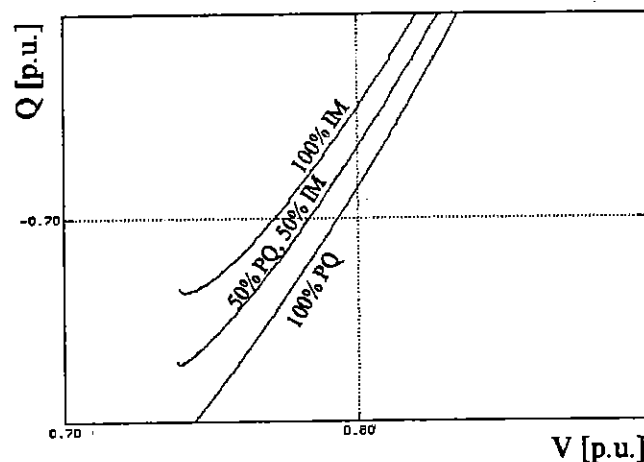


Fig.15 - Zoom of the left side of V-Q curves at bus 19

b1. Mechanical load margin for a single motor

This analysis is performed on the two-bus system of Fig.5 assuming a mechanical load independent on rotor speed ($P = T_m \omega_o$). Fig.16 shows the motor P-V curve compared to the P-V curve of a constant power factor load.

If the induction motor is modeled as a constant power factor load and if the base operating point is chosen near the rated conditions (high power factor), the assessment of the voltage collapse critical conditions leads to higher power values with respect to the effective critical point evaluated with the auxiliary node model. This dangerously optimistic result does not occur if the base operating point is assumed at a low mechanical load level (low power factor): in this case the critical point is detected for a load power lower than the one in the effective critical point and, although there is no risk of overestimating the critical conditions, again the result is incorrect.

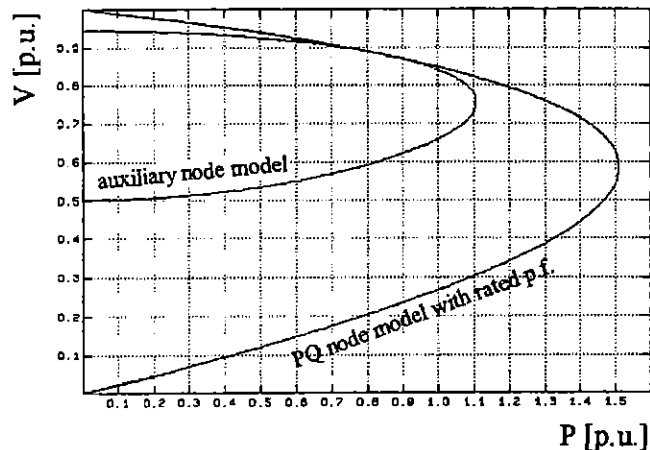


Fig.16 - P-V curves comparison:

- a) equilibrium points of the linearized dynamic system
- b) PQ node with constant power factor(rated value)
- c) PQ node with constant power factor(value assumed in critical conditions)

b2. Maximum number of motors connected to the node

The aggregation of several induction motors into a single model is still very controversial [7-12]. We consider the simple case of a progressive increase in the number of equal motors connected to the same bus. The auxiliary node model is used (Fig.17), but in this case the reactances depends on the number k of equal motors and the load-flow equations must be modified to take into the variations of k .

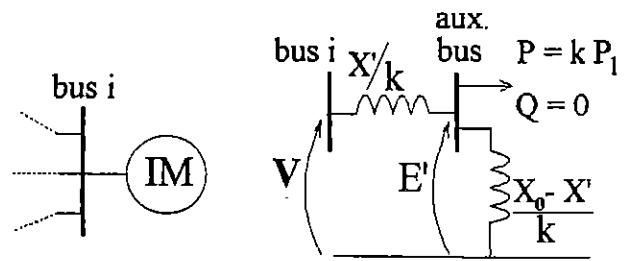


Fig.17 - Auxiliary node model for an aggregation of induction motors

In this case the power factor is almost constant for a wide range of load variation, with the exception of the neighbourhood of the critical conditions, as shown in Fig.18. Hence the PQ load model seems to be an acceptable representation of several induction motors, at least for normal operating conditions.

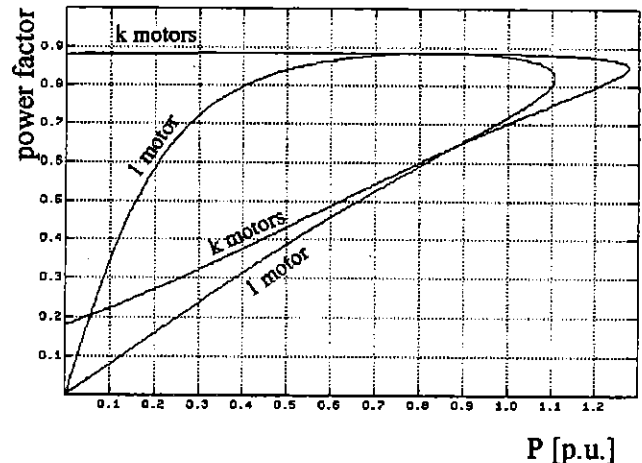


Fig.18 - Power factor characteristic

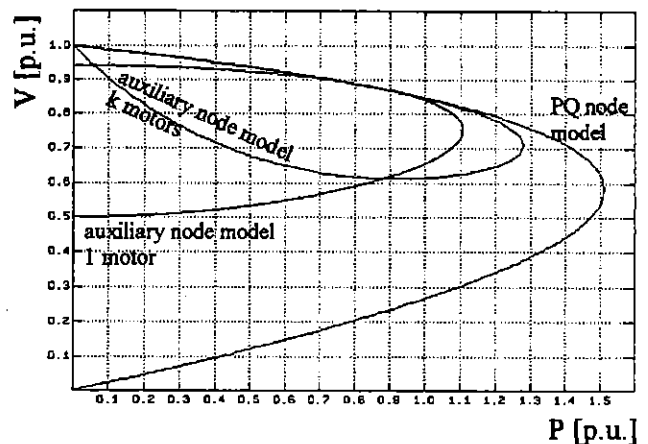


Fig.19 - P-V characteristic

Fig.19 shows the P-V curves of the two-bus system for three different load models:

- *single motor with variable mechanical torque (auxiliary node model)* : the critical point corresponds to the smallest power value;
- *constant power factor load*: corresponds to the highest critical load power, thus confirming that this kind of modeling is inadequate, as it overestimates the critical conditions;
- *aggregation of equal motors with fixed mechanical torque (auxiliary node model with variable reactances)*: the critical point is reached at an intermediate power value.

Conclusions

The main points of this paper are:

- In voltage stability studies the induction motor loads cannot be adequately represented with the usual load models because of the voltage dependence of the motor reactive power.
- The small disturbance stability limit (null eigenvalue) coincides with the static limit (singularity of the load-flow Jacobian matrix) if the motor steady-state behavior is exactly modeled.
- The proposed steady-state motor model is fully compatible with the standard load-flow equations and allows to exactly represent motor loads in any available program.
- The reduction of various voltage stability margins, due to induction motor loads, has been shown with several examples on two test systems.

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Appendix I

List of symbols

t	time
ω_0	synchronous speed
$s=(\omega_0 - \omega)/\omega_0$	slip
T_m	mechanical torque
H	inertia constant
X_0	magnetising reactance
X'	transient reactance
R_2	rotor equivalent resistance
T'_0	open circuit time constant
$T' = T'_0 \frac{X' + X_L}{X_0 + X_L}$	short circuit time constant (network included)
$I = I_d + j I_q$	motor current
$E' = E'_d + j E'_q$	transient e.m.f.
$V = V_d + j V_q$	voltage at the motor terminals
$V_R = V_R$	generator voltage
X_L	(Thévenin equivalent of the network) network reactance (Thévenin equivalent of the network)

Appendix II

The parameters used for the two-bus system in Fig.5 are:

Induction motor	X' 0.2	T'_0 1.	H 1.	X_0 3.3
Network	V_R 1.	X_L 0.2		

Appendix III

The standard IEEE 30-bus network (see [21] for data and base case) has been modified by adding three induction motor loads at buses 14, 19 and 26 (Fig.A1).

In the motor per unit system the values $X'_{pu} = 0.25$ and $X_{0pu} = 3$ have been considered for all induction motors. Stator and rotor losses are neglected. The values of X' and X_0 in the network per unit system are:

$$X_0 = L_{mot}/(K_{bus} P_{bus}) X_{0pu}$$

$$X' = L_{mot}/(K_{bus} P_{bus}) X'_{pu}$$

where:

P_{bus} total bus active power
 K_{bus} portion of the total bus active power
 represented by induction motors
 L_{mot} induction motor mechanical load level

Therefore, the following p.u. values have been used for
 reactances in the auxiliary node motor model:

bus	K_{bus}	L_{mot}	X'	X_0
14	100%	100%	4.0323	48.3871
19	100%	100%	2.6316	31.5789
26	100%	100%	7.1429	85.7143
14	50%	100%	8.0646	96.7752
19	50%	100%	5.2632	63.1579
26	50%	100%	14.2857	171.4286
14	100%	50%	2.0162	24.1935
19	100%	50%	1.3158	15.7895
26	100%	50%	3.5714	42.8571

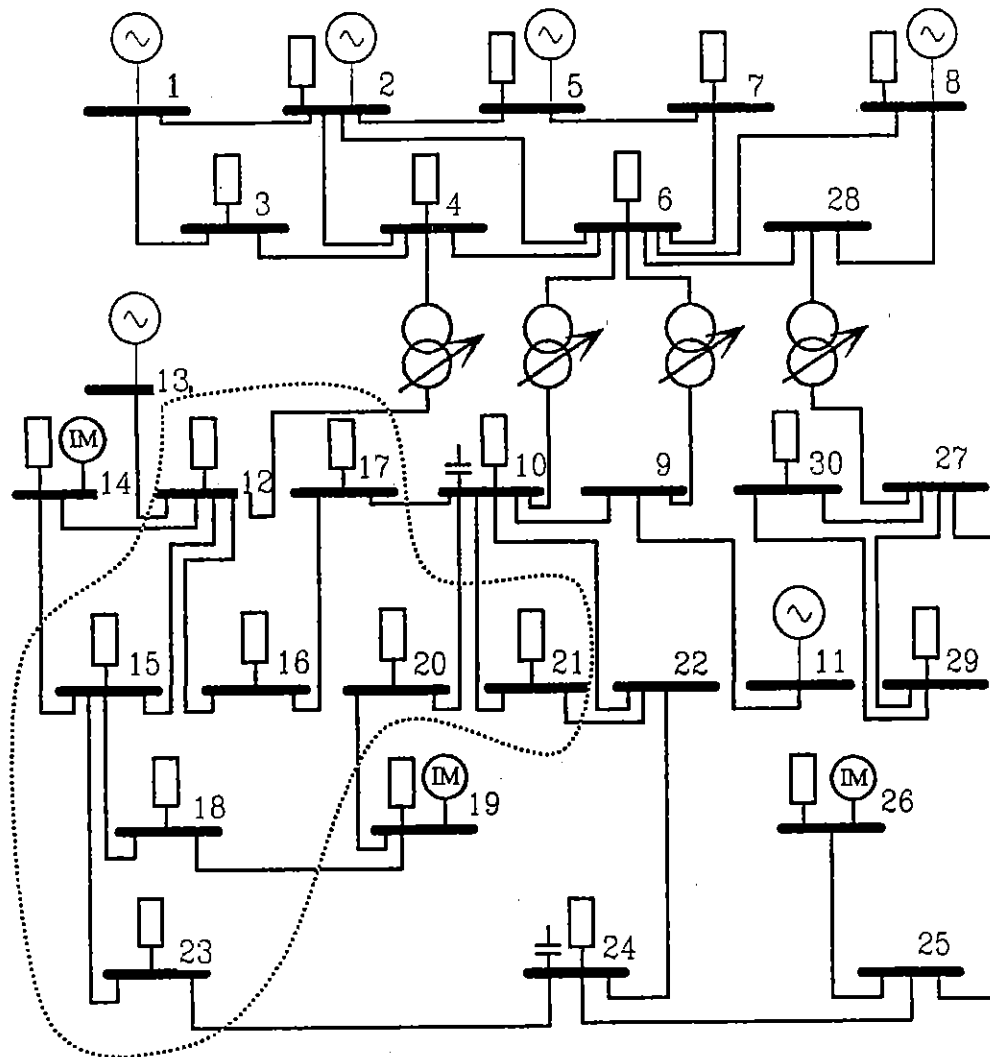


Fig.A1 - Modified IEEE 30-bus test network

MODELLING OF DYNAMIC LOAD BEHAVIOUR

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Abstract

The paper develops a system model which incorporates generator dynamics, and static and dynamic loads. The model has a differential-algebraic form. Conditions governing the algebraic solvability of this model are considered. It is shown that solvability is closely related to the static load characteristics and the transient response of the dynamic loads. For some load model structures, solvability can never be guaranteed, independent of the choice of parameters. However, for other load model structures, parameters can be chosen which effectively ensure global solvability of the model.

Keywords: load modelling, differential-algebraic systems

1 Introduction

Loads have an important influence on the dynamic behaviour of power systems. This is particularly so in the case of voltage stability phenomena, where the interaction between loads and tap changing transformers tends to be the dominant feature [1, 2, 3, 4]. However loads can also play an important role in intermachine (angle) behaviour [5, 6, 7]. The modelling of loads can therefore have a significant effect on the accuracy of studies of power system dynamic behaviour.

It is not practical, or indeed even possible, to model every load that is connected to a power system. Aggregation and hence approximation is necessary. Appropriate load models that adequately describe generic behaviour of aggregated loads are required. Significant work is emerging in this area. This paper will focus on the models proposed by Hill, Karlsson, Van Cutsem and Mansour [8, 9, 10, 11]. Their models feature nonlinear functions which capture the steady state and transient behaviour in load responses, and parameters which relate to load recovery time. In particular, load at a bus can be expressed as an input-output relationship between (real and reactive) powers and voltage, or via a state space realisation.

These load models are special cases of a class of load models introduced in [8] (modulo scaling) of the form

$$e_p(P_d, V)\dot{P}_d + f_p(P_d, V) = g_p(P_d, V)\dot{V} \quad (1)$$

The relationships between the various models will be reviewed in this paper. Interestingly, models which fit the general form of (1) can exhibit some subtle but important differences. In particular, the form of $g_p(P_d, V)$ can alter conditions governing the solvability of algebraic equations which form part of the model. As a result, load models which exhibit qualitatively similar behaviour in response to a voltage step, may have significantly different influences on the large disturbance behaviour of a power system.

Power systems are typically modelled as a set of differential-algebraic (DA) equations. Solvability of the

algebraic equations is an issue which has attracted much interest recently, see for example [13, 14, 15]. At surfaces where the algebraic equations are not uniquely solvable, system behaviour becomes fast and complex. The DA model can no longer be used to describe system behaviour. It has been suggested that these difficulties can be overcome by including in the model a description of the faster transient processes [15, 16]. An alternative approach to overcoming algebraic singularity is proposed in this paper. It turns out that for certain forms of the load model function $g_p(P_d, V)$, the algebraic equations are globally uniquely solvable. The DA model can then be treated as purely differential, with no possibility of model breakdown. Load models can therefore provide an appealing, and somewhat natural way, of overcoming difficulties caused by the presence of algebraic equations in the model.

The paper is structured as follows. Section 2 considers the form of aggregate load models. These load models are incorporated into a system model in Section 3. Section 4 explores conditions for solvability of the system model. Particular attention is given to the part played by load models. Those conditions are investigated further in Section 5 in terms of specific load model structures. Section 6 provides conclusions.

2 Load models

In this section, we review the general aggregate load models proposed in [8, 10, 12]. Connections between the input-output form of the models and the state space form are explored. Two specific models which fit the general form are considered.

2.1 General load models

The general form of the response of a dynamic load to a step change in voltage is shown in Figure 1. A voltage step produces a transient jump of power followed by recovery to a steady-state [9]. A load model was sought in [8] which captured this generic behaviour. A special case of the class of load models described by (1) was proposed. That particular model is considered further in Section 2.2. The ideas were extended to the more general model (1) in [12].

For clarity of results, this paper will consider a slightly modified form of (1), viz.,

$$\dot{P}_d + f_p(P_d, V) = g_p(P_d, V)\dot{V} \quad (2)$$

There is no loss of generality with the removal of $e_p(P_d, V)$. A similar equation relates Q_d and V . Higher order input-output models can also be considered [17]. Such models would be necessary for the capture of oscillatory load behaviour [18]. However we shall restrict our attention to first order load recovery. Note we assume throughout that $P_d, Q_d \geq 0$.

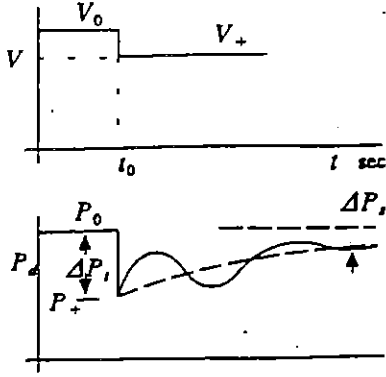


Figure 1: Generic Load Response

From (2) it can be seen that:

- Steady-state behaviour is given by $\dot{P}_d = 0$, $\dot{V} = 0$, i.e.,

$$f_p(P_d, V) = 0 \quad (3)$$

- Transient behaviour is given by integrating

$$dP_d = g_p(P_d, V)dV \quad (4)$$

From the Implicit Function Theorem, if $\frac{\partial f_p}{\partial P_d} \neq 0$, (3) can be solved to give $P_d = P_s(V)$. Integrating (4) gives $h_p(P_d, V) = c$, where c is a constant of integration. Similarly to (3), this can provide a function $P_d = P_t(V)$. These functions may only be defined locally. Figure 1 illustrates the role of the functions P_s and P_t in the steady-state and transient parts of the load response respectively.

There are many ways which the functions (f, g) of (2) can be parameterised. Two possibilities are considered in Sections 2.2 and 2.3.

It is often convenient to transform the IO form of (2) into an equivalent state version. To achieve this, assume P_d can be expressed in terms of some state variable x_p

$$P_d = \phi(x_p, V) \quad (5)$$

via function ϕ . Then differentiating (5) and substituting into (2) gives

$$D_{x_p} \phi \dot{x}_p = -f_p(\phi(x_p, V), V) \quad (6)$$

if we choose

$$D_v \phi = g_p(\phi(x_p, V), V) \quad (7)$$

The IO model (2) and the state model (5)-(7) are too general to be useful for applications. Structures for (f, g) must be found which can be parameterised and identified from field measurements. Further, the resultant model should be verified within the range of operation of variables (x, V) . This paper considers two load models which provide such structure. The first is an exponential recovery model proposed by Hill and Karlsson [8, 9]. The second model was suggested by Xu and Mansour [11]. It is an extension of the adaptive load [10] to allow independent real and reactive power recovery, and voltage dependent steady state load. These two models are now considered.

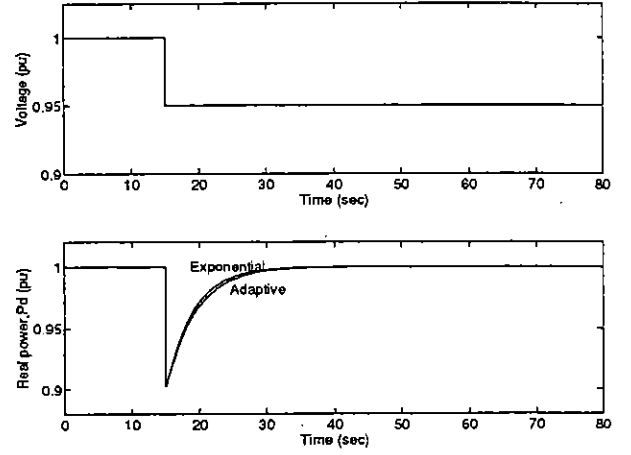


Figure 2: Step Response of Load Models

2.2 Exponential load model

If the load recovery is assumed to have an exponential form, then the following model can be derived,

$$T_p \dot{P}_d + P_d = P_s(V) + k_p(V)\dot{V} \quad (8)$$

where T_p is the recovery time-constant; P_s and $k_p(V) = T_p P'_t(V)$ define the steady-state and transient load characteristics.

Equations (5)-(7) can be used to transform this model to state form. Let

$$P_d = \phi(x_p, V) = x_p + P_t(V) \quad (9)$$

Then the DA state model is given by

$$T_p \dot{x}_p = -x_p + P_s(V) - P_t(V) = P_s(V) - P_d \quad (10)$$

$$P_d = x_p + P_t(V) \quad (11)$$

Note from (7) that $P'_t(V) = \frac{1}{T_p} k_p(V)$.

The response of this model to a voltage step is shown in Figure 2.

2.3 Adaptive load model

The load model suggested by Xu and Mansour [11] has the state form

$$T_p \dot{x}_p = P_s(V) - P_d \quad (12)$$

$$P_d = x_p P_t(V) \quad (13)$$

with a similar model for Q_d . (Recall the assumption $P_d, Q_d \geq 0$. If P_t, Q_t take their usual form, then (13) indicates that $x_p, x_q \geq 0$.) It can be easily shown that the IO version of this model is

$$T_p \dot{P}_d + P_t(V)(P_d - P_s(V)) = T_p P_d \frac{P'_t(V)}{P_t(V)} \dot{V} \quad (14)$$

This fits the general IO form (2). Response of this model to a voltage step is also shown in Figure 2.

2.4 Comparison of models

It is interesting to compare the state forms of the exponential model (10)-(11) and the adaptive model (12)-(13). The differential equation driving the state x_p is the same in both cases. But the algebraic equations differ. In the exponential model, x_p is an additive term, whilst for the adaptive model it is a multiplier. Figure 2 indicates that this difference does not reflect greatly in the step behaviour of the models. However, in Section 5 it will be shown that this difference in the algebraic equations can have a significant effect on the solvability of the system model, i.e., on the conditions under which the model breaks down.

3 System model

In this section, the load models described in Section 2 are combined with appropriate models of generators and the network. This system model is adapted from the models given in [12, 17, 19]. Generator dynamics have been included, but regulating transformers and generator reactive power limits are not modelled. These latter features have been removed to enable a clearer examination of the algebraic solvability issues.

3.1 System component models

Because we are predominantly interested in load behaviour, it is sufficient to represent machines using the classical machine model. Other generator models could be substituted if necessary, without affecting the conclusions of the investigation. However the clarity of the presentation would be reduced. In accordance with this model, machines are represented by a constant voltage E_i in series with its transient reactance.

Consider a network of m generators and n_b buses, i.e., there are $n_b - m$ buses which have no generation of real power. Denote the i th bus voltage magnitude and angle by V_i and θ_i respectively. Let the generator buses be the last m buses, i.e., generator i is attached to bus $n_b - m + i$.

Let the load at each of the first d buses be dynamic, i.e., $P_{di}, Q_{di}, i = 1, \dots, d$ are given by a dynamic load model of the form (2). The loads at the other $n_b - m - d$ non-generator buses are static functions of voltage, i.e.,

$$\begin{aligned} P_{di} &= P_{di}(V_i) \\ Q_{di} &= Q_{di}(V_i) \end{aligned} \quad i = d+1, \dots, n_b - m \quad (15)$$

Note that these functions of voltage may be constants, or in fact zero. Define

$$\begin{aligned} P_d &= [P_{d,1}, \dots, P_{d,n_b-m}]^t = [P_{d1}^t, P_{d2}^t]^t \\ Q_d &= [Q_{d,1}, \dots, Q_{d,n_b-m}]^t = [Q_{d1}^t, Q_{d2}^t]^t \end{aligned}$$

where P_{d1}, Q_{d1} are d -vectors, P_{d2}, Q_{d2} are $(n_b - m - d)$ -vectors.

We shall use a machine reference angle model, with the n_b th bus as the reference. By setting $\theta_{n_b} := 0$, the elements of the angle vector $\theta = [\theta_1, \dots, \theta_{n_b-1}]^t$ are effectively referred to the phase angle of the m th generator. It is convenient to define the following vectors,

$$\begin{aligned} V &= [V_1, \dots, V_{n_b}]^t = [V_d^t, V_s^t, E^t]^t \\ \theta &= [\theta_1, \dots, \theta_{n_b-1}]^t = [\theta_d^t, \theta_s^t, \theta_g^t]^t \end{aligned}$$

where subscripts d, s, g refer to dynamic load buses, static load buses and generator buses respectively. Therefore

V_d, θ_d are d -vectors, V_s, θ_s are $(n_b - m - d)$ -vectors, and E, θ_g are $m, (m-1)$ -vectors respectively. Note that generator voltage magnitudes E are constant.

For simplicity, assume the system is lossless. Thus the bus admittance matrix Y is purely imaginary, i.e., its elements can be written as $Y_{ij} = jB_{ij}$.

Let P_{bi} and Q_{bi} denote the total real and reactive power leaving the i th bus via transmission lines or transformers. Then

$$\begin{aligned} P_{bi}(\theta, V) &= \sum_{j=1}^{n_b} V_i V_j B_{ij} \sin(\theta_i - \theta_j), \\ i &= 1, \dots, n_b - 1 \end{aligned} \quad (16)$$

$$\begin{aligned} Q_{bi}(\theta, V) &= -\sum_{j=1}^{n_b} V_i V_j B_{ij} \cos(\theta_i - \theta_j), \\ i &= 1, \dots, n_b \end{aligned} \quad (17)$$

Define

$$\begin{aligned} P_b &= [P_{b,1}, \dots, P_{b,n_b-1}]^t = [-P_{11}^t, -P_{12}^t, P_g^t]^t \\ Q_b &= [Q_{b,1}, \dots, Q_{b,n_b}]^t = [-Q_{11}^t, -Q_{12}^t, Q_g^t]^t \end{aligned}$$

where P_{11}, Q_{11} are d -vectors, P_{12}, Q_{12} are $(n_b - m - d)$ -vectors, and P_g, Q_g are $(m-1), m$ -vectors respectively.

Assuming the usual form of the swing equation, the dynamics of each generator can be described by

$$\dot{\omega}_i = -\frac{D_i}{M_i} \omega_i - \frac{1}{M_i} (P_{gi} - P_{gi}^o) \quad (18)$$

$$\dot{\theta}_{gi} = \omega_i \quad (19)$$

where P_{gi}^o, M_i, D_i are the machine's mechanical power, inertia and damping constants respectively. Define

$$\begin{aligned} \omega &= [\omega_1, \dots, \omega_m]^t \\ P_g^o &= [P_{g1}^o, \dots, P_{g,m-1}^o]^t \end{aligned}$$

The swing equations can then be written

$$\dot{\omega} = -M^{-1} D \omega - M^{-1} T^t (P_g(\theta, V) - P_g^o) \quad (20)$$

$$\dot{\theta}_g = T \omega \quad (21)$$

where M, D are diagonal matrices of inertia and damping constants, and T is a specially structured matrix of ± 1 entries. (Note that some transformation of powers is often required to achieve the precise form (20)-(21) [20].)

Combining the generator swing equations and the dynamic load equations with the equations describing the power balance at buses gives the total system representation. The system model is now developed for both the IO and state forms of the load model.

3.2 IO load formulation

In vector form, the general IO load model becomes

$$\dot{P}_{d1} + f_p(P_{d1}, V_d) = g_p(P_{d1}, V_d) \dot{V}_d \quad (22)$$

$$\dot{Q}_{d1} + f_q(Q_{d1}, V_d) = g_q(Q_{d1}, V_d) \dot{V}_d \quad (23)$$

where g_p, g_q are $d \times d$ diagonal matrices.

The power balance equations for the load buses can be written as

$$P_{d1} = P_{l1}(\theta_d, V_d, \theta_s, V_s, \theta_g) \quad (24)$$

$$Q_{d1} = Q_{l1}(\theta_d, V_d, \theta_s, V_s, \theta_g) \quad (25)$$

$$0 = P_{l2}(\theta_d, V_d, \theta_s, V_s, \theta_g) - P_{d2}(V_s) \quad (26)$$

$$0 = Q_{l2}(\theta_d, V_d, \theta_s, V_s, \theta_g) - Q_{d2}(V_s) \quad (27)$$

Differentiating these equations gives

$$\begin{bmatrix} \dot{P}_{d1} \\ \dot{Q}_{d1} \end{bmatrix} = J_{dd} \begin{bmatrix} \dot{\theta}_d \\ \dot{V}_d \end{bmatrix} + J_{ds} \begin{bmatrix} \dot{\theta}_s \\ \dot{V}_s \end{bmatrix} + J_{dg} \dot{\theta}_g \quad (28)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = J_{sd} \begin{bmatrix} \dot{\theta}_d \\ \dot{V}_d \end{bmatrix} + J_{ss} \begin{bmatrix} \dot{\theta}_s \\ \dot{V}_s \end{bmatrix} + J_{sg} \dot{\theta}_g \quad (29)$$

where the $J_{..}$ are submatrices of the power flow Jacobian matrix. Note that J_{ss} includes the effects of the voltage dependent static loads $P_{d2}(V_s), Q_{d2}(V_s)$.

Assuming J_{ss} is nonsingular, (29) can be used to eliminate $\dot{\theta}_s, \dot{V}_s$ from (28), giving

$$\begin{bmatrix} \dot{P}_{d1} \\ \dot{Q}_{d1} \end{bmatrix} = (J_{dd} - J_{ds} J_{ss}^{-1} J_{sd}) \begin{bmatrix} \dot{\theta}_d \\ \dot{V}_d \end{bmatrix} + (J_{dg} - J_{ds} J_{ss}^{-1} J_{sg}) \dot{\theta}_g \quad (30)$$

$$= J'_{dd} \begin{bmatrix} \dot{\theta}_d \\ \dot{V}_d \end{bmatrix} + J'_{dg} \dot{\theta}_g \quad (31)$$

It can be seen that the requirement of nonsingularity of J_{ss} is crucial in the development of the model. This is discussed further in Section 4. Notice though that if loads at all buses are dynamic, then J'_{dd}, J'_{dg} revert to J_{dd}, J_{dg} respectively.

Combining (31) with the load model (22),(23) gives

$$J_t(\theta, V) \begin{bmatrix} \dot{\theta}_d \\ \dot{V}_d \end{bmatrix} = g_s(\theta, V) - J'_{dg} T \omega \quad (32)$$

where

$$J_t(\theta, V) = J'_{dd} - \begin{bmatrix} 0 & g_p(P_{l1}(\theta, V), V_d) \\ 0 & g_q(Q_{l1}(\theta, V), V_d) \end{bmatrix} \quad (33)$$

$$g_s(\theta, V) = - \begin{bmatrix} f_p(P_l(\theta, V), V_d) \\ f_q(Q_l(\theta, V), V_d) \end{bmatrix} \quad (34)$$

The system model can now be formulated as

$$\dot{\omega} = -M^{-1} D \omega - M^{-1} T^t (P_g(\theta, V) - P_g^o) \quad (35)$$

$$\dot{\theta}_g = T \omega \quad (36)$$

$$\begin{bmatrix} \dot{\theta}_d \\ \dot{V}_d \end{bmatrix} = J_t(\theta, V)^{-1} (g_s(\theta, V) - J'_{dg} T \omega) \quad (37)$$

$$0 = P_{l2}(\theta, V) - P_{d2}(V_s) \quad (38)$$

$$0 = Q_{l2}(\theta, V) - Q_{d2}(V_s) \quad (39)$$

This model is a DA system. The dynamic states are $\omega \in \mathbb{R}^m, \theta_g \in \mathbb{R}^{m-1}$, and $\theta_d, V_d \in \mathbb{R}^d$. The algebraic states are $\theta_s, V_s \in \mathbb{R}^{n_s-m-d}$. The algebraic equations (38),(39) typically define a $(2d+m-1)$ -manifold called the *constraint manifold*.

Recall that this model is dependent upon nonsingularity of J_{ss} and J_t . Conditions governing the nonsingularity of these Jacobians are explored in Section 4.1.

3.3 State load formulation

Formulating the system model in terms of the state version of the load model is more straight forward. In this case, assuming $D_{x_p} \phi_p, D_{x_q} \phi_q$ are nonsingular, (5),(6) can be used directly to give

$$\dot{\omega} = -M^{-1} D \omega - M^{-1} T^t (P_g(\theta, V) - P_g^o) \quad (40)$$

$$\dot{\theta}_g = T \omega \quad (41)$$

$$\dot{x}_p = -(D_{x_p} \phi_p)^{-1} f_p(\phi_p(x_p, V_d), V_d) \quad (42)$$

$$\dot{x}_q = -(D_{x_q} \phi_q)^{-1} f_q(\phi_q(x_q, V_d), V_d) \quad (43)$$

$$0 = P_{l1}(\theta, V) - \phi_p(x_p, V_d) \quad (44)$$

$$0 = Q_{l1}(\theta, V) - \phi_q(x_q, V_d) \quad (45)$$

$$0 = P_{l2}(\theta, V) - P_{d2}(V_s) \quad (46)$$

$$0 = Q_{l2}(\theta, V) - Q_{d2}(V_s) \quad (47)$$

Again the model is a DA system. In this case the dynamic states are $\omega \in \mathbb{R}^m, \theta_g \in \mathbb{R}^{m-1}$, and $x_p, x_q \in \mathbb{R}^d$. The algebraic states are $\theta_d, V_d \in \mathbb{R}^d$ and $\theta_s, V_s \in \mathbb{R}^{n_s-m-d}$. The algebraic equations (44)-(47) again typically define a constraint manifold of dimension $(2d+m-1)$.

The conditions for nonsingularity of $D_{x_p} \phi_p, D_{x_q} \phi_q$ are discussed in Section 4.2.

4 Solvability of system models

The system models obtained using the IO and state forms of the general load model are both DA systems, i.e., they are of the form

$$\dot{x} = f(x, y) \quad (48)$$

$$0 = g(x, y) \quad (49)$$

Such systems are well defined where ever $D_y g$ is nonsingular. Under such conditions the Implicit Function Theorem can be used to obtain the algebraic variables as $y = \varphi(x)$. The system can then be treated locally as a set of differential equations, $\dot{x} = f(x, \varphi(x)) = \tilde{f}(x)$. However the model breaks down at points where $D_y g$ is singular. Such behaviour is explored further in [13, 14], and numerous other papers.

In this section we wish to explore the influence of load models on algebraic singularity of the complete system model. The general IO and state load models will be considered. The more structured exponential and adaptive load models will then be investigated in Section 5.

4.1 General IO load model

We shall consider the system model formed using the IO load model (35)-(39). The algebraic equations are (38)-(39). From (26)-(27) and (29), it can be seen that the algebraic Jacobian is J_{ss} . A scaled version of this Jacobian was considered in [14], where the influence on singularity of the static load model (15) was explored. (The reactive power balance equations in [14] were divided by voltage.) The load was modelled as voltage dependent, i.e.,

$$P_{di}(V_i) = P_{di}^o V_i^{\zeta_i} \quad (50)$$

$$Q_{di}(V_i) = Q_{di}^o V_i^{\eta_i} \quad (51)$$

The following result was suggested.

Conjecture Consider the network consisting of load buses $i = d + 1, \dots, n_b - m$. Suppose

- The admittance matrix of this load bus network is negative definite,
- $P_i, Q_i \geq 0$,

and for all non-zero real and reactive power loads, either

$$\zeta_i \geq 1 \quad (52)$$

$$\eta_i = 2 \quad (53)$$

or

$$\zeta_i = 2 \quad (54)$$

$$\eta_i \geq 1 \quad (55)$$

Then $\det J_{ss} \neq 0$ at all points on the constraint manifold where $V > 0$. $\square$

A closely related result was considered in [21], where it was proven that for load indices satisfying $\zeta_i, \eta_i > 1$, the constraint manifold exists globally, i.e., for any given value of the dynamic state $(\theta_d, V_d, \theta_g)$, there will always exist at least one value of the algebraic state (θ_s, V_s) that satisfies the algebraic equations. This result still allows folds in the constraint manifold though, so there may be points where J_{ss} is singular. However it does lend support to the idea that for a certain class of load indices, J_{ss} is nonsingular at all points on the constraint manifold where voltages are non-zero.

Two special cases deserve mention:

- $\zeta_i, \eta_i = 2$, i.e., constant admittance loads. In this case the loads can be absorbed into the self admittances of each bus. Assuming bus voltages are non-zero, normal network reduction techniques can always be used to eliminate the algebraic equations. This implies global nonsingularity of J_{ss} .
- $\zeta_i, \eta_i = 0$, i.e., constant power loads. If the reactive power balance equation for each bus i was scaled by V_i , then J_{ss} would be symmetric in this case. Therefore J_{ss} is symmetrizable, so all its eigenvalues are real. It is therefore possible for a real eigenvalue of J_{ss} to become zero as the dynamic states vary. Hence global nonsingularity of J_{ss} cannot be guaranteed.

At points where J_{ss} is nonsingular, then by the Implicit Function Theorem, we obtain locally

$$\begin{bmatrix} \theta_s \\ V_s \end{bmatrix} = \varphi(\theta_g, \theta_d, V_d) \quad (56)$$

The algebraic variables θ_s, V_s can then be eliminated from (35)-(37), leaving the system described (locally) by a set of differential equations.

However, it can be seen from (37) that model validity is also dependent upon nonsingularity of J_t . In exploring this condition on J_t , it is useful to define the functions ψ_p, ψ_q such that

$$D_{v_d} \psi_p(P_d, V_d) = g_p(P_d, V_d) \quad (57)$$

$$D_{v_d} \psi_q(Q_d, V_d) = g_q(Q_d, V_d) \quad (58)$$

We observe from (4) that ψ_p, ψ_q describe the transient response of the load model.

We define the transient power balance equations at load buses (refer (24)-(27)) as $h(z, \theta_g, P_{d1}, Q_{d1}) = 0$, where $z = [\theta_d^t \ V_d^t \ \theta_s^t \ V_s^t]^t$ and

$$h(z, \theta_g, P_{d1}, Q_{d1}) = \begin{bmatrix} P_{11}(\theta_d, V_d, \theta_s, V_s, \theta_g) - \psi_p(P_{d1}, V_d) \\ Q_{11}(\theta_d, V_d, \theta_s, V_s, \theta_g) - \psi_q(Q_{d1}, V_d) \\ P_{12}(\theta_d, V_d, \theta_s, V_s, \theta_g) - P_{d2}(V_s) \\ Q_{12}(\theta_d, V_d, \theta_s, V_s, \theta_g) - Q_{d2}(V_s) \end{bmatrix} \quad (59)$$

Then, referring to (28)-(29), we obtain

$$D_z h = \begin{bmatrix} J_{dd} - \begin{bmatrix} 0 & g_p(P_{d1}, V_d) \\ 0 & g_q(Q_{d1}, V_d) \end{bmatrix} & J_{ds} \\ J_{sd} & J_{ss} \end{bmatrix} \quad (60)$$

If $\det J_{ss} \neq 0$, the Schur complement of $D_z h$ gives

$$\begin{aligned} \det D_z h &= \det J_{ss} \det \left(J_{dd} - \begin{bmatrix} 0 & g_p(P_{d1}, V_d) \\ 0 & g_q(Q_{d1}, V_d) \end{bmatrix} - J_{ds} J_{ss}^{-1} J_{sd} \right) \\ &= \det J_{ss} \det \left(J'_{dd} - \begin{bmatrix} 0 & g_p(P_{d1}, V_d) \\ 0 & g_q(Q_{d1}, V_d) \end{bmatrix} \right) \\ &= \det J_{ss} \det J_t \end{aligned} \quad (61)$$

So, assuming J_{ss} is nonsingular (this was discussed earlier), J_t is singular when $D_z h$ is singular. But, referring to (59), we see that the transient load response ψ_p, ψ_q plays a role at the dynamic load buses which is similar to the voltage dependent loads at the static load buses. So singularity of $D_z h$ and hence J_t is dependent on the form of ψ_p, ψ_q . For example, if ψ_p, ψ_q took the form of voltage dependent loads (50)-(51), with indices given by (52), (53) or (54), (55), then the above conjecture suggests that J_t would be nonsingular at all points on the constraint manifold where voltages were nonzero. The model would therefore be solvable at all such points.

Further investigation of these results is largely dependent on the form taken by g_p, g_q . We shall therefore return to these ideas in Section 5, when considering the exponential and adaptive load models. In those cases we have specific forms for the load's transient response.

4.2 General state load model

We now turn our attention to the system model developed using the state form of the dynamic load model (40)-(47). The first thing to notice is that the model solvability is dependent on nonsingularity of $D_{x_p} \phi_p, D_{x_q} \phi_q$. This condition comes about because the state model is formed from the IO model by effectively adding an extra algebraic constraint (5) for each load, and an extra algebraic variable x_p or x_q . This DA model is then manipulated to yield system behaviour in terms of the x_p, x_q variables. To illustrate this, consider the real power dynamic load description with the extra algebraic equations

$$\dot{P}_{d1} + f_p(P_{d1}, V_d) = g_p(P_{d1}, V_d) \dot{V}_d \quad (62)$$

$$P_{d1} = \phi_p(x_p, V_d) \quad (63)$$

We see in this model that P_{d1} and V_d are possible dynamic states, whilst the x_p are algebraic states. It was

shown in the discussion of the general DA system (48)-(49) that the model may break down if $D_y g$ was singular. For the DA system (62)-(63), that condition translates to model breakdown occurring if $D_x \phi_p$ is singular. Structure is required on (f, g) to explore singularity of $D_x \phi_p$, $D_x \phi_q$ further.

The validity of the state model is also dependent on solvability of the algebraic equations (44)-(47). Let

$$h'(z, \theta_g, x_p, x_q) = \begin{bmatrix} P_{11}(\theta_d, V_d, \theta_s, V_s, \theta_g) - \phi_p(x_p, V_d) \\ Q_{11}(\theta_d, V_d, \theta_s, V_s, \theta_g) - \phi_q(x_q, V_d) \\ P_{12}(\theta_d, V_d, \theta_s, V_s, \theta_g) - P_{d2}(V_s) \\ Q_{12}(\theta_d, V_d, \theta_s, V_s, \theta_g) - Q_{d2}(V_s) \end{bmatrix} \quad (64)$$

where $z = [\theta_d^t V_d^t \theta_s^t V_s^t]^t$. Note the similarity between (64) and (59). Equation (59) involved ψ_p, ψ_q which are functions of P_{d1}, Q_{d1} respectively, whereas (64) involves ϕ_p, ϕ_q which are functions of x_p, x_q respectively. Note further the definitions of ψ_p, ψ_q given in (57)-(58), and ϕ given at (5),(7). We see therefore that

$$D_z h' = D_z h = \begin{bmatrix} J_{dd} - \begin{bmatrix} 0 & g_p(P_{d1}, V_d) \\ 0 & g_q(Q_{d1}, V_d) \end{bmatrix} & J_{ds} \\ J_{sd} & J_{ss} \end{bmatrix} \quad (65)$$

So, referring to (61), if $\det J_{ss} \neq 0$,

$$\det D_z h' = \det J_{ss} J_t \quad (66)$$

Hence there is a direct correlation between the conditions governing solvability of the IO model formulation of the system model, and the state model formulation. In this case, singularity of $D_z h'$ is dependent on the form of ϕ_p, ϕ_q . This is considered further in the next section.

5 Model singularity for specific load models

As indicated in Section 4, further investigation of model solvability is dependent on knowing the form of load model functions (f, g) of (2). In this section we shall therefore use the ideas developed previously for the general load model to investigate solvability of two specific load models, the exponential recovery and adaptive models.

It was shown in Section 4 that there was direct correspondence in the conditions of solvability of the IO and state forms of the system model. Therefore we shall only consider the state forms of the models here.

5.1 Exponential load model

It can be seen from (8),(9) that for the exponential recovery load model

$$f_p(P_{d1}, V_d) = T_p^{-1}(P_{d1} - P_s(V_d)) \quad (67)$$

$$g_p(P_{d1}, V_d) = T_p^{-1}k_p(V_d) \quad (68)$$

$$\phi_p(x_p, V_d) = x_p + P_t(V_d) \quad (69)$$

where T_p, k_p are now diagonal matrices. Reactive power equations f_q, g_q, ϕ_q are similar. We see from (69) that $D_x \phi_p = I$, i.e., the identity matrix. So $D_x \phi_p, D_x \phi_q$ are always nonsingular.

In Section 4, the influence of ϕ_p, ϕ_q on the singularity of $D_z h'$ was discussed. It was conjectured that if ϕ_p, ϕ_q took the form of voltage dependent loads (50),(51), with indices satisfying (52),(53) or (54),(55), then $D_z h'$ would be nonsingular at all points on the constraint manifold where voltages were nonzero. However it can be seen from (69) that the x_p part of ϕ_p is like a constant power load term, i.e., it is independent of voltage. The form of $P_t(V_d)$ does not matter. A similar observation can be made regarding ϕ_q . It was noted in Section 4.1 that if loads were constant power, then nonsingularity of the algebraic Jacobian could not be guaranteed. Therefore, because ϕ_p, ϕ_q both have components which behave like constant power loads, nonsingularity of $D_z h'$ cannot be guaranteed, for any choice of $P_t(V_d), Q_t(V_d)$. System trajectories can therefore encounter points where the model breaks down.

Note that loads commonly respond transiently as constant admittance. In such cases, the transient load terms $P_t(V_d), Q_t(V_d)$ of ϕ_p, ϕ_q can be absorbed into the admittance matrix, leaving ϕ_p, ϕ_q as effectively constant loads x_p, x_q respectively.

A simple example, given in Section 5.3, illustrates these results.

5.2 Adaptive load model

From (13),(14) we find that for the adaptive load model

$$f_p(P_{d1}, V_d) = T_p^{-1}P_t(V_d)(P_{d1} - P_s(V_d)) \quad (70)$$

$$g_p(P_{d1}, V_d) = P_t'(V_d)P_t(V_d)^{-1}[\text{diag } P_{d1}] \quad (71)$$

$$\phi_p(x_p, V_d) = P_t(V_d)x_p \quad (72)$$

where T_p, P_t are now diagonal matrices, and $[\text{diag } P_{d1}]$ refers to a diagonal matrix with the elements of P_{d1} down the diagonal. Reactive power equations f_q, g_q, ϕ_q are similar. From (72) we obtain $D_x \phi_p = P_t(V_d)$. So nonsingularity of $D_x \phi_p, D_x \phi_q$ is dependent on all $P_{ti}(V_i), Q_{ti}(V_i)$ being nonzero. If any of these transient load values become zero, then the model breaks down.

As noted above, ϕ_p, ϕ_q influence singularity of $D_z h'$, and hence model solvability. In this case we see from (72) that because the state x_p, x_q multiplies the voltage dependent transient load response $P_t(V_d), Q_t(V_d)$, functions ϕ_p, ϕ_q can take the form (50),(51). (Recall from (69) that for the exponential load model, the state was additive, so ϕ_p, ϕ_q could never take the form (50),(51).) Therefore, if $P_t(V_d), Q_t(V_d)$ were of the appropriate form, with indices given by (52),(53) or (54),(55), then we conjecture that $D_z h'$ would be nonsingular at all points where voltages were nonzero. The model would be effectively globally solvable.

For example, it was shown in Section 4.1 that if all $\zeta_i = \eta_i = 2$, then global nonsingularity of the algebraic Jacobian could be guaranteed. So, if all $P_t(V_d), Q_t(V_d)$ had indices $\zeta = \eta = 2$, i.e., constant admittance transient load response, then we could guarantee that the DA system model (40)-(47) (and likewise the IO model (35)-(39)) would only experience model breakdown problems at points where voltages went to zero. Certainly, under normal conditions, the models would be solvable. As noted earlier, this form of transient load response is quite common.

5.3 Example

To illustrate the difference between the exponential and adaptive load models, we shall consider the simple system of Figure 3. This system consists of a single dynamic load supplied from an infinite bus. Therefore there are no generator dynamics or static load buses to consider. Also, we shall assume that real power load is zero.

If the load is modelled using the exponential load model, then the system description is

$$T_q \dot{x}_q = -x_q + Q_s(V) - Q_t(V) \quad (73)$$

$$0 = Q_t(\theta, V) - (x_q + Q_o(V)) \quad (74)$$

$$0 = P_l(\theta, V) \quad (75)$$

From (75) and (16) we find that $\theta = 0$. Let $Q_t(V) = Q_o V^\eta$. Then using (17), (74) becomes

$$h'(V, x_q) = BEV - BV^2 - (x_q + Q_o V^\eta) = 0 \quad (76)$$

where $B = 1/X$.

We are interested in finding points on the constraint manifold, i.e., that satisfy (76), at which $D_v h' = 0$. Now,

$$D_v h' = BE - 2BV - \eta Q_o V^{\eta-1} = 0$$

So,

$$BE = 2BV + \eta Q_o V^{\eta-1}$$

Substituting this back into (76) gives

$$\begin{aligned} 2BV^2 + \eta Q_o V^\eta - BV^2 - x_q - Q_o V^\eta &= 0 \\ \Rightarrow BV^2 + (\eta - 1)Q_o V^\eta - x_q &= 0 \end{aligned} \quad (77)$$

It is possible that points satisfying (77) could be encountered along a trajectory of the system. The model would break down at such points.

Now consider the adaptive load model,

$$T_q \dot{x}_q = Q_s(V) - x_q Q_t(V) \quad (78)$$

$$0 = Q_t(V) - x_q Q_t(V) \quad (79)$$

If we again set $Q_t(V) = Q_o V^\eta$, then we obtain

$$h'(V, x_q) = BEV - BV^2 - x_q Q_o V^\eta = 0 \quad (80)$$

So,

$$D_v h' = BE - 2BV - \eta x_q Q_o V^{\eta-1} = 0$$

giving

$$BE = 2BV + \eta x_q Q_o V^{\eta-1}$$

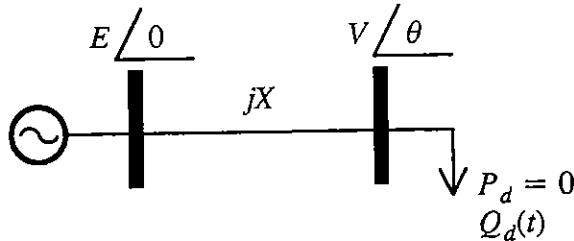


Figure 3: Two Bus System

Substituting back into (80) gives

$$\begin{aligned} 2BV^2 + \eta x_q Q_o V^\eta - BV^2 - x_q Q_o V^\eta &= 0 \\ \Rightarrow BV^2 + (\eta - 1)x_q Q_o V^\eta &= 0 \end{aligned} \quad (81)$$

If $\eta \geq 1$, then (81) can only be satisfied if $V = 0$. (Recall the assumption $Q_d \geq 0$, which ensures that $x_q \geq 0$.) Therefore the model cannot break down, except at points where $V = 0$.

6 Conclusions

The power system model which captures generator and load dynamic behaviour has a differential-algebraic (DA) form. Such models are susceptible to difficulties in solving the algebraic equations. Model break-down is closely linked with singularity of the Jacobian of the algebraic equations. The paper explores the influence of load models on solvability.

It is shown that by appropriate choice of load model structure and parameters, solvability problems of DA models of power systems can be overcome. For example, if loads were modelled using the adaptive load model [11], with a constant admittance form of transient response, then the DA power system model would be globally solvable. Thus load models provide an appealing way of overcoming such DA system difficulties.

The steady state load response does not influence solvability. Loads can have constant power steady state response, for example, and not encounter DA problems; provided the transient response has the appropriate form.

Acknowledgements

This work was sponsored in part by an Australian Electricity Supply Industry Research Board project grant "Voltage Collapse Analysis and Control", and an Australian Research Council project grant "Security Analysis and Enhancement of Large Power Systems".

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THE IRRELEVANCE OF LOAD DYNAMICS FOR THE LOADING MARGIN TO VOLTAGE COLLAPSE AND ITS SENSITIVITIES

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Abstract: The loading margin to a saddle-node or fold bifurcation is a useful and basic index of proximity to voltage collapse. Sensitivities of the loading margin can be used to select controls to avoid voltage collapse. This paper justifies the use of static power models to compute loading margins and their sensitivities. In particular, dynamic load models may be reduced to static equations and then simplified without affecting loading margins and their sensitivities. Alternatively, if dynamic models are not well known, the computations may be done with static models and conclusions drawn about a general and sensible class of underlying dynamic models. Explicit assumptions and mathematical underpinnings are presented.

1. Introduction

Dynamic voltage collapse due to loss of a stable operating equilibrium in a saddle-node or fold bifurcation is an important mechanism for power system instability [11]. Therefore it is useful to compute the location of fold bifurcations of power system models and their proximity to current operating conditions. The proximity to fold bifurcation can be measured by computing the loading margin; that is, the increase in loading required to obtain fold bifurcation. If the loading margin is dangerously small, then controls to steer the power system away from the fold bifurcation can be selected by computing the sensitivity of the loading margin to control actions [14]. The sensitivity of the loading margin to any power system parameter is also easy to compute.

Here the terms saddle-node bifurcation and fold bifurcation may be regarded as interchangeable, but a distinction is made between them in Appendix A. In the particular case of the power system loading being expressed in terms of real and reactive load powers, the loading margin is also called the load power margin.

Section 3 of the paper explains and proves the following observation:

Loading margins to fold bifurcations and their sensitivities depend only on the static parts of dynamic power system models.

For example, a dynamic power system model

$$\dot{x} = f(x, \lambda)$$

may be reduced to the static equations

$$0 = f(x, \lambda)$$

without affecting the loading margin to a fold bifurcation. Although voltage collapse due to fold bifurcation is dynamic and requires dynamic power system models to be properly understood, the computation of loading margin and its sensitivities only requires knowledge of the static parts of the dynamic models. This simplification

is of great practical value because the dynamic parts of load models are often poorly known. We can compute loading margins and their sensitivities using the better known and simpler static load models, while assuming a very general form for the poorly known dynamics which underlie the static equations. This observation generalizes more limited results in [13, 14, 5] and supports the reductions in [27] of differential-algebraic power system equations to static models.

Once static power system models $0 = f(x, \lambda)$ are obtained, they can be advantageously simplified by algebraic manipulation. Section 4 describes useful algebraic manipulations which do not affect the loading margins and their sensitivities. Section 5 states the further assumptions needed to conclude that the loading margin to fold bifurcation is the minimum loading margin to any instability.

We choose to measure proximity to voltage collapse with a loading margin because it is an accurate and basic index, appreciable by all, which takes full account of system limits and nonlinearities. Observe that every paper on voltage collapse indices not explicitly using loading margin implicitly acknowledges loading margin by using it as the horizontal scale when the performance of the proposed index is graphed.

2. Fold bifurcation and geometry

The power system state vector x typically includes bus voltages and angles as well as other system states. The parameter vector λ includes measures of system loading such as real or reactive power demand at a bus or more general loading parameters for loads with more elaborate load models. Suppose the power system is operating with state x_0 and parameter λ_0 . In order to measure the proximity to voltage collapse, the effect of increasing the loading parameters in λ in some given pattern is computed by a continuation method [e.g. 4,8,1,20] or a quasistatic simulation method [27,21] until a fold bifurcation is encountered at (x_*, λ_*) . (Recall that continuation solves for a succession of equilibria as the loading is increased and takes account of power system limits as they occur.) The loading margin is then the distance $|\lambda_* - \lambda_0|$.

We make the following assumption

Assumption 1. Near the bifurcation at (x_*, λ_*) , the system is assumed to have dynamics specified by parameterized, smooth differential equations $\dot{x} = f(x, \lambda)$.

Assumption 1 allows the voltage collapse to be analyzed and understood and the sensitivities needed to avoid the bifurcation to be computed. In a fold bifurcation, two equilibria coalesce and disappear. If one of the equilibria

is stable, then the dynamic consequences of the fold bifurcation are well approximated by movement, slow then fast along a particular dynamic trajectory called the unstable part of the center manifold of x_* [11].

There is also much information available from eigenvectors of the Jacobian $f_x|_*$ about the geometry of the fold bifurcation. The essential information about the fold bifurcation that needs to be determined is its location (x_*, λ_*) , the right eigenvector v and the normal vector N . (The right eigenvector v corresponds to the zero eigenvalue of the singular Jacobian $f_x|_*$. The normal vector $N = w f_\lambda|_*$, where w is the left eigenvector corresponding to the zero eigenvalue of $f_x|_*$.) These have the following engineering uses and interpretations:

(a) The critical loading λ_* determines the loading margin $|\lambda_* - \lambda_0|$ which describes the proximity of the current loading λ_0 to voltage collapse.

(b) The right eigenvector v describes both the direction in state space in which the equilibrium moves just before the bifurcation and the initial direction in state space along which the dynamic voltage collapse occurs. In particular, v quantitatively predicts the relative amounts of voltage decline at buses as the bifurcation is approached and as the initial part of the dynamic collapse occurs [11].

(c) N is the normal vector to the set of critical loadings in parameter space at which fold bifurcations occur. N determines the optimum direction to move in parameter space to avoid fold bifurcation. In particular, it determines the combination of loads to shed to optimally move away from bifurcation [12]. More generally and importantly, N can be used to easily compute the first order sensitivity of the loading margin to any power system parameters or controls [14,3]. This allows effective controls to be selected to optimally steer the power system to a condition with a larger loading margin to voltage collapse. For example, the effect on the loading margin of adding 1 MVAR reactive power support at any system bus can be easily computed. Thus the buses at which reactive power support is most effective can be selected. The computation of the sensitivity of the loading margin also allows the critical and noncritical power system parameters to be identified in terms of their quantitative effect on the loading margin.

3. Reduction to static equations

Since the location of the bifurcation (x_*, λ_*) and the useful quantities v and N are computed from the dynamic equations $\dot{x} = f(x, \lambda)$, one might expect that details of the dynamics of f would affect the computations. However, we explain below and prove in appendix A that this is not the case: (x_*, λ_*) , v and N only depend on the static equations $0 = f(x, \lambda)$. This not only simplifies the required modeling but is of great practical value because some power system dynamic equations, particularly load power dynamics, are poorly known. However, we retain the notion that there are some dynamics underlying the static equations so that we can understand the voltage collapse. We need only assume a very general form for these dynamics to compute (x_*, λ_*) , v and N for these dynamic equations from the corresponding static equations.

First we explain these ideas in a simple case of a single differential equation f in one state variable x and one parameter λ :

$$\dot{x} = f(x, \lambda) = -x^2 - \lambda + 1$$

The set Z is the nose curve parabola (or bifurcation diagram) $\lambda = -x^2 + 1$

$$Z = \{(x, \lambda) \mid 0 = -x^2 - \lambda + 1\}$$

and the dynamics for each fixed value of the parameter λ are indicated by the arrows in Figure 1. There is a generic fold bifurcation at the tip of the nose curve $(x, \lambda) = (0, 1)$. The most important of the conditions confirming this to be a fold bifurcation is that the derivative $f_x = -2x$ vanishes at $(x, \lambda) = (0, 1)$. That is, $f_x|_{(0,1)} = 0$. Since this condition involves f , it would seem to depend on the dynamical equations $\dot{x} = -x^2 - \lambda + 1$. However, an equivalent condition is that Z has a tangent parallel to the state space (that is, vertical in Figure 1) at the bifurcation. This shows that Z has a fold at the bifurcation when Z is viewed along the state space (vertical) axis. Thus the location of the bifurcation only depends on the set Z which is defined by the static equations $-x^2 - \lambda + 1 = 0$. To emphasize this, note that there are many other differential equations with different dynamics but with the same set Z . Examples are:

$$\begin{aligned}\dot{x} &= 2(-x^2 - \lambda + 1) \\ \dot{x} &= (x^2 + 2)(-x^2 - \lambda + 1)\end{aligned}$$

or more generally,

$$\dot{x} = h(-x^2 - \lambda + 1)$$

where h is any smooth function with $h(0) = 0$ and no other zeros and nonzero gradient at zero.

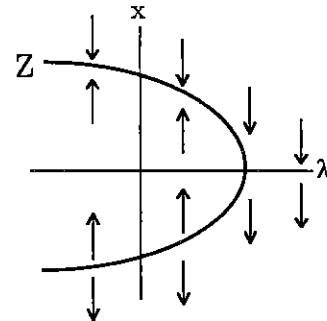


Figure 1. The set Z plus dynamics.

Thus we can compute Z and the location of the bifurcation with any of the static equations $0 = h(-x^2 - \lambda + 1)$ and the dynamics specified by h are irrelevant to this computation. Note that the eigenvalues or gradients f_x of these equations are completely different except that they all vanish at the bifurcation.

Now let the state vector x have n dimensions and the parameter vector λ have m dimensions. The n dimensional state space is denoted by X and the m dimensional

parameter space is denoted by Λ . The power system is modeled as n differential equations

$$\dot{x} = f(x, \lambda)$$

Consider the state and parameter values for which f has an equilibrium:

$$Z = \{(x, \lambda) \mid f(x, \lambda) = 0\}$$

The set Z contains hypersurfaces of dimension m in the $m + n$ dimensional combined state and parameter space $X \times \Lambda$. The essence of a continuation method is to compute a curve in Z . Appendix A characterizes the fold bifurcation and its sensitivities in terms of the shape and position of Z . Since the shape and position of Z does not depend on dynamics, the location of the fold bifurcation only depends on the static equations $0 = f(x, \lambda)$ or other static equations yielding the same equilibrium set Z . Moreover, since the vectors v and N can be computed from the geometry of the fold of Z , v and N also do not depend on dynamics and may be computed from static equations. These observations can be stated as follows (for lemma 1 and a more precise version and proof of lemma 2 see appendix A):

Lemma 2. Suppose $0 = f(x, \lambda)$ are static equations with underlying dynamics specified by

$$\dot{x} = \bar{f}(x) = h(f(x, \lambda))$$

where $h(0) = 0$ and this zero is unique and simple. Then

- (a) f has a fold bifurcation at $(x_*, \lambda_*) \iff \bar{f}$ has a fold bifurcation at (x_*, λ_*) .
- (b) When suitably scaled, $v = \bar{v}$ and $N = \bar{N}$.

In the simplest application of lemma 2, h is chosen to be the identity map and the differential equations $\dot{x} = f(x, \lambda)$ are reduced to the static equations $0 = f(x, \lambda)$. If the differential equations contain nonzero time constants $T_1, T_2, \dots, T_n$ so that $\dot{x} = T^{-1}f(x, \lambda)$, where the matrix $T = \text{diag}\{T_1, T_2, \dots, T_n\}$, then choosing h in lemma 2 to be multiplication by T^{-1} again yields static equations $0 = f(x, \lambda)$. This shows that the location and sensitivities of fold bifurcations are independent of the value of the time constants. (That this holds for the location of fold bifurcations should be obvious upon noting that multiplying the k th row of the Jacobian f_x by T_k multiplies the determinant of f_x by T_k but does not affect where the determinant vanishes. However, the nonzero eigenvalues or singular values of f_x generally depend on the time constants.) One might also want to exploit the freedom to scale the static equations by multiplying them by various constants to improve the conditioning of the Jacobian f_x for numerical computation.

Often static equations are used because the dynamic equations are poorly known. Then lemma 2 gives a very general class of dynamic equations which can be assumed to underlie the static equations. For example, suppose we have power system equations of the form

$$\begin{aligned} \dot{x}_k &= f^k(x, \lambda), & k &= 1, \dots, n_g, \\ 0 &= f^k(x, \lambda), & k &= n_g + 1, \dots, n. \end{aligned}$$

where the algebraic equations express real or reactive power balance at the loads. Consider the load numbered j first and suppose that f^{n-1} and f^n represent real and reactive power balance at load j with state variables voltage

magnitude V and angle θ . Suppose that the underlying dynamics at load j are of the form

$$\begin{pmatrix} \dot{V} \\ \dot{\theta} \end{pmatrix} = h^j \left(\begin{pmatrix} f^{n-1}(x, \lambda) \\ f^n(x, \lambda) \end{pmatrix} \right)$$

where $h^j : U \rightarrow \mathbb{R}^2$ is a vector valued vector function on U , where $0 \in U \subset \mathbb{R}^2$. We suppose that the Jacobian of h is nonsingular at the origin. (This is reasonable since independent first order variations in real and reactive power balance would be expected to give independent first order variations in $\dot{V}$ and $\dot{\theta}$.) Then if all the loads have similar form to load number j , a function h suitable for lemma 2 can be constructed from the functions similar to h^j for the loads and identity functions for the first n_g equations. Thus lemma 2 shows that for understanding the problem, we can assume general and sensible underlying dynamics for each load, but for purposes of computing fold bifurcations, the static model

$$0 = f^k(x, \lambda), \quad k = 1, \dots, n.$$

can be used with no loss of accuracy. This example shows how differential-algebraic power system models can be reduced to static (algebraic) equations for computation of loading margins and their sensitivities. The computations are valid for a general class of dynamics underlying the static equations. For another example of the use of this reduction see [27, section 3.2].

4. Simplifying the algebraic equations

Once static equations are obtained, it is often useful to algebraically manipulate them to simplify them or gain some computational advantage. The following lemma shows that algebraic manipulations which preserve the set Z and the rank of the equations do not affect fold bifurcations (for a more precise and complete version and proof see appendix A):

Lemma 4. (informal version) Let $f : X \times \Lambda \rightarrow X$ have $Z = \{(x, \lambda) \mid f(x, \lambda) = 0\}$. Let $\bar{f} : X \times \Lambda \rightarrow X$ have $\bar{Z} = \{(x, \lambda) \mid \bar{f}(x, \lambda) = 0\}$. Suppose that $Z = \bar{Z}$ and that at $z_* \in Z$, $\text{rank } f_z|_* = \text{rank } \bar{f}_z|_*$ where $z = (x, \lambda)$. ($f_z|_*$ is the Jacobian of f with respect to z evaluated at z_* .) Then

- (a) f has a fold bifurcation at z_* $\iff \bar{f}$ has a fold bifurcation at z_* .
- (b) When suitably scaled, $v = \bar{v}$ and $N = \bar{N}$.

Sometimes load dynamics are specified in an implicit form and algebraic manipulation is required to show how the dynamics can be removed using lemma 2. For example, consider the following power system dynamic model with a single dynamic load model shown explicitly:

$$\dot{y} = f^1(y, V, \lambda) \quad (4.1)$$

$$0 = g(\dot{P}^d, \dot{V}, P^d, V, \lambda) \quad (4.2)$$

$$P^d = P(y, V) \quad (4.3)$$

Here V is the load voltage, y are the other state variables, P^d is the real power demanded by the load and the function P evaluates the real power supplied to the load by the electrical network. Equation (4.2) is a general form due to Hill; it includes as a special case his exponential recovery model [19]. Equation (4.3) is real power balance at the load and equation (4.1) are the other dynamic equations. Appendix B shows that if (4.1-4.3) specify a well posed differential equations in the state variables (y, V) , then lemmas 2 and 4 can be applied to yield static equations

$$0 = f^1(y, V, \lambda) \quad (4.4)$$

$$0 = g(0, 0, P(y, V), V, \lambda) \quad (4.5)$$

Note that equation (4.5) is the static part of the dynamic load model (4.2), since it corresponds to setting $\dot{P}^d = \dot{V} = 0$ in (4.2).

It is often useful to reduce the number of equations by solving some of the equations for some of the variables and substituting the solved variables in the remaining equations. The inverse of this process, in which new variables are defined and more equations are written, is also sometimes useful because it can improve system sparsity. When changing the static equations in this way, we want to be assured that the fold bifurcations and their geometry are essentially unchanged. This is the substance of the following lemma (for a more precise version and proof see appendix A):

Lemma 5. (informal version) *Let $x \in \mathbb{R}^n$ be written as $x = (a, b)$ where $a \in \mathbb{R}^{\bar{n}}$ and $b \in \mathbb{R}^{n-\bar{n}}$. Write the n equations $0 = f(x, \lambda)$ as $0 = \begin{pmatrix} f^1(a, b, \lambda) \\ f^2(a, b, \lambda) \end{pmatrix}$ where f^1 consists of $\bar{n}$ equations and f^2 consists of $n - \bar{n}$ equations. Suppose that $f_b^2|_*$ is invertible so that the equations $0 = f^2(a, b, \lambda)$ are solvable (at least locally) for b so that $b = h(a, \lambda)$. Define the $\bar{n}$ equations $\bar{f}(a, \lambda) = f^1(a, h(a, \lambda), \lambda)$. Then f has a fold bifurcation at $z_* = (a_*, b_*, \lambda_*) \iff \bar{f}$ has a fold bifurcation at $\bar{z}_* = (a_*, \lambda_*)$. Moreover, for suitable scaling of $\bar{n}$ and $\bar{v}$, $N = \bar{N}$ and $v = \begin{pmatrix} \bar{v} \\ h_a|_* \bar{v} \end{pmatrix}$.*

A simple example of the use of lemmas 2 and 5 occurs when the dynamic equations include $\dot{\delta} = \omega$, where δ is the power angle of a generator. Lemma 2 reduces $\dot{\delta} = \omega$ to the algebraic equation $0 = \omega$ and lemma 5 allows ω to be set to zero in all the other power system equations and omitted from the system state.

Lemma 5 also allows equation (4.4-4.5) to be rewritten with an additional state P^d as

$$0 = f^1(y, V, \lambda)$$

$$0 = g(0, 0, P^d, V, \lambda)$$

$$P^d = P(y, V)$$

if desired, without affecting the fold bifurcation.

A more substantial application of lemma 5 can be seen to yield the Liapunov Schmidt reduction (choose a of lemma 5 to be a coordinate along v and define $f^1 = wf$, etc.) [10].

Note that each of the lemmas which reduce dynamic equations to static equations or simplify the static equations states the relation between v and n before and after the reduction or simplification. Therefore, if we compute

v and n with the simplified static equations, we can work back through the applicable lemmas to deduce v and n for the dynamic equations. That is, the geometry of the fold of the dynamic equations is calculable from the geometry of the fold of the simplified static equations and vice versa. The most striking instance of this property is that the initial direction v of movement of the dynamic voltage collapse can be determined from static equations!

Cañizares [5] proves by direct calculation that a set of differential-algebraic equations modeling an AC/DC power system can be reduced to static equations while preserving the loading margin and the bifurcation geometry. Lemmas 2 and 5 apply to the AC/DC power system model and provide alternate proofs of Theorems 1 and 2 of [5] (use the versions of lemmas 2 and 5 in Appendix A to obtain Theorem 2).

5. The context of other instabilities

So far we have only considered the computation of loading margin to a fold bifurcation of an equilibrium. The previous assumption 1 suffices if we are only concerned with this narrow focus. There are other possible instabilities and transient dynamics to be considered in determining the loading margin to the first instability of the power system as the loading is increased. We state some additional assumptions necessary if we are to conclude that the fold bifurcation is this first bifurcation and hence the applicable instability mechanism for the given loading increase. The reader should refer to [27], especially the second scenario of section 2.2, for a previous account of some of the ideas in this section.

Assumption 2. The system is assumed to be governed by smooth differential equations parameterized by λ as the loading increases. However, the differential equations may undergo discrete changes when limits or structural changes are encountered. Limits important to voltage collapse include generator reactive power limits and tap changing transformer limits. A line tripping or a generator outage are examples of structural changes.

Assumption 3. The initial operating condition (x_0, λ_0) is assumed to be asymptotically stable. This is usually known either by assuming a zero loading λ_0 at which the asymptotically stable equilibrium x_0 is known or by previous or current operating experience at a higher loading λ_0 . (If the power system operates at (x_0, λ_0) without transients, then (x_0, λ_0) must be stable.)

Then there are three typical ways in which stability of the equilibrium can be lost as the loading is increased from λ_0 :

(1) *Fold bifurcation instability.* The stable equilibrium disappears in a fold bifurcation. This instability occurs generically and locating and avoiding this bifurcation is the focus of this paper. Note that a fold bifurcation *must* occur eventually because all solutions disappear for sufficiently high loading.

(2) *Instability due to discrete change.* A stable equilibrium may disappear or become unstable when the differential equations undergo discrete changes. For example, when a generator encounters a reactive power limit, the system equations change in such a way that the equilibrium persists but can possibly change stability [13]. If a line or generator is tripped, then the system equations change in such a way that equilibrium can possibly be lost.

(3) *Hopf bifurcation instability.* The stable equilibrium persists, but becomes unstable in an oscillatory instability associated with Hopf bifurcation. This instability occurs generically. Complete dynamic models are required to detect or analyze a Hopf bifurcation. We do not consider oscillatory instabilities in this paper. There are interesting possible mechanisms for voltage collapse involving Hopf bifurcations [29] but we have not seen convincing operating experience of oscillations playing a significant role in voltage collapse incidents. In general, leaving aside the thrust of this paper, the importance of power system oscillations and the possibilities of more exotic power system dynamical behaviors are strong incentives to develop good dynamic models including dynamic load models.

It is also possible in exceptional cases for combinations of instabilities (1) (2) and (3) to occur simultaneously, but we ignore these nongeneric possibilities.

If the discrete change or Hopf bifurcation instabilities do not occur as the loading is increased, then the equilibrium will always lose stability in a fold bifurcation. However, an additional assumption is needed to ensure that the system state stays near enough to the equilibrium as the loading is increased to guarantee that stability is not lost due to large signal transient effects:

Assumption 4. The system state tracks the system equilibrium sufficiently closely as the parameters change. This assumption is known as the quasistatic assumption [11]. Deviation of the state from the stable equilibrium can occur when the parameter rate of change becomes large enough or when discrete changes to the equations occur. In either case, transients occur and these transients are assumed to restore the state near to the stable equilibrium in such a way that the particular form and size of the transient can be neglected.

The ideas of this section so far can be summarized as follows:

Continuation methods can be applied to compute the fold bifurcation starting from a known stable condition at low loading. If stability is not lost due to oscillatory instabilities or limits or structural changes as the loading is slowly increased, then the stability must eventually be lost in a fold bifurcation and the loading at this fold bifurcation will determine the minimum system loading margin to any instability.

As regards checking whether instability occurs due to oscillatory instabilities (Hopf bifurcations), we suggest that it is useful to analyze and avoid oscillatory instabilities separately from the fold bifurcation. Although Hopf bifurcations and fold bifurcations can occur in the same power system models [9,2,24,7,29,15] there is not yet evidence that their instability problems are closely related. For

example, one might argue that if excitation systems are well designed, Hopf bifurcation instabilities encountered at higher loadings when studying voltage collapse models would vanish. (The parameter sensitivities for Hopf and fold bifurcations can be studied and compared using the techniques of [15].) From a practical modeling point of view, the Hopf bifurcation requires a complete dynamic model to be known and so is better analyzed separately. (We note that eigenvalue, singular value and Hopf analyses of differential-algebraic equations are only valid if the underlying dynamics enforcing the algebraic constraints are assumed to be both stable and fast.)

We hope to develop analytic tools for checking for instabilities due to limits or structural changes in future work.

Even with the assumptions above, the continuation approach to computing loading margins and their sensitivities has at present significant limitations. For example, the timing and complex interaction of discrete and continuously evolving events are not suitably represented. However, a time domain simulation can represent dynamic models in great detail and readily reproduce complex series of events. The loading margin and sensitivity approach is complementary to time domain simulation. While the loading margin approach must make some restrictive assumptions, it does provide useful sensitivity information and insight not available from time domain simulations.

6. Conclusions

This paper explains and justifies how static power system models can be used to compute the loading margin to voltage collapse due to fold or saddle node bifurcation and its sensitivities to parameters and controls. The static models are understood to have underlying dynamics which are responsible for the dynamic voltage collapse as a consequence of the fold bifurcation.

If power system dynamic equations are given, then we show that reducing the dynamic equations to static equations by setting the derivatives of the state variables to zero does not affect the loading margin or its sensitivities. Once the static equations are obtained, they may be algebraically simplified in several ways, again without affecting the loading margin or its sensitivities. Conclusions about the geometry of the fold bifurcation of the dynamic equations are available from the simplified static equations. This is illustrated by showing how a general class of dynamic load models due to Hill [19] can be reduced to static models. We do not advocate any particular static load models in this paper, but we do point out that it is essential to make an explicit choice of the load parameters to be included in the parameter vector λ when specifying a load model.

More commonly, the complete dynamic equations of the power system are not well known. However, we can compute load power margins and their sensitivities with

static power system equations and the paper shows that the results are accurate for a general and sensible class of underlying dynamic equations.

It is a substantial advantage for a voltage collapse index to be independent of dynamics both in simplifying and optimizing the power system model for computations and in allowing the computation to be done accurately despite poorly known dynamics. The paper shows that the load power margin is independent of dynamics and we suspect that this useful property is also shared by the energy function index. (Note that [23] gives an interpretation of an energy function index as the area enclosed by the intersection of Z with a plane spanned by the real power and angle at a bus; this interpretation is independent of the dynamics.)

Indices based on eigenvalues [e.g. 16] or singular values [e.g. 22, 25] depend on the linearization of the system at an operating equilibrium. Therefore these indices require for a power system model either a full set of differential equations or differential-algebraic equations with the assumption or knowledge that the algebraic equations are enforced by underlying dynamics that are both fast and stable. In particular, voltage collapse indices based on eigenvalues or singular values depend on load dynamics.

This paper provides justification and explicit assumptions for computations of loading margins and their sensitivities with static power system models. There are some technical differences between the mathematical definitions and methods appropriate for fold bifurcations of static and dynamic equations. The appendices of the paper contribute definitions and methods appropriate for fold bifurcations of static equations.

Acknowledgments

We thank Scott Greene and Rajesh Rajaraman for useful comments. Support in part by NSF grants ECS-9009079 and EPRI contracts RP 8010-30 and RP 8050-03 is gratefully acknowledged.

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Appendix A: Definitions and lemmas

A definition of a fold bifurcation of equations f is presented and its relation to the standard definition of a

saddle node bifurcation is discussed. The fold bifurcation of equations f is characterized in terms of a fold of the set

$$Z = \{(x, \lambda) \mid f(x, \lambda) = 0\}$$

which only depends on static parts of the power system model. Lemmas needed for sections 3 and 4 are stated and proved.

Let the state space X be $\mathbb{R}^n$ and the parameter space Λ be $\mathbb{R}^m$. Dynamic equations are $\dot{x} = f(x, \lambda)$ where $f : U \rightarrow TX$ is a smooth set of differential equations and U is an open set of $X \times \Lambda$ containing (x_*, λ_*) . (TX is the tangent space of X .) We will also define a fold bifurcation for smooth static equations $0 = f(x, \lambda)$ where $f : U \rightarrow X$.

It is useful in this appendix to distinguish between saddle-node bifurcations and fold bifurcations. We make this distinction by defining the bifurcations slightly differently.

Definition: Let $f : U \rightarrow X$ or $f : U \rightarrow TX$ be a smooth function and let the dimension of X be n . f has a **fold bifurcation** at $(x_*, \lambda_*) \in U$ if it satisfies conditions F(a)-F(d):

- F(a) $f(x_*, \lambda_*) = 0$
- F(b) $f_x|_*$ has rank $n - 1$
- F(c) $wf_\lambda|_* \neq 0$
- F(d) $wf_{xx}|_*(v, v) \neq 0$

where v and w are nonzero vectors satisfying $f_x|_*v = 0$ and $wf_x|_* = 0$.

Condition F(a) states that (x_*, λ_*) is a root or equilibrium of f . Condition F(b) states that the Jacobian f_x is singular at the bifurcation and that the kernel $\langle v \rangle$ of $f_x|_*$ is one dimensional. The transversality conditions F(c) and F(d) help to ensure that the fold bifurcation is not degenerate.

The standard definition for a generic saddle-node bifurcation can be obtained by adding another condition to F(a)-F(d):

Definition: f has a **saddle-node bifurcation** at (x_*, λ_*) if f has a fold bifurcation at (x_*, λ_*) and

- (e) $f_x|_*$ has a simple zero eigenvalue.

There are two possibilities for f to satisfy F(b). The generic possibility is that $f_x|_*$ has rank $n - 1$ and a unique simple zero eigenvalue. In this case f satisfies the conditions for a generic saddle node bifurcation at (x_*, λ_*) and the dynamics near (x_*, λ_*) are as described in [11]. The exceptional possibility is that $f_x|_*$ has rank $n - 1$ and a nonsimple zero eigenvalue. That is, there is a nontrivial Jordan block with zero eigenvalues of geometric multiplicity one and algebraic multiplicity greater than one. The consequence of the nontrivial Jordan block is that the dynamics of f near (x_*, λ_*) are more complicated. For example, in the case of Jordan block $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, there are Hopf and saddle connection bifurcations nearby (see [18, section 7.3]). This bifurcation is codimension 2 but can occur in power system models (see point A in [28]).

However, the conditions F(a)-F(d) are sufficient to guarantee that near λ_* there is locally a critical hypersurface in parameter space at which two equilibria coalesce. On one side of the hypersurface near λ_* there are two

equilibria and on the other side near λ_* there are none. This is proved in [10, section 6.2]. Those skeptical about this fact in the exceptional case should look at Figure 7.3.1 of [18].

One theme of this appendix is that if we are concerned with algebraic equations and their equilibria, then F(a)-F(d) are the right conditions for fold bifurcation. This is the context of singularity theory and in this context no distinction is made between the dynamics at a generic saddle node and the dynamics at a fold bifurcation with a nonsimple eigenvalue, because these phenomena are indistinguishable by examining the equilibria only. Indeed it can be inappropriate in this context to discuss eigenvalues of algebraic equations because changes to the equations which preserve their equilibria generally alter the eigenvalues. For example, multiplying one of the algebraic equations by two does not affect the equilibria but generally does alter the eigenvalues.

The fold bifurcation defined by F(a)-F(d) is a generic codimension 1 event in the singularity theory context. In the dynamic vector context, we must additionally require simplicity of the zero eigenvalue in order to obtain a generic saddle node bifurcation and exclude the exceptional codimension 2 event of a nonsimple eigenvalue.

Now we define a fold of a set $Z \subset U \subset X \times \Lambda$. The natural projection $X \times \Lambda \rightarrow \Lambda$ is given by $(x, \lambda) \mapsto \lambda$ and we write $\pi : Z \rightarrow \Lambda$ for the restriction to Z of the natural projection $X \times \Lambda \rightarrow \Lambda$.

Definition: Let Z be a subset of U and $z_* \in Z$. Z has a **fold** at $z_* \in U$ if it satisfies conditions ZF(a)-ZF(c):

- ZF(a) Z is a submanifold of U of dimension m
- ZF(b) $\pi_z|_*$ has rank $m - 1$
- ZF(c) $N\pi_{zz}|_*(v, v) \neq 0$

where v and N are nonzero vectors satisfying $\pi_z|_*v = 0$ and $N\pi_z|_* = 0$.

ZF(b) is equivalent to the set Z having a tangent parallel to the state space or having a fold when viewed along this tangent. This tangent direction is given by the right eigenvector v . The hypersurface of critical parameters at which there is a fold bifurcation is the projection of the fold in Z onto the parameter space Λ and the normal vector to this hypersurface is N .

Lemma 1 is the key lemma which proves that fold bifurcations of f correspond to folds of Z :

Lemma 1. Suppose $Z = f^{-1}(0)$. Then

- (a) f has a fold bifurcation at $z_* \iff Z$ has a fold at z_* and rank $f_z|_* = n$.
- (b) Suppose conditions F(a)-F(d) and ZF(a)-ZF(c) are satisfied. Choose the scalings of v, w, V, N such that $|v| = |V|$ and $|wf_\lambda|_* = |N|$. Then $v = (v, 0)$, $N = wf_\lambda|_*$ and $N\pi_{zz}|_*(v, v) = -wf_{xx}|_*(v, v)$.

Proof (a) $\Rightarrow$: Since $f_x|_*$ has rank $n - 1$ and $wf_\lambda|_* \neq 0$, $f_z|_* = (f_x, f_\lambda)|_*$ has rank n . Then there is an subset

$U_1 \ni z_*$ open in $X \times \Lambda$ on which f has rank n . Then $Z = (f|_{U_1})^{-1}(0)$ is a submanifold of U of dimension m and ZF(a) is proven.

Since $(f_x|_*, f_\lambda|_*) \begin{pmatrix} v \\ 0 \end{pmatrix} = 0$ and the only x for which $(f_x|_*, f_\lambda|_*) \begin{pmatrix} x \\ 0 \end{pmatrix} = 0$ are multiples of v , $TZ|_* \cap TX|_* = \langle (v, 0) \rangle$. Since the natural projection $X \times \Lambda \rightarrow \Lambda$ has kernel TX , its restriction to Z , π has $\ker \pi_z = TZ \cap TX$. Hence $\ker \pi_z|_* = TZ|_* \cap TX|_* = \langle (v, 0) \rangle$ and $\pi_z|_*$ has rank $m-1$ and ZF(b) is proven. Write $v = (v, 0)$ and note that $\pi_z|_* v = 0$ and $v \neq 0$. Also write $N = wf_\lambda|_*$ and note that $N \neq 0$.

The natural projection $X \times \Lambda \rightarrow X$ is given by $(x, \lambda) \mapsto x$. Write $\chi : Z \rightarrow \Lambda$ for the restriction to Z of the natural projection $X \times \Lambda \rightarrow X$. Note that $\chi_* v = \chi_*(v, 0) = v$. For any $z = (x, \lambda) \in Z$,

$$0 = f(\chi(z), \pi(z))$$

Differentiate to obtain

$$0 = f_x \chi_z + f_\lambda \pi_z \quad (A1)$$

Multiply by w on the left and evaluate at z_* to obtain

$$0 = wf_\lambda|_* \pi_z|_* = N \pi_z|_* \quad (A2)$$

Differentiate (A1) with respect to Z to obtain

$$0 = f_{xx} \chi_z \chi_z + f_{x\lambda} \chi_z \pi_z + f_{\lambda\lambda} \pi_z \pi_z$$

Multiply on the left by w and "on the right" by (v, v) , evaluate at z_* and rearrange to obtain

$$N \pi_{zz}|_*(v, v) = -wf_{xx}|_*(v, v) \quad (A3)$$

Hence $N \pi_{zz}|_*(v, v) \neq 0$ and ZF(c) is proven.

(a) $\Leftarrow$: $f(z_*) = 0$ and F(a) follow immediately from $z_* \in Z = f^{-1}(0)$. Since $\langle v \rangle = \ker \pi_z|_* = TZ|_* \cap TX|_*$, $v \in TX|_*$ and can be written as $v = (v, 0)$ where $v \neq 0$. $v \in TZ|_*$ implies that $(f_x|_*, f_\lambda|_*) \begin{pmatrix} v \\ 0 \end{pmatrix} = f_x|_* v = 0$ and $v \in \ker f_x|_*$. Moreover, if $v_1 \in \ker f_x|_*$, then $0 = f_x|_* v_1 = (f_x|_*, f_\lambda|_*) \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} v_1 \\ 0 \end{pmatrix} \in TZ|_* \cap TX|_* = \ker \pi_z|_*$. Since $\ker \pi_z|_* = \langle v \rangle = \langle (v, 0) \rangle$, v_1 must be a multiple of v . Hence $\ker f_x|_* = \langle v \rangle$ and $\text{rank } f_x|_* = n-1$ and F(b) is proven.

Choose $w \neq 0$ such that $wf_x|_* = 0$. $f_x|_* = (f_x|_*, f_\lambda|_*)$ rank n and $f_x|_*$ rank $n-1$ imply that $wf_\lambda|_* \neq 0$ and F(c) is proven. Obtain (A2) as before and use the single dimension of the kernel of $\pi_z|_*$ to deduce that w can be rescaled so that $N = wf_\lambda|_*$. Obtain (A3) as before and deduce that $wf_{xx}|_*(v, v) = -N \pi_{zz}|_*(v, v) \neq 0$ and F(d) is proven.

(b): Part (b) follows from the proofs of part (a). $\square$

Lemma 2. Suppose $0 = f(x, \lambda)$ are static equations with underlying dynamics specified by

$$\dot{x} = \bar{f}(x) = h(f(x, \lambda))$$

where $h(0) = 0$ and the zero of h is unique and simple. That is, $h(x) = 0 \iff x = 0$ and $h_x|_0$ is nonsingular. Then

- (a) f has a fold bifurcation at $(x_*, \lambda_*) \iff \bar{f}$ has a fold bifurcation at (x_*, λ_*) .
- (b) Choose the scalings of $v, \bar{v}, w, \bar{w}$ such that $|v| = |\bar{v}|$ and $|wf_\lambda|_* = |\bar{w}\bar{f}_\lambda|_*$. Then $v = \bar{v}$, $N = wf_\lambda|_* = \bar{w}\bar{f}_\lambda|_* = \bar{N}$ and $wf_{xx}|_*(v, v) = \bar{w}\bar{f}_{xx}|_*(\bar{v}, \bar{v})$.

Proof (a): The uniqueness of the zero of h implies that $0 = \bar{f}(x) = h(f(x)) \iff 0 = f(x)$ and $Z = f^{-1}(0) =$

$\bar{f}^{-1}(0) = \bar{Z}$. Since $\bar{f}_x|_* = h_x|_0 f_x|_*$ and $h_x|_0$ is invertible, $\text{rank } f_x|_* = n \iff \text{rank } \bar{f}_x|_* = n$. Now Lemma 1(a) yields f has a fold bifurcation $\iff Z$ has a fold and $\text{rank } f_x|_* = n \iff \bar{Z}$ has a fold and $\text{rank } \bar{f}_x|_* = n \iff \bar{f}$ has a fold bifurcation.

(b): Lemma 1(b) yields (b). $\square$

The essentials of a fold in π are not changed when Z is changed by a diffeomorphism affecting only the state space only as proved in lemma 3. Note that lemma 3 applies when a new state space of different dimension is used.

Lemma 3. Let Z be a submanifold of $X \times \Lambda$ of dimension m and $\pi : Z \rightarrow \Lambda$ be the restriction to Z of the natural projection $X \times \Lambda \rightarrow \Lambda$. Suppose π has a fold at $z_* \in Z$ so that $\pi_z|_*$ has rank $m-1$ and $N \pi_{zz}|_*(v, v) \neq 0$ where v and N are nonzero vectors satisfying $\pi_z|_* v = 0$ and $N \pi_z|_* = 0$. Let $\bar{Z}$ be a submanifold of $\bar{X} \times \Lambda$ and $\bar{\pi} : \bar{Z} \rightarrow \Lambda$ be the restriction to $\bar{Z}$ of the natural projection $\bar{X} \times \Lambda \rightarrow \Lambda$. Suppose that there is a diffeomorphism $\phi : Z \rightarrow \bar{Z}$ which is onto and which preserves Λ so that $\bar{\pi} \phi = \pi$. Let $\bar{v} = \phi_z|_*(v)$ and $\bar{N} = N$ and $\bar{z}_* = \phi(z_*)$. Then $\bar{\pi}_{\bar{z}}|_*$ has rank $m-1$, $\bar{\pi}_{\bar{z}}|_* \bar{v} = 0$, $\bar{N} \bar{\pi}_{\bar{z}}|_* = 0$, $\bar{N} \bar{\pi}_{\bar{z}\bar{z}}|_*(\bar{v}, \bar{v}) = N \pi_{zz}|_*(v, v)$, and $\bar{\pi}$ has a fold at $\bar{z}_*$.

Proof: ϕ a diffeomorphism and $\bar{\pi} \phi = \pi$ implies $\bar{\pi} = \pi \phi^{-1}$ and

$$\bar{\pi}_{\bar{z}} = \pi_z(\phi_z)^{-1} \quad (A4)$$

Hence $\text{range } \bar{\pi}_{\bar{z}}|_{\phi(z_1)} = \text{range } \pi_z|_{z_1}$. In particular, $\text{rank } \bar{\pi}_{\bar{z}}|_* = \text{rank } \pi_z|_* = m-1$. Also

$$\bar{\pi}_{\bar{z}}|_* \bar{v} = \pi_z|_*(\phi_z|_*)^{-1} \phi_z|_* v = \pi_z|_* v = 0$$

$$\bar{N} \bar{\pi}_{\bar{z}}|_* = N \pi_z|_*(\phi_z|_*)^{-1} = 0$$

Rearranging (A4) and differentiating yields

$$\bar{\pi}_{\bar{z}\bar{z}} \phi_z \phi_z + \bar{\pi}_{\bar{z}} \phi_{zz} = \pi_{zz}$$

Multiplying by $N = \bar{N}$ and v and evaluating at z_* gives

$$\bar{N} \bar{\pi}_{\bar{z}\bar{z}}|_*(\bar{v}, \bar{v}) = N \pi_{zz}|_*(v, v) \quad (A5)$$

The results obtained imply that $\bar{\pi}$ has a fold at $\bar{z}_*$. $\square$

Note that equation (A5) only holds when the scaling of $\bar{v}$ and $\bar{v}$ is such that $\bar{v} = \phi(v)$. However, (A5) is usually used to ensure that $N \pi_{zz}|_*(v, v) \neq 0 \iff \bar{N} \bar{\pi}_{\bar{z}\bar{z}}|_*(\bar{v}, \bar{v}) \neq 0$ and in this case the scalings of $\bar{v}$ and v are irrelevant.

Lemma 4. Let $f : X \times \Lambda \rightarrow X$ be smooth and have zero set Z . Let $\bar{f} : \bar{X} \times \Lambda \rightarrow \bar{X}$ be smooth and have zero set $\bar{Z}$. Suppose that $Z = \bar{Z}$ and that at $z_* \in Z$, $\text{rank } f_x|_* = \text{rank } \bar{f}_x|_*$ where $z = (x, \lambda)$. Then

- (a) f has a fold bifurcation at z_* $\iff \bar{f}$ has a fold bifurcation at z_* .
- (b) Choose the scalings of $\bar{v}, \bar{w}, \bar{N}$ such that $|v| = |\bar{v}|$ and $|wf_\lambda|_* = |\bar{w}\bar{f}_\lambda|_*$. Then $v = \bar{v}$, $N = wf_\lambda|_* = \bar{w}\bar{f}_\lambda|_* = \bar{N}$ and $wf_{xx}|_*(v, v) = \bar{w}\bar{f}_{xx}|_*(\bar{v}, \bar{v})$.

Proof: Follows from lemmas 1 and 3 with ϕ as the identity map. $\square$

Lemma 5. Let $f(x, \lambda) = \begin{pmatrix} f^1(a, b, \lambda) \\ f^2(a, b, \lambda) \end{pmatrix}$ be a smooth function with component functions f^1, f^2 where

$$\begin{aligned} f &: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n \\ f^1 &: \mathbb{R}^{\bar{n}} \times \mathbb{R}^{n-\bar{n}} \times \mathbb{R}^m \rightarrow \mathbb{R}^{\bar{n}} \\ f^2 &: \mathbb{R}^{\bar{n}} \times \mathbb{R}^{n-\bar{n}} \times \mathbb{R}^m \rightarrow \mathbb{R}^{n-\bar{n}} \end{aligned}$$

Suppose that $\text{rank } f_b^2|_* = n - \bar{n}$ so that $f_b^2|_*$ is invertible.

$$\begin{aligned} \text{Define } \bar{f} &: \mathbb{R}^{\bar{n}} \times \mathbb{R}^m \rightarrow \mathbb{R}^{\bar{n}} \\ \bar{f}(a, \lambda) &= f^1(a, h(a, \lambda), \lambda) \end{aligned}$$

where h is the unique smooth function $U \rightarrow \mathbb{R}^{n-\bar{n}}$ satisfying $f^2(a, h(a, \lambda), \lambda) = 0$ and $h(a_*, \lambda_*) = b_*$ for some neighborhood U of (a_*, λ_*) . Write $z = (x, \lambda)$ and $\bar{z} = (a, \lambda)$. Then

- (a) $\text{rank } f_z = n \iff \text{rank } \bar{f}_{\bar{z}} = \bar{n}$.
- (b) f has a fold bifurcation at $z_* = (a_*, b_*, \lambda_*) \iff \bar{f}$ has a fold bifurcation at $\bar{z}_* = (a_*, \lambda_*)$.
- (c) For suitable scaling of $\bar{N}$ and $\bar{v}$,
 $N = \bar{N}$ and $v = \begin{pmatrix} v^1 \\ v^2 \end{pmatrix} = \begin{pmatrix} \bar{v} \\ h_{a|_*} \bar{v} \end{pmatrix}$.

Proof (a): Observe that $\text{rank } f_z = \text{rank } f_{\bar{z}, b}$ since f_z and $f_{\bar{z}, b}$ differ only in the order of their columns. Note that $\bar{f}_{\bar{z}} = f_{\bar{z}}^1 - f_b^1 (f_b^2)^{-1} f_{\bar{z}}^2$ and

$$\begin{pmatrix} f_{\bar{z}}^1 & f_b^1 \\ f_{\bar{z}}^2 & f_b^2 \end{pmatrix} \begin{pmatrix} I_{\bar{n}+m} & 0 \\ -(f_b^2)^{-1} f_{\bar{z}}^2 & I_{n-\bar{n}} \end{pmatrix} = \begin{pmatrix} \bar{f}_{\bar{z}} & f_b^1 \\ 0 & f_b^2 \end{pmatrix} \quad (\text{A6})$$

The rank of the LHS of (A6) = $\text{rank } f_{\bar{z}, b}$ since the second matrix is invertible and the rank of the RHS of (A6) = $\text{rank } \bar{f}_{\bar{z}} + \text{rank } f_b^2 = \text{rank } \bar{f}_{\bar{z}} + n - \bar{n}$. Hence $\text{rank } f_z = \text{rank } \bar{f}_{\bar{z}} + n - \bar{n}$ and (a) follows.

(b): By lemma 1(a), f has a fold bifurcation at z_* implies that its zero set Z has a fold at z_* and $\text{rank } f_z|_* = n$. In particular, Z is a submanifold of $U_1 \subset \mathbb{R}^n \times \mathbb{R}^m$ where U_1 is a neighborhood of z_* . By part (a), $\text{rank } f_{\bar{z}}$ has rank $\bar{n}$ and it follows (see proof of lemma 1(a)) that the zero set $\bar{Z}$ of $\bar{f}$ is a submanifold of $\bar{U}_1 \subset \mathbb{R}^{\bar{n}} \times \mathbb{R}^m$ where $\bar{U}_1$ is a neighborhood of $\bar{z}_*$. Similarly $\bar{f}$ has a fold bifurcation at $\bar{z}_*$ implies that $\bar{Z}$ is a submanifold of $\bar{U}_1$ and Z is a submanifold of U_1 . Thus either f has a fold bifurcation at z_* or $\bar{f}$ has a fold bifurcation at $\bar{z}_*$ imply that both Z and $\bar{Z}$ are submanifolds.

$$\begin{aligned} \text{Define } \zeta &: U \rightarrow \mathbb{R}^n \times \mathbb{R}^m \\ \zeta(a, \lambda) &= (a, h(a, \lambda), \lambda) \\ \eta &: \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^{\bar{n}} \times \mathbb{R}^m \\ \eta(a, b, \lambda) &= (a, \lambda) \end{aligned}$$

We can find open sets $V, \bar{V}$ with $z_* \in V \subset U_1, \bar{z}_* \in \bar{V} \subset \bar{U}_1$ such that

$$\begin{aligned} (a, \lambda) \in \bar{Z} \cap \bar{V} &\iff \bar{f}(a, \lambda) = 0 \\ &\iff f^1(a, h(a, \lambda), \lambda) = 0 \\ &\iff f^1(a, h(a, \lambda), \lambda) = 0 \\ &\text{and } f^2(a, h(a, \lambda), \lambda) = 0 \\ &\iff f(a, h(a, \lambda), \lambda) = 0 \\ &\iff (a, h(a, \lambda), \lambda) \in Z \cap V \end{aligned}$$

Therefore $\zeta|_{\bar{Z} \cap \bar{V}}$ maps $\bar{Z} \cap \bar{V}$ onto $Z \cap V$. Since $\bar{Z} \cap \bar{V}$ is a submanifold of U and $Z \cap V$ is a submanifold of $\mathbb{R}^n \times$

$\mathbb{R}^m$ and ζ and η are smooth functions, $\zeta|_{\bar{Z} \cap \bar{V}}$ and $\eta|_{Z \cap V}$ are smooth functions. Moreover, it is easy to verify that $\zeta|_{\bar{Z} \cap \bar{V}} \circ \eta|_{Z \cap V}$ and $\eta|_{Z \cap V} \circ \zeta|_{\bar{Z} \cap \bar{V}}$ are identity functions. Therefore $\zeta|_{\bar{Z} \cap \bar{V}}$ is diffeomorphism from $\bar{Z} \cap \bar{V}$ onto $Z \cap V$. Now use part (a) and lemmas 1(a) and 3 to deduce result (b).

(c): From part (b) and lemmas 1(b) and 3, we obtain

$$N = \bar{N} \text{ and } v = (\zeta|_{\bar{Z} \cap \bar{V}})_*|_{z_*}(\bar{v}) = \begin{pmatrix} \bar{v} \\ h_{a|_*} \bar{v} \end{pmatrix} \quad \square$$

Appendix B: Manipulation of a load model

The dynamic load model is specified by

$$\dot{y} = f^1(y, V, \lambda) \quad (\text{B1})$$

$$0 = g(\dot{P}^d, \dot{V}, P^d, V, \lambda) \quad (\text{B2})$$

$$P^d = P(y, V) \quad (\text{B3})$$

We apply lemma 1 by first eliminating P^d and solving g to obtain an expression for $\dot{V}$ to put the equations in state space form. Suppose that $g_{P^d} P_V + g_{\dot{V}} \neq 0$. As shown below, this condition is sufficient for equations (B1-B3) to define well posed differential equations in state variables (y, V) . Differentiating (B3), substituting in (B2) and using (B1) yields

$$0 = g(P_y f^1(y, V, \lambda) + P_V \dot{V}, \dot{V}, P(y, V), V, \lambda) \quad (\text{B4})$$

Let (y_*, V_*, λ_*) be an equilibrium of equations (B1-B3). Then (y_*, V_*, λ_*) also satisfies (B4). Since $g_{P^d} P_V + g_{\dot{V}} \neq 0$, the implicit function theorem implies that there is an open set $U \ni (y_*, V_*, \lambda_*)$ and a unique smooth function $f^2: U \rightarrow \mathbb{R}$ such that $f^2(y_*, V_*, \lambda_*) = 0$ and

$$0 = g(P_y f^1(y, V, \lambda) + P_V f^2(y, V, \lambda), f^2(y, V, \lambda), P(y, V), V, \lambda) \quad (\text{B5})$$

Now (B1-B3) can be written in state space form in U :

$$\dot{y} = f^1(y, V, \lambda) \quad (\text{B6})$$

$$\dot{V} = f^2(y, V, \lambda) \quad (\text{B7})$$

Applying lemma 2 yields the static equations

$$0 = f^1(y, V, \lambda) \quad (\text{B8})$$

$$0 = f^2(y, V, \lambda) \quad (\text{B9})$$

Now we use lemma 4 to deduce that fold bifurcations of equations (B8-9) are the same for the equations

$$0 = f^1(y, V, \lambda) \quad (\text{B10})$$

$$0 = g(0, 0, P(y, V), V, \lambda) \quad (\text{B11})$$

Equations (B8-B9) imply equations (B10-B11) via (B5). Conversely, equations (B10) and (B5) imply that for $(y, V, \lambda) \in U$,

$$0 = g(P_V f^2(y, V, \lambda), f^2(y, V, \lambda), P(y, V), V, \lambda) \quad (\text{B12})$$

But f^2 is unique so we can deduce from (B11) and (B12) that $0 = f^2(y, V, \lambda)$. Hence equations (B8-B9) and (B10-B11) are equivalent and have the same zero set Z .

Write $z = (y, V, \lambda)$. To apply lemma 4, it remains to show that the ranks of the Jacobians with respect to z of equations (B8-B9) and (B10-B11) are the same at a point $z_* \in Z$. The Jacobian of (B10-B11) is $\begin{pmatrix} f_z^1|_* \\ a \end{pmatrix}$ where $a = [g(0, 0, P(y, V), V, \lambda)]_z|_*$. Differentiating (B5) with respect to z and evaluating at z_* yields

$$0 = (g_{Pd}P_V + g_V)\Big|_* f_z^2|_* + (g_{Pd}P_y)\Big|_* f_z^1|_* + a$$

and since $g_{Pd}P_V + g_V \neq 0$, we can deduce that $\text{rank} \begin{pmatrix} f_z^1|_* \\ a \end{pmatrix} = \text{rank} \begin{pmatrix} f_z^1|_* \\ f_z^2|_* \end{pmatrix}$.

Appendix C: Concerning the definition of fold

This appendix shows that the definition of fold point of Z in Appendix A is standard by proving it equivalent to the definition 4.1 of fold point in [17, Chapter 3 page 87]. The proof also shows a geometric meaning of the transversality condition $n\pi_{zz}|_*(v, v) \neq 0$.

It is convenient to work in a fixed chart of Z with open set $U_1 \ni z_*$ and to neglect the distinction between objects associated with Z and their coordinates. Define the matrix function

$$M : U_1 \rightarrow \mathbb{R}^{m \times m} \\ z_1 \mapsto \pi_z|_{z_1}$$

and define S_{m-1} to be the matrices in $\mathbb{R}^{m \times m}$ of rank $m - 1$. S_{m-1} is a submanifold of $\mathbb{R}^{m \times m}$ (see proof of lemma 6 below or [17, page 60]). If M intersects S_{m-1} transversally at $M(z_*)$, let U_2 be a neighborhood of z_* on which M intersects S_{m-1} transversally and define the submanifold F of U_1 by $F = M^{-1}(S_{m-1}) \cap U_2$.

Then definition 4.1 of [17] may be rewritten and specialized to

$z_* \in U_1$ is a fold point of π iff

- (a) $\pi_z|_* = M(z_*) \in S_{m-1}$
- (b) M intersects S_{m-1} transversally at $\pi_z|_*$
- (c) $TF|_* + \ker \pi_z|_* = TZ|_*$.

Define $\gamma : \mathbb{R} \rightarrow Z$ to be a smooth curve in Z with $\gamma(0) = z_*$ and $\gamma_s|_0 = v$. Since $\ker \pi_z|_* = \langle v \rangle$, condition (c) is equivalent to

(c') γ intersects F transversally at z_* .

Let W_1 be a neighborhood of 0 in $\mathbb{R}$ so that $\gamma(W_1) \subset U_1$. Define the curve of matrices $A = M \circ \gamma$:

$$A : W_1 \rightarrow \mathbb{R}^{m \times m} \\ s \mapsto M(\gamma(s))$$

Now we claim that the condition

(d) A intersects S_{m-1} transversally at $\pi_z|_*$ is equivalent to conditions (b) and (c'). For (c') implies $\gamma_s|_0 \notin TF|_*$ and this, together with (b) implies $M_z(\gamma_s|_0) = A_s|_0 \notin TS_{m-1}|_{M(z_*)}$ and hence (d). Conversely, (d) implies (b). Moreover, (d) implies (c') because γ not transverse to F at z_* implies $\gamma_s|_0 \in TF$ implies $A_s|_0 = M_z(\gamma_s|_0) \in TS_{m-1}|_{M(z_*)}$ implies that A is not transverse to S_{m-1} at $\pi_z|_*$.

Lemma 6 below proves that condition (d) is equivalent to

$$nA_s|_0v \neq 0$$

But $A_s|_0 = (M(\gamma(s)))_s|_0 = M_z|_*\gamma_s|_0 = \pi_{zz}|_*v$ so condition (d) is equivalent to

$$n\pi_{zz}|_*(v, v) \neq 0,$$

which is the condition ZF(c). But conditions (a) and (b) are equivalent to ZF(a) and ZF(b). Therefore the conditions ZF(a), ZF(b), ZF(c) of the fold definition in Appendix A are equivalent the definition 4.1 of [17].

It remains to state and prove the following:

Lemma 6. Let $A : \mathbb{R} \rightarrow \mathbb{R}^{m \times m}$ be a smooth function so that $A(s)$ is an $m \times m$ matrix parameterized by s . Suppose $A(0)$ has rank $m - 1$ and let v and n be nonzero vectors such that $A(0)v = 0$ and $nA(0) = 0$. Let S_{m-1} be the submanifold of matrices in $\mathbb{R}^{m \times m}$ with rank $m - 1$. Then

$$A \text{ transversal to } S_{m-1} \text{ at } A(0) \iff nA_s|_0v \neq 0.$$

Proof : Write B^{ij} for the cofactor of the (i, j) element of a matrix $B \in \mathbb{R}^{m \times m}$. Since $A(0)$ has rank $m - 1$, i and j can be chosen so that $(A(0))^{ij} \neq 0$. Define the smooth functions $\tilde{n}(B) = (B^{1j}, B^{2j}, \dots, B^{mj})$ and $\tilde{v}(B) = (B^{i1}, B^{i2}, \dots, B^{im})^T$. Let $U \subset \mathbb{R}^{m \times m}$ be a neighborhood of $A(0)$ such that $B \in U \Rightarrow B^{ij} \neq 0$. Define

$$\beta : U \rightarrow \mathbb{R} \\ B \mapsto \tilde{n}(B)B\tilde{v}(B)$$

Write e_i for the column vector with 1 in the i th coordinate and 0 elsewhere. Recall that multiplying elements of a matrix row by their corresponding cofactors and adding yields the matrix determinant while multiplying elements of a matrix row by the cofactors corresponding to another row of the matrix and adding yields zero. Hence

$$B\tilde{v}(B) = \det B e_i \text{ and } \tilde{n}(B)B = \det B e_j^T \quad (C1)$$

and it follows that

$$\beta(B) = B^{ij} \det B$$

Then $S_{m-1} \cap U = \beta^{-1}(0)$ because, in U , $B^{ij} \neq 0$ so that $\text{rank } B = m - 1$ iff $\det B = 0$. β is regular because the expansion of a determinant in terms of cofactors implies that $[\det B]_{b_{ij}} = B^{ij}$ and hence that $\beta_{b_{ij}} = (B^{ij})^2 \neq 0$. (Here $[\det B]_{b_{ij}}$ and $\beta_{b_{ij}}$ denote the gradients of $\det B$ and β with respect to b_{ij} .) $S_{m-1} \cap U = \beta^{-1}(0)$ and β regular imply that $TS_{m-1} = \ker \beta_B$.

Let $V \subset \mathbb{R}$ be a neighborhood of 0 with $A(V) \subset U$. Define $\alpha : V \rightarrow \mathbb{R}$ by $\alpha(s) = \beta(A(s))$. Then, since $TS_{m-1} = \ker \beta_B$ and β regular, A transversal to S_{m-1} at $A(0) \iff \alpha_s|_0 = \beta_B|_{A(0)}A_s|_0 \neq 0$.

It remains to prove that $\alpha_s|_0 = nA_s|_0v$ for suitable rescalings of n and v . Equation (C1) implies that $A(0)\tilde{v}(A(0)) = \det A(0)e_i = 0$. Moreover, $\tilde{v}(A(0)) \neq 0$ since it has a nonzero component $(A(0))^{ij}$. Since $A(0)$ has a one dimensional kernel, a suitable rescaling of v yields $v = \tilde{v}(A(0))$. Similarly $n = \tilde{n}(A(0))$. Now

$$\begin{aligned} \alpha_s|_0 &= [\tilde{n}(A(s))A(s)\tilde{v}(A(s))]_s|_0 \\ &= [\tilde{n}(A(s))]_s|_0 A(0)v + nA_s|_0v \\ &\quad + nA(0)[\tilde{v}(A(s))]_s|_0 \\ &= nA_s|_0v \end{aligned}$$

□

Discussion for Session 6

Edited by Tom Overbye, University of Illinois

Questions that address Carson Taylor's presentation

Nelson Martins (CEPEL)

This question concerns over-excitation limiter models to use in voltage stability studies. My understanding is it makes an enormous difference for voltage stability whether it is a hard limiter, that goes to manual control so that you lose active performance from your generator (no longer in a feedback control loop), or a soft limiter with a continuous control of IFD. This makes your system much better and much more stable. In this case control effort continues in order to drive your system stable, rather than the hard limited case where the generator acts as a PQ bus, which is awful for voltage stability. You can see this just from a normal power flow. When you change from a PV bus to a PQ bus you increase the rank of the Jacobian by one and change the spectra. You may go from stability to instability. This is why in the beginning (before voltage stability came into the scenario studies) people didn't know that if you start with your x' vector in the convergence process from the load flow, but change the nature of buses from PV to PQ, you no longer converge. It is a problem with the rank of the matrix.

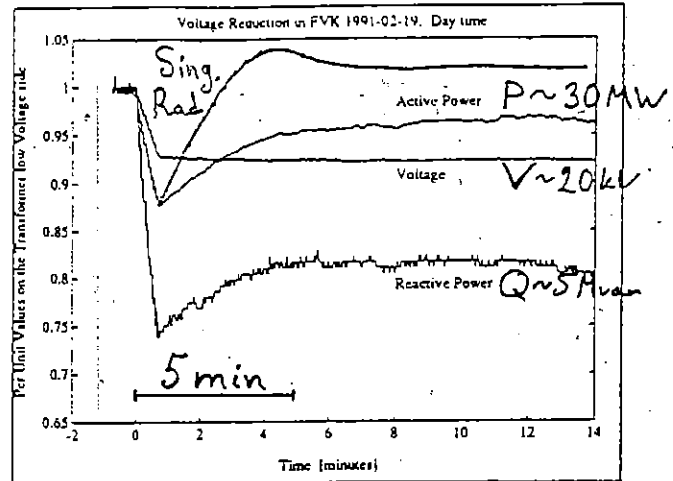
Carson Taylor, BPA

I guess the most dramatic case related to your first question is that, in some of the older type limiters, there is just a relay type action for tripping to manual, which depends upon the what the current field rheostat setting is. In some cases there is a follower so it trips when it is near rated field current. In other cases it depends on the operator action. If he were initially near a unity power factor when the relay operated and tripped to manual, it would go back to a setting close to unity power factor which would be disastrous. Instead of being run back to rated field current it gets run back to near unity power factor. In these sum limiters there is a question of how much influence the normal regulation would have; I think the feeling is that they would not have very much influence. Concerning the PQ bus model, you can see that the model is completely wrong, because two machines could have different synchronous reactances and that effect would not be captured with a PV to PQ bus switching.

Daniel Karlsson (Sydkraft AB)

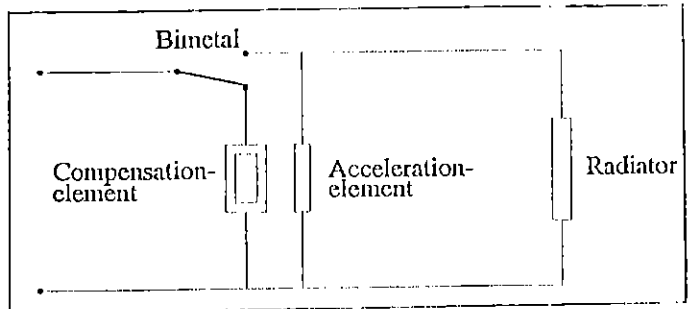
This is more like a comment. Carson Taylor mentioned the feeder measurements in Sweden, and he also mentioned that the recovery time constant, which was on the order of 3-5 minutes, was much shorter than expected. We were really surprised since we would have expected 20-30 minutes. This time constant actually has nothing to do with the temperatures in the houses or the outside temperature, since within this short time the temperature in the house has not changed significantly. Figure 1 is from a recording performed a few years ago of the per unit voltage, active power, and reactive power values from an extremely residential load of about 10,000 households (30 MW, 5 Mvar, 20 kV). The voltage was decreased about 6-7%; you can see the recovery in real and reactive power. The origin of this recovery must be sought somewhere else, so we looked into thermostats and found that a very common type of thermostat in Sweden is the bimetallic design (Figure 2). We checked the construction and found that the bi-metal was heated by a compensation element and an acceleration element, with the compensation element heated during the off period and the acceleration element during the on period. The heating of these elements is, of course, voltage dependent, with a time constant that could be on the order of 3-5 minutes. We studied a number of single heater radiators and found that by lowering the voltage the temperature in the house would actually increase. From this experience we learned two things: (1) experiments

are extremely necessary; (2) design details need to be considered.



Per unit values of the quantities recorded after a voltage reduction in the substation FVK.

Figure 1



Modern bimetallic thermostat equipped with acceleration and compensation elements.

Figure 2

Questions that address Fernando Alvarado's presentation

Constantine Vournas (National Technical Univ. of Athens)

My comment has to do with the maximum loadability problem. I found the distinction made by Fernando Alvarado's paper, of replacing one curve by two curves, very important. Then the critical point is no longer the tip of the curve but is the point of tangency between these two curves (Figure 3). The two curves are the well known network PV curve and the load curve. So if the bottom curve is the transient load characteristic and the top curve is the network characteristic, the critical point (point "C") will be an algebraic singularity point. For example, if the load characteristic is the equilibrium points of induction motors, then the critical point is a saddle node bifurcation point. So I think that with using two curves to specify exactly the type of load/network we are using, it is easy to see why the bifurcation is not at the tip of the curve. Now the vertical line is a steady state load characteristic (referenced in the presentations by Ian Dobson and Ian Hiskens). With LTCs to restore load, then point "A" is the steady-state load characteristic. Point

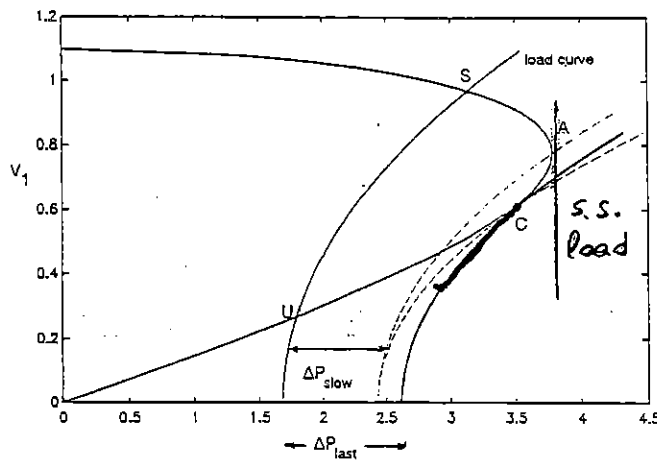


Figure 3

"A" which is the tip of the curve, is the saddle node bifurcation of the system including full load restoration. For induction machines this is the load characteristic for machines which are constant power. But if you have an induction machine that is lower voltage, you can have equilibrium; Therefore your equilibrium curves start from a certain load level and then behave like constant power.

Fernando Alvarado (University of Wisconsin)

That was really more of a comment than a question, so I will simply say that, for the most part I agree - perhaps at the very end I would say that a characterization of an induction motor load as a constant power may not be quite the thing to do. I suggest you may want to go into some of the mechanical characteristics of the load as well.

Vournas: I was speaking about the steady state characteristic in the sense that Ian Dobson made in his last paper. You do have a steady state characteristic of the equilibrium point.

Nicholas Miller (General Electric Co.)

I would like to introduce a slightly different way of looking at these eyeball wrenching nose curves that Fernando introduced. I think this will help clarify some ideas in some peoples minds. Referring to Figure 4, the upper left curve is a PV curve with unity power factor. If we change the power factor to zero, we get a VQ curve, shown below the PV curve. In the PV space this curve is just a vertical line. In fact as we continuously change the power factor we get a whole family of curves, shown in the bottom 3D representation. Some people like this curve and some people hate it. We have what you call an onion curve in PQV space, with the curve independent of the load voltage dependence, constrained for any load to be on that surface. For just a simple lossless curve the critical voltage of the unity power factor is about the square of two over two while the critical voltage for the Q curve is at 0.5 so what I have done (with the horizontal lines) is kind of eyeball a locus of critical voltages on the face of that onion. These are the points at which we would see singularity of the Jacobian for constant MVA class loads. It is easy to see the voltage drops some as we go from 1.0 power factor to 0.0 power factor.

Now considering the algebraic equations of an induction [motor], we can come up with something that looks a little bit like the top right figure. We have more or less constant MVA power down to some point, more or less constant Q down to some point, then we start to get up towards the top of the speed torque characteristic. We may have done that by increasing the mechanical load, or by decreasing the AC voltage, or by some kind of combination thereof, or by adding more load. We can see that we get to a point where we get a stall. So if we imagine adding progressively more motors,

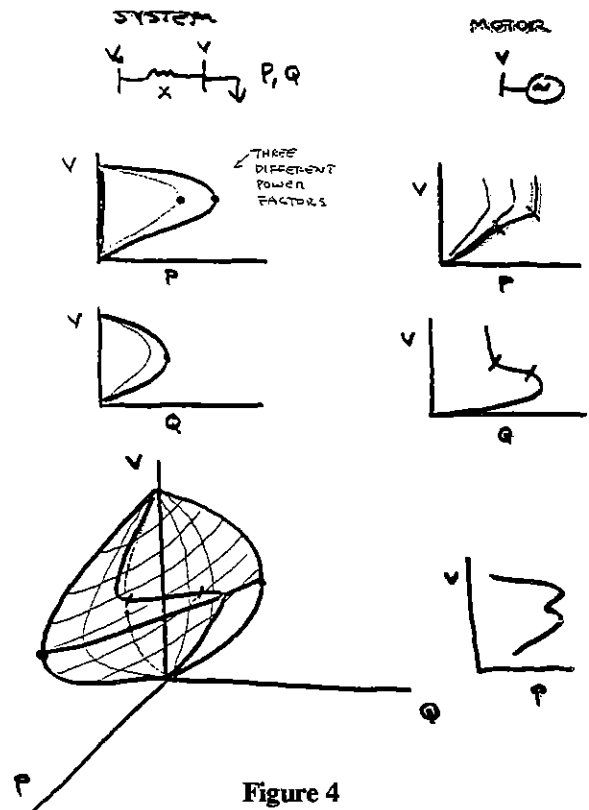


Figure 4

each one loaded at the same point on its speed torque curve, we start moving down the surface of this PQV curve, with the voltage beginning to sag. Obviously it is not a unity power factor. Then we get to the point where we snap over the top of the speed torque characteristic, either because the voltage has come down or the load has gone up or any combination thereof. We zip across a space that is algebraically, but not dynamically, meaningful as we go down the backside of the speed torque characteristic and end up at zero speed or some creep speed.

The system has not gone away, of course, but the reactive power consumption now has gone zipping up rather dramatically. Well, what happened is that we got sucked across the face of this onion curve, and now we are way in the back with a very poor power factor, since the active power has actually dropped down while the reactive power has actually gone up. If you take the projection of this trajectory, which was constrained to the PQV surface, and project it back on to just the PV curve, which is what Fernando plotted, you get the bottom right figure. I have a little trouble with considering the nose point as the bifurcation point. Yes indeed, that is the point at which it crosses the waist, but the boundary conditions for that don't care. The only question is the relative intersection of the load characteristic with this whole surface.

Ian Hiskens has raised a very interesting point which we have been wrestling with for years, and I would like to extend his digression to a postulate: as long as the signature load characteristic, no matter how complicated it is, pierces the onion surface from the outside in with a positive slope, you have a solution and will not reach bifurcation. In the general statement the saddle node bifurcation is going to occur whenever the load characteristic becomes tangential with the surface. For a real complicated system, you can have multiple points of tangency resulting in two stable equilibrium points, one of which you probably don't want.

Alvarado: I really have no basic disagreement. I want to point out a couple of things in here (referring to Figure 4). One is that, indeed, this stuff, that resulted in the double

nose, happens. The P power goes down; actually, from studies I've done, it tends to go down even more steeply than shown here, and it is the reactive power increase that is responsible for this. About my calling that a bifurcation -- it all depends on the relative size and importance of the motors. There is no questions that in many systems you might actually end up stalling a very small, rather distant motor when the voltage is still rather high. At what point do you care? The fact that somebody's refrigerator stalled way up here (on the PV curve) is not a bifurcation. Well, in a sense it is, and in a sense it isn't. That is the only point I was making. In theory, the system at large can still continue to deliver more and more power, but there gets to be a point where you can't do it anymore. That doesn't take away from the fact that you have already suffered, that already something else happened. The other thing I was saying is, it is very likely that a discrete event, switching off the stalled motor, has occurred. If that happens, then you are resetting the curve, and of course, once the motor tries to get restarted, you see where the dynamics come in now.

Questions that address Miroslav Begovic's presentation

David Hill (University of Sydney)

I think Miroslav [Begovic] brought up an important issue. We should think more about the identifiability of the model structures we are proposing. The question I have is, why he chose to interpret the dynamic model as a quasi-static one. The model you end up with you will only be able to use relative to dynamics that are much faster than the dynamics in the load model. Whereas if you identify the full dynamic model, then you can use that model in the voltage stability studies.

Miroslav Begovic (Georgia Institute of Technology)

The answer to this is simple. The objective that we had for this identification was to provide tools for steady state analysis. There was no intent of dealing with dynamic models, nor with the results that would involve calculations with dynamic models. So the presentation of a nonlinear static model was a starting point. The Karlsson-Hill dynamic model that we used for illustration was taken just as an example of how the model structure can change during a voltage disturbance involving the relative distribution of the various model components. Dealing with the issues of how many parameters can we identify and how much we can go in to detail -- that depends on what we have in terms of data. For normally operating machines, there is very little data that can be drawn from to obtain useful information about models. So we are left with very restricted input to deal with, which drove the decision to use a static model of the simplest possible form and to take the time variance of that model as the driving force for results.

Questions that address Gianfranco Chicco's presentation

Martins: Concerning the induction motor model for the power flow problem, I have been using the same model since 1976. I copied it from Brian Stott, who had sold his power flow program to British Petroleum for their refineries to do a starting power flow with exact PQ loadings of the various induction motors in the systems. Since you have around 100 induction motors in a refinery, you need to get all the initial conditions right, so that you don't have to put in some mismatch adjustments. In this way you can model 1000 induction motors; the only thing you get is 1000 extra buses in your problem, but it is a power flow exactly like any other. The model given in the paper calculates additionally this lip for every motor, but you must provide all the parameters for each motor. Brian Stott copied this model from GEC England, who had used it since about 1970. This model is implemented in CEPPEL's power flow, but not using the extra buses. They prefer doing it with an injection current, and thus not having to add extra buses. I don't know which is the best way, but since an extra bus only requires one line

that won't substantially affect sparsity, I would prefer to have the extra buses. I haven't seen this model very much in the literature.

Regarding Ian Dobson's comments on theories, I agree with everything that I can understand out of his talks; I also agree with everything I don't understand. That's the dynamic model I suggested in the paper for the QV sensitivity matrix used with eigenvalue analysis. The benefit of the model for the load is that I can make the saddle node bifurcation point exactly equal to the point of Jacobian singularity. For the Rio area model, in a 1600 bus system we have a 5,000 MW load with 272 load points. I substituted these load points by induction load models, so that there is a matrix giving me the dynamics of these 272 induction motors. This is also in line with Ian Hiskens' presentation. With doing this you can add dynamic characteristics into your stability study.

Questions that address Ian Dobson's presentation

Eyad Abed (University of Maryland)

There is a motivating example in the paper that looks like this:

$$\dot{x} = (x^2 + 2)(-x^2 - \lambda + 1) \quad (1)$$

$$\dot{x} = h(-x^2 - \lambda + 1) \quad (2)$$

where h is a function. I just want to note that you can't write (1) as (2). So I would like to know if it is possible to generalize your results in the following way:

$$\dot{x} = G(x, \lambda, h(-x^2 - \lambda + 1)) \quad (3)$$

For this kind of modeling you have a function of your knowledge of equilibrium points multiplied by another function of the state and parameter. I think this is more in tune with what your aiming at. Thank you.

Ian Dobson (University of Wisconsin)

I agree with the comment that you can't put the first equation in the form of the second; that is something I'd missed. The motivation for the second equation is, for example, in a simple case where what is in the brackets is something like real power balance at the load; you might think that if you have a mismatch then dynamics would be a function of that mismatch. That is what was motivating having that choice of h. I agree with Eyad that it might be useful to generalize it further, but you need to write down the appropriate conditions for this more general function G. I thank him for his observations.

Peter Sauer (University of Illinois)

I always get a little nervous when people say that dynamics are irrelevant, and so I want to add some caution to the things that Ian has said here. If I understand correctly, you are proposing that the equilibrium point is the only thing that matters for that type of a bifurcation and that type of a margin, but that in my opinion is a pretty narrow view of a loading margin. So you are restricting your definition of a loading margin purely to the fold bifurcation, and that is where your point applies. Is that correct?

Dobson: Yes - I had to make assumptions to get the nice results, and I hope I have made the assumptions very clear. Saddle node bifurcation is very important in voltage collapse. If you look at the actual incidents and traces, along with a lot of the simulation work, you continually see this monotonic decrease, which is the signature of the dynamic consequence of saddle node or fold bifurcation. I think the fact that you do see this decrease in actual voltage collapses motivates looking at fold bifurcations extremely carefully.

I'm also a big fan of studying oscillations in power systems, but am suggesting that it be ignored for the focus, which I outline in the paper, of doing quasi-static analysis, looking at loading margins; then, if the loading margin is too small, trying to steer away from the bifurcation using controls. After all, if you do have a possible fold bifurcation, it is definitely going to get you into trouble, and you definitely need to understand it. I fully accept that there are other problems you want study and correct for as well.

Claudio Cañizares (University of Waterloo)

What motivated me originally to write this down was some of the issues that Ian Dobson raised. After listening to the presentation, and talking with him, I just wanted to make sure that I understood correctly what he said. I understand that once you have your dynamic model, you can play with your static equations and find the saddle node or fold bifurcation points. I also understood that you can get your maximum loadability, equivalent to bifurcation, if you use the correct bifurcation diagram. That contradicts one of the conclusions that Fernando had at the end of his presentation, when he said that maximum loadability doesn't necessarily represent bifurcation. I believe this is because we are not looking at the correct bifurcation diagram, or at least the correct projection of the bifurcation diagram in state space.

Consider the example I obtained from some colleagues at the University of Toronto; they presented it in a paper at the 1994 PES Winter meeting (Figures 5). When they have nine induction motors they use aggregate load models with taps (just for the sake of argument) -- don't worry about the generators. So if you look at the equilibrium points, whether you have a linear torque variation of the load or constant torque, you can see that you have several crossing points which are the equilibrium points. For instance for the linear case you

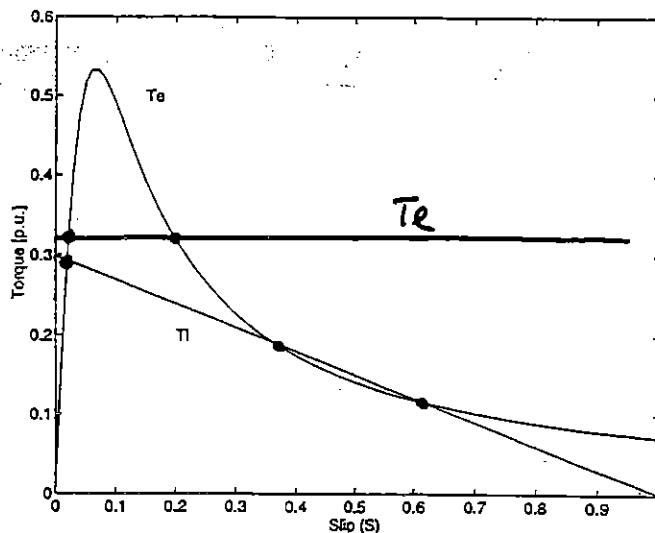


Figure 5

have either three or only one equilibrium points, so you are going through several bifurcations. When torque is constant then you only have two. A comment here regarding Mr. Chicco's presentation in which he said that the maximum torque is equal to the saddle node bifurcation point. That is true if the constant torque assumption is made. Steady-state does not tell where the saddle nodes are at. Also, something Pete Sauer mentioned is, if you go to a higher order model then Hopf bifurcations begin to appear.

Alvarado: I just want to clarify the terminology I have proposed. What I was proposing for this loadability definition is that active power is not always the bifurcation

parameter. I would like to make the distinction between loadability in active power space vs. bifurcation space.

Vournas: In my opinion maximum loadability has to do with the way load increases. I would agree exactly if you say: maximum power transfer is for the maximum amount of megawatts. However in my paper I used loadability in a different sense.

Alvarado: Perhaps maximum power transfer is the better terminology, but you have to distinguish between the two notions very clearly.

Christopher DeMarco (University of Wisconsin)

Certainly power is the traditional quantity that was measured, so it is something that is natural to focus on, even if the underlying parameters that are changing may be more inclusive.

Dobson: I am in agreement with the comments. It is of course very important to choose the right parameter on your horizontal axis when you are doing bifurcation analysis, and in specifying the static load model. One point made by Fernando [Alvarado] is you need to specify what is the parameter. You can't just write down a model; you need to specify what is actually varying with the parameter in order to complete the model. I think everybody is consistent on this point, so I need say no more.

Robert Schlueter (Michigan State University)

My question is to Carson Taylor. I thought his paper was really excellent, but my questions regard three points. The first is in regard to the field current limiters that totally disable the generator exciter. What percentage of current exciter limiters would be the ones that would go to manual? And in relation to that question, when you lose stability by disabling the exciter, we have done some work showing that stability is lost well before hitting the load flow nose point. Is that what you would expect from long term dynamic simulations? Second, if you had the more modern exciters, and you increase gain K2 to move away, does that cause loss of stability? That is, you have some caution there in terms of how fast you unload the machine. Third, in moving close to collapse, do the tap changers' actions of bringing load back become more severe discontinuous disturbances? Does the system response to the discontinuities get larger as you move towards collapse?

Taylor: I would say that pre-1970 machines have the trip to manual control. Second, when you lose voltage regulation you can go back to the pre-1950 conditions, when aperiodic angle instability was possible, as well as the possibility of voltage instability -- it depends on the situation. When you go into current limiting on one generator, you quickly go into current limiting on other generators. Of course, in general there is a combination of angle and voltage instabilities. Third, a faster run back would cause a faster collapse. Fourth, the real important contribution of LTC transformers is the hitting of their limits. Of course, for voltage sensitive loads, once you hit the LTC limit you have a much more stable situation.

Ian Hiskens (University of Newcastle)

At Deep Creek Lake (in 1991), I presented a paper that showed that, for the same sized tap, as the system progressively tapped, it got weaker and weaker. The oscillations after each tap became more significant, until finally synchronizing torque was lost. Each subsequent tap was a greater disturbance.

Yakout Mansour (BC Hydro)

Excitation current action of various limiters have very high capability for a very short period of time (only a few cycles). One type of limit (bridge limit) allows 3 per unit for about

30 seconds. As Carson explained, then it switches to the field winding limit, down to nominal rating of the field (2 - 2.5 per unit). Some of them have smoother inverse characteristics. When this model is put in a long term simulation, the response looks like Figure 6, with the field current increasing for about 30 seconds, and then it decreases. Unfortunately, the timing of this decrease matches the setting of the deadband for LTC transformers. This causes a fast collapse. This brings me back to an important point from the first day -- coordination of all the controls is very important to prevent collapse. We can't have controls responding individually.

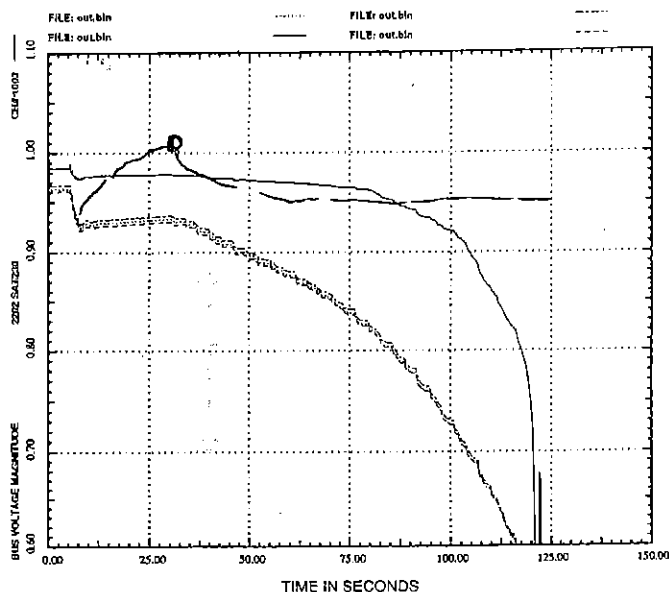


Figure 6

Sauer: I wanted to talk about Bob Schleuter's question because he brought up a very interesting point, which you can sketch out quickly. If you plot the PQ capability of a lossless transmission line connecting a single machine to an infinite bus, the maximum power transfer is roughly a circle. With a regulator (fixing the generator's voltage), this is due to the angle limit ($\delta = 90^\circ$). The mode of the eigenvalue that hits zero, limiting the loadability in any direction, is the shaft (electromechanical) mode that pairs δ and ω . When you do a fixed excitation analysis (getting rid of the exciter so you get a 3rd order model with δ , ω , and field current), you can show analytically that the new limitation surface in the PQ plane is inside the other circle. Again the loadability is limited in any direction by another zero eigenvalue which also occurs at $\delta = 90^\circ$. However in this case the mode is the E' mode, not δ/ω ; it is the field mode that limits the system when you are on manual control, and the electromechanical mode when you have excitation. But in both cases the angle is 90° .

Hill: I would like to make some comments on load modeling that tie together all the papers in the session. As an analyst, I do not aim to answer practical problems, but just to influence how you think about these problems. Generic modeling is a tool to be used carefully. Remember that the commonplace experimental model is a generic model which only captures static behavior. We should use dynamic generic models with much the same care as we do with the static version. We can think of three levels of modeling: generic or black box, gray box, detailed or physical modeling. A clear point is that we must use some generic modeling, i.e., we are not going to model down to every refrigerator, heater and so on. The question becomes where to aggregate.

Suppose that at a 330 kV bus there are two levels of tap changing feeding an aluminum smelter, a coal mine, and an

urban load respectively. Depending on the analysis needed, we could aggregate at any of the three main voltage levels. Of course, it is easier to do P,Q,V measurements based on field tests at lower voltages. Further, in voltage stability analysis, as pointed out in Carson's paper, the limits and delays in the higher tap changers are important. So they should be modeled explicitly. Separate generic models would be done for the smelter, the coal mine (a network of induction motors), and the urban load. But maybe we could consider combining the latter two.

At issue here is the investigation of other generic structures tailored to a load mix, of the various polynomial static generic models. The closer we can get to physical features for a given complexity, the better. Fernando's results emphasize this point emphatically. Ian Hiskens' paper with me points to the influence the chosen structure has on the behavior of the overall system model. If we can justify a load model that leads to absence of singularity in the differential-algebraic equations, this seems worthwhile. Recall that this singularity may represent a real phenomenon as described in Thierry's [Van Cutsem] Tuesday paper [Session 3], and it should not be eliminated artificially. This begs the question of sensitivity to load model structure.

Miroslav's [Begovic] paper points to another important issue, of identifiability of the chosen generic structure from the field data. In the work I did with Daniel Karlsson, the recovery behavior was quite clear, and the parameter estimation would be done with minimal algorithmic sophistication. The results reported by Carson [Taylor] indicate a need for more care in this regard with some measurements. There is a large body of knowledge on this subject which could be useful. Finally, when load system models are written down with tap changer load dynamics and (if necessary) the swing equation dynamics, it is clear that the distance to saddle node bifurcation depends on steady-state load characteristics. Putting derivatives to zero in the two (experimental or adaptive) generic models discussed today puts the loads directly equal to these characteristics. I agree however with Pete, that this is a very limited view; though a stable equilibrium exists after a system change, the trajectory may not settle to it. To answer that part of the question requires the transient load characteristics.

George Knight (Imperial College)

We need to remember that we are looking at a wide range of different conditions. We don't know under what conditions we may get voltage instability, and if we're a planner we're looking far into the future. This leads me to the view that we need to rely primarily on the load synthesis approach. We can then use the testing to substantiate the load synthesis approach. When we come to the load synthesis, particularly the EPRI approach, we find that we need the mix of demand, the composition of demand, and the characteristics of individual components. Now the characteristics of individual components are readily available, but we find the information we need from the utilities is provided in energy rather than power (we want it in power), and it is usually provided for the whole system rather than for parts (which is what we want). And quite often it only talks about residential, commercial and industrial. It doesn't say anything about the mix in percentage of heating, lighting or motor load. We have to make do with information that is not directed towards the purpose we want.

Session 7

Control

"A Systems Approach to Controlling the Onset of Voltage Collapse"
Robert J. Thomas: Cornell University

"Foliations and Power System Controllability"
Chen-Ching Liu, Mingguo Hong
University of Washington
Khoi T. Vu: ABB Transmission Technology Institute
Shaheen Ahmed: University of Capetown

"Computation of Corrective Control Actions to Prevent Dynamic Voltage Instability"
Thomas J. Overbye: University of Illinois

"Effective and Economic Improvement of Voltage Stability by a SMES"
Edmund Handschin, C. Damschen, Ch. Rehtanz, Th. Stephanblome
Universität Dortmund

"Transmission Line Impedance Control by VIPS Apparatus
for Static Voltage Stability Enhancement"
Akihiko Yokoyama, Akira Yoshida: University of Tokyo
Yasuji Sekine: Science University of Tokyo

"Load Recovery in Voltage Stability Analysis and Control"
David J. Hill: University of Sydney
Ian A. Hiskens, Dragana Popovic: University of Newcastle

A Systems Approach to Controlling the Onset of Voltage Collapse

Robert J. Thomas
Cornell University, School of Electrical Engineering
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To be presented at:
The International Seminar on Bulk Power System Voltage Phenomena - III
Voltage Stability, Security and Control
Davos, Switzerland
August 23-27, 1994

Abstract

Researchers have been working on the problem of voltage collapse for the past ten or so years. The instability characterized as voltage collapse is always associated with a heavily loaded system and has a path to system-wide blackout characterized by declining voltage magnitudes. In the beginning the problem was studied solely as one associated with a loss of equilibria. Mathematically this is characterized by the singularity of the system Jacobian at the onset of the phenomenon. In early studies, as today, the nearness to the onset of a voltage collapse can be studied as a quasi-static bifurcation occurring in response to a slowly varying increase or decrease in load. However, as the system approaches the saddle node bifurcation point where the current system operating point will disappear, load dynamics play the dominant role in determining what path the system will ultimately take. In fact, as shown in [17] the collapse may begin earlier than when the bifurcation point is reached because of dynamic excursions due to perturbations in the load. Dynamic load characteristics have long been known to have an effect on system stability and in fact, assumptions about the load model are known to significantly affect any predictions about system behavior.

In this paper, we discuss issues associated with voltage stability as a function of the load model. We note that as the system operating point approaches a collapse or bifurcation point it behaves as though it is

a "strange attractor" (i.e., an equilibrium that characterizes unpredictable motion when there are small changes in initial conditions) due to load model uncertainties. We revisit two quasi-static control methods, one a planning tool and the other an on-line maneuvering tool. Both are based on the notion of improving the value of the minimum singular value of the system Jacobian. We describe three candidate algorithms or approaches for controlling voltage dynamics. These are the "entrainment control" of Hubler, Lusher and Jackson [22-24], the OGY control of Ott, Grebogi and Yorke [25-28], and the ODS (Optimal Decision Strategy) control of Thomas [29-31]. We show that all three are equivalent control methodologies and in fact the first two are subsets of the third. We show by way of a simple example that a pointwise implementation of the ODS is useful for controlling voltage dynamics. As an example we use a simple representation of a static VAR compensator as the control mechanism and implement the control on a small, simple, basic configuration consisting of a generator, a static load, transmission lines, and an aggregate dynamic load.

Introduction

Today, the blackout problem of an electric power system must be studied as either a transient stability problem or a problem of voltage collapse or both. That is, one mechanism for loss of a major portion of a grid is the inability of certain generators to

maintain synchronism in the face of a severe disturbance. However this type of stability problem, while important, is being controlled to a great extent by the advent of faster and more reliable protection systems. In recent years the instability usually termed a voltage instability has been responsible for several major network collapses world-wide [2-4]. In some cases angle and frequency remained constant while voltage continued to decay to a critical value causing protection equipment to dismantle the network. In other cases, frequency and angle swings accompanied the voltage decay. The phenomenon was not always in response to a contingency such as a three-phase fault, but rather in response to what is generally characterized as inadequate VAR support at critical network busses. The fundamental research issue has been the identification of control or support techniques which will cause the system to maintain the necessary voltage profile when an increase in power flow is required. The actual cases of blackouts characterized by voltage depressions reported in the literature indicate that standard practice procedures such as transformer tap changing, capacitor switching, synchronous condenser adjustment, and load shedding may aggravate an already unstable voltage profile [4,5]. As with any nonlinear system, there appears to be more than one way to cause the instability. The most commonly investigated phenomenon involves relating steady-state stability and voltage instability with neighboring multiple equilibria [6-9]. Steady-state stability involves maintenance of a stable equilibrium as certain parameters are allowed to vary. In fact, Venikov [1] showed that it is the Jacobian of the load-flow equations that characterizes the classical steady-state stability limits and therefore eigenvalues of the linearized system matrix have a direct bearing on any bifurcation of the equilibrium state. In [10], Glavitch et.al. developed a voltage stability index based on the feasibility of solution to power-flow equation for each node. A comparison of several proposed performance indices is given in [11]. Using an extension of Venikov's singularity of the Jacobian of the load-flow equations characterization of a voltage collapse, we proposed the minimum

singular value of the Jacobian as a security index and discuss ways to improve the system operating condition if given the resources to do so [12-15]. Lof, Smed, Anderson and Hill have extended and improved the use of the minimum singular value as an index [37-38].

The dynamics of the voltage collapse process is apparently very complicated and has not been thoroughly understood. Problems of voltage collapse can be directly attributed to the inability of the generation to deliver adequate power (real and/or reactive) to the load. The problem is easy to understand in terms of the so-called nose curve and the bifurcation of equilibrium that occurs when excess demand, or alternatively, a generation shortage, occurs. The problem is perhaps more difficult to understand when dynamics are involved. This may be due to the fact that dynamic load models are neither well understood nor universally accepted. It is clear that the collapse dynamics cannot be described solely by generator dynamics which is traditionally believed to be responsible for transient instability. In [16], voltage instability is characterized in association with tap-changing transformer dynamics by defining the voltage stability region in terms of allowable transformer tap setting. In [17], the effect of incorporating induction motor characteristics on the voltage stability region is discussed. In this paper we suggest that there are two control problems that need to be addressed. The first is the control needed to maneuver or posture the system so that it is least sensitive to voltage problems. This is the so-called steady-state control problem mathematically defined by nonlinear algebraic equations that are parameterized in an appropriate way. The second is the problem associated with the control of system dynamics when the system is inadvertently experiencing a collapse situation. In this paper we revisit an approach to the quasi-steady-state control problem and discuss several apparently new approaches to controlling system voltage dynamics. As an example we examine a simple representation of a static VAR compensator as the control mechanism and applying a pointwise control strategy as the algorithm for implementing the control.

The Quasi-Steady-State Control Problem

Suppose $f(\underline{x}, \underline{p}) = 0$ represent the set of nonlinear algebraic equations (i.e., the load-flow equations) whose state solutions $\underline{x}$ are equilibria or operating points of the system for given values of the parameter vector $\underline{p}$. Generally for the steady-state voltage stability problem we treat the loads as independent parameters. It is often very difficult to accurately determine an appropriate load model. However, given a load model different from the real and reactive injection model usually assumed, the methods can usually be modified to accommodate it.

As shown in [14] and elsewhere [32], the sensitivity of parameter variations to voltage magnitude changes is embedded in the Jacobian, $\underline{f}_{\underline{x}}$. In fact, $\Delta \underline{p} / \Delta \underline{V} = 0$ when $\underline{f}_{\underline{x}}$ is singular and the system is at a saddle node bifurcation point.

The question is "how close is $\underline{f}_{\underline{x}}$ to being singular?". One answer to the question involves the minimum singular value of the Jacobian $\underline{F} = \underline{f}_{\underline{x}}$ since

$$\|\underline{F}^{-1}\|^{-1} = \sigma_{\min}(\underline{F})$$

where $\sigma_{\min}(\underline{F})$ is the smallest singular value of $\underline{F}$. Hence the minimum singular value is a natural security index for monitoring how close a power system is operating to a voltage instability.

In [14] we examined two control techniques for improving system security, one a planning tool and the other an operational control tool, based on $\sigma_{\min}$. In the next section, we revisit a method for locating favorable busses for injecting VAR's in support of voltage and based on $\sigma_{\min}$. The derivation reveals the relation between the minimum singular value and the system parameter vector. A detailed discussion of this method is given in [15]. We then revisit a generation dispatch control that moves the system equilibrium to the optimal operating condition with the smallest minimum singular value thus ensuring maximum distance from the bifurcation boundary while respecting generator VAR limits.

Effect Of Parameter Changes On The Minimum Singular Value

Let $r = 2(m+n)$ be the dimension of $\underline{x}$, $\underline{p}$ and the range of $\underline{f}$. Suppose that $(\underline{x}^*, \underline{p}^*)$ is on a trajectory of $f(\underline{x}, \underline{p}) = 0$ with

$$[\underline{f}_{\underline{x}}]_{\underline{x}^*} = \underline{U} \underline{\Sigma} \underline{V}^T \quad (1)$$

the singular value decomposition of the Jacobian matrix. We would like to know how changes in $\underline{p}$ affect $\sigma_{\min}$. Note that $\underline{x}$ must change according to

$$\Delta \underline{x} = -\underline{f}_{\underline{x}}^{-1} \Delta \underline{p} \quad (2)$$

where $\underline{f}_{\underline{x}}$ is assumed to be nonsingular. Writing a first order Taylor series on the Jacobian, we obtain

$$[\underline{f}_{\underline{x}}]_{\underline{x}^* + \Delta \underline{x}} - [\underline{f}_{\underline{x}}]_{\underline{x}^*} = [\underline{f}_{\underline{x}\underline{x}}]_{\underline{x}^*} \Delta \underline{x} \quad (3)$$

where $\underline{f}_{\underline{x}\underline{x}}$ is a three-dimensional array. Let

$$[\underline{f}_{\underline{x}}]_{\underline{x}^* + \Delta \underline{x}} = (\underline{U} + \Delta \underline{U})(\underline{\Sigma} + \Delta \underline{\Sigma})(\underline{V} + \Delta \underline{V})^T \quad (4)$$

be the singular value decomposition of $[\underline{f}_{\underline{x}}]_{\underline{x}^* + \Delta \underline{x}}$ where $\Delta \underline{U}$, $\Delta \underline{V}$, and $\Delta \underline{\Sigma}$ are small perturbations on $\underline{U}$, $\underline{V}$ and $\underline{\Sigma}$ respectively. Substitution (1), (2), and (4) into (3), expanding the matrix multiplication's, and disregarding second and third order perturbations, we obtain

$$\underline{U} \Delta \underline{\Sigma} \underline{V}^T + \Delta \underline{U} \underline{\Sigma} \underline{V}^T + \underline{U} \Delta \underline{\Sigma} \Delta \underline{V}^T = \underline{f}_{\underline{x}\underline{x}} \underline{f}_{\underline{x}}^{-1} \Delta \underline{p} \quad (5)$$

with $\underline{x}$ dependent terms evaluated at $\underline{x}^*$. This equation specifies r^2 constraints on $2r^2 + r$ entries of $\Delta \underline{U}$, $\Delta \underline{V}$, and $\Delta \underline{\Sigma}$. Additional constraints are given by orthogonality of $\underline{U} + \Delta \underline{U}$, and $\underline{V} + \Delta \underline{V}$ and

$$(\underline{U} + \Delta \underline{U})(\underline{U} + \Delta \underline{U})^T = \underline{I} \quad (6)$$

and

$$(\underline{V} + \Delta \underline{V})(\underline{V} + \Delta \underline{V})^T = \underline{I} \quad (7)$$

Equations (6) and (7) each impose $r(r+1)/2$ constraints on $\Delta \underline{U}$ and $\Delta \underline{V}$. Together, equation (5), (6), and (7) represent $2r^2+r$ algebraic scalar equations in $2r^2+r$ variables. Furthermore, these equations are linear since the second order terms in (6) and (7) are neglected. Any numerical technique can be used to solve this set of linear equations, however, this is not necessary if we are only interested in how $\sigma_{\min}$ changes with $\underline{p}$. As shown in [14] the relation between changes in $\sigma_{\min}$ and changes in the parameters is given approximately by the simple linear relation

$$\Delta \sigma_{\min} = \underline{c}^T \Delta \underline{p} \quad (8)$$

for a vector $\underline{c} \in \mathbb{R}^r$ that is readily computed as the smallest entry in the diagonal matrix

$$-U^T f_{xx} f_x^{-1} \Delta \underline{p} \underline{V} \quad (9)$$

. Equation (8) provides an explicit relation between changes in $\sigma_{\min}$ and changes in real and reactive load and generator input power. It also indicates the effect VAR's at each load bus will have on $\sigma_{\min}$. That is, the system will be moved to the most secure state with respect to a voltage instability by adding VAR's at the bus corresponding to the largest coefficient in $\underline{c}$ thus providing the largest increase in $\sigma_{\min}$.

Generation Dispatch as a Voltage Control

In the previous section we discussed the use of the minimum singular value as a planning tool for VAR placement. Here we use it as an on-line steady-state control performance index. Again, we define an optimal operating point against voltage instability to be the system setting which yields the largest minimum singular value of the system Jacobian. Hence, the optimization problem can be stated as: given a load setting, find the input power distribution to generators and condensers that maximizes the minimum singular value. Embedded in the statement are the following system operating constraints: 1) total input power balances demand plus transmission losses,

2) generators and condensers are not asked to supply more than their limits, 3) voltages magnitudes at generator and condenser busses are within specified ranges and 4) losses in transmission lines do not exceed line thermal limits. The first constraint is an equality constraint while the other three are inequality constraints. We assume that the power system that we wish to optimize has been operating at a state in which all these constraints are met. The proposed technique will, at each step, increase the minimum singular value while make certain that the system remains in the allowable region.

Here $\underline{x}$ is the state vector of generator angles and load bus voltage angles and magnitudes, and $\underline{p}$ is now the parameter vector of generator input powers and real and reactive load powers that are available for maneuvering the system operating point. Again, suppose that $(\underline{x}^*, \underline{p}^*)$ is on a trajectory of the load-flow equation $\underline{f}(\underline{x}, \underline{p}) = 0$ satisfying all the stated constraints. From the previous section, $\underline{x}$, $\underline{p}$, and $\sigma_{\min}$ must change according to

$$\Delta \underline{x} = -\underline{f}_x^{-1} \Delta \underline{p} \quad (10)$$

and

$$\Delta \sigma_{\min} = \underline{c}^T \Delta \underline{p} \quad (11)$$

where again the vector $\underline{c}$ is a function of the Jacobian, a three-dimensional array which is the derivative of the Jacobian, and the left and right singular vectors resulting from a singular value decomposition of the Jacobian.

For the equality constraint, note that since the input power to the slack machine is not included in the parameter vector $\underline{p}$, the demand will always be met regardless of $\Delta \underline{p}$ since the slack machine will supply the necessary mismatch. As for the inequality constraints, define those which are satisfied with equality as active constraints, and the rest will be regarded as inactive. First we will present a procedure for increasing $\sigma_{\min}$ when the active constraints remain active and the inactive constraints remain inactive. The active constraints are then linearized about each operating point resulting in two vector equations:

$$\underline{A} \Delta \underline{p} = \underline{0} \quad (12)$$

for generator and condensers power supplying VAR capability limits, and

$$\underline{B} \Delta \underline{x} = \underline{0} \quad (13)$$

for generator and condenser voltage magnitudes and transmission loss limits. Substituting expression (2) for $\Delta \underline{x}$ into (13) yields

$$-\underline{B} \underline{f}_x^{-1} \Delta \underline{p} = \underline{0} \quad (14)$$

Next, partition vector $\underline{p}$ as $\underline{p}^T = [\underline{p}_1^T, \underline{p}_2^T]$ with $\underline{p}_1$ containing input powers while real and reactive loads are in $\underline{p}_2$. Define $\underline{c}_1^T$, $\underline{A}_1$ and $\underline{B}_1$ to be the parts of $\underline{c}^T$, $\underline{A}$, and $-\underline{B} \underline{f}_x^{-1}$ multiplied by $\Delta \underline{p}_1$ in equations (11), (12), and (14) respectively. As a further simplification suppose the load is fixed so that $\Delta \underline{p}_2 = \underline{0}$. Equations (11), (12), and (14) can then be written as

$$\Delta \sigma_{\min} + \underline{c}_1^T \Delta \underline{p}_1 = \underline{0} \quad (15)$$

$$\underline{A}_1 \Delta \underline{p}_1 = \underline{0} \quad (16)$$

$$\underline{B}_1 \Delta \underline{p}_1 = \underline{0} \quad (17)$$

Define a vector function $\underline{G}(\Delta \sigma_{\min}, \Delta \underline{p}_1)$ to be the concatenation of the left hand side of (15), (16), and (17). A continuation solution to the optimization problem is to repeatedly find a $\Delta \underline{p}_1$ that increases $\sigma_{\min}$ subject to $\underline{G}(\Delta \sigma_{\min}, \Delta \underline{p}_1) = \underline{0}$. Note that for small changes $\Delta \sigma_{\min}$ is related to $\Delta \underline{p}_1$ by

$$\underline{G}_{\Delta \underline{p}_1} \Delta \underline{p}_1 = -\underline{G}_{\Delta \sigma_{\min}} \Delta \sigma_{\min} \quad (18)$$

where by the definition of $\underline{G}$

$$\underline{G}_{\Delta \underline{p}_1} = \begin{bmatrix} \underline{c}_1^T \\ \underline{A}_1 \\ \underline{B}_1 \end{bmatrix} \quad (19)$$

and

$$\underline{G}_{\Delta \sigma_{\min}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

Choosing $\Delta \sigma_{\min}$ positive, solving equation (18), determining the new state variables from (10), recomputing $\underline{c}_1$, $\underline{A}_1$, and $\underline{B}_1$, and repeating this cycle until the solution is no longer possible, we have an algorithm.

If we assume the constraints are independent, a necessary and sufficient condition for equation (18) to have a solution is that $\underline{G}_{\Delta \underline{p}_1}$ be full-row rank. It should also be noted that when the dimension of $\underline{G}$ is less than dimension of $\underline{p}_1$, solutions to equation (18) are not unique. One which minimizes the 2-norm of $\Delta \underline{p}_1$ is given by

$$\Delta \underline{p}_1 = -\underline{G}_{\Delta \underline{p}_1}^T \left[\underline{G}_{\Delta \underline{p}_1} \underline{G}_{\Delta \underline{p}_1}^T \right]^{-1} \underline{G}_{\Delta \sigma_{\min}} \Delta \sigma_{\min}$$

To incorporate changes in constraint status, the following procedures may be used; first, note that an active constraint is to stay active if the step computed with that constraint removed violates the inequality. This means that, at each step, solve equation (18) with active constraints removed one at a time, and change the status to inactive those which yield steps with no violation of the inequalities. Lastly, at each step check violations of the inactive constraints, and change the status to active those not satisfying the inequalities.

We should note that the solution satisfies Kuhn-Tucker conditions [16], hence a local maxima of the minimum singular value is guaranteed. The proof of this is given in [17].

Dynamic Load Models

While one may argue that it is appropriate to model loads as constant injections for the quasi-steady-state control problem, it clearly is not appropriate for the

dynamic control problem. There are several models are currently in use and improved models are being proposed on a regular basis. There are several excellent references on the topic including [18-21]. Some of the ones currently in use will be discussed next.

The equations of the often used ZIP (constant impedance, current and power factor, and real and reactive power) load model are given by

$$P_L(V) = A_0 + A_1 V + A_2 V^2$$

$$Q_L(V) = B_0 + B_1 V + B_2 V^2$$

where the constants $A_0, A_1, A_2, B_0, B_1, B_2$ are determined by measurement for a specific situation. Sometimes these quadratic representations are approximated by the relations

$$P_L(V) = AV^\alpha$$

$$Q_L(V) = BV^\sigma$$

where again the constants A, B, α and σ need to be specified. While these models have utility, they are static and do not capture the dynamic aspects of load swings.

In [17] Thomas and Tiranuchi added induction motor dynamics with a differential equation of the form

$$\dot{S} = \frac{1}{M\omega^2} \left[\frac{as^2 + bs + c}{ds^2 + es + f} \right]$$

to the ZIP model where s is the usual induction motor slip variable. While the use of this model was important in exposing the fact that voltage collapse is indeed a dynamic phenomena, it is insufficient to capture the broad range of aggregate dynamic behavior seen by the bulk power system. In addition, most researchers working on load modeling problems agree that it is unrealistic to expect that a relevant aggregate load model can be built by combining the characteristics of disparate devices connected to the bus.

In [18] Hill developed a nonlinear dynamic model that is intended to capture the essential behavior of the load and one for which parameters could be identified from measurements. The development of this

model begins by assuming that there is a (nonlinear) relationship between voltage and its n -derivatives and real and reactive power demand and their n -derivatives. A model, called a recovery model, that preserves steady-state relations is developed. It admits equations of the form

$$\frac{dP_L}{dt} + f(P_L, V) = k(P_L, V) \frac{dV}{dt}$$

with a linear recovery model having the specific form

$$T_P \dot{P}_L + P_L = P(V) + k_p(V) \dot{V}$$

$$T_Q \dot{Q}_L + Q_L = Q(V) + k_q(V) \dot{V}$$

Here $P(V)$ and $Q(V)$ could be the ZIP representation, for example and the recovery time constants T_P and T_Q are to be determined from measurements. Xu and Mansour [19] modified the Hill model by considering the real and reactive power demand to be represented by the quantities ρP_L and ξQ_L . ρ and ξ are now considered to be state variables governed according to the differential equations

$$T_\rho \frac{d\rho}{dt} = (P_0 - \rho) P_L(V)$$

$$T_\xi \frac{d\xi}{dt} = (Q_0 - \xi) Q_L(V)$$

Both the Hill model and the Xu and Mansour model capture the effects of aggregate induction motor and other dynamics through specifying the appropriate parameters.

In [33-34] we explored the use a feed-forward artificial neural network (ANN), trained on real data, as a substitute for a conventional first-principle dynamic load model. This approach belongs to the class of so-called measurement-based approaches [35]. Our objective was to utilize ANN's fast computation capability to represent dynamic load behavior on-the-fly. This approach avoids the difficulties and costs of collecting and updating load

component data, such as is involved in the component-based approaches [35]. The results in [33] show that ANN's can emulate some known load models well. Moreover, given a limited amount of data, the ANN load model exhibits a satisfactory ability to approximate load behavior over a suitable range beyond that described by the data. With such encouraging preliminary results, we continued to explore the theoretical explanation of ANN's approximation capability and the justification of using ANN's as an appropriately accurate model of power system dynamic load.

In [34] we described the results of training a feedforward network with a single hidden layer. We used field measurement data gathered from a Taiwan Power Company primary substation to train and test the network. The network has a standard feedforward functional relationship between input z and output $f_n(z)$ given by

$$f_n(z) = \sum_{k=1}^n \alpha_k \sigma_k(w_k z + b_k)$$

where the sigmoidal function is

$$\sigma(\xi) = \frac{1}{1 + e^{-\xi}}$$

The functional relationship describing the load dynamic behavior is usually expressed as a differential equation of the form:

$$dz/dt = f(z, p) \quad (21)$$

where z is a state variable such as P and/or Q and p is a parameter such as V or $\dot{V}$. Since field measurements are obtained at discrete times, we used the discrete-time version of (21):

$$z_{k+1} = g(z_k, p_{k+1}, p_k) \quad (22)$$

where z_k and p_k are the measurements of z and p , respectively for $k=1,2,\dots,n$. The load modeling problem then is to approximate g given the measurements of z and p .

In most modeling approaches the form of g is chosen and the task is to compute the parameters associated with the selected functions. More specifically, we used the neural network trained on real data to approximate the relations

$$(P/P_0)_{k+1} = f_p((P/P_0)_k, V_{k+1}, V_k) \quad (23)$$

$$(Q/Q_0)_{k+1} = f_q((Q/Q_0)_k, V_{k+1}, V_k) \quad (24)$$

where f_p and f_q are unknown functions to be approximated by the neural network.

It is a fact that the amount and type of data needed to train networks of this type is currently difficult to obtain. However, the results presented in [34] are encouraging and the data will become available as more measurement systems are installed.

Deterministic models of load are appropriate but are complicated and fraught with uncertainty. A model that may show promise in terms of accounting for uncertainty might include a bounded-but-unknown time-varying component in the differential equation. In terms of the Hill model this could take the following form for the real power

$$\begin{aligned} \frac{dP_L}{dt} + f(P_L, V) + \Delta f_1(t)P_L + \\ \Delta f_2(t)V = k(P_L, V) \frac{dV}{dt} \end{aligned}$$

where $\Delta f_1(t)$ and $\Delta f_2(t)$ are both bounded-but-unknown Lebesgue measurable functions. A linear recovery model might take the form

$$\begin{aligned} T_P \frac{dP_L}{dt} + (I + \Delta f_1(t))P_L = \\ P(V) + k_p(V) \frac{dV}{dt} \end{aligned}$$

for example. This type of model would be appropriate for capturing model uncertainty and is compatible with models used for control purposes. It is currently under investigation

Candidate System Control Algorithms

In searching for a suitable dynamic control algorithm it is important to recognize the need to implement them on-line and in real-time. Also, the control should take account of system-wide conditions, be tolerant of model uncertainties and be able to accommodate a wide range of nonlinear behavior. These are severe requirements. We offer the following three methods as candidates.

Entrainment [22-24]

The word entrain conjures up the notion of a host dragging or drawing something along, for example, particles in a fluid flow. The notion of entrainment as a control concept for systems with complicated dynamics is described in a series of papers by Hubler, Hubler, Luscher, and Jackson [22-24].. In their work they assume system dynamics of the form

$$dx/dt = f(x) + Bu$$

where $f(x)$ is assumed differentiable and solutions are assumed to exist for all $t > 0$ and the system possesses attracting limit sets for trajectories. The control problem is then to find a suitable control (or perturbation of $dx/dt = f(x)$) such that the perturbed solutions are "entrained" to what are termed user specified "goal dynamics" $g(t)$. That is, choose $u(t)$ such that $\lim_{t \rightarrow \infty} |x(t) - g(t)| = 0$. The solution is to apply a control of the form $Bu(t) = E(g, \dot{g})S(t)$ where $S(t) = 0$ for $t < 0$ and $S(t) = 1$ for $t > 0$. In order to achieve the goal dynamics $g(t)$ for large t function E should have the form $E(g, \dot{g}) = \dot{g} - f(g)$. Control can be achieved for given initial conditions in some set $\Omega(g)$ called the basin of entrainment. We note that a basin of entrainment $\Omega(g_0)$ exists for all systems when the goal dynamics is a fixed point $g(t) = g_0$.

OGY Control [25-28]

The Ott-Grebogi-Yorke or OGY approach is meant to attack the control of systems with strange attractors, that is, systems where unpredictable motion occurs when there are small changes in initial conditions. Strange attractors are associated with fractal sets and chaos. Reference [36] discusses the existence of chaos in power system models even without load models containing uncertain dynamics. OGY control relies on the existence of a dense set of unstable periodic orbits in the attractor but works as well when the attractor is "regular". Following [25-28] the system dynamics are assumed to be of the form

$$X_{n+1} = F(X_n, p)$$

which may be constructed from a continuous-time differential equation representation. p is an accessible parameter or control value that can be changed in some neighborhood of its nominal value p^* . p^* also defines the nominal parameter about which we would like to stabilize the system. With X_d a desired fixed-point satisfying

$$X_d = F(X_d, p^*)$$

we may approximate the dynamics in the neighborhood of X_d by

$$X_{n+1} - X_d = H(X_n - X_d)$$

If g_s and g_u are contravariant eigenvectors of H corresponding to eigenvalues λ_s and λ_u (i.e., $g_s e_s = g_u e_u = 1$ and $g_s e_u = g_u e_s = 0$) then for small $|p_n - p^*|$

$$X_{n+1} = p_n h + [\lambda_u e_u g_u + \lambda_s e_s g_s] [X_n - p_n h]$$

where

$$h = \left. \frac{\partial X_d(p)}{\partial p} \right|_{p=p^*}$$

Since we want X_{n+1} to fall on the stable manifold of X_d choose control parameter p_n such that

$$\frac{\langle g_u, X_{n+1} \rangle}{\|g_u\| \|X_{n+1}\|} = 0$$

or

$$p_n = \frac{\lambda_u X_n g_u}{(\lambda_u - 1) h g_u}$$

This amounts to aiming the velocity vector of the system at the stable manifold of X_d . In the case of a saddle fixed-point, when the n^{th} iterate falls near the fixed-point for $p = p^*$, the control p_n is turned on to effectively move the fixed-point in such a way that the next iterate X_{n+1} is forced onto the stable manifold of X_d .

Optimal Decision Strategies (ODS) [29-31]

Optimal Decision Strategies (ODS) are procedures that optimize the present position of the system on the basis of present state and/or possibly "near future" events. These procedures are "short sighted" but have highly desirable implementation properties. Certain specific strategies to be discussed have been proposed by the author and other investigators. These strategies are special cases of the following general structure.

To begin the discussion of this structure, assume the system to be controlled has dynamics represented by the set of control dependent equations

$$\dot{x} = f(x, u) \quad (25)$$

There is assumed to be a specified control interval $T = [t_0, t_1]$ and a point $t \in T$ is a decision or observation time or point. The set $S_t = \{v \in \mathbb{R}^1 \mid t_0 \leq t \leq t + s\}$ is termed the time-increment set and is assumed to be a fixed, specified time-shift increment. We

assume specified value constraints on the state and control functions x and u . Also assumed specified is the following ordered (possibly infinite) sequence of scalar functions $\{d_j\}$ with values $d_j(x(t), x(t+s), s(t), t)$ depending on the present state value, its derivative, and future state values for fixed t and s . A more general form would include dependence on higher order derivatives at both present and future end points. The sequence $\{d_j\}$ is termed an optimizing decision sequence with the extremalization of each d_j being at most a two-variable optimization over admissible values of the symbols $x(t)$ and $x(t+s)$ for each fixed $x(t)$, t , and s . Thus, for each t and each $j = 1, 2, \dots$, a set of candidates for the optimal instantaneous derivative-value and future state-point pair $x(t)$ and $x(t+s)$ is found with the collection decreasing for increasing j until a unique value is determined. In this way we sift out that pair of values of $x(t)$ and $x(t+s)$ which minimizes the decision sequence. Note that in some cases, the explicit dependence of future-state points on present derivative values, or vice-versa, may be known and the problem may be further reduced to one-variable searches over $\mathbb{R}$ for sequence-minimizing values.

The value of seS_t identifies the decision process as one of two basic types. If $s = 0$, then decisions are based on the present trajectory values and indicate the optimal escape path from the present position and hence are termed pointwise decision strategies. If $s > 0$, decisions are made at the present state based on future state predictions via the model (25), indicating the optimal approach to the next state point and hence are termed pointwise prediction decision strategies (p.p.d.s). A pointwise decision strategy (p.d.s.) is, less formally, an "approach" to stabilizing a nonlinear system implemented through the application of a sequence of rules $\{d_j\}$ designed to determine the instantaneous input value $u(t)$ from the current data $\{x(t), f(x(t), \dot{x}(t)), f_x(x(t), \dot{x}(t))\}$. Several specific p.d.s. examples have appeared in the literature during the past several years.

[29-31] provide reference to some of the earlier applications.

Relationships Between the Methods

In what follows system dynamics are again assumed to be represented by nonlinear differential equations of the form (25). It is interesting to note that the first application of an ODS involved a prediction or one-step look-ahead in much the same fashion as OGY. The performance functional was

$$d_1 = \sum_{i=1}^n |x_i(t+s)|$$

The aim was to drive the system trajectory to the equilibrium $x_s=0$. This is done by choosing the control at $x(t)$ that minimizes d_1 at $x(t+s)$ using controls bounded in magnitude by relations of the form

$$-u_{\min} \leq u(t) \leq u_{\max}$$

It worked well for the swing equations.

The second index to be used was a criterion that measured the angle between the velocity of the system, dx/dt , at time t and the desired velocity of the system as represented by some model dynamics

$$\dot{x}_d = V(x_d)$$

The angle condition is then

$$d_1 = \begin{cases} \frac{\langle \dot{x}, \dot{x}_d \rangle}{\|\dot{x}\| \|\dot{x}_d\|} & , \dot{x} \neq 0 \\ \|\dot{x}_d\| & , \dot{x} = 0 \end{cases}$$

which is much the same as the OGY criterion. In fact it appears that if we choose the model dynamics to be those of the stable manifold and unconstrain the control we have the OGY procedure.

Finally, ODS index

$$d_1 = \|\dot{x} - \dot{x}_d\|$$

has been used extensively. When this index is used with model dynamics

$$\dot{g} = V(g)$$

the choice of V defines the entrainment goal dynamics $g(t)$ during the transient interval.

It is thus apparent that Optimal Decision Strategies capture entrainment, OGY and gives an indication of how to handle control constraints when they are present.

A Power System Example Involving an ODS Pointwise Control Algorithm

It is important to include both generator dynamics and load dynamics in problems of control because, due to the interconnected nature of the system, controlling one will have an effect on the other. In the case of dynamic control of generator dynamics we place the controls near the generator for maximum effect. It is prudent to place them near the load if load dynamics are the issue. In effect, the strength of the interconnections between the generators and the load will determine whether controls placed at the load will have much of an effect on the generator swings and vice versa. Here we will adopt the usual models for the generator (i.e., swing equation, flux decay, Parks, etc.), one of the relevant dynamic load models for the loads and a Y-matrix interconnection structure. For control we assume that a dynamically controllable static VAR compensator is available at relevant busses. We model the SVC by a simple time varying susceptance $B_s(t)$ with magnitude constraints of the form

$$|B_s(t)| \leq \eta$$

for all t . In terms of the Xu and Mansour load model the equations for load demand would be given by

$$\begin{aligned} \rho P_L(V) &= \rho[A_0 + A_1 V + A_2 V^2] \\ \xi Q_L(V) &= \xi[B_0 + B_1 V + (B_2 + B_3(t))V^2] \end{aligned}$$

The form of the equations for the interconnected system are then

$$\begin{aligned}\dot{X} &= f(X, Y) \\ 0 &= g(X, Y, W, u) \\ \dot{W} &= h(Y, W, u)\end{aligned}\quad (26)$$

Where, for example, X is the vector of generator variables, Y is the vector of load variables and u and W are vectors containing the controls. These equations may be written compactly as in (25).

The control problem we have in mind can be stated as follows: Given the system described by (26) and a desired system response $Y_d(t) = Y_d$ we wish to minimize the difference between Y_d and $Y(t)$ at each instant t . In our case Y is the portion of the vector Y equal to the magnitude of the bus voltage. The desired voltage is chosen to be its pre-disturbance value. It is assumed that the system is controllable and the full system state vector can be accessed. One way to approach this problem is to use the control, u , to align the velocity vectors $f(X, Y)$ and $h(Y, W, u)$ appropriately by adjusting $u(t^i)$ at successive time intervals (not necessarily equally spaced in time) based on the criterion

$$\min \|Y(t) - Y_d(t)\|$$

$$\text{s.t. } g(X, Y, W, u) = 0$$

$$|u(t)| \leq \eta$$

Provided that functions $f(X^i, Y)$, $g(X^i, Y, W^i, u)$ and $h(Y, W^i, u)$ are known, the process implies that we are simply attempting to arrive at a set of control values based upon the knowledge of a single pair of points, (X^i, W^i) . Obviously, one may not conclude that this choice of control will yield a solution for all (X, W) . On the other hand, if such a method, when iterated, were to provide convergence to the desired final state, the advantages of this "pointwise" approach then becomes clear. Since this approach requires only one point at each time step, real-time adaptive control using this process becomes feasible. The algorithm for

such an approach would go as follows:

Given an initial condition and a convergence algorithm (e.g., Newton-Raphson, etc.) for each time t^i

- 1) Measure $(X, W) = (X^i, W^i)$
- 2) Use the convergence algorithm to arrive at a set of control values, $\underline{u}$
- 3) Let the system evolve for time $(\Delta t)^i$ using control value $\underline{u}$.
- 4) Measure (X^{i+1}, W^{i+1})
- 5) Go to 2.

A Simple Application Example

In this section the pointwise algorithm is applied a single generator, three bus power system (see Figure 1). For this case, $N=M=K=1$. Furthermore, simulations are performed using a generator inertia $M=1$ and damping $D=.05$. The mechanical power is assumed to $P_s=1$. The line reactance is $X_l=.8$, and the internal resistance is $X_d=.5$.

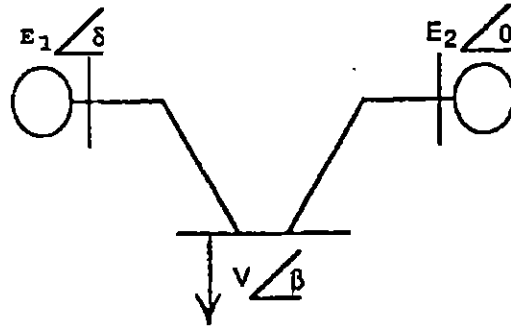


Figure 1. Three Bus Example

The model parameters are as follows:

$$A_0=A_1=B_0=B_1=0, A_2=1.2, B_2=.45, \eta=1, P_0=Q_0=1, T_p=T_q=30, E_1=1, E_2=1.1$$

Given these values and with the control $B_s(t)=0$, system (1) possesses a stable equilibrium at:

$$(\delta^0, \omega^0)=(.7682, 0), (V^0, \beta^0)=(.9342, .2033), (p^0, \xi^0)=(P_0, Q_0).$$

The control is applied every $\Delta t=.1$ sec. Figure 2 shows the resulting load voltage. The dashed line shows that the initial

condition is unstable. Application of the pointwise control shows that the voltage collapse can be avoided if Δt is chosen properly.

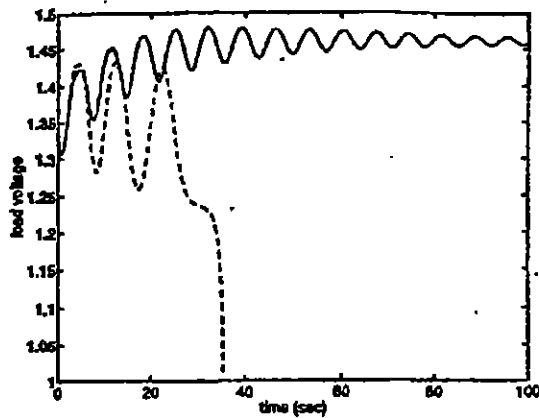


Figure 2. Load Voltage, V, versus time without the control (dashed line) and with the control (dotted line) given the same initial condition.

Conclusion

In preparing this paper it was my objective to collect my thoughts about the voltage stability problem and present them in the context of the work I am most familiar with. As I hope is apparent from the paper, I as others, believe the problem spans a range of issues from planning for avoidance to on-line control, from operating based on current information to real-time control of the events. It is not surprising to me that it has taken about twenty years of work by competent investigators world-wide to get where we are today, and there is more to do. The problem is rich and complex. It is reminiscent of the continued work on transient stability that occupied the professional lives of many people over decades of time.

Today we have a rare opportunity. We may realistically consider implementing real-time on-line system-wide control because of advances in computing, communications, sensors and other technologies. And because we are beginning to understand the dynamics of large and small scale systems. The range of issues are

enormous. I have tried in this paper to expose my own view of them. I am anxious to learn the views of others at this workshop as we continue to strive for understanding and solutions.

Acknowledgment

This paper could not have been written without contributions from two former students, Piak Tiranuchit and Bih-Yuan Ku, and one current student Eric Sakk. Not all of you have met each other. Piak is in Thailand, Bih-Yuan is in Taiwan and Eric is in Ithaca. This paper represents my view of how our collective work fits together.

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Foliations and Power System Controllability

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Abstract

This paper is concerned with the voltage controllability issue. A simple power system model with reactive power control is used. Our main result is the derivation of a voltage controllability region. The construction of the region is based on techniques from the theory of foliation. Inside the controllability region the system can be steered from one state to another. Practically, the concept is significant because the region helps identify limitations of the available controls.

I. Introduction

The voltage collapse phenomena have attracted much attention from utilities and power system researchers for more than a decade so far. The basic mechanisms for instability, steady-state or dynamic, are now understood [1]. Some research issues remain, however. Examples of the issues are load modeling for slow but large voltage variations and on-line voltage controllability. Reference [2] reports the results of a recent investigation on dynamic load modeling.

This paper is concerned with the voltage controllability issue. A simple power system model with reactive power control is used for the theoretical study. The main result here is the derivation of a voltage controllability region. This region is constructed using techniques from the theory of foliation. Essentially the controllability region is a set of power system states inside which the system can be steered from one state to another. Practically, the controllability region concept is significant because the region helps identify limitations of the available controls.

This research is an extension of our previous work on the 'leaf' theory of voltage collapse [3-4]. Reference

[5] is a very readable book on foliations in general. The most relevant reference for our controllability study so far is [6], where a general class of systems is dealt with. Some techniques developed in our proofs are inspired by the results of [7-8].

II. Power System Model

Consider the simple power system consisting of a generator connected to a load through a transmission line (Figure 1). The generator is modeled as a constant voltage source E . The load bus voltage is represented by $V\angle\theta$ and the bus load is given by $P_D + jQ_D$.

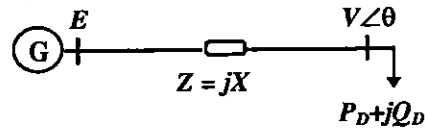


Figure 1: A simple power system

The purpose of this study is to investigate the controllability of a nonlinear power system model using the foliation theory. This paper uses a simple model so that the mathematical concepts and results can be better understood from a physical point of view. The following model includes only the dynamics from the load side of the power system; the well-known generator dynamics are ignored for our first study:

$$P_D = P_S(V) + k_P \dot{\theta}, \quad (1)$$

$$Q_D = Q_S(V) + k_Q \dot{V}, \quad (2)$$

where $P_S(V)$ and $Q_S(V)$ are functions of voltage magnitude V representing, respectively, the steady-state real and reactive load demands. Functions $P_S(V)$ and

$Q_s(V)$ satisfy $P_s(V) = Q_s(V) = 0$ for $V = 0$. The symbols k_p and k_Q are positive constants.

Equating the real and reactive load demands (1) and (2) to the power delivered through the transmission line, the following system is obtained:

$$\begin{bmatrix} \dot{\theta} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} f_1(\theta, V) \\ f_2(\theta, V) \end{bmatrix}, \quad (3)$$

where functions $f_1(\theta, V)$ and $f_2(\theta, V)$ are given by

$$f_1(\theta, V) = -\frac{(1/x)EV \sin \theta}{k_p} - \frac{P_s(V)}{k_p}, \quad (4)$$

$$f_2(\theta, V) = \frac{(1/x)(-V^2 + EV \cos \theta)}{k_Q} - \frac{Q_s(V)}{k_Q}. \quad (5)$$

The 2-dimensional vector $[f_1(\theta, V) \ f_2(\theta, V)]^T$ is called f .

Consider a capacitor bank on the load bus as a control device. The capacitor bank serves as a reactive power source for this power system. Let the susceptance of the capacitor bank be u . The susceptance u can vary from a lower limit a to a higher limit b , depending on how many capacitor units are connected. The dynamics of the power system with the additional capacitor bank can be described by:

$$\begin{bmatrix} \dot{\theta} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} f_1(\theta, V) \\ f_2(\theta, V) \end{bmatrix} + \begin{bmatrix} 0 \\ g(V) \end{bmatrix} u, \quad a \leq u \leq b, \quad (6)$$

where functions $f_1(\theta, V)$ and $f_2(\theta, V)$ are given by (4), (5) respectively and $g(V) = V^2$.

For system (6), it is assumed that:

- 1) Functions $f_1(\theta, V)$ and $f_2(\theta, V)$ are C^1 , that is, $f_1(\theta, V)$ and $f_2(\theta, V)$ are functions with continuous first order partial derivatives;
- 2) Control u is a piecewise C^1 function of time, and $a \leq u \leq b$.

For the simple power system in Eq. (6), it is reasonable to assume that $P_s(V)$ and $Q_s(V)$ are C^1 functions. Control u is a piecewise continuous function of time when it is a switched capacitor bank. Realistically, in this case, u takes on discrete values between a and b . In this study, it is assumed that control u is able to change

discretely and/or continuously between a and b . This would allow continuously changing reactive power sources such as synchronous condensers.

With the above assumptions, system (6) is identified as a *controlled dynamical system* or *CDS*, u being the control. Among the issues of study for *CDS* (6) is the (nonlinear) controllability. In the next few sections, the controllability of *CDS* (6) will be discussed using techniques from the foliation theory.

III. Foliations

Consider *CDS* (6) on the 2-dimensional manifold $\Omega = \mathbb{R}^2/\{0\}$, which is obtained by removing the origin from $\mathbb{R}^2$. Suppose the state of *CDS* (6) is represented in polar coordinates on Ω , as Figure 2 illustrates. The concept of foliations of a manifold will be described in this section. Rigorous definitions of terms related to foliations are given in the Appendix.

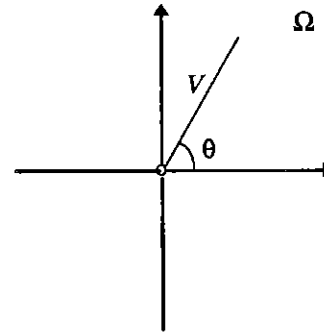


Figure 2: Representation of *CDS*

In general, a p -dimensional, class C^r *foliation* of an n -dimensional manifold M^n can be described as a family of disjoint subsets of M^n . Each of the subsets is a connected C^r submanifold of equal dimension $p < n$. The union of all these subsets is M^n . Each subset is called a *leaf* of the foliation. A p -dimensional foliation of an n -dimensional manifold M^n is also said to be of *codimension* $(n-p)$.

On manifold Ω , two codimension-1 foliations Φ_1 and Φ_2 can be obtained. As shown in Figure 3(a), leaves of foliation Φ_1 are radial lines, which exclude the origin. Leaves of foliation Φ_2 are concentric circles centered at the removed origin, as illustrated in Figure 3(b). Both foliations Φ_1 and Φ_2 are of class C^∞ . It is also seen that each leaf of foliation Φ_1 can be identified by a point on a circle S^1 . S^1 is itself a 1-dimensional smooth manifold, and will be called the *quotient space* of foliation Φ_1 . The *coorientation* of the foliation Φ_1 can

be either of the two possible directions on the quotient space S' , namely, the clockwise or the counter-clockwise direction.

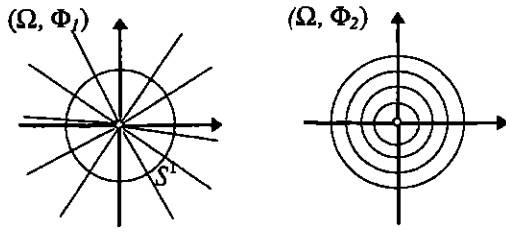


Figure 3(a): Foliation Φ_1 Figure 3(b): Foliation Φ_2

There are many ways to obtain foliations on a manifold. For a dynamic system, it has been shown that the family of integral trajectories of the corresponding C^r vector field on Ω is a 1-dimensional C^r foliation of Ω , as long as the vector field is nonsingular.

For foliation Φ_1 of Ω , each leaf corresponds to a constant voltage angle, whose value can be identified on the quotient space S' . It is also seen that the coorientation of foliation Φ_1 can be either of the two possible directions in which the voltage angle can change, i.e., increase or decrease.

For CDS (6), the main interest here is in the system behavior for the (load-side) angle range $(-\pi/2, 0)$ and voltage range $(0, V_0)$. In other words, it is assumed that the generator is always supplying power to the load. The limit V_0 is the upper limit for the voltage at the load bus. Therefore, CDS (6) will be studied on Ω' , which is a 2-dimensional manifold (without boundary) obtained by taking the intersection of the fourth quadrant of Ω with the open disk centered at the origin. Refer to Figure 4 for the open disk with radius V_0 .

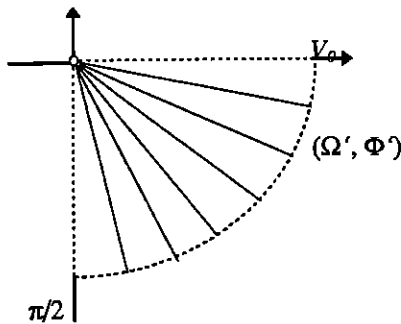


Figure 4: Manifold Ω' and foliation Φ'

Let Φ' be a codimension 1 foliation of Ω' defined in the following way: Leaves of Φ' are the intersection of leaves of Φ_1 and manifold Ω' . Also let the state of CDS (6) be represented by polar coordinates on Ω' . The quotient space S' of foliation Φ' is then (almost) a quarter of a circle, illustrated in Figure 4. The coorientation of foliation Φ' is defined by the two possible directions on S' . Foliation Φ' on Ω' will be used later to study the controllability of CDS (6).

IV. Controllability of CDS (6) with Unbounded Controls

For CDS (6) on manifold Ω' where foliations can be identified, two types of controllability are studied, i.e., the *complete controllability* and the *controllability on a foliation*. Let CDS $\dot{x} = f(x, u)$ be defined on a smooth manifold M with U as the set of permissible controls. Given two points $x, y \in M$, y is said to be reachable from x if there exists a sequence of controls $\{u_1, u_2, \dots, u_k\} \subset U$, k being finite, so that the system can be steered from x to y . Complete controllability refers to the notion that the system can be steered from one point to another by a suitable control sequence.

On the other hand, controllability on a foliation is the ability of a dynamic system to move from a leaf to another leaf. A leaf L in the foliation F of manifold M is reachable from another leaf L' if there is a point on L' reachable from some point on L . A system is said to be controllable on foliation F if, for any pair of leaves in foliation F , one leaf is reachable from the other. See the Appendix for definitions of the complete controllability and the controllability on a foliation.

In [6], a sufficient condition under which the two types of controllability are equivalent is provided. For systems on a 2-dimensional manifold, the results of [6] can be summarized in the following theorem.

Theorem 1 Let

$$\dot{x} = f(x) + g(x)u, \quad -\infty \leq u \leq \infty. \quad (7)$$

be a CDS defined on the 2-dimensional smooth connected manifold M . Control u is a piecewise continuous function of time. Let F be the continuously differentiable foliation defined by $\dot{x} = g(x)$. Let Σ_f be

the system on foliation F with $\dot{x} = f(x)$. If F is of codimension 1 and locally coorientable, then

1) CDS (7) is completely controllable if and only if system Σ_f is controllable on foliation F ;

2) System Σ_f is controllable on F if on each leaf $L \in F$, there are two points $x_1, x_2 \in L$ such that vectors $f(x_1), f(x_2)$ define different coorientations on F . The condition is necessary if the set F/L (i.e., the set of leaves of F without L) is not connected.

To apply Theorem 1 to CDS (6), it is assumed that the control is unbounded, i.e. $a = -\infty, b = \infty$. The systems $\dot{x} = f(x)$ and $\dot{x} = g(x)$ from (6) can be identified.

The foliation defined by $\dot{x} = g(x)$ where $g(x) = [0 \ V^2]^T, x = [\theta \ V]^T \in \Omega'$ is Φ' . Since the first component θ remains constant, the foliation is a set of radial lines as shown in Figure 3(a). In other words, leaves of Φ' are the radial lines corresponding to solutions of $\dot{x} = g(x), x \in \Omega'$. The foliation Φ' is of codimension 1 and coorientable.

Consider CDS (6) on Ω' . The P locus is defined as the solution set of $f_1(x) = 0, x(\theta, V) \in \Omega'$. As shown in Figure 5, the P locus is a 1-dimensional submanifold embedded in Ω' . The P locus separates Ω' into two open regions. Denote by A the upper region and by B the lower region. It can be shown that $f_1(\theta, V) < 0$ for $x(\theta, V) \in A$ and $f_1(\theta, V) > 0$ for $x(\theta, V) \in B$.

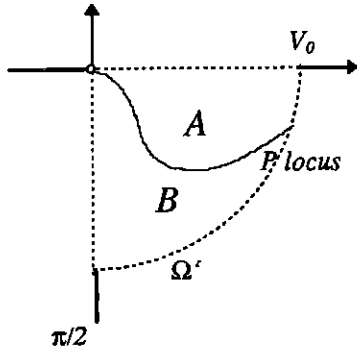


Figure 5: The P locus on Ω'

Let C be the union of all leaves that intersect both regions A and B , as shown in Figure 6. For a leaf L in C , one can select points $x_1, x_2 \in L$ such that $x_1 \in A, x_2 \in B$. Then vector $f(x_1)$ indicates a clockwise coorientation, which results from a negative value in the first component. Similarly, $f(x_2)$ corresponds to a counter clockwise coorientation, as Figure 7 shows.

Note that vector $f(x_1)$ is the direction of motion of system Σ_f , which is defined by $\dot{x} = f(x)$. It can be shown that set $C \subseteq \Omega'$ is a 2-dimensional connected manifold without boundary. As it appears in Figure 7, C is a cone in Ω' , consisting of leaves of Φ' that cut through the P locus.

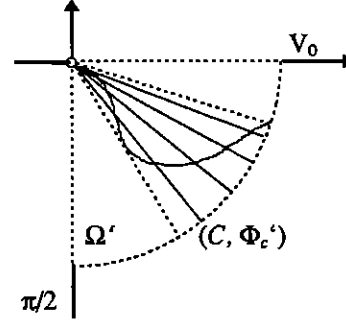


Figure 6: Manifold C and foliation Φ'_c

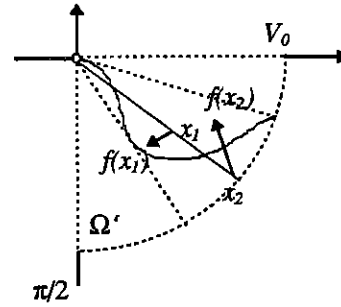


Figure 7: A representative leaf of foliation Φ'_c

Now, for CDS (6) on manifold C , foliation Φ'_c of C defined by $\dot{x} = g(x)$ is of codimension 1 and coorientable. By definition, on each leaf of Φ'_c , there are two points at which vectors given by $f(x)$ define different coorientations. Moreover, by excluding a leaf L from foliation Φ'_c , Φ'_c/L is disconnected. As a direct application of Theorem 1, the following result is obtained for CDS (6).

Theorem 2 Given CDS (6) on Ω' with $a = -\infty, b = \infty$, let $C \subseteq \Omega'$ be the open connected manifold defined above and Φ'_c be the foliation of C defined by $\dot{x} = g(x)$. Also denote by Σ_f the system on Φ'_c with $\dot{x} = f(x)$. Then

1) System Σ_f is controllable on foliation Φ'_c ;

2) CDS (6) is completely controllable if and only if CDS (6) is on C . That is, when control u is unbounded, C is the complete controllability region of CDS (6).

The significance of Theorem 2 is the complete controllability, which allows one to steer the system from one state to any other state in C with proper controls. For CDS (6), control u can be chosen so that the system can either move along the radial line (voltage control) or along a circle (angle control). A sequence of movements can be planned so that the system arrives at the target state in finite time.

As defined earlier, control u of CDS (6) corresponds to the susceptance of the capacitor bank in the simple power system. The lower and upper bounds of u (i.e., a and b) in the practical environment are finite. The complete controllability region for this case, if it exists, must be a subset of C .

The next section provides a necessary and sufficient condition under which the complete controllability region exists for CDS (6) with bounded control. Construction of complete controllability regions on Ω' will also be discussed.

V. Controllability of CDS (6) with Bounded Controls

The discussion begins with the following lemma.

Lemma 3 The solution set of equation $f_1(\theta, V) = 0$ is one of the following:

- 1) The empty set;
- 2) A set of discrete points;
- 3) A 1-dimensional C^1 submanifold of Ω' , which transversely intersects leaves of foliation Φ' except at some discrete points.

Lemma 3 is a consequence of the implicit function theorem and the C^1 function $f_1(\theta, V)$ given by (4). For CDS (6) with realistic parameter values, the solution set of equation $f_1(\theta, V) = 0$ is a connected 1-dimensional manifold embedded in Ω' . As mentioned previously, this 1-dimensional connected manifold is the P locus.

For CDS (6) on Ω' , by specifying $u = a$ and $u = b$, two vector fields X_a and X_b can be obtained. Vector fields X_a and X_b have at most finitely many singular points, which lie only on the P locus. At these singular points, the corresponding vectors of X_a and X_b are zero vectors. Singular points of X_a and X_b , together with the points on the P locus where the P locus fails to

intersect leaves of Φ' transversely, are called the sectioning points of the P locus. Denote the sectioning points by $s_1, s_2, \dots, s_i, \dots$. Sectioning points $s_1, s_2, \dots, s_i, \dots$ partition the P locus into curve segments $S_1, S_2, \dots, S_m, \dots$. (The sectioning points are not included on these curve segments.) In Figure 8, two sectioning points are identified on the P locus. The 2 sectioning points divide the P locus into 3 curve segments, S_1, S_2 and S_3 .

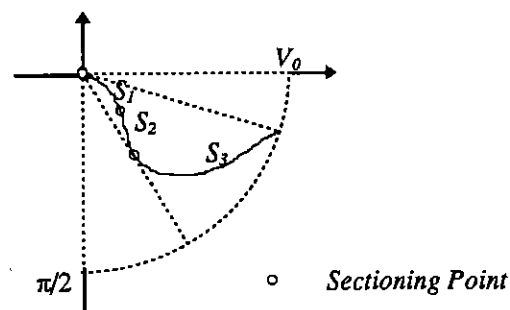


Figure 8: Three curve segments on the P locus

Let $V_a(x)$, $V_b(x)$ denote the vectors at $x \in \Omega'$ such that $V_a(x) \in X_a$, $V_b(x) \in X_b$. Let $S_i \subset P$ be a curve segment defined in the last paragraph. Then it is true that the angles of vectors $V_a(x)$ and $V_b(x)$ are either identical for all $x \in S_i$ or different by π for all $x \in S_i$. This is so because on S_i , $f_1(V, \theta) = 0$, and functions $f_2(V, \theta) + g(V)a$ and $f_2(V, \theta) + g(V)b$ do not change sign. A curve segment S_i is called a G segment if the angles of vectors $V_a(x)$ and $V_b(x)$, $x \in S_i$ differ by π . The G segments on the P locus are represented by $G_1, G_2, \dots, G_r, \dots$. For CDS (6), the existence of G segments depends on the values of a and b . Figure 8 shows an example when S_3 is identified as a G segment.

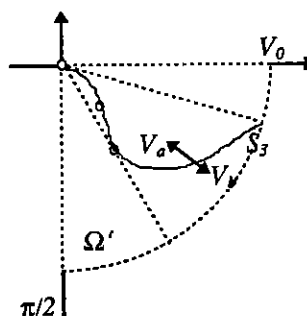


Figure 9: Curve segment S_3 as a G segment

Let Φ_a, Φ_b be the (integral trajectory) foliations defined by vector fields X_a, X_b . Then

Theorem 4 For CDS (6) with bounded a and b , a complete controllability region is nonempty if and only if there exist G segments. Given a G segment G_i , a complete controllability region Γ_i for CDS (6) is given by

$$\Gamma_i = \{\cup L_x / L_x \in \Phi_a, x \in G_i\} \cap \{\cup L_x / L_x \in \Phi_b, x \in G_i\},$$

where L_x denotes the leaf such that point $x \in L_x$. $\Gamma_i \subset \Omega'$ is an open and connected manifold.

To avoid excessive mathematical technicalities, the proof of Theorem 4 is not included in this paper. The complete controllability region given in the theorem is an intersection of two sets. Each set is the union of integral trajectories crossing the G segment. The difference between the two sets is the control magnitude. Figure 10 shows a complete controllability region on Ω' with S_j being a G segment.

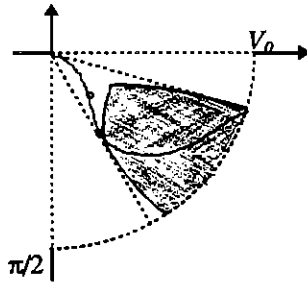


Figure 10: A controllability region for CDS (6) with bounded controls

The complete controllability region can be constructed piece by piece for all the G segments. For a given G segment, take any two points in the controllability region. One can use control a or b to move from one of the points to a neighborhood of the G segment. The union guarantees that the neighborhood is large enough to intersect the integral trajectory. To steer the system to the other point, one can follow the trajectory that takes the system from the neighborhood of G segment to the destination by control a or b . Our proof of the theorem involves a technique for calculation of the control necessary to move the system anywhere within the neighborhood of the G segment.

VI. Conclusion

This paper reports some preliminary results of our investigation on the application of foliation theory to problems on power system dynamics. Even though the voltage controllability region has been derived for a simple model, the extension to general multi-bus power systems is yet to be carried out. For the controllability regions to be practical, both rotor and voltage dynamic mechanisms should be incorporated.

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VIII. Acknowledgment

This research is sponsored by the Power System Program of National Science Foundation, USA, through Grant No. ECS-9307149.

IX. Appendix - Definitions

i. Foliation

A topological space M is called an n -dimensional manifold if it is locally homeomorphic to R^n . Denote the n -dimensional manifold by M^n . For M^n , there is an open cover $U = \{U_i\}_{i \in \Lambda}$ such that for each $i \in \Lambda$ there is a map $\varphi_i: U_i \rightarrow R^n$ which maps U_i homeomorphically onto an open subset of R^n . The pair (φ_i, U_i) is called a *chart* (or *coordinate system*) with domain U_i . The set of charts $\Phi = \{(\varphi_i, U_i)\}_{i \in \Lambda}$ is an *atlas*. Manifold M^n is called an n -dimensional differentiable manifold of class C^r or a C^r n -manifold if the coordinate change on the overlap of two charts $(\varphi_i, U_i), (\varphi_j, U_j)$

$$\varphi_j \varphi_i^{-1}: \varphi_i(U_i \cap U_j) \rightarrow \varphi_j(U_i \cap U_j)$$

is of differentiable class C^r . A maximal atlas (with respect to set inclusion) of the C^r n -manifold M^n is called a C^r *atlas*.

Let $F = \{L_\alpha; \alpha \in A\}$ be a family of arcwise connected subsets L_α of the manifold M^n with a C^r atlas $P^{(r)}$. F is a k -dimensional C^r *Foliation* of M^n if F satisfies the following.

- (1) $L_\alpha \cap L_\beta = \emptyset$ for $\alpha, \beta \in A$ with $\alpha \neq \beta$.
- (2) $\bigcup_{\alpha \in A} L_\alpha = M^n$.
- (3) Given a point p in M^n , there exists a chart $(U_\lambda, \varphi_\lambda) \in P^{(r)}$ about p such that for L_α with $U_\lambda \cap L_\alpha \neq \emptyset$, $\alpha \in A$, each arcwise connected component of $\varphi_\lambda(U_\lambda \cap L_\alpha)$ is of the form

$$\{(x_1, x_2, \dots, x_n) \in \varphi(U); x_{k+1} = c_{k+1}, x_{k+2} = c_{k+2}, \dots, x_n = c_n\},$$

where $c_{k+1}, c_{k+2}, \dots, c_n$ are constants determined by the (arcwise) connected component. L_α is a leaf of the foliation F . A k -dimensional C^r foliation is also referred to as a C^r codimension $n-k$ foliation. An n -dimensional manifold M coupled with a C^r codimension q foliation F is called *foliated* and denoted by (M^n, F) . A chart which satisfies condition (III) is called a *foliated chart*. The set of all foliated charts is called a *foliated atlas*.

It is well known that the family of trajectories of a nonsingular C^r vector field on a n -dimensional C^r ($s \geq r+1$) manifold M^n is a 1-dimensional C^r foliation of

M^n . Trajectories of a C^r singular vector field on a n -dimensional C^s ($s \geq r+1$) manifold are not of equal dimension since the singular points are of zero dimension. The family of the trajectories of a singular vector field is classified as a singular foliation in some books. The concept of singular foliation is used in Theorem 4.

ii. Controllability on a foliation and complete controllability

Given a dynamical system $\dot{x} = f(x)$ on manifold M and foliation F on manifold M , let $\varphi(x, t)$ denote the integral trajectory of the dynamical system with $x = \varphi(x, 0)$ and F_x denote the leaf such that $x \in F_x$ for arbitrarily given $x \in M$. By $\Sigma_f^t(F_x) = F_y$ it is meant that there exists a point $x' \in F_x$ such that $F_{\varphi(x', t)} = F_y$. Then the sequence of mappings $\Sigma_f^{t_n}(\dots(\Sigma_f^{t_1}(F_x))) = \Sigma_f^T(F_x)$ with $T = t_1 + \dots + t_n$ and $t_1, \dots, t_n > 0$ is defined as a *system on the foliation F* , denoted by Σ_f . System Σ_f is said to be *controllable on foliation F* if for every F_x and F_y , there exist $t_1, \dots, t_n > 0$ such that $\Sigma_f^T(F_x) = F_y$, $T = t_1 + \dots + t_n$.

A CDS given by $\dot{x} = f(x, u(t))$ on manifold M is said to be *completely controllable* on M if for every $x_1, x_2 \in M$, there exist a piecewise continuous control $u(t)$ and a finite time $T \geq 0$ such that $x_2 = \varphi(x_1, u(t), T)$, where $\varphi(x, u(t), t)$ is the integral trajectory of the CDS satisfying $x = \varphi(x, u(0), 0)$ and $t \geq 0$. A set $M' \subseteq M$ on which a CDS is completely controllable is called a *complete controllability region* of the CDS on M . When the term *complete controllability regions on M* is used, it implies that there are more than one connected components on which the CDS is completely controllable.

Computation of Corrective Control Actions to Prevent Dynamic Voltage Instability

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Abstract

One of the mechanisms by which dynamic voltage instability can originate is through the occurrence of a contingency which causes the system to lose its equilibrium point. In this context the static power flow equations would become unsolvable. If corrective control action is not taken quickly enough following the contingency, the system state will vary as dictated by system dynamics with the usual consequence of voltage collapse. This paper presents a method to determine these corrective control actions, with particular attention given to determining the allowable time frame for corrective control. To this end, a structure-preserving energy function is used to define a potential energy well in voltage space for the post-control system, with the stable equilibrium point corresponding to the bottom of the well, while unstable equilibrium points correspond to saddle points on the boundary of the well. Then a sufficient condition for the system to retain stability following the contingency is that the corrective control action be completed before the system state has gained sufficient energy to escape the well. The paper addresses the practical considerations of applying this method to a realistically sized system which contain a large number of potential controls. The method is demonstrated on both small systems and the IEEE 118 bus test system.

I. Introduction

Optimal transmission system operation requires adequate control strategies for the prevention of voltage instability. The lack of such strategies can result in either a widespread blackout due to voltage collapse, or more commonly, unduly constrained system economic operation to avoid an imprecisely defined threat of voltage instability. Voltage instability generally arises from two types of system events: gradual parameter variation (such as due to load/generation variation), and contingencies. In the first case the system state is assumed to move from a point of relative security to one of increased vulnerability to voltage instability through a quasi-static variation (time scale of tens of minutes to hours). The system then ultimately experiences voltage collapse as a result of either a saddle node bifurcation in which the system's operating equilibrium disappears [1], or as the result of a Hopf bifurcation in which a pair of eigenvalues of the system Jacobian cross the imaginary axis with nonzero imaginary part [2].

In the second case, a previously stable system loses voltage stability due to the occurrence of a large disturbance, such as a line outage contingency. Then, in the absence of effective control intervention, the system would eventually experience voltage collapse. For such a case there are three mechanisms by which the system could experience collapse. First, the system has an equilibrium point but it is unstable. Second, following the contingency the system does not have an equilibrium point. Mathematically this corresponds to lack of a real solution to the power flow equations. Third, a stable equilibrium exists, but following the contingency the system state is not within the region of attraction (ROA) of this equilibrium. This paper will be primarily concerned with developing control strategies for dealing with the second and, to some extent, the third possibilities.

The paper is organized as follows. First, the static measure from [3] for quantifying the unsolvability of such power flow cases, along with the method from [4] for developing statically optimal control responses to restore such a case to solvability are reviewed. Then, the full dynamic consequences of unsolvability are considered. The method is demonstrated on both small systems and the IEEE 118 bus system.

II. Static Considerations of Unsolvability

As is known to most planning and operations engineers, there are often contingent (or sometimes basecase) situations for which the Newton-Raphson (N-R) power flow does not converge. Since convergence is not guaranteed, this situation could be due to either a poor initial voltage guess, or because the case does not have a real solution. Examples of the former can be reduced through various methods to prevent power flow divergence. The concern in this paper is with the latter. This section reviews a Euclidean measure to quantify the unsolvability of such cases, and a method to compute the statically optimal control responses to restore the case to solvability [3], [4].

Consider the two bus system shown in Figure 1, where bus 1 is the slack bus and bus 2 is a load bus. The buses are connected through the two transmission lines as shown with the indicated per unit reactances (100 MVA base). The system power flow equations can then be written as

$$S = f(x) \quad (1)$$

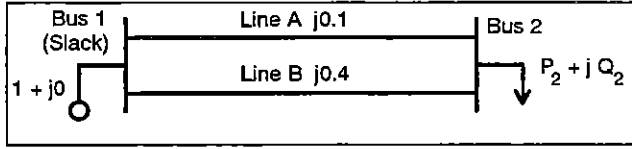


Figure 1: Two Bus System

where S is a vector of the constant real and reactive power load minus generation at all buses except the slack

$$S = [P_2, Q_2]^T \quad (2)$$

x is the nonslack bus voltage vector in rectangular notation, $V_i = e_i + jf_i$,

$$x = [e_2, f_2]^T \quad (3)$$

f is the function of the real and reactive power balance constraints

$$f = [f_{p2}(x), f_{q2}(x)]^T \quad (4)$$

$$f_{pi}(x) = - \sum_{j=1}^n [e_i(e_j G_{ij} - f_j B_{ij}) + f_i(f_j G_{ij} + e_j B_{ij})] \quad (5a)$$

$$f_{qi}(x) = - \sum_{j=1}^n [f_i(e_j G_{ij} - f_j B_{ij}) - e_i(f_j G_{ij} + e_j B_{ij})] \quad (5b)$$

and $G + jB$ is the network bus admittance matrix.

Depending upon the values of P_2 and Q_2 , the power flow equations (1) can have either two, one, or no real solutions [5]. The region in load parameter space with two solutions is defined as the solvable region, while the region with no solution is defined as the unsolvable region; the two regions are separated by a hypersurface Σ where the equations have a single solution and upon which the Jacobian of (4) is singular.

The boundary Σ is dependent, of course, upon the system topology. For example, Figure 2 shows this boundary in load parameter space for the two bus basecase with a constant power load S of 500 MW and 100 Mvar (dashed line), and for a contingent case with line B removed (solid line). Note that the contingency has caused the operating point to change from being within the solvable region to the unsolvable region. The problem then is to first quantify the unsolvability of the case, and then to determine the optimal method to return to solvability.

The unsolvability of the case can be quantified using the Euclidean distance in load parameter space from the point S (in the unsolvable region) to the closest point on the boundary Σ . This closest boundary point is determined by first defining a cost function of one half the square of the power flow mismatch equations:

$$F(x) = \frac{1}{2} [f(x) - S]^T [f(x) - S] \quad (6)$$

Note that $F(x)$ is greater than or equal to zero for all x , and is equal to zero only at the power flow solutions.

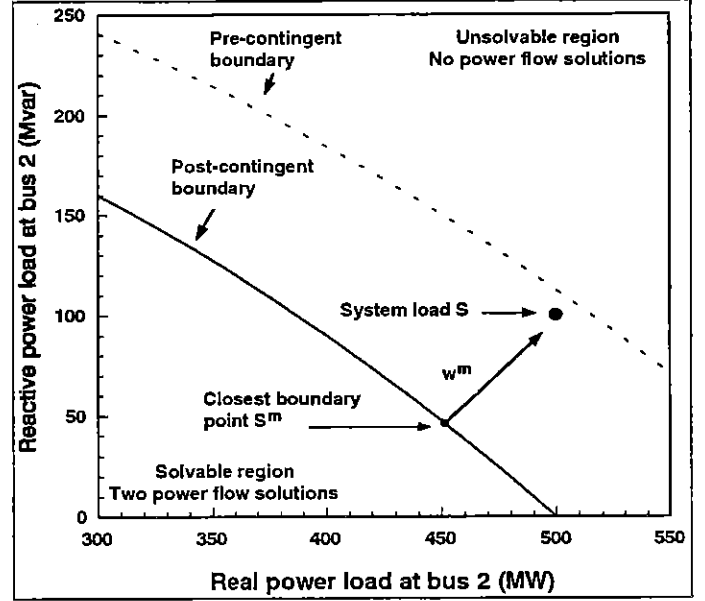


Figure 2: P_2/Q_2 Parameter Space Relationships

Since the case is unsolvable, there is no x such that $F(x) = 0$. Rather, let x^m be defined as the value of x corresponding to the minimum of the cost function $F(x)$; thus x^m can be thought of as the "best possible" solution to the power flow equations. Define $S^m = f(x^m)$ to be the point in parameter space corresponding to x^m . In [3] it is shown that S^m is the closest point on Σ to S . The Euclidean distance between these points, given by:

$$\delta = \|S^m - S\| = \sqrt{[S^m - S]^T [S^m - S]} \quad (7)$$

is then used to measure the unsolvability of S . A power flow based algorithm to determine x^m and hence S^m is presented in [3]. As an example, for the two bus case the closest boundary point is determined to be 451.4/46.2 MW/Mvar with $\delta = 0.725$ per unit (see Figure 2); at this point $x^m = [0.500 - 0.451]^T$.

The direction in parameter space to the closest boundary point is then just $-(S - S^m)$, which is parallel to the normal vector of Σ at S^m . At S^m the Jacobian of $f(x^m)$, $J(x^m)$, has (generically) a single zero eigenvalue. In [6], [7] it is shown that the left eigenvector associated with the zero eigenvalue of $J(x^m)$, w^m , is parallel to the normal vector to Σ at S^m .

Since S is an element of the normal ray emanating from S^m , the "optimal" direction (at least in a Euclidean sense) to move back to Σ is in the opposite direction to the normal, that is $[S - S^m]$. However this direction is seldom practical since it commonly involves changes in power injections at a large percentage of system buses, and in its determination neither cost not the availability of such controls has been considered. Rather, to calculate the sensitivity of δ to practical controls, let u be defined as the vector of such controls. Then, using a result from [7], the sensitivities of δ with respect to u are given by

$$\delta_u = -w^m [f(x^m) - S]_u \quad (8)$$

where $[f(x^m) - S]_u$ is the Jacobian of the power flow equations with respect to the system controls u , and w^m is the normalized left eigenvector associated with the zero eigenvalue of $J(x^m)$. The sign ambiguity of the eigenvector w^m (outward or inward from Σ) is resolved by choosing it to always be directed outward (i.e., sign of w^m is chosen so that $(S - S^m) \cdot w^m > 0$).

The values of Δu then provide a linear estimate of the effect a particular control has upon β . Neglecting nonlinearities, the necessary change in a single control u_i to just return the case to solvability is given by $-\beta/\Delta u_i$. For the case of power injection controls (i.e., elements of S), for which $[f(x^m) - S]_{u_i}$ (the column of the Jacobian corresponding to u_i) is constant, the accuracy of the linearization depends only upon the distance of S from Σ and the curvature of Σ in the vicinity of S^m . The error is the difference between Σ and its tangent plane approximation at S^m . For other controls the linearization accuracy also depends upon how well $[f(x^m) - S]$ is approximated by the Jacobian linearization.

As an example, Table 1 compares the estimated and actual changes needed for different controls to return the two bus case to solvability. Which control(s) to use depends upon their relative cost and availability, with the switching in of shunt capacitance being undoubtedly preferable to shedding load. The last line in the table represents the more realistic load shed scenario in which both real and reactive power are simultaneously shed to maintain the same power factor (in this case $\text{pf} = 0.98$). Of course one would usually want to move the control by more than the actual value given in the table to provide a margin of security. The effects of these load controls in load parameter space are shown in Figure 3. There is of course nothing in the method which would preclude using multiple controls simultaneously. Additionally, application of the method to larger systems is straightforward, and because of the sparsity of $[f(x^m) - S]_u$, the sensitivities can be computed with minimal computation. More details and an example using the IEEE 118 bus system are contained in [4].

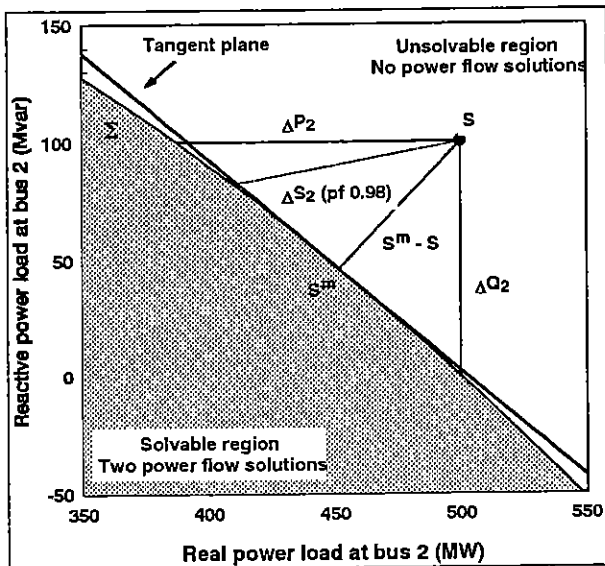


Figure 3 : Load Changes to Return Case to Solvability

P = 5.0, Q = 1.0 (per unit), $\beta = 0.725$			
Control (per unit)	Estimate	Actual	% Error
ΔC_2	2.15	1.80	16.3
ΔP_2	-1.082	-1.127	4.0
ΔQ_2	-0.976	-1.000	2.5
ΔS_2 (0.98 pf)	-0.904	-0.922	2.0

Table 1 : Two Bus Control Changes to Return to Solvability

III. Dynamic Considerations of Unsolvability

The previous section examined the unsolvable power flow problem using purely static analysis. However to fully consider this problem, the dynamic consequences of the loss of the power flow solution must also be addressed. The unsolvable power flow problem is, by definition, one for which the system dynamic equations no longer have an equilibrium point. Thus, if control action is not taken sufficiently quickly following the contingency which pushed the system into the unsolvable state, the system would typically experience voltage instability and eventual collapse.

Thus a method is needed to determine how quickly the controls must be applied to avoid voltage collapse. This information could then be used to determine if the proposed controls possess the necessary time rate of response. A practical example of this situation is mentioned in [8], where, in order to avoid voltage collapse, utilities need to decide between installing fast and expensive SVC's, or the more inexpensive but slower switched capacitors. Note that this problem is akin to the transient stability problem in which the question is to determine how quickly a control (typically circuit breaker operation to clear a fault) must be applied to avoid angular instability. The main difference between these two problems is the much shorter time frame for the transient stability problem.

To study the dynamic consequences of unsolvability, the static power flow equations need to be augmented to include system dynamics. This will be done initially by just considering load dynamics, with generators represented using the standard power flow PV bus model (i.e., constant real power and voltage magnitude) provided their reactive power is within limits, and by the PQ bus model if the reactive output attempts to exceed these limits. A number of different dynamic load models have been proposed, with one popular model motivated by the fact that many loads are equipped with regulators (including down-stream LTC transformers) which, when modeled in aggregate, vary the load according to a generic first order model [8], [9], [10], [11], [12]:

$$T_{pi} \dot{\lambda}_{pi} = P_{oi} V_i^a - \lambda_{pi} V_i^\alpha := \Delta P_i(x) \quad (9a)$$

$$T_{qi} \dot{\lambda}_{qi} = Q_{oi} V_i^b - \lambda_{qi} V_i^\beta := \Delta Q_i(x) \quad (9b)$$

with $T_{pi}, T_{qi} > 0$. The first term on the right-hand side represents the static load demand at the bus, while the second term is the dynamic load demand. A necessary condition for

an equilibrium point is that the static demand equal the dynamic demand. During the course of a dynamic simulation, the dynamic power balance equations must always be satisfied:

$$\lambda_{pi} V_i^\alpha - f_{pi}(x) = 0 \quad (10a)$$

$$\lambda_{qi} V_i^\beta - f_{qi}(x) = 0 \quad (10b)$$

where x is the vector of the voltage angles at all system buses (except the slack) and the voltage magnitudes at all buses except the PV generator buses (x is now expressed in polar coordinates for better clarity of system dynamics). At buses with no dynamic load (such as buses with no load, PV generator buses, or purely static load buses) the values of λ_{pi} and λ_{qi} are constant, and the steady state load demand is identical to the dynamic load demand:

$$P_{oi} V_i^a = \lambda_{pi} V_i^\alpha \quad (11a)$$

$$Q_{oi} V_i^b = \lambda_{qi} V_i^\beta \quad (11b)$$

At all other buses λ varies according to (9) in an attempt to reach an equilibrium point.

To illustrate this problem, again consider the two bus case, but with line B's impedance changed to $j0.1$. Using the load model from (9), let $P_{o2} = 5.0$, $Q_{o2} = 1.0$, $a=0$, $b=0$, $\alpha=2$, $\beta=2$, and $T_{p2} = T_{q2} = 5$ seconds. From Table 1 it is seen that following the outage of line B the case would be unsolvable, with $\beta = 0.725$. At some point after the line outage assume a 220 Mvar shunt capacitor is switched in at the load bus. The Table 1 results show that from a purely static point of view this should be sufficient restore solvability. Since the static load demand has no voltage dependence (i.e., a and b are zero), the desired equilibrium point x^s is the power flow solution after the capacitor has been inserted; the load bus voltage of this solution is $0.962 \angle -31.3^\circ$. The actual dynamic response of the load bus voltage magnitude is shown in Figure 4 for different capacitor switching times. As was shown in [8] and in this figure, it is clear that the time of

control application is of critical importance to whether the system eventually reaches the equilibrium x^s .

The traditional method of solving this type of problem is to perform a series of computationally expensive time domain simulations. This is necessary because while each simulation can determine whether a case is stable or not, it does little to quantify the degree of stability. In contrast, energy function methods provide a direct means to quantify system stability. The use of energy function methods for the related problem of transient stability is well documented [13], [14], while their use for quasi-static voltage stability is more recent [15], [16]. Since the present problem combines aspects of both, this suggests the use of a unified energy framework [17].

Using the structure-preserving energy function from [15], $\theta(x)$, define a "potential energy well" in voltage space for the system after controls have been applied (i.e., the post-control system). The SEP (high voltage power flow solution) x^s corresponds to the bottom of the well, while a subset of the UEPs (low voltage power flow solutions) correspond to saddle points on the boundary of the well. Then, provided the energy derivative along trajectories of the system, $\dot{\theta}(x)$, is less than or equal to zero within a bounded region Ω_c about x^s , a sufficient condition for the system to retain stability following the contingency is that the control action be completed before the system has gained sufficient energy to escape the well, with the minimum value equal to the energy of a low voltage solution on the boundary of the well, defined as θ^{cr} . The region Ω_c is defined as the set of all x about x^s for which $\theta(x) \leq \theta^{cr}$. For $\theta(x)$ to formally define a Lyapunov function some restrictions would need to be placed on system models, including not allowing voltage dependence in $P_s(V)$ and restricting the method to networks with no transfer conductance terms. In the realistic systems considered here, these restrictions are relaxed, and the term "energy function" is used rather than Lyapunov function.

Before applying energy methods to the problem, it is necessary to develop conditions for the load model from (9) such that $\dot{\theta}(x) \leq 0$ for $x \in \Omega_c$. This is done by first developing the relationship between x and λ . By dividing both sides of (10a) by V_i^α and (10b) by V_i^β , λ can be written as an explicit function of x :

$$\lambda = g(x) \quad (12)$$

from which it is straightforward to show that

$$\dot{\lambda} = J(x) \dot{x} \quad (13)$$

$$\dot{x} = J(x)^{-1} \dot{\lambda} \quad (14)$$

where $J(x)$ is the Jacobian of $g(x)$. Also recall the definition of the energy derivative

$$\dot{\theta}(x) = \nabla \theta(x) \dot{x} \quad (15)$$

and that for $\dot{\theta}(x)$ from [15] (with a lossless system) for the post-control system

$$\nabla \theta(x) = [\Delta P(x) \quad \Delta \tilde{Q}(x)] \quad (16)$$

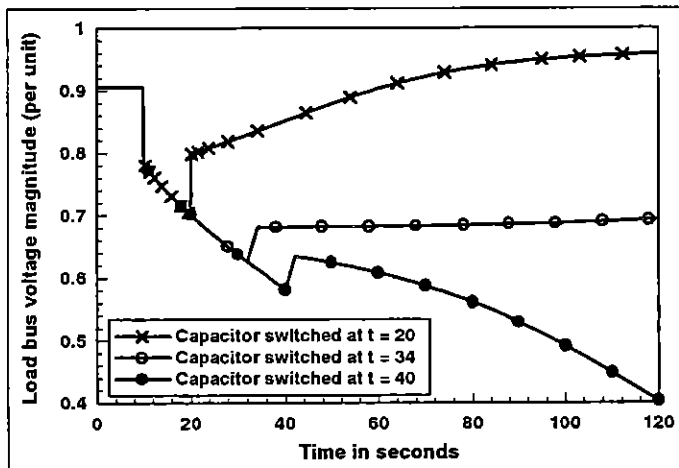


Figure 4 : Load Bus Voltage for Different Switching Times

where $\Delta\tilde{Q}_i(x) = \Delta Q_i(x) / V^i$. Then using (14) and (9) we get

$$\dot{x} = J(x)^{-1} \begin{bmatrix} \frac{V^\alpha}{T_p} & 0 \\ 0 & \frac{V^{\beta+1}}{T_q} \end{bmatrix} \begin{bmatrix} \Delta P(x) \\ \Delta\tilde{Q}(x) \end{bmatrix} \quad (17)$$

Finally, defining the diagonal matrix as A , $\dot{\vartheta}(x)$ can then be rewritten as

$$\dot{\vartheta}(x) = [\Delta P(x) \quad \Delta\tilde{Q}(x)] J(x)^{-1} A \begin{bmatrix} \Delta P(x) \\ \Delta\tilde{Q}(x) \end{bmatrix} \quad (18)$$

A sufficient condition for $\dot{\vartheta}(x) \leq 0$ is that $J(x)^{-1}A$ be negative definite. Whether this condition is met depends both upon $J(x)$ and A . For the case where all the time constants T_{pi} and T_{qi} are identical (i.e., $A = \alpha I$ with α equal to the time constant), this condition is met if all of the eigenvalues of $0.5(J(x) + J(x)^T)$ are negative. Otherwise the condition depends upon the relative values of T_{pi} and T_{qi} ; it is strongly dependent upon the ratio of these two values at a particular bus i . In actuality the size of the region about x^s for which $\dot{\vartheta}(x) \leq 0$ is typically larger than that for which $J(x)^{-1}A$ is negative definite. This is due to constraints on allowable values of $[\Delta P(x) \quad \Delta\tilde{Q}(x)]$ imposed by (9) and (10). The size of the region for which $J(x)$ is negative definite depends upon the load parameters α_i and β_i with, in general, the region enlarging for higher values. Numerical testing indicates that for realistic load models the condition that $\dot{\vartheta}(x) \leq 0$ is usually met along system trajectories.

The maximum time to apply control following a contingency is then determined by calculating the energy for the contingent system trajectory assuming no control action (analogous to the transient stability fault-on trajectory). Since this system has no equilibrium point, the system will eventually gain sufficient energy to exit the region of attraction about x^s . The time at which $\vartheta(x)$ increases to ϑ^{cr} is then the critical time -- control must be applied before this time to guarantee stability. For example Figure 5 shows the contours of the post-control energy well for the two bus case, with the SEP at $0.962 \angle -31.3^\circ$ and the UEP at $0.679 \angle -47.4^\circ$;

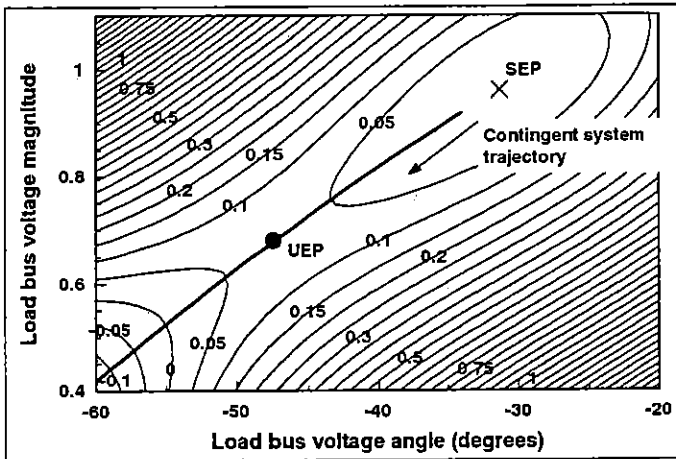


Figure 5 : Two Bus System Energy Well

the energy associated with the low voltage solution saddle node is $\vartheta^{cr} = 0.0593$. Superimposed on the well is the contingent system trajectory assuming no control action. To retain stability, the capacitor must be switched in before the trajectory exits the region of attraction of SEP. For this case, in which the trajectory passes almost through the UEP, the critical switching time is when the energy is equal to that of the UEP. Figure 6 plots the value of this energy as a function of time. From the figure it is directly seen that the trajectory passes through this point at about 34 seconds. The result demonstrating the switching of the capacitor at this critical time is shown by the second curve in Figure 4.

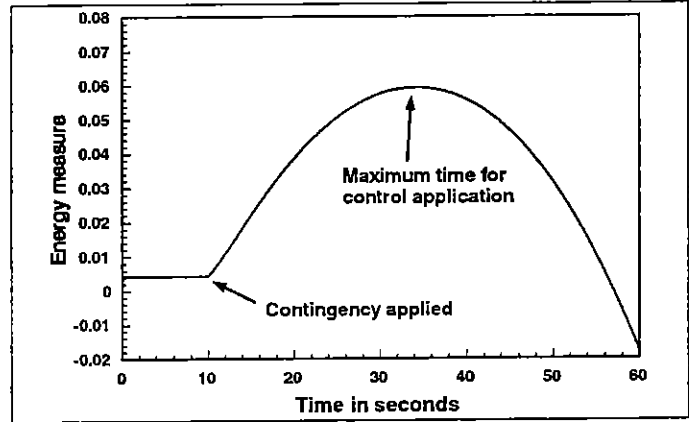


Figure 6 : Contingent System Energy with No Control

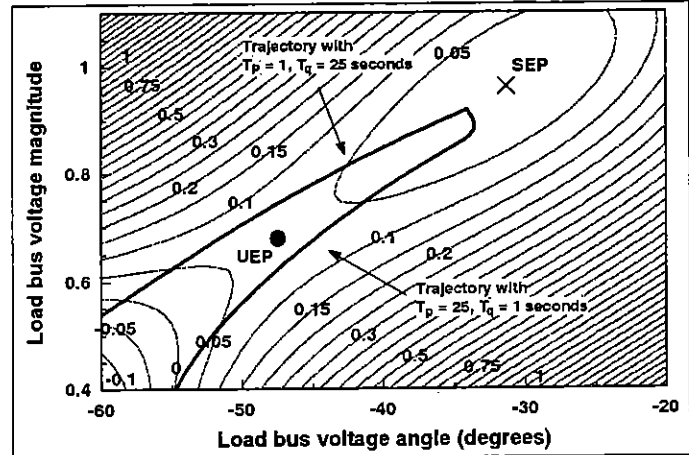


Figure 7 : Two Bus System Trajectories

The point at which the trajectory exits the region of attraction of the SEP is dependent of course on T_{pi} and T_{qi} . Figure 7 shows the two bus trajectories first for a case in which the reactive power responses much faster than the real power (lower trajectory in the figure), and then for the opposite case (upper trajectory). Note that the trajectories no longer exit through the UEP. Similar to the transient stability problem, there are two choices for the critical energy value: the energy associated with the controlling UEP [18], or the potential energy boundary surface method (PEBS) which determines the maximum potential energy along the uncontrolled system trajectory [19]. For the first trajectory the UEP method estimates a maximum control time of 85

seconds, while the PEBS method gives 127 seconds. The actual time is 115 seconds. For the second trajectory the UEP method estimates a maximum control time of 13.1 seconds, while the PEBS method gives 16.1 seconds. The actual for this case is 20.4 seconds. This discrepancy arises because the wide ratios in the time constants result in a situation where $\dot{\vartheta}(x)$ actually becomes positive slightly after the trajectory has passed through the PEBS.

To demonstrate the method on a system with more than a single UEP, consider the three bus system shown in Figure 8. For a contingency resulting in the loss of one of the lines from bus 2 to bus 3 the case becomes unsolvable with $\beta = 0.09$. One method to restore solvability is to switch in both of the 100 Mvar capacitor banks at buses 1 and 2. The post-control system then has the SEP solution shown in Table 2, along with three UEP solutions. UEPs A and B are type-one solutions, while UEP C is type-two. How quickly the capacitors must be switched depends upon the load model. Assume that the loads are modeled using the dynamic model from (9) with $a=0$, $b=0$, $\alpha=2$, $\beta=2$ at both buses.

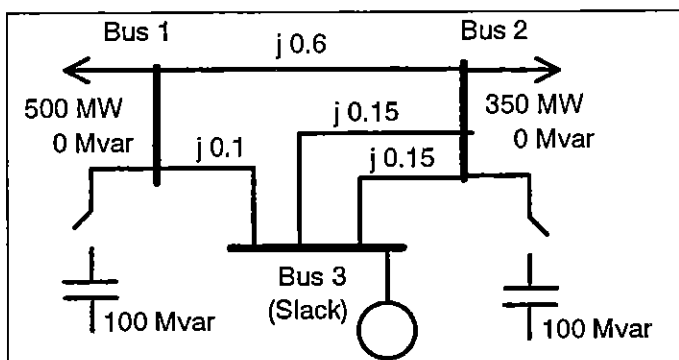


Figure 8: Three Bus System

Voltage	SEP	UEP A	UEP B	UEP C
V_1	0.952	0.560	0.747	0.574
θ_1	-31.8°	-61.0°	-44.5°	-59.7°
V_2	0.987	0.687	0.538	0.646
θ_2	-32.1°	-51.9°	-65.3°	-55.4°
$\dot{\vartheta}(x)$	--	0.297	0.274	0.297

Table 2: Three Bus System Post-Control Solutions

If $T_{p1} = T_{p2} = 2$ seconds, with no capacitor switching the system will experience voltage collapse along trajectory 1 shown in Figure 9. Note that this trajectory does not pass particularly close to either of the UEPs. Figure 10 shows the energy as a function of time. The maximum time to switch the capacitors can be estimated either from when the trajectory passes through the PEBS or when the trajectory energy is equal to that of the controlling UEP. The trajectory passes through the PEBS at about 90.5 seconds with an energy value of 0.289, which is very close to the actual maximum of 90.1 seconds (found using a series of time domain simulations). The controlling UEP could be

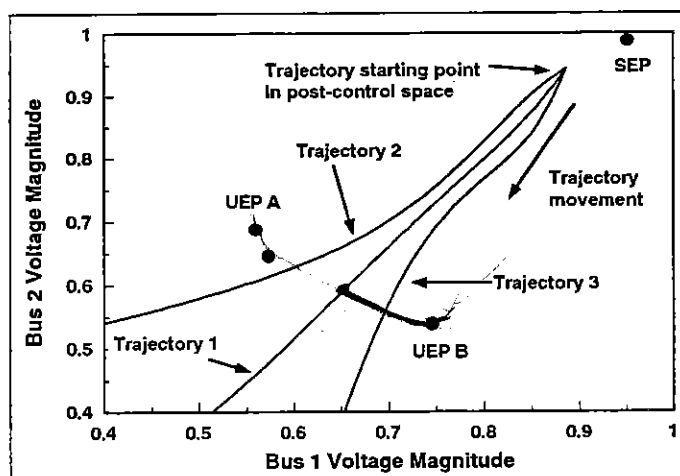


Figure 9: Three Bus Contingent System Trajectories

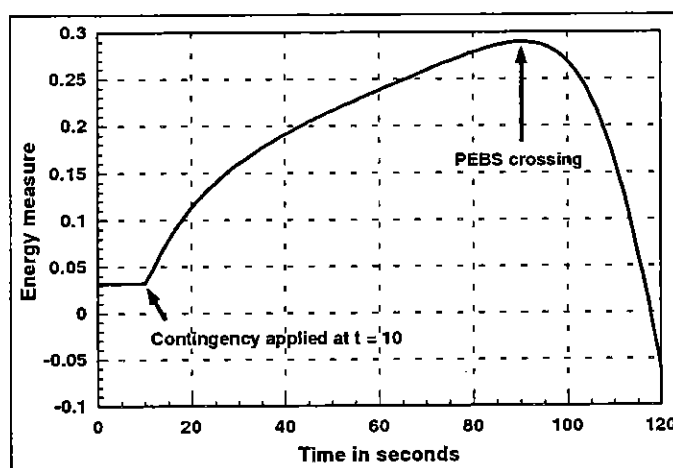


Figure 10: Trajectory 1 Energy

determined using a method similar to that of [20] of integrating the trajectory starting from the PEBS crossing point. It is also usually possible to determine this UEP by just initializing the power flow with the PEBS voltage values. Both methods indicate that UEP B is the controlling UEP. The trajectory energy value is equal to the UEP value at 77.5 seconds. Figure 11 shows the voltage recovery for a switching time slightly less than the maximum (90.0 seconds).

The dependence of the direction of voltage collapse on the load parameters is demonstrated in Figure 9 with trajectories 2 and 3. To get trajectory 2, T_{p1} was decreased to 1 second, while T_{p2} was increased to 5 seconds. The faster load recovery at bus 1 now causes that portion of the system to be more heavily loaded. Consequently, the trajectory crosses the PEBS in the vicinity of UEP A, which is now the controlling UEP. The PEBS energy of 0.297 is almost identical to that of the controlling UEP (0.296). This value is reached at about 127 seconds, which is also the actual maximum control time for this case. Conversely, for the trajectory 3 case T_{p1} was now increased to 5 second, while T_{p2} was decreased to 1

second. The faster load recovery at bus 2 now caused the trajectory to exit close to UEP B with an energy of 0.279 at 91 seconds while the controlling UEP energy level was reached at 89.8 seconds. This was also quite close to the actual of 90.8 seconds.

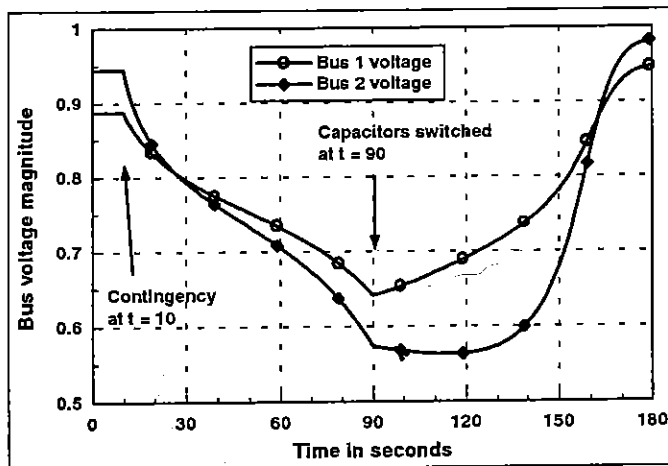


Figure 11: Three Bus Voltage Recovery

As a final example the method is demonstrated on the 118 bus case for the line 5-8 contingency. At a load of double the basecase value this contingency is unsolvable with $\beta = 0.0648$. One method (of last resort) to restore solvability is to shed load. Assume that 30 MW and 7.7 Mvar are to be shed at bus 3 (maintaining the original power factor of 0.969). The method from Section II indicates that, from a static point of view, this is sufficient to restore solvability. Again the dynamic question is to determine the maximum allowable time after the contingency to do this control. With the contingency occurring at $t = 10$ seconds, Figure 12 plots the energy of the system using the dynamic load models from (9) at a number of buses with $\alpha_i = \beta_i = 2$ and time constants T_{pi} & T_{qi} of 5 seconds. As was the case for the smaller systems, a sufficient condition for stability is that the load be shed before the system energy climbs to the value equal to that of the controlling UEP. For this case the UEP energy of 0.0357 is reached at 133 seconds, while the trajectory crosses the PEBS at 143 seconds with an energy of 0.0370. These values are quite close to the actual maximum of about 134.5 seconds. The controlling UEP was quickly determined by initializing a power flow solution using the PEBS crossing point values. The power flow had an initial maximum mismatch of just 0.02 per unit, and converged in 2 iterations.

The practical result of calculating the maximum allowable time frame for control application after a contingency is the determination of what controls can be used to mitigate the contingency. If the time frame is just a few seconds then either fast-acting automatic controls must be installed, or (more likely) system operation is constrained to avoid operating in that region. With a longer time frame, coupled with a plan detailing which controls to change should the contingency occur, effective intervention by the power system operator becomes a possibility. The advantage of the energy approach is that fairly good estimates of the maximum time for control application can be determined directly.

Furthermore, since it is straightforward to calculate the sensitivity of the controlling UEP to various control actions [21], the energy approach could also tell which controls would be most effective in increasing this time.

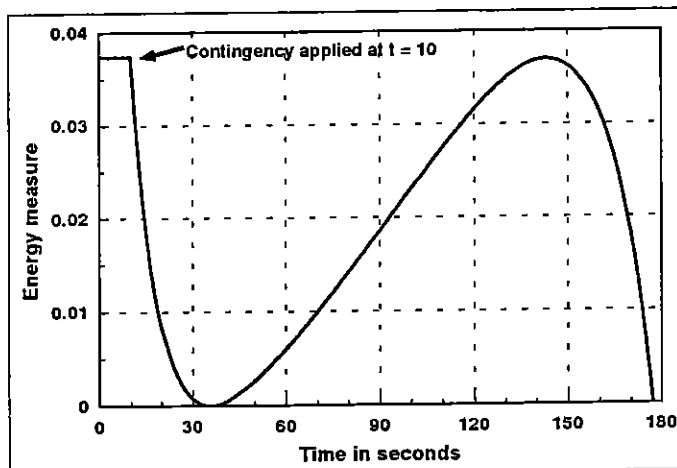


Figure 12: 118 Bus Contingency with No Control

IV. Conclusion

This paper addresses the practical problem of power flow cases which have no real solution. It has been shown that it is straightforward to statically determine a set of controls which could restore a case to solvability. However system dynamics must also be considered to ascertain whether the controls can be applied quickly enough following the contingency to avoid voltage collapse. The paper has shown that energy methods offer the potential for directly determining the maximum allowable time frame for control application. This application of energy methods is similar to their use for the transient stability problem, with the critical energy value determined using the controlling UEP concept. Future research includes further investigation of a composite energy function suitable for both voltage and transient stability assessment, along with more detailed dynamic models.

V. Acknowledgments

The author would like to acknowledge the support of NSF through its grant NSF ECS-9209570 and the Power Affiliates program of the University of Illinois at Urbana-Champaign.

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Discussion of papers presented by Thomas, Liu, and Overbye

Edited by Lester H. Fink, ECC, Inc.

Eyad H. Abed (University of Maryland):

Bob [Thomas] I enjoyed your talk. Being in the same building of at least two of the OGY names, Ott and Grebogi, I am familiar with their method. I don't think that their method is a special case of what Dr. Thomas presented, because in their method they turn off the control most of the time except when they're in the neighborhood of a periodic orbit or an equilibrium that they would like to converge to. I believe that your control is similar to the optimal aiming control strategy where the control is on all the time. So that is one question. The other question is regarding control of the saddle node bifurcation. I wasn't really clear on how your method addresses the saddle node bifurcation control problem: whether you are controlling after the saddle node bifurcation has occurred, just before or what.

Robert J. Thomas (Cornell University):

The methods, I think, are equivalent in the sense that in the ODS technique you get to choose the direction or preferred dynamics. Those dynamics can be along a stable trajectory of the saddle point. So as long as you are on that trajectory you will always get as a solution a zero control in the same way that the OGY control gets a zero control, and also you do that by the sort of angle condition between the velocity vector; you use that angle condition so I think it is equivalent. The performance indices are the same. It is a question of the choice of the dynamics, the direction dynamics in the ODS procedure. One thing I didn't mention because things went very quickly is that I really like the philosophy of the ODS control in the saddle node area because it is applying little bits of control for long periods of time, in order to keep the system kind of contained until you can do something in our context. In their context, I am not exactly sure about their philosophy regarding the periodic orbit or the physical problems to which they are applying it.

As to the second question, just after the saddle node bifurcation occurs, and you are into the dynamic region, you want to control back to the saddle point. It's kind of a holding strategy at the maximum power point, so that you can try to at least maneuver to that bifurcation point, back to that maximum power point.

Abed: So you are reintroducing an equilibrium once it has disappeared?

Thomas: I am going to reintroduce that saddle node - yes.

Constantine Vournas (National Technical Univ. of Athens):

Two simple questions - one is for the neural networks. How is this method applied in load identification? You have a special bus, you train the neural network, and then you have this ANN on all the time? - or how do you do it in different buses in case this is going to be applied? The other question is for the control of bifurcations - what are the available inputs which one can maneuver to change the equilibrium point?

Thomas: On the neural network, the idea is again this generic load that you would apply to get data at individual buses, and you would train the neural network about the disturbances that occur in the network. What the open issues are in all neural nets is whether they can take training data, or how much training data you actually need in the system so that the neural network can actually extrapolate beyond what it has already learned beyond the training set. Clearly, if you had all the data in the world, you could train the neural net to respond exactly to everything that happened at that bus or anywhere else. The issue is that you can't do that because

you can never get that kind of data, so there are some really interesting questions about how much data that you need, the richness of the data set, and how much training do you really need to do. Also, the training algorithms are a problem. If you start getting lots and lots and lots of data, it begins to take longer and longer to really get the networks trained well. So that is the first question.

I guess your second question is, what inputs are available to maneuver to reintroduce the saddle node. Discrete supplementary controls might be one - things like dynamic brakes; in the time frame we are talking about here, it would probably have to be network kinds of inputs.

Marija D. Ilic (Massachusetts Institute of Technology):

Bob [Thomas], I am very interested in your views on the coordination issues when you have multiple controls. On all of these really specialized control approaches, including sigma min and ODS strategies, what are the issues that you see, because in the closed loop you still end up with a nonlinear model.

Thomas: By coordination I assume you mean the systemwide issues, and those are tough ones. I tried to gloss over those in the beginning by saying that communication systems are getting better, and that sensors are getting better, and that we will be able to do computations very fast. What "very fast" means I am not exactly sure, but at least from where I am, in terms of the control algorithms themselves, you need to be able to do some fairly fast systemwide computations. Whether you can actually decentralize those computations is a real open issue and it is a good problem.

Christopher L DeMarco (Univ. of Wisconsin, Madison):

Chen-Ching [Liu], some of what you suggested, with control on a manifold or a lower dimensional space in the system, suggests some of the things that are used in sliding mode control. I'd be curious what comparisons you might offer with the sliding mode ideas.

Chen-Ching Liu (Univ. of Washington):

If you just look at the case with bounded control, then that control trajectory is indeed quite similar to sliding mode control. The big difference is that this foliation based control has the manifold theory behind it. While they use the sliding mode controllers in our end user space, it is much easier to work on a manifold if you go with the foliation base.

Marija D. Ilic:

I have several questions [for Tom Overbye] on the static approach under the assumption that there are no dynamic problems. I am just talking about the first part of the problem. The first thing I would like to mention is that around 1990, I don't know the exact issue in the IEEE PAS, Stobart published a very similar approach, and that approach has been carried out for New England utilities and is now at the implementation level. In light of that part that has been done, I have a specific question. The performance criteria that you are using for going from the S to F(x) has quadratic form, a sort of scalar product of mismatches, so it's one number. The Stobart approach uses something that is like a vector of all the mismatches at bus loads, and the performance criteria was to minimize the largest deviation as you apply the control so that you have the largest mismatch at that bus. When developing Stobart's vector criterion, we compared it to the scalar one used by you, and it really turns out that the vector criterion is, by an order of magnitude, less conserva-

tive (we can talk about it a little bit more). Next, I am puzzled by your comment with regard to using the developed method only when a load flow solution does not exist. Why? Conceptually, if you have the voltage deviations from whatever the optimal is, I think you can use it so that you don't run yourself actually into nonexistence of the solution. This is what is being done now at REMVEC rather than waiting until you really run into the problem, and so you activate the controls as you need them.

Thomas J. Overbye (Univ. of Illinois at Urbana-Champaign):

I would certainly agree that if you know that you are coming into a problem that you would activate the controls ahead of time. What I was looking at is, how do you apply controls if your system all of a sudden becomes unsolvable, or a contingency causes it to be unsolvable? I differentiate that because a lot of people have done a lot of work with working inside a sigma - how do we move away from the sigma? There are lots of methods for doing that but I haven't seen any work in moving from the unsolvable back to solvability, and certainly when you use controls you don't just want to move your controls to get you right back to sigma - that would be silly. You'd keep going to get away from the boundary.

Ilic: Just as a practical matter I don't think you will see too many operators waiting until that happens. They'll do things as they approach that area.

Nelson Martins (CEPEL):

Tom, congratulations for using u , the vector of practical controls rather than a full eigenvector. Vectors should, however, not be forgotten. I'll not let this happen, you know. Actually, we have used part of an augmented eigenvector, and so you have the optimal control vector direction within this augmented eigenvector. So we are still in business there, and you can put as many elements, as many inputs as you wish. This is very good for optimal control - you can specify the control action.

Overbye: I should mention that the calculation of sensitivities is very fast since the Jacobean is sparse; the eigenvector is a full vector, but you multiply it by a sparse matrix.

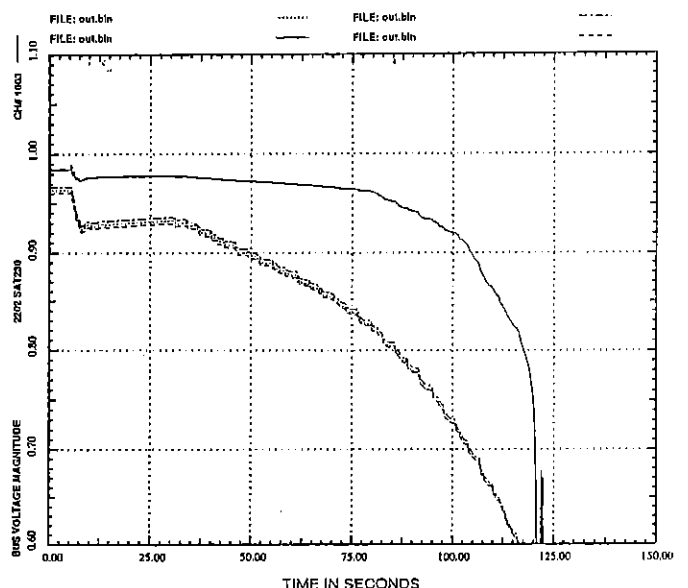
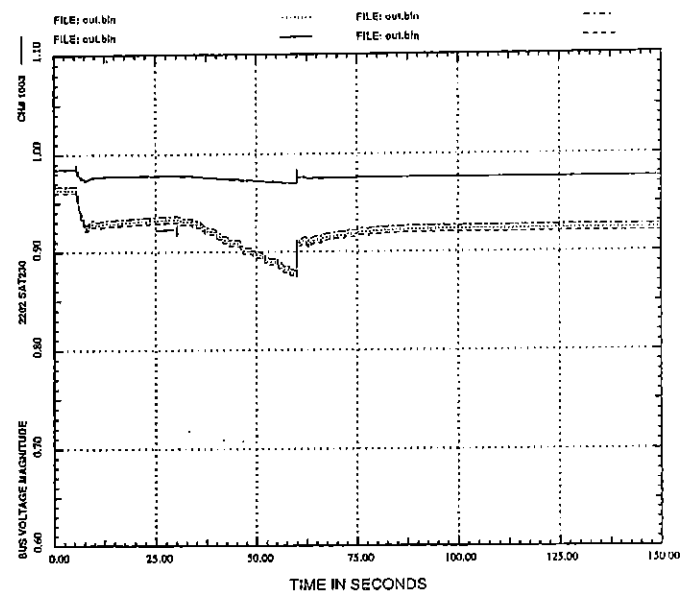
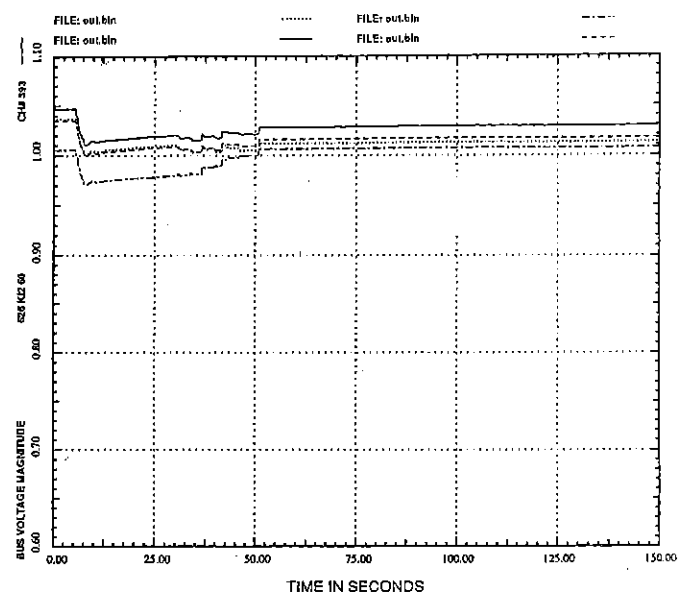
Yakout Mansour (BC Hydro):

First of all, by the way Maria [Ilic], you usually run into these cases that he [Overbye] was talking about in a study mode. You don't necessarily run into unsolvable cases in the status quo, but sometimes you study contingencies, and then it becomes unsolvable.

I just want to mention a point to show the significance of what these gentleman were talking about - to show that, even though the "Illinois" system is only 2 buses, systems that have about 4,000 times as many as Illinois, like ours, have the same concept applied, so the "Illinois" effect goes all the way to the far West. [FIGURE] These are 3 cases with an outage, where a different level or different margins as you load the system up, get to the point where it is referred to as a saddle point bifurcation. I showed in the morning the full dynamic analysis of that large system where, if you don't do anything, you do collapse. Then at some time we switch something, or take a preventive or corrective measure like

Case	Description	No outage margin (MVA)	5L42 outage margin (MVA)
1	Modified basecase	1000	408
2	LM Load incread by 6%	454	25
3	LM load increased by 10%	217	negative

Three Base Cases for Simulation



Tom was talking about; in this case it happened to be a capacitor. In this case we have full dynamic load models like they were talking about, so there is everything in there (now I don't want to give the impression that we have solved everything). The piece that is missing here is what those people are talking about. This thing was determined by trial and error. As a matter of fact, as I came here, two of my people were working on determining the strategy on the timing and the magnitude of this, but I wanted to show in concept that, yes, slow controllers could work and they could make a correction, and you could get away from bifurcation. The importance of your work is well appreciated and this is where it fits in a system larger than Illinois.

Fernando Alvarado (Univ. of Wisconsin, Madison):

I have seen a practical method for obtaining solutions in unsolvable cases. It is simply to declare that no voltage should go below 0.9, and add as much Q as necessary to hold that. I have seen people do that as a practical way of obtaining something you are doing. If 0.9 doesn't work, you can use 0.91, or 0.8, or whatever.

Overbye: Another approach is that you can convert all your loads from constant power to constant impedance when you go below some voltage level. People have used these approaches. I'd have to think about it more to know exactly what is going on there, but I think that what you would want to do is to actually come up with some nice norm, such as minimum distance from the boundary, to quantify that, rather than arbitrarily adding some reactive power somewhere. I don't know the details so I cannot really comment.

Chen-Ching Liu:

This is part 2 of the answer to Chris's [DeMarco] question earlier, I hope you don't mind. I think the unbounded cases are really different from sliding mode control. You see, that piece of pie that comes with every region is actually spanned by radial lines; the way it is constructed is to start by showing you can move from one line to another line via any finite number of lines in between, and it does not have to be a connected trajectory. That is really very unusual to control theory concepts, but then somehow you add unbounded control and you allow a jump, then you will be able to get the complete control.

Discussion of Overbye's paper submitted subsequently by Alvarado

This paper addresses in a formal manner an important problem. It is, in fact, a strong desire of all to replace "unsolvability" with "conditions for solvability." That is, no power flow would be unsolvable. For "unsolvable" cases distances to a solvable case would be used. In practice, of course, experienced engineers always do this. However, this paper deals with a method for automating this notion.

Quite recently, a student of this discussor studied the same problem and came up with a simple but slightly different approach to its solution [1]. The method amounts to the following:

Reduce loads and generators to the origin.

Expand load in a direction to restore original load until the point of collapse is reached. This is point 1 in the Figure.

A more interesting variant is to also try to find a minimum L^1 norm distance. Such a distance can be found from the following algorithm:

Reduce loads and generators to the origin.

Expand load in a direction to restore original load until the point of collapse is reached. This is point 1 in the Figure.

From the left eigenvector at this point, determine the direction normal to the boundary. This direction defines a hyperplane. Its largest direction cosine also identifies a location for most effective load shedding.

From the intended operating point, reduce demand in this most effective direction until it intersects the hyperplane defined above.

This new point defines a new direction for demand increase. Starting once again at the origin, determine the new point of collapse in this new direction. This is point 2 in the Figure.

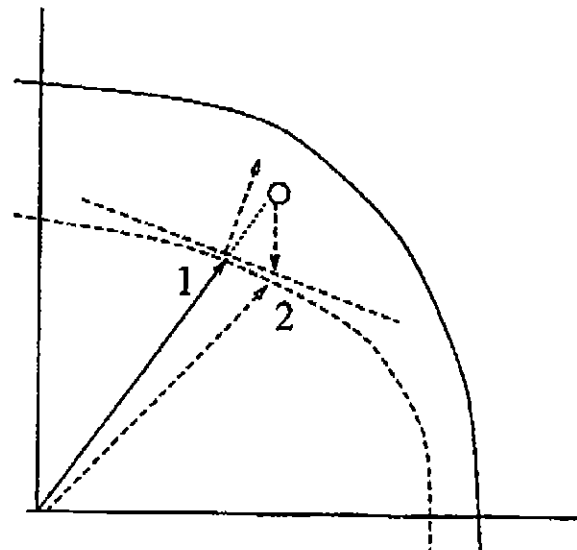
The last few steps can be repeated to refine the solution further. Several other variants are possible.

Reference

J. Wallace: Megawatt (MW) Marginal Limits to a Secure Power System; Master's Thesis, Department of Electrical and Computer Engineering, The University of Wisconsin, Madison, Wisconsin, May 1994.

Closure by Overbye

I would like to thank the discussor for his interest in the paper. Reducing loads and generators to the origin and then expanding load (and generation, I assume) will give a boundary point, although it will not (generically) be the closest boundary point using either a Euclidian or L^1 norm. I would agree with the discussor that it would be interesting to reformulate the problem using an L^1 norm. The Euclidian norm was originally chosen for quantifying the degree of unsolvability both because its implementation is straightforward, and because distance is usually measured using a Euclidian norm. Since the "closest" boundary point is dependent upon the norm employed, this point could also change, although probably not substantially in realistic problems. The challenge would be to determine this point for a non-Euclidian norm. Note that the actual change required in a particular set of controls to return a case to solvability is independent of the norm employed. Since the selection of this set of controls also includes practical considerations such as cost and available control range, I suspect the recommended controls to return a case to solvability would probably be fairly independent of the type of norm. However, this is an area for additional investigation.



Effective and economic improvement of voltage stability by a SMES

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Abstract

There is an increasing interest in applying superconducting magnetic energy storage (SMES) within electric energy systems. This paper deals with SMES application for the provision and activation of seconds reserve power and the improvement of voltage quality. A system engineering approach is used for optimizing and evaluating the SMES locations and control concepts for enhancing short and mid term dynamics. The results are illustrated by means of a 2 GW modell power system within an interconnected system.

Introduction

Recent trends with regard to economic and reliable supply of electrical energy are:

- necessity of an improved quality of voltage and frequency in the expanding interconnected power systems,
- constraints with respect to the expansion of power plants and transmission lines,
- an increasing demand for a more efficient use of primary energy in an ecological desirable way.

These trends require fast power changes due to control actions. These requirements may be met by energy storage systems. A new approach in this field caused by developements of superconducting materials, deals with the use of superconducting magnetic energy storages (SMES). SMES may be characterized by high current and fast rate of changes.

A SMES stores electrical energy within the magnetic field of a superconducting coil. The stored energy is given by the integral over the volume of the field distribution according to:

$$W_{SM} = \int_{V_{SM}} B_{SM} H_{SM} dV = \frac{1}{2} L_{SM} I_{SM}^2 \quad (1)$$

The principle of the SMES-construction independent of operational specifications can be represented by the block diagram shown in figure 1.

The grid voltage is transformed by a transformer into a large secondary-current. The connected converter works according to the operating mode as rectifier or inverter. By means of gate-turn-off thyristors (GTO) for connecting the SMES with

the network it is possible to control active and reactive power simultaneously and independently from each other. In this case there is no need for a reactive power compensation. According to the voltage adjusted by means of the firing angles of the thyristors at the converter the superconducting magnetic coil (SC-coil) will be charged or discharged. The firing angles are set by a bridge circuit (SCR firing circuit), which gets the rated value of a controller. The cooling system guarantees the required operating temperature of the SC-coil (4 K). In the case of a quench - the uncontrolled change of the SC-coil from superconducting to normalconducting state - a coil protection ensures the discharge of the SC-coil with maximal speed to prevent the destruction of the system.

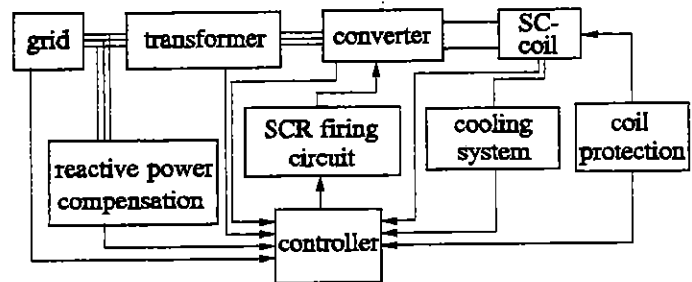


Fig. 1: Block diagram of a SMES-system

Depending on the different power system operational requirements the use of SMES with a capacity in the range of a few MWh as well as the use of SMES with a capacity in the range of about 1000 MWh ... 5000 MWh is discussed [1].

With respect to the possibilities of SMES the following requirements may be met by SMES with a low or medium storage capacity:

- allocation of seconds reserve power,
- improving the dynamic and transient stability because of the immediate allocation of synchronising resp. damping power by the SMES,
- an improved voltage quality by means of reactive power compensation of the converter,
- life-time prolongation of the components of primary controlled power plants by smoothing the operation of the primary control,
- damping of low frequency power oscillations ($f \leq 1$ Hz) on the transmission lines,
- increasing of the maximal stable transmission of load with long transmission lines.

An additional feature of a large SMES is:

- smoothing of the daily load-level by consuming and providing of electrical energy with very high efficiency.

This paper only deals with SMES with a capacity of a few MWh according to the requirements mentioned before, because the operation of such units seems to be technological and economical reasonable at the near future.

Because of the allocation of power within a few milliseconds, the use of SMES is suitable to cover the demanded increase of load according to short-term load variations for a certain period of time. With the example of a 10 GW network within the UCPT-Interconnected network it is shown in [2] that the SMES is of economical interest for the allocation of seconds reserve power. The primary control can operate without throttling the control- and step-valve, because of the active power allocation of the SMES taking place until it is replaced by secondary control (minutes reserve). The degression of costs with increasing scale of the SMES suggests the economic dimensioning of SMES by installing the seconds reserve power centrally in the network using one or two SMES. Further increase of economical attractiveness can be achieved by using the SMES simultaneously for primary control as well as for ensuring the voltage stability.

The conception of the storage control design is essential for the performance of the mentioned requests. The specification and illustration of the procedure for SMES-installation occurs according to the special consideration of an improved voltage stability.

Requirements on SMES with respect to voltage control

Depending on the load situation, the topology and the concept of power system control the phenomenon of voltage collapse may occur during power system operation. Especially in a heavily loaded system a small increase of the load can yield an undue voltage level if the nearest generators can not allocate enough reactive power because of its operational limits and reactive power must be transported over long distances. This power transportation leads to a certain transmission line voltage drop. Thus the decrease of voltage will be spread over the entire power system. Due to this decrease of voltage the provision of reactive power from the transmission line capacities is reduced. The reactive power deficit may be further increased by the action of the tap changing transformers. Thus a period of slowly decreasing voltage is followed by an accelerating voltage collapse because of the inability of the power system to supply the reactive power demand.

In steady state operation voltage stability can be achieved until the bus load P_i meets the so called critical load $P_{i,crit}$. If the

entire power system is modelled by its Thevenin equivalent (U_{Th} , x_{Th}) with respect to bus i , the critical load $P_{i,crit}$ can be calculated according to formula (2).

$$P_{i,crit} = \frac{U_{Th}^2}{2 |x_{Th}|} (\sqrt{\tan^2 \phi_i + 1} - \tan \phi_i) \quad (2)$$

In formula (2) ϕ_i characterizes the load angle of the complex bus load S_i .

In contrast to active power allocation, which is limited by the SMES-capacity, the allocation of reactive power is not limited in time nor in quantity. Thus the SMES may compensate for the reactive load Q_i and hence decrease the load angle ϕ_i (2). Due to this voltage stability can be achieved even if the load P_i increases.

Concerning the dynamic and transient voltage stability the reactive power control of the SMES shall influence the course of voltage of the system after a switching event or a fault. Therefore the excitation control will be relieved i.e. its operational range will be expanded. The disadvantages of a very fast excitation control can be avoided by this and the requirements for the excitation control can be reduced. Hence a greater power factor may be possible.

Especially for faults within the network combined with great voltage drops the SMES can support the voltage-level, which reduces indirectly by the voltage dependent loads the unbalance between power generation and consumption.

Large gain factors and short response times of the electronic excitation systems used today can deteriorate the stability of the whole system. In this context a power system stabilizer (PSS) has the function to build a damping torque by an additional input signal for the voltage controller. The feasible damping of the rotor oscillations by the use of a PSS is in opposite to the quality of the voltage control. It is possible using the SMES to reduce these oscillations directly by charging resp. discharging of active power by the SMES. Because of the very short access time for the charged energy the SMES active power control is predestined to produce phase swinging damping torques. Within this direct influence on the torque balance the reaction on the voltage control is minimal.

Integration of SMES into power system control

The system engineering approach for integration of SMES into the power system control is applied to the model power system shown in figure 2.

Due to the interconnected operation a certain amount of seconds reserve power has to be allocated within the model power system. Depending on the complex power S_1 a station-

ary reactive power compensation at this bus may prevent voltage collapse. On the other hand dynamic and transient stability problems can arise according to the entire system load P_1 and the contribution of the interconnected system in the short circuit power $S_{SC,IS}$ at the bus where the modell power system is connected to the interconnected system.

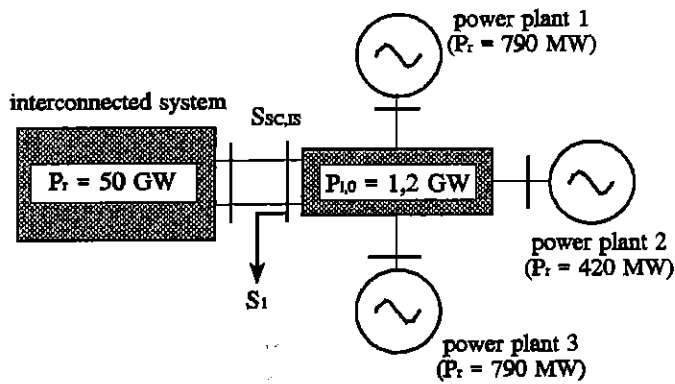


Fig. 2: Modell power system with a load of $P_{I,0} = 1,2$ GW in the border system

The use of SMES within the modell power system shown in figure 2 shall be illustrated starting from the economical providing and allocation of seconds reserve power, the damping of electromechanical oscillations and especially the improvement of voltage stability. Starting from these demands the dimensioning and the location of the SMES will be determined and the control scheme will be designed.

Dimensioning of SMES

Within the border system shown in figure 2 the seconds reserve power is only provided by SMES taking into account the recommendations of the DVG /3/. From this the energy of the SMES immediately yields $W_{SM,max} = 11$ MWh and the maximal power is $P_{SM,max} = 100$ MW. In a first approach the converter is to be dimensioned for providing $|Q_{SM}| = |P_{SM}|$ even if $P_{SM} = P_{SM,max}$. This yields $S_{SM,max} = 141$ MVA. It must be emphasized that a SMES with such an oversized converter with regard to providing of seconds reserve power is still of high economic attractiveness.

Synthesis of SMES control scheme

The SMES control has to provide the seconds reserve power as well as to improve the voltage quality. Whereas for allocation of the seconds reserve power the SMES is to be adapted to the other primary controllers within the power system, for the improvement of voltage stability a suitable allocation of reactive power is necessary. Therefore an event driven control scheme according to figure 3 yields an optimal operation /6/.

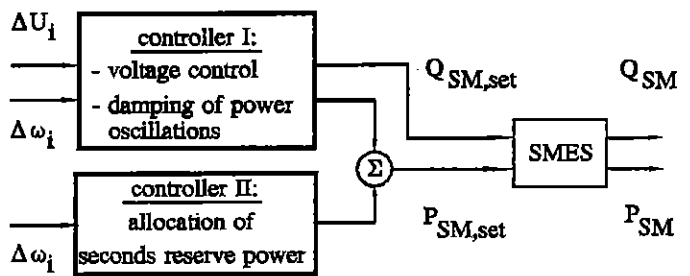


Fig. 3: Event-driven SMES control scheme

Load changes are combined with low-frequency or aperiodic changes of system frequency. In this case the system frequency is a useful input signal for the SMES-controller I shown in figure 3, whereas the reactive power control is characterized by the voltage U_i at the bus where the SMES is connected to the power system. With respect to a high economic attractiveness the SMES controller I includes also an active power control loop for damping of electromechanical oscillations (input signal: $\Delta\omega_G$, $P_{acc}/5$) but the examination of this control loop is not the subject of this paper.

So that a SMES can provide a predefined part of the seconds reserve power according to the agreements of the interconnection partners it is to be adapted with a suitable control to the behaviour of the other primary controllers used in the interconnected system. With respect to the coordinated provision of seconds reserve power by the interconnected network and the SMES according to the DVG-recommendations the SMES-controller I is realized by means of a proportional controller. The gain factor arises from the provision of the maximal power $P_{SMES,max}$ of the SMES in the moment of the loss of the biggest plant within the system on the one hand and by the droop of the other primary controllers within the system on the other hand.

With respect to the realization of the SMES-controller II the synthesis of the controller based on the linear system theory limits the desired effect of control to a bounded area around the system operating point. Furthermore the use of the linear system theory needs a detailed mathematical description of the complex energy systems. In this case the use of a control scheme based on the fuzzy set theory enables an event driven voltage control by the SMES adapted to the requirements of the actual system state.

The fuzzy controller represents a nonlinear mapping from the input to the output variables. The performance is established by rules. In contrast to a mathematical model of the controlled system knowledge of experts is used for the modelling of input/output characteristic. The fuzzy logic which allows probable values within the intervall $[0;1]$ enables a quantitative assessment of the linguistic attributes. By this the transitions between different reactions of the controller can be smoothly

designed avoiding discontinuous input/ output characteristics. The general structure of a fuzzy controller is displayed in figure 4.

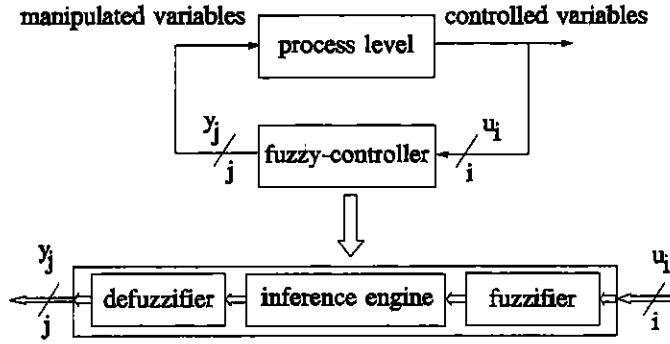


Fig. 4: General structure of a fuzzy controller

The fuzzyfier maps the numerical values of the measured quantities according to their degree of membership to the linguistic attributes called fuzzy sets by means of nonlinear membership functions. The inference process evaluates the linguistic rules and determines a linguistic description of the output variables from which a numerical value has to be derived for the control action by means of the defuzzifier. The fuzzifier, the inference and the defuzzifier will be designed in the time domain of the controlled system.

With regard to an appropriate reaction of the SMES reactive power control for the damping of voltage oscillations the input signal ΔU_i and the output signal $Q_{SM,set}$ are to be described by means of fuzzy sets like:

negative:	positive:
large, medium, small;	small, medium, large

With regard to a continuous course of the output signal $Q_{SM,set}$ the fuzzy sets of the input signal ΔU_i are to be realized with equal absolute values of the edge steepness of neighbored sets.

The use of only one input signal for the reactive power control causes the number of fuzzy sets of the output signal $Q_{SM,set}$ equal to the number of the sets of the input signal ΔU_i . The centers of gravity of the fuzzy sets of the output signal with a membership of $\mu = 1$ are defined with consideration of the dimensioning of the converter as:

- $|Q_{SM,set}(\mu_{small}=1)|=10 \text{ MVar},$
- $|Q_{SM,set}(\mu_{medium}=1)|=50 \text{ MVar},$
- $|Q_{SM,set}(\mu_{large}=1)|=100 \text{ MVar}.$

The reactive power allocation of the SMES is controlled by the relating of similar named fuzzy sets of the input signal ΔU_i and the output signal $Q_{SM,set}$ within the inference, like: IF (ΔU_i =negative large) THEN ($Q_{SM,set}$ =negative large).

Location of SMES and influence on voltage stability

Concerning the economical installation the SMES location must be determined with regard to providing seconds reserve power and maximal improvement of voltage stability and voltage quality. Because of the existing transmission capacities within electrical supply systems the providing of seconds reserve power causes no restrictions to the SMES-location. Thus the SMES location can be exclusively determined with respect to an improvement of voltage stability and voltage quality.

First the use of SMES for increasing the critical load $P_{1,crit}$ is examined. Within the power system shown in figure 2 the bus load S_1 should be increased without violating the voltage stability. With respect to an optimal compensation of the reactive load Q_1 the SMES has to be located at bus 1. The entire power system is modelled by its Thevenin equivalent with respect to bus 1. From figure 5 the plot of the bus voltage U_1 depending on the load P_1 ($\phi_1=\text{const.}=30^\circ$) can be obtained.

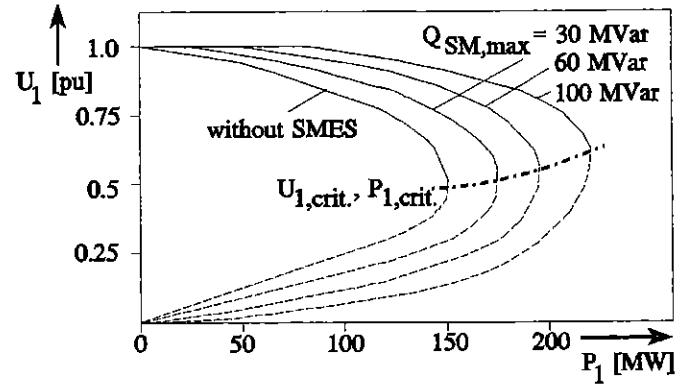


Fig. 5: Critical load $P_{1,crit}$ depending on SMES reactive power Q_{SM}

Figure 5 illustrates that for an increasing load P_1 the bus voltage U_1 decreases until a critical state, which can be characterized by $U_{1,crit}$ and $P_{1,crit}$. By the use of a SMES dimensioned above ($Q_{SM,max}=100 \text{ MVar}$) it becomes possible to increase $P_{1,crit}$ significantly. Even if the rated power $S_{SM,max}$ of the converter unit is decreased ($P_{SM,max}=\text{const.}=100 \text{ MW}$; $Q_{SM,max}=60 \text{ MVar}$ resp. $Q_{SM,max}=30 \text{ MVar}$) with respect to economical reasons the shifting of $P_{1,crit}$ is of a considerable amount.

In a second case the power plant number 2 in figure 2 shall be operated with a large power factor. Due to this a SMES ($P_{SM,max}=100 \text{ MW}$; $Q_{SM,max}=100 \text{ MVar}$) located at power plant 2 should relieve the excitation control of this power plant.

Referring to the damping of power and hence rotor oscillations the SMES active power control as well as the PSS have influence on the system-eigenvalues correlated with these oscillations. In the case of a simultaneous use of PSS and

SMES at the same power plant it is possible that the fast voltage control of the SMES reactive power control compensates for the voltage modulation of the generator terminal voltage by the PSS. With respect to the advantages of the four-quadrant-operation of SMES the simultaneous use of SMES and PSS at power plant 2 will not be examined.

In order to study the impact of the SMES on voltage stability and voltage quality the dynamic system performance after a three phase short circuit on a double line ($T_{SC} = 0.18$ s) with the following fault clearance by switching off the fault line will be analysed. Caused by the choice of the fault position there are transient phenomena of a considerable amount, which have to be controlled by the power system control. A steady operation point with a relative low static stability is used representing a peak load situation of the modell power system $P_1 = 1,5 P_{1,0}$ and a relative weak link to the interconnected power system ($S_{SC,IS} = 2$ GVA). Figure 6 shows the course of the terminal voltage U_{G2} of generator 2 as an example.

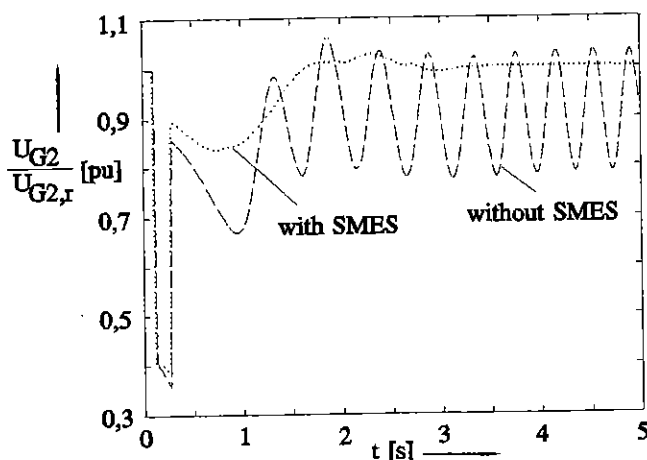


Fig. 6: Terminal voltage U_{G2} of generator 2 as a result of a three phase short circuit

It can be obtained from figure 6, that without the use of SMES the modell power transient instability occurs after the three phase short circuit with respect to the investigated operation point. During the short circuit time T_{SC} occurs a large decrease of the terminal voltage because of a relative weak coupling to the interconnected system. Thus the load demand $P_1 \cdot (U/U_0)^2$ of the voltage dependent loads is reduced. With respect to the investigated peak load situation and the constant driving torques of the generators such an acceleration of the turbogenerator set occurs, that after switching off the fault line the synchronism is not to be restored and the system becomes unstable.

In this operation point the additional destabilizing effect of the PSS in the transient time range is not essential for the unstable system behaviour. Therefore the only switching off of the PSS

do not guarantee the stabilization of the system in this special situation.

During the fault time T_{SC} the SMES reduces the longitudinal voltage caused by the large reactive part of the short circuit current by allocating inductive reactive power Q_{SM} and with this already supports the voltage during the fault time. In the case of this voltage support the SMES further reduces mediatly by the voltage dependent loads the unbalance between power generation and consumption. Following the switching off of the fault line the SMES damped the voltage oscillations by the allocation and charging of reactive power Q_{SM} .

It should be mentioned, that the SMES-active power control loop, which is not under examination within this paper, is working for a immediate reduction of the unbalance between power generation and consumption. Thus the SMES reactive control loop as well as the SMES active power control loop is working for a stabilization of the modell power system.

With regard to the permanent variation of the parameters and changes of network topology caused by switchings within an electrical supply system and in addition with a maximal use of capacity of existing generators and transmission lines as well as with regard to constraints of planning of interconnections, for use of SMES within the modell power system the realization of the SMES voltage control loop will be investigated referred to the robustness of control quality depending on system load P_1 and the contribution of the interconnected system in the short circuit power $S_{SC,IS}$ at the bus where the modell power system is connected to the interconnected system.

Figure 7 shows for the assessment of the transient stability and the voltage quality with the example of generator 2 the marked in time, squared and integrated deviation of the terminal voltage ΔU_{G2} from the steady operation point (ITAE-criterion) for the described three phase fault dependent on $P_1/P_{1,0}$ and $S_{SC,IS}$.

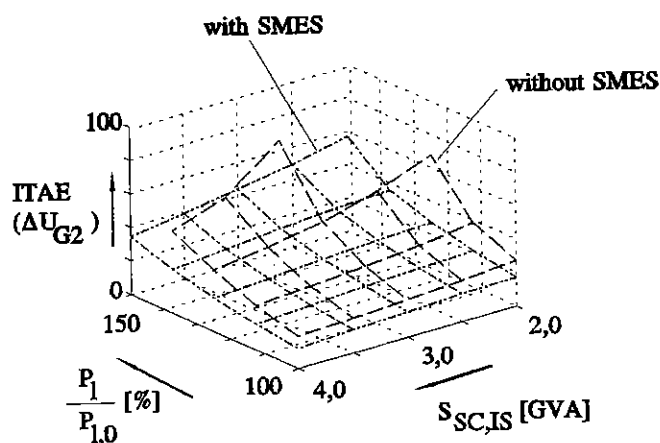


Fig. 7: ITAE-criterion of the deviation ΔU_{G2} of the terminal voltage of generator 2

Figure 7 illustrates that by means of SMES the use of generator and transmission capacities can be increased without instabilities even in the case of largest assuming system fault. Moreover the use of SMES allows larger degrees of freedom for planning the interconnection coupling of the border system. In this case the requirements resulting of stability aspects for the short circuit power $S_{SC,IS}$ at the connecting nodes in the interconnected system can be reduced. Apart from this in the case of stable system operation without the use of SMES the application of SMES may lead to an increase of voltage quality.

Conclusions

Through the use of SMES it becomes possible to meet requirements and quality improvements which cannot be realized by means of conventional techniques. Starting with the specification of the requirements on the SMES a system engineering approach is used, which considers these requirements from the beginning of the design. Starting from this the dimensioning, the synthesis of the controller and the location of the SMES is described.

The allocation of seconds reserve power by SMES is of economic interest and therefore a criterion for the dimensioning of the SMES. The economic attractiveness of the SMES can be increased by adding further tasks to it within the system. Because of this an event driven control scheme is represented which enables the coordinated allocation of the seconds reserve power by SMES and conventional primary controlled power plants as well as the improvement of voltage stability and voltage quality by sufficient allocation of reactive power by the SMES.

The design of the SMES voltage control based on the fuzzy set theory shows robust stability properties, since its knowledge base takes the actual operating conditions into consideration. Therefore a high insensitivity against topological changes is achieved.

Within this paper the use of SMES is examined for both at a load bus as well as near a generator. Installing the SMES within a load centre the emergency of a voltage collapse can be avoided due to an increase of the critical load $P_{1,crit}$. Using the SMES in the vicinity of a generator the dynamic voltage performance can be improved and transient instabilities may not occur.

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Transmission Line Impedance Control by VIPS Apparatus for Static Voltage Stability Enhancement

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Abstract

This paper presents new static control methods using stabilizing switching of transmission lines and variable series capacitor (VSRc) for enhancing static voltage stability. The proposed method based on OPF with minimization of total loss of reactive power over the whole power system can find an optimum network configuration or optimum compensation levels of multiple VSRcs for a given operating condition. Numerical results are shown for model power systems.

1 Introduction

Since new technology such as synchronized switching and power electronics makes it possible to change the configuration of transmission network, the impedance of transmission line and the phase angle of voltage in the future power systems [1-4], it will be very important to design comprehensive control schemes of such variable impedance apparatus with multi-purpose stabilizing functions. Possibilities are under study from various viewpoints in Japan to introduce such power system series apparatus at long transmission lines or tie lines.

New control methods of variable series capacitor (VSRc), high speed phase shifter (HSPS) and stabilizing switching of transmission lines for the first swing stability and damping enhancement have been investigated from a dynamic viewpoint by the authors [5-7]. The power system incorporating dynamic control of VSRc, HSPS and stabilizing switching for the main purpose of power system stability improvement is called Variable Impedance Power System (VIPS) by the authors.

In this paper, as one of the multiple purposes, from a static viewpoint, a new control scheme using stabilizing switching of transmission lines or VSRc is presented for enhancing static voltage stability, which means to find an optimum network configuration and optimum compensation levels of multiple VSRcs for a given operating condition.

First, it is discussed that total loss of reactive power is closely related to the well-known active power margin of P-V curve in terms of the static voltage stability. Second, the optimum operating conditions are obtained

by solving an optimization problem where the total loss of reactive power is minimized subject to the upper and lower limits of each and total VSRc capacities in addition to constraints in a usual OPF problem. Then, the optimality is checked by using P-V curve. Numerical examples for 4-bus and 11-bus model systems are given to show the effectiveness of the proposed control scheme.

2 Static Voltage Stability Index

2.1 Load Demand Margin

In this paper, static voltage stability is tested by total load demand margin between the current operating point and the critical load flow point for a scenario of load demand increase as shown in P-V curve of Fig.1. It is well known that the larger the margin becomes, the more stable the voltage is judged to be. Therefore, the static voltage stability here is not influenced by dynamic characteristics of generators, loads and control systems but only by specified load flow conditions and network parameters such as line impedance and susceptance. The critical load flow point required for checking the static voltage stability of the present operating condition is obtained by multiple load flow calculations under the following assumptions [8, 9]:

1. Power factor at each bus is fixed to be 0.95 independently of load demand.
2. Load demand at each load bus increases at the same (percentage) rate.
3. Active output power at each generator also increases at the same (percentage) rate except at a slack generator. The increment of transmission loss is absorbed by the slack generator.
4. Bus voltage of each generator is kept constant.
5. The installed capacity of shunt capacitor at each load bus is also kept constant.

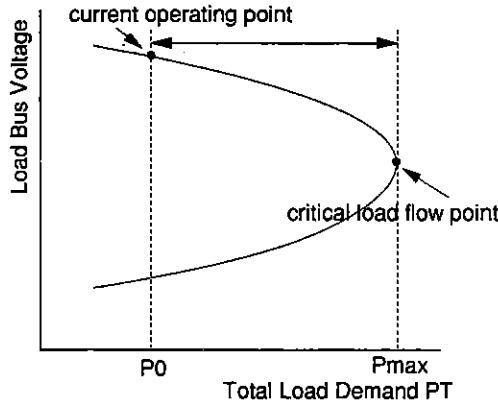


Figure. 1: P-V curve

2.2 Reactive Power Loss

Since the voltage stability is strongly related to reactive power balance as well as active power, the total loss of reactive power in the network can be an assessment index of the static voltage stability. In [10], the following index using the total reactive power loss was proposed:

$$L_Q = \frac{\sum Q_{loss}}{P_{total}} \quad (1)$$

where Q_{loss} [MVar] is reactive power loss of a transmission line or a transformer, and P_{total} [MW] is the total load demand.

In this study, the total loss of reactive power is minimized by changing the transmission line impedance and the network configuration using VSrC or stabilizing switching. In our previous paper [11], we proposed a preventive control method for the static voltage stability enhancement, where the critical load flow is maximized directly by changing tap ratios, shunt capacitors and generator bus voltages. As a problem to be solved in the above preventive control scheme becomes more complicated if the line impedance control is taken into account, the proposed optimization technique based on the conventional OPF is adopted. Here, it is noted that the results obtained by minimizing the total reactive power loss need to be evaluated by the static voltage stability index of the total load demand margin.

3 Minimization of Total Reactive Power Loss

3.1 Optimal Power Flow without Impedance Control

In this subsection, a method for determining an optimum operating condition under a specified load flow condition at generator and load buses without taking into account transmission line impedance control is described. The optimum operating condition is defined as

state variables such as the magnitude and the phase angle of voltage and control variables such as the capacity of shunt capacitor, the tap ratio of transformer and the generator bus voltage which minimize the total loss of reactive power under a given load flow condition.

This optimization problem is expressed as the following equations:

$$\begin{aligned} \min. & f(z) \\ \text{sub.to} & g(z) = 0 \\ & h(z) \leq 0 \end{aligned} \quad (2)$$

where objective function f is the total loss of reactive power over the whole power system, equal constraints of vector function g represent active power balance at each generator bus and active and reactive power balances at each load bus, and unequal constraints of vector function h represent the lower and upper limits of the voltage magnitudes at generator and load buses and those of the capacity of shunt capacitor at each load bus. z is a vector of control variables such as the generator bus voltage and the capacity of shunt capacitor.

The solutions of the above optimization problem will not only provide the most stable operating condition from a viewpoint of static voltage stability but also make it possible to relatively compare the static voltage stability before and after the network configuration change.

3.2 Transmission Line Impedance Control

As VSrC is aiming mainly at improving the first swing stability and damping after disturbance in Japan, the proposed quasi-static control plays a role of setting the steady-state operating condition of VSrC. In other countries, this quasi-static control corresponds to the power flow scheduling for power wheeling. The switching-over control is originally useful for the first swing stability enhancement. From a viewpoint of optimum network configuration, however, it is important to discuss the static voltage stability.

In this study, VSrC is considered as variable reactance changing between $-X_c$ and 0 as shown in Fig.2(a). Therefore, the transmission line reactance with VSrC varies between $(X_l - X_c)$ and X_l , where X_l is the transmission line reactance and $-X_c$ is the maximum reactance in the negative value of the series capacitor. If locations of VSrCs are given, the reactances of the transmission lines with VSrCs are added to the control variables and the following unequal constraint for the reactance is added to the optimization problem described by eq.(2) in order to improve the static voltage stability:

$$\underline{X}_i \leq X_i \leq \bar{X}_i \quad (3)$$

where X_i is reactance of transmission line i with VSrC and $\underline{X}_i$ and $\bar{X}_i$ are the lower and upper limits of the reactance respectively.

In case of VSrC, $\underline{X}_i$ and $\bar{X}_i$ are set to be $(X_{li} - X_{ci})$ and X_{li} respectively. In case of switching-over control,

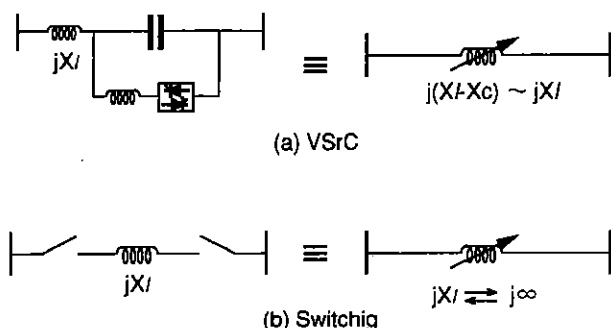


Figure. 2: Equivalent Model

Table. 1: Transmission line impedance

line	impedance [p.u.]
1-3	$1 \times 10^{-5} + j0.1720$
2-4	$1 \times 10^{-5} + j0.1720$
1-2	$1 \times 10^{-5} + j0.3440$
3-4	$1 \times 10^{-5} + j0.3440$
1-4	$1 \times 10^{-5} + j0.3845$

1000MVA base

on the other hand, as transmission line switching is considered approximately as a step change of its reactance from the steady-state value to a very large value vice versa as shown in Fig.2(b), X_i and $\bar{X}_i$ are set to be X_{li} and a very large positive value, respectively.

4 Application to Simple Model Systems

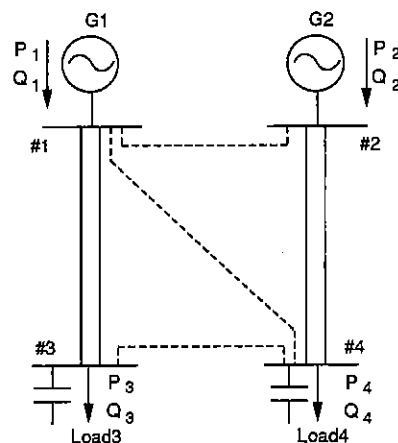
In this section, the relationship between the minimum total loss of reactive power and the static voltage stability index of total load demand margin is discussed using simple model systems and it is made clear that transmission line impedance control by VSrC is effective for the static voltage stability enhancement.

4.1 Simple Model Systems

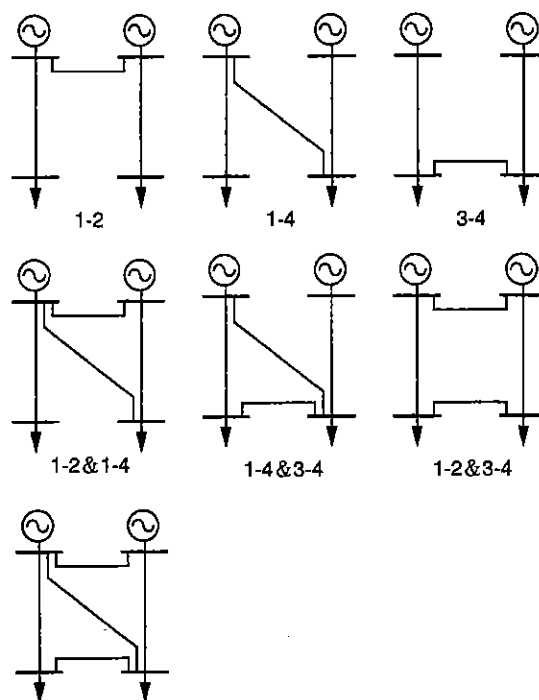
A 2-machine 2-load model system shown in Fig.3(a) is used, where each transmission line is of two circuits and dotted lines denote interconnecting lines. Seven model systems are considered depending on the switched-on interconnecting lines shown in Fig. 3(b). Line constants are shown in Table 1. Generator 1 is assumed to be a swing generator.

4.2 Comparison of two static voltage stability indices

In this subsection, it is checked for four cases of load distribution shown in Table 2 using seven model systems without VSrC, whether the total loss of reactive power over the whole system can evaluate the static voltage stability accurately. The upper and lower limits



(a) model system



1-2&1-4&3-4 interconnection

(b) interconnection

Figure. 3: 2-machine 2-load model system

Table. 2: Load distribution

case	P_3	Q_3	P_4	Q_4
1	1.0	0.329	3.0	0.986
2	1.6	0.526	2.4	0.789
3	2.0	0.657	2.0	0.657
4	2.4	0.789	1.6	0.526

unit:[p.u.]

Table. 3: Minimum total loss of reactive power

interconnection	case1	case2	case3	case4
1-2	2.334 (7)	1.398 (7)	1.316 (7)	1.551 (6)
1-4	1.225 (3)	1.124 (4)	1.314 (5)	1.779 (7)
3-4	1.618 (6)	1.349 (6)	1.301 (4)	1.355 (3)
1-2& 1-4	1.234 (4)	1.106 (2)	1.214 (3)	1.548 (5)
1-2& 3-4	1.527 (5)	1.341 (5)	1.315 (6)	1.365 (4)
1-4& 3-4	1.148 (2)	1.109 (3)	1.175 (2)	1.308 (2)
1-2& 1-4& 3-4	1.131 (1)	1.105 (1)	1.151 (1)	1.255 (1)

unit:[p.u.],():order

Table. 4: Margin of total load demand

interconnection	Case1	Case2	Case3	Case4
1-2	0.070 (7)	0.947 (7)	1.303 (5)	0.419 (5)
1-4	1.201 (4)	2.222 (3)	1.303 (5)	0.419 (5)
3-4	0.770 (6)	1.355 (6)	1.468 (4)	1.236 (3)
1-2& 1-4	1.221 (3)	2.222 (3)	1.303 (5)	0.419 (5)
1-2& 3-4	0.933 (5)	1.389 (5)	1.478 (3)	1.215 (4)
1-4& 3-4	1.871 (2)	2.258 (2)	2.044 (2)	1.541 (2)
1-2& 1-4& 3-4	1.896 (1)	2.374 (1)	2.104 (1)	1.577 (1)

unit:[p.u.],():order

of bus voltage V at each bus and susceptance Y of shunt capacitor at each load bus are as follows:

$$0.8 \leq V[\text{p.u.}] \leq 1.2 \quad (4)$$

$$0.0 \leq Y[\text{p.u.}] \leq 1.2 \quad (5)$$

The output active power from generator 2 is fixed to be 2.0 [p.u.]. Results of the minimum total reactive power loss and the total load demand margin are shown in Tables 3 and 4 respectively. It is seen from Tables 3 and 4 that the network configuration with the smaller total reactive power loss is the more stable from a viewpoint of static voltage stability.

4.3 Transmission Line Impedance Control by VSrC

Suppose that VSrCs are installed at all interconnecting lines in seven model systems shown in Fig.3(b). Compensation ratio of VSrC for the transmission line reactance varies between 0 [%] and 70 [%], which means that reactance X_i of interconnecting line i with VSrC varies as follows:

$$0.3X_{i1} \leq X_i \leq X_{i1} \quad (6)$$

where X_{i1} is reactance of transmission line i without VSrC.

The total load demand and the output active power from generator 2 are fixed to be 3.0 [p.u.] and 1.5[p.u.] respectively. The optimization problem is solved for seven cases of load distribution shown in Table 5.

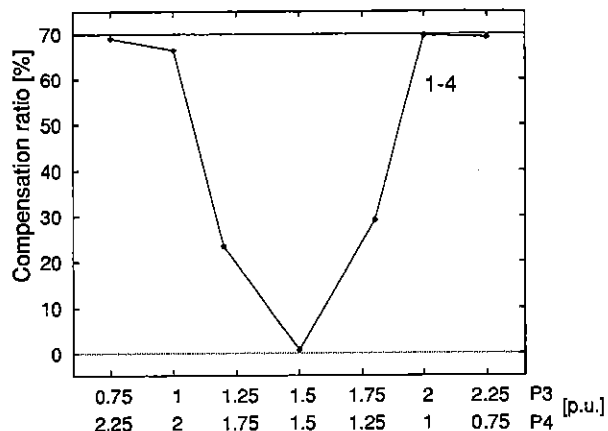
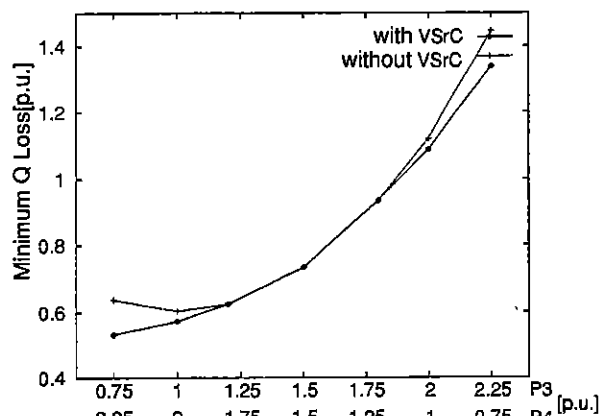
Fig.4 shows the minimum total loss of reactive power with and without VSrC and the compensation ratio of

Table. 5: Load distribution

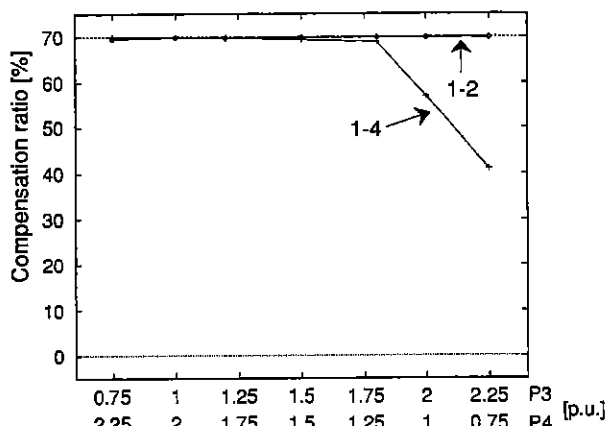
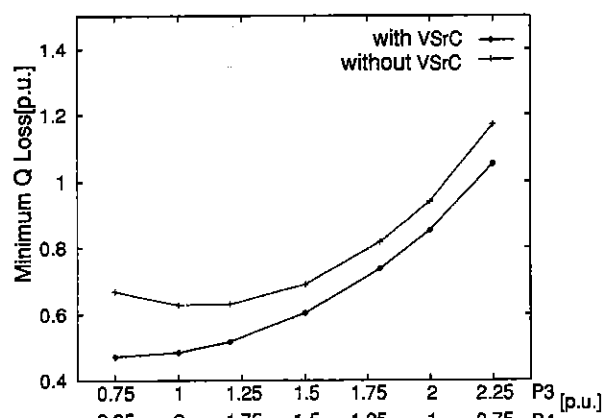
Case	$P_3 : P_4$	P_3	Q_3	P_4	Q_4
1	3:1	2.25	0.740	0.75	0.247
2	2:1	2.00	0.657	1.00	0.329
3	3:2	1.80	0.592	1.20	0.394
4	1:1	1.50	0.493	1.50	0.493
5	2:3	1.20	0.394	1.80	0.592
6	1:2	1.00	0.329	2.00	0.659
7	1:3	0.75	0.247	2.25	0.740

unit:[p.u.]

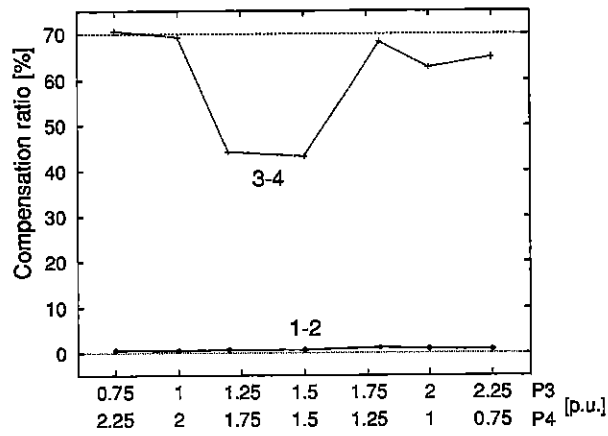
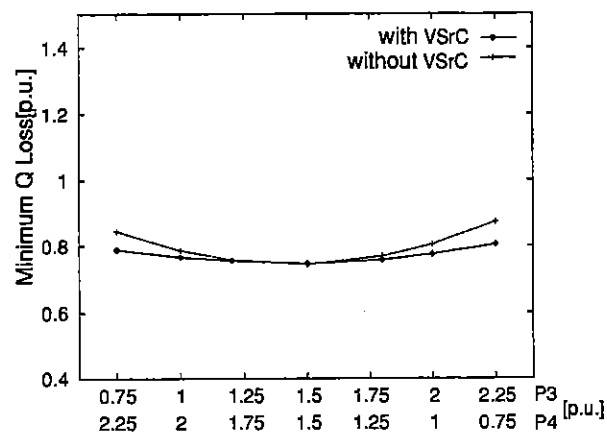
each interconnecting line in three 1-4, 1-2 & 1-4 and 1-2 & 3-4 interconnected model systems. It is seen from Fig. 4 that the minimum total loss of reactive power is reduced by VSrC installed at the interconnecting line in most of cases and that the reduction is large when the load demand at one bus is larger than that at the other bus, for example, in cases 1 and 7. In case 4 of four 1-2, 1-4, 3-4 and 1-2 & 3-4 interconnected model systems, where little active power flows through the interconnecting line, the impedance control by VSrC has little effect on the reduction of the minimum total loss of reactive power. To make it clear that the reduction of the minimum total loss of reactive power leads to the static voltage stability enhancement, P-V curves drawn from the optimum operating point of case 7 under the assumptions mentioned in section 2.1 are shown in Fig.5. It should be noted that the reduction of the total reactive power loss is not necessarily proportional to the increment of the total load demand margin. Fig.6



(a) 1-4 interconnection



(b) 1-2 & 1-4 interconnection



(c) 1-2 & 3-4 interconnection

Figure. 4: Results of impedance control by VSrC

illustrates typical behaviors of shunt capacitors at both load buses.

5 Optimization of Impedance Control in Multi-Machine Power System

5.1 Constraint of Total Capacity of VSrCs

In a multi-machine power system which has many candidates of VSrC installation locations, in order to find some optimum locations, that is, transmission lines which are very effective for the static voltage stability improvement, the following unequal constraint of total capacity of VSrCs is added to the optimization problem described by eq.(2) in section 3.1:

$$0 \leq \sum_{i \in I} \Delta X_i \leq \alpha \sum_{i \in I} X_{ci} \quad (7)$$

$$0 \leq \Delta X_i \leq X_{ci} = \beta_i X_{li} \quad \text{for all } i \quad (8)$$

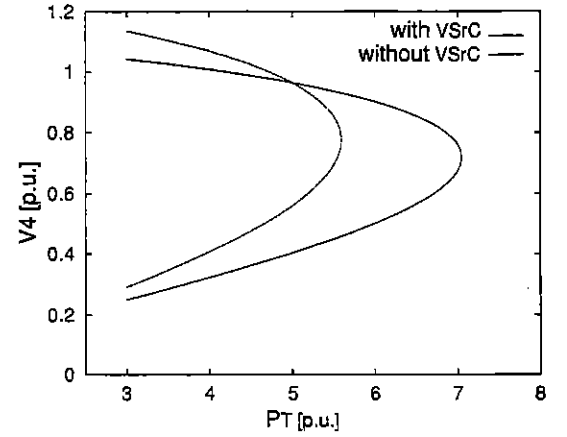
$$(X_{li} - X_{ci} \leq X_i \leq X_{li})$$

where ΔX_i is the amount of compensation for each transmission line reactance and I is a set of all transmission line number. $\alpha (0 < \alpha \leq 1)$ is the maximum ratio of the total amount of possible compensations for all transmission line reactances to the total amount of maximum compensations of VSrCs assumed to be installed at all transmission lines. $\beta_i (0 < \beta_i \leq 1)$ is the maximum compensation ratio of each VSrC.

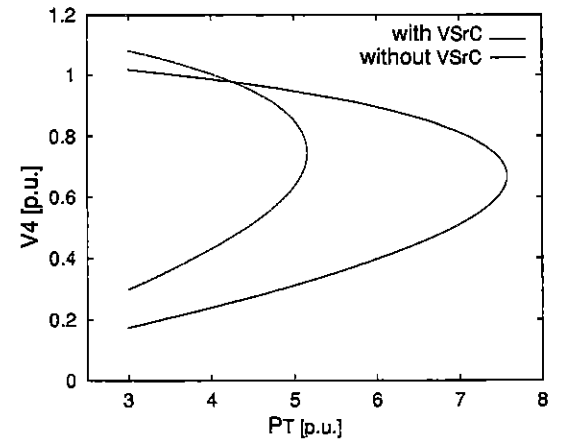
To solve the proposed optimization problem makes it possible to install VSrCs at the transmission lines in order of effectiveness for the static voltage stability up to the upper limit of total capacity of VSrCs. ΔX_j less than a very small positive value obtained above means that VSrC is unnecessary on transmission line j under the condition of the limited total capacity of VSrCs.

5.2 Optimum Locations of VSrCs in 11-bus Model System

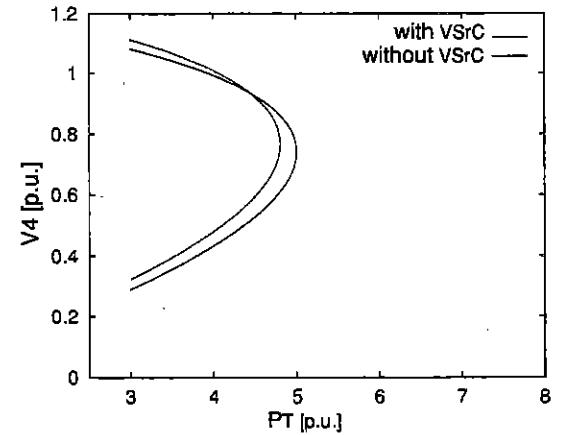
A 11-bus model system in Fig.7 is used for numerical examples. First, the optimization problem is solved under the heavily loaded condition shown in Table 6, where $\beta_i = 0.7$ for all i . It is seen from numerical results for $\alpha = 0.6, 0.4$ and 0.2 shown in Tables 7,8 and 9 respectively that the number of transmission lines compensated by VSrC becomes small as α decreases. In case of $\alpha = 0.2$ in Table 9, the transmission lines which are effective for reduction of the total reactive power loss are five lines 8-9, 1-2, 4-11, 2-3 and 7-8, through which heavy active power flow from generators to loads as shown in Fig.8. Fig.9 illustrates the relationship between the minimum total loss of reactive power and α , which makes it clear that the effect of α on the reduction of the minimum reactive power loss is very small in the range of α over 0.4. This fact is also guaranteed by P-V curves in Fig.10. It is noted that the similar



(a) 1-4 interconnection



(b) 1-2 & 1-4 interconnection



(c) 1-2 & 3-4 interconnection

Figure. 5: P-V curve in case 7

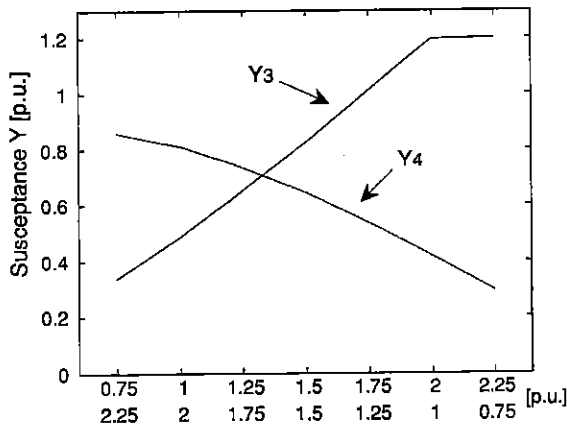


Figure 6: Behavior of shunt capacitor

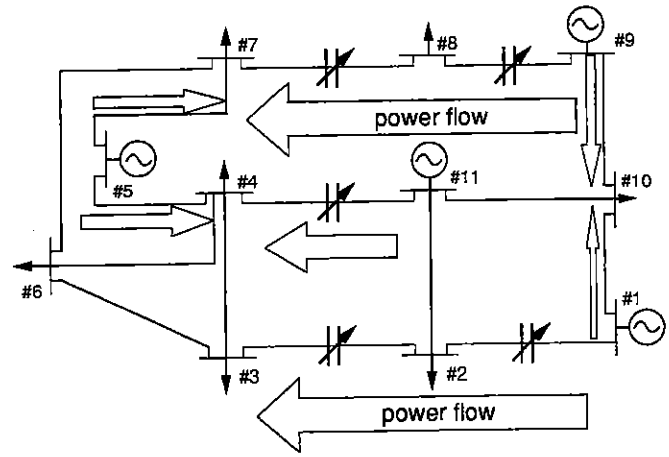


Figure 8: Optimum locations of VSrCs and power flows ($\alpha = 0.2$)

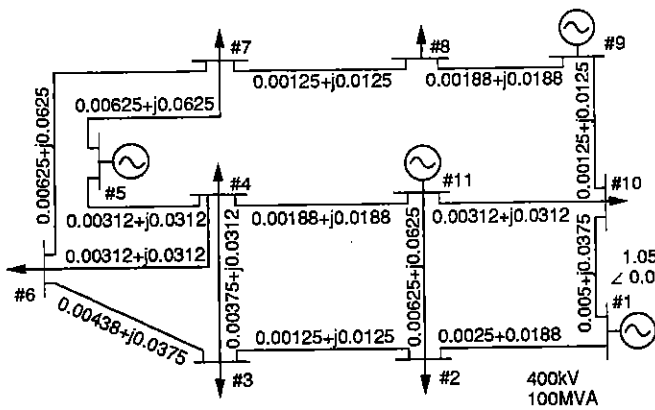


Figure 7: 11-bus model system

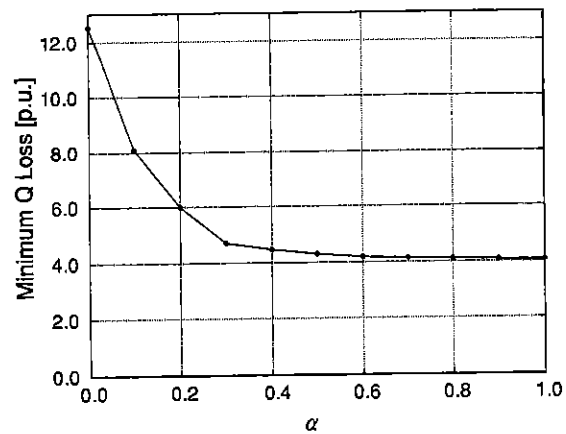


Figure 9: Minimum Q loss vs. α

Table 6: Load flow condition

bus	P	Q
1	swing bus	
2	-4.20	-1.380
3	-6.50	-2.136
4	-8.50	-2.794
5	8.50	—
6	-10.00	-3.287
7	-8.50	-2.794
8	-6.50	-2.136
9	16.50	—
10	-4.20	-1.380
11	10.00	—

unit: [p.u.]

results are obtained under the lightly loaded condition of a half of the previous heavy load flow.

Next, the optimization problem is solved under the uniformly distributed load flow condition of Table 10 different from Table 6, however the total load demand in Table 10 is the same as that in Table 6. Numerical results for $\alpha = 0.6$, 0.4 , and 0.2 are shown in Tables 11, 12 and 13 respectively. Optimum locations of VSrCs and power flows in case of $\alpha = 0.2$ are illustrated in Fig. 11.

It should be noted that shunt capacitors at all load buses have little effect on the reduction of the minimum total loss of reactive power, because the upper limits of the shunt capacitors have been installed at all load buses in all cases.

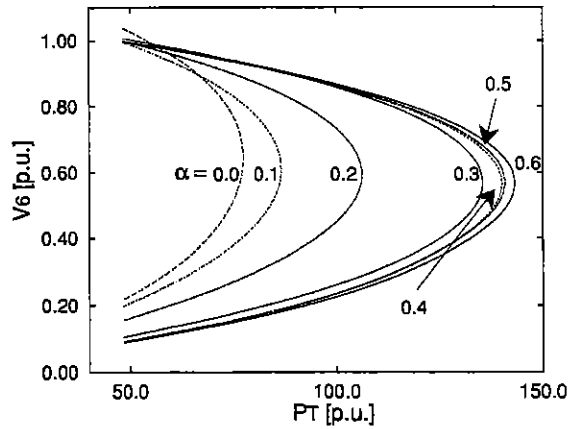


Figure. 10: P-V curve with α

Table. 8: Change of transmission line reactance ($\alpha = 0.4$)

transmission line	reactance [p.u.] before comp.	reactance [p.u.] after comp.	comp. ratio [%]
8 - 9	0.0188	0.0058	69.1
1 - 2	0.0188	0.0058	69.0
4 - 6	0.0312	0.0097	68.9
4 - 5	0.0312	0.0097	68.8
4 - 11	0.0188	0.0059	68.7
2 - 3	0.0125	0.0040	67.9
7 - 8	0.0125	0.0041	67.4
3 - 6	0.0375	0.0123	67.3
9 - 10	0.0125	0.0104	17.0
1 - 10	0.0375	0.0362	3.5
3 - 4	0.0312	0.0301	3.5
10 - 11	0.0312	0.0301	3.4
5 - 7	0.0625	0.0606	3.1
2 - 11	0.0625	0.0614	1.7
6 - 7	0.0625	0.0614	1.7

Table. 7: Change of transmission line reactance ($\alpha = 0.6$)

transmission line	reactance [p.u.] before comp.	reactance [p.u.] after comp.	comp. ratio [%]
8 - 9	0.0188	0.0058	69.0
1 - 2	0.0188	0.0058	69.0
5 - 7	0.0625	0.0195	68.8
4 - 6	0.0312	0.0097	68.8
4 - 11	0.0188	0.0059	68.8
4 - 5	0.0312	0.0098	68.6
3 - 6	0.0375	0.0120	68.0
2 - 3	0.0125	0.0040	68.0
7 - 8	0.0125	0.0041	67.1
9 - 10	0.0125	0.0053	58.0
10 - 11	0.0312	0.0258	17.3
1 - 10	0.0375	0.0317	15.4
6 - 7	0.0625	0.0532	14.9
3 - 4	0.0312	0.0289	7.5
2 - 11	0.0625	0.0600	4.0

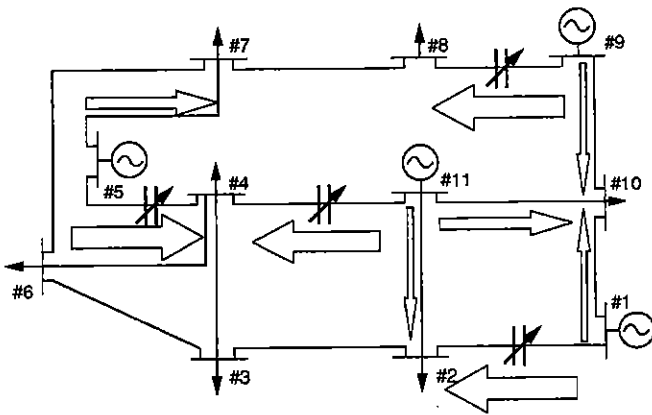
Table. 9: Change of transmission line reactance ($\alpha = 0.2$)

transmission line	reactance [p.u.] before comp.	reactance [p.u.] after comp.	comp. ratio [%]
8 - 9	0.0188	0.0058	69.0
1 - 2	0.0188	0.0059	69.0
4 - 11	0.0188	0.0059	68.5
2 - 3	0.0125	0.0041	67.3
7 - 8	0.0125	0.0043	65.8
4 - 6	0.0312	0.0235	24.6
9 - 10	0.0125	0.0121	3.1
4 - 5	0.0312	0.0305	2.3
3 - 4	0.0312	0.0308	1.2
10 - 11	0.0312	0.0308	1.2
3 - 6	0.0375	0.0371	1.2
1 - 10	0.0375	0.0371	1.0
5 - 7	0.0625	0.0621	0.7
6 - 7	0.0625	0.0621	0.6
2 - 11	0.0625	0.0621	0.6

Table 10: Uniformly distributed load flow condition

bus	P	Q
1	swing bus	
2	-6.800	-2.235
3	-6.800	-2.235
4	-7.000	-2.301
5	12.40	—
6	-7.000	-2.301
7	-7.000	-2.301
8	-6.800	-2.235
9	12.40	—
10	-6.800	-2.235
11	12.40	—

unit:[p.u.]

Figure 11: Optimum locations of VSrCs and power flows under uniformly distributed load flow condition ($\alpha = 0.2$)Table 11: Change of transmission line reactance under uniformly distributed load flow condition ($\alpha = 0.6$)

transmission line	reactance [p.u.] before comp.	reactance [p.u.] after comp.	comp. ratio [%]
5 - 7	0.0625	0.0192	69.2
4 - 5	0.0312	0.0097	68.9
1 - 2	0.0188	0.0059	68.8
4 - 6	0.0312	0.0098	68.7
8 - 9	0.0188	0.0059	68.7
4 - 11	0.0188	0.0059	68.6
3 - 4	0.0312	0.0103	66.9
10 - 11	0.0312	0.0104	66.7
2 - 3	0.0125	0.0043	65.9
9 - 10	0.0125	0.0047	62.7
7 - 8	0.0125	0.0048	61.8

Table 12: Change of transmission line reactance under uniformly distributed load flow condition ($\alpha = 0.4$)

transmission line	reactance [p.u.] before comp.	reactance [p.u.] after comp.	comp. ratio [%]
1 - 2	0.0188	0.0059	68.7
4 - 11	0.0188	0.0060	68.4
4 - 6	0.0312	0.0099	68.3
4 - 5	0.0312	0.0099	68.3
8 - 9	0.0188	0.0060	68.3
5 - 7	0.0625	0.0198	68.3
2 - 3	0.0125	0.0062	50.5

Table 13: Change of transmission line reactance under uniformly distributed load flow condition ($\alpha = 0.2$)

transmission line	reactance [p.u.] before comp.	reactance [p.u.] after comp.	comp. ratio [%]
4 - 5	0.0312	0.0098	68.8
8 - 9	0.0188	0.0059	68.6
1 - 2	0.0188	0.0059	68.5
4 - 11	0.0188	0.0061	67.5

5.3 Preventive Control against Voltage Collapse by VSrC

In this subsection, the proposed optimization procedure is considered as preventive control against voltage collapse. When the voltage magnitude of a load bus reaches the lower limit as the load demand increases, the optimization calculation is carried out and the generator bus voltages and the VSrCs and the shunt capacitors are reset to be new optimum values for the static voltage stability improvement.

A numerical simulation on the 11-bus model system is carried out with increase of load demand starting from the load flow condition of Table 6. VSrCs are installed at five effective transmission lines 1-2, 2-3, 4-11, 7-8 and 8-9 obtained in the previous section, where parameters α and β_i ($i = 1, \dots, 5$) in the optimization problem are assumed to be 0.5 and 0.7 for five VSrCs respectively. The lower limit of the bus voltage is set to be 0.8[p.u.]. Constraints for voltage V and shunt capacitor susceptance Y are as follows:

$$0.8 \leq V \leq 1.2 \quad (9)$$

$$0.0 \leq Y \leq 1.2 \quad (10)$$

The behaviors of the voltage of bus 6 and VSrCs with increase in total load demand are shown Figs.12 and 13 respectively. The preventive control at the first step in Fig.13 corresponds to the optimization at the total load demand $P = 48.4$ [p.u.] under the load flow condition of Table 6. It is noted that the upper limits of

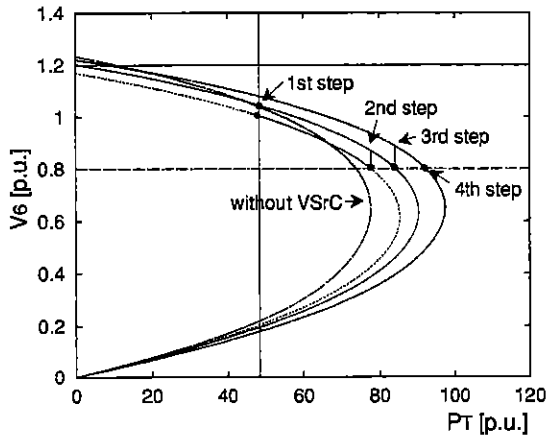


Figure. 12: Voltage behavior by preventive control

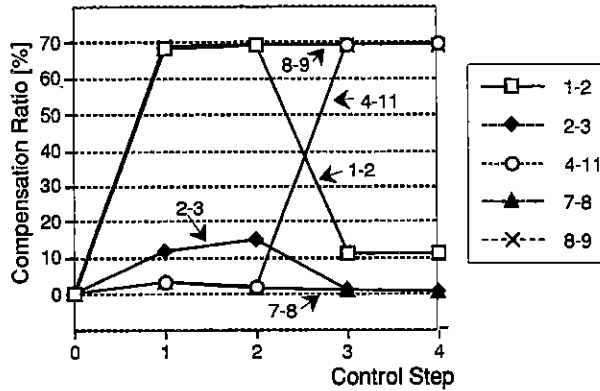


Figure. 13: Behaviors of VSrCs

the shunt capacitors are also installed at all load buses at any time in this simulation.

6 Switching-Over Control of Transmission Line

6.1 Optimization Problem

In section 4, it has been made clear that the static voltage stability is influenced by the network configuration and the load distribution. In this section, an optimization method for finding an optimum network configuration for the static voltage stability is presented. The optimum network configuration is realized by the switching-over control of transmission lines proposed by the authors [6].

As mentioned in section 3.2, in case of the switching-over control, the following constraint is added to the optimization problem with minimization of the total reactive power loss represented by eq. (2):

$$X_{li} \leq X_i \leq \gamma X_{li} \quad (i \in I, \gamma \gg 1) \quad (11)$$

X_j near the upper limit obtained by solving the above optimization problem means that transmission line j can be switched off for the reduction of the total reactive power loss.

6.2 Numerical Examples in Simple Model Systems

The 2-machine 2-load simple model system shown in Fig.3(a) is used, where all of three interconnecting lines with VSrCs are taken into account. The optimization problem with $\gamma = 5.0$ is solved for seven cases of load distribution of Table 5. The results are shown in Table 14, where interconnecting lines 1-2 or 1-2 & 3-4 should be switched off in case 5, interconnecting lines 1-2 & 3-4 in case 6, interconnecting line 1-2 in case 7, and no interconnecting line in other cases. Table 15 obtained from the optimization represented by eq.(2) for seven interconnected systems shows that 1-4 interconnected or 1-4 & 3-4 interconnected system is optimum in case 5, 1-4 interconnected system in case 6, 1-4 & 3-4 interconnected system in case 7 and 1-2 & 1-4 & 3-4 interconnected system in other cases. It is made clear, therefore, that the proposed optimization method can be used for finding optimum network configurations for the static voltage stability.

7 Conclusion

In this paper, an OPF method based on minimization of total loss of reactive power over the whole power system with unequal constraints of transmission line reactance has been proposed for setting the optimum operating conditions of voltages at generator buses, shunt capacitors at load buses and variable series capacitors (VSrCs) on transmission line to improve the static voltage stability. The proposed method is also applicable to selection of optimum locations of VSrCs and an optimum network configuration for the static voltage stability. The effectiveness of the proposed method has been made clear by numerical examples using model systems.

It is under study from a viewpoint of computation efficiency in a large-scale power system whether the total reactive power loss should be minimized over the whole power system or that in its local subsystem where the voltage stability is severe is enough to be minimized.

Since VSrC and switching-over control of transmission lines aim mainly at improving the first swing stability and damping after disturbance in our previous studies, it will be necessary to coordinate the proposed static control for the static voltage stability and the dynamic control for the first swing stability, damping and torsional oscillation of turbine shaft by subsynchronous resonance (SSR) in the future study.

References

- [1] E. W. Kimbark. "Improvement of system stability by switched series capacitors". *IEEE Trans. on*

Table 14: Interconnecting line impedance by negative compensation ($\gamma = 5.0$)

Case	line no. 1-2	comp. ratio [%]	line no. 1-4	comp. ratio [%]	line no. 3-4	comp. ratio [%]
1	0.345	-0.3	0.386	-0.5	0.344	-0.1
2	0.345	-0.3	0.386	-0.3	0.345	-0.2
3	0.346	-0.5	0.386	-0.3	0.345	-0.3
4	0.348	-1.1	0.385	-0.2	0.346	-0.5
5	1.705	*-395.6	0.386	-0.4	1.312	*-281.4
6	1.715	*-398.5	0.385	-0.2	1.712	*-397.8
7	1.714	*-398.2	0.385	-0.1	0.347	-0.9

*:switched-off possible transmission line

unit:[p.u.]

Table 15: Minimum total loss of reactive power

interconnection	case1	case2	case3	case4	case5	case6	case7
1-2	1.186	0.933	0.817	0.736	0.761	0.832	0.986
1-4	1.445	1.120	0.934	0.733	*0.624	*0.604	0.638
3-4	0.861	0.775	0.731	0.707	0.732	0.775	0.861
1-2& 1-4	1.171	0.940	0.817	0.690	0.631	0.629	0.669
1-4& 3-4	0.909	0.803	0.736	0.663	*0.624	0.618	*0.633
1-2& 3-4	0.873	0.805	0.768	0.745	0.758	0.786	0.844
1-2& 1-4& 3-4	*0.858	*0.769	*0.714	*0.657	0.631	0.631	0.651

*:optimum interconnection

unit:[p.u.]

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LOAD RECOVERY IN VOLTAGE STABILITY ANALYSIS AND CONTROL

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Abstract: This paper presents an overview with some extensions of recent results on mid to long-term voltage stability analysis and control in power systems. The results are based on use of generic dynamic load modelling, i.e., using aggregate nonlinear structures. In general, the model for the transmission network and supply end dynamics is of the hybrid differential - algebraic - discrete kind. The various stability questions are analysed in terms of network and load characteristics (steady-state and transient). The results are shown to be a useful framework for deriving criteria for various control actions. An analysis of tap-locking is given as an illustration.

1 Introduction

The area of voltage stability analysis and control for power systems has yielded an extensive and diverse array of analytical contributions. These are represented in several recent publications [1-5]. It is now well-accepted that the basic problem has static (based on power flow equations) and dynamic (based on differential equations) aspects. This paper addresses these static and dynamic aspects in the mid and longer terms, i.e. a time-scale of a minute or so and beyond assuming generator stability is no longer an issue.

The results hinge on a novel approach to load modelling which captures the nonlinear transient recovery behaviour. Between generators and low voltage customers, power flows through step-up transformers, transmission lines and several stages of step-down transformers; some of the transformers have on-load tap-changers (OLTC). Generally speaking, all these devices can be modelled accurately. However, if we are concerned with voltage behaviour at subtransmission levels or higher, it is not sensible to model every cus-

tomers device in the same detail. Aggregation and so approximation is used. This leads often to a serious lack of uniformity between precise generator and transmission models and crude aggregate load models. Depending on the level at which the load is defined, it could contain several levels of OLTC transformers, numerous electrical motors, switched capacitance and other load devices. For angle stability studies, this is (dubiously) summarised by expressing load powers as simple functions of voltage, i.e. without dynamics. In voltage stability studies, the dynamic behaviour of loads cannot be ignored, but some form of aggregation is still clearly needed to get uniform modelling.

The approach to load modelling used here follows previous work by Hill, Karlsson and Van Cutsem [9-12], where models are developed which feature nonlinear functions capturing the steady-state (static sense) and transient behaviour in load responses and parameters which relate to load recovery time. Various extensions have been reported in [17,18,25]. Load at a bus can be expressed as an input-output relationship between (real and reactive) powers and voltage or via a state-space realisation where the internal state variables can be interpreted physically if no aggregation is used. The connection between these various models is very close. Two special cases of interest are the exponential recovery [9-11] and adaptive loads [12,18].

Earlier papers by the authors and others [9,16-18,25] have developed basic stability analysis for system models based on the aggregate recovery load models. In a single-load analysis using the exponential recovery model, the results in [9] suggest that regular operating points in the sense $\frac{dV}{dQ} > 0$, i.e. increasing injected reactive power at a bus also increases the voltage, are (dynamically) stable, but it is also possible for an irregular point to be stable. This connection was extended

to multi-load systems in [16,17]; small disturbance stability is seen to depend on power flow Jacobians derived from the steady-state and transient load characteristics respectively. These results are generalised here to more general classes of load models. Analysis is viewed in three clear steps: computation of equilibria, small disturbance analysis and large disturbance analysis. Equilibria are determined by the power flow equations with steady-state load characteristics. Static voltage behaviour is described by the sign properties of the Jacobian matrix; this relates to various results on determining proximity to static voltage collapse [19-22]. Small disturbance analysis is based on the steady-state power flow Jacobian and another derived from transient load characteristics. For special load structures, the stability criteria are given in terms of the eigenstructure of these Jacobians and the recovery times. Large disturbance analysis depends on the global form of the manifolds related to the various system characteristics.

The control part of the paper deals with actions to prevent voltage collapse. Xu and Mansour [18] have studied critical timing of capacitor switching with ideas reminiscent of using the equal-area criterion for critical fault clearing to prevent transient instability. Load shedding based on generic load models has been studied in [24]; some analytical basis for the criteria is given in [25] for a single-load system. Here we focus on tap-locking. The above analytical framework can be used to give precise criteria on when to take control action. These criteria are related to various bifurcation points in the manifold structure [23].

The structure of the paper is as follows. Section 2 gives a review and extension of aggregate nonlinear dynamic load modelling. The hybrid differential-algebraic-discrete model for the supply system is derived in Section 3. (In special cases, this reduces to differential-algebraic or purely differential forms.) Section 4 gives a review of definitions which cover both static and dynamic properties of voltage behavior. Detailed voltage stability analysis of the multi-load case is presented and illustrated in Section 5. Section 6 summarizes extensions to general multi-load systems with OLTC transformers. Derivation of tap-locking control action is discussed in Section 7. Section 8 gives conclusions and comments on possible extensions.

Many of the ideas in this paper were previously summarised in [25] where emphasis was given to the single load case; this paper gives multi-load results. A companion paper [30] explores model structure in greater detail.

2 Load Modelling

The aim in this section is to review and extend the ideas presented in [9-12,17,18] on generic load mod-

els, i.e. general purpose nonlinear dynamic structures which represent the aggregate effect of all load connected to the bus. The treatment is extracted from [25].

2.1 Examples and State Models

Firstly, we consider two examples of specific loads [9,12].

Example 1 Induction Motor

A motor connected to a power system bus is shown in Figure 1a); the usual equivalent circuit is given in Figure 1b).

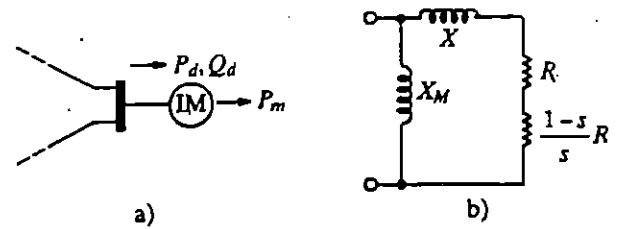


Figure 1: Induction Motor Load

The dynamics are described by

$$\frac{ds}{dt} = \frac{1}{I\omega_o^2} \left(\frac{P_m}{1-s} - P_d \right) \quad (1)$$

where I is the moment of inertia, ω_o is the stator frequency (in rad/sec), P_m is mechanical load power and $s = \frac{\omega_o - \omega_m}{\omega_o}$ the slip where ω_m is induction motor speed. Assuming the magnetising inductance is large, i.e. $X_M \rightarrow \infty$, we have

$$P_d = \frac{s R V^2}{R^2 + s^2 X^2} \quad (2)$$

$$Q_d = \frac{s^2 X V^2}{R^2 + s^2 X^2} \quad (3)$$

□

Example 2 Implicit OLTC

The load consists of an OLTC transformer feeding a static voltage dependent load as shown in Figure 2.

For a time-scale of many minutes, a reasonable approximation to the tap-changer operation is given by

$$\dot{n} = \frac{1}{T} (V_1^\circ - n V) \quad (4)$$

where V_1° is the voltage set-point and T determines the speed of tap changing.

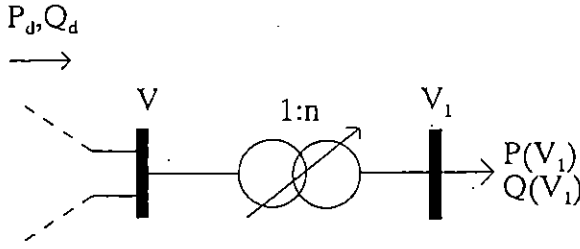


Figure 2: OLTC and Static Load

The load powers are given simply by

$$P_d = P(nV) \quad (5)$$

$$Q_d = Q(nV) \quad (6)$$

□

In both these examples, we have the overall bus load powers expressed as functions of voltage and a state variable, i.e. s and n respectively. The general form of model equations (1)-(3) and (4)-(6) is

$$\dot{x} = a(x, V) \quad (7)$$

$$P_d = b_p(x, V) \quad (8)$$

$$Q_d = b_q(x, V) \quad (9)$$

This form is too unstructured to pursue in analysis; however, in general, it also does not make sense to model high voltage aggregate load according to any one device.

With more detailed modelling, we would get the same form as (7)-(9) with x as a vector including other physical variables, eg. electrical dynamics in the motor. Van Cutsem [12] suggested using this general form (7)-(9) for load models in voltage stability studies. Physical special cases and one generic special case were considered. The latter so-called adaptive load (after change of notation and scaling) is

$$\dot{x} = K(P_d^0 - P_d) \quad (10)$$

$$P_d = x f_p(V) \quad (11)$$

$$Q_d = x f_q(V) \quad (12)$$

with K a constant. This represents aggregate load with a non physical state variable and has adequate structure to be useful.

2.2 Input-Output Models

Motivated by the idea that physical models do not make sense for high voltage aggregate load, a dynamic generalisation of the usual static model, i.e.

$$P_d = P_s(V) \quad (13)$$

$$Q_d = Q_s(V) \quad (14)$$

was sought in [9]. Figure 3 shows a generic load response with some significant parameters. A load voltage step produces a transient jump of power followed by recovery to a steady-state level.

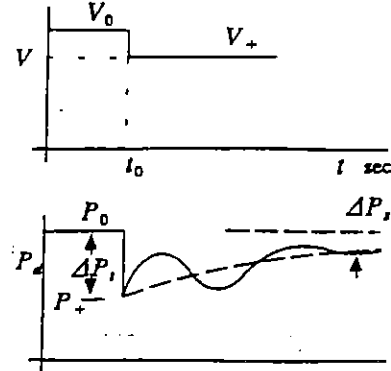


Figure 3: Generic Load Response

A class of models of the following form was considered for monotone recovery [9]

$$\dot{P}_d + f_p(P_d, V) = g_p(P_d, V)\dot{V} \quad (15)$$

A similar equation relates Q_d, V . Higher order input-output models can also be considered usefully [9,17]; for instance, capture of oscillatory recovery would require this.

Restricting attention to first-order load recovery, we note that in (15):

- Steady-state behaviour is given by $\dot{P}_d = 0, \dot{V} = 0$, i.e.

$$f_p(P_d, V) = 0 \quad (16)$$

- Transient behaviour is given by integrating

$$dP_d = g_p(P_d, V)dV \quad (17)$$

From the Implicit Function Theorem, if $\frac{\partial f_p}{\partial P_d} \neq 0$, (16) can be solved to give $P_d = P_s(V)$. Integrating (17) gives $h_p(P_d, V) = c$, where c is a constant of integration; then similarly to (16), this can provide a function $P_d = P_t(V)$. These functions may only be defined locally. Figure 3 illustrates the role of the functions P_s and P_t in the steady-state and transient parts of the load response respectively. (As usual, Δy denotes a change in variable y .)

The possibilities within general models like (7)-(9) and (15) have not been fully explored. There are many ways which the functions (a, b) and (f, g) can be parameterised.

In [9], one useful special case arises naturally: assume that the recovery is exponential; then it is easy to derive the model as the form

$$T_p \dot{P}_d + P_d = P_s(V) + k_p(V)\dot{V} \quad (18)$$

where T_p is the recovery time-constant; P_s and $k_p(V) = T_p P'_t(V)$ define the steady-state and transient load characteristics. Karlsson [10,11] used field data derived from various voltage signals to identify parameters in model (18) when P_s , P_t , Q_s and Q_t take the power-law form $P = P_o \left(\frac{V}{V_o}\right)^\alpha$, $Q = Q_o \left(\frac{V}{V_o}\right)^\beta$. Thus the load is completely described by parameters V_o , P_o , Q_o , T_p , T_q , α_s , α_t , β_s and β_t . (α_s and β_s are the usual static load indices.)

2.3 Connections

In [9], the models in Examples 1,2 are shown to give the general input-output (IO) form (15) when the intermediate state variable is eliminated. Also, it is seen to be convenient to transform the exponential IO form (18) into an equivalent state version

$$\dot{x}_p = P_s(V) - P_d \quad (19)$$

$$P_d = \frac{1}{T_p} x_p + P_t(V) \quad (20)$$

(Substitution of (20) into (19) gives the form (7),(8).) Equation (20) defines the state variable as a function of (P_d, V) . While IO models are motivated by parameter identification, state models are more convenient for numerical solution. It turns out that the general state and IO model forms are equivalent (at least locally).

Starting with the general IO form (15), we aim to find a state equation. Suppose P_d can be expressed in terms of some state variable x_p

$$P_d = \phi(x_p, V) \quad (21)$$

via function ϕ . Then differentiating (21) and substituting into (15) gives

$$D_{x_p} \phi \cdot \dot{x}_p = -f_p(\phi(x_p, V), V) \quad (22)$$

if we choose

$$D_v \phi = g_p(\phi(x_p, V), V) \quad (23)$$

Note for the exponential recovery model, equation (23) becomes $P'_t(V) = \frac{1}{T_p} k_p(V)$. The general equivalent state model is defined by

$$a(x, V) = -f_p(\phi(x, V), V) / D_{x_p} \phi(x, V) \quad (24)$$

$$b_p(x, V) = \phi(x, V) \quad (25)$$

For the converse, we start with the general state model (7)-(9). Differentiating (8) and substituting (7) gives

$$\dot{P}_d = D_x b_p a(x, V) + D_v b_p \cdot \dot{V}$$

Assume that (8) can be solved (at least locally) for x as a function (P_d, V)

$$x = \psi(P_d, V) \quad (26)$$

The equivalent IO model is defined by

$$f_p(P_d, V) = -D_x b_p(x, V) a(x, V) |_{x=\psi(P_d, V)} \quad (27)$$

$$g_p(P_d, V) = D_v b_p(\psi(P_d, V), V) \quad (28)$$

At the level of generality in the IO form (15) or state form (7)-(9), these models are not useful for applications. Structures for (f, g) and (a, b) respectively must be found which can be parameterised and identified from field measurements; moreover, the resultant model should be verified within the range of operation of variables (x, V) later used in analysis procedures. The above mentioned work for the exponential model [10,11] makes a start on this important path.

A further special generic model has been suggested by Xu and Mansour [18] as

$$T_p \frac{dx_p}{dt} = P_s(V) - P_d \quad (29)$$

$$P_d = x_p P_t(V) \quad (30)$$

with a similar model for Q_d . This is an extension of the adaptive load [12] to allow independent real and reactive power recovery and voltage dependent steady-state load. It is easy to show that the IO version of this model is

$$T_p \dot{P}_d + P_t(V)(P_d - P_s(V)) = T_p P_d \frac{P'_t(V)}{P_t(V)} \dot{V} \quad (31)$$

which is of the general IO form (15). It is interesting to compare the steady-state and transient load characteristics. From the state models (19)-(20), (29)-(30), (7)-(9) and IO versions (18), (31), (15) it is easy to see the results of Table 1.

Table 1 Comparison of Generic Load Models
(for real power)

	Steady-state	Transient
Exponential recovery	$P_d = P_s(V)$	$P_d = \frac{1}{T_p} x(o-) + P_t(V)$
Adaptive	$P_d = P_s(V)$	$P_d = x(o-) P_t(V)$
General State	$a(x, V) = 0$ $b_p(x, V) = P_d$	$P_d = b_p(x(o-), V)$
General IO	$f_p(P_d, V) = 0$	$dP_d = g_p(P_d, V) dV$

Comparing the exponential recovery and adaptive load models with the examples, we see that neither capture the behaviour of these loads over large excursions of the state variable. So there is still plenty of room for improvement in the development of special structures.

Other interesting aspects of generic load modelling are contained in references [9-12,17,30,25]. A recent paper [26] considers some IO and state connections and block diagram representations.

3 Power Supply System Model

This section shows how the load models developed in Section 2 can be combined with appropriate models of the generators, network and transformers. Earlier versions of the model were given in [13,16,23]. For more details, refer to [23].

Consider a network of m generators and n_b buses, and thus there are $n_b - m$ buses which have no generation of real power. Suppose there are t on-load tap-changing transformers, at buses $1, \dots, t$, and assume that for each transformer the voltage of the tap bus is being regulated. Denote the i th bus voltage and angle by V_i and θ_i respectively. Let the generator buses be the last m buses, i.e., generator i is attached to bus $n_b - m + i$. It will be convenient to write the vector of non-generator bus voltages $V = [V_1, \dots, V_{n_b-m}]^t = [V_r^t \ V_u^t]^t$ where ' t ' denotes matrix transpose, and V_r, V_u are $t, (n_b - m - t)$ -vectors referring to regulated and unregulated bus voltages respectively.

The generators are modelled as constant real power sources of $P_{gj}, j = 1, \dots, m$, and have terminal voltages $E_j \angle \theta_j, j = 1, \dots, m$. Define $E = [E_1, \dots, E_m]^t = [V_{n_b-m+1}, \dots, V_{n_b}]^t$. We shall take the n_b th bus as the reference bus (i.e., $\theta_{n_b} := 0$) which means that elements of the angles vector $\theta = [\theta_1, \dots, \theta_{n_b-1}]^t$ are referred to the phase angle of the m th generator. The generator reactive outputs are $Q_{gj}, j = 1, \dots, m$ and they have upper limits of $Q_{lim,j}, j = 1, \dots, m$. For each generator, the terminal voltage and the reactive power have the following characteristic:

$$\begin{aligned} E_j &= E_{oj} & \text{if } Q_{gj}(E_{oj}) \leq Q_{lim,j} \\ Q_{gj}(E_j) &= Q_{lim,j} & \text{if } Q_{gj}(E_{oj}) > Q_{lim,j} \end{aligned}$$

Note that at the point where the limit is first encountered, both the above constraints (on E and Q_g) are satisfied.

For simplicity, assume the system is lossless. Thus the bus admittance matrix Y is purely imaginary, i.e., its elements can be written as $Y_{ij} = jB_{ij}$. The turns ratios $n_i, i = 1, \dots, t$ are included in the susceptance elements of the admittance matrix, i.e., B_{ij} are regarded as functions of the vector of tap ratios $n = [n_1, \dots, n_t]^t$.

Let P_{bi} and Q_{bi} denote the total real and reactive power leaving the i th bus via transmission lines or transformers. Then

$$\begin{aligned} P_{bi}(\theta, V, E, n) &= \sum_{j=1}^{n_b} V_i V_j B_{ij}(n) \sin(\theta_i - \theta_j), \\ i &= 1, \dots, n_b - 1 \end{aligned} \quad (32)$$

$$\begin{aligned} Q_{bi}(\theta, V, E, n) &= - \sum_{j=1}^{n_b} V_i V_j B_{ij}(n) \cos(\theta_i - \theta_j), \\ i &= 1, \dots, n_b \end{aligned} \quad (33)$$

We are not modelling any swing dynamics of generators. Therefore the n_b th bus shall be chosen as a slack bus to absorb any real power mismatch which may occur as system conditions alter. This corresponds to deleting the n_b th real power equation. Note that it is a simple extension to a distributed slack formulation where all generators absorb some predetermined proportion of the power mismatch. Define $P_b = [P_{b,1}, \dots, P_{b,n_b-1}]^t = [-P_r^t \ P_g^t]^t$ and $Q_b = [Q_{b,1}, \dots, Q_{b,n_b}]^t = [-Q_r^t \ Q_g^t]^t$ where P_r, Q_r are $(n_b - m)$ -vectors referring to non-generator buses, and P_g, Q_g are $(m - 1)$, m -vectors respectively referring to generator buses.

Let D_p be the set of d_p buses,

$$D_p = \{i : i \leq n_b - m; \text{ a real power dynamic load is attached to bus } i\}$$

and D_q be the set of d_q buses,

$$D_q = \{i : i \leq n_b - m; \text{ a reactive power dynamic load is attached to bus } i\}$$

Then there are $(n_b - m - d_p)$ real power loads which are static functions of voltage $P_{di} = P_{si}(V_i)$. Also, there are $(n_b - m - d_q)$ static reactive power loads, $Q_{di} = Q_{si}(V_i)$. Note that these functions of voltage could be constant, or in fact zero.

Define

$$\begin{aligned} 0 &= P_{di} - b_{pi}(x_{pi}, V_i) &:= g_{1i}(V, x_p, P_d) &\text{if } i \in D_p \\ 0 &= P_{di} - P_{si}(V_i) &:= g_{1i}(V, x_p, P_d) &\text{if } i \notin D_p \end{aligned} \quad (34)$$

and

$$\begin{aligned} 0 &= Q_{di} - b_{qi}(x_{qi}, V_i) &:= g_{3i}(V, x_q, Q_d) &\text{if } i \in D_q \\ 0 &= Q_{di} - Q_{si}(V_i) &:= g_{3i}(V, x_q, Q_d) &\text{if } i \notin D_q \end{aligned} \quad (35)$$

Now consider the modelling of tap-changers. Tap-changers are a special type of transformer whose turns-ratio is automatically adjusted in order to regulate the voltage of a specified bus, i.e., usually the voltage at the secondary side of the transformer. Each tap-changer has a reference voltage value $V_i^o, i = 1, \dots, t$, which is the target value of an automatic voltage regulator. The tap movements are discrete and can be modelled by

$$n_i(t) = n_i(t^-) - d f(V_i - V_i^o) \quad (36)$$

where $d > 0$ is the size of each tap-step,

$$f(x) = \begin{cases} -1 & \text{if } x \leq -\varepsilon \\ 0 & \text{if } |x| < \varepsilon \\ 1 & \text{if } x \geq \varepsilon \end{cases} \quad (37)$$

and $(-\varepsilon, \varepsilon)$ is the deadband [27]. A simpler alternative to (36) is a continuous-state model of tap-changing. This model could be written as,

$$\frac{dn_i}{dt} = \frac{1}{T_i} (V_i^o - V_i) \quad (38)$$

where T_i is the tapping time constant of transformer i . This model provides an acceptable approximation over

the longer term and so forms a convenient basis for the analytical study of voltage collapse behaviour.

Combining the transformer tap dynamic equations (38) with the equations describing the power balance at buses $1, \dots, n_b - 1$, gives the total system representation

$$\dot{n} = T^{-1}(V^o - V_r) := f_1(V_r) \quad (39)$$

$$\dot{x}_p = a_p(x_p, V) := f_2(x_p, V) \quad (40)$$

$$\dot{x}_q = a_q(x_q, V) := f_3(x_q, V) \quad (41)$$

$$0 = g_1(V, x_p, P_d) \quad (42)$$

$$0 = -P_t(\theta, V, n) + P_d := g_2(\theta, V, n, P_d) \quad (43)$$

$$0 = g_3(V, x_q, Q_d) \quad (44)$$

$$0 = -Q_t(\theta, V, n) + Q_d := g_4(\theta, V, n, Q_d) \quad (45)$$

$$0 = P_g(\theta, V, n) - P_g^o := g_5(\theta, V, n) \quad (46)$$

where x_p, x_q denote d_p, d_q vectors referring to the real and reactive power dynamic load buses respectively; T is a diagonal $(t \times t)$ matrix; P_d and Q_d denote $(n_b - m)$ -vectors with components P_{di} and Q_{di} respectively.

This model is a set of differential-algebraic (DA) equations. The dynamic states of the system are the tap positions n and load dynamic variables x_p and x_q whilst the algebraic variables are bus quantities θ, V, P_d, Q_d . Define $g = [g_1^t \ g_2^t \ g_3^t \ g_4^t \ g_5^t]^t$. Then $g(n, x_p, x_q, \theta, V, P_d, Q_d) = 0$ defines a $(d_p + d_q + t)$ -manifold called the constraint manifold. The system always remains on that manifold and is driven over that manifold by the dynamics of (39)-(41).

The model (39)-(46) assumes that all generator voltages are fixed at $E_{oj}, j = 1, \dots, m$, i.e., in vector form, $E = E_o := [E_{o1}, \dots, E_{om}]^t$. Based on this assumption, the m reactive power equations at the generator buses are deleted. But, if any of the generator reactive powers Q_{gj} becomes fixed (at its limit), the corresponding generator voltage E_j becomes a variable, and a reactive power equation at the generator bus j given by

$$0 = Q_{gj}(\theta, V, E, n) - Q_{lim,j} := g_{6j}(\theta, V, E, n) \quad (47)$$

should be added to the system model (39)-(46). Suppose that there are h generators whose reactive powers are at limits. Then, the constraint manifold will be augmented by equations $g_6 = [g_{6,1}, \dots, g_{6,h}]^t$, and will involve h more variables, the elements of E corresponding to the generators that are on limits. The constraint manifold will still have dimension $(d_p + d_q + t)$.

Note that generator dynamics are not modelled here. This simplification is based upon the fact that generally swing dynamics are faster than the load dynamics of interest in mid to long-term voltage stability studies; this suggests that the traditional "swing equations" may be omitted in the study of these voltage collapse scenarios. The inclusion of swing dynamics is considered in [30].

If tap-changers are modelled in the discrete form (36), the final model has a hybrid differential - algebraic -

discrete form [28]

$$\dot{x} = f(x, y, z) \quad (48)$$

$$0 = g(x, y, z) \quad (49)$$

$$z(k+1) = h(x, y, z(k)) \quad (50)$$

A complication is that tap-changers do not act synchronously although one approach to simplification is to assume they do for purposes of analytical insight [27].

Assuming the continuous OLTC model is adequate, equations (48)-(50) reduce to a DA model (48),(49). There are still analytical and numerical difficulties with DA systems [29]. At surfaces where the algebraic equation (49) is not uniquely solvable, the system behaviour becomes fast and complex. It turns out that for certain types of transient load characteristics, the algebraic equation is uniquely solvable and the DA model (48),(49) becomes purely differential. This issue is studied in detail in [30].

An analytical study of the model described above has only been partially completed. Progress so far is reported in [9,16-18,23,25]. Of course, numerous other results are relevant as described in these papers. In particular, the results of Venkatasubramanian et.al. [29] form a sophisticated basis for many stability considerations in DA models of power systems.

4 Stability Definitions

Proposals for stability definitions have been given in [4,14,15] building off a discussion in [13] and some other ideas, e.g. [19]. A brief review will be given here. The notation $x \geq y (\leq y)$ for vectors x and y means that all components of x and y satisfy $x_i \geq y_i (\leq y_i)$.

We have several interesting concepts related to voltage V :

- Viability

The system voltages lie in an acceptable band

$$\underline{V} \leq V \leq \bar{V}$$

- Regularity

A power system is at a voltage regular operating point if: i) for any PQ bus, $\Delta Q_i > 0$ gives $\Delta V > 0$; ii) for any PV bus, $\Delta E_j > 0$ gives $\Delta V > 0$.

This definition describes strong QV regularity. A weaker local regularity condition can also be stated:

A power system is at a locally voltage regular operating point if: i) for any PQ bus, $\Delta Q_i > 0$ gives $\Delta V_i > 0$; ii) for any PV bus, $\Delta E_j > 0$ gives $\Delta Q_j > 0$.

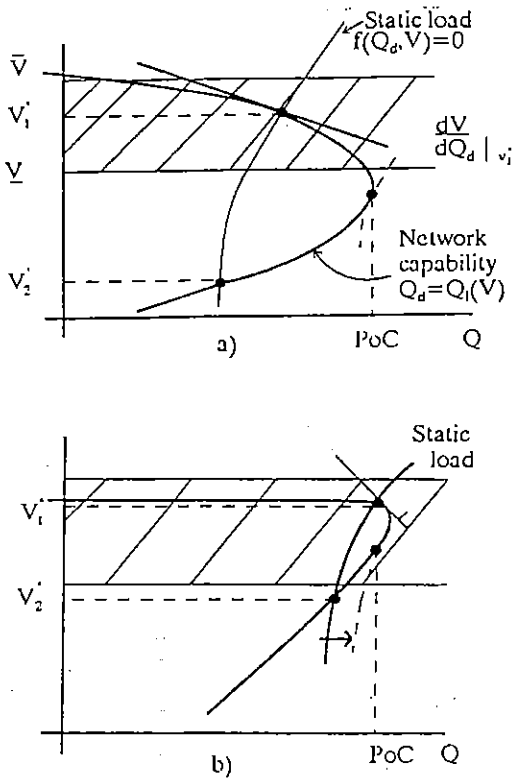


Figure 4: Static Voltage Stability Concepts

This weaker condition corresponds to satisfaction of the Venikov criterion [35] at each bus. In Section 5.2, these concepts are interpreted analytically in terms of properties of Jacobians.

These concepts relate to static behaviour. They are illustrated in Figure 4 for the simple case of a single dynamic reactive load on a single line system [25]. The point where $\frac{dV}{dQ}$ becomes infinite is in general only significant as the Point of Maximum Loadability (PML). In Figure 4a), the constraint manifold and load steady-state characteristic are drawn. Their intersections at V_1^*, V_2^* are the equilibria. V_1^* is shown as viable and V_2^* not viable. The Point of Collapse (PoC) is defined as the point where bifurcation occurs between these curves due to some parameter variation. The PoC and PML coincide only in the case where we have constant power steady-state load characteristic. Viability, QV regularity and proximity to collapse are in general independent properties. For instance, if the PoC lies in the viable band, viable voltages may be unacceptably close to collapse - see Figure 4b). Constant power loads are often assumed and then closeness to the PoC corresponds to high $Q-V$ sensitivity. Otherwise, sensitivity with respect to the bifurcation parameter must be considered. A more general concept of regularity which gives sensitivity behaviour of angles and voltages to

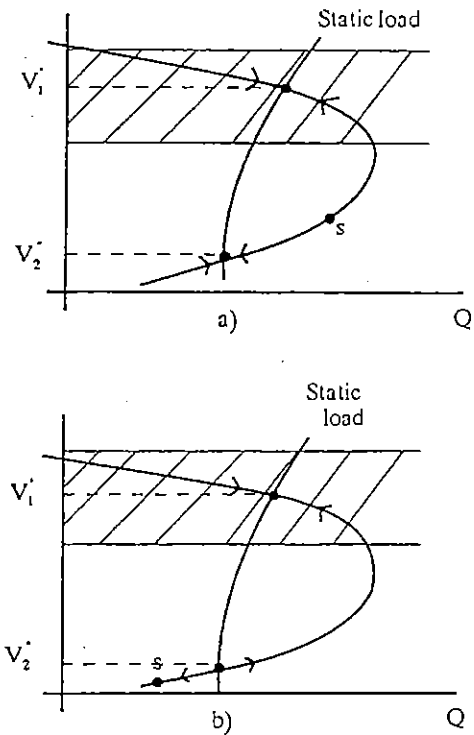


Figure 5: Dynamic Voltage Stability Concepts

real and reactive powers can be useful.

The remaining concepts relate to dynamic behaviour:

- Small disturbance voltage stability

A power system at a given operating state is small disturbance voltage stable if following any small disturbance, its voltages are identical to or close to their pre-disturbance values.

- Large disturbance voltage stability

A power system at a given operating state and subject to a given large disturbance is large disturbance voltage stable if the voltages approach post-disturbance equilibrium values.

- Voltage collapse

A power system at a given operating state and subject to a given large disturbance undergoes voltage collapse if it is voltage unstable or the post-disturbance equilibrium values are non-viable.

It may be found necessary to extend the definition of large disturbance stability to cater for small oscillations around an equilibrium point.

In Figure 5 a) b) two cases for a single line and load system [25] are illustrated. Typically the high voltage

equilibrium is stable, but the low voltage solution can be small disturbance stable or unstable. Depending on the disturbance and the transient load characteristic, voltage collapse can occur by the voltage approaching a nonviable equilibrium or a singular region - see points in Figure 5 b). Looking at Figures 4 and 5, we see many scenarios. For instance, if the load curve is pushed further to the right, there seems to be the possibility that V_2^* can be viable, irregular and stable. This in fact is a situation that has been deliberately designed in practice with an SVC included in the load [8].

We emphasize that these definitions require precise mathematical interpretation before embarking on analysis, eg. voltage stability can be translated to asymptotic stability (in the sense of Lyapunov) [33] of the post-disturbance equilibrium.

5 Voltage Stability Analysis for Multi-load System

The model developed in Section 3 attempts to represent uniformly the devices which practical observations declare relevant to voltage instability. The interaction of generator limits, OLTC dynamics and load dynamics through a network needs to be better understood analytically. In [9,25] it is shown that even the simplest case of a single dynamic load power on a single line system contains nontrivial stability questions. The following discussion generalises that analysis to the multi-load network situation.

5.1 Multi-load System Model

The voltage stability analysis for the single dynamic load case using the general IO load model (15) is given in [25]. Analysis of multi-load systems using specific state load models is given in [16-18]. We will consider here the multi-load system where loads are modelled by their general IO model. For convenience, consider the case where all loads are dynamic and generator buses have been eliminated from the power flow. With load bus voltages $V_i \angle \theta_i$, $i = 1, 2, \dots, N$, where $N = n_b - m$, equations (43), (45) give power flow equations as

$$P_d = P_l(\theta, V) \quad (51)$$

$$Q_d = Q_l(\theta, V) \quad (52)$$

where all quantities are N -vectors.

In vector form, the general IO load models are

$$e_p(P_d, V)\dot{P}_d + f_p(P_d, V) = g_p(P_d, V)\dot{V} \quad (53)$$

$$e_q(Q_d, V)\dot{Q}_d + f_q(Q_d, V) = g_q(Q_d, V)\dot{V} \quad (54)$$

where e_p, g_p, e_q, g_q are $N \times N$ matrices. (We add matrices e_p, e_q for convenience [26]; of course, they can be removed without loss of generality.)

Differentiating (51), (52) gives

$$\begin{bmatrix} \dot{P}_d \\ \dot{Q}_d \end{bmatrix} = J \begin{bmatrix} \dot{\theta} \\ \dot{V} \end{bmatrix} \quad (55)$$

where J is the power flow Jacobian matrix. Combining with (53), (54) gives

$$C_d(\theta, V) \begin{bmatrix} \dot{\theta} \\ \dot{V} \end{bmatrix} = g_s(\theta, V) \quad (56)$$

where

$$C_d = -EJ_t \quad (57)$$

$$E(\theta, V) = \begin{bmatrix} e_p(P_l(\theta, V), V) & 0 \\ 0 & e_q(Q_l(\theta, V), V) \end{bmatrix} \quad (58)$$

$$J_t(\theta, V) = \begin{bmatrix} -\frac{\partial P_l}{\partial \theta} & e_p^{-1}g_p(P_l(\theta, V), V) - \frac{\partial P_l}{\partial V} \\ -\frac{\partial Q_l}{\partial \theta} & e_q^{-1}g_q(Q_l(\theta, V), V) - \frac{\partial Q_l}{\partial V} \end{bmatrix} \quad (59)$$

$$g_s(\theta, V) = - \begin{bmatrix} f_p(P_l(\theta, V), V) \\ f_q(Q_l(\theta, V), V) \end{bmatrix} \quad (60)$$

It is clear that g_s is essentially a power flow function based on static characteristics and J_t is derived from the network and transient load characteristics. The dynamics of (56) are well-defined when both $E(\theta, V)$ and $J_t(\theta, V)$ are nonsingular. We define points where $\det C_d(\theta, V) = 0$ as dynamically singular. Such points are investigated in companion paper [30].

5.2 Regularity Analysis

We will analyse the multi-load system via the model formed from (51) to (54). The steps to (56) interpreted V, θ as dynamic states and P_d, Q_d as the algebraic states. (Alternatively, we could obtain a form locally where the dynamic states were P_d, Q_d and the algebraic states were V, θ . This is less natural though as the power flow equations (51), (52) give V, θ implicitly.)

For static analysis, we set derivatives in (56) to zero which gives

$$g_s(\theta, V; \mu) = 0 \quad (61)$$

For convenience, the vector of power flow and load parameters μ is made explicit here. Well-known sensitivity analysis leads to

$$D(\theta, V; \mu)J_s(\theta, V; \mu) \begin{bmatrix} d\theta \\ dV \end{bmatrix} = F_\mu(\theta, V; \mu)d\mu \quad (62)$$

where

$$D(\theta, V; \mu) = \begin{bmatrix} \frac{\partial f_p}{\partial P_d} |_{P_d=P_l(\theta, V; \mu)} & 0 \\ 0 & \frac{\partial f_q}{\partial Q_d} |_{Q_d=Q_l(\theta, V; \mu)} \end{bmatrix} \quad (63)$$

$$J_s(\theta, V; \mu) = \begin{bmatrix} -\frac{\partial P_l}{\partial \theta} & -\frac{\partial P_l}{\partial V} - \left(\frac{\partial f_p}{\partial P_d}\right)^{-1} \frac{\partial f_p}{\partial V} \\ -\frac{\partial Q_l}{\partial \theta} & -\frac{\partial Q_l}{\partial V} - \left(\frac{\partial f_q}{\partial Q_d}\right)^{-1} \frac{\partial f_q}{\partial V} \end{bmatrix} \quad (64)$$

$$F_\mu(\theta, V; \mu) = \begin{bmatrix} \frac{\partial f_p}{\partial \mu} \\ \frac{\partial f_q}{\partial \mu} \end{bmatrix} \quad (65)$$

Note that if $\mu = (P_d, Q_d)$, i.e., constant power, then $F_\mu = D$.

The following result is a generalisation of one in [25].

Result 1 Suppose $\det D(\theta, V; \mu) \neq 0$. The system at equilibrium point $(\theta^*, V^*; \mu^*)$ is regular iff

$$[J_s(\theta^*, V^*; \mu^*)^{-1} D(\theta^*, V^*; \mu^*)^{-1} F_\mu(\theta^*, V^*; \mu^*)]_{ij} > 0 \quad (66)$$

where $[\cdot]_{ij}$ refers to the ij th element of the matrix.

This criterion (66) includes many special cases aimed at generalising the Venikov criterion [35,9].

In the special case of constant power, regularity is determined by the inverted static power flow Jacobian J_s . Regularity is lost when the static power flow Jacobian becomes singular.

Considering QV regularity, it is useful to write the Jacobian matrix J_s described in (64) as

$$J_s = \begin{bmatrix} G_{p\theta s} & G_{pvs} \\ G_{q\theta s} & G_{qvs} \end{bmatrix}$$

Subscript 's' is added to denote the use of the static load functions.

Result 2 Suppose $\det G_{p\theta s} \neq 0$. The system at equilibrium point (θ^*, V^*) is QV regular if

$$G_{qvs}^* = G_{qvs} - G_{q\theta s} F_{p\theta s}^{-1} G_{pvs} \quad (67)$$

is an irreducible M-matrix.

Matrix G_{qvs}^* denotes the Schur complement of matrix $G_{p\theta s}$ within a partitioned matrix. The condition $\det G_{p\theta s} \neq 0$ is reasonable for situations where there are no angle stability problems.

Special Case Exponential Recovery [16]

From the active and reactive power counterpart of (18), we have

$$f_p(P_d, V) = P_d - P_s(V) \quad (68)$$

$$f_q(Q_d, V) = Q_d - Q_s(V) \quad (69)$$

which gives

$$D(\theta, V) = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, J_s(\theta, V) = \begin{bmatrix} -\frac{\partial P_s}{\partial \theta} & \frac{\partial(P_s - P_d)}{\partial V} \\ -\frac{\partial Q_s}{\partial \theta} & \frac{\partial(Q_s - Q_d)}{\partial V} \end{bmatrix} \quad (70)$$

where I is the identity matrix.

It can easily be verified that for the adaptive load model (29),(30), which is given in IO form at (31), $J_s(\theta, V)$ is the same as in (70), and

$$D(\theta, V) = \begin{bmatrix} P_t(V) & 0 \\ 0 & Q_t(V) \end{bmatrix} \quad (71)$$

Result 2 is closely related to earlier ones by Abe and Isono [39] and Schleuter [19]. However, the result is conservative; in general, it is enough to check the signs of the elements of G_{qvs}^{*-1} . According to the definition of voltage regular operating points, strong QV regularity corresponds to G_{qvs}^{*-1} having all positive elements. Local QV regularity requires that the diagonals of G_{qvs}^{*-1} are positive. Note that if G_{qvs}^* is diagonally dominant and symmetric (a common property of power systems), then local QV regularity is equivalent to G_{qvs}^* being positive definite.

The main point being made here is that static regularity properties can be precisely related to the static model (61) which depends on the static load characteristics given by functions $f_p(P_d, V)$, $f_q(Q_d, V)$.

The Jacobian matrix J_s determines static regularity properties. J_s and various submatrices (and their Schur complements) have featured in much recent work on static Voltage Collapse Proximity Indicators (VCPI) [19-22]. For instance Löf et al. [20,21] look at the profile of the minimum singular value $\sigma_{\min} G_{qvs}^*$ with respect to loading. The situation $\sigma_{\min} G_{qvs}^* = 0$ corresponds to the PML, i.e., the system loadability limit, where QV sensitivity becomes infinite.

5.3 Small Disturbance Stability

Linearizing the dynamic model (56) about the operating point (V^*, θ^*) gives

$$C_d(\theta^*, V^*) \begin{bmatrix} \Delta \dot{\theta} \\ \Delta \dot{V} \end{bmatrix} = D(\theta^*, V^*) J_s(\theta^*, V^*) \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (72)$$

where D and J_s are defined at (63) and (64) respectively, $\Delta V = V - V^*$ and $\Delta \theta = \theta - \theta^*$.

A general stability criterion is as follows.

Result 3 Assume $\det E(\theta^*, V^*) \neq 0$ and $\det J_t(\theta^*, V^*) \neq 0$. The load system at operating point (θ^*, V^*) is small disturbance stable iff the matrix $C_d^{-1}(\theta^*, V^*) D(\theta^*, V^*) J_s(\theta^*, V^*)$ is (asymptotically) stable.

Result 3 immediately suggests dynamical voltage stability indices based on the eigenstructure of $C_d^{-1}(\cdot) D(\cdot) J_s(\cdot)$. Recall from Section 5.2 that static indices were based on J_s and its submatrices. So this simple result neatly connects the static regularity analysis of J_s with dynamic characteristics of loads in C_d to determine overall stability. The static aspect is represented by J_s derived from steady-state load behaviour

(68-69); the dynamic question (depending on J_s and J_t) is determined by the interaction of both static and transient load characteristics.

Special Case (cont) Exponential Recovery

In the exponential recovery case, reference [16] presents several explicit criteria. J_t becomes the Jacobian corresponding to the dynamic power flow defined in terms of the transient load characteristics

$$P_o + P_i(V) - P_i(\theta, V) = 0 \quad (73)$$

$$Q_0 + Q_i(V) - Q_i(\theta, V) = 0 \quad (74)$$

whilst

$$E(\theta, V) = T_d = \begin{bmatrix} T_p & 0 \\ 0 & T_q \end{bmatrix} \quad (75)$$

Also, from (70), $D(\theta, V) = I$, the identity matrix. The linearised dynamics are then given by

$$-T_d J_t(\theta^*, V^*) \begin{bmatrix} \Delta \dot{\theta} \\ \Delta \dot{V} \end{bmatrix} = J_s(\theta^*, V^*) \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (76)$$

Corollary Assume $\det J_t(\theta^*, V^*) \neq 0$. The system at equilibrium point (θ^*, V^*) is small-disturbance stable iff the matrix $-J_t^{-1}(\theta^*, V^*)T_d^{-1}J_s(\theta^*, V^*)$ is (asymptotically) stable.

In [9,16], it is shown that static stability typically implies dynamic stability for load systems of this type, but the converse need not be so, i.e. irregular and stable operating points appear reasonable.

As indicated above, typically the high voltage operating point is regular and stable. The low voltage solution is irregular; it is stable or unstable according to the position of singularities on the constraint manifold. Figure 5 illustrates this for the case of a single dynamic load. In [16], this issue is discussed for example systems.

More specific stability criteria for special types of loads are studied in [16,17]. The formulation of equations for stability studies of the adaptive load case is given in [18].

Large disturbance behaviour in multi-load systems has been considered to some extent in [16]. Example behaviour and preliminary results on Lyapunov analysis are presented there.

5.4 Large Disturbance Stability

Firstly, we take a brief look at the single dynamic reactive load on a line case [9,18,25]. Model (56) becomes scalar of the form

$$C_d(V)\dot{V} = g_s(V) = -f_g(Q_I(V), V) \quad (77)$$

Recall that the transient load characteristic has the form $h_q(Q_d, V) = c$ where c is a constant of integration determined by the initial conditions, i.e. prefault operating point. Referring to Figure 6, we consider the system in prefault equilibrium at point a . The fault affects the line capability, so the post fault equilibria are points b, c corresponding to voltages V_1^*, V_2^* respectively. Suppose the transient load characteristic (dashed curve) intersects the post fault constraint manifold at points d, e . At the fault (and any associated protection action), the system will move to either point d or e . Analysis yields the following scenarios:

1. Transient jump to point d ; subsequent trajectory moves along constraint manifold to stable equilibrium b - voltage stability and no collapse.
2. Transient jump to point e ; singular point at 1; subsequent trajectory moves along constraint manifold to stable equilibrium c - voltage stability and collapse (since V_2^* is not viable).
3. Transient jump to point e ; singular point at 2; equilibrium c is unstable; subsequent trajectory moves along constraint manifold to singular point-voltage instability and collapse.

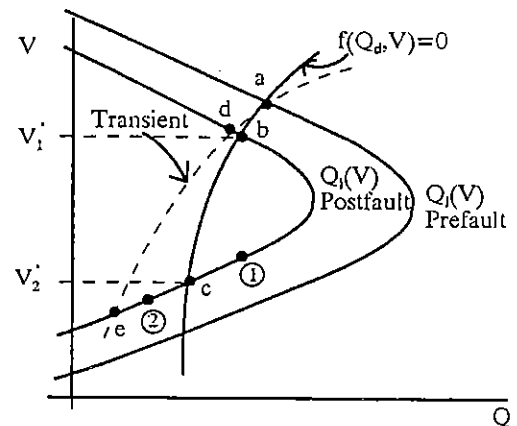


Figure 6: Large Disturbance Behaviour

The regions of attraction of the stable equilibria can be described precisely and qualitative insights about voltage stability easily obtained, cf. equal area criterion for angle stability. The analytical basis for such results is Lyapunov stability theory [33]. This suggests that we look for energy related Lyapunov functions for system (77). Some progress is reported in [17]. One result there achieves a potential energy type function which is a valid Lyapunov function. That result is generalized in [25].

These previous results are for a single load case. We now propose a Lyapunov function for the multi-load

system described by (56)-(60).

Result 4 For the multi-load system (56)-(60) under the conditions of Result 3, the potential energy function

$$W(x, x^o) = \int_{x^o}^x -(C_d(x)^{-1}g_s(x))^t dx \quad (78)$$

where $x^t = [\theta^t \ V^t]$, is a valid Lyapunov function.

The proof follows the usual Lyapunov function argument. The conditions for small disturbance stability ensure that W is locally positive definite since $\frac{\partial^2 W}{\partial x^2}|_{x^o} = C_d^{-1}(x^o)D(x^o)J_s(x^o)$. Also,

$$\begin{aligned} \dot{W}(x, x^o) &= -(C_d(x)^{-1}g_s(x))^t \dot{x} \\ &= -(C_d(x)^{-1}g_s(x))^t (C_d(x)^{-1}g_s(x)) \end{aligned}$$

The result follows from standard Lyapunov theory [33]. One important consideration in the extension from the single load case to multi-load systems is that the integral (78) must be path independent for the energy function to be useful. To ensure this, the Jacobian of $C_d(x)^{-1}g_s(x)$ must be symmetric. This implies that the matrix $C_d^{-1}(x)D(x)J_s(x)$ of the linearized system (72) must be symmetric. There is some freedom in the choice of load models which may help to achieve the required symmetry. The application of these ideas to stability assessment is still under investigations.

It can be seen from (56) that stability properties are influenced by where $C_d(\theta, V)$ (or $J_t(\theta, V)$) goes singular. This is considered further in [30]. Note that the factors governing singularity are the transient load characteristics and line capability. Points of singularity may repel or attract trajectories. The behaviour of the system near such points is complex and will not be further entered into here.

5.5 Example

The multi-load system shown in Figure 7 shall be used to illustrate the preceding voltage stability analysis. This multi-load case was considered in [16]. The system and its constraints are the same as those used by Abe and Isono in [39], except for the reactive power loads being of the recovery type described in Section 5.1.

In particular, it is assumed that P_{d1}, P_{d2} are constant, and the reactive power loads are described by the exponential recovery model (18). The system has two equilibria; the high voltage operating point 1 in Figure 8 is viable, whilst the low voltage operating point 2 is non-viable.

It is shown in [16] that in the case of the load system with static real power load, Corollary reduces to the stability of the matrix $-G_{qvi}^{-1}T_q^{-1}G_{qvs}^* = C_{qd}^{-1}G_{qvs}^*$ evaluated at equilibrium point, where G_{qvi}^* is the

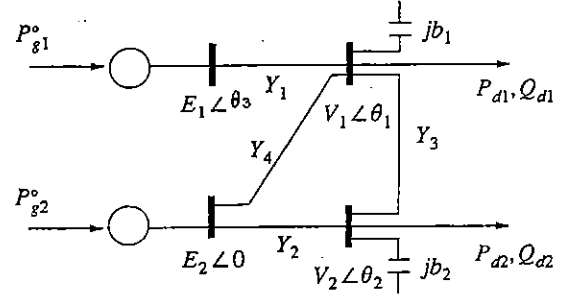


Figure 7: Model System

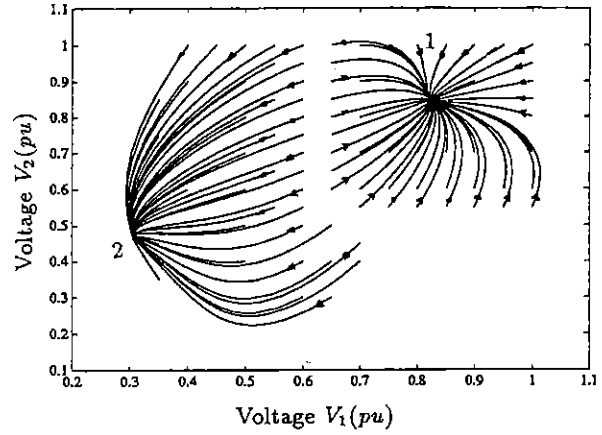


Figure 8: Voltage trajectories

corresponding Schur complement of the dynamic Jacobian J_t . Matrix $G_{qvs}^{*-1}(\theta^*, V^*)$ and eigenvalues of $C_{qd}^{-1}(\theta^*, V^*)G_{qvs}^*(\theta^*, V^*)$ for the system of Figure 7 are given in Table 2.

Table 2

Operating point	G_{qvs}^{*-1}	Eigenvalues of $C_{qd}^{-1}G_{qvs}^*$
1	0.4885 0.2423 0.2494 0.6942	-0.1758 -0.1268
2	0.0410 -0.2445 -0.3734 1.0030	-0.1022 -0.1261

Regularity analysis based on $G_{qvs}^*(\theta^*, V^*)$, i.e. signs of the components of $G_{qvs}^{*-1}(\theta^*, V^*)$, indicates that only operating point 1 is QV regular (G_{qvs}^{*-1} has all positive elements); the other equilibrium point 2 is irregular. But based on the eigenvalues of $C_{qd}^{-1}(\theta^*, V^*)G_{qvs}^*(\theta^*, V^*)$, both operating points are dynamically stable. Figure 8 shows that voltage trajectories starting from points where $V \neq V^*$ stop when either one of the operating points is encountered. So for the chosen static and transient load characteristics, each equilibrium point is stable. However, for other transient load characteristics, we can have small disturbance instability, which causes voltage collapse in some situations.

6 Extension to Tap-Changing Transformers

Section 5 presented a methodology for load stability analysis. That analysis is now extended to include OLTCs. Consider the system of Section 5.1, but now with t on-load tap-changing transformers at the first t buses. (Recall the model developed in Section 3.) The vector of load bus voltages is then $V = [V_r^t V_u^t]^t$ where V_r, V_u are $t, (N-t)$ -vectors referring to regulated and unregulated bus voltages respectively.

Using the continuous-state tap-changing model, the dynamical model is

$$\dot{n} = T^{-1}(V^o - V_r) \quad (79)$$

$$e_p(P_d, V) \dot{P}_d + f_p(P_d, V) = g_p(P_d, V) \dot{V} \quad (80)$$

$$e_q(Q_d, V) \dot{Q}_d + f_q(Q_d, V) = g_q(Q_d, V) \dot{V} \quad (81)$$

$$0 = P_d - P_l(\theta, V, n) \quad (82)$$

$$0 = Q_d - Q_l(\theta, V, n) \quad (83)$$

Within tap limits, the equilibria satisfy $V_r = V^o$, together with

$$g_s(\theta, V, n) = 0 \quad (84)$$

where g_s is defined at (60), but now with the n dependence of P_l, Q_l taken into account. The equilibria (θ^*, V^*, n^*) have stability properties dependent on the load characteristics, network and tapping time delay T . Regularity in the sense of the influence of bifurcation parameters on the unregulated voltages V_u can be analyzed using similar steps to Section 5.1.

Proceeding as for the multi-load case in Section 5, the model can be written as

$$\dot{n} = T^{-1}(V^o - V_r) \quad (85)$$

$$C_d(\theta, V, n) \begin{bmatrix} \dot{\theta} \\ \dot{V} \end{bmatrix} = g_s(\theta, V, n) + t_n(\theta, V, n) \quad (86)$$

where C_d is defined as before but with n dependence added, and t_n is an additional term given by

$$\begin{aligned} t_n(\theta, V, n) &= \begin{bmatrix} t_p(\theta, V, n) \\ t_q(\theta, V, n) \end{bmatrix} \\ &= \begin{bmatrix} e_p \frac{\partial P_l}{\partial n} T^{-1}(V_r - V^o) \\ e_q \frac{\partial Q_l}{\partial n} T^{-1}(V_r - V^o) \end{bmatrix} \end{aligned} \quad (87)$$

The dynamics are well-defined away from the singular surface $\det C_d(\theta, V, n) = 0$.

Denote $x = \begin{bmatrix} \theta \\ V \end{bmatrix}$. For small-disturbance stability, we use a linearized version of (85), (86) around the equilibrium point (x^*, n^*) ,

$$\begin{bmatrix} \Delta \dot{n} \\ \Delta \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & -T^{-1}K \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \Delta n \\ \Delta x \end{bmatrix} \quad (88)$$

i.e.

$$\begin{bmatrix} \Delta \dot{n} \\ \Delta \dot{x} \end{bmatrix} = A \begin{bmatrix} \Delta n \\ \Delta x \end{bmatrix} \quad (89)$$

where

$$K = [0_{t \times N} \ I_t \ 0_{t \times (N-t)}] \quad (90)$$

$$A_{21} = C_d^{-1}(x^*, n^*) \frac{\partial(g_s + t_n)}{\partial n}(x^*, n^*) \quad (91)$$

$$A_{22} = C_d^{-1}(x^*, n^*) \frac{\partial(g_s + t_n)}{\partial x}(x^*, n^*) \quad (92)$$

Note that $0_{i \times j}$ refers to an $i \times j$ matrix of zeros, and I_t is the t -dimensional identity matrix.

At the equilibrium point (x^*, n^*)

$$\frac{\partial g_s}{\partial x} = D(x^*, n^*) J_s(x^*, n^*) \quad (93)$$

$$\frac{\partial t_n}{\partial x} = -E(x^*, n^*) J_n(x^*, n^*) T^{-1} K \quad (94)$$

$$\frac{\partial g_s}{\partial n} = D(x^*, n^*) J_n(x^*, n^*) \quad (95)$$

$$\frac{\partial t_n}{\partial n} = [0_{2N \times 1}] \quad (96)$$

where

$$J_n(\theta, V, n) = \begin{bmatrix} -\frac{\partial P_l}{\partial n} \\ -\frac{\partial Q_l}{\partial n} \end{bmatrix} \quad (97)$$

$E(\cdot), D(\cdot)$, and $J_s(\cdot)$ are defined by (58), (63), and (64) respectively.

Result 5 Assume $\det C_d(x^*, n^*) \neq 0$. The load-tap changer system of (85), (86) at operating point (x^*, n^*) is small disturbance stable iff the matrix

$$\begin{bmatrix} I_t & 0 \\ 0 & C_d^{-1} \end{bmatrix} \begin{bmatrix} 0 & -T^{-1}K \\ DJ_n & (DJ_s - EJ_n T^{-1}K) \end{bmatrix} \quad (98)$$

with all matrices evaluated at (x^*, n^*) , is (asymptotically) stable.

Recall from (57) that C_d is dependent on the transient Jacobian J_t . Result 5 therefore establishes the connection between small disturbance stability and the Jacobians J_s, J_t and J_n . It is easy to check that if the tap ratios n are fixed, then the stability conditions of Result 5 reduce to those of Result 3.

Result 5 can be used to classify the equilibria in terms of small disturbance stability. Just as in Section 5.4, we can explore the large disturbance stability in terms of pre- and post-fault static capability, transient load behaviour and the singular surface. For a simple single load system model of the exponential recovery form (18) with zero recovery times $T_p = T_q = 0$, Vu and Liu [31] have given a compelling explanation of voltage collapse in this simple system. The scenario includes generator limits and a concept of a 'shrinking leaf' to describe the behaviour of the $\dot{V} = 0$ surface ($g_s(n, V) = 0$) at a generator limit. A generalisation

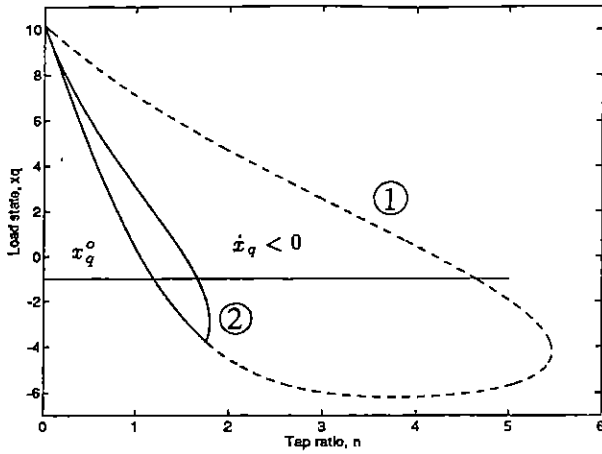


Figure 9: Excess load region as viewed in $n - x_q$ plane

to more general recovery models is given in [23]. The results in [23] identify a so-called excess load region (to replace the 'shrinking leaves' of [31,42]), i.e. the region where the load demand is (transiently) greater than the steady-state load requirement. Referring to (77) and (69), this corresponds to $Q_d > Q_s(V)$. Figure 9 illustrates the excess load region as seen in the $n - x_q$ plane. The equilibria of the system are intersections of the line $x_q = x_q^o$, which corresponds to the set point voltage $V = V^o$, and the boundary of the excess load region. Curve 1 represents the boundary of the region obtained for fixed generator terminal voltage, whilst curve 2 takes into account generator operation at its limit. As seen, the effect of the generator reactive limit is to contract the $\dot{x}_q < 0$ region. Insights into the mechanism of voltage collapse revealed the importance of this (contracted) excess load region and the load state behaviour during voltage collapse. Actually, in voltage collapse scenarios, the system trajectory either exits/passes by the excess load region from its low side, or exits from its upper side. In [23], it is shown that this system trajectory behaviour plays a very important role in the voltage control problem (particularly tap-locking) aimed at arresting voltage collapse.

An unsolved problem is to identify valid Lyapunov functions for (nonlinear) OLTC systems.

7 Voltage Control

7.1 Introduction

The subject of voltage control in power systems covers many subproblems which can be classified as primary (local), secondary (regional) or tertiary (national/system) [41]. In this section, we will restrict attention to techniques of emergency state control aimed at arresting voltage collapse. Xu and Mansour [18] give an interesting analysis of the critical switching time for capacitive support after a fault. A criterion for load

shedding based on generic load models is presented in [24]. Supporting analytical arguments are given in [25]. Both contributions are for a single dynamic load. Here we focus on tap-locking where some progress has been made for multi-load systems.

7.2 Tap-locking

The analysis of tap-locking as a means of preventing voltage collapse has been studied precisely in the work of Vu and Liu [31,42] and extensions to more general load models have been made in the report [23] where the single load and tap-changer case is considered.

Detailed analysis of the timing of the tap changer control has pointed out the importance of the system trajectory behaviour. The results can be summarised as follows [23]:

- Where the system trajectory passes below, or exits from the low side of the excess load region, then a bifurcation point of the system equations determines the critical time for tap-locking;
- Non-existence of a bifurcation point means that tap-locking can be performed at any time; this is characteristic of situations where excess load region is unbounded so that bifurcation can not occur;
- Where the system trajectory exits from the upper side of the excess load region, the exit point is the last opportunity to lock taps; later tap-locking will not alleviate voltage collapse.

Similar results hold in multi-load systems too. But the greater number of dynamic loads and tap-changing transformers makes a tap-locking strategy more complicated. To illustrate this, the system of Figure 7 is extended to include two tap-changers, each of which feeds a dynamic reactive power load modelled by the exponential recovery model (18). It is also assumed that the generators have reactive power limits as was indicated in Section 3. We shall analyse this tap-locking problem by locking one tap at some point of time, then determining when the other tap should be locked to alleviate voltage collapse.

The excess load region is defined in a similar way to the single load case, and again plays an important role. But note that this region now represents the intersection of two manifolds, each of which corresponds to the excess load region of the particular load. Figure 10 illustrates the case where the system trajectory passes underneath the excess load region, as viewed in $n_2 - x_{q1}$ plane. The bifurcation point of the system equations (point A in Figure 10) determines the last possible time for locking of the regulating tap-changer. That is, locking the tap n_2 at any point up to point A means that the trajectory on its subsequent vertically movement will

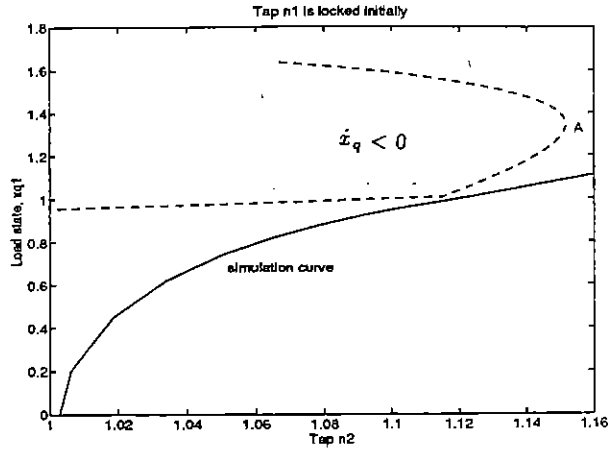


Figure 10: Excess load region and system trajectory

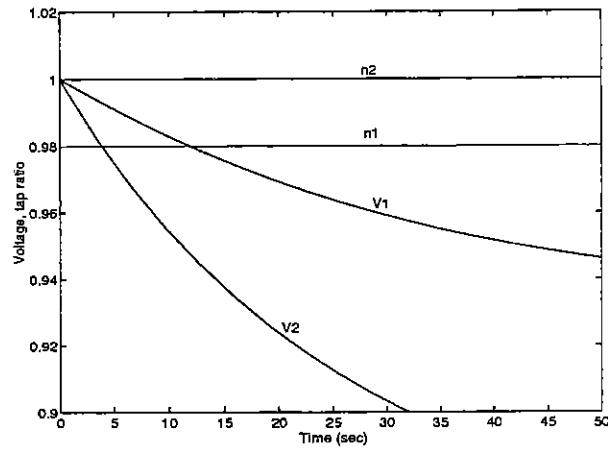


Figure 11: Tap changers are locked initially

encounter the boundary of the excess load region and stop.

It is important to observe that in multi-load systems the successful tap-locking interval does not always include the time origin. Figure 11 illustrates a case where both tap-changers are locked initially. Voltage collapse takes place. The situation of Figure 11 is shown in Figure 12 as seen in $n_2 - x_{q1}$ plane. The system trajectory passes below the excess load region. It can be seen that the first successful time to lock tap n_2 is at the bifurcation point B.

In some situations, a bifurcation point of the system is not the appropriate value at which to lock the tap. Actually, in situations where the system trajectory exits the excess load region from its upper side, i.e., above the bifurcation point, it is hard to analytically determine the latest time to lock the tap. But a conservative criterion can be given, as locking of the tap will certainly be successful provided it occurs before the tap increases above the exit point value. Later tap-locking may preserve the load voltages, but this is greatly influenced by the dynamics of the system.

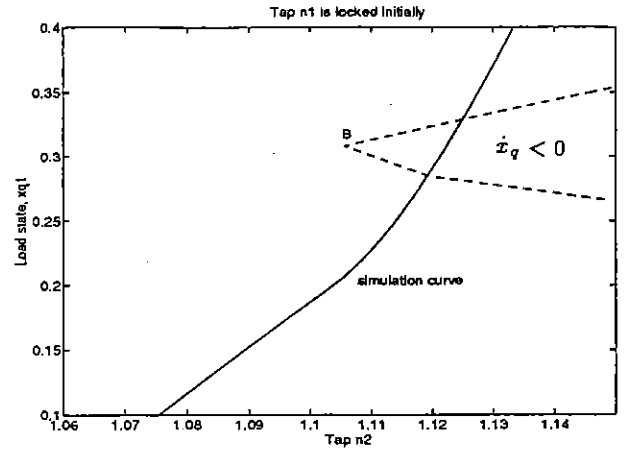


Figure 12: Illustration of the first appropriate point to lock the regulating tap

Even though this example has been successfully analysed, the complexity in the behaviour indicates that there is still a need to derive more practical (but approximate) guidelines for tap locking in multi-load systems with numerous OLTCs.

Finally, a message that comes through clearly from the studies reported above is that control actions which are based on load voltage level (or load voltage derivative behaviour) per se, i.e., with respect to viability, are in general without theoretical foundation. In both static and dynamic senses, voltage level is a poor indicator of voltage security! The voltage scale must be calibrated against system and load characteristics for derivation of effective control.

8 Conclusions

This paper has given an overview and some extensions of recent results on mid to long-term voltage stability analysis and control in power systems. This work has its origins in results on OLTC systems [38-40], and work sponsored by Swedish utilities on voltage collapse [13,15] and load modelling [9-11]. The emphasis is on clearly identifying the static and dynamic aspects of the analysis of power supply systems in the mid to long-term. It has been shown that clear results for static regularity (sensitivity), small-disturbance stability, large-disturbance stability and voltage collapse control can be expressed in terms of steady-state and transient load characteristics, and network properties including tap-changer time delays. These results can provide guidelines for control design; this has been illustrated for tap-locking. It is made clear that theoretically schemes for voltage collapse analysis and control must rely on more than just deservation of voltage level in a viability sense.

There are several directions in continuing work. The behaviour of cascaded tap-changing transformers with

nonlinear loads can be quite exotic; it is clear that nonlinear control can be usefully considered to deal with this complexity [43]. The load models used here have been first order. In [9,17], use of higher order models to capture oscillatory recovery [34] is suggested. Studies of oscillatory phenomena with cascaded transformers systems and higher-order loads will be reported. A theoretically challenging direction is to develop energy function techniques for the large disturbance behaviour; some progress has been given in [17]. Also further work along the lines of [29] is needed to explore behaviour near bifurcation surfaces in multi-load systems. Finally, the results presented here have assumed the generator dynamics can be ignored. This is valid after the transient period and is a common assumption. However, generator dynamics can be excited by later events such as sharp voltage dip (affecting synchronising power) and load shedding. Related studies which have a more generator dynamics emphasis are given in [17,30,44-47] and the references of those papers, i.e. the importance of network structure and load modelling in power system stability.

ACKNOWLEDGEMENT

This work was sponsored in part by an Australian Electricity Supply Industry Research Board project grant "Voltage Collapse Analysis and Control".

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Discussion of papers presented by Rehtanz, Yokoyama, and Hill

Edited by Lester H. Fink, ECC, Inc.

Hiroyuki Kita (Hokkaido University):

I would like to congratulate the author [Rehtanz] for his fine paper, but I am a bit surprised to find that you have used a fuzzy control. I suppose you have tried another control concept. I would like to hear your comments about the reason for choosing this kind of control.

Christian Rehtanz (University of Dortmund):

We have tried another control concept, but the fuzzy theory is very simple, you can realize it very quickly. Because of this, we presented it here.

Fernando Alvarado (University of Wisconsin):

I noticed that this method gives you, of course, active power and reactive power capability, and it seems to me that you use the active power capability mostly for angle stability and the reactive power for voltage stability, which is very natural. The question that comes to mind is, did you really need this SMES, or could you have done it with the static VAr compensator?

Rehtanz: The SMES was used for active power, and it gives an economic advantage; we want to enlarge this economic advantage by using this SMES for other tasks, and because of this we use it for voltage stability. It is correct that, because the reactive power comes only from the convertor, which is made of GTO transistors, a static VAr compensator could be used.

Haria Crisciu (TRACTEBEL):

I want to ask Professor Yokoyama two questions. The first, how is the reactive output power at each generator taken into account in the optimization process? Is it a control variable like a shunt capacitor, or is it a dependent variable? The second question, why did you choose as the objective function the mismatch of reactive losses rather than active losses? Generally the goal in the power system is to minimize active losses; in this case active losses goes by the transfer of reactive power.

Akihiko Yokoyama (University of Tokyo):

For the first question, reactive power flow out from the generator is a dependent variable in this case, under normal operating conditions between the upper and lower reactive power limits, because the generator bus voltage is a control variable. For the second question, we use total reactive power loss as the objective function, not the active power loss. As I described in this paper, first, the Central Research Institute of the Electric Power Industry in Japan reported that the total reactive power loss divided by the total active power demand can be used as a suitable static voltage stability index; second, minimization of the total reactive power loss can realize the optimal distribution of current flow in the network, which may lead to the maximum loadability - total active power loss is not used as the voltage stability index; third, the objective function is easy for the system operator to understand; and fourth, the OPF method is available and it is easy to incorporate this idea into an OPF program.

Nicholas W. Miller (General Electric Company):

I have a question and a comment. I always get excited when I see people promoting series capacitors. The comment first: from an economic point of view, as we go through the exercise of sizing and siting series capacitors, it is always important to be very careful to recognize that the cost of series capacitors isn't a linear function of dollars per kVA. The

first ohm is a doozie; a lot of dollars are associated with the first ohm, and then it goes up from there. When you are writing optimization types of things, you need to be very careful to take that into account, or you always end up with lots of little installations, which isn't very practical.

The question: it was not clear to me what motivated the switching off of banks as the loading went up. Our experience suggests that usually the power systems that are getting even moderately stressed, and up to heavily stressed, like to have all their series capacitors in all the time, as long as there aren't voltage or SSR issues, and even with controlled compensation - at least as we control compensation - the SSR issue goes away.

Yokoyama: The possibility to introduce variable series capacitors, the self-commutated HVDC system using GTO etc. into the power system is under study by the government, utilities, and universities in Japan. In this project, we learned that we have to add multiple control functions to variable series capacitors and something like the power electronics based series apparatus. Now, we are doing research on dynamic stability improvement by using variable series capacitors, not by using shunt capacitors. In addition to the function of this dynamic stability improvement, we are going to add another control function of static voltage stability enhancement, because the cost is very expensive, as you said. To improve only the static voltage stability or loadability without improving dynamic stability, it is necessary not only to switch on the series capacitors but also to switch them off, as shown in this analysis. At the next stage, we will have to consider coordination of both control functions.

Daniel Cheng (National Grid Company):

I would like to make a comment and ask a question. First the comment: the optimization problem is very interesting. Basically it is trying to change the system impedance to redistribute MVAr so as to get to a system state that is more stable than in a critical state. I can see good potential in using these techniques, especially for meshed networks like NGC, where we have a lot of double circuit lines, and switching out one line of a double circuit line is a possibility. So there is a high potential for using this technique.

Second, the question. The $h(z)$ in your equation 2, the voltage constraints - I would like to add a constraint which is the MVA or the MVAr flow through the lines. In your formulation you seem to have missed that line flow constraint, which to me seems to be very important.

Yokoyama: I missed that constraint because of the simplicity of the OPF calculation. If we take into account the line flow constraints, the problem size becomes large, but we have to add them for practical use, as you said.

Peter W. Sauer (Univ. of Illinois at Urbana-Champaign):

My question to David [Hill] relates to model reduction in Section 5.1. We retain at least one generator when we do our model reduction. You have eliminated all generators from your model. First, why do you eliminate all of the generation, and then, how did you do that? Did you assume constant voltage for the generators you eliminated in Section 5.1.

David J. Hill (University of Sydney):

The handling of generators is done just as in many previous papers. We have one slack and the rest ($n-1$) PV buses, just as in a regular power flow. So there are $n-1$ algebraic power

balance equations which relate the generator angles, load angles and load voltages. Conceptually, the implicit function theorem can be used to eliminate the generator angles in terms of other variables. The assumption we have is that generator dynamics are in equilibrium and are not re-excited. The angles are not constant, but buried in the reduced model. This is just for cosmetic simplicity. The extra algebraic equations can be carried along in our analysis if you prefer. If you don't like the absence of δ and δ' in a power system model, I regard this as generator fixation. It is just as valid to complain about absence of s , s' (induction modors) or n , n' (TCUL) in our traditional angle stability calculation.

Thomas J. Overbye (Univ. of Illinois at Urbana-Champaign):

My question is on Figure 8, where the trajectories sort of appear out of nowhere. I was wondering where those trajectories came from. Could you comment where they came from, and also, if we increase the loading and bring those two equilibrium points closer together, what would happen?

Hill: Obviously, in the middle of the figure that you are referring to, there is some kind of singular surface. There has got to be a saddle node bifurcation here and a singularly induced bifurcation such as studied in the work of Prof. Zaborsky. Those sorts of considerations are here. If I can just describe it in the case of the "Illinois" system, we have to look at what goes on in the transitions. The key thing is the singularity of the coefficient which relates to the transient load characteristics. You've got a saddle node bifurcation in the normal way with a load flow, and then you have got the singular surface associated with singularity, and where that occurs here is related to whether you've got, in this special case of the "Illinois" system, two stable equilibria or one stable and one unstable; but you can see that there is a singularly induced bifurcation sitting there in the middle somewhere. So you have to take those two bifurcations into account to understand what happens when these things come together, and that depends on the interplay between static and dynamic load characteristics. In this particular example, we didn't study it, so I really can't say much more about it, but those are the things that you have to consider.

Carson W. Taylor (Bonneville Power Administration):

I have a question on one of your load models, which showed up in three papers today, involving the load loss voltages and angles being state variables. This would mean that you cannot have discontinuities, but in real life, except for very fast transients, voltages and angles can have discontinuities.

Hill: This is an important feature of the newer generic models. It is instructive to look at this in detail. In system theoretic terms, the load model has relative degree zero. This is easiest to see in IO form. Any switching in the network can produce a jump in voltage (and angle). This passes through the load model to give a jump in power. We have a DA model, and the state model in load angles and voltages is valid only locally.

Nelson Martins (CEPEL):

Bill Price has presented a discussion on voltage stability where the aggregate load models are all in block diagram form. Van Cutsem, David Hill, Yakout, and others - all of those first were equation based, and now come the ones that are block diagram based: Yakout, Bill Price, and others. I think that having all these in block diagram form makes the comparison between the different models much easier for me, at least (a control oriented guy). So I suggest that all of you that work in this area try to have a standard block diagram representation.

Hill: Yes, we do it in other papers. You might remember that in the stability paper I presented block diagram versions.

Robert A. Schlueter (Michigan State University):

In my paper earlier this week, I call the model that you are talking about, without generator dynamics (and what perhaps Pete Sauer, Ajjarapu, and others have done with generator dynamics and ignoring the load dynamics), our type 3 model. Now from my point of view, everyone of those models has a sequence of bifurcations. Much has been done in the generator dynamics, and really what usually brings about non-safe or destructive bifurcations is really the exciter dynamics going unstable. Now in working with Type 3 models - and much of the work that you are doing and others in this room have done have been on Type 3 models - really you cannot say either one [generator or exciter dynamics] is the issue. I would think that, in terms of doing these kinds of things in the future, it is smarter to go to a Type 4 model that has both dynamics in it, because then you are really looking at sequences. The other thing is that one really has to take into account what happens in real life, especially in the load models. What are your thoughts in regard to that?

Hill: As I said earlier, I am an analyst who is aiming to change the way people think about a certain problem. It doesn't answer all practical problems right away, and you have to crawl before you walk. There has been a lot of attention given to the generator end of the system, and the load end of the system has been ignored. So this is kind of a reaction the other way. Let's focus on the load end of the system, get as much as we can out of that exercise, and then go back and put the two together. I think people are thinking about putting the two back together. We just had a paper on damping studies which does that. Tom Overby has been considering the two together, as have you. So this was just an exercise focussed on the load end of the system, the main aim being to explain the interplay between static and transient load characteristics. Obviously it is not the complete answer.

Relay action is important and your idea of a sequence of equilibria/stability region assessments should be pursued. But we have not explicitly done this yet. However, as explained to Carson, the model allows easily for such investigation.

Miraslav M. Begovic (Georgia Institute of Technology):

If you look at Figure 6 you see that load characteristics are identified and used in the analysis extending into the low voltage solutions. My question is, how confident can we be at this stage that the model is accurate at such voltages, given that it is not being identified? Do you have any ideas of how that confidence could be reinforced?

Hill: This is a conceptual picture just like the other papers. The field tests that were done in Sweden are really the only model variation that I am aware of, and the voltage variations were about 10%, so obviously we have no validation for low voltages. But if you go back to the device oriented models, we can explore them in detail down to low voltage levels, as Fernando did earlier. And this is really a conceptual picture, to say that if you have that characteristic and it is validated, these are the sorts of things that can happen. The validation is a separate problem. Hopefully, you will help us with that.

Ian A. Hiskens (University of Newcastle):

If you pose equations in the state model form, then it becomes a DA system. The V-dot model you can think of as being the model that comes about when you differentiate the algebraic equations with respect to time and substitute them back into the original differential equations to eliminate the states.

Now when there are discontinuities, then the algebraic equations take a discontinuity. If you are thinking in terms of the state model, the states of the load remain continuous, but the voltages, which are algebraic variables, take a step change. The Θ -dot, V-dot form of the model is valid as long as there

are no structural changes occurring. As soon as a structural change occurs, then this implicit model needs to be altered accordingly.

NOTE: The following additional discussion on Prof. Hill's presentation took place during the discussion period of Session 8.

Chair (Paul van Son, ECC, Europe)

I would like to ask Mr. David Hill who asked me for some 5 minutes, because he wanted to come back to a discussion of yesterday where he thought that there was a misunderstanding.

David Hill (Univ. of Sydney)

Thank you for the opportunity to give an answer to last night's questions after I have had a bit of sleep. I believe Pete still wants to have the last word and I am sure he will going by past performance. This slide is a little busy but it is the only one that I have left to put the answers on so let's take it in 2 parts. Pete said what do you do with the generator buses and I made a vague answer something like well we do the usual thing of writing power flow equations and then eliminating ones corresponding to known power so it really is just that. If we have a generator feeding on a bus, we can write a power flow equation in the usual way; we have one slack bus, we have $n-1$ PV buses, we can carry those algebraic equations all the way through, but it just looks a little bit prettier in the mathematics if you make some sort of an elimination. It is the same sort of elimination that Yakout's done in his paper, and Chen-Ching Liu's done in his paper, and I've done, where you can, for every given power, eliminate an angle in the power flow. The physical side of this is that you are just assuming the generator dynamics to be in equilibrium; you are not exciting those generator dynamics. If you do excite the generator dynamics, you need to add the dynamic parts of the swing equation to this, and include that in the analysis. What I feel you have is what I call generally a generator fixation. People just don't accept a model which doesn't have a δ -dot in it related to the angle of the generator, but I feel it's just as valid to say: why, in transient stability, don't we have N -dot or S -dot or other terms related to the load dynamics. It's the same - we are just eliminating some dynamics that we don't regard as important to the current analysis.

Let me go on to Carson's question. I didn't understand that what he was questioning was really the validity of the load model; I thought he was referring to just stability analysis like Bob Schlueter talks about. Really, the stability analysis question is a sequence of stability analyses related to switchings on the network, and we haven't touched that properly yet. Now I've had a discussion with Carson, and find that the validity of the load model is at question here. You can see that, throughout the paper, we keep talking about the nose curve after various switchings (and it is in Yakout's paper as well) - you have a prefault nose curve, and a post fault one, and then a post control one, so that switching is going on all over the place. I just want to emphasize that the whole point of that load model was precisely to cater for those jumps. If you take away the term you questioned, you get a load model that other people have used. The load normally is a nonlinear function of voltage plus a V -dot term. If you pass a voltage step through that, you get an impulse, so there is a problem. But with the load model that we use, if you look at it in the input/output form, for those of you that are familiar with systems theory, it is a relative degree 0 system. So you can pass a voltage step through that, and you get a voltage step; it leads to a jump in power, which is exactly the response that we want. So that term, which really is the new part, is vital to cater for what Carson is talking about.

Now you can look at this either in the input/output form or you can look at it in the state space form. Van Cutsem,

Yakout, myself and Ian, we take both forms, while most people use just the state space form. I think it is a little more complicated to see in the state space form that there is no problem with voltage steps. What happens then, through a transient, is that the state variable X is constant. You can't have that jumping. If you put X equal to a constant, you then track that through the model and you will get different load characteristics for steady state and for transients, but there is no problem with the voltage step. Any switching in the network that you like to come up with can be accommodated by the model, and as Ian explained last night, sometimes you have to go back to the basic DA model.

The reduced model that I talked about is really only valid between switchings, because it is dependent on which manifold you are on. When you switch to a new manifold, you've got to change the reduced model. You certainly shouldn't use the reduced model throughout all the switchings, but the original DA load model is ok. If you can impose a high voltage step at the bus, really the load response is going to be of a form such that there will be some quick time constant, then a slower one, and then an even slower one. If you can think of the recovery response broken up into different time zones, where over about one second you will be looking at the effect of induction motor recovery, the normal slip recovery, then over a period of a few minutes (if you analyze a tap changer with a load, you can derive this response) you get a recovery related to tap changes, and over a longer term you get a recovery related to heating, and so on. You just pick the time frame that you are interested in and do the system identification related to that.

Peter Sauer (University of Illinois)

I have not necessarily a fixation for generators, but a very strong fixation for model reduction. I think maybe it was at Davos 1, maybe it was at Henniker, I don't remember where it was, but the interesting theory that got me involved in model reduction came out of one of these conferences. I am convinced that there is really only one legitimate way to do model reduction - one honest way. There are many dishonest ways. Start, for example [Figure], with the generator dynamics as the X variables and the load dynamics as the Y variables, with no algebraic equations just to make it easy. If you don't want the generator dynamics, if you want to get rid of those, you can just get rid of the equations - don't use them - but then you have to decide what you are going to do with the remaining X . And what you do with X is what model reduction is all about. Based on the ideas of singular perturbation and integral manifolds, there are two extremes that are very useful and quite easy and commonly used. They are justifiable, and so I have no problem with them.

My question was, which of these two things did you do? If you take the time constant T to be very small, which means that the dynamics are very fast (much faster than the load dynamics, which sounds like what you would like to say) then T times the derivative goes to zero, and you are left with the algebraic equations $0 = g(x,y)$, which hopefully, with your implicit function theorem, can be solved for X , X as a function of Y and then substitute that X back into g , and you have legitimately eliminated all of the generator dynamics. Maybe that is what you did.

The alternative is to say that the X time constant T is very large, in which case it is infinitely slow, and so the derivative of that the dx/dt is equal to zero. This is not the same result: $dx/dt = 0$ means that X is a constant. So these are two extremes of the possible model reductions based on the time scales.

Now your statement that you were using load flow thinking, using the term PV bus, makes me nervous, because I don't know that either of these two model reductions will give you a PV bus. A PV bus is a phenomenon of load flow. I am not saying that it isn't, but I am not convinced that it is, and so I am pleased to have this to work on when I get back, to

are no structural changes occurring. As soon as a structural change occurs, then this implicit model needs to be altered accordingly.

NOTE: The following additional discussion on Prof. Hill's presentation took place during the discussion period of Session 8.

Chair (Paul van Son, ECC, Europe)

I would like to ask Mr. David Hill who asked me for some 5 minutes, because he wanted to come back to a discussion of yesterday where he thought that there was a misunderstanding.

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$$\begin{array}{l}
 X = \text{Generation} \\
 Y = \text{Load}
 \end{array}$$

$$\begin{array}{ccc}
 T_x \ll T_y & & T_x \gg T_y \\
 T_x \frac{dx}{dt} = g(x, y) & \Rightarrow & 0 = g(x, y) \\
 & \Downarrow & \\
 T_y \frac{dy}{dt} = f(x, y) & & x = h(y) \\
 & & \Downarrow \\
 & & x = x^0
 \end{array}$$

$$X = \text{"PV" bus?}$$

Figure

My question was, which of these two things did you do? If you take the time constant T_x to be very small, which means that the dynamics are very fast (much faster than the load dynamics, which sounds like what you would like to say)

then T times the derivative goes to zero, and you are left with the algebraic equations $0 = g(x,y)$, which hopefully, with your implicit function theorem, can be solved for X , X as a function of Y and then substitute that X back into g , and you have legitimately eliminated all of the generator dynamics. Maybe that is what you did.

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Session 8

Simulation

"Experience in the Dynamic Analysis of Voltage Phenomena"

C. A. Roa-Sepulveda: Universidad de Concepcion

U. G. Knight: Imperial College

"Stochastic Evaluation Indices of Dynamic Voltage Stability

Based on Manifold Equilibrium Point"

Keiichiro Yasuda, Yicheng Zhou, Ryuichi Yokoyama

Tokyo Metropolitan University

"Use of Multiple Load Flow Solutions to Approximate a Closest Loadability Limit"

Naoto Yorino, Shigemi Harada, Hiroshi Sasaki

Hiroshima University

Minoru Kitagawa: Chugoku Electric Power Company

"Influence of Load Modeling Assumptions
on the Dynamics of Large System Voltage Collapse Simulations"

Nicholas W. Miller, William W. Price

General Electric Co.

"A Comparison of Discrete vs. Continuous Dynamic Models
of Tap-Changing-Under-Load Transformers"

Peter W. Sauer, M. A. Pai

University of Illinois at Urbana-Champaign

"Voltage Stability Studies with PSS/E"

Stefan G. Johansson, Fredrik G. A. Sjögren, Jaap E. Daalder

Chalmers University of Technology

Daniel Karlsson: Sydkraft AB

EXPERIENCE IN THE DYNAMIC ANALYSIS OF VOLTAGE PHENOMENA

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Abstract

A detailed model DYVOLC (Dynamic Voltage Collapse) developed at Imperial College for the dynamic analysis of voltage phenomena on a multi-machine, up to the point of voltage collapse or oscillation, is outlined along with some of the results obtained using the model on single and 3 machine systems. Particular aspects of simulation of a 14 machine system are mentioned. Views on the role of accurate modelling of components for dynamic analysis is also described. Saddle-node and Hopf bifurcations were observed in some of the results and there is comment on the behaviour of eigenvalues close to bifurcations.

Particular points of difficulty (including the modelling of the system external to the part of the system under study) and the repercussions of these on the models used for the system and the component plant are discussed.

Keywords: Voltage control, reactive power, stability, dynamic modelling, long-term dynamic simulation, bifurcations, eigen-analysis.

1. Introduction

The risk of voltage instability, which in some circumstances can occur at near normal operating voltages, is of considerable practical importance since it may be the key factor determining power transfer limits quite apart from the impact on the system and the customers.

This has led to considerable work on the estimation of maximum power transfer limits set by voltage conditions and on the detection of critical conditions. Most of the applied analyses have used a steady-state approach using approximate, readily computable and sometimes empirical measures in system planning, operations and control.

There have already been two International Workshops dedicated to this problem [1,2]. The two Proceedings, and associated literature show that much of the effort has been devoted to developing steady-state performance indexes (VCIs) to help operators in monitoring voltage-related problems. Profs. Fischl and Mercede [3] asked the following: Can VCIs be found correctly to predict an impending voltage

collapse? We can add to this: Can VCIs be found correctly to predict any voltage instability? To the authors knowledge, steady-state VCIs to answer those questions have not been described yet.

Methods of study can be grouped as either steady-state (static) or dynamic methods. The steady-state methods imply that a steady-state model or a linearised dynamic model of the power system has been used, whereas dynamic models use a set of differential equations subject to a set of algebraic equations. The bulk of research has been done on the steady-state formulation of the problem. One incentive has been to develop a voltage collapse criterion which can be readily used in on-line operation.

Steady-state methods can be subdivided into those which test for the existence of an acceptable voltage profile across the power system (Load Flow Feasibility methods) [4-13] and those which test for the existence of a stable equilibrium point of the power system (Steady-state Stability methods) [14-24]. Both methods attempt to propose a criterion based on a scalar Performance Index, PI, and a threshold value to decide if the power system is close to voltage instability.

The dynamic approach is concerned with the long-term dynamic simulation of voltage instabilities. A literature review shows that the dynamic simulation of voltage instabilities started with consecutive load flows modifying the voltage support device settings at every load step increase [26, 27]. Analyses were then carried out taking one voltage supporting device at a time [28-30] or doing sensitivity analyses for different load models [31]. Commercially available dynamic simulation programs, e.g. [25], use different integration routines depending on the scale of the phenomena under study.

There has also been great interest in more theoretical issues -what are the dynamics of voltage collapse and apart from fully understanding the phenomenon, how necessary and advantageous is it to use a dynamic approach. This has led research workers into complex areas of bifurcation and chaos [32-36]. Although those approaches include a well-founded mathematical formulation from which it is possible to analyse all possible bifurcations, it is not clearly shown how

the transition of power system from one operating point to another under the influence of voltage supporting devices is considered.

This paper presents the experience gained in performing dynamic simulation for voltage instability studies. Areas such as system and demand modelling, equation solving for long-term simulation together with modal analysis are presented and comments on their selection and use are also given.

2. Modelling

The system conditions which may exist in voltage studies extending into the collapse situation will differ substantially from those expected in power flow or transient stability studies in the following respects:

- The range of voltage may be substantially greater than in power flow studies.
- The periods of time for which voltages substantially outside normal limits may exist will be much longer than in transient stability studies.

This affects the type of information required on demand and network characteristics, particularly the sensitivity of active and reactive power demands and losses to off-nominal voltages. The fact that transmission level voltages may fall by 20% or more in extreme cases means that transformer tap ranges may be exceeded and hence the quite usual assumption of constant power and reactive power will not be valid. The constant impedance assumption will be also suspect; it will not be correct when the transmission voltage is within the overall tap range of transmission to distribution voltage transformers nor will it be for many types of load when outside the tap range. Whichever model is adopted the voltage sensitivities should be modelled.

The computing required for a dynamic model is much greater than that for a steady state model, not least because the dynamic model effectively estimates many more states than the static model. This disparity will be critical in real- and near real-time applications for control and operational planning and may be an embarrassment in planning studies. Hence it is important to define as far as possible the technical limitations of the steady state methods.

There seem to be broadly three classes of models: the steady state (snapshot), dynamic with static load models and dynamic with dynamic load models this being the order of increasing complexity, increasing computing needs and hopefully increasing accuracy of simulation.

The following judgements are offered in regard to the role of the three basic approaches:

- A full dynamic model with dynamic demand should be used if detailed interaction between control mechanisms is to be investigated.
- A steady state model should be adequate for run of the

mill planning and operations studies provided:

- points of discontinuity in the operation of control mechanisms can be recognised and the effect included.
 - the model is adequate for the network configuration being studied.
 - sample results from a steady state model should be validated against a full dynamic model.
- A full dynamic model should be used for analysis of the effect of different demand characteristics on system voltage stability.

The two methods available to estimate demand characteristics are direct measurement and synthesis by aggregating the elementary components of the different types of load, with known characteristics, into composite totals at the various nodes for each area.

Direct Measurement

In this method, the changes in power and reactive power taken by the demand at selected points following a sudden change in voltage on the busbar supplying the demand is measured. The voltage change may be caused by setting paralleled transformers on different taps and switching one out, switching out an unloaded line, etc. Tap changers on the networks subjected to the changed voltage will usually be locked.

The problems of this method are:

1. it is difficult to obtain voltage depressions greater than say 10% because of consumer reaction
2. it requires quite extensive planning, instrumentation and manpower and because of this it is only likely to be used on a small scale
3. it will only give data for the times of the tests; these would probably include several over a day, perhaps weekday and weekend, but repeating for other seasons and years could be difficult.

On the other hand it does, within its limitations, give directly the information required. The tests are likely to require direct involvement and support of the utility concerned.

Synthesis of Demand Characteristics

In this method, the PQ/V characteristics are synthesised from knowledge of the characteristics of the elementary components of the demand (lights, heating, freezers/refrigerators, different motor loads) at each of the nodes represented. These techniques have been studied in detail by EPRI. The first proposal was to aggregate the demands at nodes on the distribution network, carry out load flows on the network for various values of voltage at the supply node from the transmission network and hence obtain the relationship between the value of this voltage and the P, Q supplied. This technique was apparently not taken up by

utilities and a simpler proposal was put forward. In this, the *PQ/V* characteristics are estimated at the bulk supply points by aggregating the elementary components of demand (as just described), at these points and adding to this an overall approximation for the distribution networks themselves.

A spreadsheet has been constructed based on the EPRI models which permits aggregate demand models to be estimated rapidly and flexibly. The major problems are in obtaining the demand mix and composition. The decision to adopt the synthesis approach to modelling demand in this project, using the spreadsheet, has not been regretted.

Sometimes it is desirable to analyse voltage instabilities by using a reduced system in which an instability is likely to occur. This brings the problem and importance of determining an equivalent for the remainder of the system. An equivalent which will be robust enough to suit both voltage and angle stability is needed.

Retaining the external connections from the test area to the remainder of the system, several alternatives for the capacities of equivalent generation and demand at the external nodes may be studied. These include effectively treating the test system as isolated or with the external system defined by the transfers between external and test systems. The authors tried several approaches in studies on a 14 generator test system connected by three double circuit lines to the main system, which was some 8 times larger in generation capacity than the test system. The most satisfactory equivalent appeared to be the most physically based one.

Assuming the voltage behaviour of the network is to be analysed by studying the effect as demand increases, a further question is how the generation and demand in the external system should be increased as the system demand increases. There will be three main possibilities:

1. Generation and demand increase in the test system; generation and demand are held constant in the equivalent system. This is probably the least realistic scenario.
2. Generation and demand increase in the test system; demand is constant in the equivalent system but generation increases in the equivalent system to meet its appropriate proportion of the test system demand increase. This scenario corresponds to a localised increase in demand in the test area, but with system wide generation regulation. In practice such a demand increase would probably be small and voltage problems would be most likely under outage conditions.
3. Generation and demand increase in both systems. This scenario corresponds to a system wide increase in demand.

Assumptions will probably have to be made on the unit commitment in the equivalent system. If the demand increase is less than the spinning reserve and allowing some

synchronising of gas turbines it will be reasonable to assume that the unit commitment does not change substantially during the study run. If this does not hold, the unit commitment will change and the study run should be done in stages.

Although it seems obvious, care must be taken that the load flow program used at the start of the simulation can model accurately the effects of the various control mechanisms and demand characteristics down to the voltage at which the dynamic simulation starts.

A significant problem encountered was in the tracking of eigenvalues when discontinuities occurred.

3. Long-term Dynamic Simulation

The overall solution of the model requires the solution of a set of differential equations subject to an algebraic set. This can be done in several ways and with different mathematical methods.

The breakdown of the set of differential equations needed to simulate voltage instabilities is arranged to take account of large differences in time constants ranging from 100s for OLTC's to 50 ms for SVC's.

The mathematical model for voltage instability simulation is similar to the transient stability problem. However, when considering slow dynamics, OLTC's for example, the size and complexity of the problem is considered significantly greater. Both the amount of data and the computation required to perform the simulation of one specific operational condition in any voltage problem is much larger than for a transient stability study.

The solution task is made even more difficult because of the uncertainty of knowing when, how and/or where any voltage instability is likely to develop. This situation shows the need to find mathematical tools which are both robust and stable enough to perform dynamic simulations well beyond accepted long-term transient stability analyses.

The methods for solving the equation set of differential and algebraic equations can be classified in three groups: (1) iterative solution, (2) combining both sets of equations to form one large differential set and (3) implicit integration. The decision is not always clear and strongly depends on the investigator and his ability to develop purpose built software and to apply standard packages. In any case, the situation described in the preceding paragraph always applies.

Network Solution

Two points need to be stated before choosing any network solution method. Firstly, the method has to solve the matrix equation $Ax = b$ and be computationally efficient and stable to withstand sudden changes in the system state. Secondly, the method has to be able to cope with any network switching and/or tap changes without extensive calculations.

The solution has been traditionally done through a Gauss-

type formulation. This technique was also used, at the early stages of this study, in solving the network solution. However, unexpected numerical instabilities were found when dynamic simulations were performed. These oscillations were coming from the network solution and not from the integrator. Moreover, the network solution took many iterations to converge in every integration step. The cause of this situation was located and identified to be the technique used to correct for those voltages outside the tolerance.

To overcome this problem, a relaxation technique applied to compensate injected currents provides fast voltage calculation and is stable enough to handle sudden voltage drops. It did not show instability and achieved the solution point quickly.

Solution of Differential Equations

One of the most important issues encountered during static analyses of voltage collapse is when the power system is near the point of collapse. At this point, one or several of the eigenvalues of the state matrix become(s) very small, if not zero. The range of eigenvalues is a measure of the 'stiffness' of the set of equations.

Accuracy and stability are key properties to be achieved in the numerical solution of any set of differential equations. Achievement of both properties is strongly dependent on the method chosen. Being Σ -stable ensures that the method is unconditionally stable. This means that the method will be stable for large integration steps. Backward Euler and the trapezoidal method are Σ -stable and single-step method.

Although these methods could solve a set of stiff equations, there is another important point to be considered before finally adopting a method. In voltage instability studies, it is unknown when, where and/or how the instability will occur. This uncertainty means that the time span for analysis may be quite large with an unpredictable end point. Moreover, at each integration step taken towards collapse, the overall state matrix will become stiffer.

Considering the above points, a stiff differential equation solver was used based on the Gear method. This method is said to be Σ -stable, multi-step and multi-order. This characteristic is of extreme importance because the method will thereby achieve a compromise between accuracy and stability and use the maximum integration step length to speed up solution at each integration step. This compromise is done automatically by the method.

The maximum integration step length of this method needs to be carefully managed. If demand increase is used to initiate voltage collapse, any voltage instability will occur after a period of time which can be long and, therefore, the integrator will see the power system as quasi steady-state and will choose a large integrator step length. This situation

could cover some instabilities at early stages of the simulation. Restricting the maximum integration step length to be equal to the period of change of the independent variable, i.e. load increase rate, will avoid this but will produce longer simulation times. A compromise has to be adopted.

4. Preventing Voltage Instabilities

If any change is produced in the power system, the full state matrix is recalculated, and after a stabilising period, new eigenvalues are found. This new set of eigenvalues is then plotted in the initial complex plane. The new eigenvalue positions give an indication of the improvement or worsening of the overall system stability due to these changes, see for example Figure 1. The plot will show which eigenvalues are producing instabilities found through a dynamic simulation and/or by the technique itself.

In addition to knowing which eigenvalues are producing a voltage instability situation. It is also necessary to know where to act in order to find suitable control strategies.

Investigators have used the information given by the right eigenvectors of the unstable modes to determine which state variables are producing the instability. This process requires considerable expertise in power system modelling so that a meaningful interpretation of the right eigenvector can be made. To complement the information given through eigenanalyses, techniques such as "Participation Factors" or "Residuals" can be used to find those states affecting the critical mode shift.

A **dynamic boundary** between stable and unstable operational states can be found by analysing the power system during any disturbance. This boundary can be found through an eigenvalue analysis by plotting the values in a complex plane with time as a parameter. When any (one or more) eigenvalue reaches the imaginary axis, the power system will suffer voltage instability and the time, at which this occurs, will be known as the "Voltage Collapse Critical Time (VCCT)" measured from the initiating event. The problem in defining this boundary lies in the fact that it depends not only on the kind of disturbance but also on the time co-ordination of the actions of the voltage control devices.

Knowing the eigenvalues producing the voltage instability situation, a modal decomposition is carried out to determine which power system component(s) is (are) affecting these eigenvalues. The voltage supporting devices attached to those components or area and their time delays will define the dynamic boundary as the difference between VCCT and the largest time constant of those controls. This boundary can be expressed in time terms and can be called the "Dynamic Boundary Time (DBT)".

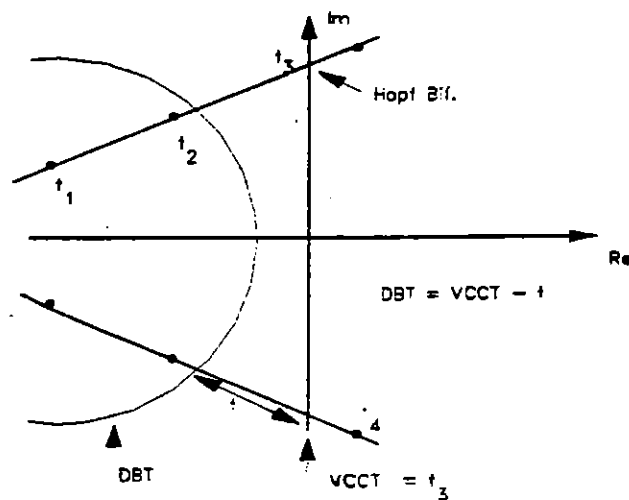


Figure 1 The Dynamic Voltage Instability Boundary
 VCCT = Voltage collapse critical time
 DBT = Dynamic boundary time

Knowing VCCT and DBT, suitable control procedures for voltage control can be found. These procedures should be simulated and studied in order to check their viability. Effects of delays in operator actions could also be analysed, assuming suitable time constants.

Defining a geographical area in which actions can be co-ordinated to solve and/or prevent voltage collapse is of great use because this area can be made reactive power sufficient, with spare capacity to supply reactive power to its neighbours. The principal characteristics of this area concept are :

- An area is a zone where the voltage swings at all nodes are coherent for an outside disturbance.
- All the voltage control devices, belonging to the area, should be co-ordinated.
- Areas should be virtually independent of each other: e.g. the transmission boundaries that separate voltage control areas may be weak.
- Reactive reserve on the boundaries between areas should not be used until reactive sources within the area are nearly exhausted.
- Local reactive supply may have to be activated automatically because the reactions of operators may be too slow to prevent voltage instabilities, even in the initial stages.
- To reduce the risk of voltage collapse, each area should mainly use slow acting reactive reserves, e.g. capacitor banks, to support a voltage profile, holding adjustable fast acting reserves, e.g. generators and SVC's, to meet sudden changes.

5. Models for System Components

The models used for the main power system components of the voltage instability simulator considering the above comments are outlined below [37].

5.1. Synchronous machine and controllers

5.1.1. Machines. A two-axis model is used with the following assumptions:

- One (paralleled) equivalent generator at each node.
- Saturation effects due to high stator currents are neglected.
- System average frequency is assumed constant.
- Limits are placed on maximum power output and excitation.
- Flux decay effects are included.

The equations used considering the above requirements can be found in [37].

5.1.2 Automatic voltage regulator. The model used corresponds to the IEEE Type 1 regulator. The calculation method for the lead-circuit parameters can be found in [38].

5.1.3. Speed governor. In voltage collapse situations, there is often little change in the system generation demand balance until the point of collapse. After reviewing the governor models given in the IEEE standard [39], the speed governor model used consists of a gain and a time constant.

5.2. Static VAR compensators (SVCs)

The model used is given in [40], the blocks associated with the measuring and the fire-angle control however being neglected due to their small time constants.

5.3. Mechanically-switched capacitors (MSCs)

Although MSCs are essentially discrete devices, a dynamic continuous model similar to that of the SVC model is used.

5.4. Transformers and tap changers

A continuous acting model is used as given in [41]. Dead band in the voltage measurements and time delay in the tap changer control and mechanism are included. It is likely that on-load tap changers (OLTCs) will operate in the time frame of a voltage collapse study. To minimise recalculation of the network model with tap change, the model given in [42] is used.

5.5. Demand modelling

Demand characteristics have been studied in some detail, mainly following an EPRI approach [43].

5.5.1. Static characteristics of demand. The aggregate static model of demand at a node is built up from voltage sensitivities of power and reactive power of the individual types of demand at the node [43-44]. Frequency terms are neglected.

5.5.2. Dynamic characteristic of demand. Induction motors (IM) form the main dynamic component of demand. In the IM model, stator flux dynamics are normally neglected due to the small time constants. It is necessary to include rotor flux dynamics in order to account for magnetising effects in the machine as the terminal voltage changes. The induction motor load at each node is aggregated into equivalent machines, using the equations given in [45] and the principles given in [43]:

6.5.3 The final model. The final model is a combination of static and dynamic models with undervoltage relay modelling to include time delayed motor disconnection resulting from voltage drops. The motor load is reconnected on voltage recovery.

6. Examples

6.1. The single-machine test system

The single-machine test system (Figure 2) was used in sensitivity analyses in order to find those parameters which will affect or produce Hopf bifurcation scenarios in power systems. The steady-state and dynamic data are given in Appendix A.

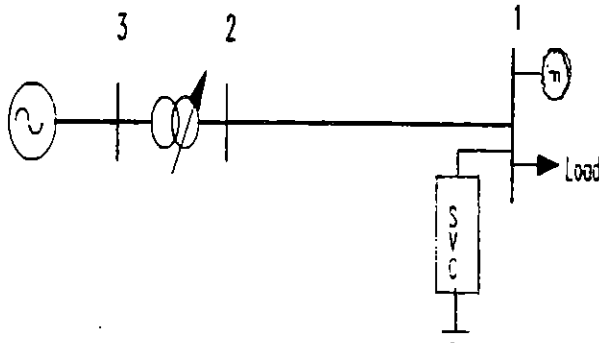


Figure 2 Single-machine test systems

A stressed test system is also considered by opening one of the two parallel lines between nodes 1 and 2 and assuming that all transient phenomena have settled down. In the remaining of the paper, the system with two lines will be referred to as the full system and the system with one line as the reduced system. A 1% per-min. ramp load increase (a higher rate than normal, hopefully, to shorten computing times) was applied to both base operating conditions until voltage instabilities due to Hopf bifurcation were produced.

System Behaviour for Full and Reduced Test System

The demand increase is applied to each test system and continued until voltage instabilities are found. These are sudden voltage collapses due to AVR reaching maximum field voltage limits, as it can be seen in Figures 3 and 4.

It can be clearly seen that with this single machine system, after the AVR saturation, the collapse of the system is very fast not allowing time for operator actions. The full system can be loaded at about 6% more than the reduced system and both curves show a coherent voltage behaviour, as expected. The full system collapses when the load has been increased by 30% and the reduced system for a load increase of 24%. No maximum stator current was violated during these tests.

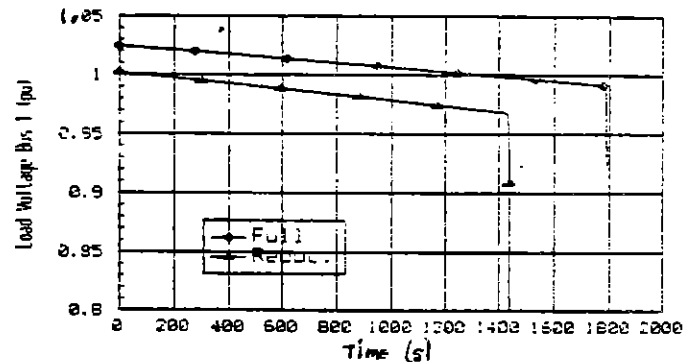


Figure 3 Demand voltage for voltage collapse

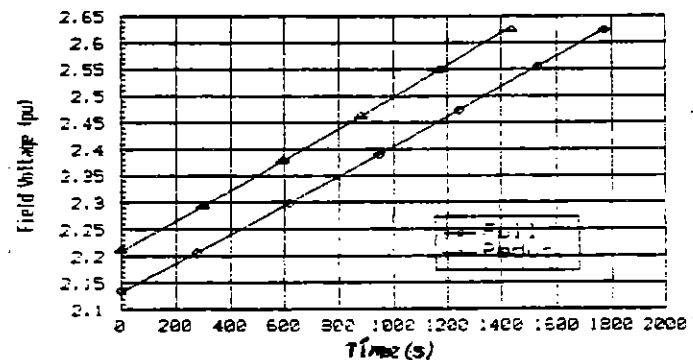


Figure 4 Generator field voltage for voltage collapse

The following tests were performed assuming that the test system has a large AVR capability to support the generator terminal voltage.

Sensitivity analyses to generator parameters

The data given in Appendix A shows transient and steady-state quadrature reactances of equal value. This assumption corresponds to a salient-pole rotor (SPR). Figures 5 to 8 show the dynamic behaviour of the reduced test system assuming a round rotor (RR) with the transient quadrature reactance of 30% of the steady-state one. The stated load increase was applied for TG = 20, 30 and 50. The generator AVR was the only voltage supporting device allowed to act.

It is interesting to note again that the lower the TG, the larger the load increase proportion before reaching Hopf bifurcation scenarios. These levels correspond to 59%, 56% and 52% for TG = 20, 30, 50, respectively. However, all cases violated maximum stator current at a load increase of 47%.

Figures 5 and 6 show that oscillations are likely to disappear when the AVR transient gain is of low value. Figure 7 shows demand voltage oscillations for SPR and RR generators. It can be clearly seen that by using a RR generator the system can be loaded some 1.6% more heavily than the system with a SPR generator.

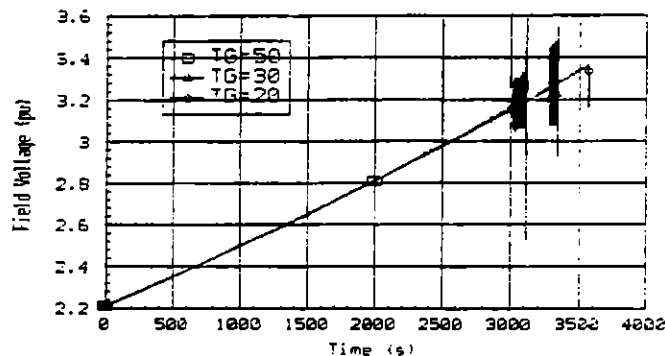


Figure 5 Field voltage for a RR generator
TG = 20, 30, 50

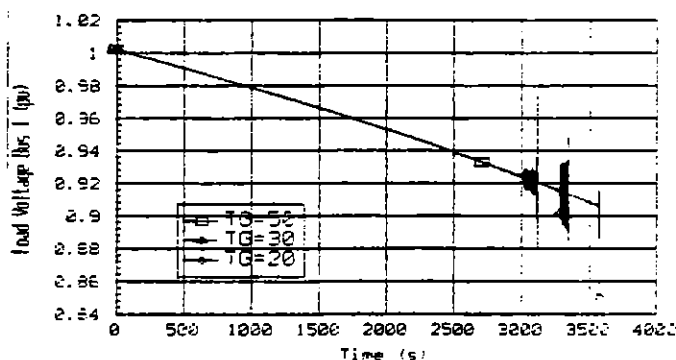


Figure 6 Demand voltage for a RR generator
TG - 20, 30, 50

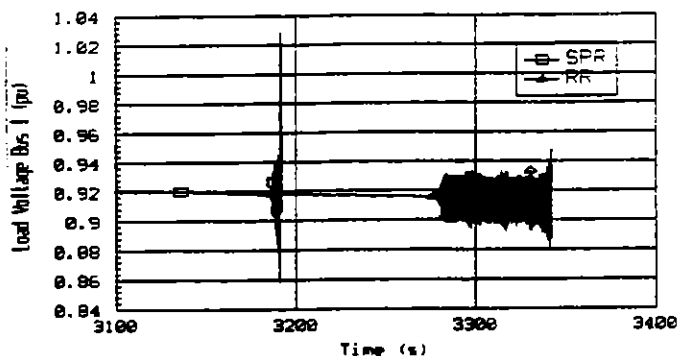


Figure 7 Demand voltage for RR and SPR generators
TG = 30

Figure 7 also indicates strong damping in voltage oscillations when assuming a RR generator. This phenomenon calls for more exact generator models when Hopf bifurcations are possible. Generator saturation could significantly affect power system behaviour in scenarios likely to produce voltage oscillations.

It was noticed that the slope of the eigenvalue shift for the SPR generator is bigger than that of a RR generator, Figure 8. However, the time to reach the imaginary axis is longer for a RR generator than for a SPR one. This will allow operators and controls more time to act, 100s in this case.

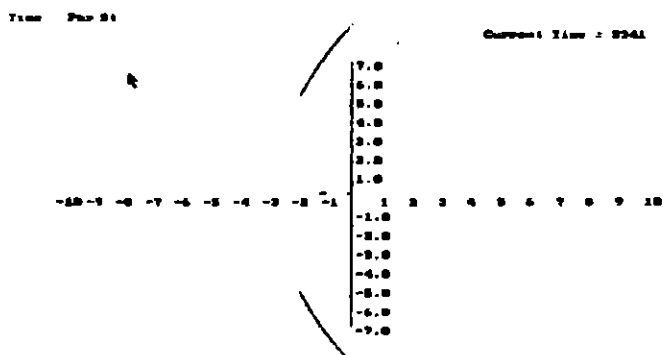


Figure 8 Eigenvalue shift for a RR and TG = 30

Sensitivity analyses to load type

All previous analyses were carried out assuming the demand to be constant-power. This section analyses system behaviour for a mixed demand of constant-power characteristic, but including a proportion of induction motors (IM). Further studies were done by selecting other types of demand and published in [47]. The reduced test system is used with AVR TG = 30, SPR generator and no SVC action.

A mix of IMs and a static constant-power load is considered. The IM proportion was set alternatively at 1% , 10% and 25% of the total demand. The reduced test system is used with a SPR generator, TG = 30 and no SVC action.

From Figures 9 and 10, a highly non-linear system behaviour is observed when the IM component is 25%. For IM demand proportions of 1% and 10%, the test system behaves almost in a linear fashion.

For an IM proportion of 1%, the test system collapses at the same power level as the case without IM. The system loadability is increased in 1.5% when the IM component increases to 10%. This is explained by the proportion of demand which responds with a voltage sensitive characteristic due to the IM model. The demand voltage magnitude for a demand 10% dynamic is slightly lower than for a demand with 1% IM load.

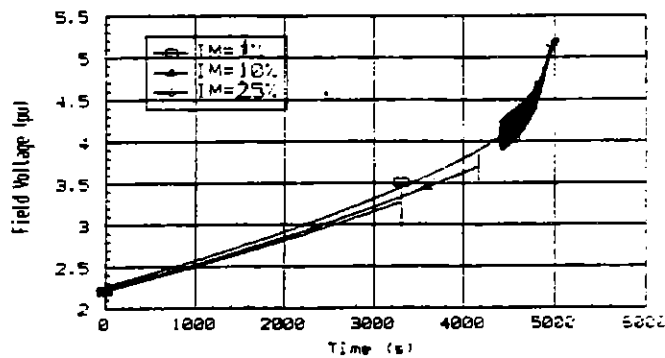


Figure 9 Field voltage for dynamic load

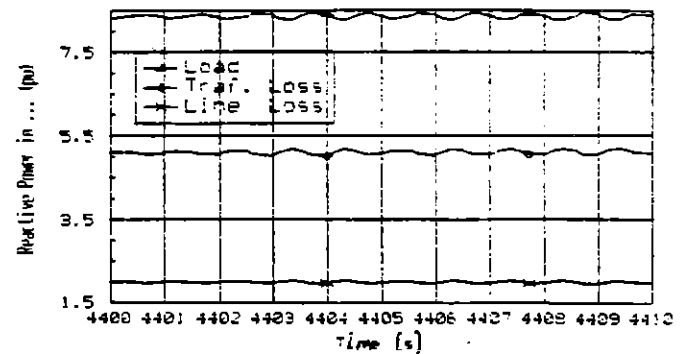


Figure 12 Q oscillations, IM = 25% of demand

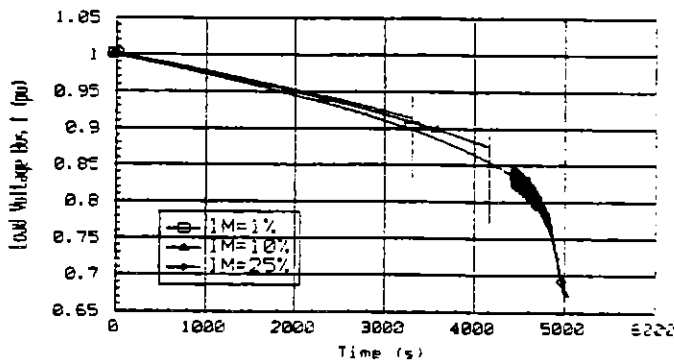


Figure 10 Demand voltage for dynamic demand

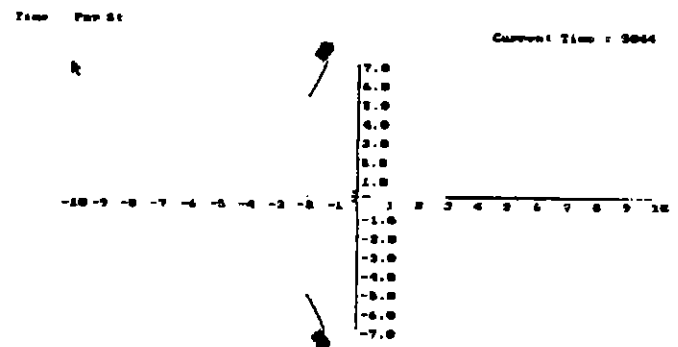


Figure 13 Eigenvalue shift for IM = 25% of total demand

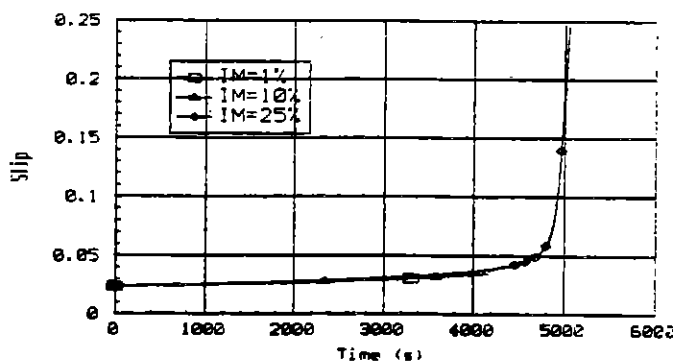


Figure 11 IM slip

Figure 12 shows the reactive power oscillations of the test system. A close view of this curve depicts oscillations of opposite phase between the load and the transformer and line reactance power losses. This transfer of energy in the system may explain the various oscillations observed. Moreover, the IM slip, Figure 11, increases rapidly at the moment that the system instability starts.

Figure 13 shows the shift of the dominant mode of this phenomenon. A non-linear movement is observed for this mode at about 4400s. A real positive eigenvalue appears at 4800s, moving to the left hand side of the complex plane. This is the point in time when oscillations start vanishing and the overall system enters a phase of graceful voltage degradation.

In all cases, the generator maximum stator current was violated at 47%, 45% and 41% demand increases for IM load proportion of 1%, 10% and 25%, respectively.

6.2. A multi-machine test system

The multi-machine test system, shown in Figure 14, consist of 3 equivalent generators with AVRs and governors, 10 busbars, 2 lines (the 500 kV line consists in 5 parallel circuits), 6 transformers and 3 capacitors. This is the test system proposed in 1991 by CIGRE Task Force 38.02.1 (Mr. C.W. Taylor convener)).

Two test conditions are considered. The full system assumes all 5 500 kV lines are connected and the reduced system considers one 500 kV line tripped. The 890 MVar and 1500 MVar capacitor banks are assumed to be SVCs. Generators are also assumed to have both SPR and RR characteristics. A 2% per-minute load increase was applied to every node.

Figures 15 to 21 show the system dynamic behaviour for different combinations of control mechanisms and machine types. The demand was taken to be constant power with no dynamic component.

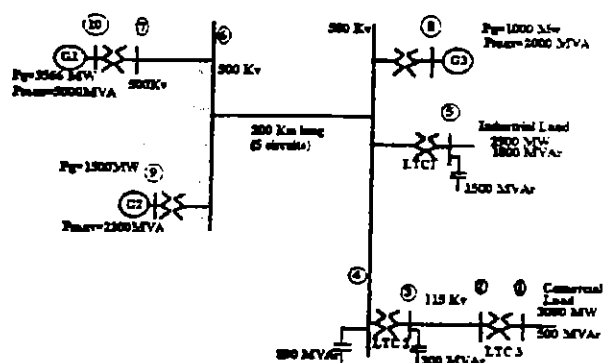


Figure 14 The CIGRE test system

The maximum stator current of Gen.2 was violated in the full system with a 40% demand increase whereas maximum stator current of Gen. 3 was violated in the reduced system at 35% demand increase, AVR only acting (these figures apply for both SPR and RR cases). With AVR's plus SVC's at buses 4 or 5, the maximum stator current limit was violated at a demand increase of 37% with both SPR and RR machines.

The system loadability is decreased by 5% when the test system is subject to the specified contingency, see Figure 15. However, by allowing SVC action on the reduced system it is possible to achieve the same load as with the intact system, the demand voltage levels being some 3% lower however.

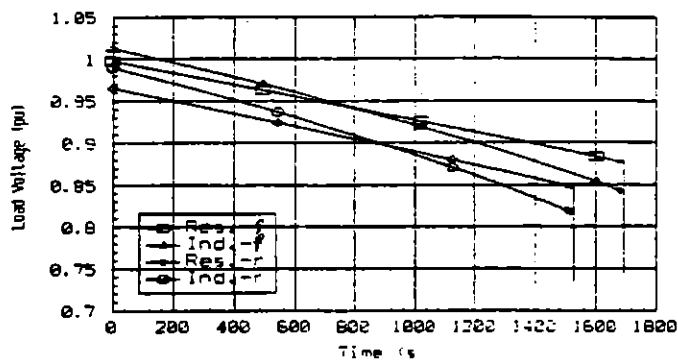


Figure 15 Demand voltage for full (f) and reduced (r) systems

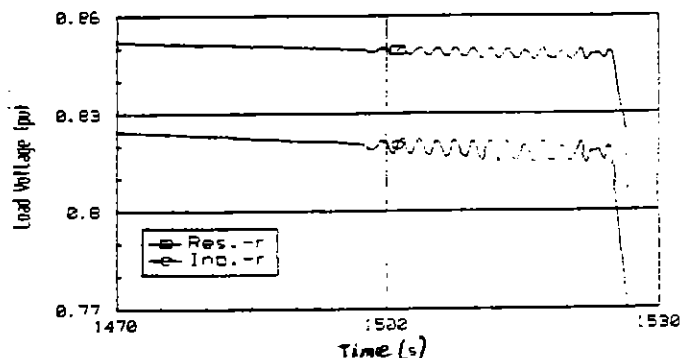


Figure 16 Demand voltage oscillation for Reduced system

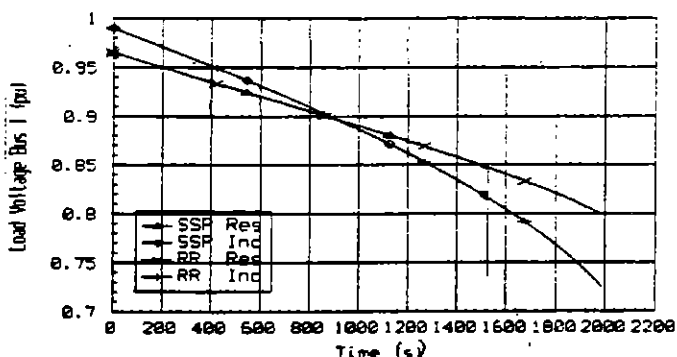


Figure 17 Demand voltage for SPR and RR generators

Figure 16 shows an expanded (in time) trace of the voltage oscillation before a sudden voltage collapse. This phenomenon is corroborated by eigenvalue analysis. Figure 19 depicts the shift of the dominant mode. It can be clearly seen that oscillations starts when the eigenvalue is on the imaginary axis and the sudden collapse is signalled by the appearance of a real positive eigenvalue moving fast to the left hand side complex plane.

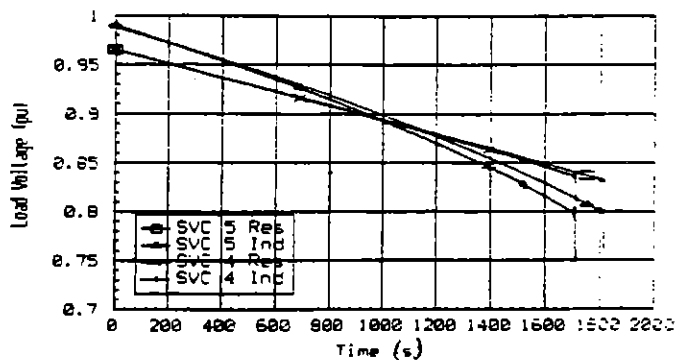


Figure 18 Demand voltage for SVC at Bus 4 and 5, SPR

Comparative analyses with respect to generator rotor type are shown in Figure 17. The collapsing time is extended by

some 500s (i.e. a demand some 16% higher) with a RR machine. Figure 20 shows a clear eigenvalue crossing and going even more unstable; a real positive eigenvalue appears travelling towards the imaginary axis. This combined effect is reflected on the network as a graceful voltage degradation. Further analysis needs to be done on this point because it suggests that it may not be appropriate to consider the imaginary axis on a multi-machine system as a "hard" stability limit of significance over the whole system [46].

Finally, Figure 21 shows the eigenvalue shift for the reduced system with generation AVR's and a SVC acting at busbar 4. Strong damping is observed whilst the SVC is acting, but this is rapidly lost when the SVC saturates. The trajectory is then the same as if the SVC does not operate (compare Figures 19 and 21).

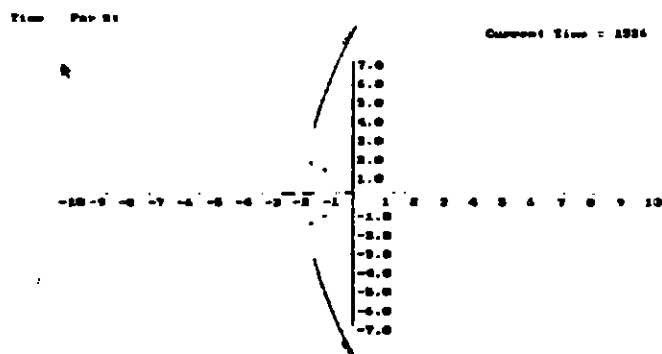


Figure 19 Eigenvalue shift for reduced system, SPR

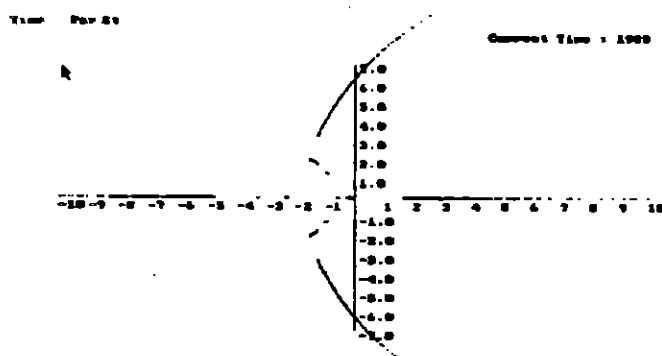


Figure 20 Eigenvalue shift for reduced system, RR

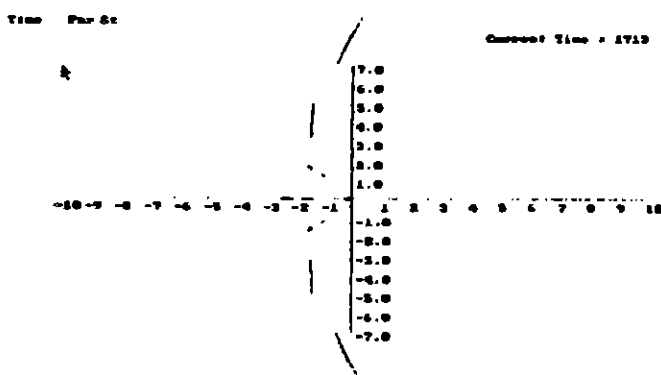


Figure 21 Eigenvalue shift for SVC in Bus 4, SPR

Work was also done on a 14 machine test system, the studies including the effects of different demand characteristics and methods of modelling the system external to the test system. It was evident that a production version of the program would require attention to coding to speed up the computation and that a more sophisticated version of bringing the test system close to the point of severe voltage problem than that employed by the authors would be helpful.

7. Conclusions

Particular points of difficulty (including the modelling of system externals to the part of the system under study) and the repercussions of these on the models used for the system and the component plant have been discussed. Demand modelling is still one of the most uncertain areas and its impact in the determination of control actions is of great importance.

The development of a voltage instability tracking method based on modal analysis to assist in locating areas of the power system with voltage problems and their causes is of paramount practical importance. It is not sufficient to raise an alarm in control centres to warn of potential system instabilities but it is also necessary to identify the causes of these problems quickly. In the light of the results obtained with the multi-machine test system, it is also necessary to clarify the concept of having the imaginary axis as a "hard" stability limit of system wide significance in these multi-machine systems. In addition the contribution of the analytical technique employed - small disturbance analysis with the system conditions sequentially linearised at the operating point - needs further study.

It has been long reported that the closer to voltage instability the power system is, the more non-linear is its behaviour. Therefore, new and more robust numerical techniques need to be used. Steady-state and dynamic analyses are complementary and should be carried out for every voltage instability found in order to understand the voltage stability problem fully. A unified methodology to incorporate voltage and angle stability studies should also be a short-term priority (see e.g. papers by J. Zaborsky and co-authors).

8. Acknowledgements

The authors would like to thank the National Grid Company plc, UK for providing support and their colleagues at Imperial College, in particular Drs Cory and Short, for helpful discussions during earlier stages of this work.

Dr. Roa-Sepulveda would also like to thank University of Concepcion, Chile; Rotary Club International, USA; University of London, UK; and the Committee of Vice Chancellors and Principals, UK for their combined support during his period of study in England.

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Appendices

Appendix A

Topology Location		Admittance B	Impedance R X	
1	2	0.2384	0.0002	0.0030
2	3	0.0000	0.0002	0.0117

OLTC Node (wot)	Node (wt)	tap op.	tap max	tap min	time (s)
3	2	1.1	1.1	0.8	100

SVCs node	Max. Cap. MVar	Max. Reac. MVar	Slope (%)	time (ms)
1	150	75	5	50

Generator name	Rating MW	#units	pf	r	xd	xq	xd'	xq'	td'	tq'
G1	372.3	4	0.9	0.0024	2.33	2.21	0.308	2.21	(s)	0.60 0.60

H	max pu	Ef	TG	Reg (%)	Time ct. (s)	Dead band (%)
5.37	2.6	30	5	0.1	1	

Load Flow Solution						
Node	Pg	Qg	Pi	Qi	Voltage	Angle
1	0.00	0.00	7.49	4.34	1.0248	-6.45
2	0.00	0.00	0.00	0.00	1.0385	-5.29
3	7.52	4.99	0.00	0.00	1.0000	0.00

Appendix B

The parameters of the voltage sensitive load model are as follows:

Cl:	Kpv2=1.80, Kqv1=3.0, Kqv2= 2.00, Qa1=0.5
Res:	Kpv2=1.04, Kqv1=0.0, Kqv2= 0.11, Qa1=0.0
Ind:	Kpv2=0.86, Kqv1=0.0, Kqv2=-0.08, Qa1=0.0

Appendix C

The assumed dynamic parameters of the multi-machine test system not included in the CIGRE paper are as follows:

SVCs

Location at bus	3	4	5
Max. Injec. (MVar)	450	1050	2100
Vol. cap. (pu)	0.95	0.05	0.95
Max. Abs. (MVar)	225	525	1050
Vol.ind. (pu)	1.025	1.025	1.025
time const. (ms)	50	50	50

Generators

	G1	G2	G3
Max. field voltage (pu)	3.0	3.2	4.6
Reg (%)	5.0	5.0	5.0
Dead Band (%)	1.0	1.0	1.0
Time const. (s)	0.1	0.1	0.2

The maximum stator current is calculated using the rated conditions of the equivalent generator.

Biographies

C.A. Roa-Sepulveda graduated with Honours from University of Concepcion, Chile in 1984. In 1993, he obtained a Ph.D. in Electrical Engineering at Imperial College of Science, Technology and Medicine, London. He became a Lecturer at University of Concepcion and a professional engineer in 1984. Research activities include power systems modelling and simulation, operational planning, EMS and the use of advanced mathematical and computational tools for power system analysis.

U.G. Knight whilst with the CEGB obtained his Ph.D. degree from the University of London in 1969 for a thesis on the application of optimisation methods to the planning and operation of power systems. He retired from CEGB in 1985 having worked in the research and development, planning and system operation fields and since then has done consultancy work in these areas. He is currently an honorary research fellow at Imperial College, London and a private consultant. His research interests include the application of expert systems and modelling techniques to power system planning and operations, and aspects of voltage stability.

STOCHASTIC EVALUATION INDICES OF DYNAMIC VOLTAGE STABILITY BASED ON MANIFOLD EQUILIBRIUM POINT

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Abstract—In this paper, the concept of "manifold equilibrium point" which expands the concept of the "stable-unstable equilibrium point" used in the past stability evaluations is newly defined and the stochastic evaluation technique of voltage stability which takes note of the disturbance at the manifold equilibrium point is proposed. Furthermore, the existence of the manifold equilibrium point and the possibility of application of the technique proposed are verified by means of simulations.

1. INTRODUCTION

Power systems in recent years are being greatly increased in service distances and in capacities, and the accompanying increase in the complexity of load characteristics and in the mismatching of the control function have become one of the leading causes of the voltage instability phenomenon. The effect of heavier load and long-distance and high-density transmission of electric energy on the voltage stability has been studied from the viewpoint of the static or quasi-dynamic analysis, but since the dynamic factors like the dynamics of the generator, the dynamics of the load and the mismatching of the control functions are considered as greatly affecting the voltage stability, the study of dynamic voltage collapse has lately become an important theme[1].

Several techniques and indices, such as 1)proximity index of voltage instability[5]-[7], 2)evaluation techniques of voltage instability based on the bifurcation theory[8] and 3)the relationship between the multiple solutions of the power flow and the voltage instability[2]-[4], have been developed and proposed in order to analyze the voltage instability problem in power systems. In the studies of these subjects, the system constituent elements and the dynamics of the generator, the dynamics of the load and the dynamic behavior of the controller are treated principally as the definitive factors.

While the rarely observed phenomenon of voltage collapse in the complex power system composed of diverse elements is controlled by the above-mentioned definitive factors, it is, at the same time, subject to marked influence of the factors of uncertainty, such as the disturbance the system is subjected to

at a certain operating point. For instance, the critical load flow point in the static analysis known as the structure of multiple load flow solutions become the singular operating point, when the high voltage solution and the low voltage solution agree[2]-[4], and when the operating state of the system is at such a singular operating point, the disturbance it suffers gives rise to three possibilities, namely, the voltage recovery, which reverts the operating point to the high-voltage solution, the voltage lowering, which sets back the operating point to the low-voltage solution and the voltage collapse, which makes the system inoperable. Also, according to the bifurcation theory, it is shown that the behavior of the system at the bifurcation point is determined by where the system trajectory shifts to[8].

Furthermore, in the dynamic analysis, where dynamic voltage collapse taking the dynamics of the on-load tap changer into account is studied, for instance, it is shown that the stability of a certain operating point (B point in the equilibrium points shown in Fig. 1) is determined by the direction of the disturbance inflicted and that the direction of the disturbance exerts marked influence on the voltage stability[9],[13].

Considered in light of these phenomena, the state of the system at the certain operating point is likely to be stable at some points and unstable at other points, depending upon the disturbance it sustains. In other words, analysis of the relationship between the unstable phenomenon known as voltage collapse and the dynamic factors by a deterministic method and how the stochastic aspects of the problem like what sort of influence is exerted on the voltage instability by the uncertainty-causing disturbance given to the system should be treated have become important keys.

In this paper, the concept of "manifold equilibrium point" which expands the concept of the "stable-unstable equilibrium point" used in the past stability evaluations is newly defined from the foregoing viewpoints and the stochastic evaluation technique of voltage instability which takes note of the disturbance at the manifold equilibrium point is proposed. Furthermore, the existence of the manifold equilibrium point

proposed is verified by means of simulations using the model that takes the dynamics of the on-load tap changer (hereinafter abbreviated as OLTC) into consideration and the model that takes the dynamics of the load into consideration. In addition, analysis and evaluation of voltage collapse will be undertaken in connection with these models using the manifold equilibrium point concept in order to show the compatibility and the possibility of application of the technique herein concerned.

This paper is composed of the following parts. In Chapter 2, the concept of manifold equilibrium point is proposed. In Chapter 3, the stochastic evaluation indices at the manifold equilibrium point are discussed. And in chapter 4, the existence of the proposed manifold equilibrium point and the applicability of the respective evaluation indices are shown by means of the results of simulations using power system models respectively taking the dynamics of OLTC and the dynamics of the load into consideration.

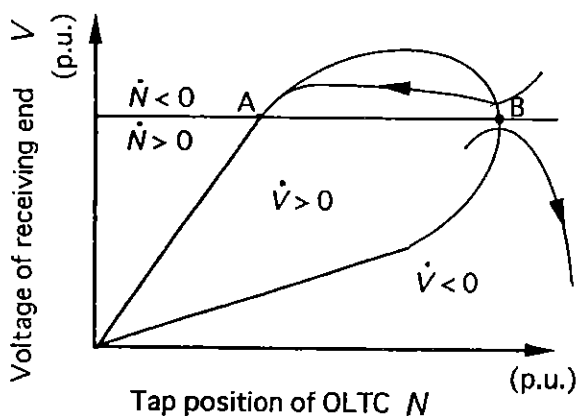


Fig. 1. Dynamic voltage collapse mechanism taking the dynamics of OLTC into consideration

II. PROPOSAL OF MANIFOLD EQUILIBRIUM POINT

In general the concept of stable-unstable equilibrium point is made use of in the study of the instability problems[10]. At the stable equilibrium point, the state of the system is at the stable equilibrium point, and it returns to the original stable equilibrium point even when it is subjected to a small disturbance (Fig. 2). Under the unstable equilibrium point, on the other hand, the system state is also at the equilibrium point, but when it is subjected to a small disturbance, it directly falls into an unstable state (Fig. 3). In short, with the models like the stable-unstable equilibrium point type, the destination of the system is inevitably the stability or instability regardless of the "direction" of the small disturbance. However, in the complex dynamic phenomena encountered in actual engineering, the stability of a certain equilibrium point is considered to be dependent upon the definitive input-output relations with the configuration elements of itself and outside, and, at the same time, to be closely related to the direction of the disturbance the system is subjected to. At this juncture, therefore, the following "manifold equilibrium point" is proposed.

The "manifold equilibrium point" means

- 1) The system is in a state of equilibrium without any disturbance.
- 2) The system becomes stable or unstable, or returns to its manifold equilibrium point itself in accordance with the small disturbance the system is subjected to.

Fig.4 represents the concept of the "manifold equilibrium point" proposed and shows the peculiar fact that the system stability differs according to the "direction" of the small disturbance it is subjected to. Fig.5, moreover, shows the distribution of behaviors in the vicinity of the manifold equilibrium point. The "Dead band" in the figure, incidentally, means that under certain constraint conditions the disturbance does not arise in this direction.

The "manifold equilibrium point" is the expanded version of the normal "stable-unstable equilibrium point" concept. In Fig.5, $c=2\pi$ makes the "manifold equilibrium point" a "stable equilibrium point" and $a=2\pi$ results in the unstable equilibrium point. Furthermore, $d=2\pi$ makes the operation impossible and $b=2\pi$ makes the operating point of the system move to another stable equilibrium point (which is defined in this paper as the "transient equilibrium point").

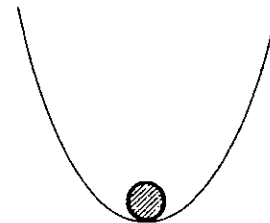


Fig.2. Stable equilibrium point

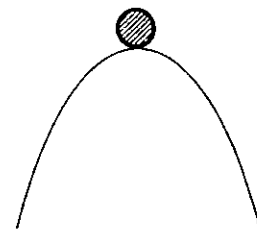


Fig.3. Unstable equilibrium point

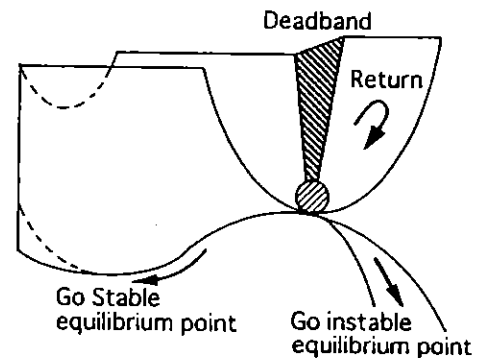


Fig.4. Manifold equilibrium point

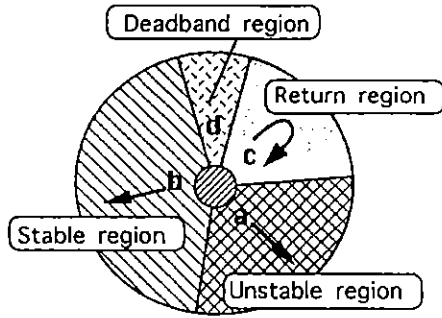


Fig. 5. Diagram showing the behavioral distribution of the manifold equilibrium point

III. STOCHASTIC EVALUATION INDICIES AT THE MANIFOLD EQUILIBRIUM POINT

The "manifold equilibrium point" proposed in this study is the expanded version of the concept of "stable-unstable equilibrium point", and the stability of the manifold equilibrium point differs in accordance with the direction of the small disturbance acting on the system. The small disturbance acting on the system is generally considered to be composite actions of many factors of uncertainty, and the direction of small disturbance sustained at the manifold equilibrium point is uncertain. In short, the stability of the "manifold equilibrium point" is uncertain and is thought to have some connection with the stochastic factor.

In this paper the stability of "manifold equilibrium point" will be analyzed on the basis of the postulates given below.

Postulate 1. When the state of the system is at the manifold equilibrium point, the behavior of the system relative to subsequent stability is determined by the direction of the small disturbance it sustains.

Postulate 2. The direction of the system disturbance at the manifold equilibrium point is given by the probability density function $P(\theta)$ related to the angle θ .

Postulate 3. The generation probability of the event known as system disturbance complies with the ergodic hypothesis, which regards "the time probability and the set probability as being identical" [11].

If the direction of the disturbance at the manifold equilibrium point is given by the probability density function relative to the angle θ , the probability of disturbance generation in the various regions of Fig. 5 becomes as given below,

$$\begin{aligned}\sigma_a &= \int_a P(\theta) d\theta, & \sigma_b &= \int_b P(\theta) d\theta \\ \sigma_c &= \int_c P(\theta) d\theta, & \sigma_d &= \int_d P(\theta) d\theta\end{aligned}$$

where σ_a , σ_b , σ_c , and σ_d represent probabilities of disturbance generation in the unstable region, stable region, return region and dead band, respectively. and $0 \leq \sigma_a, \sigma_b, \sigma_c, \sigma_d \leq 1$. Normally, as $\sigma_d = 0$, $\sigma_a + \sigma_b + \sigma_c = 1$. In the next section, the stochastic evaluation indices relative to the generation probability of various events at the manifold equilibrium point will be defined.

A. The Generation Probability of Various Events at Manifold Equilibrium point

The event generation probability at the manifold equilibrium point is calculated on the basis of the ergodic hypothesis, which regards the time probability and the set probability as being identical.

"The time probability and the set probability are identical" means that the generation probability, is identical even when the event generation probability relative to a system is considered for an infinite time, or when the generation probability relative to the similar events of similar systems of infinite numbers is considered at the same time. In this paper this concept of time probability based on the ergodic hypothesis will be used in order to facilitate calculations of the various event generation probabilities (set probability) at the manifold equilibrium point.

In the analyses made in the next section, σ_a , σ_b and σ_c are assumed to be more than 0 but less than 1.

(a) Generation Probability P_a of Instability Phenomenon at the Manifold Equilibrium Point

When the effect of the return region is considered using the concept of time probability, the generation probability of unstable events at the manifold equilibrium point can be obtained with the following equations:

$$\begin{aligned}P_a &= \sigma_a + \sigma_c \cdot \sigma_a + \sigma_c \cdot \sigma_c \cdot \sigma_a + \dots \\ &= \sigma_a [1 + \sigma_c + \dots + \sigma_c^k + \dots] \\ \sigma_c &< 1 \\ \therefore P_a &= \frac{\sigma_a}{(1 - \sigma_c)} = \frac{\sigma_a}{(\sigma_a + \sigma_b)}\end{aligned}\quad (1)$$

(b) Generation Probability P_b of Stability Phenomenon at the Manifold Equilibrium Point

$$\begin{aligned}P_b &= \sigma_b + \sigma_c \cdot \sigma_b + \sigma_c \cdot \sigma_c \cdot \sigma_b + \dots \\ &= \sigma_b [1 + \sigma_c + \dots + \sigma_c^k + \dots] \\ \therefore P_b &= \frac{\sigma_b}{(1 - \sigma_c)} = \frac{\sigma_b}{(\sigma_a + \sigma_b)}\end{aligned}\quad (2)$$

As shown in equations (1) and (2), if the disturbance generation probability in the return region is taken as $\sigma_c = 0$, $P_a = \sigma_a$ and $P_b = \sigma_b$ since $\sigma_a + \sigma_b = 1$, and if the disturbance generation probability in the return region is $\sigma_c \neq 0$, $P_a > \sigma_a$ and $P_b > \sigma_b$ since $\sigma_a + \sigma_b < 1$. In other words, if no return region is available, P_a and P_b become respectively identical with σ_a and σ_b , and if the return region is available, it is evident that P_a and P_b respectively become larger than σ_a and σ_b .

(c) Residence Probability P_c in the Return Region at the Manifold Equilibrium Point

$$P_c = \lim_{k \rightarrow \infty} \sigma_c^k = 0 \quad (3)$$

In the generation probability of the events in the manifold equilibrium point described above, the disturbance generation probability in the return region does exist, but since its residence probability P_c is zero when viewed in light of infinite time, the generation probability P_a of the instability phenomenon and the generation probability P_b of the stability phenomenon are determined solely by the disturbance generation probabilities σ_a and σ_b in the stable and unstable regions. As $P_a + P_b = 1$, consideration over an infinite time gives the final system stability as stable or unstable respectively at probability P_a or P_b .

B. Time Ratios in the Generation of Various Events

The generation probability of various events at the manifold equilibrium points evaluates the system stability as the set probability. One other problem that should be considered in connection with voltage stability is the temporal evaluation of unstable events occurring at identical probabilities, such as whether the events are generated immediately or after the passage of a lone time. This temporal evaluation is particularly important for the voltage recovery control. At this juncture, therefore, the following evaluation indices are proposed for evaluating the temporal speed of the various events generated by the probabilistic method.

(a) Time Ratio T_a of Instability Phenomenon Generation at the Manifold Equilibrium Point

$$T_a = \frac{(\sigma_b + \sigma_c)}{(\sigma_a + \sigma_b + \sigma_c)} = (\sigma_b + \sigma_c) \quad (4)$$

(b) Time Ratio T_b of Stability Phenomenon Generation at The Manifold Equilibrium Point

$$T_b = \frac{(\sigma_a + \sigma_c)}{(\sigma_a + \sigma_b + \sigma_c)} = (\sigma_a + \sigma_c) \quad (5)$$

(c) Time Ratio T_{a+b} of Stability-Instability Phenomenon Generation at the Manifold Equilibrium Point

$$T_{a+b} = \frac{\sigma_c}{(\sigma_a + \sigma_b + \sigma_c)} = \sigma_c \quad (6)$$

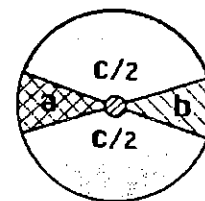
Here the time ratios of various phenomena generation take (0, 1) values. When the value is close to zero, the phenomenon is shown to arise stochastically immediately, but when it is close to 1, the phenomenon is shown to arise stochastically after the passage of an infinite time. At the manifold equilibrium point proposed, only two possibilities, the stability and instability, are available as the ultimate results of system behavior, but the lengths of the stochastic

time of various phenomena generation is subject to the influence of the return region. The fact that the time ratio T_{a+b} of the stability-instability phenomenon generation at the manifold equilibrium point is equal to the disturbance generation probability σ_c in the return region is evident from Fig. (6).

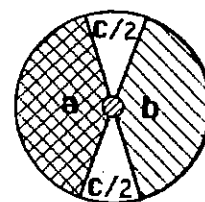
The time ratio T_{a+b} is pertinent for overall evaluation thereof. For instance, in the two manifold equilibrium points shown in Fig.6, the evaluation indices would be as given below if the system disturbance probability density $P(\theta)$ is uniformly distributed against the angle θ .

$$\begin{aligned} P_{a1} &= P_{a2} = 1/2 & P_{b1} &= P_{b2} = 1/2 \\ T_{(a+b),1} &= \frac{C_1}{2\pi} \rightarrow 1 & T_{(a+b),2} &= \frac{C_2}{2\pi} \rightarrow 0 \end{aligned}$$

The instability phenomenon generation probabilities P_{a1} and P_{a2} and the stability phenomenon generation probabilities P_{b1} and P_{b2} at the manifold equilibrium point 1 and the manifold equilibrium point 2 are identical respectively, but the time ratios $T_{(a+b),1}$ and $T_{(a+b),2}$ of stability-instability phenomenon generation are entirely different. As the return region of the manifold equilibrium point 1 is wide (the probability of disturbance being generated in the return region is high), the stability-instability phenomena are not stochastically generated till late ($T_{(a+b),1} \rightarrow 1$). Against this, as the return region of the manifold equilibrium point 2 is small (the probability of disturbance being generated in the return region is low), the stability-instability phenomenon is stochastically generated immediately ($T_{(a+b),2} \rightarrow 0$). The instability phenomena generation probabilities may be identical when viewed from the voltage recovery control viewpoint, but more urgent control is required at the manifold equilibrium point 2. In this paper the following two are adopted as the stability evaluation indices at the manifold equilibrium points from the foregoing arguments.



(a) Manifold equilibrium point 1



(b) Manifold equilibrium point 2

Fig. 6. Comparison of manifold equilibrium points

Generation probability of instability phenomenon :

$$P_a = \frac{\sigma_a}{(\sigma_a + \sigma_b)} \quad (7)$$

Time ratio of stability-instability phenomena generation:

$$T_{a+b} = \sigma_c \quad (8)$$

When the stability of the manifold equilibrium points given in Fig.6 is evaluated using the above two indices, the instability phenomenon generation probability $Pa2$ of the manifold equilibrium point 2 and the stability phenomenon generation probability $Pa1$ of the manifold equilibrium point 1 are identical, but when the time ratios of stability-instability phenomenon generation at that time is evaluated, the stability-instability phenomenon at the manifold equilibrium point 2 is construed to arise immediately at a higher generation probability when compared with the probability of the manifold equilibrium point 1.

IV. VERIFICATION OF THE MANIFOLD EQUILIBRIUM POINT UNDER DYNAMIC VOLTAGE STABILITY

In this chapter the existence of the manifold equilibrium point under dynamic voltage stability and the applicability of the proposed stochastic evaluation methods will be verified by means of two simulations using the model that takes the dynamics of OLTC into consideration and the model that takes dynamics of the load into consideration.

A.Verification of the Manifold Equilibrium Point with the Model Taking the Dynamics of OLTC into Consideration

(a) Dynamics of OLTC[12],[15]

As the reverse action phenomenon of OLTC(i.e., the voltage drops despite raised tap setting value), which is a voltage- reactive power controller, has become clarified, the fact that it plays an important role in voltage stability has become known. In this section, the existence of the proposed manifold equilibrium points and the applicability of the stochastic evaluation indices will be verified by means of simulations using a simple model system having 2 units of OLTC, as shown in Fig. 7.

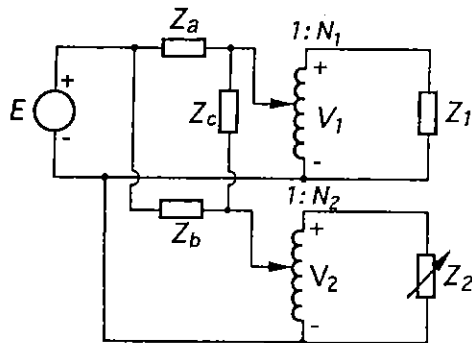


Fig. 7. A simple mode system including two units of OLTC

The Dynamics of OLTC is obtained with the equations given below.

$$\frac{dN_1}{dt} = \frac{1}{T_1} (V_{10} - V_1(N_1, N_2)) \quad (9)$$

$$\frac{dN_2}{dt} = \frac{1}{T_2} (V_{20} - V_2(N_1, N_2)) \quad (10)$$

The meanings of the symbols used in equations (9) and (10) in Fig.7 are as given below.

- N_1, N_2 : Tap position of OLTC.
- V_{10}, V_{20} : Preset voltage.
- V_1, V_2 : Secondary voltage of OLTC.
- T_1, T_2 : Time constant of OLTC.
- E : Generator terminal voltage.

Table 1. Values [p.u] of the constants of the model given in Fig.7

Z_a	0.3	Z_l	0.260275
Z_b	0.2	V_{01}/V_{02}	0.8/1.0
Z_c	0.25	E	1.3122142

The model system developed in [13] is used in the simulation. All values of impedance are assumed to be inductive for simplicity.

$$\begin{aligned} \dot{Z}_a &= j Z_a & \dot{Z}_b &= j Z_b & \dot{Z}_c &= j Z_c \\ \dot{Z}_1 &= j Z_1 & \dot{Z}_2 &= j Z_2 \end{aligned}$$

The equations for the secondary voltages of OLTC are obtained from the circuit equations, and these substituted into equations (9) and (10) and adjusted give the equations given below[12],[15].

$$\frac{dN_1}{dt} = \frac{f_1(N_1, N_2)}{f(N_1, N_2)} \quad (11)$$

$$\frac{dN_2}{dt} = \frac{f_2(N_1, N_2)}{f(N_1, N_2)} \quad (12)$$

provided, however,

$$\begin{aligned} f_1(N_1, N_2) &= (Y_2 + N_2^2/Z_2) V_{10} N_1^2/Z_1 \\ &\quad - (H + N_2^2/(Z_2 Z_a)) E N_1 \\ &\quad + (H + Y_1 N_2^2/Z_2) V_{10} \end{aligned} \quad (13)$$

$$\begin{aligned} f_2(N_1, N_2) &= (Y_1 + N_1^2/Z_1) V_{20} N_2^2/Z_2 \\ &\quad - (H + N_1^2/(Z_1 Z_b)) E N_2 \\ &\quad + (H + Y_2 N_1^2/Z_1) V_{20} \end{aligned} \quad (14)$$

$$\begin{aligned} f(N_1, N_2) &= N_1^2 N_2^2/(Z_1 Z_2) + Y_1 N_2^2/Z_2 \\ &\quad + Y_2 N_1^2/Z_1 + H \end{aligned} \quad (15)$$

(b) The Manifold Equilibrium Point of OLTC Model and the Simulation Result

In this section the differential equations (11) and (12) showing the dynamics of two OLTC units will be used as concerned models at this time and the existence of the proposed manifold equilibrium point and the applicability of the stochastic evaluation indices proposed will be examined. The values of the constants are as shown in Table 1. Here Z_2 as parameter will be varied. The tap position of the OLTC in the state of equilibrium points can be obtained from the intersection points of two curved lines indicated by means of simultaneous algebraic equations that use the left-hand side members of equations (11) and (12) as zero. Fig. 8-A shows the two curved lines and the two intersection points ($Z_2 = -j1.0$). The left equilibrium point is the normal stable equilibrium point, while the right equilibrium point is the proposed manifold equilibrium point. The behavior of the system around the manifold equilibrium point faces two possibilities, the stable and unstable. In Fig. 8-B the distribution of the behaviors at the manifold equilibrium point is shown. The behavior distribution figure, using the various point on the circumference of 0.04 radius serving as the center of the manifold equilibrium point as the starting point, can be obtained by solving the simultaneous differential equations (11) and (12).

Figs.9-A and 9-B show the manifold equilibrium point and the distribution of behaviors at $Z_2 = -j3.0$, while Figs.10-A and 10-B show the manifold equilibrium point and the distribution of behaviors at $Z_2 = -j4.196$. As the parameter Z_2 increases in the negative direction (i.e., in the direction in which the capacitance of the reactive load declines), the stable equilibrium point and the manifold equilibrium point gradually draw closer to each other and ultimately become unified to form a single manifold equilibrium point. Moreover, the stable region of the manifold equilibrium point also gradually become smaller as parameter Z_2 increases, and it finally becomes zero, as is shown in the figure. At the point (the so-called fold bifurcation point), where the stable equilibrium point and the manifold equilibrium point converge, the stable region vanishes and only the return region and the unstable region exist. For this reason, the generation probability of the instability phenomena P_a is 1. In short, it can easily be understood that at this point the instability phenomenon known as voltage collapse inevitably arises at the probability of $P_a = 1$.

When the probability density function $P(\theta)$ of system disturbance is assumed to be uniformly distributed against the angle θ , the relationship between parameter Z_2 and the generation probability P_a of instability phenomenon arising at the manifold equilibrium point is shown in Fig.11. It can be seen that when the parameter Z_2 increases in the negative direction, the generation probability of instability phenomenon P_a rises at the manifold equilibrium point and it finally become 1. In other words, as the capacitance part of the reactive power load decreases, the generation probability of voltage collapsed at the manifold equilibrium point rises higher.

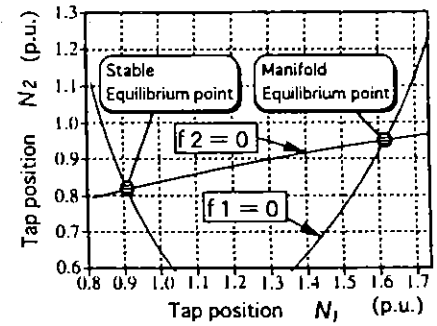


Fig.8-A. Manifold equilibrium point ($Z_2 = -j1.0$)

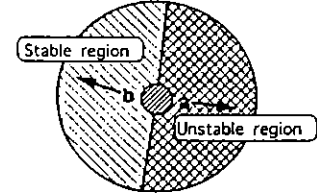


Fig.8-B. Distribution of behaviors ($Z_2 = -j1.0$)

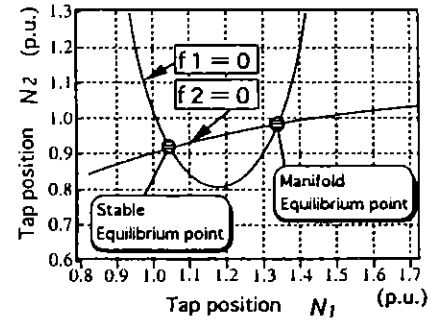


Fig.9-A. Manifold equilibrium point ($Z_2 = -j3.0$)

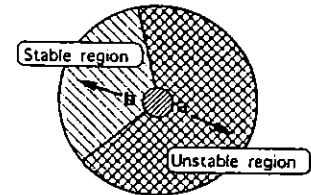


Fig.9-B. Distribution of behaviors ($Z_2 = -j3.0$)

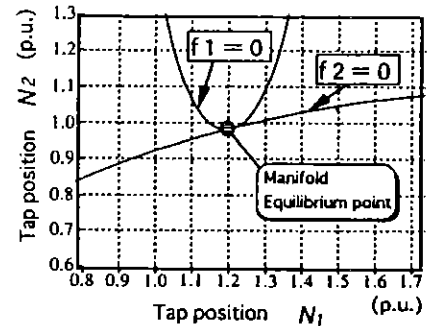


Fig.10-A. Manifold equilibrium point ($Z_2 = -j4.196$)

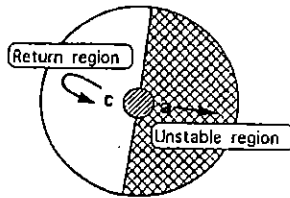


Fig.10-B. Distribution of behaviors ($Z_2=-j4.196$)

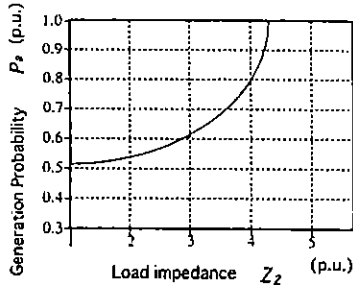


Fig.11. The relation between P_a and Z_2

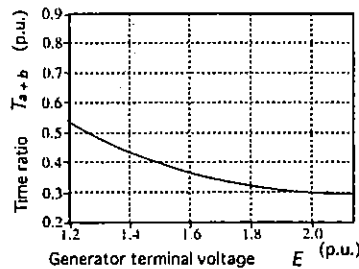


Fig.12. The relation between T_{a+b} and E

The time ratio of stable-unstable phenomenon generation at the manifold equilibrium point using the generator terminal voltage E as parameter is shown in Fig.12. As can be seen from Fig.12, higher the generator terminal voltage E rises, the lower the time ratio T_{a+b} of the stable-unstable phenomenon generation at the fold bifurcation point becomes, and the possibility of voltage collapse taking place within a short time becomes higher. In short, rise in the generator terminal voltage exerts exceedingly strong influence on the dynamical voltage stability. In the simulations conducted at this time, it has been seen that the return region existed only at the fold bifurcation point where the stability equilibrium point and the manifold equilibrium point converge, but the existence of the return region at the normal manifold equilibrium point of the system using more than two OLTC units could also be anticipated.

B. Verification of the Manifold Equilibrium Point by Means of the Two-Bus Power System Taking the Dynamics of the Load into consideration

In this section the existence of the manifold equilibrium point under the dynamic voltage stability and the applicability of the proposed evaluation indices will be verified by using the two-bus model system (see Fig. 13). Incidentally, the definitions of the variables are as given in Table 2.

Table 2. Definitions of the variables of the model shown in Fig. 13(p.u.)

E	:	The voltage of the sending end(to be constant).
V_2, V_3	:	The voltage of the receiving end.
V_{2d}, V_{3d}	:	Preset voltage of receiving end.
θ_1, θ_2	:	Phase angle of preset voltage at the receiving end.
X_{12}, X_{13}	:	Reactance of line.
N_2, N_3	:	The tap ratio of OLTC.
QC_2, QC_3	:	Rotary condenser of the receiving end.
YC_2, YC_3	:	Admittance of the phase modifier equipment of the receiving end.
KQ_2, KQ_3	:	Time constant of the dynamics of the reactive load of the receiving end.
$\alpha_{P2}, \beta_{P2}, \gamma_{P2}$	:	Constant of the active load static characteristics of the receiving end 2.
$\alpha_{P3}, \beta_{P3}, \gamma_{P3}$	:	Constant of the active load static characteristics of the receiving end 3.
$\alpha_{Q2}, \beta_{Q2}, \gamma_{Q2}$	:	Constant of the reactive load static characteristics of the receiving end 2.
$\alpha_{Q3}, \beta_{Q3}, \gamma_{Q3}$	:	Constant of the reactive load static characteristics of the receiving end 3.
$PR_{12}, QR_{12}, PR_{23}, QR_{23}$	:	Active-reactive power of the lines.
PL_2, QL_2, PL_3, QL_3	:	Active and reactive power supplied to the load.

Table 3. Values(p.u.) of various constants of the model given in Fig. 13.

α_{P2}	0.0700	β_{P2}	-0.112	γ_{P2}	0.742
α_{P3}	0.0600	β_{P3}	-0.0900	γ_{P3}	0.6300
α_{Q2}	2.6200	β_{Q2}	-2.7400	γ_{Q2}	0.3583
α_{Q3}	0.1268	β_{Q3}	-0.7350	γ_{Q3}	1.2500
X_{12}	0.0150	X_{23}	0.4515	YC_3	3.0000
N_2	1.0100	N_3	1.0100	E	1.0100

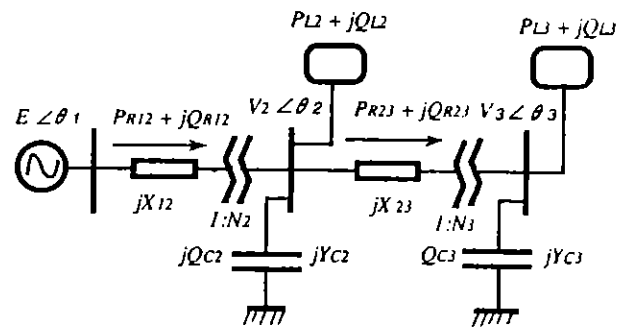


Fig.13. Two-bus mode system including the dynamics of load

(a) Formulation of the 2-Bus Power System Model

The relationship between system conditions and load conditions of various types will be shown below.

Dynamics of Load[14]

The dynamics of load has become one of the main causes of the voltage instability phenomenon. As the characteristics of the users are of infinite variety, the characteristics of the load of power systems are also diverse. In this paper the following models of Load dynamics proposed in [14] will be used from the viewpoint that the dynamics of reactive power strongly affect the voltage stability.

$$P_{Li}(V_i) = P_{si}(V_i) \quad (16)$$

$$Q_{Li}(V_i, \dot{V}_i) = Q_{si}(V_i) + K_{Qi}(\dot{V}_i) \quad (17)$$

provided,

$$P_{si}(V_i) = \alpha_{Pi}V_i^2 + \beta_{Pi}V_i + \gamma_{Pi} \quad i=2,3 \quad (18)$$

$$Q_{si}(V_i) = \alpha_{Qi}V_i^2 + \beta_{Qi}V_i + \gamma_{Qi} \quad i=2,3 \quad (19)$$

Reactive Power Balance Equations.

$$Q_{R12} + Q_{C2} = Q_{L2} + Q_{R23} \quad (20)$$

$$Q_{C2} = Y_{C2}V_2^2 \quad (21)$$

$$Q_{R23} + Q_{C3} = Q_{L3} \quad (22)$$

$$Q_{C3} = Y_{C3}V_3^2 \quad (23)$$

Active Power Balance Equations.

$$P_{R12} = P_{L2} + P_{R23} \quad (24)$$

$$P_{R23} = P_{L3} \quad (25)$$

Load Flow Equations of the Line.

$$P_{R12} = \frac{EV_2}{X_{12}N_2} \sin(\theta_1 - \theta_2) \quad (26)$$

$$Q_{R12} = \frac{EV_2}{X_{12}N_2} \cos(\theta_1 - \theta_2) - \frac{V_2^2}{X_{12}N_2^2} \quad (27)$$

$$P_{R23} = \frac{V_2V_3}{X_{23}N_3} \sin \theta_2 \quad (28)$$

$$Q_{R23} = \frac{V_2V_3}{X_{23}N_3} \cos \theta_2 - \frac{V_3^2}{X_{23}N_3^2} \quad (29)$$

Dynamic Equations Relative to Voltages.

When the equations from (16) and (29) are arranged in order, dynamic equations relative to the following voltages are obtained.

$$\begin{aligned} \frac{dV_2}{dt} = \frac{1}{K_{Q2}} & \left(\frac{EV_2}{X_{12}N_2} \cos(\theta_1 - \theta_2) \right. \\ & - \frac{V_2^2}{X_{12}N_2^2} - \frac{V_2V_3}{X_{23}N_3} \cos \theta_2 + \frac{V_3^2}{X_{23}N_3^2} \\ & \left. + Y_{C2}V_2^2 - \alpha_{Q2}V_2^2 - \beta_{Q2}V_2 - \gamma_{Q2} \right) \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{dV_3}{dt} = \frac{1}{K_{Q3}} & \left(\frac{V_2V_3}{X_{23}N_3} \cos \theta_2 - \frac{V_3^2}{X_{23}N_3^2} \right. \\ & \left. + Y_{C3}V_3^2 - \alpha_{Q3}V_3^2 - \beta_{Q3}V_3 - \gamma_{Q3} \right) \end{aligned} \quad (31)$$

(b) The Manifold Equilibrium Point of the 2-Bus Model and the Results of Simulations.

In the following the existence of the manifold equilibrium point and the applicability of the proposed stochastic evaluation indices will be studied using differential equations (30) and (31), which indicate the dynamic characteristics of the 2-bus model that takes the dynamics of the load into consideration. The values of the constants are given in Table 3. Y_{C2} will be varied as a parameter. The state of equilibrium point of the system is determined by solving the simultaneous equations having zero for the left-hand members of equations (30) and (31), making it possible to grasp it as the intersection point of the two curved lines indicated in the equations. Fig.14-A indicates the curved lines at ($Y_{C2}=4.35$) and the intersection point of the two. The right equilibrium point is a normal stable equilibrium Point, but the left equilibrium point is the proposed manifold equilibrium point, and two possibilities, i.e., stability and instability, are available for the behaviors of the system around the manifold equilibrium point. The distribution of the behaviors at the manifold equilibrium point is shown in Fig. 14-B.

Figs.15-A and 15-B show the manifold equilibrium point and the distribution of behaviors at $Y_{C2}=2.35$, while Figs. 16-A and 16-B show the manifold equilibrium points and the distribution of behaviors at $Y_{C2}=0.35$. As the parameter Y_{C2} declines, the stability equilibrium Point and the manifold equilibrium point gradually draw closer to each other and finally converge to form a single manifold equilibrium point. Moreover, as the parameter Y_{C2} declines the stable region of the manifold equilibrium point gradually grows smaller and finally becomes zero. This fact can be seen from the figure. As given in the example cited in (4.1), it can be seen that where the stability equilibrium point and the manifold equilibrium point converge (the so-called fold bifurcation point), the unstable phenomenon known as voltage collapse inevitably arises at the probability of $P_a=1$.

When the probability density function $P(\theta)$ of the system disturbance is assumed to be uniformly distributed against the angle, the relationship between parameter Y_{C2} and the generation probability P_a of instability phenomenon at the manifold equilibrium point is shown in Fig.17. As is evident from Fig.17, when parameter Y_{C2} (reactive compensation

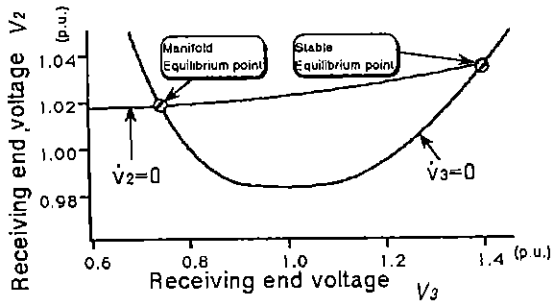


Fig.14-A. Manifold equilibrium point ($Y_{c2}=4.35$)

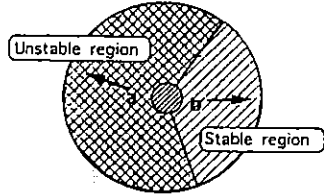


Fig.14-B. Distribution of behaviors ($Y_{c2}=4.35$)

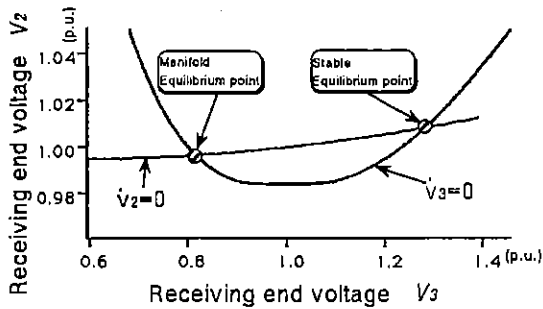


Fig.15-A. Manifold equilibrium point ($Y_{c2}=2.35$)

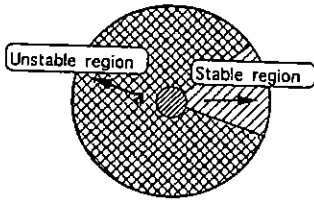


Fig.15-B. Distribution of behaviors ($Y_{c2}=2.35$)

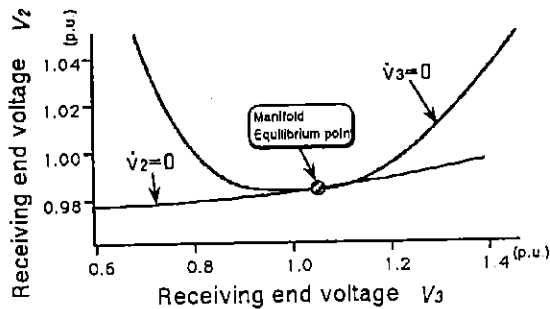


Fig.16-A. Manifold equilibrium point ($Y_{c2}=0.35$)

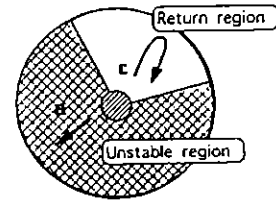


Fig.16-B. Distribution of behaviors ($Y_{c2}=0.35$)

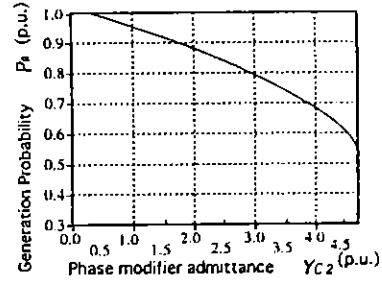


Fig.17. The relations between Y_{c2} and P_a

capacity) declines, (1) the voltage at the stability equilibrium point gradually drops and (2) the generation probability P_a of instability phenomenon at the manifold equilibrium point rises and it finally reaches 1. It can thus be seen that the parameter Y_c exerts influence on the dynamic voltage stability from two aspects.

V. CONCLUSION

1) In this paper the manifold equilibrium point, which expands the concept of stable-unstable equilibrium point involved in the past stability evaluation, is proposed from the viewpoint that the rarely seen phenomenon of voltage collapse is under control of deterministic factors but also has some relationship with the factor of uncertainty known as small disturbance the system sustains at some operation point. The stability of this manifold equilibrium point is evaluated on the basis of the direction of the small disturbance the system sustains.

2) Assuming that the distribution of disturbance at the manifold equilibrium point against the angle θ is controlled by the probability density function $P(\theta)$, probability evaluation indices for the generation probability P_a of the unstable phenomenon at the manifold equilibrium point indicating the possibility of the generation of the instability phenomenon known as voltage collapse at the manifold equilibrium point, and the stability-instability phenomenon generation time ratio T_{a+b} at the manifold equilibrium point, which indicates the length of time till generation takes place, have been defined. Furthermore, these indices can easily be obtained on the basis of the ergodic hypothesis of noise analysis.

3) Verification of the existence of the manifold equilibrium point relative to voltage stability has been undertaken using the model that takes the dynamics of OLTC into account and the model that takes the dynamics of the load into account.

Furthermore stochastic evaluation relative to the dynamic voltage stability has been made using the proposed manifold equilibrium point theory. The result of simulation cited in this paper and the physical causal relationship of voltage instability phenomenon are in agreement, showing the usefulness of the manifold equilibrium point.

4) The models used for simulations are both expressed in differential equations having two variables, but the use of differential equations having more two variables is also possible. However, when application of the manifold equilibrium point theory is to be made using differential equations having more than two variables, multidimensional stochastic variables must be used.

In the future studies of the calculations in the analysis of the various regions (stable region, unstable region, return region) relative to the manifold equilibrium point, selection of the probability density functions of disturbance at the manifold equilibrium point and of the application of the manifold equilibrium point theory to power systems of practical scales are to be undertaken.

VI. ACKNOWLEDGMENT

In conclusion, deep gratitude is due Professor Yasuo Tamura of Waseda University for the simulation models provided and for the guidance on all aspects of the present study he has kindly provided.

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Use Of Multiple Load Flow Solutions To Approximate A Closest Loadability Limit

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Abstract

This paper proposes the use of a multiple load flow solution pair to approximate a closest loadability limit (voltage collapse point or saddle-node bifurcation). By the solution pair is meant, an operating solution and a low voltage solution, which coalesce one another at a loadability limit. The paper discusses several features of the solution pair, which may have been overlooked in previous studies, and then proposes a method to estimate a closest loadability limit. The left and right eigenvectors corresponding to the zero eigenvalue of load flow Jacobian are also approximated in terms of the solution pair. Most of the computation time of the proposed method is taken to calculate the load flow solution pair. Additional computation time is less than that of an ordinary load flow.

Introduction

During the last few years various methods for detecting loadability limits of power systems have been proposed. A simple method to find the loadability limits is to repeatedly use conventional load flow under increase in loading condition until convergence is no longer obtained. This method inherently has convergence problem near the loadability limits since the load flow Jacobian becomes singular at this limit. A practical extension of this method is to interpolate or to extrapolate a series of the load flow solutions to estimate a loadability limit. Continuation methods [3-6] are a reliable way to avoid the singularity of the Jacobian and are free from the convergence problem; therefore, even low voltage load flow solutions can be computed to obtain the P-V curve if needed. Point of collapse methods [3,6-10] are a good way to directly compute the loadability limits. An advantage of these approaches is that the left or right eigenvector corresponding to the zero eigenvalue of the load flow Jacobian can be obtained as a by-product of its computation. These eigenvectors are useful for identification of weak spots and for examination of possible control actions to avoid voltage collapse [11]. There is another approach to utilize an special analytical feature of load flow equations [12]. All of these approaches are common in that the loadability limits are computed at a specific operating point assuming that the loads are increased towards a specified direction most likely to occur.

A closest loadability limit from an operating point may be of

importance from the viewpoint of secure monitoring. This limit, for example, corresponds to the case where the loads are increased towards the worst direction. One way to obtain this method is to utilize one of the above approaches. That is, after calculating a loadability limit for some specified direction, the left eigenvector is computed, which is used to determine a new direction; this process is repeated until the direction is converged [13]. This process is, however, expensive. Direct computation method has also been proposed in [13], and its application to large scale systems may be possible but has not yet been done as long as the authors know.

Low voltage load flow solutions have often been discussed in voltage stability studies. Analytical features of such solutions of load flow problems have been discussed in [1]. One application of a low voltage solution is the voltage instability proximity index (VIPI), in which normal load flow solution (the high voltage solution) and the corresponding low voltage solution is utilized to estimate the proximity to voltage collapse [14-16]. A low voltage solution is also used in energy based security index [17,18]. Computational burden to compute low voltage solutions is permissible; in fact, it is done in an existing security monitoring system [19], where a series of high voltage and low voltage solution pairs are used to describe the P-V curve on line.

This paper proposes the use of the multiple load flow solution pair (the high voltage solution, x_0 , and low voltage solution, x_1) to approximate the closest loadability limit. It has been shown in [1] that the mean value of the solution pair, $a=(x_0+x_1)/2$, corresponds to a singular point of the load flow equations, and this feature was utilized to construct VIPI [14-16]. However, the exact meaning of this singular point may have been overlooked. In this paper, we emphasize that the difference of the solution pair, $b=(x_0-x_1)/2$, exactly agrees with the saddle-node right eigenvector at this singular point, a . That is, the point, a , corresponds to the loadability limit at which the right eigenvector, b , direct the operating voltage, x_0 . Thus, these vectors themselves are meaningful targets to be monitored. An analysis will show that the solution pair having these characteristics further can satisfy conditions for a closest loadability limit under certain assumptions. Then, the solution pair will be utilized to approximate the closest limit and the corresponding left eigenvector. Most of the

computation time of the proposed method is taken to calculate the load flow solution pair. Additional computation time is less than that of an ordinary load flow.

Theoretical Background and Proposed Method

A closest loadability limit

Let $x \in \mathbf{R}^n$ and $c \in \mathbf{R}^2$ be vectors of normal bus and slack bus voltages in rectangular coordinates, respectively, and let $y \in \mathbf{R}^n$ be a vector of nodal bus injections. Load flow equations in rectangular coordinates can be represented in a quadratic form as follows [1,2].

$$y = \tilde{x}^T \mathbf{H} \tilde{x} \quad (1)$$

where $\tilde{x} = [x^T, c^T]^T$. In this paper, we rewrite the load flow equations in the following form to observe their nature.

$$y = \frac{1}{2} J(x, c) \cdot x + \frac{1}{2} g(x) \cdot c \equiv f(x, c) \quad (2)$$

where $J(x, c)$ represents the load flow Jacobian. Each row of $J(x, c)$ and $g(x)$ can be represented as

$$\begin{cases} J_i(x, c) = (H_i \cdot x + h_i \cdot c)^T \\ g_i(x) = (h_i^T x)^T \end{cases}$$

where $H_i = H_i^T$ for $i=1..n$. The above expression of load flow Jacobian $J(x, c)$ can easily be confirmed by differentiating (2) with respect to x . A closest loadability limit is defined as the loadability limit when load power consumptions increase towards the worst direction. This problem is formulated in [13], where two solution methods (iterative and direct methods) are presented as well as the conditions for a closest loadability limit. A reduced form of the conditions for a closest loadability limit from an operating point y_0 may be written as

$$y_* = y_0 + Mw = f(x_*, c) \quad (3)$$

$$w^T J(x_*, c) = 0 \quad (4)$$

$$\|w\| \neq 0 \quad (5)$$

where

$$M = \begin{bmatrix} I_m & 0 \\ 0 & 0 \end{bmatrix}$$

where we assume that $y^T = [y_L^T \ y_G^T]^T$, $y_L \in \mathbf{R}^m$, $y_G \in \mathbf{R}^{n-m}$ with y_L representing load bus injection vector and that I_m is the identity matrix of dimension m . Thus, Mw represents the possible deviations from the operating condition y_0 , where only load power deviations are considered to be variables while generator injections are assumed as constants. Vectors y_* and x_* represent the closest loadability limit. Equation (3) states that y_* and x_* must satisfy load flow equations (2). Equation (4) states that the Jacobian $J(x_*, c)$ is singular with left eigenvector w , corresponding to the zero eigenvalue of $J(x_*, c)$. A necessary condition for a "closest" limit is represented as that direction of load power deviations must be identical with

(or parallel to) the left eigenvector w with respect to the elements corresponding to load bus injections Mw . The left eigenvector indicates the closest direction to the loadability boundary and can be used to determine an optimal control strategy to avoid voltage collapse [11,13].

Multiple Load Flow Solutions

Problems concerning multiple load flow solutions have often been discussed so far in relation to voltage stability problems. Solutions except an operating solution correspond to unstable equilibria when a loadability limit agrees with a saddle node bifurcation where the system loses stability. It is known that the number of the existing multiple solutions decreases as an operating point approaches a loadability limit; only a pair of solutions remains near the boundary [20]. We call such a solution pair high voltage solution (an operating solution) and low voltage solution in this paper. Voltage Stability Proximity Index (VIPI) is a security measure to estimate the proximity to the boundary based on the solution pair [14-16]. Denoting the high and low voltage solutions respectively as x_0 and x_1 , VIPI is written in terms of our expression (2) as

$$VIPI = \cos^{-1} \frac{y_0^T f(a, c)}{\|y_0\| \cdot \|f(a, c)\|} \quad (6)$$

where

$$x_0 = a + b, \ x_1 = a - b \quad (7)$$

or

$$a = \frac{x_0 + x_1}{2}, \ b = \frac{x_0 - x_1}{2} \quad (8)$$

In order to emphasize an important meaning of VIPI, we will carry out the same procedure as in [1], but using our formulation. Substituting (7) into (2), we can easily obtain

$$y_0 = f(a, c) + J(a, c) \cdot b + f(b, 0) \quad (9)$$

$$y_0 = f(a, c) - J(a, c) \cdot b + f(b, 0) \quad (10)$$

Therefore,

$$J(a, c) \cdot b = 0 \quad (11)$$

$$y_0 = f(a, c) + f(b, 0) \quad (12)$$

The above equations clarify interesting characteristics of the multiple load flow solution pair. Equation (11) states that vector, a , indicate a singular point at which load flow Jacobian becomes singular, and b is the right eigenvector corresponding to the zero eigenvalue of the Jacobian. Equation (12) states that $f(a, c)$ represents bus injection at the loadability limit and $-f(b, 0)$ is the power margin evaluated at the operating point, y_0 . (See Fig. 1)

Properties of the right eigenvector has been discussed in [11]. The right eigenvector approximates the initial voltage collapse direction and therefore it can be used to identify weak nodes. Vector b may be used for this purpose.

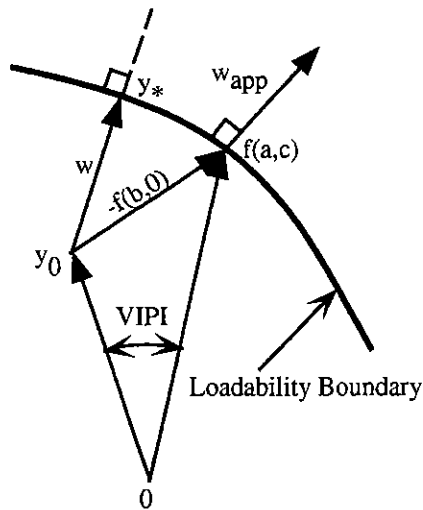


Fig. 1 Concept of Proposed Method

Approximation Of A Closest Loadability Limit

In this section, we propose an approximation of a closest loadability limit using the multiple load flow solution pair. First, we try to make the following assumptions.

$$J(a, c)^T \cdot b \equiv 0 \quad (13)$$

$$\mathbf{b} \equiv \mathbf{s} \cdot \mathbf{f}(\mathbf{b}, 0) \quad (14)$$

where $s \in \mathbb{R}^1$. Then, from (11)-(14), we have

$$f(\mathbf{a}, \mathbf{c}) = y_0 - f(\mathbf{b}, 0) \quad (15)$$

$$\mathbf{f}(\mathbf{b}, 0)^T \mathbf{J}(\mathbf{a}, \mathbf{c}) = 0 \quad (16)$$

$$\|f(b, 0)\| \neq 0 \quad (17)$$

The above equations correspond to (3) to (5) with $M=I_n$. Therefore, $f(a,c)$ is a closest loadability limit under the assumptions (13) and (14). Although assumption (13) may not be satisfied in general, symmetric structure of $J(a,c)$ may verify its validity for some extent. Equation (14) assumes that b is parallel to $f(b,0)$. This assumption is equivalent to

$$J(\mathbf{x}_0, \mathbf{c}) \cdot \mathbf{b} \cong \lambda \mathbf{b} \quad (18)$$

since $J(x_0, c)b = J(a, c)b + J(b, 0)b = 2f(b, 0) = \lambda b$, where $\lambda = 2/s$. Equation (18) assumes that b is the left eigenvector corresponding to λ , which may be a minimum eigenvalue of $J(x_0, c)$. When an operating point x_0 is close to a loadability boundary, $J(x_0, c)$ is near singularity with small $|\lambda|$, and (18) becomes almost exact. Thus, assumption (14) may be valid when a loadability limit is close.

We have examined the numerical characteristics of the loadability limit $f(a,c)$ through comparison with the solution of (3)-(5). Then, we finally proposes the following

approximation for a closest loadability limit.

$$y_{*app} = y_0 + \frac{Mw_{app}}{\|Mw_{app}\|} \|Mf(b, 0)\| \quad (19)$$

where w_{app} is the left eigenvector of $J(a,c)$, and is obtained by solving

$$\mathbf{w}_{\text{app}}^T \mathbf{J}(\mathbf{a}, \mathbf{c}) = 0 \quad (20)$$

Meaning of the proposed approximation is that an improved direction w_{app} is used instead of $-f(b,0)$, but its magnitude is retained. Deviations of generator bus injections are excluded from (15) by using M. The reason of this exclusion comes from an undesirable numerical property of $f(a,c)$ for generator bus injections. That is, as seen from (15)-(17), $f(a,c)$ approximates a closest limit which allows change in generator injections. In this sense, the subvector of $f(a,c)$ corresponding to generator buses approximates the worst case dispatch, and is omitted in (19) in accordance with formula (3)-(5). This characteristics of generator injections also degenerate the approximation of the loadability limit of (19). However, the approximation error was not so large in our numerical examinations as will be shown in the next section.

We should add some comments on low voltage solutions to be computed. There exist multiple low voltage solutions at an operating condition, and the number of low voltage solutions decreases as the operating point moves towards a loadability boundary. Finally, one of them remains near a boundary, and then meet the high voltage solution at the boundary [20]. However, as is examined in [16,17,18], which solution remains finally is depends on the pattern of load power increase. Therefore, in [16], multiple VIPI's are suggested to used for different load increase patterns. Also in [17,18], multiple energy based indices are examined to take into account such situations. From the view point of equations (15)-(17), such situations may be interpreted as that each $f(a,c)$ calculated from different x_1 approximates a different local minimum of distance to a loadability limit for different direction of load power increase. Several computation methods for low voltage solutions have been presented in [14-22]. An effective way to find out the low voltage solutions to be monitored is proposed in [18].

Numerical Simulations

Numerical simulations are carried out to examine the proposed approximation in the previous section. The Ward and Hale six bus system in Fig. 2 [24] was selected as an example system. Loading condition of the original system is listed in Table 1, which will be referred to as base case loading condition hereafter. The following two different patterns of load power increase will be used in this examination.

- Case 1 The case where all real and reactive loads are increased as a linear function of parameter k . ($k=1$ for base case)
- Case 2 The case where real and reactive powers at load bus 3 are increased as a linear function of parameter k_3 , while the remaining loads keep constant. ($k_3=1$ for base case)

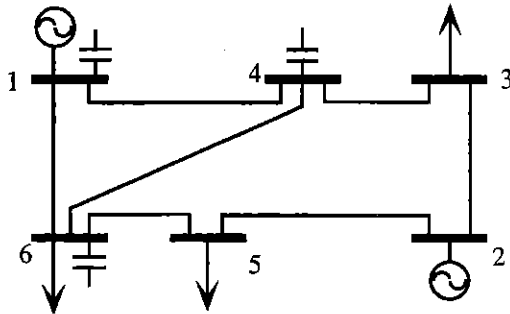
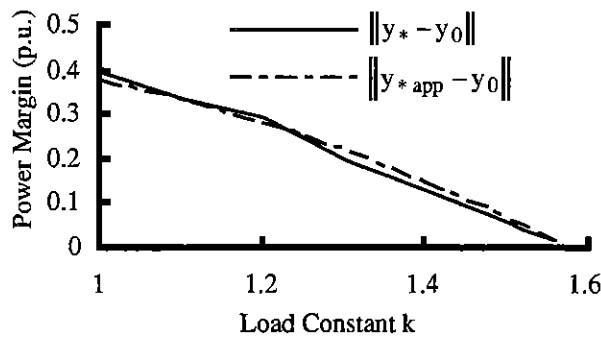
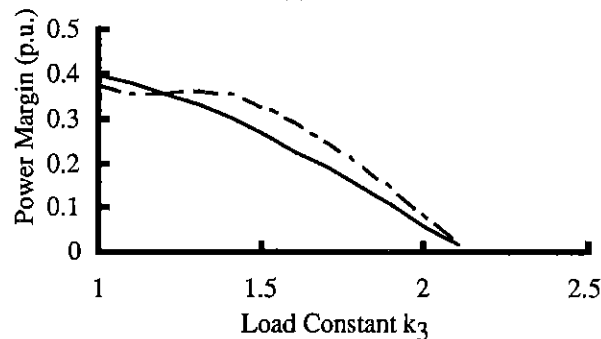


Fig. 2 An Example System

In this numerical simulations, performance of the proposed approximation (19) will be examined through comparison with the exact solution of (3) - (5). The equations (3) - (5) have been solved by N-R method with an initial estimate given by (19) and (20).



(a) Case 1

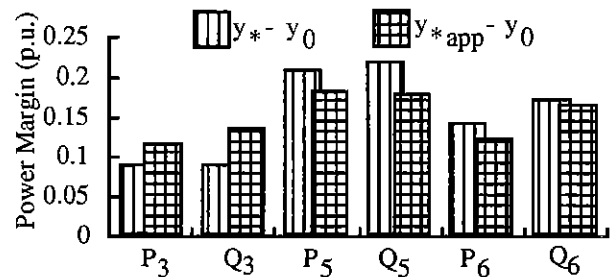


(b) Case 2

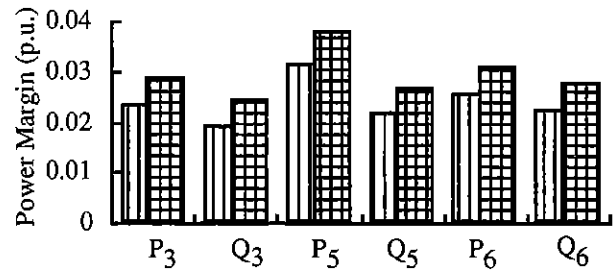
Fig. 3 Comparison of Total Load Power Margins

Fig. 3 plots the variations in load power margins with respect to k for case 1 and k_3 for case 2, where the solid lines represent the exact distances to the closest loadability limit obtained from (3) - (5), whereas the dashed lines by the proposed method.

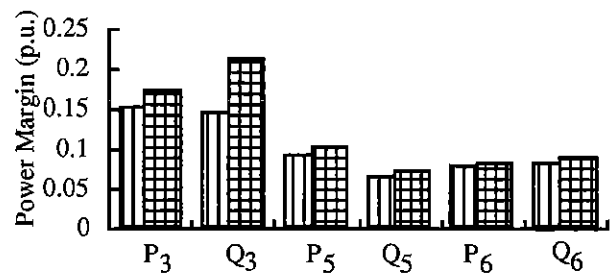
Fig. 4 shows the load power margins for individual load buses for the base load case ($k=k_3=1$) and increased loading conditions ($k=1.5$ for case 1 and $k_3=1.5$ for case 2). As seen from these Figures, the proposed method yields fairly good approximations.



(a) $k=k_3=1$ (Base Case)



(b) $k=1.5$



(c) $k_3=1.5$

Fig. 4 Load Power Margins for Individual Loads

Fig. 5 plots the angle between the exact and approximated left eigenvectors, w and w_{app} . As is observed from the Figure, the approximation error is less than 15 degree, which tends to decrease as the operating point become close to a loadability boundary.

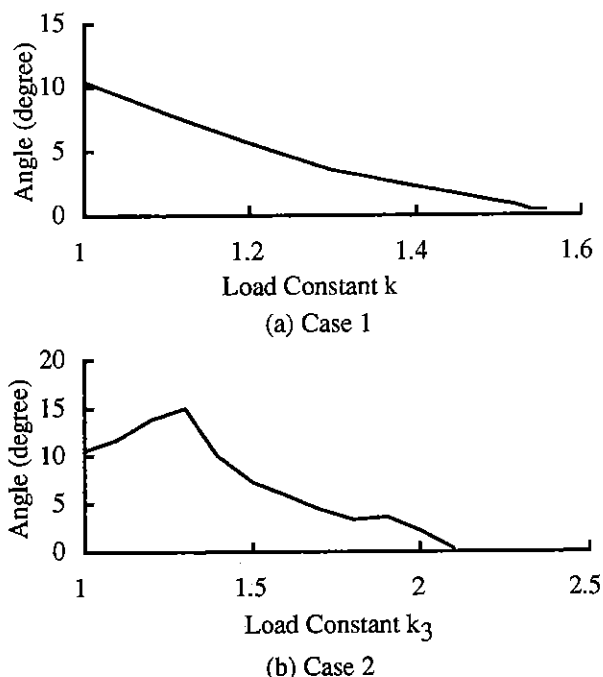


Fig. 5 Angle Between the Exact and Approximate Left Eigen Vectors, M_w and $M_{w_{app}}$

As seen from the above examinations, the approximation error of (19) may be acceptable. However, we should mention that the performance of $f(a,c)$ as an index is not satisfactory as we have expected. Table 2 compares loadability limits in terms of $f(a,c)$, y^*_{app} and y^* , in which a few undesirable features of $f(a,c)$ are observed. Firstly, generator output P_2 and voltage V_2 are reduced at $f(a,c)$ compared with those values at y_0 . Instead, increased active and reactive powers are supplied from slack bus at $f(a,c)$. This may be interpreted as that the above values approximate the worst case load dispatch as mentioned previously. However, such a dispatch may contradict normal policy of power system operations. In addition, there is no guarantee that these values satisfy operational requirements such as reactive power limitations of generators. Secondly, it

is observed in Table 2 that load power consumptions given by $f(a,c)$ are not satisfactory either. This may come from the approximation errors of generator injections or from assumptions (13) and (14), which are unrealistic in usual situations. However, if we pay attention to loadability margin $Mf(b,0)=M(y_0-f(a,c))$, its absolute value $\|Mf(b,0)\|$ shows better approximation. In the final form of y^*_{app} , the above value $\|Mf(b,0)\|$ is retained but its direction is determined by the approximated left eigenvector w_{app} . Thus, the proposed approximation yields improved performance.

Conclusions

This paper proposes the use of low voltage load flow solutions to approximate a closest loadability limit. Analytical features of low voltage solutions are investigated to formulate necessary conditions for a closest loadability limit. Numerical examinations using the Ward and Hale six bus system show that the proposed method yields fairly good results to approximate load bus injections at the closest loadability limit.

Factors to degenerate the proposed approximation are examined. A major problem is that, in this approximation, specified generator bus injections are undesirably modified, which may not satisfy operational requirements such as reactive power limitations of generators.

The approximation error, however, was acceptably small in our examinations and tends to reduce as an operating point moves towards a loadability limit. Therefore, the proposed approximation may be able to provide useful information if it is used together with the existing methodology such as continuation methods, point of collapse methods, etc. The proposed approximation can also yield a good initial estimates for the direct solution of a set of necessary conditions. A reduced form of such conditions are also presented, and its solution is used as a bench mark in this paper.

Table 1 Base Case Loading Condition

P_2	V_2	P_3	Q_3	P_4	Q_4	P_5	Q_5	P_6	Q_6
0.5000	1.100	-0.5500	-0.1300	0	0	-0.3000	-0.1800	-0.5000	-0.0500

Table 2 Loadability Limits Evaluted at y_0 (Case 1, $k=1.5$)

	P_2	V_2	P_3	Q_3	P_4	Q_4	P_5	Q_5	P_6	Q_6
$f(a,c)$	0.3868	1.056	-0.8087	-0.2296	0	0	-0.4137	-0.2893	-0.7474	-0.1220
y^*_{app}	0.5000	1.100	-0.8538	-0.2198	0	0	-0.4881	-0.2971	-0.7810	-0.1031
y^*	0.5000	1.100	-0.8484	-0.2145	0	0	-0.4815	-0.2918	-0.7758	-0.0975

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Discussion of papers presented by Rao-Sepulveda, Zhou, and Yorino

Edited by L. H. Fink, ECC, Inc.

(The initial portion of the discussion of Dr. Rao-Sepulveda's paper was only partially recorded, and could not be reconstructed. Dr. Rao-Sepulveda had to leave early.)

Marija Ilic (Massachusetts Institute of Technology)

I would like to bring up a generic question for everybody for discussion. The speaker [Rao-Sepulveda] brought up a very interesting concern, that in looking at the problem from the operator's viewpoint, there will be many situations where the mathematical conditions will indicate that something has to be done in 10 seconds or very quickly, and the operator is just not in a position to cope with that situation.

I don't know whether you will agree, but I think that we will never be able, at the control center level, to cope with a lot of control functions that are in that timeframe. Therefore, I think the emphasis should be on revisiting the primary controllers, and the control logic at the primary controllers, so that those control functions can be made more powerful for dealing with unusual operating conditions. If one looks into the state-of-the-art of the AVR's, and governor controls, and so forth, those are decentralized constant gain controllers. For a lot of the dynamic problems that we are concerned with here, close to bifurcation, I think one has to consider very seriously the type of controls that Bob Thomas was talking about yesterday, and in addition to that, coordination of those, so that one can cope with a systemwide problem in that time frame in an automated way, and not rely on the operator.

I think that what is happening from the operator's viewpoint is that he is saying: "well, there are all these mathematics - so what do I do with them?" That is a very real concern on his side. What we are forgetting is that the control actions are really at the upper level in the typical power system. I don't know what is the thinking of people working with this, for example, Tom's dynamic concepts for dealing with bifurcations when you don't have a solution in a dynamic sense, I don't know if he had in mind something like solving that problem from the control center level. I think that that has very little chance - we really have to rely on automated actions. I think that this is a generic question that we have not addressed here, and I think it may be worthwhile discussing.

George Knight (Imperial College)

I think I agree with Maria that control action should be at the upper level. I think in fact what happens is that there is not necessarily any signal at the upper level which tells the operator that he is approaching trouble. For instance, the more reactive compensation is applied on the network, then the less appropriate is the actual voltage level of that network as an indicator of trouble. So as I said, I agree with your point, but I don't think it is necessarily any specific local controller on the system which will get you out of this trouble. That is why I think the operator (and system operation management) is so interested in indicators of proximity to voltage collapse, because he has no other mechanism necessarily which tells him he is getting into trouble. The work that Claudio and I were doing was to try to get an idea to give the operator as to how much time he's got left.

Sandro Corsi (ENEL-CRA)

Concerning improvement of the primary control, we have to take into account that the primary control faces very locally the plant balance, very locally, preservation, so that when we have a problem in the network, we can preserve the power plant. In that case the primary requirement is that you operate rapidly, but for the problem of voltage estimation in

general, the problem is that of coordination over all of the reactive power resources that you have. Typically, the problem is an area, so in my mind I don't see which way we can improve the primary control, because the primary control is already very fast, very effective locally. Conversely what I think is possible is to have fast automatic control action. We have done some simulation in which we model very carefully all the primary and the limiting controller loops, and we saw that if you take some kind of control automatically, you are able to avoid some collapse, some instability condition. Conversely, you are not able to make this stabilizing control action manually, so I am not so sure that we don't need an automatic control action. In some cases the operator is able to take some actions, calling by telephone, or pushing some button, but in other cases I think the operator has no time - is not able to face the situation.

Carson Taylor (Bonneville Power Administration)

I guess I disagree with Dr. Knight a little bit on what can be monitored at the control center for decision making. Rather than looking at voltage magnitudes, look at the reactive power output of the generators; they are a very sensitive indicator that can be used for operator actions, both preventive control where you try to maintain a large amount of spinning reactive power reserve, as much as possible, at all times by shunt compensation and application and so on, and it could be used for manual corrective control, but it would be better to have the faster corrective control automatic. You can very easily have a centralized control center where you take remote signals from generator reactive power output, and use those signals to augment local voltage measurements to switch on shunt capacitor banks. There is an example of that in my book, for our system, where it worked very well. You can switch in shunt capacitor banks before the voltage instability mechanisms even start, before the tap changers start to operate, just because the reactive power outputs of the generators are such a sensitive indicator. With regard to primary control, one method to make it more effective is simple transformer drop compensation in the automatic voltage regulator.

Knight: I don't think we are in disagreement actually, Carson. I very much agree with your concept of undervoltage relaying of demand. I think in fact the industry in total has been rather slow to adopt anything other than underfrequency relaying to control demand, and I very much agree with what you say. I still think though you have the problem: at what level should you initiate that action. You still have to give some advice to the central - somebody - as to at what voltage level you should actually take action. On your point about the generator rotors, I remember quite a few years ago now reading an article in the paper from Australia in which the point was made that one of the most sensitive indications of potential problems on the system is the field current of the generators.

Marian Sobierajski (Technical University of Wroclaw)

I would like to congratulate the author for an excellent concept, and for a well-presented paper. I have some remarks or maybe questions. First of all, I think that this is not a stochastic, but rather a probabilistic, method, because your steady state points to linearization of the differential algebraic equations, and then you look for a zero eigenvalue. You really don't consider a stochastic process, so there is no influence of time, such as autocorrelation or other conditions, on your results. The question - Have you considered eigenvalue sensitivities to calculate the probabilities of events? I think that some papers on this problem were published about 1980. This is a good idea for analyzing a large set of differential and algebraic equations. For a power system of

realistic size, you should take into account generator voltage regulators, dynamic loads, tap changing regulators, and so on. The dimensionality is very great, and using Monte Carlo methods, as you know, needs lots of CPU time, computer time. So that is why maybe an analytical approach using your method, which is an excellent idea, may be more useful.

(Shinichi Iwamoto, translating for) Yicheng Zhou (Tokyo Metropolitan University)

They have not really used eigenvalue sensitivities, but I think your advice will be well taken. The first remark that you made, that the approach is not really stochastic but probabilistic: I think that for this question you will have to talk with [the author] directly.

Thomas Overbye (Univ. of Illinois)

This [Yorino's] is a very interesting paper. Two comments and a question. The first comment is, it is nice to see smoothly varying indicators of proximity to voltage instability, and I saw that you had that in your paper. That seems to be a characteristic of methods that use low voltage solutions. The second comment is about the closest boundary points. When we talk about a closest point we have to think about what norm we are using. The work that Ian Dobson has done uses the Euclidean norm, which is a valid norm to use, but you could also argue for an L_1 norm. I have been talking with Fernando Alvarado and Ian Dobson, we have been discussing this amongst ourselves, and in some senses you might want to use an L_1 norm. I think that your point $f(a,c)$ in your paper [Figure 1] is actually very close to, if not equal to, the L_1 norm minimum point, so it is a minimum point in an L_1 norm sense. So I think it is actually a pretty good point to use.

The question is, with multiple load flow solutions, do you have any fast methods for determining the appropriate low voltage solutions. In a 3 bus system you might deal with 2; however, if you go to a 2,000 or 6,000 bus system, you've got a lot of solutions. Do you have any comments on fast ways to calculate the appropriate solutions?

Naoto Yorino (Hiroshima Univ.)

About the multiple solutions, low voltage solutions in the method, we have seen that the locus in the on-line monitoring problem is ok, because we can track low voltage solutions that are dominant and that can be obtained from the off-line computation. The problem with a contingency, or something like that. In that case it generally is difficult, and we don't have a very efficient method up to now. That is a problem to be studied. There are a lot of methods that have been proposed.

Influence of Load Modeling Assumptions on the Dynamics of Large System Voltage Collapse Simulations

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Abstract

Various static and dynamic load models have been proposed for the analysis of voltage collapse in power systems. Much of the technical development and demonstration of these models has been confined to very small system examples. In this paper, the important load model effects for voltage stability analysis are discussed and proposed models are reviewed.

Comparisons are made of the effect of these load models, with various levels of detail and model parameters, on simulations of a large, complex power system. The effects examined include different transient load-voltage characteristics, different transition times from transient to steady-state characteristic, and frequency dependence. The interaction of load modeling with representation of generator over-excitation protection is also examined.

Keywords: Load modeling, Voltage collapse, Long-term system dynamics, Digital simulation

1. Introduction

With the evolution of modern, heavily compensated power systems, voltage stability has emerged as the limiting consideration in many systems [1]. The phenomenon of voltage collapse is dynamic, yet frequently evolves very slowly, relative to transient stability phenomena. For example, the 1987 collapse of the Tokyo Electric Power Company system [2] evolved over a period of about 30 minutes. The time frame of such an event is roughly two orders of magnitude longer than either the component models or the solution algorithm of a transient stability program are designed to handle.

It is well known [1,3] that slower acting devices and phenomena, such as load-tap-change (LTC) transformers, plant boiler and reactor controls, automatic generation control (AGC), various protective devices (e.g. generator field protection) and slow load dynamics, will contribute to the evolution of a voltage collapse. In a loadflow program, these effects are taken into account, if at all, by enforcement of their steady-state response. Conversely, the transient stability program typically assumes that these phenomena are slow, and corresponding variables remain constant. In actual practice, neither of these assumptions can be relied upon, thus leaving voltage collapse in a no-mans-land between these two analytical domains. Recent developments of long-term dynamic simulation tools [4,5] provide the opportunity more accurate modeling of these devices and phenomena, and development of suitable models is proceeding.

Of the various power system components, the modeling of the system loads is one of the most critical, and subject to some of the widest debate. In this paper, we will examine some of the issues related to

modeling loads for voltage stability analysis. In order to illustrate the possible effects of various assumptions, we will show simulations of a large, highly stressed power system, subject to various events and using different load modeling assumptions. These illustrations, in conjunction with the widening body of analytical literature, can be used to help make better simulations, and consequently operations and planning decisions, which will be neither overly conservative nor overly optimistic.

2. Load Dynamics

For purposes of system studies, the term "load" refers to the equivalent representation of the aggregate effect of many individual load devices and the interconnecting distribution and subtransmission systems that are not explicitly represented in the system model. Generally, the load is represented by some combination of static and dynamic models to approximate the voltage (and frequency) sensitivity of the aggregate load. The effect of the series impedance of feeders and transformers between the system bus and the loads is usually neglected or included as a lumped impedance and tap ratio.

Many papers have been written describing the nature of the load and various approaches to modeling it. Recent IEEE Task Force papers have attempted to summarize this information [6], recommend standard load models [7], and provide a bibliography [8]. However, the representation of loads for voltage stability analysis involves several aspects not required for conventional stability analyses, including: longer-term dynamics due to thermostatically-controlled loads and due to voltage regulating devices; and nonlinearities in the voltage characteristics at low voltages (e.g., due to motor stalling and tripping, discharge lighting, and inverter or switching power supplies). The modeling of these effects is not well-established and is still the subject of ongoing investigation.

In this section, we discuss the key load dynamic characteristics affecting voltage stability performance and models that have been proposed to represent these effects.

2.1 Load Dynamic Characteristics

Thermostatic Effects. An aspect of load behavior that contributes significantly to the voltage stability problem is the effect of thermostatic controls. The voltage dependence of loads in a system, particularly loads such as resistive space and water heating, can give considerable actual load power relief following a voltage depression induced by a system disturbance. However, this reduction in power does not remove the need to deliver energy, e.g. to maintain constant temperature. Eventually, the reduced power consumption of the individual loads results in thermostats leaving loads connected longer. The aggregate effect is to push the nominal load power up

towards a level which will produce the pre-disturbance actual power at the depressed voltage. The time constant associated with this resetting action is open to investigation, but values between 10 and 30 minutes have been suggested. There will be some permanent relief, since not all of the load will reset. In the nearly steady-state realm, this is the basis for conservation voltage reduction. Values of 0.5% steady-state actual load reduction per 1% voltage reduction have been observed in winter peaking systems [9].

Voltage Control Devices. The consideration of load voltage dependence and thermostatic effects is further complicated by the actions of voltage control devices in the underlying distribution and subtransmission network, including load tap-changing (LTC) transformers and feeder voltage regulators. In many systems, voltage regulating devices are set to maintain distribution or subtransmission voltages within a specified range. Depending on the equipment involved, this action occurs over a range of tens of seconds to a few minutes following a significant change in voltage. Of course, if the control range of these devices is exhausted, the regulating action stops. If and when the voltage near the loads (i.e. on the regulated side of the device) is restored to its pre-disturbance level, the voltage dependence *as viewed from the bulk system* is eliminated. While the system voltage may have changed significantly, the voltage seen by the load has returned to its pre-disturbance state. Therefore the power consumed, regardless of the load voltage dependence, is relatively constant.

Following bulk system upsets, a typical result is an immediate drop in the power consumption and the system voltages. Over the next few minutes, the action of voltage control devices will typically drive the system into conditions of still lower voltage and possibly collapse. Sometimes, when a system settles down to a condition of steadily declining voltages on the bulk system, it is due to the fact that while most of the voltage control devices have run out of regulating range, the thermostatic effects continue to push the voltage down, possibly into voltage collapse.

If cases such as these are run on a loadflow using constant power load modeling, the post-contingency loadflow would simply fail to converge. In a conventional loadflow, the finite range of the voltage regulating devices between the transmission system and the loads is ignored. This is reflected in the constant power load model connected at transmission buses. The effect of this modeling is to cause the voltage to cascade downward, without hitting the limit imposed by the maximum tap range of the LTCs. These effects can be included either within the load model, or by explicit modeling of the voltage control devices in the analysis. Dynamic simulation also permits consideration of the timing of voltage control actions, wherein some devices will act before others, thereby changing the course of the scenario.

2.2 Dynamic Load Models

Several recent references [10-13] have proposed simplified dynamic load models intended to capture the essential behavior of loads with different transient and steady-state characteristics, such as thermostatically-controlled loads and some motor-driven loads. While the form in which these models are presented appears quite different, it can be shown that all, except for [12], can be generalized to the block diagram shown in Figure 2.1. The only difference in the model proposed in [12] is that the final summation is replaced by a multiplication.

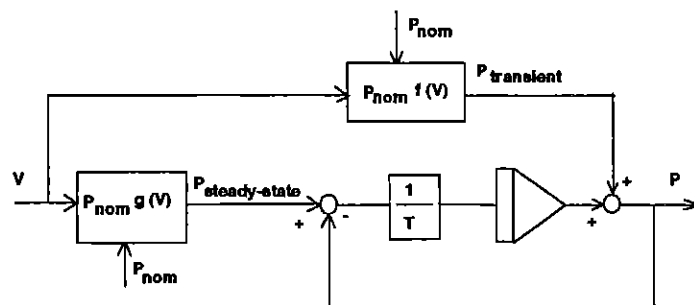


Figure 2.1 Simplified Dynamic Load Model

In this model, The steady-state load-voltage characteristic is represented by the function $g(V)$ which may be an exponential or polynomial in V . For a thermostatic load, this would normally be represented as constant power. The transient characteristic is represented by the function $f(V)$, which will often be constant impedance. Frequency sensitivity can also be included in both of these functions.

The language used to describe loads, when including voltage and frequency considerations is not well standardized. Recent IEEE and CIGRE documents have suggested the use of *nominal load power* as the amount of MW the load would consume at nominal conditions, i.e. 1.0 pu voltage and frequency. This is designated as P_{nom} in Figure 2.1. (It is initially equal to the MW consumed by the load divided by $g(V)$ for the initial voltage). The *actual or consumed load power*, also measured in MW, is the power consumed by the load under the existing conditions of voltage and frequency, i.e. the power measured by a meter. For this type of analysis it is vitally important to recognize that the *nominal load* is the independent variable, and *not* the actual load power.

To illustrate the behavior of this model, a simple radial system was simulated, with a load, represented by the model in Figure 2.1, connected through a transformer and transmission line to an infinite bus. A constant-impedance transient characteristic was used, with a constant-power steady-state characteristic, and a transition time constant (T) of 1.0 seconds. (Actual values of T for heating loads are much longer - up to several hundred seconds according to ref. 11.) Reactive power was modeled as constant impedance.

Figure 2.2 shows the response of this model (both with the summation and with the multiplication) to a 10% upward change in the high-side tap, followed 10 seconds later by a return to the original tap position. The response of the two forms of the model is similar. Because the load bus is not infinitely stiff, the bus voltage changes in response to the changing load power consumption.

Figure 2.3 illustrates the response of the dynamic load model to a 10% downward change in the *nominal load power*, followed after 10 seconds by a return to the initial value. There is an initial response due through the transient characteristic, which however is modified by a change in the bus voltage due to the finite system stiffness. In the steady-state, the actual power matches the nominal power due to the constant power steady-state characteristic.

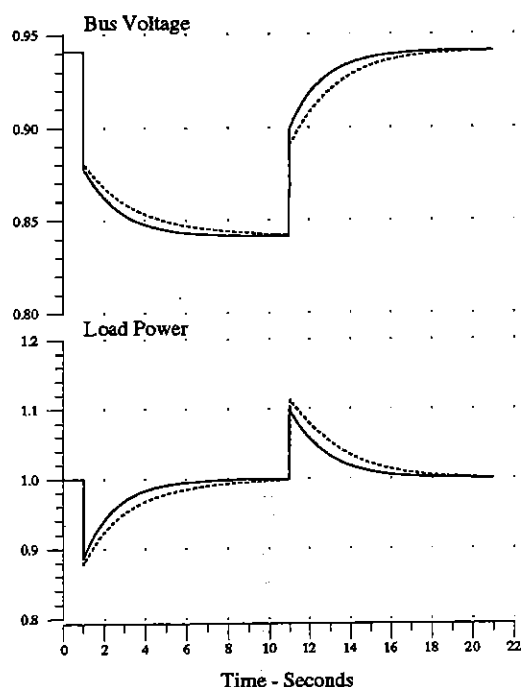


Figure 2.2 Dynamic Load Model Response to 10% Tap Change.
(Solid line - summation form; Dotted - multiplication form)

In order to represent in a simplified way the effect of limited voltage control device tap range and the time required to change taps, the model shown in Figure 2.4 is suggested. The quantity labeled V_{bus} is to system bus voltage, while V_{load} is the voltage that is applied to the load model. This voltage and V_{ref} are normally set to 1.0 p.u. initially.

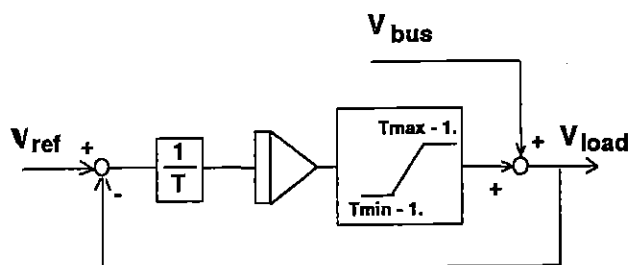


Figure 2.4 Simplified Voltage Control Device Model

The application of these models to realistic system simulations is discussed and illustrated in the next section.

3. Comparison of Results for a Large Scale System

In this section, the impact of various load modeling assumptions is investigated using a simulation of a generation and bulk transmission system, based on an actual large utility system. The simulation model has 95 buses, 114 branches, and 28 generating units, with a mixture on hydro, thermal and nuclear power plants. Excitation systems, power system stabilizers, turbine/governor systems, power

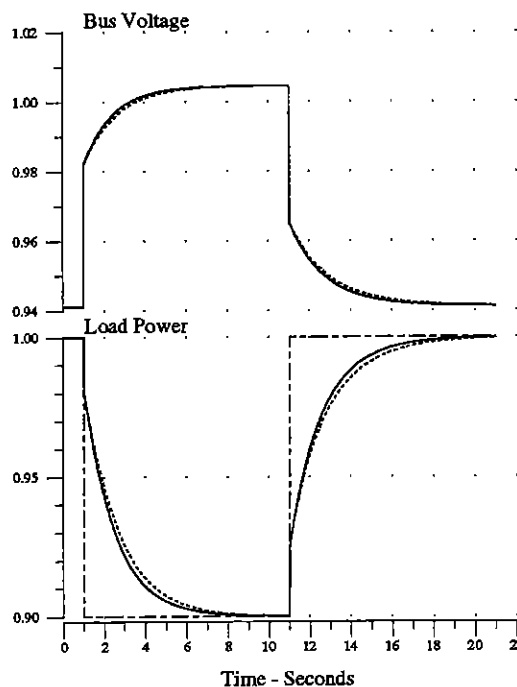


Figure 2.3 Dynamic Load Model Response to Change in P_{nom}
(Solid line - summation form; Dotted - multiplication form;
Dash/dot - nominal load power)

plant dynamics, generator protection functions, including field current limiters, high/low frequency and voltage protection are represented. Two control areas are represented with automatic generation control (AGC) with a 15 sec cycle time. Some mechanically switched compensation and transmission level LTCs are included.

The bulk system has a 500kV backbone, which carries large amounts of power from a relatively less densely loaded region to a heavy industrial region. Disturbances along the bulk transmission system, especially those which have the potential to knock large generating units off-line are of particular concern.

This system has many of the characteristics observed in large modern utilities, including heavy levels of transfer, large amounts of shunt compensation, a variety of voltage control elements, and generating plants of various ages and capabilities. Of course, each system will have its own signature behaviors, and it can be strongly argued that variations such as the ones presented here would be appropriate to help system planners and operators to determine which particular modeling considerations are most critical for their system.

For these simulations, several of the major loads in the receiving portion of the system are represented by the dynamic load model described in section 2 (summation form). The steady-state, characteristic of the real and reactive power load is always represented as constant MVA, but different models are used for the transient real power load characteristic (transient reactive power characteristic is assumed to be constant impedance) and for the transition time between transient and steady-state characteristics.

3.1 Transient Response

This first example illustrates the effect of variation in the transient response of the system loads. The disturbance is an EHV line fault and line trip, cleared in primary time, which results in a single large generator unit being knocked off-line. This represents one of the more severe, single contingency cases for this system. Figure 3.1 shows some selected system variables for the first 20 seconds of an extended term stability simulation in which the majority of the system loads are modeled with either constant current (solid trace) or constant admittance (dotted trace). The voltage plotted in the figure is a representative transmission node voltage in the vicinity of the region under the worst voltage stress. The load transition time constant is 30 seconds, so it has relatively little influence in this time frame. The resultant oscillatory behavior of the system is basically what would be expected for a highly stressed system, i.e. the constant admittance system shows much better electro-mechanical damping of the power swings than does the constant current model. It is worth noting that even though the disturbance includes tripping of a power plant, the system frequency swings to above nominal, rather than below, due to the transient load relief from the voltage depression.

As we examine the behavior of this system over a longer time, the influence of the transition time constant becomes apparent. Figure 3.2 illustrates the same quantities over about a three minute period. Here we see that the inter-machine electro-mechanical oscillations die out, and the load attempts to transition back towards constant MVA. This transition results in further depression of the system voltage. It is interesting to note that the effect of the transition is roughly offset by the further depression in voltage, with the net result being that the actual system load appears to stay roughly constant. Nevertheless, the system is becoming progressively more stressed as the transition proceeds. After a few minutes, the machine over-excitation protections begin to remove machines from automatic voltage control and to runback the excitation. Ultimately, after about three minutes, the dropping out of excitation results in a widespread voltage collapse and system breakup. The somewhat higher stress level associated with the constant current transient model results in a slightly faster system breakup.

3.2 Transition Time

In this example we examine the effect of the transition time between the transient and steady-state characteristic. Constant current transient response and constant MVA steady-state response are the two most common modeling assumptions for the aggregate system load. In Figure 3.3 we show three cases, with variations in the transition time constant between 3 seconds (solid trace), 30 seconds (dotted trace) and 300 seconds (dot-dash trace). Again, these are the first few seconds of the simulation.

The case with the very fast transition, which is intended to very roughly approximate a high concentration of motor loads, exhibits negative damping. In Figure, 3.4, we see that this behavior results in the system breaking up quite quickly. The difference between the 30 second and the 300 second time constant could be ascribed to different aggregate responses of loads and LTCs. Reference [11] determined time constants with a range from 83 seconds to 364 seconds for the active power term, and 0.1 second to 1,025 seconds for the reactive power term, for the same feeder depending on time of day and season.

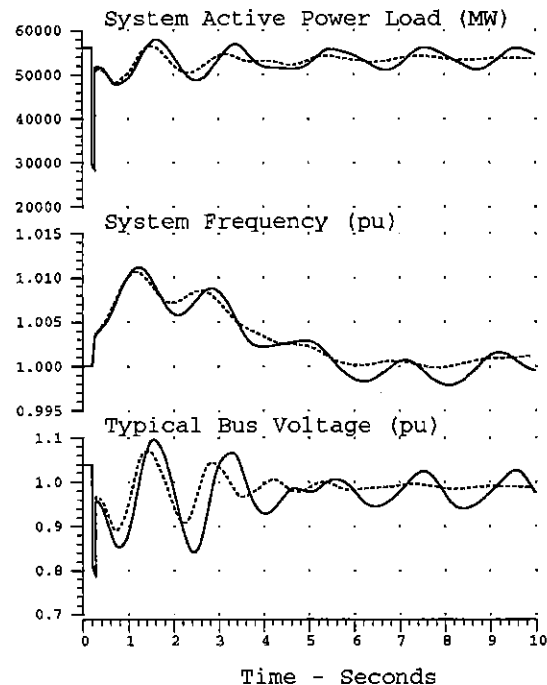


Figure 3.1 Fault & Line Trip with Accompanying Plant Trip:
Constant I Transient Load vs. Constant Z Transient Load
(Initial 20 seconds.)

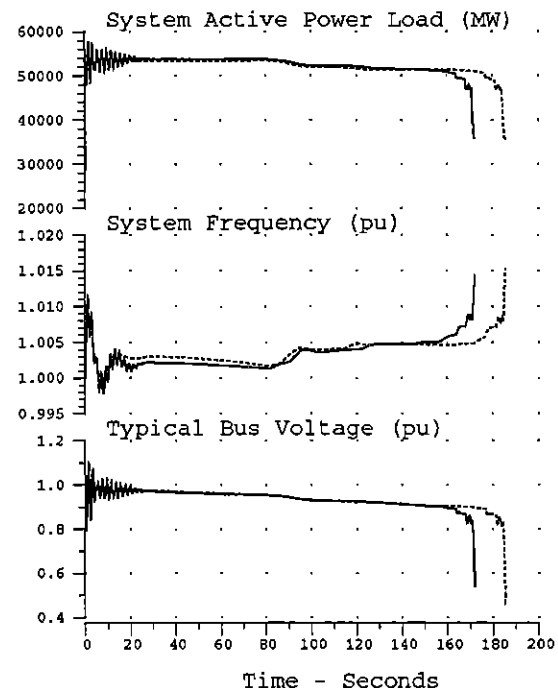


Figure 3.2 Fault & Line Trip with Accompanying Plant Trip:
Constant I Transient Load vs. Constant Z Transient Load

In this case, the slower transition time resulted in a later system separation. The difference in timing is somewhat less than might be expected, owing to both cases stressing the generator excitation systems enough to start machines towards OEL operation and reactive power runback.

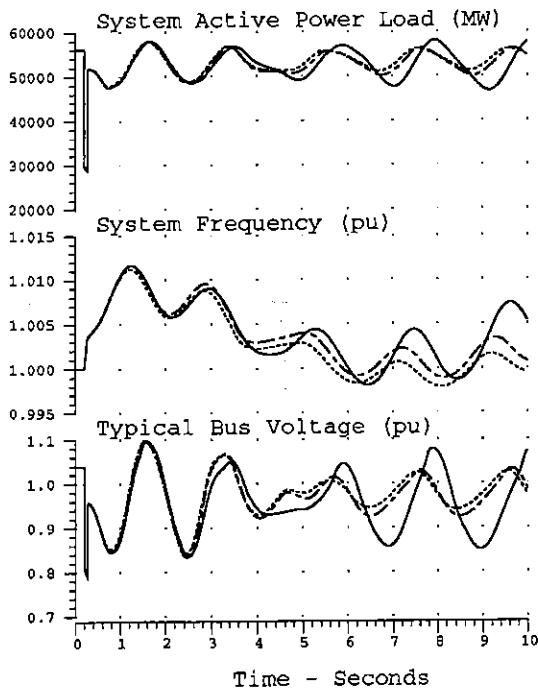


Figure 3.3 Fault & Line Trip with Accompanying Plant Trip -
Constant I Transient Load:
3 sec vs. 30 sec vs. 300 sec transition time constant
(Initial 10 seconds)

3.3 Effects of Over-excitation Limiters (OELs)

This case, while not strictly a variation in load modeling, illustrates some the ways in which other modeling assumptions interact with the load model. In this case, we compare the results of the previous case, using a 30 second transition time constant, to the same case with the machine over-excitation limiters disabled, but with all other machine protections still enabled. Figure 3.5 shows that the case with the OELs (solid trace) is driven to voltage collapse much more quickly than the case without (dotted trace). In the latter case, the most severely affected units are ultimately knocked off-line by their armature current protection. In our experience, it is relatively unusual for this protection to be activated before the OEL during voltage collapse events.

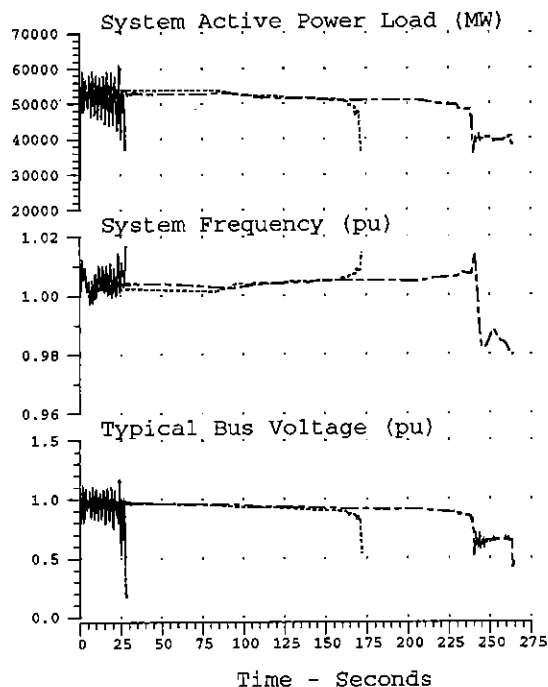


Figure 3.4 Fault & Line Trip with Accompanying Plant Trip;
3 sec vs. 30 sec vs. 300 sec transition time constant;
Constant I Transient Load:

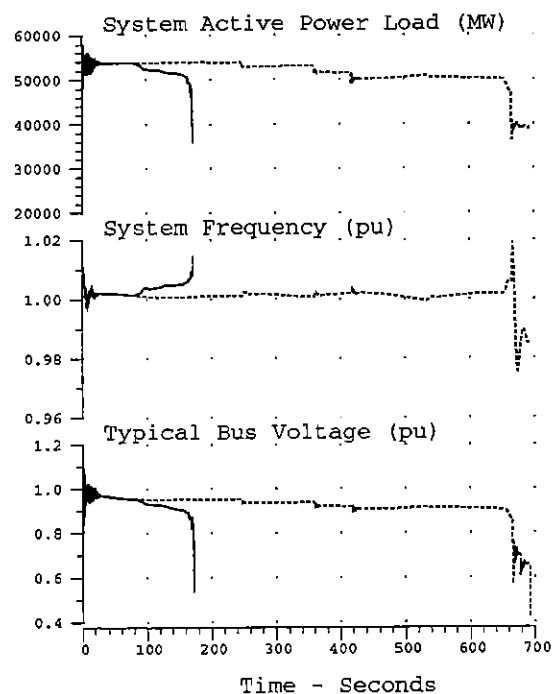


Figure 3.5 Fault & Line Trip with Accompanying Plant Trip;
w/ OELs vs. w/o OELs

3.4 Frequency Effects

In systems suffering from very severe disruptions, it is common for significant frequency excursions to be observed simultaneously with voltage excursions. For this example, the disturbance causes a deficiency in active power resulting in a frequency excursion. For this disturbance, a large power plant complex in the exporting region of the system is tripped off-line. Figure 3.6 shows the first ten seconds of the event. The plots compare the effect of including a frequency dependent term in the load model. The case including a second order dependence ($1. + 2 \Delta f$, solid trace) exhibits slightly better damping than the case with no frequency dependence (dotted trace). Because the loss of generation is in the exporting system, the voltages stay relatively healthy in the importing region. Consequently, beyond the deviations due to the electro-mechanical oscillations, there is little change in the actual power due to voltage depression. However, it is clear by the end of 10 seconds that the frequency dependent case is starting to arrest its frequency decline due to the load relief.

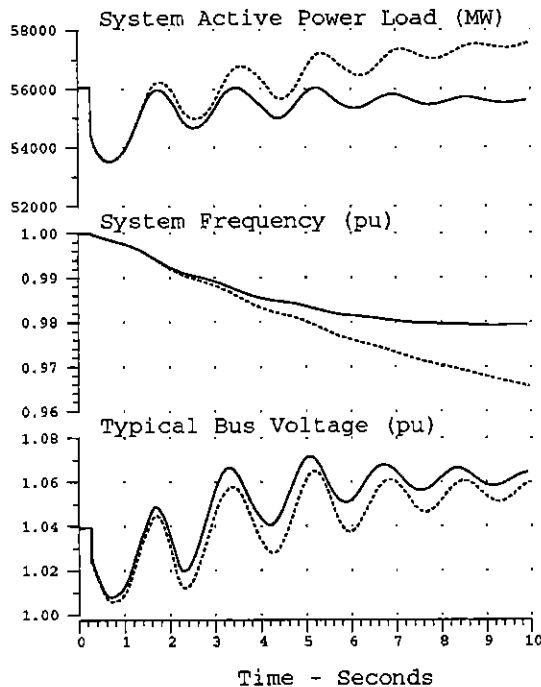


Figure 3.6 Exporting System Multiple Plant Trip:
w/ Frequency Dependent Load Model vs. without
(Initial 10 seconds)

Figure 3.7 shows the longer term consequences of that load relief. In the case with frequency dependence, the frequency decay is ultimately arrested and reversed first by the action of the governors, and then by the response of the AGCs. The case without frequency dependence, continues a monotonic decline, resulting in widespread generator tripping on low frequency, and complete blackout. About four minutes after the initial event, the case with the frequency dependent load model, a number of machine OELs activate resulting

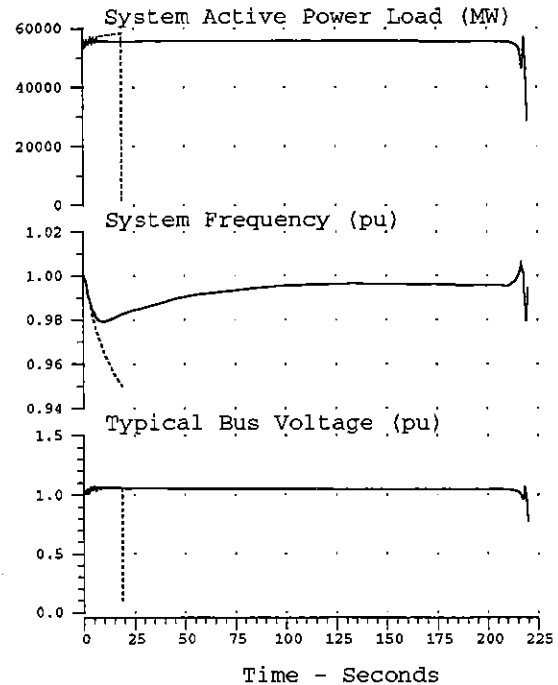


Figure 3.7 Response to Exporting System Multiple Plant Trip:
w/ Frequency Dependent Load Model vs. without

a voltage collapse. This case illustrates a phenomenon we have observed in a number of systems. That is, for systems under a very high degree of stress, the character of a system breakup can be radically altered by modifications in the system load model. In this case, we see that for one set of assumptions, the result is a frequency collapse, and for another, the result is voltage collapse.

3.5 Transformer Tap Range

As was noted above, the amount of available range for regulation by the subsystem LTCs can have a major impact on the post-contingency viability of a system. In this example, the potential impact is illustrated by modifying the minimum voltage above which the steady-state constant MVA characteristic is enforced. Figure 3.8 shows the results of a base case (solid trace) with constant current transient response and constant MVA steady-state response enforced down to 80% voltage. This represents an assumption of an equivalent of two layers of voltage regulation residing in the equivalenced subsystem. This system exhibits the same behavior as was observed in Figure 3.2. The second trace is for the same case, with only one layer of underlying voltage regulation being imposed, corresponding to a minimum voltage for enforcement of the constant MVA characteristic at 90% voltage. In this case, the load relief in the most severely stressed part of the system is more permanent (thermostatic effects are not modeled in this case). This prevents activation of the machine OELs, and the system remains intact for the duration of the simulation.

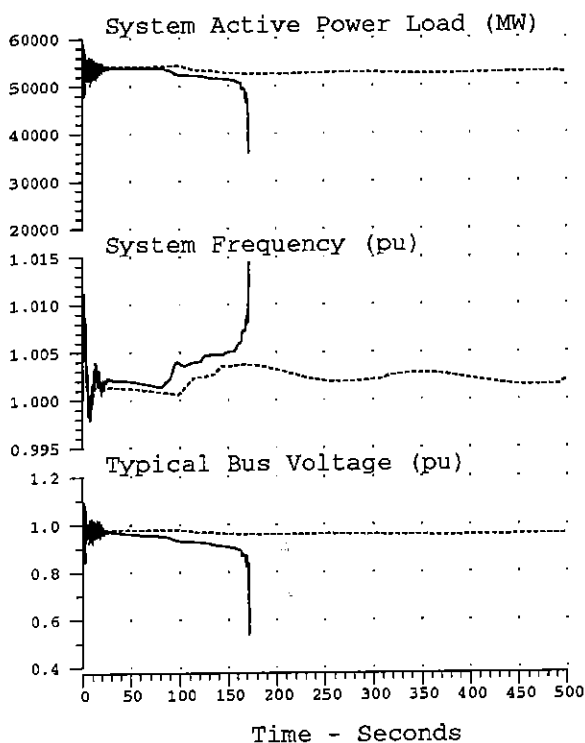


Figure 3.8 Fault & Line Trip with Accompanying Plant Trip:
Minimum equivalent LTC range 90% vs. 80%

4. Conclusions

The dynamic nature of the aggregate system load, that is, the change in characteristics between the transient and long-term period, has important consequences for analysis of voltage stability. In systems where traditional concerns about transient stability and damping aren't a major issue, the load transition time is more likely to be an important consideration than the transient load characteristic, itself. Extremely fast transition times will result in less damping of electromechanical oscillations.

Big disturbances frequently involve large frequency as well as voltage excursions. Modeling of frequency dependence of the loads may therefore be as important as voltage dependence. Variations in assumptions regarding frequency dependence can produce qualitatively different responses.

Care should be exercised in establishing the voltage range over which particular voltage dependent behaviors will be enforced, recognizing the action and limits of voltage regulating devices. Consideration of changes in load character at depressed voltages can radically change overall system response.

There is a strong interrelationship between the load modeling and the modeling of other system components, in terms of the overall system response. High fidelity simulations are dependent on good representations of all of these elements.

The simple dynamic load model of Figure 2.1, with appropriate parameters based on field measurements or load composition

analysis, provides a good first step in improving load models for voltage stability analysis. Addition of a series impedance to represent transformer and feeder impedance, together with representation of the effect of voltage regulating devices by a simple model such as shown in Figure 2.4, provides a corresponding first approximation to modeling the intervening distribution system. More detailed models are hard to justify given the limited availability of accurate load data.

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A Comparison of Discrete vs Continuous Dynamic Models of Tap-Changing-Under-Load Transformers

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Abstract

This paper deals with the issue of dynamic modeling of Tap-Changing-Under-Load (TCUL) transformers. As key voltage control devices, their dynamic behavior can be important in the network and load modeling for various dynamic studies. This paper provides derivations of various discrete and continuous dynamic models with respect to time and tap variation. It discusses the required assumptions, provides a comparison of the responses of these different models and comments on the possible impact on simulation studies.

Introduction

While the steady-state behavior of TCUL transformers has been studied for a long time in the context of load flow, analysis of the dynamic behavior is more recent. One of the first detailed descriptions of TCUL dynamics and their controls was given in [1] (and abbreviated in [2]). This work presented discrete-tap dynamic models based on traditional TCUL construction and philosophy of voltage control. Combining this with standard U.S. technology [3-5], the basic operation of TCUL transformers is described below.

Typical TCUL transformers have 32 tap positions (16 above and 16 below center) with each tap change corresponding to 5/8% of nominal or per-unit voltage. This corresponds to nominal plus or minus 10%. The taps are changed by a raise/lower motor which cranks a spring to provide the physical force needed to make a very rapid tap change. The time for this raise/lower motor to complete one tap change cycle is about 10 sec. This time delay is normally unavoidable for each tap change since it is the physical mechanism which precedes tap movement.

Typical TCUL controls have the following features:

- a. Inputs for: sensed voltage through a potential transformer for control power and voltage control; sensed line current through a current transformer for line-drop compensation; sensed circulating current through a current transformer for parallel operation.
- b. Outputs for: driving a raise/lower motor starter relay.
- c. Voltage center of band setting range from nominal to plus or minus 12.5%.
- d. Total voltage bandwidth setting between 0.83% to 5.0% (0.415% to 2.5% above and below band center). The minimum total bandwidth is usually considered to be twice that of one tap change - i.e. 1.25%.
- e. Timer: adjustable from 0 to 120 sec. The timer starts when the voltage goes outside the band and resets upon return to the band or when reset by contact in the non-sequential mode. A typical timer delay is between 30 and 60 seconds to allow for motor load starting.
- f. Mode of operation: non-sequential mode resets the timer after a tap change, regardless of voltage; sequential mode resets the timer only after the sensed voltage is back within the control band; blocking mode disables automatic operation.
- g. Reverse power flow logic: many TCUL controls include the capability to reverse the raise/lower logic when sensed load power is reversed in direction.

When the sensed voltage as modified by the line-drop compensation is within the bandwidth of the

control, the TCUL is a fixed-tap device for small enough disturbances. When the error signal goes outside the bandwidth of the control, the timer is initiated. The reports of [1,2] indicate that the time delay may be of two types: fixed setting or inversely proportional to the error signal. References [3-5] discuss fixed-setting operation only. In either case, the timer determines the delay before a tap change is initiated. Once the timer sends a signal to the tap-changer relay, tap changing will occur after the necessary time to setup the mechanical switching mechanism. When a control is set in the non-sequential mode, the timer is invoked after every tap change. When a control is set in the sequential mode, the timer is only reset when the voltage returns to the bandwidth setting. In the sequential mode, there is still a delay for each tap change associated with the setup time for each mechanical switching.

A large number of references have discussed the stability properties of this type of voltage control [6-18]. For the most part, these references address the issue of when the TCUL raise/lower logic results in a tap change which aggravates the voltage condition rather than improves it. The models used in these references range from discrete to continuous with respect to tap value and time. The following sections derive and analyze various TCUL dynamic models.

TCUL circuit representation

To start the analysis, the network notation of Figure 1 is given. This uses the quasi-steady-state model for network transients [19,20]. The TCUL transformer is modeled with a pure reactance (independent of the tap value) in series with an ideal transformer. The nominal value of the TCUL tap "a" is 1.0.

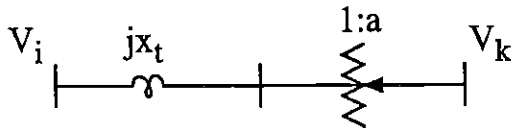


Figure 1: TCUL circuit representation

In this paper, the voltage to be controlled is bus k , with no line-drop compensation. More accurate models would include a tap-dependent reactance [1].

Discrete-tap model D1

Regardless of time frames, the unique feature of TCUL transformers is the ability to change tap values during operation. To capture this phenomenon in a mathematical model, there are essentially two aspects which must be studied. The first is the physical mechanism which makes this tap change possible. The second is the control law which is used with this tap changing capability.

While modern devices are being proposed for very-rapid tap changing, most of the existing TCUL mechanisms rely on some type of mechanical switching hardware. Once the decision has been made to change a tap, this mechanical switching takes a fixed amount of time - denoted here as T_m . The decision to change taps is determined by the control logic being used. Central to this logic is the timer which delays the tap-change decision by T_d sec. In this paper we will consider only the non-sequential mode of control. This assumes that the timer is reset after each individual tap change. In addition, we neglect the bandwidth hysteresis discussed in [1]. The total time between each tap change is then $T_T = T_m + T_d$.

We start with a discrete-time, discrete-tap dynamic model by introducing a fixed time increment h , assumed to be much smaller than the total delay time T_T . Beginning at t_o , we propose the following model for the value of the tap "a" at successive time increments:

$$a(t) = a(t - h) + \Delta a$$

where $a(\tau) = a(t_o)$ for $\tau \leq t_o$ and Δa is determined by the following logic:

1. Start with timers τ_+ and τ_- equal zero.
2. Increment time by h and:
 - (a) Compute $V_{err} = |V_k| - V_{ref}$, T_d and T_m .
 - (b) If $V_{err} \leq -DB/2$ and $\tau_- = 0$, start τ_- .
 - (c) If $V_{err} \geq +DB/2$ and $\tau_+ = 0$, start τ_+ .
 - (d) If $V_{err} > -DB/2$ and $\tau_- < T_d$, reset τ_- .
 - (e) If $V_{err} < +DB/2$ and $\tau_+ < T_d$, reset τ_+ .
 - (f) If $\tau_- \geq T_d + T_m = T_T$ and $a(t - h) + \Delta n \leq a_{max}$, then reset τ_- and set $\Delta a = \Delta n$.
 - (g) If $\tau_+ \geq T_d + T_m = T_T$ and $a(t - h) - \Delta n \geq a_{min}$, then reset τ_+ and set $\Delta a = -\Delta n$.
 - (h) Otherwise set $\Delta a = 0$.
3. Go to 2.

For model D1, the delays T_d and T_m are fixed constants. This is a discrete-tap dynamic model because the value of "a" can only take on a finite number of values and so the value of "a" can change instantaneously. While this does describe the fundamental dynamics of the TCUL, it does not describe more subtle dynamics such as the shorting of windings during the tap change [1].

Discrete-tap model D2

If the timer delay (T_d) is dependent on voltage error and deadband as discussed as an option in reference [1],

$$T_d = T'_{do} \frac{DB/2}{||V_k| - V_{ref}|}$$

with the changer mechanism delay (T_m) fixed, the model is the same as for model D1 except T_d (and therefore also T_T) will be different at every instant of time as $|V_k|$ varies.

Discrete-tap model D3

If the total delay (T_T) is dependent on voltage error and deadband (assumes some modified mechanical mechanism),

$$T_m = T'_{mo} \frac{DB/2}{||V_k| - V_{ref}|}, T_d = T'_{do} \frac{DB/2}{||V_k| - V_{ref}|}$$

the model is the same as for model D1 except T_d and T_m (and therefore also T_T) will be different at every instant of time as $|V_k|$ varies.

Discrete-tap model D4

If the timer delay time (T_d) is a combination of fixed and variable time,

$$T_d = T_{d1} + T'_{do} \frac{DB/2}{||V_k| - V_{ref}|}$$

with the changer mechanism delay (T_m) fixed, the model is the same as for model D1 except T_d (and therefore also T_T) will be different at every instant of time as $|V_k|$ varies. This is similar to model D2.

The transition to continuous-tap dynamic models

If the TCUL transformer is modeled in full detail with fast network transients, it may be possible to create a continuous-tap model which accounts for all winding transients during the switching sequence. This approach is not taken in this paper.

An alternative way to convert the dynamic model from discrete-tap to continuous-tap is to formulate a continuous-time tap derivative as a limit. Working with the discrete-tap dynamic model D1, clearly,

$$\frac{a(t) - a(t-h)}{h} = \frac{\Delta a}{h}$$

For $h \geq 0$, the left-sided continuous-time derivative of a is,

$$\frac{da}{dt} = \lim_{h \rightarrow 0} \frac{a(t) - a(t-h)}{h} = \lim_{h \rightarrow 0} \frac{\Delta a}{h}$$

provided this limit exists. With models D1-D4, the value of Δa is bounded and can only take on 3 values. Thus with these models, the limit becomes either unbounded or undefined. To avoid this, consider the simultaneous limit of reducing the time delays and tap size Δn . As a discrete device, $a(t)$ will only change after a total delay time T_T and it will change in increments of Δn . To make a continuous-time, continuous-tap model, this delay time and incremental tap change must be reduced as a limiting process. For example, rather than waiting 30 seconds for a .00625 change, wait 15 seconds for a .003125 change twice. That is, reducing the time delay and tap size by the same percentage results in a model which captures the correct overall time delay for the actual tap change. To implement this limit process we define T_m , T'_m , T_d , T'_d , T_T , T'_T , Δn and $\Delta n'$ with the parameter ϵ ($\epsilon \geq 0$) as,

$$\begin{aligned} T_m &= \epsilon T'_m \quad (T'_m = \text{actual } T_m) \\ T_d &= \epsilon T'_d \quad (T'_d = \text{actual } T_d) \\ T_T &= \epsilon T'_T \quad (T'_T = \text{actual } T_T) \\ \Delta n &= \epsilon \Delta n' \quad (\Delta n' = \text{actual } \Delta n) \end{aligned}$$

with $T_T = T_m + T_d$ as before. In addition, modify the model to ensure a constant tap when $|V_k| - V_{ref}$ is within the deadband. This is done by redefining Δa in the "otherwise" condition as ϵ^2 rather than zero. Substituting these expressions for T_m , T_d , T_T , Δn into model D1 gives a mechanism to make the transition from the discrete-tap model ($\epsilon = 1.0$) to the continuous-tap model ($\epsilon = 0$).

Continuous-tap model C1

By examining the control logic given for model D1, it is clear that as $\epsilon T'_m$, $\epsilon T'_d$, $\epsilon T'_T$ are reduced far enough, they become smaller than h . At this point the limiting process destroys the assumption that $h \ll T_T$ and the value of $a(t)$ must be computed over increments small enough to capture each reduced tap change using:

$$h = \epsilon T'_T$$

Assuming that for these cases,

$$\lim_{\epsilon \rightarrow 0} \frac{a(t) - a(t - \epsilon T'_T)}{\epsilon T'_T} = \lim_{h \rightarrow 0} \frac{a(t) - a(t - h)}{h}$$

then,

$$\frac{da}{dt} = \lim_{\epsilon \rightarrow 0} \frac{\Delta a}{T_T}$$

which gives the first continuous-tap dynamic model:

$$\frac{da}{dt} = \begin{cases} \frac{\Delta n'}{T_T} & \text{if } |V_k| - V_{ref} \leq -\frac{DB}{2} \\ -\frac{\Delta n'}{T_T} & \text{if } |V_k| - V_{ref} \geq \frac{DB}{2} \\ 0 & \text{otherwise} \end{cases}$$

and $a_{min} \leq a \leq a_{max}$.

While $|V_k|$ is dependent on "a" through the network, the derivative of "a" is independent of the magnitude of voltage error.

To simplify the model further, consider reducing the deadband in the limit by defining DB and DB' with the parameter ϵ as,

$$DB = \epsilon DB' \quad (DB' = \text{actual } DB)$$

In the limit as $\epsilon \rightarrow 0$, the model is,

$$\frac{da}{dt} = \begin{cases} \frac{\Delta n'}{T_T} & \text{if } |V_k| - V_{ref} < 0 \\ -\frac{\Delta n'}{T_T} & \text{if } |V_k| - V_{ref} > 0 \\ 0 & \text{otherwise} \end{cases}$$

and $a_{min} \leq a \leq a_{max}$.

Continuous-tap model C2

If the actual timer delay (T'_d) is dependent on voltage error and actual deadband [1],

$$T'_d = T'_{do} \frac{DB'/2}{||V_k| - V_{ref}|}$$

with the actual changer mechanism delay (T'_m) fixed, the model is,

$$T'_{c1} \frac{da}{dt} = -|V_k| + V_{ref}$$

$$a_{min} \leq a \leq a_{max}$$

with

$$T'_{c1} = \frac{T'_{do} \frac{DB'}{2} + T'_m ||V_k| - V_{ref}|}{\Delta n'}$$

Continuous-tap model C3

If the actual total delay time (T'_T) is dependent on voltage error and actual deadband,

$$T'_m = T'_{mo} \frac{DB'/2}{||V_k| - V_{ref}|}, \quad T'_d = T'_{do} \frac{DB'/2}{||V_k| - V_{ref}|}$$

the model is

$$T'_{C2} \frac{da}{dt} = -|V_k| + V_{ref}$$

$$a_{min} \leq a \leq a_{max}$$

with

$$T'_{C2} = \frac{(T'_{mo} + T'_{do}) \frac{DB'}{2}}{\Delta n'}$$

Continuous-tap model C4

If the actual timer delay time (T'_d) is a combination of fixed and variable time,

$$T'_d = T'_{d1} + T'_{do} \frac{DB'/2}{||V_k| - V_{ref}|}$$

with the actual changer mechanism delay (T'_m) fixed, the model is

$$T'_{C3} \frac{da}{dt} = -|V_k| + V_{ref}$$

$$a_{min} \leq a \leq a_{max}$$

with

$$T'_{C3} = \frac{T'_{do} \frac{DB'}{2} + (T'_m + T'_{d1}) ||V_k| - V_{ref}|}{\Delta n'}$$

Simulations

The discrete and continuous models presented above were examined using the network of Figure 2. In each case, the transformers began on center tap (1.0) and were suddenly loaded with a constant impedance, unity power factor load of 1.0 pu. The load was removed after 200 seconds. The network and TCUL data of Table 1 was used.

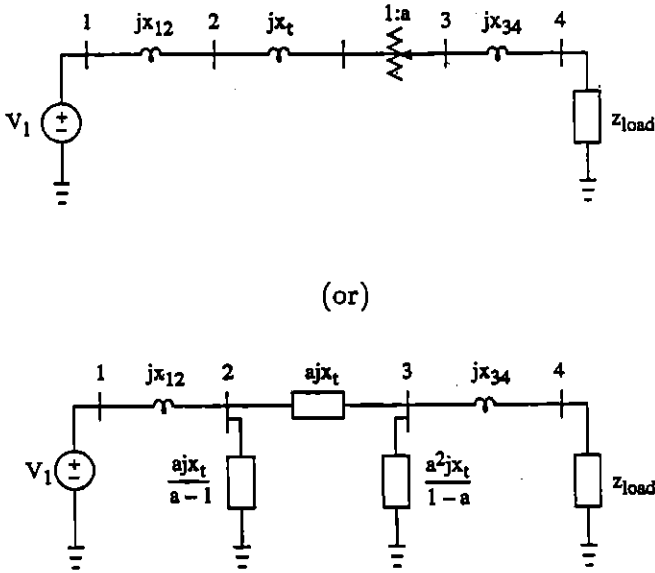


Figure 2: Simulation network

Table 1: Network and TCUL data

	$V_1 = 1.0 \angle 0^\circ$ pu, $X_{12} = 0.15$ pu
	$X_t = 0.02$ pu, $X_{34} = 0.10$ pu,
	$a_{min} = 0.9$ pu, $a_{max} = 1.1$ pu, $V_{ref} = 1.0$ pu
Model D1:	$\Delta n = 0.00625\epsilon$ pu, $DB = 0.014\epsilon$ pu $T_m = 10.0\epsilon$ sec, $T_d = 20.0\epsilon$ sec.
Model D2:	Same as D1 only $T'_{do} = 20.0$ sec.
Model D3:	Same as D1 only $T'_{mo} = 10.0$ sec., $T'_{do} = 20.0$ sec.
Model D4:	Same as D1 only $T_{d1} = 20.0\epsilon$ sec., $T'_{do} = 20.0$ sec.
Model C1:	$\Delta n' = 0.00625$ pu, $DB = 0.0$ pu, $T'_m = 10.0$ sec., $T'_d = 20.0$ sec.
Model C2:	Same as C1 with $DB' = 0.014$ pu and $T'_{do} = 20.0$ sec.
Model C3:	Same as C1 with $DB' = 0.014$ pu, $T'_{mo} = 10.0$ sec. and $T'_{do} = 20.0$ sec.
Model C4:	Same as C1 with $DB' = 0.014$ pu, $T'_{do} = 20.0$ sec. and $T'_{d1} = 20.0$ sec.

For purposes of this paper, the models D1-D4 with $\epsilon = 1$ are considered to be "exact" for each of their respective assumed control logics. The simulation responses for these four cases are shown in Figure 3. Models D2 and D3 reflect a rather strong participation by the voltage error dependent time delay. The large initial error results in a rapid tap change. This effect is much more subtle in model D4. As a transition to the continuous-tap cases, Figure 4 gives the responses for models D1-D4 using $\epsilon = 0.2$. The fundamental properties are preserved in each case. The responses of the continuous-tap models are shown in Figure 5. They very accurately predict the continuous version of the discrete-tap dynamic models in the limit as ϵ approaches zero. Model C3 is the fixed time constant model which is frequently used in the literature. Since this time constant is fixed, it does not reflect the possible longer delay which may occur when a large voltage error is encountered. Models C2 and C4 more accurately predict the delay which will occur when the changing mechanism actually has a component which does react faster to large voltage errors. Model C4 appears to give a fairly good representation of the fixed delay time of model C1.

Conclusions

One objective of this paper was to provide straightforward derivations of continuous-tap dynamic models from basic discrete-tap dynamic models. The results of this simple illustration indicate that the derivations are reasonable. A second objective of this paper was to identify the appropriate form and time constant data for continuous-tap dynamic models. This objective appears to be satisfied in the derivations and simulations. The issue of which continuous-tap dynamic model is most appropriate for power system dynamic simulations depends rather strongly on the actual timer delay property in the control logic. If the timer delay is truly fixed, then models C2 and C3 do not accurately capture the time response. While model C1 accurately captures the dynamic response, it has an awkward derivative which is not necessarily good for use in steady-state stability analysis. If the true deadband is used in C1 (rather than 0) it reflects the true nature of fixed taps for small disturbances. Model D4 also captures the fundamental fixed-time delay, and has a derivative which should be useful in detecting unstable tap movement in a steady-state stability analysis.

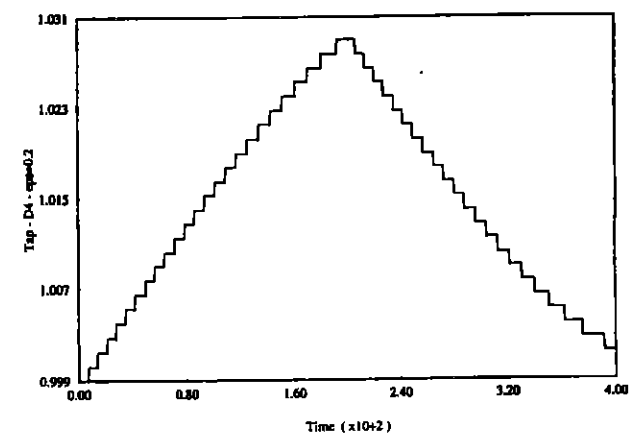
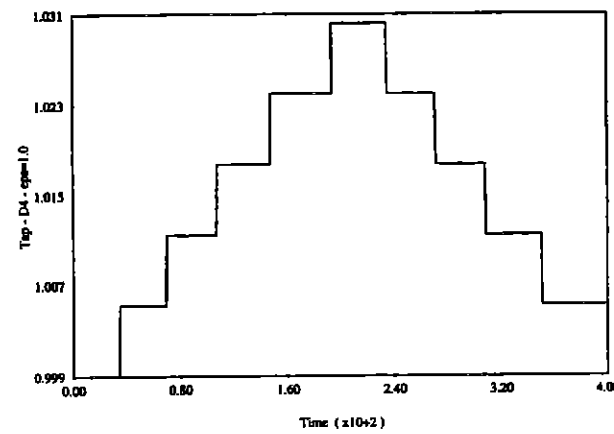
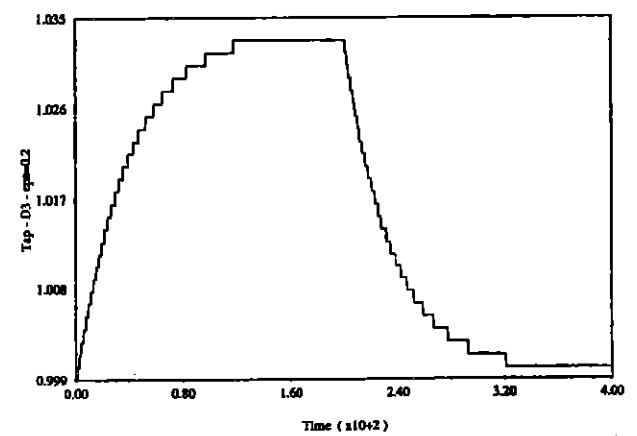
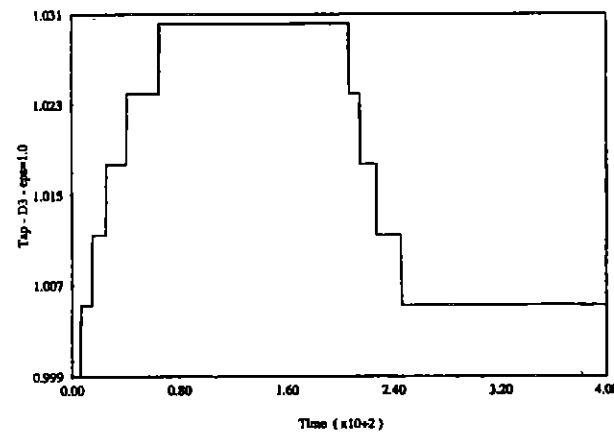
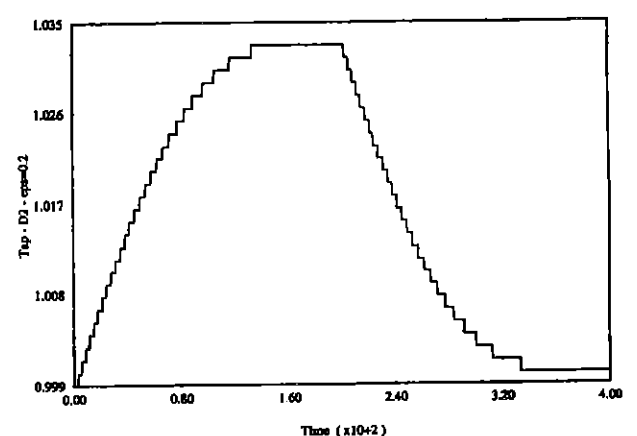
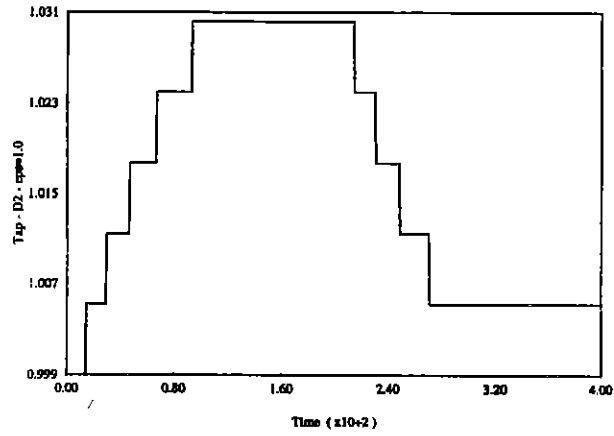
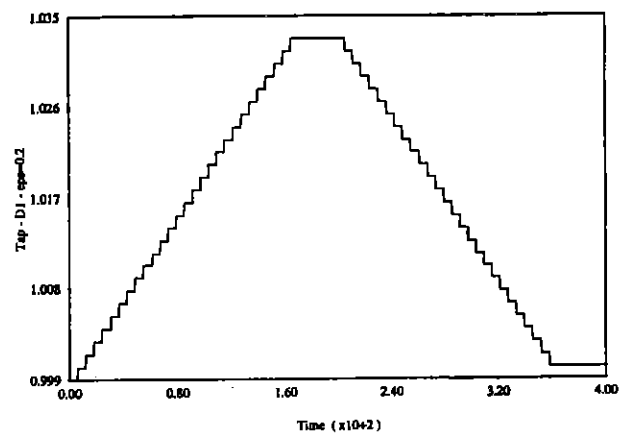
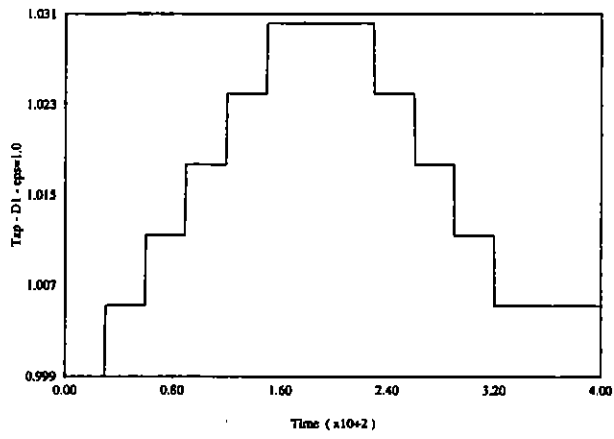


Figure 3: Models D1-D4, $\epsilon = 1.0$

Figure 4: Models D1-D4, $\epsilon = 0.2$

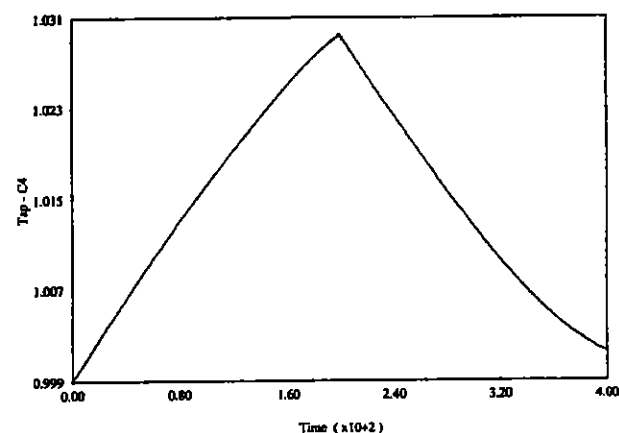
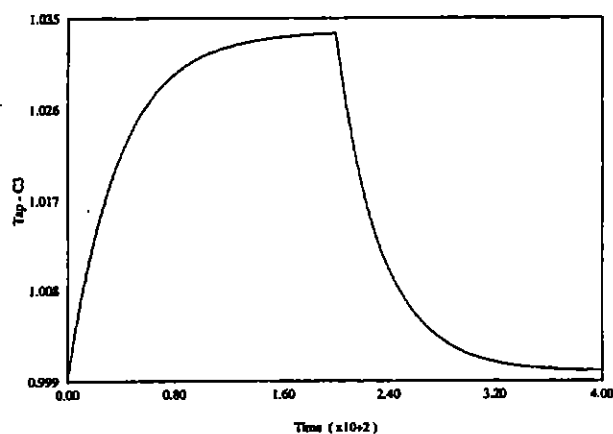
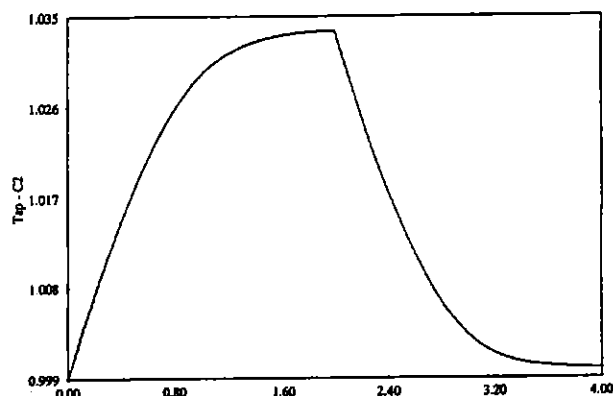
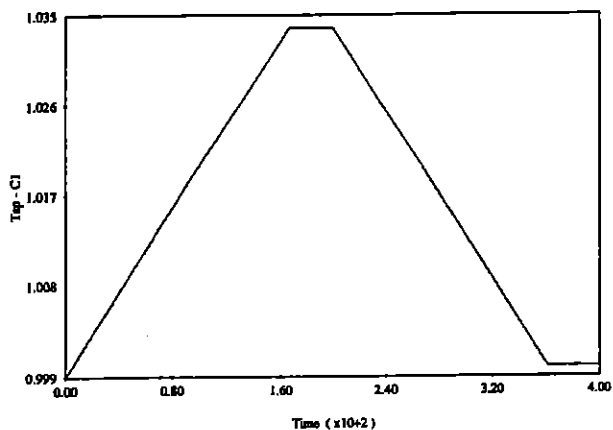


Figure 5: Models C1-C4

Acknowledgements

This work was funded in part by NSF grants ECS 93-18695 and ECS 91-19428, the University of Illinois Power Affiliates Program, and the Grainger Endowments to the University of Illinois. Thanks to Beckwith Electric Company for furnishing references [3]-[5]. Special thanks also to Prof. Felix Albrecht of the University of Illinois Mathematics Department for his valuable comments and suggestions.

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Voltage stability studies with PSS/E

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Abstract

A study of simulations of voltage stability phenomena using the PSS/E program (Power System Simulator) is presented. The objective is to explain how the interaction of different components, such as on-load tap changers, field and armature current limiters and dynamic loads, can endanger the voltage stability of a system. Special attention is given to the user-written models that have been implemented in PSS/E.

A computer model has been designed for a voltage regulator combined with field and armature current limiters. This combination is used for nuclear power plant generators in Sweden. Two different models for an on-load tap changer control unit commonly used have also been implemented in the program. The dynamic load model with load recovery is based on field measurements. Furthermore, some problems in using simulation tools are discussed, as well as the importance of parameter determination for some of the models implemented. The simulations highlight the importance of the generator current limiter and its interaction with the on-load tap changer and the type of load model chosen.

Keywords

Voltage stability, simulation, dynamic load model, on-load tap changer, armature current limiter.

1. Introduction

The problem of voltage stability and voltage collapse has been under consideration for 10 to 20 years. The approach to the phenomenon ranges from power system recording analyses, simulations of voltage collapses and incidents experienced by the power companies, to thorough theoretical mathematical studies. Researchers in the field have not yet been able to agree upon definitions of voltage collapse and voltage stability. Concordia defines voltage stability in a straightforward way, using words well known to utility companies: "...in terms of the ability to maintain voltage so that when load is increased, load power will increase, and so that both the power and voltage are controllable" [4]. Kwatny, on the other hand, uses static bifurcation theory for voltage collapse definition [12]. The authorities in this field also continue to discuss whether the voltage

collapse and stability phenomenon is a static or a dynamic problem [15]. Researchers in close co-operation with power companies often view voltage collapse as a static problem, since it can be analysed with ordinary load flow programs. In the academic world, where the bifurcation theory is predominant, voltage collapse is regarded as a dynamic problem.

In addition to study of the phenomenon of voltage stability and collapse itself, stability margins and indices have been defined and methods to derive such quantities have been developed [13]. Some authors in the field use simulation in order to include the dynamics of load, tap changer and generator reactive power capacity limits in the voltage collapse studies [5, 16]. A good survey of the voltage stability discipline can be found in reference [9], where both analytical tools and industrial experience are presented.

A voltage collapse can be initiated by either a primary fault or an unexpected load demand increase, in combination with insufficient reactive power reserves or transmission capacity. In order to avoid collapse, a detailed knowledge of the reactive power capacity in stressful situations, for large generators close to load centres, is essential.

The aim of this report is to provide models for load devices, on-load tap changer control systems and generator reactive power output capacity limits, based on data and experiments, and to compare our models with previous ones in simulation technique for voltage stability studies. The methods and models used in the report are designed to be easily accepted, both within the power industry and in the academic world. The simulation technique is well accepted in the power industry, and many of the dramatic simplifications sometimes necessary for analytical studies are avoided. Furthermore, well established and tested programs can be used to hold and solve the load flow case.

We believe that more attention should be directed to the importance of load dynamics, on-load tap changer control dynamics and generator limits in research on voltage collapse. In the variety of papers already published, detailed transfer limits, stability indices, etc., have been derived, while omitting, or only roughly modelling such important aspects as load recovery, tap-changer control systems and generator field and armature current limiters. While there is value in developing

advanced mathematical tools based on simple system component models, we have to keep in mind that the voltage levels in the post-fault load flow case and the static nose curves (P/V and Q/V curves) are still used as criteria for power transfer limit settings within many power companies. To be accepted by power companies and used as practical tools for system design and operation, research results must be related to the data and methods used within the power industry today. The results reported here are intended to be suitable for use as refined methods for voltage collapse studies within power companies.

2. Computer model implementation

In order to simulate the dynamic events that cause voltage instability, it is important to know how to specify relevant models of the equipment that affects the long-term voltage stability. A survey of components affecting the long-term voltage stability is given in [3]. It is also important to balance sufficient accuracy in the models against unreasonably long simulating time for large power systems. The test systems that have been used in this paper include dynamic load models, generator voltage controllers with field and armature current limiters, and on-load tap changing transformers. These models are all based on field and laboratory measurements.

2.1 Dynamic load model

The dynamic model proposed by Karlsson [10] is a special case of a dynamic load model given by Hill [8]. The model implemented is described by the following equations:

$$T_{pr} \frac{dP_r}{dt} + P_r = P_0 \left(\frac{V}{V_0} \right)^{\alpha_s} - P_0 \left(\frac{V}{V_0} \right)^{\alpha_t}$$

$$P_m = P_r + P_0 \left(\frac{V}{V_0} \right)^{\alpha_t}$$

where

V = supplying voltage [kV],
 V_0 = pre-fault value of supplying voltage [kV],
 P_0 = active power consumption at pre-fault voltage [MW],
 P_m = active power consumption model [MW],
 P_r = active power recovery [MW],
 α_s = steady state active load-voltage dependence,
 α_t = transient active load-voltage dependence, and
 T_{pr} = active load recovery time constant [s].

$$T_{qr} \frac{dQ_r}{dt} + Q_r = Q_0 \left(\frac{V}{V_0} \right)^{\beta_s} - Q_0 \left(\frac{V}{V_0} \right)^{\beta_t}$$

$$Q_m = Q_r + Q_0 \left(\frac{V}{V_0} \right)^{\beta_t}$$

where

Q_0 = reactive power consumption at pre-fault voltage [Mvar],
 Q_m = reactive power consumption model [Mvar],
 Q_r = reactive power recovery [Mvar],
 β_s = steady state reactive load-voltage dependence,
 β_t = transient reactive load-voltage dependence, and
 T_{qr} = reactive load recovery time constant [s].

When there is a voltage drop of 5-10% on load nodes, field measurements show that α_t is around 2. This means that the transient behaviour of the load can be regarded as a constant impedance. In most of the measurements, α_s is well below 1, which indicates a changed voltage dependence for the active power, and the load characteristic tends to be more like constant power (Figure 1). The time constant T_{pr} for this changing phase is around some hundred seconds. This phenomenon has been explained by the power characteristic in electrical domestic heating [11].

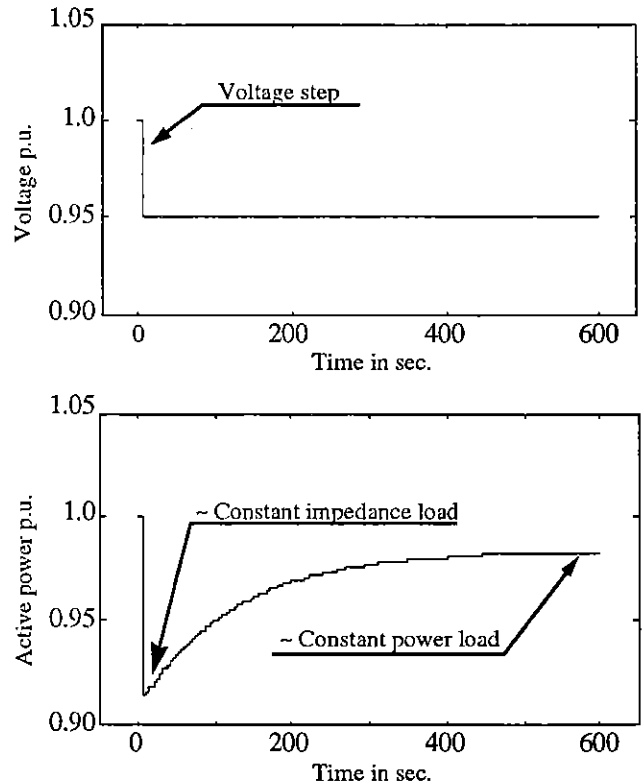


Figure 1 The active power recovery caused by a voltage step

An important experience when implementing the load model was finding that the voltage dependence had to be included during the iterations. Since the time step in the integration procedure is several thousand times smaller than the time constants in the dynamic model, the voltage dependence during

load flow iterations was not taken into account in the beginning. This did not work well all the time; numerical instabilities occurred and it was necessary to expand the implementation to include voltage dependence during the load flow calculations, see Figure 2. A delay of one time step between the iterated voltage and the corresponding load demand was eliminated by the implementation.

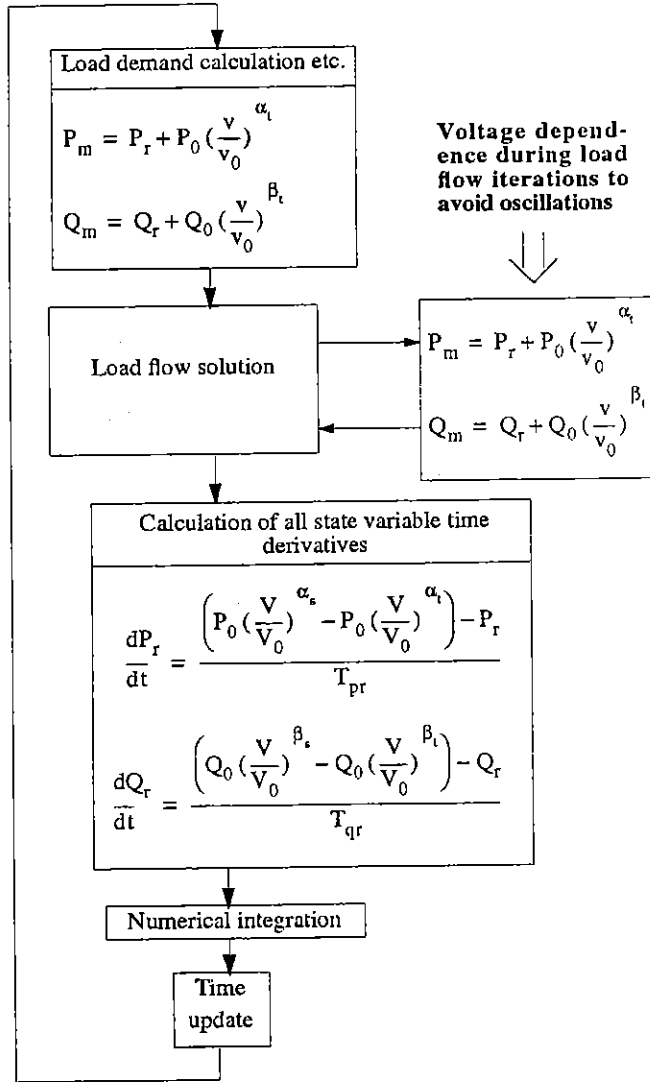


Figure 2 The solution algorithm for the dynamic load

2.2 Voltage regulator, including field and armature current limiters

A model program for the FREA¹⁾ excitation system has been written, based on a graduation thesis [6] in which the dynamics of the system were measured and identified. The FREA system consists of different units, among which the voltage regulator represents the basic one. The model of the FREA system used in the simulations contains two units, a basic one for voltage

control, illustrated in Figure 3, and an another for field and armature current limitation, illustrated in Figure 4.

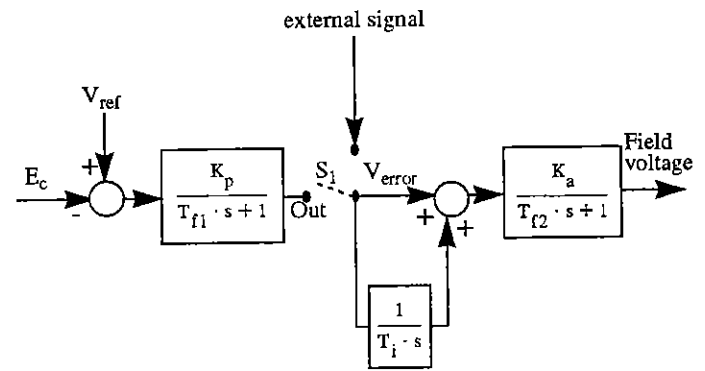


Figure 3 Block diagram of the voltage regulator

The external signal is the connection between the two units.

Typical parameters for the model are:

$K_p = 10-100$, $T_{f1} = 25$ ms, $T_i = 2-5$ s, $K_a = 1-40$ and $T_{f2} = 1-40$ ms.

Since the limitation of the field or the armature currents affect the reactive power generation, it is important to simulate these.

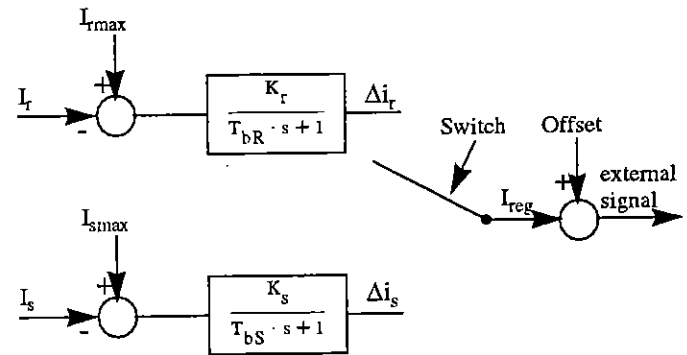


Figure 4 Block diagram of the field and armature current limiters

Typical parameters for the model are:

$K_r = 2-10$, $T_{bR} = 25$ ms, $T_{bS} = 25$ ms and $K_s = 2-10$.

Using the two units above provides a model with the following functions. The regulated field voltage is normally controlled by the difference signal $V_{ref} - E_c$. When one of the currents exceeds its permitted maximum, a timer that controls the switch S_1 starts and the current limiter takes control over the field voltage after a delay of t_1 seconds. The timer delay is a protection against undesired limiter functions i.e. short-circuits. When the timer value is greater than the delay time t_1 , the limiter controls the field voltage instantaneously when an overcurrent occurs. This condition is maintained for at least t_2 seconds or until the timer is reset.

The two conditions that activate the current limiters are:

1. $I_s > I_{smax}$ or $I_r > I_{rmax} \Rightarrow$ timer starts, and
2. Timer value $> t_1$.

The two conditions to reset the timer are:

1. $I_s < k \cdot I_{smax}$ and $I_r < k \cdot I_{rmax}$, and
2. Timer value $> t_2$.

Figure 5 illustrates the course of events when I_r is outside its limit. The sequence when I_s is outside its limits is analogous. When I_r and I_s are both outside their limits, the external signal will be assigned the largest value of $|\Delta i_r|$ and $|\Delta i_s|$.

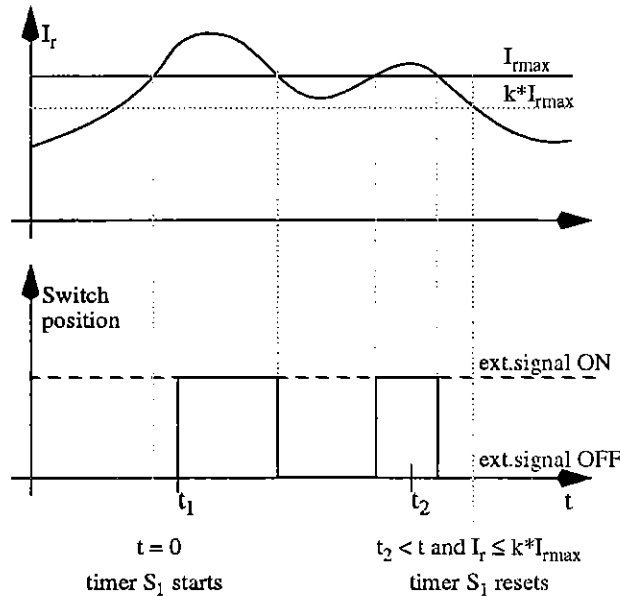


Figure 5 Conditions that control the blocking of the external signal when $I_r > I_{rmax}$

Since the generator is permitted to operate only in an over-excited mode when the current limiters are active, it was necessary to implement a blocking signal. This signal also blocks the limiter when the terminal voltage falls below a permitted value. If this were not done, the simulations showed that the regulated field current changed sign and the generator became more and more underexcited. This blocking of the current limiter is important in order to prevent underexcitation and to avoid incorrect regulation. It was noticed that the blocking was the source of large moment steps in mechanical power. Consequently, it is unlikely that this running condition can be allowed for more than a short time.

2.3 On-load tap changer models

Two different types of OLTC-relays (On-Load Tap Changer) commonly used in the Swedish utility network have been modelled [1]. The purpose of these relays is to keep the secondary voltage within a specific dead band around a set-point value. The time from the point at which the voltage deviation exceeds the dead band until the relay triggers the tap changer mechanics on the transformer is called the functional time of the relay. This time interval depends mainly on two things: a) the basic setting time of the relay, chosen by the operator, and b) whether

the relay is working in constant time mode or inverse time mode.

When the voltage deviation is large and one step change on the transformer is not sufficient to restore the voltage, then two other things must be taken into account. One is the time it takes to reenergize the tap changer mechanics after a changing. The other is whether the mechanics use a pulse or a constant control signal from the relay to trigger the tap changer. The effects of these conditions will be discussed later on. As the operation time needed for an energized tap changer to change step is within a few periods of the network frequency, it is considered negligible in these models. When the relays are exposed to a large voltage step (~several dead bands/second), the functional time becomes reduced for reasons unknown. More identification of the relays is necessary to model this phenomenon. The two relays modelled are the RXCE41¹⁾ and RV902²⁾.

2.3.1 Description of the RXCE41

The RXCE41 relay has several settings that control its behaviour. On the front panel of the relay, the operator has to tune in one of the basic setting times: 15, 30, 60, 90 or 120 seconds. All actual time delays are scaled to one of these basic settings. Also the voltage set-point value and the deadband on the relay must be tuned. Finally, one of the four working modes of the relay has to be chosen.

I) Constant time mode and pulsed control signal

The functional time is independent of the amount of the voltage deviation. A short pulse of approximately 1 second triggers the tap changer, after which the relay is restarted. This means that this mode is the slowest way to restore voltage.

II) Constant time mode and constant control signal

The functional time is independent of the amount of the voltage deviation. If several tap changing steps are necessary to restore the voltage, the reenergizing time delay of the tap changer mechanics controls the speed with which the voltage is restored.

III) Inverse time mode and pulsed control signal

The functional time is dependent on the amount of the voltage deviation. When large deviations are present and several tap changing steps are necessary in order to restore the voltage, the time delay of the first steps is controlled by the delay in the transformer. When the voltage approaches the desired voltage, the time between tap changings is controlled by the relay. The functional time of the relay, in this mode, may be shorter than the reenergizing time of the tap changer mechanics. If this is so, trigger pulses will be lost due to the fact that the mechanics have no memory. Although the voltage will be restored (if pos-

sible without running the tap changer into an end stop), but it will take a longer time.

IV) Inverse time mode and constant control signal

The functional time is dependent on the amount of the voltage deviation. A larger deviation means a shorter functional time of the relay. If several tap changing steps are necessary, the steps following the first are controlled by the mechanical delay. This is the fastest way to restore the voltage deviation.

The different characteristics of these modes are shown in Figure 6. It can be seen from the figure that the relay settings affect the time constants of the voltage restoration. This is important when a dynamic load is connected to the transformer.

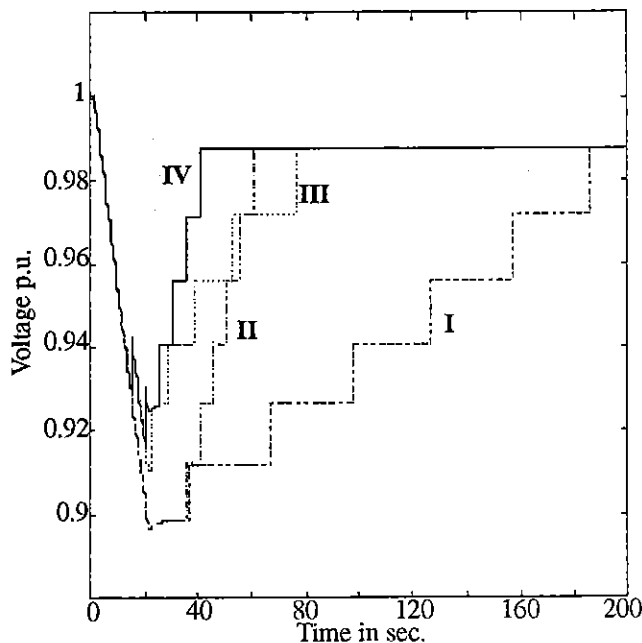


Figure 6 Different modes for the RXCE41 relay.

- I) Constant time mode and pulsed control signal,
- II) Constant time mode and constant control signal,
- III) Inverse time mode and pulsed control signal,
- IV) Inverse time mode and constant control signal.

The relay timer is started when the voltage exceeds the dead band, and reset when the voltage deviation is less than the return ratio multiplied by the dead band. A hysteresis effect on the function of the relay is thus achieved.

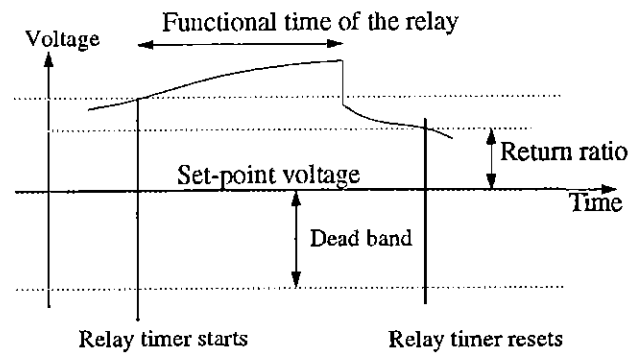


Figure 7 One tap changing manoeuvre and the corresponding timer actions. The return ratio is around 0.75 times the deadband.

In inverse time mode, the deviation is not integrated in the usual sense. It is the voltage deviation that actually gives a functional time. If the relay time is greater than this functional time, the tap changer receives an order to change. The voltage deviation divided by the dead band is defined as δ and calculated for every time step. If δ is greater than 1, the voltage is outside the dead band and the relay timer is running. When δ is below the return ratio the timer is reset. The five different basic setting times offer different functional times, T_{ds} . If the operator has chosen the basic functional time ΔT to 15 seconds, the model calculates the functional time from

$$T_{ds} = (\Delta T - 2) \cdot e^{\frac{-(\delta - 1)}{1.25}} + 2$$

If the voltage is just outside the deadband ($\delta = 1$), it will take 15 seconds for the relay to operate. Note that the formula is also valid for $1 > \delta > \text{'return ratio'}$, when the timer is running. Therefore, it is possible to have quite long delays even in the inverted time mode. Similar formulas are valid for the other basic functional times.

The calculated functional times have a maximum discrepancy of 1.5 seconds (3 seconds for $T = 120$ s) between the model and the relay.

2.3.2 Description of the RV902

The RV902 relay works as an inverse time relay. It does not use a constant control signal to the tap changer. It has no timer-function and the voltage deviation is integrated all the time.

The model integrates the voltage deviation in the following way. First, δ is calculated as the deviation from the set-point value divided by the dead band. This gives a value greater than 1 (or less than -1) when the voltage is above (or under) the dead band. Two other constants are calculated,

$$m = 1.4 \cdot (\text{dead band}), \text{ and}$$

$$k = 1.014 \cdot (1 - e^{-(1.2 \cdot (\text{dead band})^{1.5})})$$

and the model sums up the level with the following formulas:

$$\Delta \text{level} = (\delta - k \cdot \text{level}) \cdot m \cdot \frac{\text{time step}}{\text{delay time}}$$

$$\text{level} = \text{level} + \Delta \text{level}$$

When this level value is greater than the top level, the relay is functioning. The top level is usually chosen as 0.98. The function of the RV902 is similar to the RXCE41 when working in inversed time mode and pulsed control signal. Therefore the RV902 essentially works as in the III curve in Figure 6.

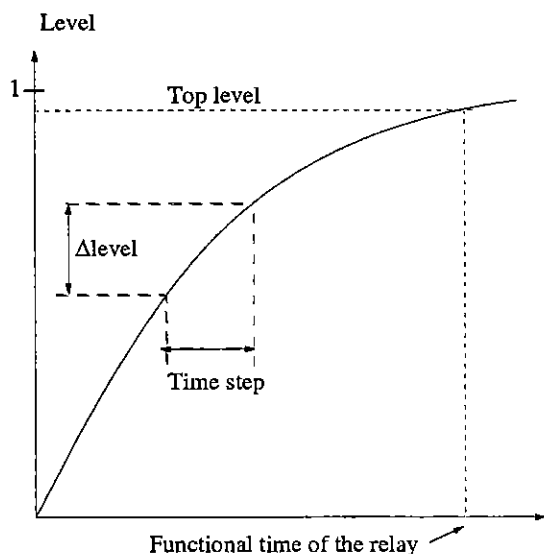


Figure 8 The integration process for the RV902 relay.

3. Simulations

The computer program used for all the dynamic simulations is the PSS/E program¹⁾ which has the modelling capacity to account for the important dynamics of voltage stability analysis.

3.1 Description of the test systems

To demonstrate dynamic analysis techniques and to illustrate the basic phenomenon of voltage instability, two test systems were used. One was a very simple radial network, System 1, and the other a meshed 27-node network, System 2. System 1, shown in Figure 9, consists of 4 buses, 4 branches and 1 generator. Most of the important factors influencing system voltage stability are readily identified in this simple system. One of the

two lines in parallel has twice the reactance of the other, in order to show distinct results.

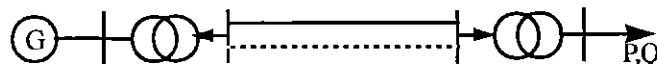


Figure 9 Test system 1

System 2, shown in Figure 10, consists of 27 buses, 33 branches and 17 generators. The simulations conducted using this representation give a better picture of the dynamics of voltage stability in a large power system.

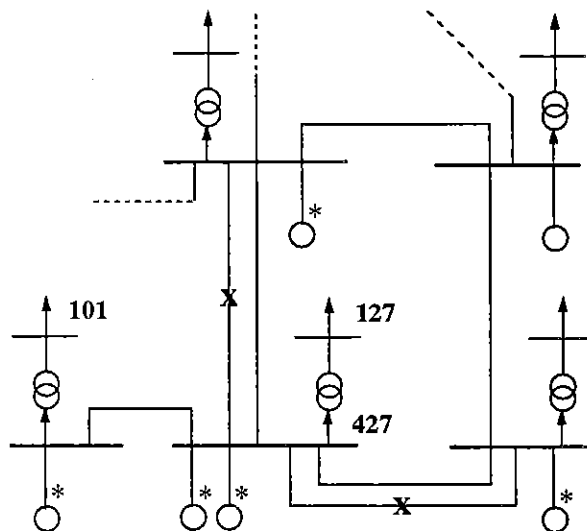


Figure 10 Part of test system 2, showing where the simulated fault was applied. The X-marked lines and one of the two identical generators at node 427 were disconnected. The asterisk (*) marks a generator with a current limiter.

When the simulations are initiated, the parameters of the different components decide the behaviour of the system. The parameters used in the following simulations are taken from field measurements [10]. Some parameters for the small network are given in the appendix.

3.2 Response of the dynamic load including the OLTCs and current limiters: System 1

A disturbance initiated by opening the line with the lowest reactance was studied for two cases, A and B. The simulations were conducted for up to 350 seconds, using the models described for dynamic load, OLTC transformers and generator current limiters.

Case A demonstrates the combined effects of one OLTC and a dynamic load. In Figure 11 it can be seen that system voltage stability is maintained though there is a low voltage level on the primary side of the transformer at the load end. The voltage

¹⁾ Manufactured by Power Technologies, Inc.

level at the secondary side is restored by the transformer in a few minutes. For every tap changing step, power demand increases, which gives a current increase on the load side of the transformer (Figure 12). This increased current is amplified on the generator side by the transformer tap changings and gives an increased voltage drop over the line.

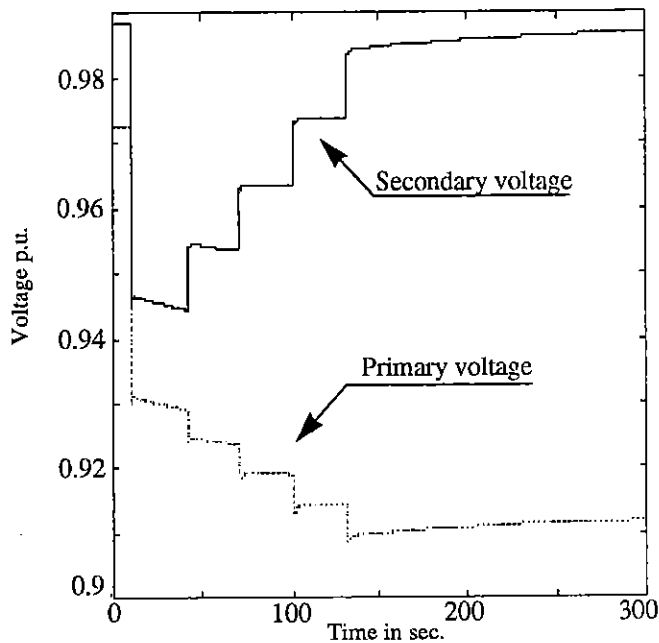


Figure 11 Case A: The voltage on both sides of the transformer at the load end.

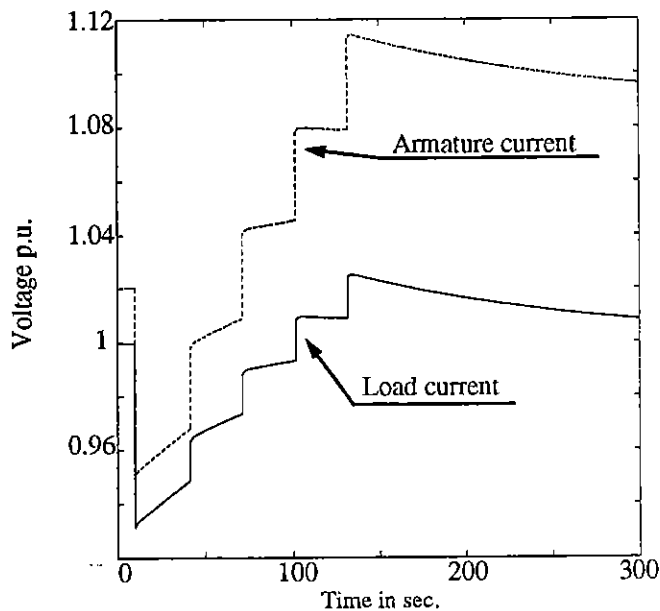


Figure 12 Case A: The armature and load current. Note the significantly larger armature current during the recovery.

When the time needed to restore the voltage level by the regulating transformer and the time constant of the load recovery

are in the same time domain, an overshoot in power demand can appear. In Figure 13 one can see the combination of these effects.

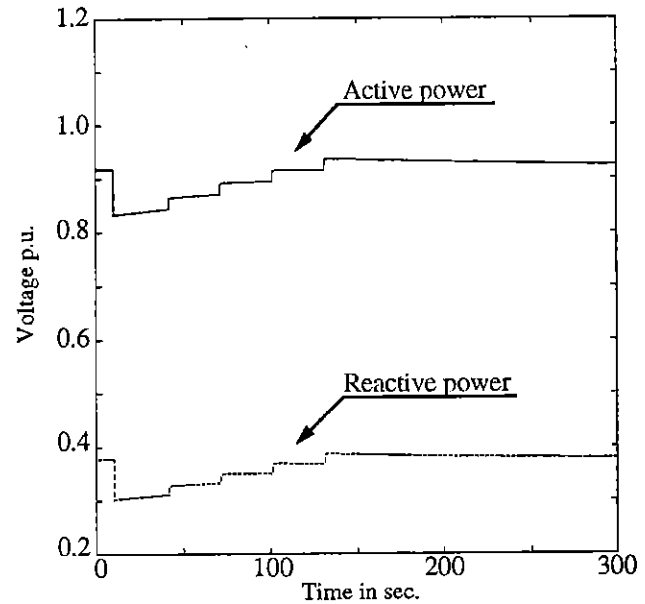


Figure 13 Case A: Active and reactive power demand for the dynamic load.

Case B is similar to case A except that the current is higher than the settings of the current limiter. The generator is limited in order to study the effects of the armature current limiter (Figure 14). It can be seen that as soon as the armature current limiter is operating the load voltage starts to decline, due to the tap changing manoeuvres and the armature current limitation. Since the delayed operation of the current limiters is usually a few seconds, just a short period of overcurrent is sufficient to activate one of them, which puts the system into a very critical situation.

When there is a disturbance in a large meshed network, this overshoot may not occur. Variation in the parameters of both transformers and loads spread out the overshoot. However, the generator current is increased due to the tap changers and load recovery, and this can, in a system under stress, cause the limiter to activate. The outcome of the current limiter action depends on the generator size and the network around the generator. An example of a collapse where the current limiters had a major impact is the French collapse of 1987 [7]. The importance of the current limiters is also mentioned in [3].

For a network without load recovery, one could get the effects described above from several levels of voltage regulating transformers, working unselectively. This could be the source of large armature currents, when a voltage disturbance is not restored sufficiently fast by the transformers near the origin of the disturbance.

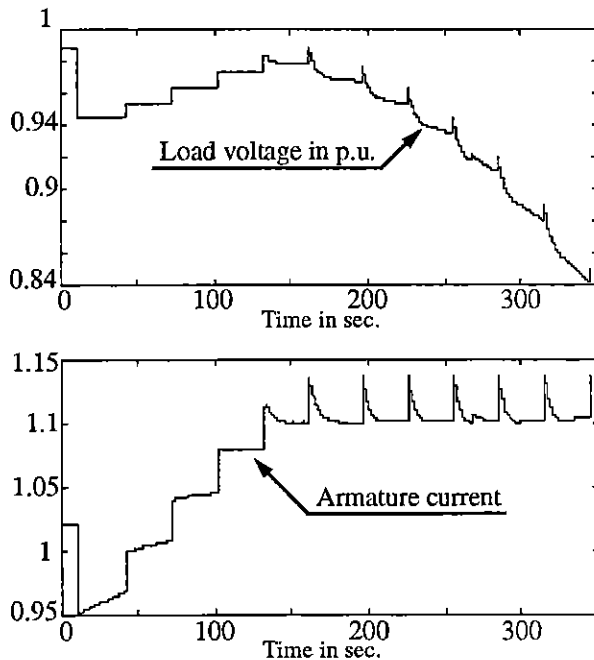


Figure 14 Case B: Voltage and armature current where the current is limited to 1.1 p.u. It can be seen that the tap changer tries to restore the voltage while the current limiter decreases the current within a few seconds to the limiting value.

3.3 The importance of the load model chosen: System 1

In the event of a disturbance, power systems, depending on their load representation, can respond with completely different characteristics.

The following simulations, on System 1, illustrate the statement above. The limiting factor in these simulations is the armature current limiter. The system is disturbed by a 5% voltage step down on the generator set-point (i.e. disconnecting reactive support at the generation end). Figure 15 shows four simulations with different load models; in Figure 16 there are the same simulations with the OLTC transformer relay at the load end activated. After the disturbance, the small system is not able to supply power to the constant power model at all (curve a in Figure 15). There is no stable operating point for the voltage. As the voltage decreases, the current would increase if it were not prevented from doing so by the current limiter. If the active power demand is nearly proportional to the constant current, the system responds nearly the same way (b in Figure 15). Curve d shows that when the load is more voltage dependent, the system is stable. With this model, current demand decreases when the voltage decreases and the current limiter is not activated. Thus, a stable working point can be found.

The dynamic load model, described in Section 2.1, has two voltage phases in time (curve c in Figure 15). At first, the load

is principally a constant impedance and follows the d curve. With time, the load recovers (Figure 1) and becomes more like a constant power load. The voltage starts to decline as for curve a and b in Figure 15. Neither of the three static load models (a, b and d), could be used to emulate the behaviour of the field measured dynamic load model.

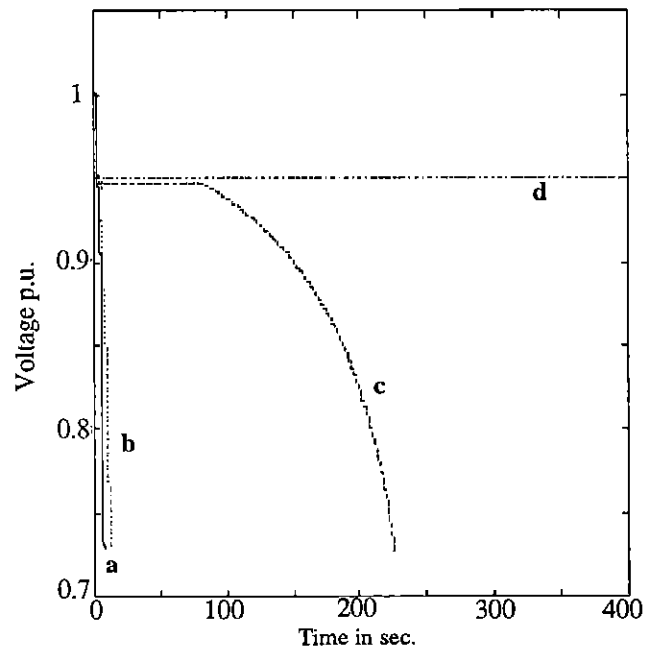


Figure 15 Four different load models and their interaction with a current limited generator. The reactive power demand is independent of the voltage in simulations a, b and d.

a) Constant power model for P.

b) $P = P_0 \cdot (V/V_0)^{0.8}$

c) A dynamic load model according to Section 2.1

d) Constant impedance model for active power

One can also see in Figure 16 that the transformer actions increase the speed of the voltage decline. All of the models chosen have an unstable voltage when the tap changer of the transformer starts to restore the voltage.

If the transformer is omitted in order to decrease the computer simulation times and the constant power load is used with the motivation of restoring the voltage, the result could be a too pessimistic simulation (curve a). If a load model that is proportional to the voltage squared is used, a completely different response is possible (curve d). By choosing models that lie between a and d, a simulation of the voltage somewhere between these could be utilized.

(Figure 16). A simulation with a similar behaviour could be found in [14].

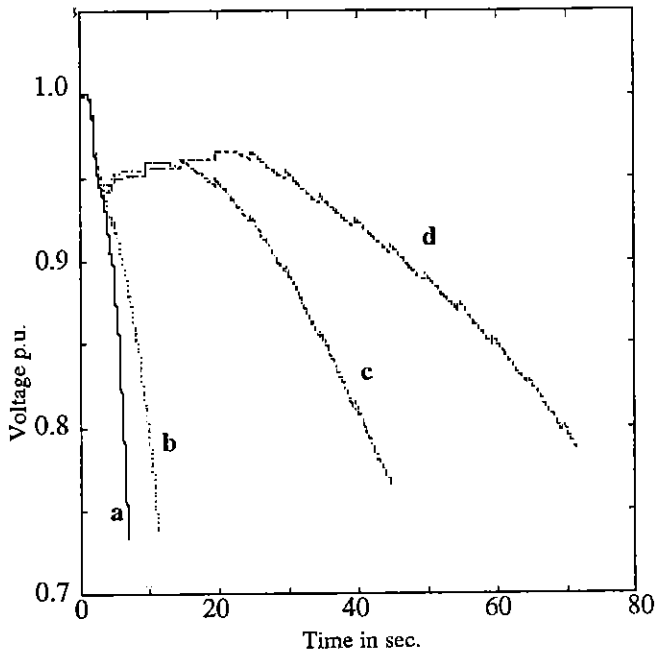


Figure 16 The same load models as in Figure 15 but with one OLTC relay active. Note the shorter time scale. The tap changings accelerate the voltage decline.

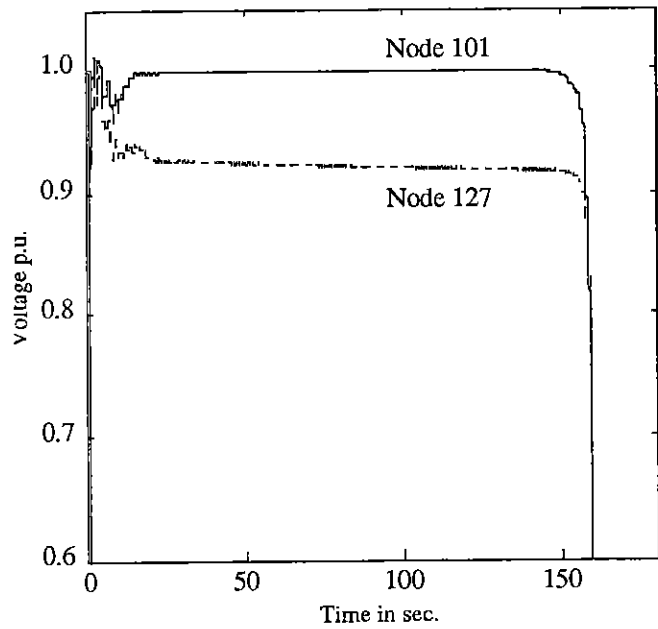


Figure 17 Voltages in System 2. After the short-circuit the voltage is rather stable in node 101 but it slowly drops due to load recovery in node 127. After 160 seconds the voltage collapses.

3.4 Response of dynamic loads including OLTCs and current limiters: System 2

To demonstrate the long term voltage phenomenon in a large power system, a short-circuit causing two lines and one generator to trip was simulated (Figure 10). Two cases were studied: C and D. In case C all the tap changers in the system were locked and in case D they were active.

Case C: When the short-circuit occurs, the generators with armature current limiters are activated and they continue to operate during the whole simulation. After the transient oscillations have died out, the delayed operation of the armature current limiters decrease the voltage. While the voltage level becomes acceptable in node 101, it is low in node 127 and slowly decreasing. After 160 seconds the load recovery in the nodes with depressed voltages pulls the voltages down, i.e. the dynamic in load characteristics causes the system to collapse. This simulation is analogous to curve c in Figure 15.

Case D: When one level of voltage regulating transformers is active the progression will go much faster. Up to the point when the first transformer changes tap position, the simulation is the same as above. After just a few tap movements of the transformers in the area, the voltage collapses. Note that the primary voltage at node 427 shows the same behaviour as the primary voltage in Figure 11. We have also seen that the transformer actions speed up the process in the simple system

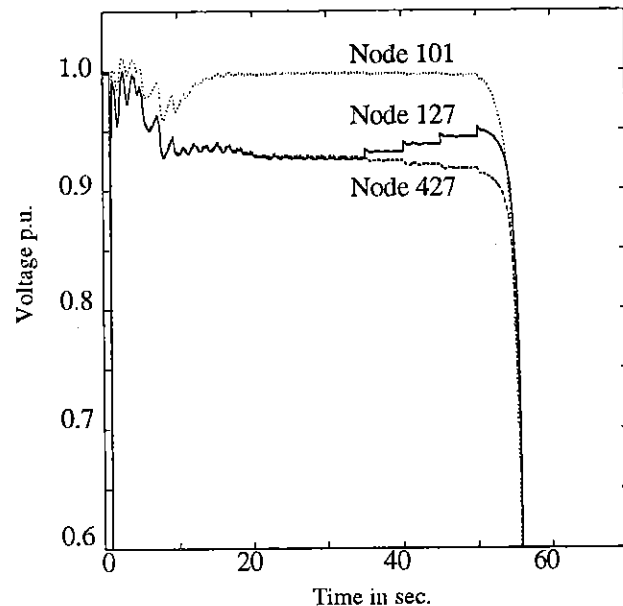


Figure 18 Voltages in System 2. The same divergence of the voltage on the different sides of the transformers can be seen in Figure 11.

3.5 Discussion of the simulations

In a weakened network with generators having limited voltage regulation capabilities, the regulating transformer behaviour is

important when the generators become current limited. When a voltage level falls below the transformer dead band, the voltage will be restored by the OLTC. Every tap changing increases load voltage and thereby increases load demand. This also increases the current on the primary side, see Figure 12. The step up transformers near the generators can not cope with the load transformer tap changings due to the larger voltage drops in the network, which affect the load transformers more than the step up transformers. The current increase, therefore must be taken from a generator that is not current limited, probably far away from the critical area, since the mechanical power to the limited generators is virtually unchanged. If this is not possible or is very difficult due to line losses in the transmission system, the voltage starts to decrease, sometimes very fast (Figure 14, 17 and 18). In these simulations it appeared to us that the OLTC-regulation speeded up the voltage decline when the armature current limiters came into action (Figure 16 and 18). In several French regions an automatic OLTC blocking system has been installed to prevent voltage collapse [2].

The system operator should be presented with the actions of the current limiters, due to their influence on the system. Therefore, this presentation is going to be implemented at the Sydkraft utility company.

4. Conclusions

The authors believe that it is important to model components with a high degree of accuracy. Many phenomena that were not expected to have an effect on voltage stability simulations turn out to be important when included. One thing that was found important was the blocking of the activated current limiter when there was a risk that the generator would become under-excited.

Accurate simulation can be a complement to more theoretical/mathematical treatment of voltage stability phenomena. These simulations can also be used to verify general theories developed on small, easier-to-understand networks.

However, it must be kept in mind that the simulation results depend on the quality of the input data. Several parameters in the load model used here are difficult to extract from a real network. The simulations reported here show clearly how different load models affect the outcome of a simulation. The lack of information in this area should be remedied.

The simulations highlight the importance of the generator current limiter and its interaction with the on-load tap changer and the type of load model chosen.

5. Acknowledgements

The authors would like to thank Sydkraft company and Sydkraft Research Foundation for their engagement in this research area and for their financial support.

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Appendix

Primary data for the simple test system:

The dynamic load model:

$\alpha_s = 0.38$, $\alpha_t = 2.26$, $\beta_s = 2.68$, $\beta_t = 5.22$ $P_0 = 0.10$ p.u., $Q_0 = 0.04$ p.u., $T_{pr} = 127.6$ s, $T_{qr} = 75.3$ s.

Line parameters:

The short line: $x = 0.75$ p.u.

The long line: $x = 1.5$ p.u.

The transformers:

$x_k = 0.1$ p.u., voltage set-point = 1.0 p.u., deadband = 0.02 p.u., tap step = 0.015 p.u.

Discussion of papers presented by Miller, Sauer, and Johansson

Edited by Lester H. Fink, ECC, Inc.

Yakout Mansour (BC Hydro)

My question might actually address all three papers, or at least two of them. I feel very nervous about that sharp drop in the simulation results at the bifurcation point. The reason I am saying this is not really to discuss the details of it, but just to caution you. We kept getting results like this for about 3 or 4 months, and we did not believe it, and we still didn't believe it, until we found that there was a bug in the modeling and the integration technique. It looks to me as though in both cases you have a numerical problem at the end point. This is not a real collapse. This looks to me, especially when I look at the frequency plot when it goes sharp, and then goes back, this is not like a real frequency plot. Maybe you may say: "but who cares, I know that it is collapsing," but I can say that if you are playing with controllers, and want to see the impact of controllers on the performance, you want to go beyond that - you don't want to stop right there.

The same goes for PSSE, which was not meant to be a long-term dynamics program. In that respect you will find a lot of deficiencies with it, whether in the computations or in some of the modeling. PSSE is a good program, by the way, but I would caution you not to stretch things beyond their limits.

Again, one of the modeling questions, whether it addresses the first or the third or even both papers, addresses some of what Pete said about tap changer models. When you use a small simple model, and you have just one tap changer, then the effect of nonlinearities from a system point of view gets amplified. Look, for instance, at what Carson said the other day, that around the Seattle area you have to model 700 tap changers. From a system point of view, if you say that they're all kind of changing randomly on the high side, you find that the increments, how the system sees tap changers, - probably the difference between one and another could be in the fraction of a second, and the impact of the tap changers collectively on the high side could be very close to continuous (and that is to be proven by the way, we are still working on that). If we can avoid updating the Jacobian every time something gets changed by having something continuous, when we don't have to change reactances, and get reasonable results, that could have a tremendous impact on the speed of simulation.

Nicholas Miller (General Electric Company)

Your point about the fidelity of the simulation at the point where it is ripping to pieces is well taken, Yakout. I can't speak for PSSE, of course, but we went to some considerable pains in developing the algorithm for this, using a variable time step integration. Whenever we look at the plots, you find things that look like numerical problems. I've got a three-minute simulation with phenomena for which some of the bandwidth is very fast, and when you zoom in, you see that there isn't any particular evidence of some of the classic marks of numerical misbehavior - things are nice and continuous, and you are doing the right thing numerically. Having said that, that doesn't make me believe the results any more. The fidelity of our modeling? - there are a zillion things going on out here. I've got machines that are going off for low frequency, high frequency, voltage, there's OELs going off, who am I kidding? Did I get the sequence right? Not a chance. The system is in dire straits, no question about it, but making a decision as to what to do - am I going to wait until this point and then say, oh my God, I have to do x, y, or z? No, not a chance. Your point about the frequency going off: system frequency when the system is doing crazy things is one of those holy grails of simulations. You get these steps. The way we do this, it's the center of inertia

speed, so when machines pop off you get discreet steps in the center of inertia speed. What does that mean? I don't know.

Peter Sauer (Univ. of Illinois at Urbana-Champaign)

The discreet model D1 is acceptable as an accurate model. The continuous model C1 is really the same, but it has an awkward form. The model C3, which is what is normally used in the literature for a continuous time model, useful for eigenvalue analysis and whatnot, has the problem that the time constant is the same regardless of the size of the disturbance, which is not realistic given that the tap changing under load transformer has fixed time increments to change from tap to tap to tap. So it depends on whether you are doing five tap changes or one tap change, the time is different. The traditional C3 model does not model that correctly. The model C1 does, and the model C4 gets close. C4 is a nicer model, because it is easy to write the derivatives and Jacobians and all that. Model C1 has constant derivatives, which is kind of strange. It does model the fact that in real life, for the small signal stability analysis of a tap changing underload transformer, the tap should not be in there at all, because they are fixed taps, and for small enough changes they are not going to change. So a TCUL should not come into any small signal stability analysis in that sense. If you really want to check the stability of TCULs, then you have to go to either a large change or a continuous model, and the continuous model of only 2, 3, and 4 will give you that stability information. C1 still won't, because it has a Jacobian that is zero.

Fredrik Sjögren (Chalmers Univ. of Technology)

Just a short comment about PSSE. The reason why we use PSSE is that - there is no good reason for why we use PSSE. The only answer I get is that Sydkraft utility uses PSSE and it is easy to exchange documents. I think PSSE is not that bad.

George Knight (Imperial College)

I found the comparisons of the frequency dependent and frequency independent models extremely interesting. What I am going to say does not really have any impact on the models you need, but it might affect computing time. If you initiate the problem by slow but steady load increase, do you still think you need to include frequency dependence, if you are only looking up to the point of collapse?

Marija Ilic (Massachusetts Institute of Technology)

This is a comment to Pete's example from Washington to Sydney. It made me think of something rather different. If you were to do that actually, to consider delivery like that one would have to have a right, which has completely different characteristics. So for your modeling, you would look into detail and find a big modeling problem. The line from wherever to Sydney would have to be very capacitive. I am saying this as a half joke, but it really made me think that as we discuss voltage problems in one area, we tend to neglect other areas. Particularly in some parts of the Northeast United States, they have very heavy cable type transmission, and the characteristics of the network are completely different than the ones that we consider in most of the modeling that we discuss, and that changes something. I would just like somebody's comment, if somebody knows whether studies of that sort are done, because I would like to have a reference for that. I think that everything is different if you have to model a network like that, particular with voltage control.

Ian Hiskens (University of New Castle)

Pete, I was wondering how you address - you have done the non-sequentials, what do you do for sequential tapping?

Sauer: I started to think about it, but it got too nasty and time was short. My feeling is that the C3 model might be more reflective of the sequential model, because for big changes it will change more quickly. The C3 model does not have the long time delay of a multiple tap changer, which is kind of like the sequential mode of operation. So it is very likely that the C3 model, which Chen-Ching has used in a lot of his work, is correct, or at least correct for the sequential case, and my reason for attacking it as being incorrect for the non-sequential case might disappear. I talked to several manufacturers, and I believe the percentage of sequential versus non-sequential is about 50/50, at least in the U.S. Half of the transformers are sequential and half of them are non-sequential. I suspect the answer will come out to be that C3 may be legitimate for that type. It makes sense.

Chen-Ching Liu (University of Washington)

I appreciate this hard work to clarify the transition from a discrete model to a continuous model. I think that is very good work. My question is more from an analytical point of view. I can see that the purpose of trying to use a continuous model instead of a discrete one is because we hope the model will be more simplified, and be more consistent with the other differential equations and so forth. But if you look at C1, C2, and C4, these absolute values really worry me, plus other nonlinearities. So if you put it together, it seems to me they become some kind of discrete modeling again, because of the absolute value and other nonlinearities; so you have to treat it in several cases, when you analyze equations. So my question for Pete is, have you thought about how you actually incorporate C1, C2, C4 into analysis?

Sauer: C1 is not a problem for numerical analysis. Actually none of them are a problem for simulation, but they can all be easily programmed. The tricky part is, if you are doing Nelson Martin's type of stuff. I had to look up in my calculus book how to take the partial derivative of an absolute value, and you are right. It does sort of come out to be a discrete thing. You have to be careful when you take partial derivatives of absolute values of the magnitude of the voltage, but it can be done, and gives you an explicit equation for the Jacobian. So it is a little messier with the absolute value in there, I agree. I haven't thought about it any more than that.

Miller: Three things. First, to answer George's question of a while ago: We do lots of simulations where we take the system and ramp it up to collapse, and as long as you choose a reasonable ramp time, which is something that we like to do anyway, the AGC will keep the system at essentially flat frequency. This raises the point that it is very important to have some isochronous function in the long term simulations, which is not something that you get with a regular transient stability program.

I really included that example though, because a lot of these cases where systems are really getting knocked into *in extremis* really do involve serious frequency disturbances, and the frequency dependence on the load can be just as important. Maybe a discussion for another day is the fact that we use an algebraic network model, and if you have a 2% frequency excursion then B is not really right, since it is really ω -C, and the amount of capacitance you are getting from these underfrequency terms is wrong, and wrong in the direction that you should worry about, since you are getting less VARs out of your banks at 98% frequency instead of 100%.

The C3 model of the transformer - I'm very interested in Pete's stuff - I guess I'm still a little worried about getting those other models besides C3 into the A matrix, but as you may know, most of that work that I showed was sponsored

by Japanese utilities, and the C3 transformers that switch on volt-second error seem to be the norm in Japan. So maybe that's where that piece of information came from.

Xin Jiang (Washington University)

I have a comment on Mr. Miller's paper and a few questions also regarding voltage drop. I thought if the system loses stability, you won't get such a vertical voltage collapse, and I thought maybe this system reached a singularity, because we have done something similar. Although it is a very small system, we get this similar voltage collapse like a vertical line, because at the singularity the speed of decrease of the voltage approaches infinity. That's why you get such a sharp vertical voltage drop. Also, around the singularity the usual phasor calculus is in question, because the usual phasor calculus uses the quasi-static assumption, it omits some derivative terms, so still there is somewhat that needs to be done.

Carson Taylor (Bonneville Power Administration)

With regard to the last point made by Nick Miller, on the cost of data gathering and so on, the bulk power delivery transformer, the main LTC transformer in many cases (they might be in the size range of 20 MVA), is one piece of data that is well known, and can be incorporated in databases for the area of interest quite easily. The impedance of 10% or so is known to the utility, and the actual time delay for the LTC and the tap settings and so on, so you might consider incorporating that model directly. With regard to Pete Sauer's paper, it's very interesting. One other possibility I don't think you discussed, is that in the North American case we have a fairly long initial delay of maybe 60 seconds. You could represent an initial long delay correctly and then switch to a continuous model once the tap changer starts moving. The other thing I might mention is that we did use the discrete model in the standard IEEE load models that we presented in San Francisco, and it is also in the EPRI ETMSP program.

Lester Fink (ECC, Inc.)

I would like to interject a question to Nick. Ordinarily I would address it to either Yakout or Taylor, but both of their systems are primarily hydro, so Nick I am addressing it to you. In all of this attention on the load, it seems to me that there is one rather critical portion of the load that has not been addressed, and that is the balance-of-plant in the thermal stations; the auxiliaries, fans, and pumps, and so forth, and how they react under off-normal conditions. Have you given any attention at all to this?

Miller: Yes, and indeed if you noticed in that example system, at hydro, nuclear, and thermal plants, and of course especially nuclear plants, the behavior of those auxiliaries is very important - as is making sure that we have the protection correct for taking the place out. We have a specific model that we put into these simulations of the auxiliaries, which is a one state model, a mixture of a big one state induction motor plus a bunch of static devices kind of lumped together in a cluster. We normally lump those right in with the machine. Indeed, that is a very important point, and we get heartburn whenever we see data sets that come into house where utilities have taken the generator, and used that feature in "the other company's" program that has the GSU lumped into the machine model, so that the machine looks like it is stuck right on the EHV bus, and there is no balance-of-plant load on the low side. It's bad news. So yes, it is very important.

Chair (Paul van Son, ECC, Europe)

If there are no other questions or comments related to the subjects of this morning, I have one in reserve for myself. It's more practical. We are very aware of the fact that it is extremely important whether you use the right algorithm, the right models, but also extremely important, as was men-

tioned by one of the speakers, is the accuracy of the data. I very often hear complaints from researchers and people that have to make stability studies that they are not able to get the right data from the field. So the question is, what is the experience in your utility, or in the area where you are working, with the data to fill the models and to make studies? How is that going normally? Who can tell me something about that, or is it no problem at all?

Ilic: This is really quoting a very popular engineer from the midwest who many know. He stated what he thought would be the most relevant problem (for utility people) for control people to solve, and this is the problem: if you have a black box that you know nothing about, how would you make it stable and control it? I think that question stated it about right - that in effect very little is known. This is why I said Thursday that parameter or model identification and data - I think we can't go further without that, even for research or for any technology transfer. It is a very important area. But I think it really puts utility people in a completely different situation, where they can't use a lot of stuff that we are talking about, and I didn't know what to make out of that question. It was again half joke, but there was a lot of truth to it.

Mansour: This is not a technical question but every day I want to propose this and I forget, so I want to make sure that it doesn't get lost. For the right reasons we started this symposium with a tribute to the memory of Professor Tamura. I want to propose to this symposium is to dedicate the Proceeding to the name of Professor Tamura, with his picture on the front. This proposal is to all the authors of the papers that were presented here, the discussors, and the organizers of the symposium.

Chair

Thank you very much for your very useful suggestion and I don't think there will be anyone to disagree.

Bulk Power System Voltage Phenomena - III
Voltage Stability, Security & Control
22-26 August 1944, Davos, Switzerland

Addendum

"Toward an Integrated Methodology for Voltage Stability Assessments
due to Slow Load Variations"

Hsiao-Dong Chiang, Alexander J. Flueck
Cornell University

(This paper was accepted, but not presented, due to absence of the author)

Toward An Integrated Methodology for Voltage Stability Assessments Due to Slow Load Variations

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Introduction

A power system is continually experiencing disturbances. These may be classified as : *event disturbances* and *load disturbances*. Event disturbances include generator outages, short-circuits caused by lightning, sudden large load changes, or a combination of such events. Event disturbances usually lead to a change in the configuration of the power system. Load disturbances, on the other hand, are the small random fluctuations in load demands. However, the practice of huge power wheeling should be viewed as a large load disturbance. A power system may become unstable after a disturbance. This instability may be angle-related instability, voltage-related instability or others.

Recently, the problem associated with voltage instability and/or collapse occurring in several power systems has attracted significant research efforts in the power industry, see for example [34,50]. Voltage collapse is generally caused by either of two types of system disturbances: load variations and contingencies (event disturbances). Among several examples of voltage collapse, the recent occurrence in Japan in 1987 [37] was due to load variations and the occurrence in Sweden in 1982 [56] was caused by a contingency. Incidentally, we prefer to explain the 'mechanism' proposed in [32] as an indirect mechanism.

Ideally, the voltage instability and/or collapse problem could be analyzed using an appropriate power system stability program on a comprehensive power system model. A recent work along this line appears in [22]. This straight-forward approach can indeed be used to assess any type of instability problems including the voltage-related problem. We feel that this approach has several merits as well as disadvantages: (i) it is directly applicable to any detailed level of power system model, (ii) all the information of state variables during transient as well as the steady-state period is available, (iii) simulation results are directly interpretable to system operators. The chief disadvantages with this approach

are: (i) it requires intensive computational effort, (ii) it does not provide information regarding the degree of stability or instability (when the system is subject to an event disturbance), (iii) it does not provide information regarding the load margin of instability (when the system is subject to a load disturbance), (iv) it does not provide information as to how to derive preventive control when the system is deemed unstable.

In this paper, we first attempt to present an integrated framework for addressing voltage-related instability problems. We present a rigorous definition of voltage stability and present two models to explain voltage collapse and one model to explain voltage instability due to slow load variations in power systems. It will be shown that one of the three models encompasses several existing models of explaining voltage collapse. The purpose of this framework is to facilitate the development of effective tools which can overcome the chief disadvantages mentioned above. We then present two specific but different tools for assessing voltage-related instability problems due to slow load variations for power systems with sufficient damping and apply them to large-scale power systems.

Classification and Definitions

As mentioned above, power system disturbances can be classified into event disturbances and load disturbances. Voltage stability or collapse can be caused by either of these two types of disturbances. For the purpose of dynamic voltage assessment, we further classify event disturbances into outage disturbances and short-circuit disturbances and load disturbances into slow load variations and fast load variations.

Dynamic voltage assessment due to a disturbance must be accompanied by an appropriate system model. For example, an extended power flow model is appropriate for voltage assessment due to load disturbances. This is because the network transfer capability and the system damping effects coming from generator control

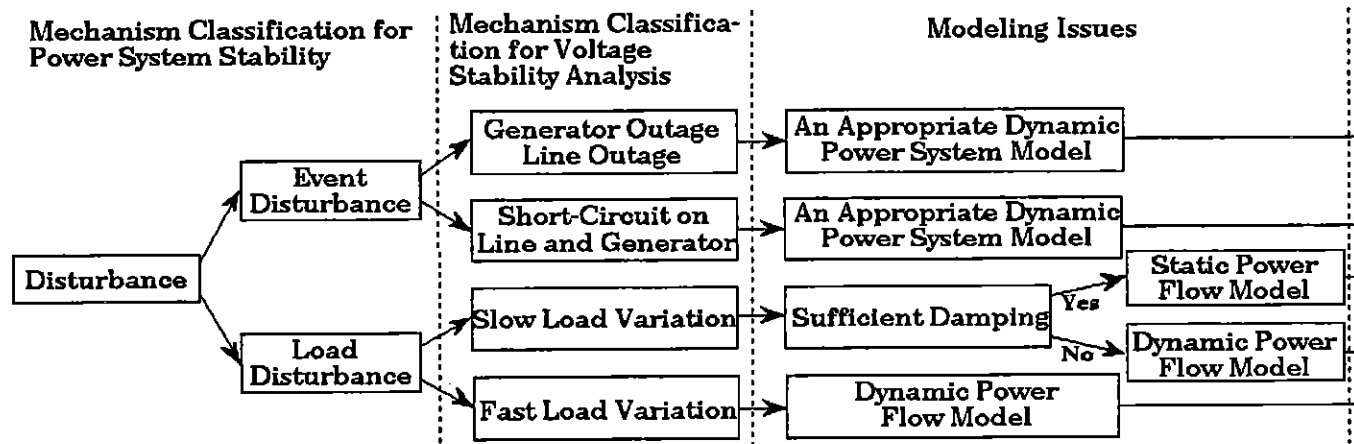


Figure 1: A chart of mechanism classifications for voltage stability analysis

devices and the network stiffness are important considerations for the analysis of the consequences of load disturbances. On the other hand, a comprehensive power system model such as the extended transient or mid-term model may be required for voltage assessment due to event disturbances (see Fig. 1).

Up to the present, there has been lacking a widely accepted and rigorous definition of voltage stability. The notion that a system is voltage stable at a given operating condition can be described by the following: if for every bus in the system, the bus voltage magnitude increases as the reactive power injection at the same bus is increased. Conversely, the following has been used by several researchers [32]: a system is voltage unstable if, for at least one bus in the system, the bus voltage magnitude decreases as reactive power injection at the same bus is increased. Calvaer [9,10] defined a bus to be voltage stable if the paralleling of a conductance results in both a real power increase and a voltage reduction and if the paralleling of a negative susceptance (inductive) results in both a reactive power increase and a voltage reduction. A system is voltage unstable if it includes at least one voltage bus that behaves in the opposite manner. In this paper, we propose the following definition of voltage stability limit: the voltage stability limit is a steady-state operating condition for which the power system is steady-state stable and the system voltage profile is within its permissible range but for which an arbitrarily small change in the operating point in an unfavorable direction causes the operating point to either lose stability or move into an unacceptable range. The definition is a slight extension of the steady-state stability limit presented in [35].

We now present a definition for voltage stability in

power systems:

Definition: A power system at a given operating condition is said to be *voltage stable* relative to a given disturbance if

- (i) the (post-disturbance) system trajectory approaches a stable steady-state whose voltage magnitudes are acceptable, and
- (ii) the transient voltage magnitudes of the trajectory are acceptable.

The steady-state stated in the above definition is very general; it can be an equilibrium point, or a limit cycle, or a quasi-periodic trajectory, or a strange attractor. The definition requires that voltage magnitudes along trajectories in the steady-state be acceptable. On the other hand, there are several types of voltage instability. We view voltage collapse as one of them.

Key Issues

The task of assessing dynamic voltage stability, due to either an event disturbance or a load disturbance, without resorting to the time-domain numerical integration approach is in general very difficult. Any attempt to do so must address the following four issues:

1. the existence of a stable steady-state after the contingency (static issue),
2. the feasibility (e.g., permissible voltage magnitudes) of the stable steady-state after the contingency (static issue),
3. an estimate of the stability region (region of attraction) of the stable steady-state after the contingency (dynamic issue), and

4. the voltage-dip problem occurring inside the stability region of the stable steady-state (dynamic issue)

In this paper, we focus on the case that the steady-state is an equilibrium point. Consequently, the above four issues become

1. the existence of a stable equilibrium point after the contingency (static issue),
2. the feasibility of the stable equilibrium point after the contingency (static issue),
3. an estimate of the stability region (region of attraction) of the stable equilibrium point after the contingency (dynamic issue), and
4. the voltage-dip problem occurring inside the stability region of the stable equilibrium point (dynamic issue)

The first two, related to the existence of an acceptable stable equilibrium point, are static issues. The third issue, related to the asymptotical behavior of the system trajectory, is concerned with governing the trajectory to the stable equilibrium point. The voltage "quality" during the transient behavior of the trajectory is the focus of the fourth issue. Apparently, the last two issues are dynamic issues. It should be pointed out that any static-based approach for solving the voltage problem due to contingencies in fact does not address the above dynamic issues (3) and (4) at all. This calls into doubt the complete effectiveness of the static-based approach. Actually, even when the underlying mathematical model is valid, a static-based approach may not be able to find (compute) a stable equilibrium point. Suppose it does, the approach just numerically finds a stable equilibrium point (in a mathematical sense). Whether or not the physical system (or the trajectory of the mathematical model) will settle down to the stable equilibrium point, found by the static-based approach, depends on (i) the initial condition of the system, and (ii) the size of the stability region of the stable equilibrium point after contingency. Several recent works, for example, [39,54,55,53] have attempted to address the dynamic issues (3) and (4).

In principle, the problem of voltage collapse due to contingencies can be investigated by using numerical integration techniques on an appropriate power system stability model. This straight-forward approach can address the above four issues directly, but at the cost of intensive computational effort and other disadvantages mentioned in the introduction.

To overcome the difficulty associated with the numerical integration approach, we propose to develop the following methods:

Static Issues

1. Develop a continuation-based method to compute a stable equilibrium point after the contingency, from which we can check its feasibility. The continuation-based method will provide the needed reliability and speed.
2. Develop an efficient eigenvalue computation method to check the stability property of an equilibrium point, which can also be used to detect the Hopf bifurcation to be discussed later.

Dynamic Issue (3) Develop an extended energy-based direct method to address the dynamic issue (3),

Dynamic Issue (4) Develop tools based on the notions of practical stability and partial stability to determine whether or not during the transient of the post-fault trajectory, the system voltage will dip to unacceptable values.

Conventional direct methods determine system (angular) stability directly based on an energy function [42,29,16,17]. These methods determine whether or not the system will remain stable once the fault is cleared by comparing the system energy (when the fault is cleared) to a calculated threshold value. Direct methods not only avoid the time-consuming calculations of step-by-step time-domain stability analysis of the post-fault system, but also provide a quantitative measure of the degree of system stability. This additional information makes direct methods very attractive when the relative stability of different plans must be compared or when stability limits must be calculated quickly. Furthermore, information regarding the degree of stability can be used to quickly determine critical clearing times for given disturbances.

Slow Load Variation

It will be shown in this section that when a power system is subject to a slow load variation, the dynamic issues (3) and (4) are usually satisfied. Suppose that a power system model is described by the specific form

$$\begin{aligned}\dot{x} &= f(x, y, \lambda) \\ 0 &= g(x, y, \lambda)\end{aligned}\quad (1)$$

where $\lambda \in R^1$ is a parameter, x is a dynamic state variable and y a static "state" variable, such as the load variables. Using terminology from the field of nonlinear dynamical systems, system (1) is a one-parameter dynamical system. In power system applications, a one-parameter dynamical system is a system together with one of the following conditions:

1. the real (or reactive) power demand at one load bus varies while the others remain fixed,
2. both the real and reactive power demand at a load bus vary and the variations can be parameterized. Again the others remain fixed,
3. the real and/or reactive power demand at some collection of load buses varies and the variations can be parameterized while the others are fixed.

The parameters in (1) are subject to variation and, as a result, changes may occur in the qualitative structure of the solutions of the static equation associated with (1). If this indeed occurs, the term bifurcation is usually used to describe such qualitative changes in the trajectory structure as the parameters of the system are varied. These changes are called *bifurcations* and the parameter values at which the bifurcations occur are called *bifurcation values*. We will focus on the so-called local bifurcation, which is concerned with the bifurcation of equilibrium points, or with situations where the problem can be cast into this form. The goal of local bifurcation theory is to investigate the nature of the static solutions as parameters vary. For example, a change in the number of solutions or in the stability properties of solutions may occur as the parameters vary, which may alter the dynamic behavior of (1). It is clear that the P-V and Q-V curves used in many utilities to analyze voltage collapse fall in the category of static models and therefore local bifurcation theory applies.

Power systems are normally operated near a stable equilibrium point. When system parameters are away from their bifurcation values and they change continuously but slowly, it is very likely that

- the stable equilibrium point changes position but remains a stable equilibrium point, and
- the old stable equilibrium point lies inside the stability region of the new stable equilibrium point.

Consequently, the dynamics starting from the old stable equilibrium point will converge to the new stable equilibrium point and will make the system state track its new stable equilibrium point, whose position is changed continuously but slowly. Hence, the dynamic issues (3) and (4) are satisfied when the system parameter varies slowly or quasi-statically with respect to the dynamics of (1).

The only typical ways in which system (1) may lose stability (under the influence of one parameter variation) are through the following:

- the stable equilibrium point $x_s(\lambda)$ and another equilibrium point $x_1(\lambda)$ coalesce and disappear in a saddle-node bifurcation as parameter λ varies, or

- the stable equilibrium point $x_s(\lambda)$ and an unstable limit cycle $x_1^l(\lambda, t)$ coalesce and disappear and an unstable equilibrium point emerges in a subcritical Hopf bifurcation as parameter λ varies,
- the stable equilibrium point $x_s(\lambda)$ bifurcates into an unstable equilibrium point surrounded by a stable limit cycle in a supercritical Hopf bifurcation as parameter λ varies.

In [24], it has been shown that for generic one-parameter dynamical systems, before a saddle-node bifurcation the equilibrium point $x_1(\lambda)$ is type-one. By type-one, we mean that the corresponding Jacobian matrix has exactly one eigenvalue with a positive real part and the rest of the eigenvalues have negative real parts. Furthermore, $x_1(\lambda)$ lies on the stability boundary of $x_s(\lambda)$. The Jacobian matrix, when evaluated at $x_s(\lambda)$, has all of its eigenvalues with only negative real parts and among them, one of the eigenvalues is close to zero. At the bifurcation occurring at say, the bifurcation value $\lambda = \lambda^*$, equilibrium points $x_s(\lambda)$ and $x_1(\lambda)$ coalesce to form an equilibrium point x^* ($= x_s(\lambda^*) = x_1(\lambda^*)$). The Jacobian matrix evaluated at x^* has one zero eigenvalue and the real parts of all the other eigenvalues are negative. If the parameter λ increases beyond the bifurcation value λ^* , then x^* disappears and there are no other equilibrium points nearby.

It can be shown that for generic one-parameter dynamical systems, before a subcritical Hopf bifurcation, the unstable limit cycle $x_1^l(\lambda, t)$ lies on the stability boundary of $x_s(\lambda)$. The Jacobian matrix, when evaluated at $x_s(\lambda)$, has all of its eigenvalues with only negative real parts and among them, a pair of complex eigenvalues are close to zero. At the bifurcation occurring at say, the bifurcation value $\lambda = \lambda^*$, the equilibrium point $x_s(\lambda)$ and the unstable limit cycle $x_1^l(\lambda, t)$ coalesce to form an equilibrium point x^* ($= x_s(\lambda^*) = x_1^l(\lambda^*)$). The Jacobian matrix evaluated at x^* has two zero eigenvalues and the real parts of all the other eigenvalues are negative. If the parameter λ increases beyond the bifurcation value λ^* , then x^* becomes a type-two equilibrium point and there are no other equilibrium points or limit cycles nearby.

As for the supercritical Hopf bifurcation, we make the following remarks. Before the bifurcation, The Jacobian matrix, when evaluated at $x_s(\lambda)$, has all of its eigenvalues with only negative real parts and among them, a pair of complex eigenvalues are close to zero. At the bifurcation occurring at say, the bifurcation value $\lambda = \lambda^*$, the equilibrium point $x_s(\lambda)$ is an equilibrium point x^* ($= x_s(\lambda^*)$). The Jacobian matrix evaluated at x^* has two zero eigenvalues and the real parts of all the other eigenvalues are negative. If the parameter λ increases beyond the bifurcation value λ^* , then x^* becomes a un-

stable equilibrium point $x^*(\lambda)$ surrounded by a stable limit cycle.

After a bifurcation occurs, the system state will evolve according to the dynamics of (1). The dynamics after bifurcation determine whether the system remains stable or becomes unstable; and the type of system instability. We analyze the dynamics after a typical local bifurcation in the next section.

Dynamics After a Local Bifurcation

In [24], Dobson and Chiang investigated a generic mechanism leading to disappearance of stable equilibrium points due to a saddle-node bifurcation and the subsequent system dynamics for one-parameter dynamical systems. A voltage collapse model was suggested based on this analysis [11]. This section briefly summarizes the model for analyzing the dynamics after a saddle-node bifurcation and slightly extends the model. Models for analyzing the dynamics after Hopf bifurcations will be also developed in this section.

Saddle-node Bifurcation

When a saddle-node bifurcation occurs at say, the bifurcation value $\lambda = \lambda^*$, equilibrium points $x_s(\lambda)$ and $x_1(\lambda)$ coalesce to form an equilibrium point $x^* (= x_s(\lambda^*) = x_1(\lambda^*))$. The Jacobian matrix evaluated at x^* has one zero eigenvalue and the real parts of all the other eigenvalues are negative. The eigenvector p that corresponds to the zero eigenvalue points in the direction along which the two vectors $x_s(\lambda)$ and $x_1(\lambda)$ approached each other. There is a curve made up of system trajectories which is tangent to eigenvector p at x^* . This curve is called the *center manifold* of x^* and is the union of a system trajectory W_-^c converging to x^* , the equilibrium point x^* and a system trajectory W_+^c diverging from x^* . Next, we consider the case that λ remains fixed at bifurcation value λ^* . Recall that before the bifurcation occurs, the system state is tracking its stable equilibrium point. Therefore, at the moment the bifurcation occurs, the system state is in a neighborhood of x^* . Hence, if the system trajectory is near W_+^c at the moment that the bifurcation occurs and if λ remains fixed at its bifurcation value λ^* , then the system trajectory after the bifurcation moves near W_+^c . The system dynamics due to the bifurcation are then determined by the position of W_+^c in state space. If W_+^c is positioned so that some of the voltage magnitudes decrease along W_+^c , then we associate the movement along W_+^c with voltage collapse. This is the center manifold voltage collapse model due to saddle-node bifurcation. This model has two advantages from a computational point of view.

1. Since p is tangent to W_+^c at x^* , the initial direction of W_+^c near x^* is determined by p which can be

computed from the Jacobian matrix at x^* .

2. Since W_+^c is a system trajectory, the dynamics of voltage collapse can be predicted by integrating system equations (1) starting on W_+^c near x^* .

Subcritical Hopf bifurcation

When a Subcritical Hopf bifurcation occurs at say, the bifurcation value $\lambda = \lambda^*$, the stable equilibrium point $x_s(\lambda)$ and a unstable limit cycle $x_1^l(\lambda, t)$ coalesce to form an equilibrium point $x^* (= x_s(\lambda^*))$. The Jacobian matrix evaluated at x^* has two zero eigenvalues and the real parts of all the other eigenvalues are negative. The subspace spanned by the two eigenvectors p_1 and p_2 that corresponds to the two zero eigenvalues points in the direction along which $x_s(\lambda)$ and $x_1^l(\lambda, t)$ approached each other. The subspace is the center manifold of x^* , say W^c . Next, we consider the case that λ remains fixed at bifurcation value λ^* . Recall that before the bifurcation occurs, the system state is tracking its stable equilibrium point. Therefore, at the moment the bifurcation occurs, the system state is in a neighborhood of x^* . Hence, if the system trajectory is near W^c at the moment that the bifurcation occurs and if λ remains fixed at its bifurcation value λ^* , then the system trajectory after the bifurcation moves along W^c . The system dynamics due to the bifurcation are then determined by the position of W^c in state space. If W^c is positioned so that some of the voltage magnitudes decrease along W^c , then we associate the movement along W^c with voltage collapse. This is the center manifold voltage collapse model due to subcritical Hopf bifurcation.

Supercritical Hopf bifurcation

When a Supercritical Hopf bifurcation occurs at say, the bifurcation value $\lambda = \lambda^*$, the equilibrium point $x_s(\lambda)$ is an equilibrium point $x^* (= x_s(\lambda^*))$. The Jacobian matrix evaluated at x^* has two zero eigenvalues and the real parts of all the other eigenvalues are negative. If the parameter λ increases beyond the bifurcation value λ^* , then x^* becomes a unstable equilibrium point $x^*(\lambda)$ surrounded by a stable limit cycle, say $x_2^l(\lambda, t)$.

Next, we consider the case that λ changes slowly after the bifurcation value λ^* and that at the moment the bifurcation occurs, the system state is in a neighborhood of x^* . Note that it is very likely that the system state will lie inside the stability region of the stable limit cycle $x_2^l(\lambda, t)$, making the system trajectory attracted to $x_2^l(\lambda, t)$. Thus, the system dynamics after a Supercritical Hopf bifurcation moves along the stable limit cycle $x_2^l(\lambda, t)$ until another bifurcation occurs on $x_2^l(\lambda, t)$ at another bifurcation value. So, the system dynamics due to the bifurcation are then determined by the position of the stable limit cycle $x_2^l(\lambda, t)$ in state space. If

$x_2^l(\lambda, t)$ is positioned so that some of the voltage magnitudes oscillate along the stable limit cycle $x_2^l(\lambda, t)$ and become unacceptable, then we associate the movement with voltage instability. This is the *limit cycle voltage instability model* due to the supercritical Hopf bifurcation.

In summary, after a saddle-node or Hopf bifurcation occurs, the system state will evolve according to the system dynamics as described above. The dynamics after bifurcation determine whether the system remains stable or becomes unstable, and the type of system instability. The analysis of a typical bifurcation of a stable *equilibrium point* in a power system with slowly varying parameters has two parts:

1. Before the bifurcation when the quasi-static model applies
2. After the bifurcation when the corresponding dynamic model applies

We stress that the static model is not applicable after the bifurcation and cannot be used to explain the dynamical behaviors after the bifurcation.

Extension

We now are in a position to extend the center manifold voltage collapse model due to either the saddle-node or Hopf bifurcation to general slow time-varying one-parameter dynamical systems. In particular, we eliminate the assumption that the system parameter “freezes” at the bifurcation point of interest and the system model (1) becomes, without loss of generality, the following form (a frozen system)

$$\begin{aligned}\dot{x} &= f(x, \lambda) \\ \dot{\lambda} &= 0\end{aligned}\tag{2}$$

The system dynamics after the bifurcation are then captured by the center manifold of the bifurcation point x^* with respect to (2). The assumption that the parameter λ freezes at a bifurcation point may be inadequate to reflect real power system behavior. A more realistic model is to allow a slowly time-varying parameter both before and after the bifurcation;

$$\begin{aligned}\dot{x} &= f(x, \lambda) \\ \dot{\lambda} &= \epsilon g(x, \lambda)\end{aligned}\tag{3}$$

where ϵ is a small number and g is a locally Lipschitz function.

In order to extend the voltage collapse model to system (3), it will be necessary to examine the adequacy of the center manifold of x^* with respect to system (2)

in capturing the dynamics after a bifurcation. In this regard, we examine the relationship between the trajectories of (2) and (3) which start at the same point near the center manifold of x^* . It will be shown that system trajectories of (3) follow the the system trajectories of (2) during each choice of time interval provided the rate of parameter variation is sufficiently small.

Proposition 1:

Let $x_1(t)$, $x_2(t)$ be the system trajectories of (2) and of (3) starting from the initial condition (x_1^0, t_0) and (x_2^0, t_0) respectively. Let U be a compact region of the state space containing the center manifold as far as it is of interest, and let K be the Lipschitz constant of (3) on U . If $x_1^0 = x_2^0 \in U$, then $\|x_1(t) - x_2(t)\| \leq \frac{\epsilon M}{K} [e^{K(t-t_0)} - 1]$, where M is a constant.

This result shows that the behavior of (2) approximates the behavior of (3) if ϵ is small enough. Consequently, when λ reaches the bifurcation value, we may approximate the system dynamics after this bifurcation point by applying (2) instead of (3). In particular, $x_2(t)$ can be made to track $x_1(t)$ arbitrarily closely for each choice of time interval, by choice of ϵ . As noted before, if the state of the system lies near the center manifold W_c^+ for the saddle-node bifurcation (or W^c for the subcritical Hopf bifurcation) at the time of bifurcation, then according to (2) which assumes that the parameter remains fixed, the state will subsequently approach a trajectory along W_c^+ (or W^c). The proper system dynamics after bifurcation are governed by (3), but Proposition 1 shows that these dynamics may be approximated by (2). Therefore, if the parameter changes slowly enough, then solutions of (3) which lie near the center manifold of (2) at the time of bifurcation, will subsequently track the center manifold trajectory of (2) in the state space. This analysis validates the center manifold model for voltage collapse in power systems with a slowly changing load.

Voltage Collapse Models

This section contains two center manifold models to explain voltage collapse in power systems due to load variations. Recall that “slow load variations” means the dynamics of load variations are relatively slower than the dynamics occurring in the state vector. The first model, called the saddle-node voltage collapse model (hereafter called the SAD voltage collapse model) was proposed in [23] which addresses power system behaviors due to slow load variations in three stages. The second model, to be presented in this section, is based on the center manifold voltage collapse model due to subcritical Hopf bifurcation, developed in the previous section.

Two Models

Stage 1: the system is in quasi-steady state and is tracking a stable equilibrium point.

Stage 2: the system reaches its "steady-state" stability limit when the stable equilibrium point undergoes a saddle-node bifurcation.

Stage 3: the system dynamics after bifurcation are captured by a center manifold trajectory.

Stage 1 is related to the feasibility of "load flow" solutions (i.e., the existence of a system operating point in a feasible region). Stage 2 determines the steady-state stability limit based on the saddle-node bifurcation. We point out that the widely used tools of P-V curve and Q-V curve in the power industry to prevent voltage collapse are typical examples involving a saddle-node bifurcation. Stage 3 describes the system dynamical behavior after bifurcation to assess whether the system, after bifurcation, remains stable or becomes unstable; and determines the types of system instability (voltage collapse and/or angle instability). Hence, the voltage collapse model describes both the static aspect (stages 1 and 2) and the dynamic aspect (stage 3) of the problem.

It can be shown that for general power system models, the saddle-node bifurcation is inevitable when the load is increased in the form of constant powers while the Hopf bifurcation may not occur. If it occurs, it must occur before the saddle-node bifurcation (see numerical examples in [4,15,1] and also Fig. 2). However, the Hopf bifurcation will not occur when the power system under study has sufficient damping. Thus, the following model is useful to analyze voltage collapse only when the system contains insufficient damping.

Hopf Voltage Collapse Model

Stage 1: the system is in quasi-steady state and is tracking a stable equilibrium point.

Stage 2: the system reaches its steady-state stability limit when the stable equilibrium point undergoes a subcritical Hopf bifurcation.

Stage 3: the system dynamics after bifurcation are captured by a two-dimensional center manifold.

The Hopf voltage collapse model also describes both the static aspect (stages 1 and 2) and the dynamic aspect (stage 3) of the problem. This model dictates that mathematically speaking, the steady-state stability limit may be determined by the subcritical Hopf bifurcation, instead of the Saddle-node bifurcation.

Abed et al. in [1] use numerical simulations to il-

lustrate the possibility of voltage collapse due to Hopf bifurcation. One implication is that the load margins will be less than one might expect if the nose point were taken as the point of voltage collapse [1]. Since detecting Hopf bifurcation requires the knowledge of the eigenvalues of the system Jacobian, the traditional repetitive power flow approach cannot detect Hopf bifurcation.

Analysis

We next show that the SAD voltage collapse model encompasses many existing models used to explain voltage collapse such as the multiple load flow model, the load flow feasibility model, the static bifurcation model, the singular Jacobian model and the system sensitivity model. Indeed, Stage 1 is related to the feasibility of the "load flow" solution [30,31]. It will be shown that stage 2 itself is a generalization of many existing models used to explain voltage collapse.

From an algebraic point of view, the point $(x(\lambda_0), \lambda_0)$ abbreviated by (x_0, λ_0) is a saddle node of (1) if the following conditions hold:

1. $f(x_0, \lambda_0) = 0$.
2. $f_x(x_0, \lambda_0)$ has a simple eigenvalue 0.
3. $f_\lambda(x_0, \lambda_0) \notin \text{Range space of } (f_x(x_0, \lambda_0))$.
4. there is a parameterization $(x(t), \lambda(t))$ with $x(t_0) = x_0, \lambda(t_0) = \lambda_0$ and $\frac{d^2\lambda(t_0)}{dt^2} \neq 0$.

We now use the above algebraic characterizations to examine several models proposed by several researchers for predicting voltage collapse. Note that the voltage collapse models based on the determinant of system Jacobian in [52,45] can be viewed as providing necessary conditions for the first two conditions for saddle nodes. We show in the following that the model based on the sensitivity of system Jacobian in [44,18] also provides a necessary condition for the first two conditions of saddle nodes. More specifically, we show that the sensitivity of state vector with respect to its parameter at a saddle node is infinity. Suppose that (x^*, λ^*) is a solution of (1) and $f_x(x^*, \lambda^*)$ is non-singular, the implicit function theorem guarantees the existence of a unique solution curve $(x(\lambda), \lambda)$ passing through (x^*, λ^*) , i.e. $x^* = x(\lambda^*)$. And we have the following identity

$$\frac{dx(\lambda)}{d\lambda} = -f_x(x(\lambda), \lambda)^{-1} f_\lambda(x(\lambda), \lambda) \quad (4)$$

Although the matrix $f_x(x(\lambda), \lambda)^{-1}$ does not exist at the saddle node $(x(\lambda^0), \lambda^0)$, the property that $f_\lambda(x_0, \lambda_0) \notin \text{Range space of } (f_x(x_0, \lambda_0))$ ensures that there is a unique solution curve passing through $(x(\lambda^0), \lambda^0)$ and $\frac{dx(\lambda)}{d\lambda}$ becomes infinite there and a small change in λ

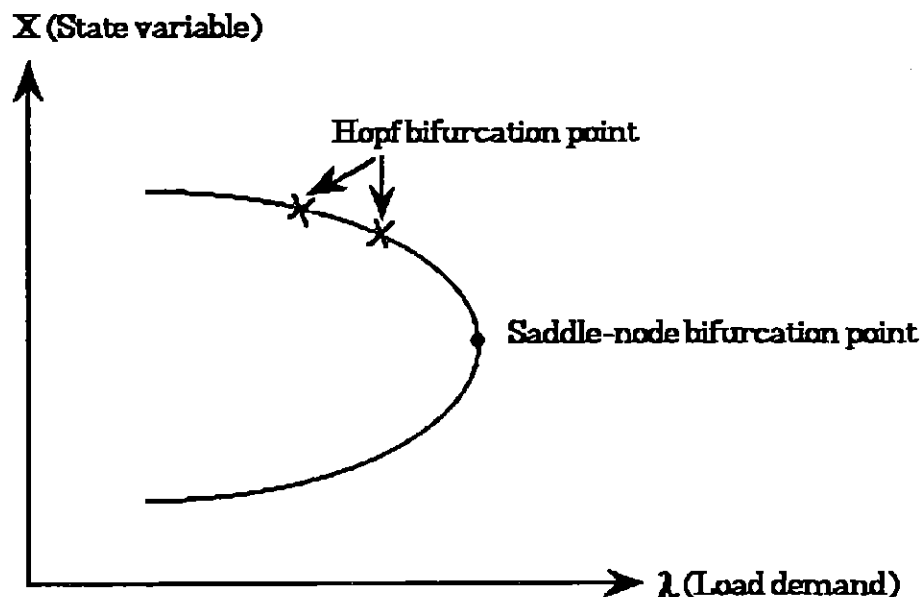


Figure 2: Hopf bifurcation, if it occurs, will be encountered before saddle-node bifurcation.

yields a large change in $\|x\|$. This result also explains the “knee” phenomenon in the P-V curve and Q-V curve.

The voltage collapse model based on multiple load flow solutions [49,46] uses the presumption that the existence of a pair of very close load flow solutions indicates that the system is about to undergo a voltage collapse. We note that before the bifurcation, there are two equilibrium points (load flow solutions) close to each other. One is stable and the other is type-one. As these two points approach each other an annihilation occurs at the saddle node bifurcation, while at the same time the system Jacobian becomes singular.

In summary, stages 1 and 2 of the SAD voltage collapse model encompass many existing voltage collapse models such as the multiple load flow solutions model, the load flow feasibility model, the static bifurcation model, the singular Jacobian model and the system sensitivity model. Stage 3 of the model goes beyond the static aspect of the problem to describe the dynamic aspect of the problem.

A Voltage Instability Model

This section presents a model to explain voltage instability in power systems due to slow load variations. The model complements the two center manifold voltage collapse models developed in the previous section.

Hopf Voltage Instability Model

Stage 1: the system is in quasi-steady state and is tracking a stable equilibrium point.

Stage 2: the system reaches its steady-state stability limit when the stable equilibrium point undergoes a supercritical Hopf bifurcation.

Stage 3: the system dynamics after bifurcation are captured by a stable limit cycle.

The voltage instability model also describes both the static aspect (stages 1 and 2) and the dynamic aspect (stage 3) of the problem. From a computational viewpoint, the modal analysis method presented in [32] can be applied to Stage 1 and 2, but not to Stage 3 of both the Hopf voltage collapse model and the Hopf voltage instability model. In this regard, we prefer to use a trajectory on the stable limit cycle (a nonlinear trajectory) to identify areas vulnerable to voltage instability instead of the eigenvectors suggested in [32].

Physically, the supercritical Hopf bifurcation point represents the operating point at which low-frequency oscillation occurs. Again, the traditional repetitive power flow approach cannot detect this supercritical Hopf bifurcation, since this requires the knowledge of the eigenvalues of the system Jacobian.

A Practical Performance Index

A number of performance indices intended to measure the severity of the voltage collapse problem have been proposed in the literature. Among them, the minimum singular value in [51], later pursued in [40], the eigenvalue pursued in [32] and the condition number in

[41] of the system Jacobian intend to provide some measure of how far the system is away from the point at which the system Jacobian becomes singular. The performance index proposed in [48] and [49] is based on the angular distance between the current stable equilibrium point and the closest unstable equilibrium point in a Euclidean sense. The performance index proposed in [20,21] measures the energy distance between the current stable equilibrium point and the closest unstable equilibrium point using an energy function. These performance indices can be viewed as providing some measure of the "distance" between the current operating point and the bifurcation point. Note that all these performance indices are defined in the state space of power system models instead of in the parameter space. Thus, these performance indices cannot directly answer questions such as: "Can the system withstand a 100 MVar increase on bus 11?" or: "Can the system withstand a simultaneous increase of 70 MW on bus 2 and 50 MVar on bus 6?"

One basic requirement for useful performance indices is their ability to reflect the degree of direct mechanism leading the underlying system toward an undesired state. In the context of voltage collapse in power systems, a useful performance index must have the ability to

1. measure the amount of load increase that the system can tolerate before collapse (when the underlying mechanism of collapse is due to load variations), or
2. assess whether the system can sustain a contingency without collapse (when the underlying mechanism of collapse is caused by a contingency) and measure the severity of the contingency.

The existing performance indices, however, generally do not exhibit any obvious relation between their value and the amount of the underlying mechanism that the system can tolerate before collapse. We assert that, in order to provide a direct relationship between its value and the amount of load increases that the system can withstand before collapse, the performance index for assessing the load margins before collapse must be developed in the parameter space (i.e., the space of load demands) rather than the state space where the existing performance indices were developed.

Determination of margins in the parameter space is still a new concept which may have been spurred by the concept presented in [23] and pursued in [25,26]. An iterative method based on sensitivity techniques for calculating load margins (to the saddle-node bifurcation value) was proposed in [28]. In [19], a method was proposed to compute the reactive power margin. The method utilizes a general optimization technique

attempting to compute the saddle-node bifurcation point by directly maximizing the reactive load that the system can supply before collapse. Another method which solves an extended $(2n+1)$ -dimensional system of equations characterizing the saddle-node bifurcation point was proposed in [5] and more recently in [6]. The methods attempt to compute the saddle-node bifurcation point directly. The success of the above two methods depends greatly on a good initial guess of the desired saddle-node bifurcation point. Otherwise, the methods may diverge or converge to an undesired saddle-node bifurcation point as explained in the following. Suppose that, we are given a power system with an initial load demand pattern and a load increase pattern which can be parameterized by a parameter, say λ , such that the initial load demand pattern can be expressed as λ_0 . Suppose that, corresponding to the load level λ_0 , there are multiple power flow solutions, say six solutions; some of them are stable some unstable and the power system is operated at the stable equilibrium point $x_3(\lambda_0)$ (See Fig. 3). As the load demand changes slowly, these six equilibrium points initially change positions but preserve their types of stability (or instability). As the load demand increases up to a load level, say at λ_3 , a pair of these equilibrium points coalesce and disappear in a saddle-node bifurcation. Likewise, another pair of these equilibrium points coalesce at the saddle-node bifurcation point x_d^2 when the load demand keeps on increasing to another load level, say at λ_2 (See Fig. 4). Finally, the pair of equilibrium points $x_1(\lambda)$ and $x_4(\lambda)$ coalesce at the saddle-node bifurcation point x_d^1 when the load parameter value is λ_1 . In this example, the load margin is $\lambda_2 - \lambda_0$ when the system is operated at $x_3(\lambda_0)$ and the load parameter λ varies slowly or quasi-statically with respect to the system dynamics. On the other hand, the load margin is $\lambda_1 - \lambda_0$ when the system is operated at $x_1(\lambda_0)$. Now, using the methods in [19,5] to estimate the load margin when the system is operated at $x_3(\lambda_0)$ may end up with a margin $\lambda_3 - \lambda_0$, a conservative estimation (in which case these methods converge to an incorrect saddle-node bifurcation point x_d^3) or with a margin $\lambda_1 - \lambda_0$, an over-estimation (in which case these methods converge to an incorrect saddle-node bifurcation point x_d^1) (See Fig. 5). The reasons that these approaches give incorrect load margins are that they are static in nature and that they do not make use of any information on the particular branch ($x_3(\lambda) - x_d^2 - x_5(\lambda)$) and they do not confine their calculations to this branch.

In [12], we have developed a new performance index that provides a direct relationship between its value and the amount of load demand that the system can withstand before a saddle-node voltage collapse. This index makes use of the information contained in the power flow solutions of the particular branch of interest. This performance index is particularly useful for power systems

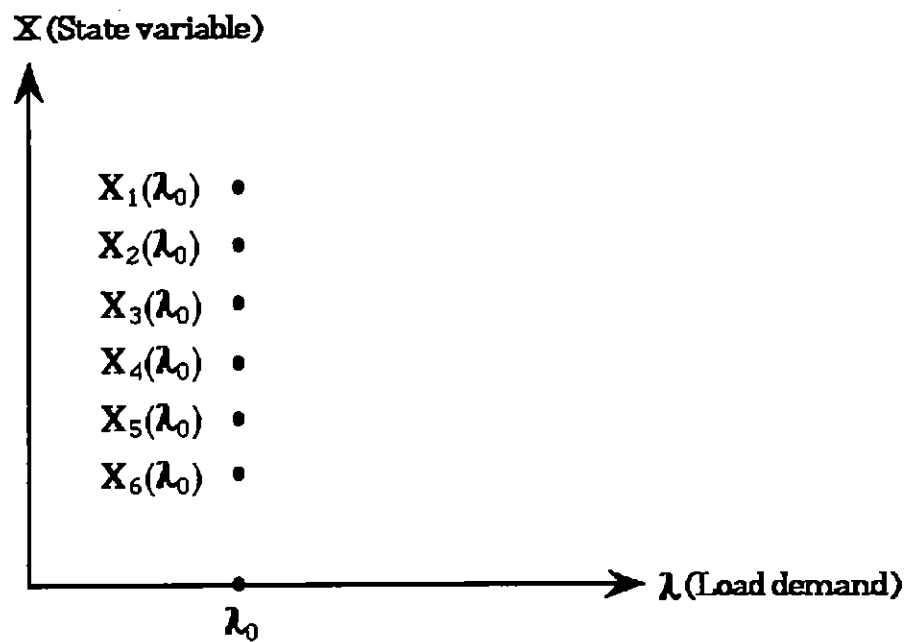


Figure 3: When the load level is λ_0 , there are six power flow solutions; some of them are stable some unstable.

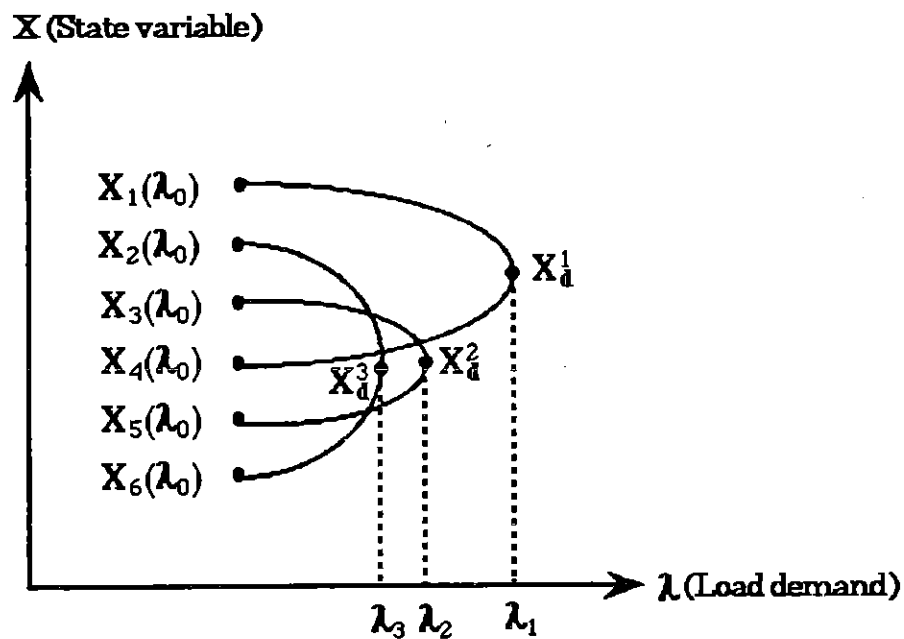


Figure 4: As the load demand increases, a pair of these equilibrium points coalesce and disappear in a saddle-node bifurcation at the load level values λ_3 , λ_2 , λ_1 respectively

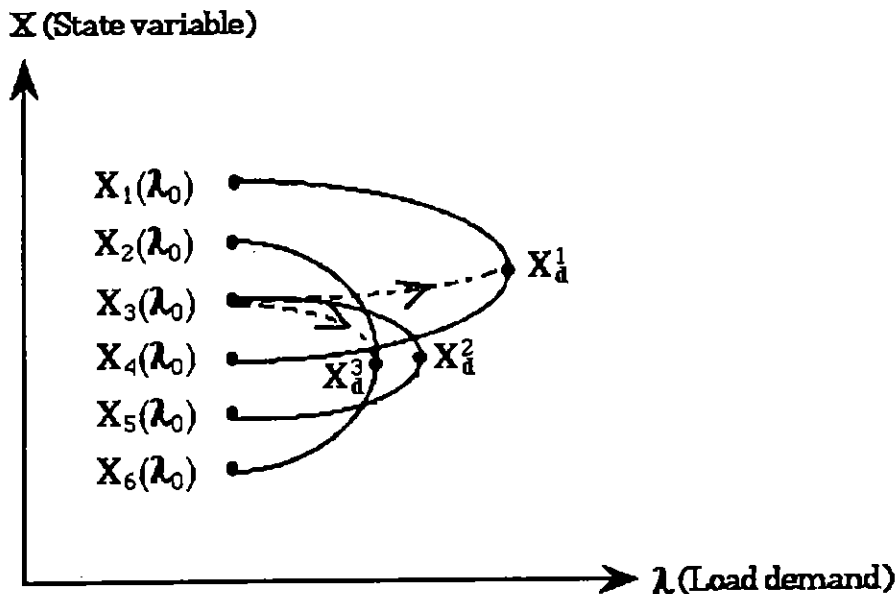


Figure 5: Using $x_3(\lambda_0)$ as an initial condition, the methods may converge to, instead of the correct bifurcation point x_d^2 , an incorrect saddle-node bifurcation point x_d^3 or x_d^1 .

with sufficient system damping. A comparison between the performance index and the existing ones is shown in Table 1. One of the features that distinguishes the proposed performance is its development in the load-demand space and its ability to answer questions such as: "Can the system withstand a simultaneous increase of 70 MW on bus 2 and 50 MVar on bus 6?". Moreover, the computation involved in the performance index is relatively inexpensive in comparison with those required in the existing ones. However, the performance index must take into account the physical limitations of typical control devices such as generator VAR limits and ULTC tap ratio limits. An improved fast performance index that accounts for all control devices (currently modeled in CPFLOW) and their physical limitations is the subject of ongoing research.

CPFLOW

Introduction

CPFLOW (Continuation Power Flow) is a comprehensive tool for tracing power system steady-state behavior due to load and generation variations [13]. In particular, CPFLOW can generate P-V, Q-V, and P-Q-V curves with the capability that the controlling parameter λ can be one of the following

- general bus (P and/or Q) loads + real power generation at P-V buses
- area (P and/or Q) loads + real power generation at P-V buses

- system (P and/or Q) loads + real power generation at P-V buses

CPFLOW, computationally based on the continuation method [47], can trace the power flow solution curve, with respect to any of the above three varying parameter, through the "nose" point (i.e. the saddle-node bifurcation point) without any numerical difficulty. The continuation methods have been used by several researchers to trace the power flow solution curve [14,33,3,7]. The main differences between the previous work and CPFLOW lie in (i) the level of modeling capability, (ii) the level of applicability (through different schemes of parameterizations), (iii) the computational speed and reliability (through different predictors, correctors and step-size control).

The main advantages of CPFLOW over repetitive power flow calculations are the following:

• Computation

1. it is more reliable than the repeated power flow approach in obtaining the solution curve; especially for ill-conditioned power flow equations
2. it is faster than the repeated power flow approach via an effective predictor-corrector, adaptive step-length selection algorithm and efficient I/O operations.

- Function – it is more versatile than the repeated power flow approach via parameterizations such that general bus real and/or reactive loads, area

	The existing indecies	The proposed one
framework	state space	load-demand space
amount of load increase before collapse	indirect relationship	direct relationship
operators	?	practical information
preventive control	?	how to derive load-shedding schemes
computational cost	more	less

Table 1: A comparison between the existing ones and the proposed performance index

real and/or reactive loads, or system-wide real and/or reactive loads, and real generation at P-V buses (e.g. determined by economic dispatch or participation factor) can vary.

CPFLOW can be used in a variety of applications such as (1) to analyze voltage problems due to load and/or generation variations (e.g. voltage dip, voltage collapse), (2) to evaluate maximum interchange capability and maximum transmission capability, (3) to simulate power system static behavior due to load and/or generation variations with/without control devices, and (4) to conduct coordination studies of control devices for steady-state security assessment.

CPFLOW uses EPRI IPFLOW [8] as a platform; making its modeling capability quite comprehensive. It can accept power flow data in the format of EPRI, BPA, IEEE, PECO, PTI, etc. The current version of CPFLOW can handle power systems up to 12000 buses. For an illustrative purpose, CPFLOW is applied to a 39-bus, a 234-bus and a 3493-bus power system with a comprehensive set of operational limits and controls.

System Model

CPFLOW has the following modelling capability:

- Generators
 - controlling the voltage between bounds
 - reactive power limits
 - real power generations due to participation factors
- Loads
 - constant power, constant current, constant impedance, or combination of them.
 - nonlinear (static load model)
- Switchable shunts and static VAR compensators
- Interchange schedules
- ULTC Transformers

- ULTC phase shifters
- Static tap changers and phase shifters
- DC transmission lines

Tracing the Solution Curve

Consider a comprehensive (static) power system model, as described in the previous section, expressed in the following form:

$$0 = f(x, \lambda) \quad (5)$$

where $\lambda \in R^1$ is a (controlling) parameter subject to variation. Using terminology from the field of nonlinear dynamical systems, system (5) is a one-parameter nonlinear system.

We next discuss an indirect method to simulate the approximate behavior of the power system (5) due to load and/or generation variation. Before reaching the “nose” point, the power system with a slowly varying parameter traces its operating point which is a solution of the following equation whose corresponding Jacobian has all eigenvalues with negative real parts:

$$f(x, \lambda) = 0, \quad x \in R^n, f \in R^n, \lambda \in R \quad (6)$$

These n equations of $n + 1$ variables define in the $n + 1$ -dimensional space a one-dimensional curve $x(\lambda)$ passing through the operating point of the power system (x^0, λ^0) . The indirect method is to start from (x^0, λ^0) , and produce a series of solution points (x^i, λ^i) in a prescribed direction, determined by participating load and generation variations, until the “nose” point is reached. However, it is known that the set of power flow equations near its “nose” point is ill-conditioned, making the Newton method diverge in the neighborhood of “nose” points. There are several possible means to resolve the numerical difficulty arising from the ill-conditioning. One effective way, adopted in CPFLOW, is as follows: First, treat the parameter λ as another state variable

$$x_{n+1} = \lambda.$$

Second, introduce the arclength s on the solution curve as a new parameter in the continuation process. This parameterization process gives

$$x = x(s), \quad \lambda = \lambda(s) = x_{n+1}.$$

The step-size along the arclength s yields the following constraint:

$$\sum_{i=1}^n \{(x_i - x_i(s))^2\} + (\lambda - \lambda(s))^2 - (\Delta s)^2 = 0$$

Third, solve the following $n + 1$ equations for the $n + 1$ unknowns x and λ

$$f(x, \lambda) = 0 \quad (7)$$

$$\sum_{i=1}^n \{(x_i - x_i(s))^2\} + (\lambda - \lambda(s))^2 - (\Delta s)^2 = 0 \quad (8)$$

It can be shown that the above set of augmented power flow equations is well-conditioned, even at the "nose" point. CPFLOW solves these augmented power flow equations to obtain the solution curve passing through the "nose" point without encountering the numerical difficulty of ill-conditioning.

Continuation Methods

Continuation methods, sometimes called curve tracing or path following methods, are useful tools to generate solution curves for general nonlinear algebraic equations with a varying parameter. CPFLOW uses continuation methods to trace power system steady-state behavior due to load and generation variation.

The theory of continuation methods has been studied extensively and has its roots in Algebraic Topology and Differential Topology. Continuation methods have four basic elements:

- Parameterization
- Predictor
- Corrector
- Step-size control

Parameterization

Parameterization is a mathematical way of identifying each solution on the solution curve so that "next" solution or "previous" solution can be quantified. There are three different types of parameterizations:

1. physical parameterization using the controlling parameter λ , in which case the step length is $\Delta\lambda$,
2. local parameterization, which uses either the controlling parameter λ or any component of the state vector x ; namely x_k to parameterize the solution curve. The step length in the local parameterization is $\Delta\lambda$ or Δx_k , and

3. arclength parameterization employing the arclength along the solution curve to perform parameterization, the step length in this case is Δs ;

$$\Delta s = \left\{ \sum_{i=1}^n (x_i - x_i(s))^2 + (\lambda - \lambda(s))^2 \right\}^{0.5}$$

The so-called pseudo arclength parameterization uses different weighting factors (instead of an equal weighting factor) in the above equation.

While using the controlling parameter to parameterize the solution curve has physical significance, it encounters numerical difficulties in the vicinity of a saddle-node bifurcation point. In order to resolve this issue and to design an effective predictor, CPFLOW utilizes the arclength parameterization.

Predictor

The purpose of the predictor is to find an approximation point for the next solution. Suppose we are at the i -th step of the continuation process and the i -th solution (x^i, λ^i) of (6) has been found. The predictor attempts to find an approximation point for the next solution (x^{i+1}, λ^{i+1}) . The quality of the approximation point by a predictor significantly affects the number of iterations required by a corrector in order to obtain an exact solution. A better approximation point yields a fewer number of iterations needed in a corrector to reach the solution. Several different predictors have been proposed in the literature of numerical analysis. They can be divided into two classes: (1) ODE based methods, which use the current solution and its derivatives to predict the next solution. The tangent method, a popular one as a predictor, is a first-order ODE-based method; (2) polynomial extrapolation based methods; which use only current and previous solutions to find an approximated solution. The secant method, a popular polynomial-based predictor, uses the current solution and the previous one to predict the next one. In order to achieve computational efficiency, CPFLOW employs the tangent method in the first phase of solution curve tracing and the secant method in the second phase. However, if the number of corrector iterations becomes too large, then the predictor switches back to the tangent since it is more accurate than the secant.

Tangent method

The Tangent method calls for the calculation of the derivatives of $x_1, x_2, \dots, x_n, x_{n+1}$ with respect to the arclength s

$$\frac{dx_1}{ds}, \dots, \frac{dx_n}{ds}, \frac{dx_{n+1}}{ds}$$

To find these derivatives, differentiate both sides of equation (6) with respect to s :

$$0 = f_x \frac{dx}{ds} + f_{x_{n+1}} \frac{dx_{n+1}}{ds} \quad (9)$$

Equation (9) is an implicit system of n linear algebraic equations in $n + 1$ unknowns

$$\frac{dx_i}{ds}, \quad i = 1, 2, \dots, n, n+1 \quad (10)$$

with the coefficients being the element s of the matrix:

$$Df = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_k} & \dots & \frac{\partial f_1}{\partial x_{n+1}} \\ \frac{\partial f_2}{\partial x_1} & \dots & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_k} & \dots & \frac{\partial f_n}{\partial x_{n+1}} \end{bmatrix} \quad (11)$$

The following equation is required to make sure that s is the arclength on the curve.

$$\left(\frac{dx_1}{ds}\right)^2 + \dots + \left(\frac{dx_n}{ds}\right)^2 + \left(\frac{dx_{n+1}}{ds}\right)^2 = 1 \quad (12)$$

Note that equations (9) and (12) form a set of $n + 1$ equations in $n + 1$ variables. Also, notice that (9) is a set of linear equations in its $n + 1$ unknowns and (12) is nonlinear. A special method to solve this $n + 1$ equations due to Kubicek [36] is as follows:

Suppose

$$dx_k/ds \neq 0 \quad (13)$$

for some k , $1 \leq k \leq n + 1$, and let Df_k be the matrix of Df with the k -th column taken out, and suppose Df_k is not singular, then equation (9) can be solved for the unknowns

$$\frac{dx_1}{ds}, \dots, \frac{dx_{k-1}}{ds}, \frac{dx_{k+1}}{ds}, \dots, \frac{dx_{n+1}}{ds} \quad (14)$$

in the form of

$$\frac{dx_i}{ds} = \beta_i \frac{dx_k}{ds} \quad i \neq k, i = 1, \dots, n+1 \quad (15)$$

The solution for the coefficients β_i can be obtained by applying Gaussian elimination to the matrix Df_k . In fact, if one performs the Gaussian elimination with pivoting to the full set matrix Df , the column index k is also found.

Substituting (15) into (12), one gets

$$\left(\frac{dx_k}{ds}\right)^2 = \left(1 + \sum_{i=1, i \neq k}^{n+1} \beta_i^2\right)^{-1} \quad (16)$$

Equations (15) and (16) constitute the explicit expression of the derivatives of the curve $\mathbf{x}(\lambda)$ passing through the j -th continuation point $(\mathbf{x}^j, \lambda^j)$ with respect to arclength s . A predictor step can be accomplished by integrating one step further in the prescribed direction with the step-size h :

$$\hat{\mathbf{x}}_j^{i+1} = \mathbf{x}_j^{i+1} + h \frac{d\mathbf{x}_j}{ds}, \quad j = 1, \dots, n+1 \quad (17)$$

In the context of computational efficiency, one has to keep in mind that the evaluation of the β_i 's involves solving a set of linear algebraic equations which could be time-consuming. Thus it is advantageous to use numerical procedures which require fewer such evaluations. This consideration prompts the use of the secant method as a predictor after the tangent method produces two approximate points.

Secant method

The polynomial extrapolation methods are based on a polynomial of varying order that passes through the current solution and previous solutions (x^i, λ^i) , (x^{i-1}, λ^{i-1}) , $\dots$, to provide an approximation point for the next solution (x^{i+1}, λ^{i+1}) . A trivial predictor is the zero-order polynomial which uses the current solution as an approximation point for the next solution; i.e.,

$$(\hat{x}^{i+1}, \hat{\lambda}^{i+1}) = (x^i, \lambda^i)$$

A slightly modified predictor based on the zero-order polynomial is

$$(\hat{x}^{i+1}, \hat{\lambda}^{i+1}) = (x^i, \lambda^{i+1})$$

A predictor, known as the secant predictor, uses a first-order polynomial (a straight line) passing through the current and previous solutions to predict the next solution;

$$(\hat{x}^{i+1}, \hat{\lambda}^{i+1}) = (x^i, \lambda^i) + h(x^i - x^{i-1}, \lambda^i - \lambda^{i-1}) \quad (18)$$

where h_i is an appropriate step-size. Predictors based on higher-order polynomial can be similarly derived. It has been experienced that lower-order predictors are more effective in the long run. The secant method is available in CPFLOW. It is interesting to note that the modified zero-order predictor is implicitly used in the repeated power flow approach.

In general, $(\hat{x}, \hat{\lambda})$ is not a solution of $f(x, \lambda) = 0$, rather it is an initial guess for the corrector iterations that will hopefully converge to a solution within the specified tolerance. The distance between (x^i, λ^i) and $(\hat{x}^{i+1}, \hat{\lambda}^{i+1})$ is called the *step length*. On the other hand, the measure of distance between (x^i, λ^i) and (x^{i+1}, λ^{i+1}) is given by the parameterization strategy, for example, arc-length.

The Corrector

After the predictor has produced an approximation $(\hat{x}^{i+1}, \hat{\lambda}^{i+1})$ for the next solution (x^{j+1}, λ^{j+1}) , the error must be corrected before it accumulates. In principle, any effective numerical procedure for solving a set of

nonlinear algebraic equations can be used for a corrector. Since a good predictor gives an approximation in a neighborhood of the next solution (x^{j+1}, λ^{j+1}) , a few iterations usually suffice for an appropriate corrector to achieve the needed accuracy.

The Newton method is chosen in CPFLOW as the corrector. This choice has an additional advantage; that is, the existing power flow computer packages based on the Newton method can be modified, with some effort, to serve as a corrector. The Newton method yields the following algorithm for computing the augmented state vector update $\Delta z^{\nu+1} \equiv (\Delta x^{\nu+1}, \Delta \lambda^{\nu+1})$

$$\begin{bmatrix} f_x(z^\nu) & f_\lambda(z^\nu) \\ p_x(z^\nu) & p_\lambda(z^\nu) \end{bmatrix} \begin{bmatrix} \Delta x^{\nu+1} \\ \Delta \lambda^{\nu+1} \end{bmatrix} = - \begin{bmatrix} f(z^\nu) \\ p(z^\nu) \end{bmatrix} \quad (19)$$

where ν represents the current Newton iteration and $p(x, \lambda)$ represents the parameterization equation. Note that the Jacobian is evaluated at each Newton iteration in the above equation.

We also have a faster, approximate method for the corrector, based on the Newton method. This approximation is sometimes called the Chord method and it can lead to faster execution of the continuation technique. The main idea is that we would like to avoid solving the large linear system (19) at each Newton iteration ν . In an attempt to save computation time the Chord method only builds the Jacobian once per continuation step. The most expensive operation in the corrector routine is factoring the large sparse Jacobian. In the Newton method, the Jacobian is evaluated and factored at every Newton iteration. In the Chord method, the Jacobian is only evaluated once (at the beginning of the corrector routine) and hence it is only factored once. Then we can use this LU decomposition to solve the "approximation" linear system used to compute the state update $(\Delta x^{\nu+1}, \Delta \lambda^{\nu+1})$

$$\begin{bmatrix} f_x(z^0) & f_\lambda(z^0) \\ p_x(z^0) & p_\lambda(z^0) \end{bmatrix} \begin{bmatrix} \Delta x^{\nu+1} \\ \Delta \lambda^{\nu+1} \end{bmatrix} = - \begin{bmatrix} f(z^\nu) \\ p(z^\nu) \end{bmatrix}$$

Here, the LU decomposition is only performed once per step, so solving the large linear system only requires a relatively inexpensive forward elimination and backward substitution. The disadvantage to the Chord method is that it converges only linearly, whereas the Newton method converges quadratically. However, the slower convergence is frequently offset by the savings from eliminating some LU decompositions.

Occasionally, the user supplied step length is too large and in the case that the Newton corrector diverges, we have an algorithm for attempting to recover from this setback. If the Newton method diverges, we know that our initial guess to the true solution is too far away. One method of recovery is to reset the state to the previous calculated solution. Then, halve the step length and

try to find the solution with the Newton method. In practice, this method also helps to choose a "better" steplength if the initial user-defined step length is too large.

Step Length Control

Ideally, the step length should be adapted to the shape of the solution curve to be traced: larger step length should be used in the "flat" part of the solution curve and smaller step-length in the "curly" part (part with high degree of curvature) of the solution curve. Of course, the shape of solution curve is unknown beforehand; making the task of designing an effective step length control difficult. Thus, good step length controls are usually custom designed for specific applications. Despite this, some general considerations may be implemented in the continuation procedure in order to improve its performance.

In general, arclength parameterization coupled with an intelligently chosen fixed step length yields an efficient and reliable computational tool. The optimal fixed step length depends on the problem that we are trying to solve, but once that step length is determined (usually done by experimentation) it can remain fixed throughout the continuation run.

CPFLOW computes the arclength in the state space, which automatically forces the predictor to take large steps on the "flat" part of the solution curve and small steps on the "curly" part. The efficiency of this approach will be demonstrated later via numerical studies.

The alternative to a fixed step length is an adaptive step length. A popular adaptive stepsize selection algorithm is based on the idea of an optimal number of Newton corrector iterations as follows:

- if $niter < nopt$ then increase Δs
- if $niter > nopt$ then decrease Δs .

However, if the number of Newton corrector iterations are dependent on anything else, in addition to stepsize, then this algorithm will lead to erratic execution. For example, if the nonlinear algebraic system is constrained, then near the constraint boundaries the number of Newton iterations tends to fluctuate wildly. In modeling real power systems, the typical control devices such as ULTCs, phase-shifters, switchable shunts, and generator excitation systems can cause this type of wild fluctuation in the number of Newton iterations at each continuation step. Therefore, the fixed statespace step length yields more efficient and reliable results.

It should be pointed out that step length controls must be designed coherently with the predictor, corrector and the problem under consideration. In the design of a predictor-corrector continuation method there is a delicate balance between speed and robustness. The key

issues here are (i) robustness - whether a predicted point produced by a low-order predictor with a step-size chosen to be as large as possible lies inside the domain of convergence of the corrector; noting that the number of iterations in the corrector process may be large, and (ii) speed - the amount of total work required to solve the given problem to a specified accuracy.

Solution Algorithm

CPFLOW uses a predictor-corrector continuation method to trace solution curves. There are two types of predictors employed in CPMFLOW. In phase one of CPMFLOW, the Tangent method serves as a predictor. After at least two solution points have been obtained, the predictor can be switched to the Secant method which requires the knowledge of the current and previous solutions (Phase two). A step-by-step exposition of the solution algorithm used in CPMFLOW is summarized in Figure 6.

Numerical Studies

We have three example power transmission systems

1. Test 39 bus network
2. Test 234 bus network
3. Test 3493 bus network

The 39 bus network is a standard test system in the power transmission system research community. In the past, this system has yielded expected results, but we will see that once again a nonlinear system displays unexpected behavior. A typical utility dataset contains thousands of buses at the high-voltage transmission system level, but even in this small example power system we find strange behavior.

The 234 bus network is an actual dataset modeling a small transmission network. With this network, we will examine the influence of some standard electric power control devices, such as tap-changing transformers and reactive power limited generators. These standard industry devices complicate the problem of tracing stationary solutions of our algebraic equations. In particular, if any device changes its control setting, then we are no longer working with a system of continuously differentiable algebraic equations. We are forced to make the assumption that pasting the piecewise continuously differentiable solution branches together will not affect our results.

The 3493 bus network is also an actual dataset modeling a transmission network. This system demonstrates the power of the continuation technique when applied to very large scale systems.

Test 39 Bus Network

The following simulation demonstrates the complexity of behavior found in general nonlinear systems. How-

Step 1: Input power system data

Step 2: Initialization

- (i) Run a power flow study to make sure the initial point to be used in the continuation process lies on the solution curve.
- (ii) Build up the current load demand pattern (P_d^0, Q_d^0) , real power generation pattern (excluding the slack bus) (P_g^0) .

Step 3: Input the buses at which loads and/or real power generations are to be varied, and also input the quantity of variations.

- (i) Build up the future load demand pattern (P_d^1, Q_d^1) , real power generation pattern (P_g^1) .
- (ii) Parameterize the load demand pattern $(P_d(\lambda), Q_d(\lambda))$, and real power generation pattern $(P_g(\lambda))$ so that (see Fig. 7)

$$(P_d(0), Q_d(0)) = (P_d^0, Q_d^0)$$

$$(P_d(1), Q_d(1)) = (P_d^1, Q_d^1)$$

Step 4: Predictor (phase-one)

- (i) calculate the derivatives $(dx_1/ds, \dots, dx_n/ds, dx_{n+1}/ds)$ via β' 's.
- (ii) choose an appropriate step-length.
- (iii) obtain the predicted point using equation (17).

Step 5: Predictor (phase-two)

- After at least two solution points have been obtained, the predictor can be switched to the secant method which requires the knowledge of the current and previous solutions.
- (i) choose an appropriate step-length.
 - (ii) obtain the approximation point using equation (18).

Step 6: Corrector (phase-one)

Solve the set of augmented power flow equations (7) and (8) using the Newton method starting from the approximation point obtained in the Predictor (phase-one).

Step 7: Corrector (phase-two)

Solve the set of augmented power flow equations (7) and (8) using the Newton method starting from the approximation point obtained in the Predictor (phase-two).

Step 8: Stop criterion

If further tracing of solution curve is of no interest, stop here. Otherwise, go to Predictor step to calculate the next point.

Figure 6: Solution Algorithm Structure

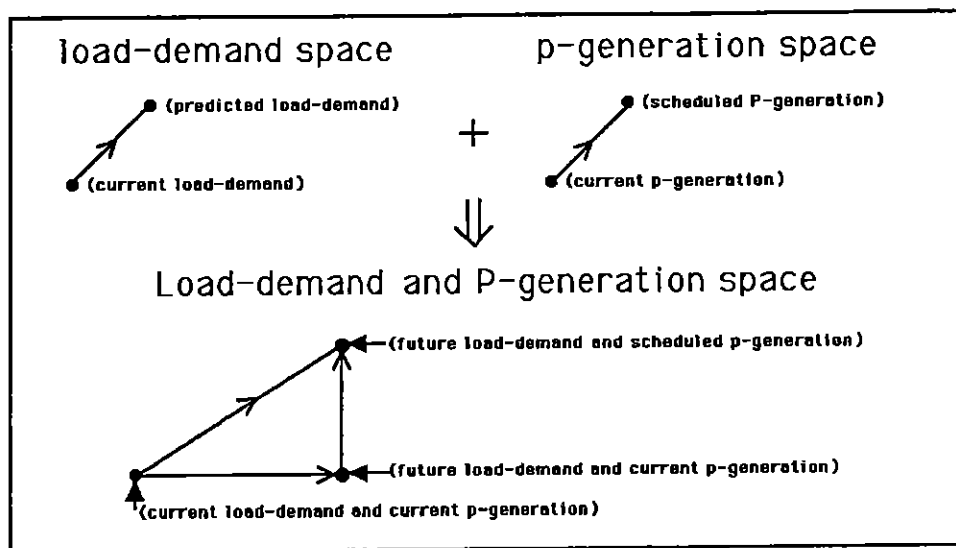


Figure 7: Parameterize both the load demand pattern and real power generation pattern

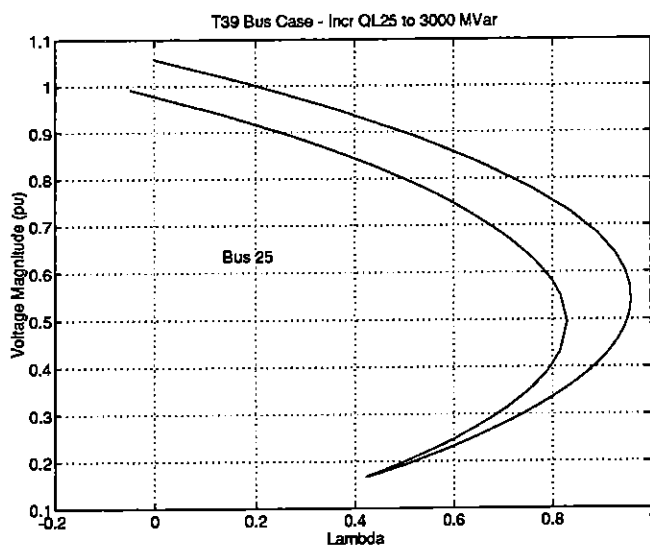


Figure 8: Bus 25 Voltage Magnitude vs. Lambda

ever, this behavior has now been discovered in a standard test power system, the New England 39 bus system. In the past, our engineering intuition has told us that a typical PV or QV curve has one and only one turning point (i.e. saddle-node bifurcation), sometimes called the maximum loading point. While the maximum loading point definition is still valid, we must now deal with the fact that our solution branches may have more than one turning point.

When examining the standard CPFLOW output, it is difficult to tell what is happening in this simulation. Since we are increasing reactive power at bus 25, it is

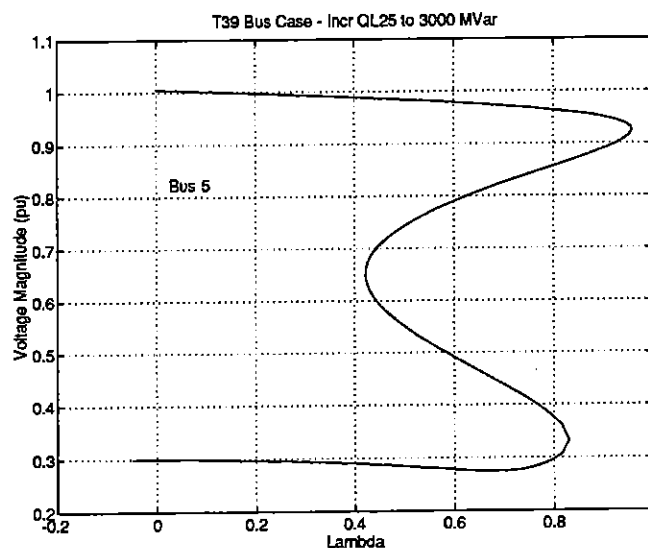


Figure 9: Bus 5 Voltage Magnitude vs. Lambda

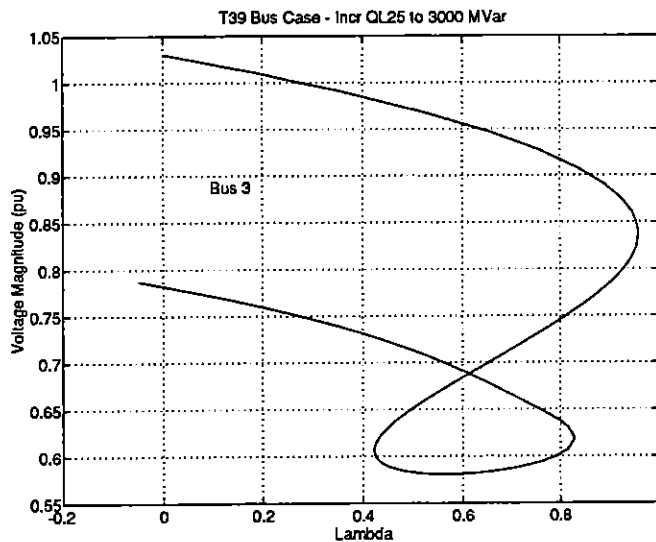


Figure 10: Bus 3 Voltage Magnitude vs. Lambda

natural to plot the voltage magnitude at bus 25 versus λ (Fig. 8). But, this strange looking “nose curve” might weaken an engineer’s confidence in the simulation results. Only after examining other buses in the network, do we realize that CPFLOW is not merely retracing its steps, but that it is actually continuing the initial solution branch (Figs. 9 and 10). This is tough to see by projecting our large state space down onto only two dimensions. For this reason, we have also plotted the entire state vector (including both magnitudes and angles) in the 2-norm versus λ (Fig. 11). In this figure, we see the multiple turning points, which assures us that CPFLOW has not retraced its steps during the continuation run.

Test 234 Bus Network

This simulation examines reactive power increases in a 234 bus case. The reactive power load is increased at bus 5047. In this case, we investigated the effect of control action on the relative position of the saddle node bifurcation point. Four separate continuation runs were made. The first run was with all controls turned off. The second simulation was with only the generator reactive power limits activated. The third simulation was with only the ULTC voltage controls activated. The final run was with all the controls turned on. In this particular data set, the generator Q limit control and ULTC V control are the only controls present.

We have provided individual plots of the voltage magnitude at the only participating load bus for each scenario on a single set of axes for comparison (Figs. 12 and 13). The case involved increasing reactive power load at only one bus, number 5047. This bus is connected to an ULTC transformer which is connected to

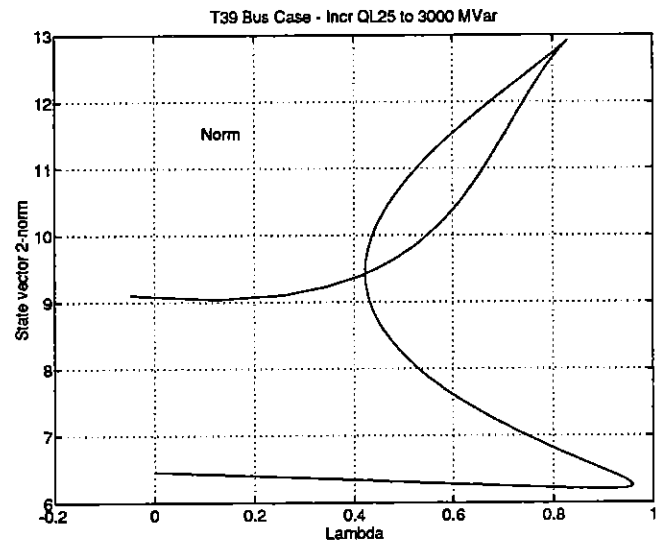


Figure 11: State Vector 2-Norm vs. Lambda

the higher voltage transmission network. The bus resides in area number zero, which is rather isolated from the other three areas in the network. Since we are investigating reactive power increases, the local area generation is vital to supply this increasing reactive load. In other words, the other three areas cannot appreciably supply reactive power to the participating bus.

In the test runs with generator reactive power controls active, we examined the list of all generators at the limit point. We noticed that the area zero reactive generation had reached its maximum limit, i.e., all generators were saturated. This causes the peculiar shape of the continuation curves. Without the control action, the curves have the familiar smooth, nose shape, but with the control action the curves are more irregular.

Test 3493 Bus Network

CPFLOW has been applied to several real power systems including a 3493 bus system with the following network components and additional real power area interchange schedule:

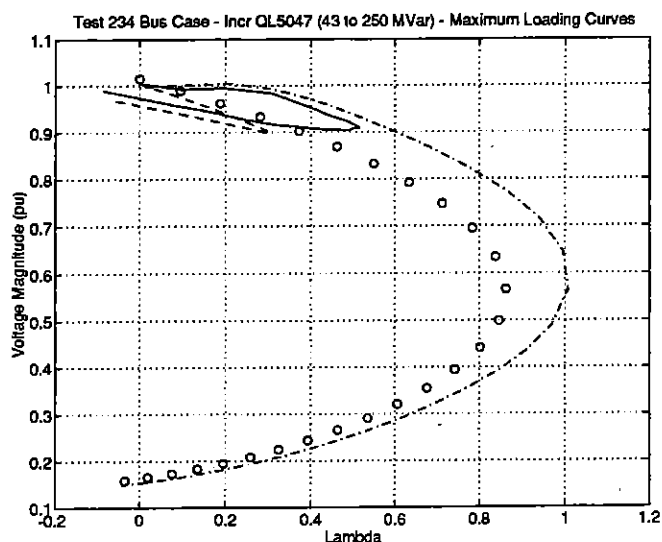


Figure 12: Simulation showing effect of control devices

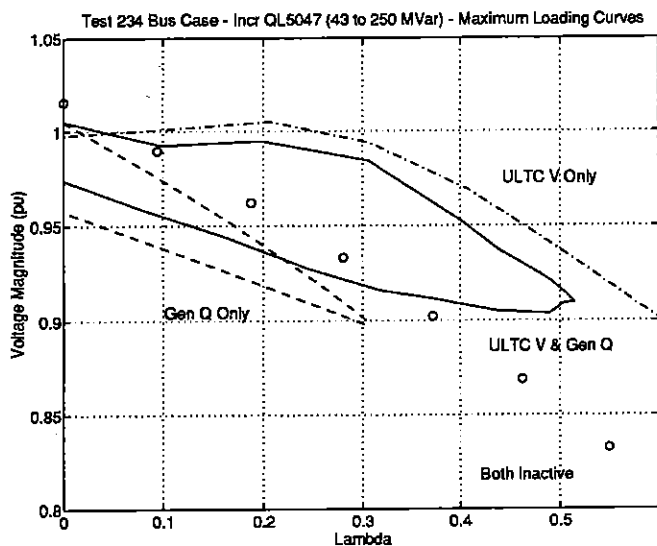


Figure 13: Expanded view showing effect of control devices

Network Data

• Buses:	3493
• Generators:	844
• Loads:	2565
• Fixed Shunts:	957
• Switchable Shunts:	125
• Lines:	5953
• Fixed Transformers:	654
• Fixed Phase Shifters:	1
• ULTC Transformers:	82
• ULTC Phase Shifters:	8
• Areas:	30

Exporting Areas

• Area 3:	482.6 MW
• Area 9:	1673.0 MW
• Area 12:	2169.0 MW
• Area 14:	1202.0 MW
• Area 15:	325.0 MW
• Area 17:	300.0 MW
• Area 19:	1919.0 MW

This case study investigates the maximum real power transfer capability for the following scenario:

- Area 1 generation decreases to zero (fourteen generators scale down to 0 MW)
- External generators increase to supply Area 1 load (nineteen generators listed in table 2)
- CPFLOW traces the entire solution curve
- Thermal loading of transmission lines is ignored

Table 2: Generator Schedule

Bus	Current Pgen	Future Pgen
507	690.0	875.0
510	817.4	1115.0
652	820.0	2104.0
681	125.0	514.0
1566	0.0	288.0
1587	1256.0	2512.0
1590	625.0	1250.0
1947	500.0	900.0
2024	131.0	854.0
2025	971.0	1050.0
2091	447.0	530.0
2092	865.0	1007.0
2112	400.0	500.0
2278	500.0	600.0
2296	530.0	630.0
2334	500.0	600.0
3011	1531.0	2100.0
3021	1100.0	1300.0
3252	1450.0	2600.0

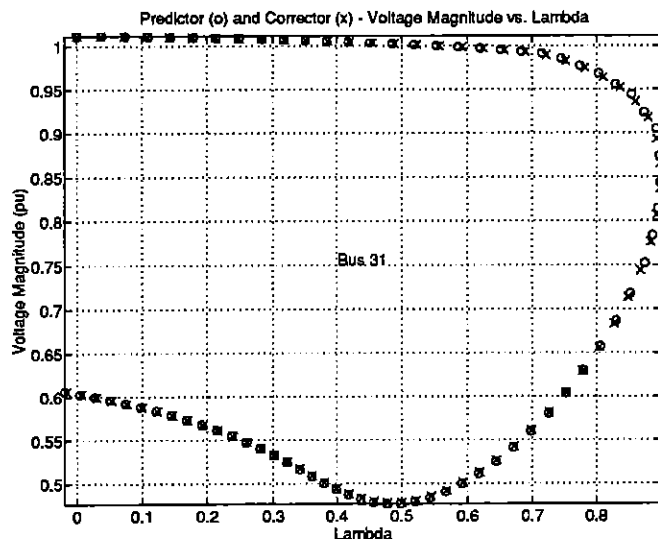


Figure 14: Predictor-Corrector plot of voltage magnitude

The following plots are taken from the CPFLOW output data files: bus.dat, gen.dat, load.dat and pred.dat. Figure 14 shows Bus 31 voltage magnitude plotted versus λ . Both predictor and corrector points are shown in this plot and in the next plot (Fig. 15) of Bus 31 voltage angle versus λ .

In the next two plots (Fig. 16 and 17), real power and reactive power generation are plotted versus λ . These plots contain all nineteen generators that are exporting power to Area 1.

Next, in Figures 18 and 19, real power and reactive power generation versus λ are shown for the importing area's generators. Here the fourteen generators in Area 1 are scaled to zero MW, while the reactive power limits of the generators remain unchanged. This practice may not be wise, since a generator has a minimum real power limit below which its reactive power output limit may not remain constant. In fact, when the real power generation is scaled down, the reactive power generation is probably affected. This level of generation modeling has been ignored so far in CPFLOW. However, improved generator models are now being considered for CPFLOW, which will include generation excitation field current limits.

The last plot (Fig. 20) shows the voltage magnitudes at a variety of load buses in Area 1 at the saddle node bifurcation point. This stem plot demonstrates the unusual voltage profile found in many real power transfer simulations. The voltage profile is typical of other cases where real power load is increasing or real power is being transferred across long distances in that the voltage magnitudes are usually near the nominal 0.9 to 1.1 p.u.

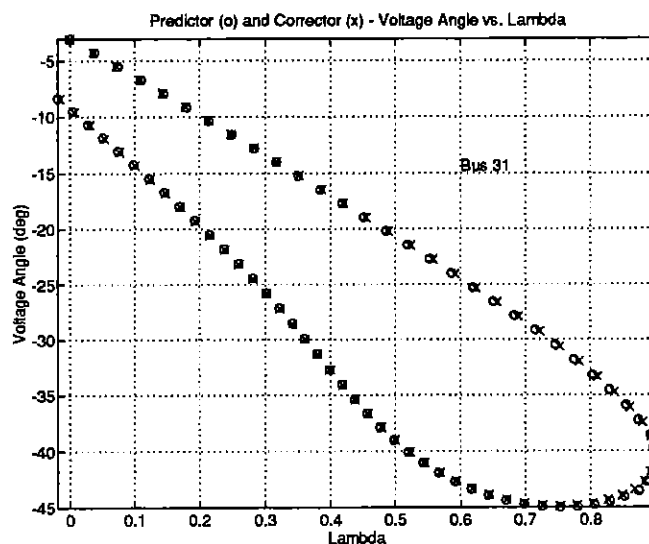


Figure 15: Predictor-Corrector plot of voltage angle

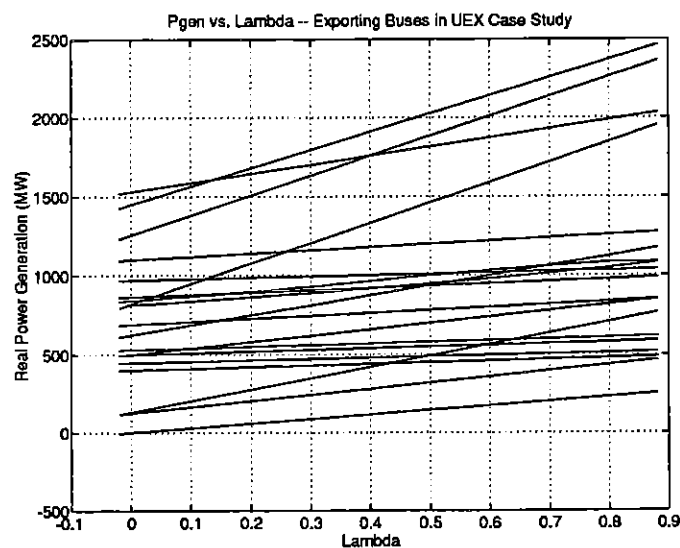


Figure 16: Real power generation for exporting generators

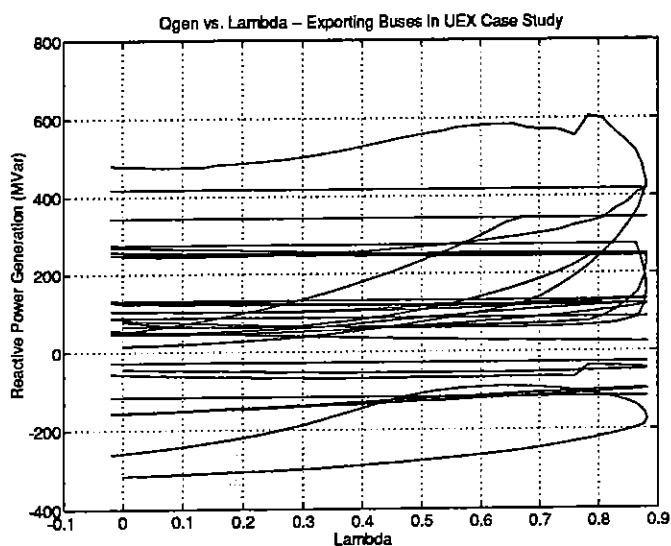


Figure 17: Reactive power generation for exporting generators

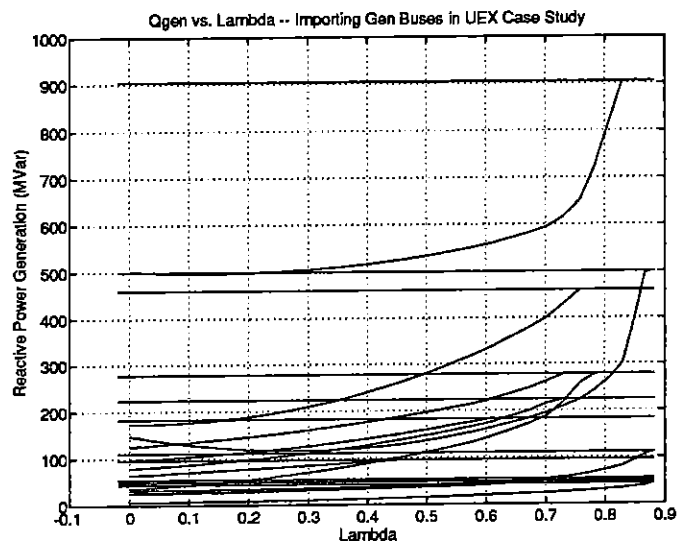


Figure 19: Reactive power output for decreasing generators in Area 1

range.

Area interchange data for Area # 1 at base case solution point

Area	MW Flow	MVar Flow
2	-1360.95	-134.75
3	34.28	44.54
8	-324.08	-25.08
12	-121.35	81.36
14	120.03	-47.63
17	1072.10	-98.89
18	43.03	62.89
Net	-536.94	-117.56

The above data describes the real and reactive power imported into Area 1 in the base case. This data has been displayed in a graphical format in Figure 21 along with the interarea power flows connecting the other nineteen generators' areas.

Area interchange data for Area # 1 at SNBP

Area	MW Flow	MVar Flow
2	-3512.39	-68.07
3	-212.90	142.24
8	-666.00	-229.72
12	-669.16	7.50
14	-69.73	-108.93
17	-991.50	-184.43
18	-730.40	-18.24
Net	-6852.09	-459.66

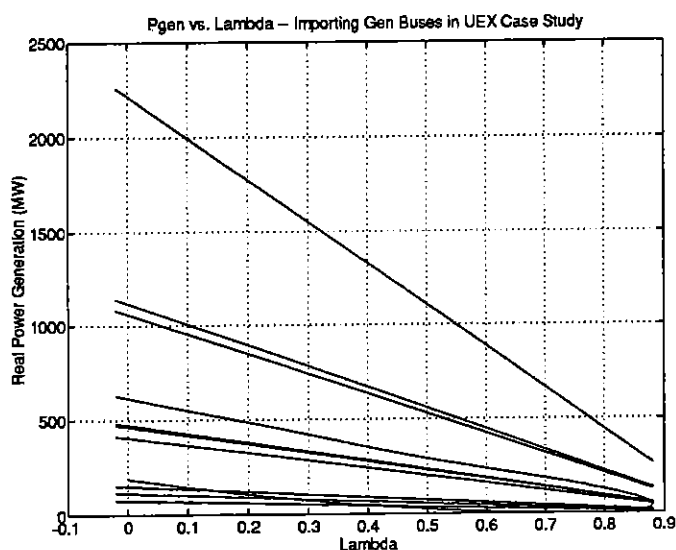


Figure 18: Real power output for decreasing generators in Area 1

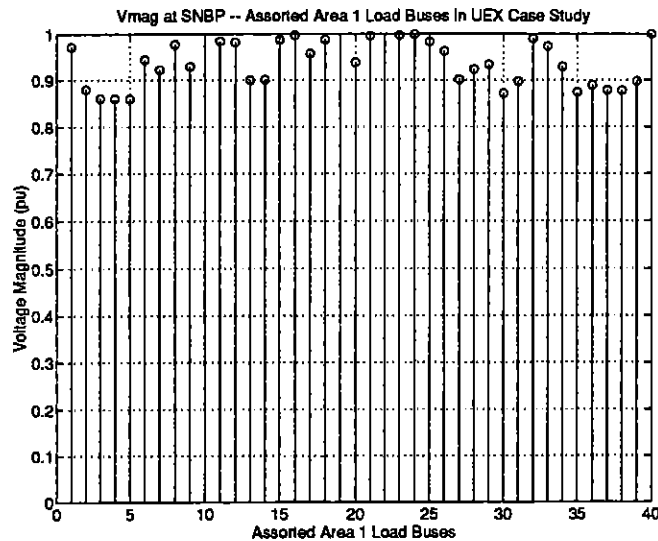


Figure 20: Voltage magnitudes at some of the load buses in Area 1 at SNB

The above table is also displayed in a graphical format in Figure 22 along with other interarea power transfers. Note that the graphical output is not complete. In particular, there is a connection between Areas 12 and 19 that could not easily be included in the diagram. The power flows between these two areas is as follows

Status	Real Power from 12 to 19	Reactive Power from 12 to 19
Base Case	736.6 MW	-170.3 MVar
SNBP	577.7 MW	-148.2 MVar

The execution time required to compute the performance index for this real power transfer capability case study with $stpsz = 0.5$ (31 steps to SNBP) was 10 minutes on a SPARC 2 workstation.

The execution time required to continue the base case solution over the entire physical parameter range for this real power transfer capability case study with $stpsz = 0.5$ (75 steps to trace entire curve) was 24 minutes on a SPARC 2 workstation.

We are now in a position to propose an integrated methodology for voltage stability assessments due to slow load variations. The methodology is based on the integration of an enhanced version of CPFLOW and an improved version of the performance index developed. The enhanced version of CPFLOW will go beyond the current version of CPFLOW to include detailed modeling of generator reactive power capability including excitation field current saturation. The improved performance index will appropriately treat any physical constraints of such devices as generator excitation systems,

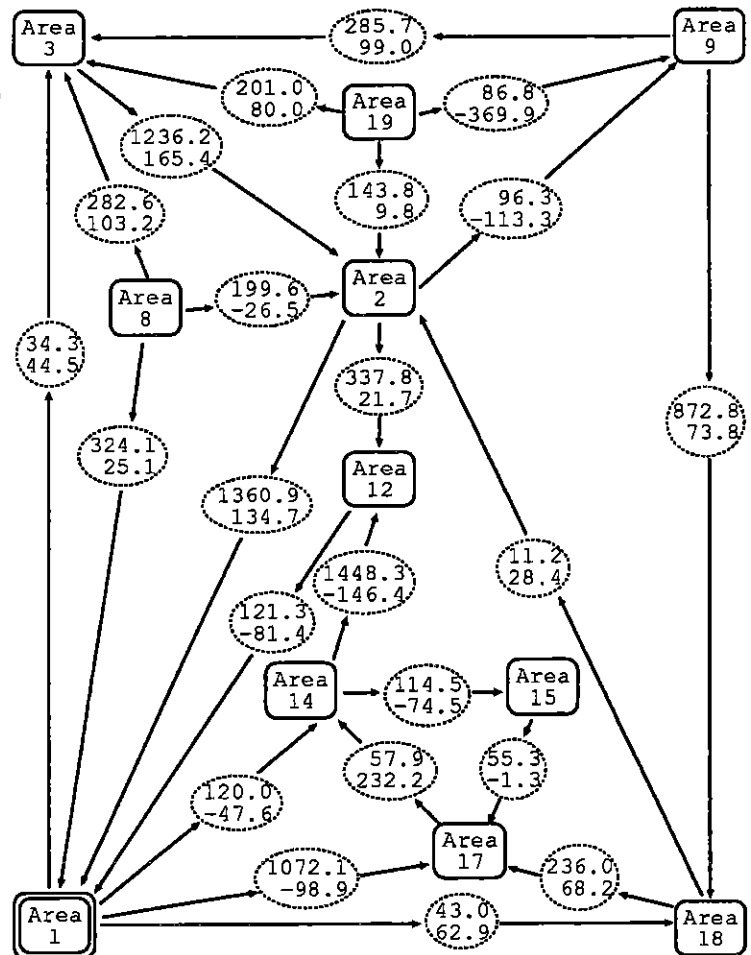


Figure 21: UEX Base Case: Real Power (MW), Reactive Power (MVar)

Concluding Remarks

Although the voltage instability and/or collapse problem could be analyzed using an appropriate power system stability program on a comprehensive power system model, this straight-forward approach has several merits as well as disadvantages. In this paper, we have attempted to present an integrated framework for addressing voltage-related instability problems using a non-time-domain approach. We have identified four issues needed to be addressed by any non-time-domain approach. This framework reveals that the existing tools for analyzing voltage instability are far from complete; especially tools for analyzing voltage instability due to event disturbances.

We have presented a rigorous definition of voltage stability and presented two models (SAD model and Hopf model) to explain voltage collapse and one model (Hopf model) to explain voltage instability due to slow load variations in power systems. It was shown that the SAD model encompasses several existing models of explaining voltage collapse. The purpose of this framework is to facilitate the development of effective tools which can overcome the chief disadvantages associated with the time-domain approach. Two specific but different tools - a (direct) performance index and CPFLOW for assessing voltage-related instability problems due to slow load variations have been demonstrated. We have proposed an integrated methodology for voltage stability assessments due to slow load variations for power systems with sufficient damping. This methodology is based on the integration of an enhanced version of CPFLOW and an improved version of the performance index developed. Another important area warranting further investigation is developing appropriate load models for stability analysis [43].

Acknowledgement

The authors wish to thank the useful discussions they had with Mr. David S. Takach of Union Electric which have resulted in several improvements in the new version of CPFLOW. The authors also thank Professor Robert J. Thomas of Cornell University for his helpful discussions.

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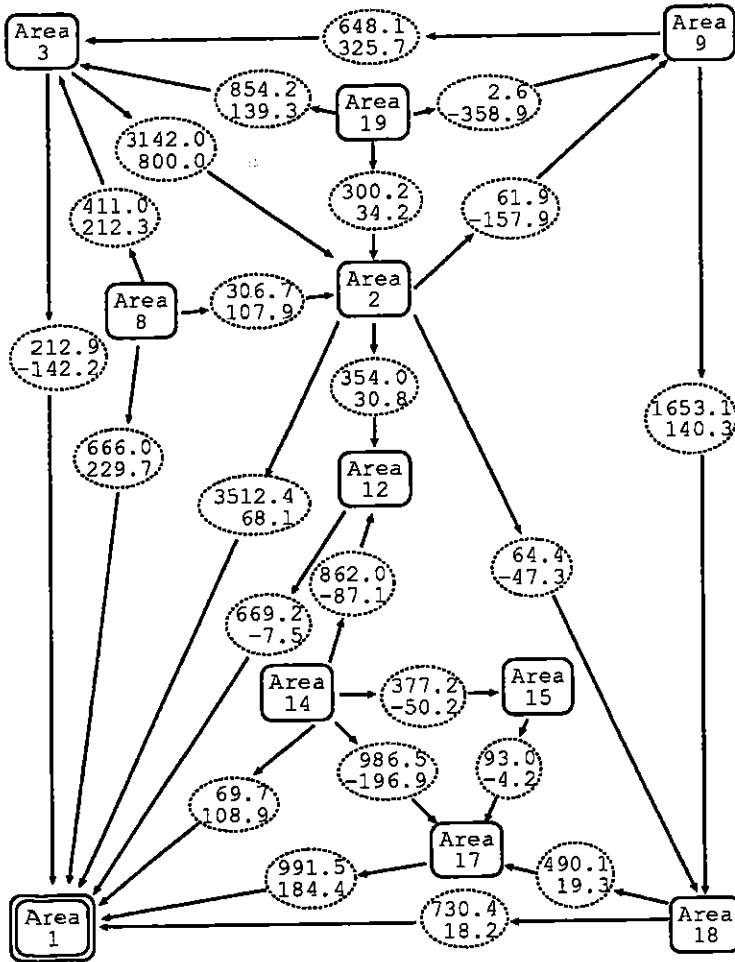


Figure 22: UEX SNBP: Real Power (MW), Reactive Power (MVar)

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