ECEN 615
Methods of Electric Power Systems Analysis

Lecture 19: Unit Commitment, Economic Dispatch, Optimal Power Flow (OPF)

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Announcements

- Homework 4 is due today.
- Homework 5 is due on November 13.
- Read Chapter 8 (Optimal Power Flow)
- Read the Chapter 3 appendices (3A covers optimization with constraints, 3B covers linear programming, 3D covers dynamic programming, and 3E convex optimization.
- An excellent book on optimization is Linear and Nonlinear Programming by Luenberger and Ye (the 5th edition came out in 2021).
- Office hours are changed to Thursdays at 4 to 5pm using the Zoom link I sent out on Canvas.

Unit Commitment: Quick Coverage (Chapter 4)

- Unit commitment is used to determine which generator units should be committed to meet the load
- The electric load varies substantially so there is almost always more generator capacity available than load
- Units have availability constraints
  - Minimum up time, time to start, cost to start
  - Minimum down time, time to shutdown, cost to shutdown
  - Ramp rates, minimum MW output
  - Scheduled and unscheduled outages
- System constraints including load, reserve, emissions, network

Solving Unit Commitment

- Unit commitment involves a potentially large number of integer and continuous variables.
  - Not just the status of each unit, but also the amount of time it has been in a particular state (i.e., off or on)
- Solved for a set of discrete time periods, which at each time period there are lots of different potential states.
- Solution approaches include
  - Dynamic programming (First edition of book)
  - Lagrangian relaxation (Second and third editions of book)
  - Mixed Integer Programming (Currently state-of-the-art; mentioned in the third edition)

Longer Term Optimization: Quicker Coverage (Chapter 5)

- Longer term optimization is a key consideration in hydro systems with significant reservoir storage.
  - Use the water when it is the most valuable taking into account potentially many other constraints
- Generator maintenance scheduling also needs to be considered.
- Building generation often involves large upfront capital costs to create an asset that will last 20 to 40 years; long-term contracts provide a way to share the risk.
- Take-or-pay contracts obligate a purchaser to purchase so much of a product over a given time period.

Example: Prairie State Energy Campus

- The Prairie State Energy Campus (PSEC) is a 1600 MW coal plant in Southern Illinois with its own coal mine that opened in 2012.
  - It is owned by municipals and coops (my former coop got >60% of the energy from PSEC)
  - While relatively efficient, it is one of the US's largest sources of CO2 emissions
  - It cost an estimate $4 billion to build; if it sells its power at $30/MWh then maximum yearly income would be $30*1600*8760=$420 million

- Illinois's new clean energy law requires PSEC to reduce carbon emissions by 45% by 1/1/35 and be 100% carbon free by the end of 2045.

Image: Pantagraph.com

Power System Economic Dispatch

- Generators can have vastly different incremental operational costs.
  - Some are essentially free or low cost (wind, solar, hydro, nuclear)
  - Because of the large amount of natural gas generation, electricity prices are very dependent on natural gas prices
- Economic dispatch is concerned with determining the best dispatch for generators without changing their commitment.
- Economic dispatch has been used by the power industry more many decades.
  - With early formulations mentioned in 1928!

1958

Theory of Economic Operation of Interconnected Areas

R. H. KERR
ASSOCIATE MEMBER AIEE

L. K. KIRCHMAYER
MEMBER AIEE

THIS PAPER extends the theory and the co-ordination equations[1] previously derived for optimum economy for a single area to obtain the co-ordination equations for optimum economic operation of a pool operated as a multiple-area system. Multiple-area operation of the pool is defined to be operation for which the interchanges between the areas are directly determined and controlled. The theory of this paper forms the basis for obtaining automatic economic operation of interconnected areas as well as the basis for multiarea dispatching computers.

The equations whose solution result in minimum cost operation for a given area are given by:

$$\frac{dF_n}{dP_n}+\lambda\frac{\partial P_L}{\partial P_n}=\lambda \quad (1)$$

where

$\frac{dF_n}{dP_n}$ = incremental production cost of plant n in \$/mwhr (dollars per megawatt-hour)

$\frac{\partial P_L}{\partial P_n}$ = incremental transmission loss of plant n
= ratio of change in transmission loss to change in particular P_n when delivering an increment of power from P_n to the hypothetical load of the area

λ = incremental cost of received power in \$/mwhr

These equations state that, for optimum economy, the incremental cost of received power should be the same from all sources. These equations would be applicable if all of the areas were treated as a single area and would involve the use of a computer representing the entire pool. This computer would require a knowledge of all plant loadings and tie-line flows to companies external to the pool, and the control system would require as a minimum a control channel to each plant.

Another approach would involve application of computer-controllers to the individual areas with means of determining automatically the most economic interchange between the areas. It would be desirable for each area to require only a knowledge of the plant loadings within the area and interconnection flows out of the area in addition to control information as to whether the area should increase or decrease its delivery to the pool. Such a decentralized approach will offer the following advantages over a centralized or single-area approach.

1. The telemetering channel requiremen are reduced as this method does not requi as much information at any given point the system.
2. Smaller decentralized computer-controllers are required rather than one centralized computer.
3. Pertinent information for accounting between areas is readily available.

The basis for such decentralized arrangements follows from the theory to be presented here. This theory has been directly applied to the design of the multiarea dispatching computer shown in Fig. 1 which has been recently installed by the Niagara Mohawk Power Corporation. Also, the General Electric automatic dispatching system may be readily extended to obtain automatic economic operation of interconnected areas according to the theory presented here.

Review of Single-Area Co-ordination Equations

The physical interpretation of the equations for a single area will be reviewed by consideration of a 2-plant system with two ties as shown in Fig. 2. The co-ordination equations become:

$$\frac{dF_1}{dP_1}+\lambda\frac{\partial P_L}{\partial P_1}=\lambda \quad (2A)$$

$$\frac{dF_2}{dP_2}+\lambda\frac{\partial P_L}{\partial P_2}=\lambda \quad (2B)$$

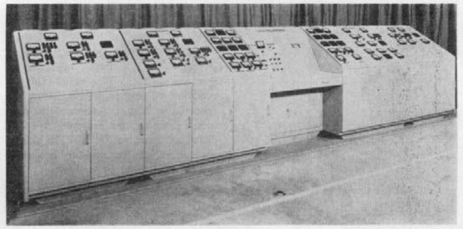
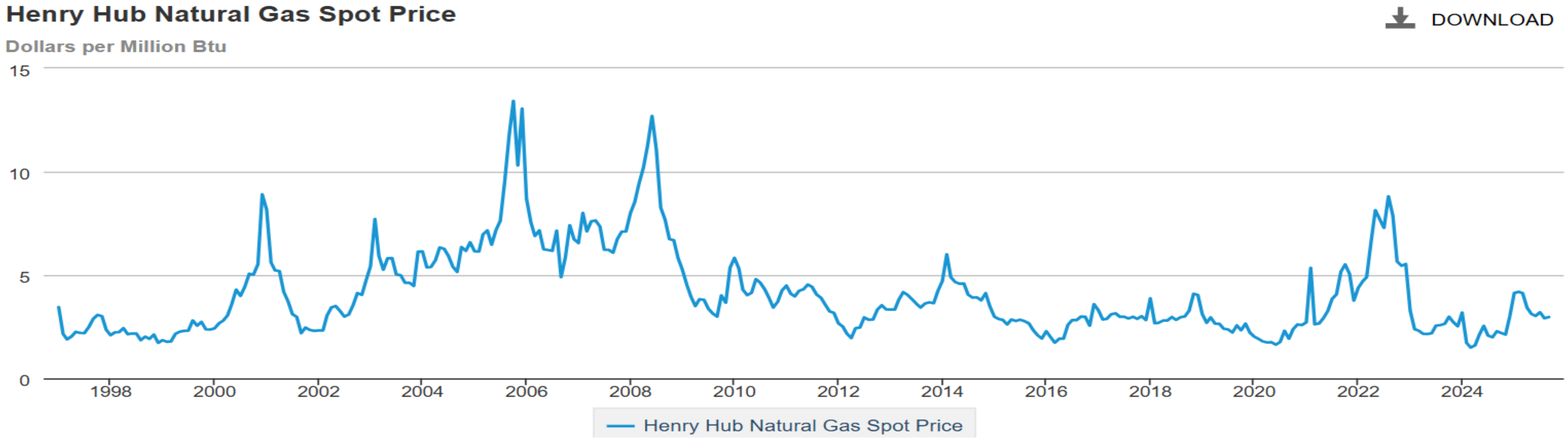


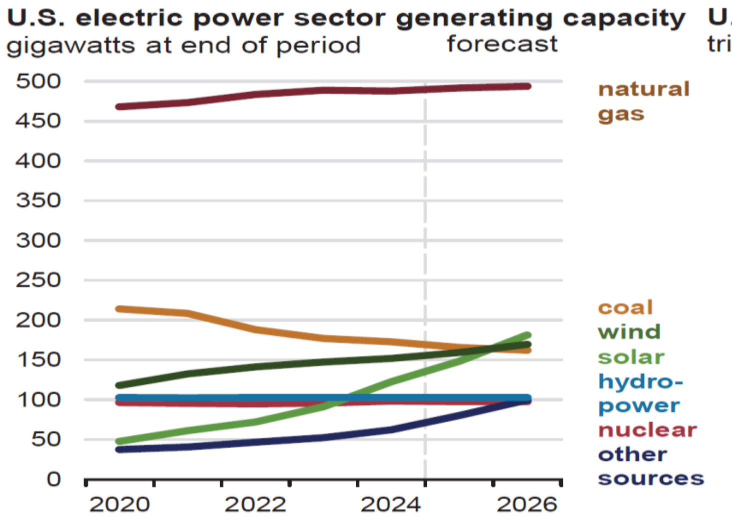
Fig. 1. Niagara Mohawk multiarea dispatching computer

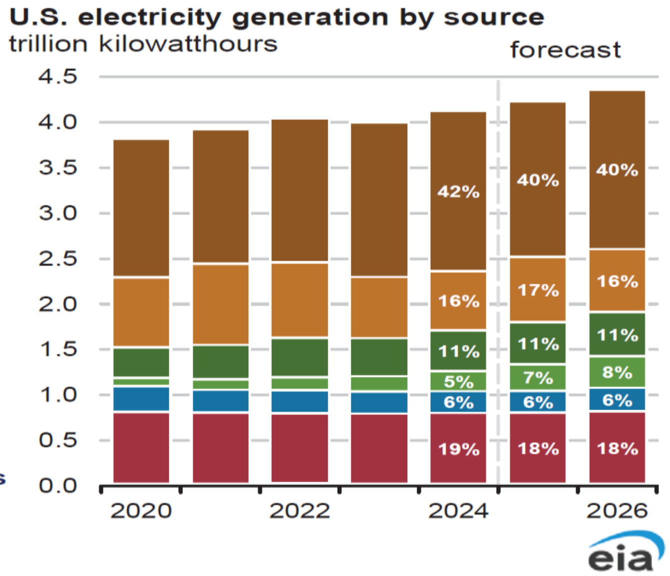
Paper 59-225, recommended by the AIEE System Engineering Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Winter General Meeting, New York, N. Y., February 1–6, 1959. Manuscript submitted November 3, 1958; made available for printing December 17, 1958.

R. H. KERR and L. K. KIRCHMAYER are with the General Electric Company, Schenectady, N. Y.

Variation in Natural Gas Prices and Generation Sources







Sources: www.eia.gov/dnav/ng/hist/rngwhhdm.htm and US EIA Short Term Energy Outlook, October 2025.

Generation Types (ERCOT)

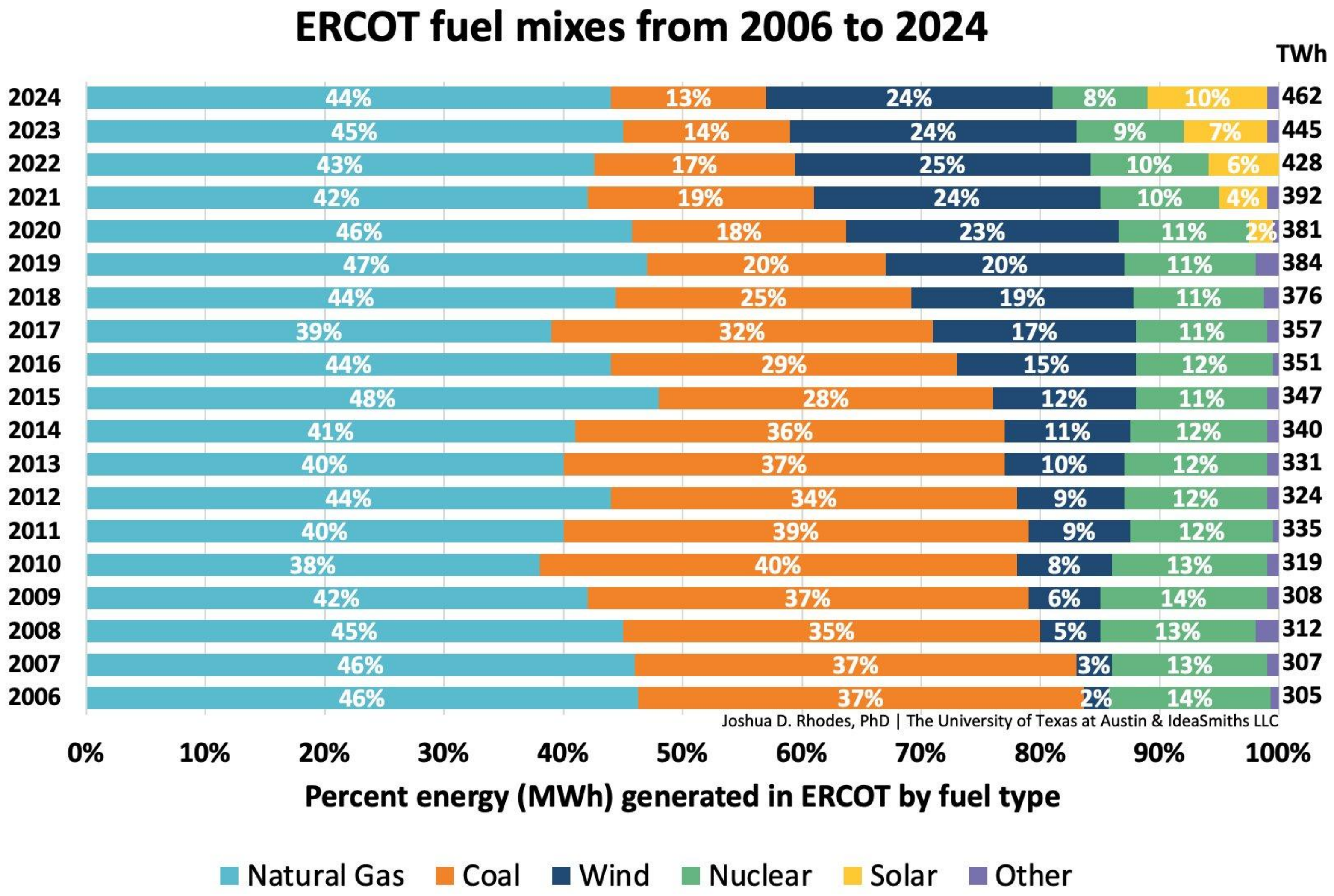
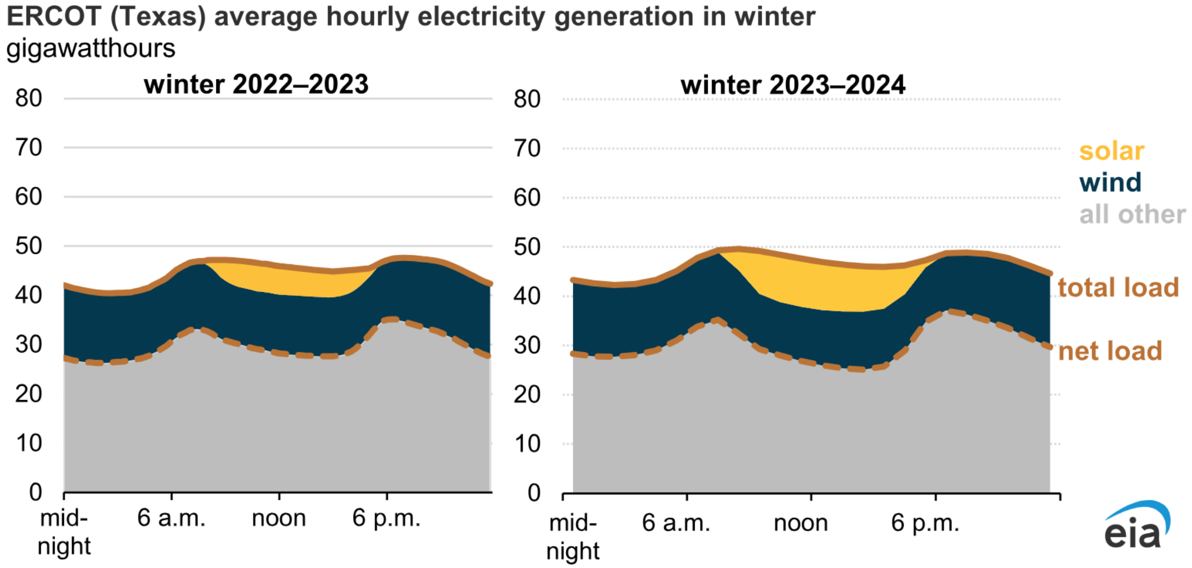
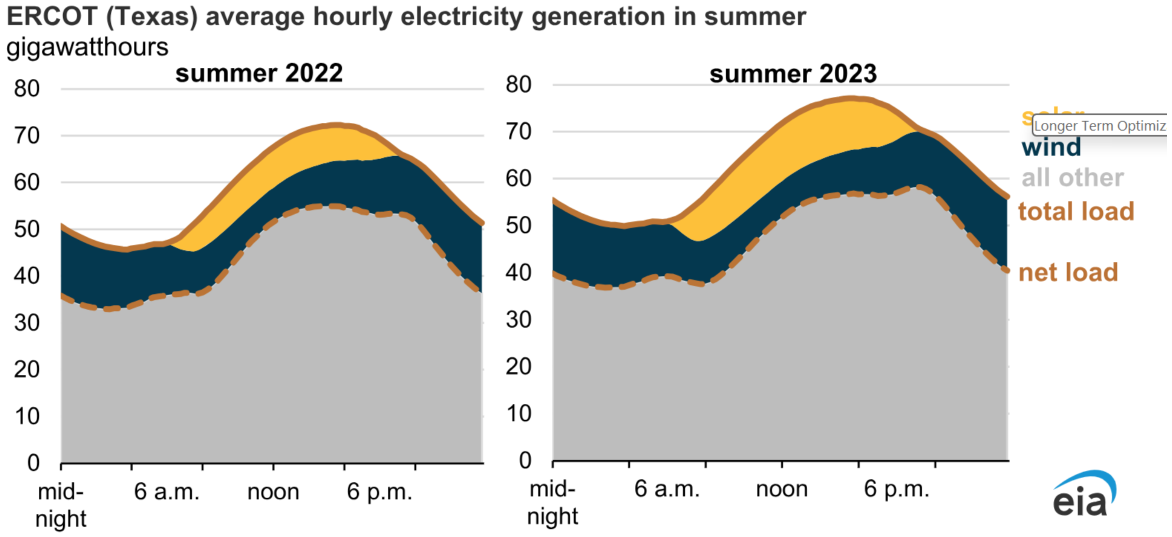


Image source: x.com/joshdr83/status/1882809452820332564?mx=2 (with the data ultimately from ERCOT)

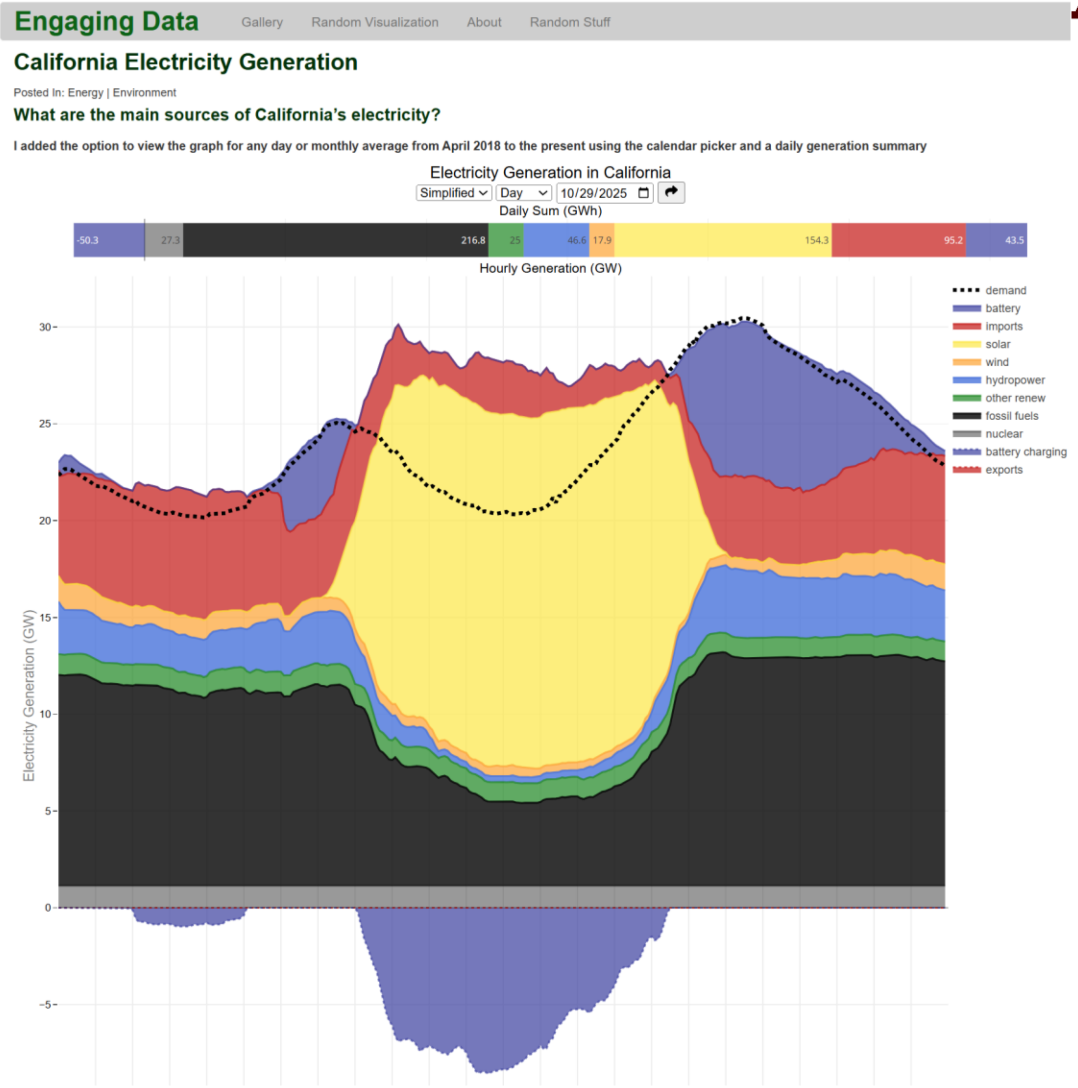
ERCOT Typical Daily Curves





Example: Nice Visualization of California Generation by Source

- The source of electricity are rapidly changing, with the image on the right showing the sources of California generation on Oct 29, 2025.



Data source: engaging-data.com/california-electricity-generation/

Power System Economic Dispatch

- Economic dispatch is formulated as a constrained minimization.
  - The cost function is often total generation cost in an area
  - Single equality constraint is the real power balance equation
- Solved by setting up the Lagrangian (with P_D the load and P_L the losses, which are a function the generation),

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}}, \lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(\mathbf{P}_G) - \sum_{i=1}^{m} P_{Gi})$$

- A necessary condition for a minimum is that the gradient is zero. Without losses this occurs when all generators are dispatched at the same marginal cost (except when they hit a limit).

Power System Economic Dispatch

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

- If losses are neglected then there is a single marginal cost (lambda); if losses are included then each bus could have a different marginal cost.

Economic Dispatch Penalty Factors

Solving each equation for λ we get

$$\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}} = 0$$

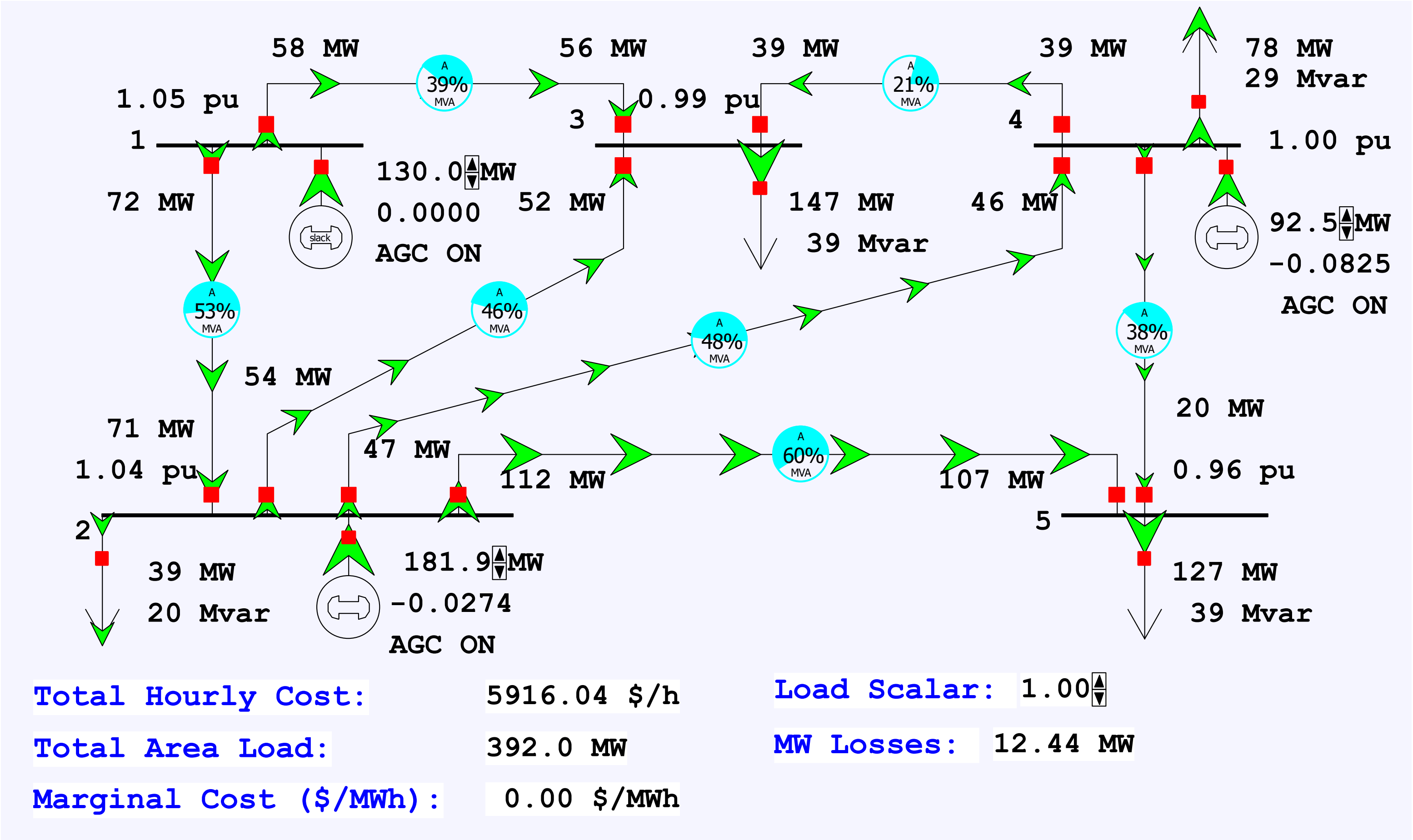
$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

Define the penalty factor L_i for the i^{th} generator

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

The penalty factor at the slack bus is always unity!

Economic Dispatch Example



Case is GOS_Example7_5; use **Power Flow Solution Options, Advanced Options** to set Penalty Factors.

Optimal Power Flow (OPF)

- OPF functionally combines the power flow with economic dispatch.
- Security Constrained OPF (SCOPF) adds in contingency analysis.
- Goal of OPF and SCOPF is to minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints.
- Equality constraints:
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

OPF, cont.

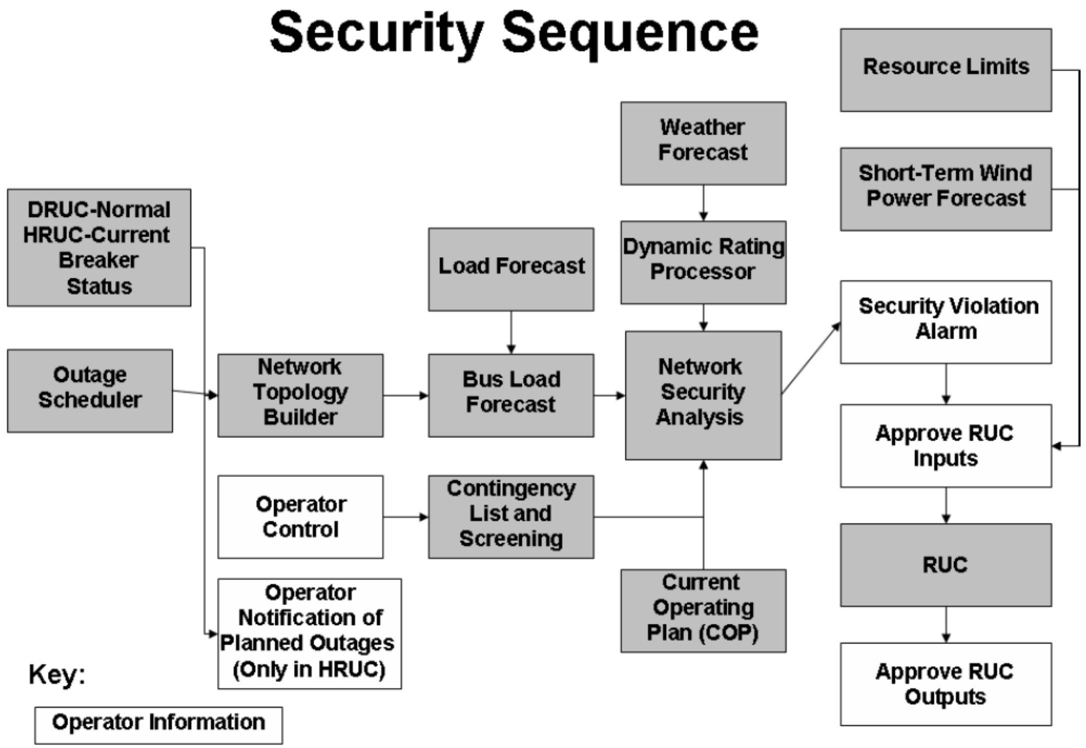
- Inequality constraints:
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls:
  - generator MW outputs
  - transformer taps and phase angles
  - reactive power controls

Key SCOPF Application: Locational Marginal Prices (LMPs)

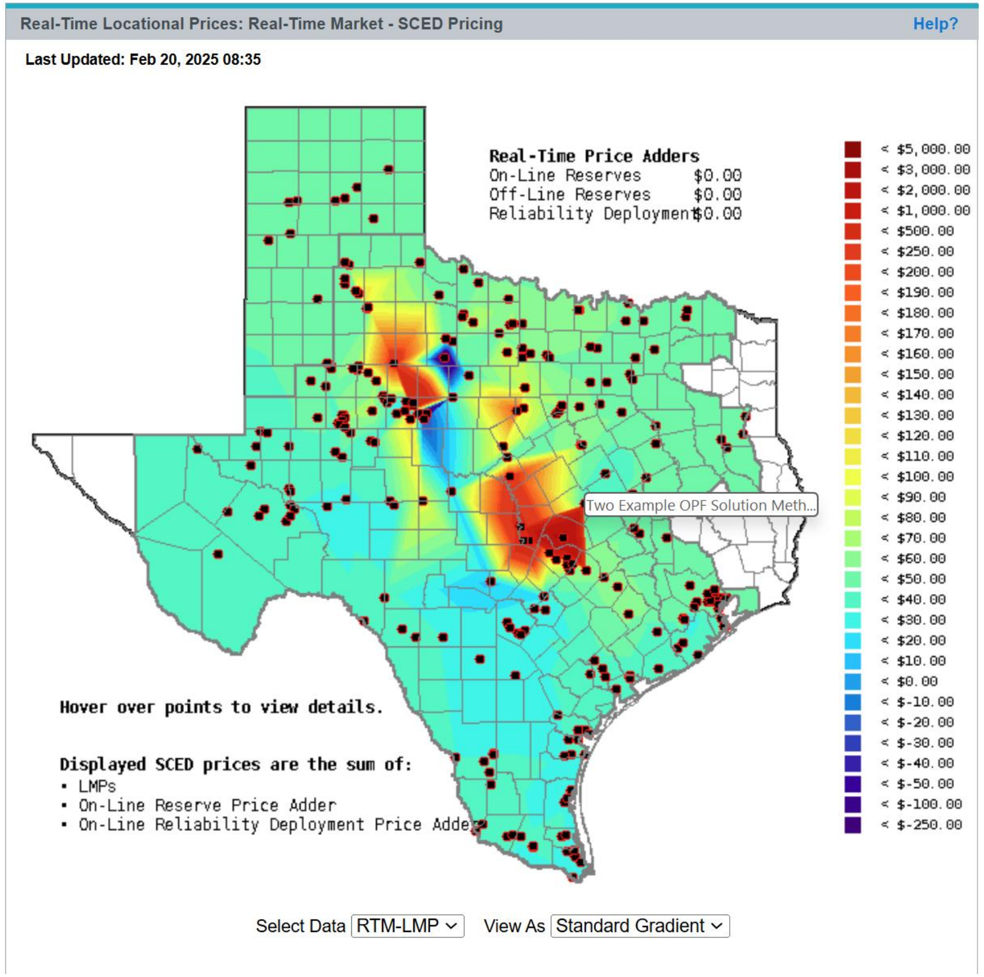
- When OPF includes contingency analysis it is known as the Security-Constrained OPF (SCOPF).
- OPF dates back to 1960's with thousands of papers
- The locational marginal price (LMP) tells the cost of providing electricity to a given location (bus) in the system.
- Concept introduced by Schweppe in 1985.
  - F.C. Schweppe, M. Caramanis, R. Tabors, "Evaluation of Spot Price Based Electricity Rates," *IEEE Trans. Power App and Syst.*, July 1985
- LMPs are a direct result of an SCOPF, and are widely used in many electricity markets worldwide both ahead and in real-time.
- The exact calculations are market specific.

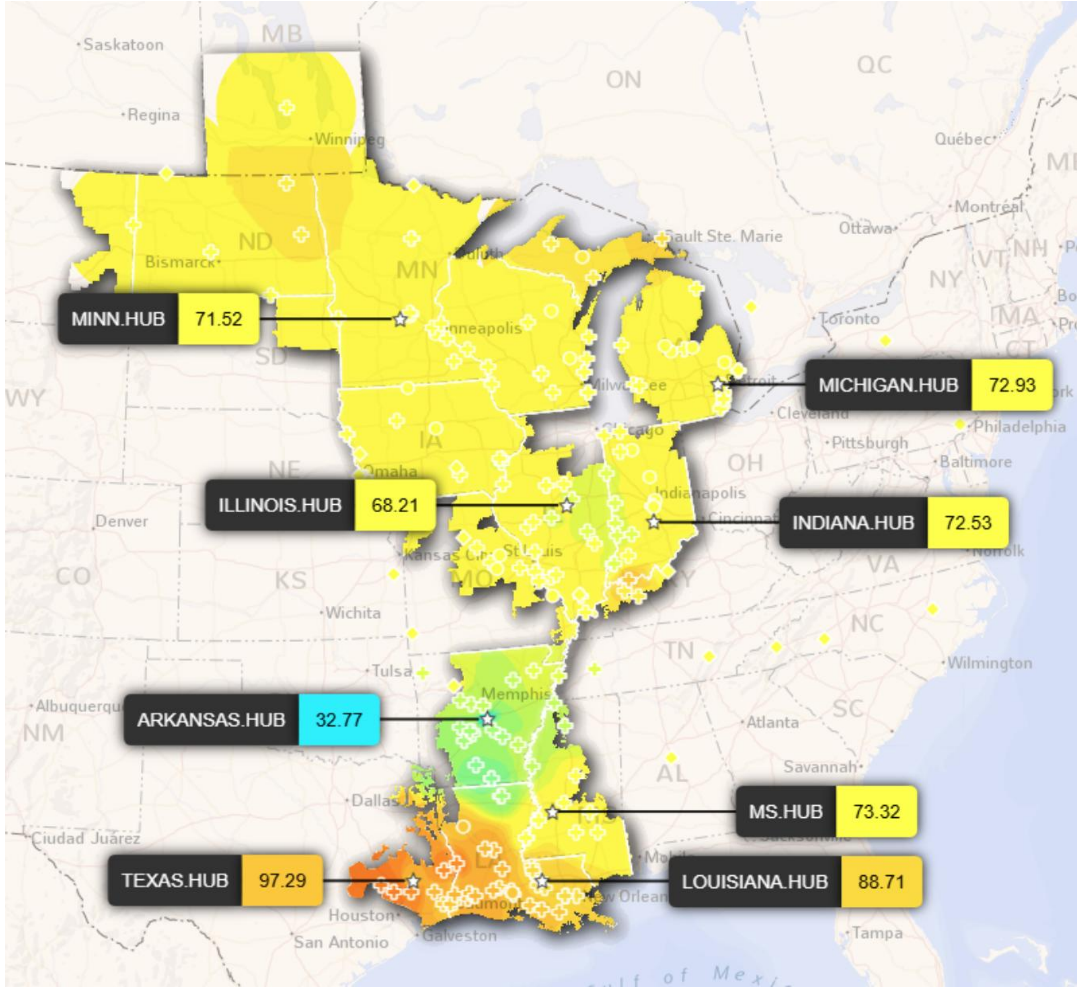
Example: ERCOT Security Sequence

- The ERCOT Nodal Protocols document details the process used by ERCOT.
  - RUC is Reliability Unit Commitment
  - DRUC is the Day-Ahead Reliability Unit Commitment
  - HRUC is the Hourly Reliability Unit Commitment
- The most recent documents are at www.ercot.com/mktrules/nprotocols/current.



ERCOT and MISO LMPs, Feb 20, 2025 at about 9am





Images: www.ercot.com/content/cdr/contours/rtmLmp.html, api.misoenergy.org/misortwd/lmpcontourmap.html

OPF Problem Formulation

- The OPF is usually formulated as a minimization with equality and inequality constraints:

$$\text{Minimize } F(\mathbf{x},\mathbf{u})$$

$$\mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0}$$

$$\mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where **x** is a vector of dependent variables (such as the bus voltage magnitudes and angles), **u** is a vector of the control variables, F(**x**,**u**) is the scalar objective function, **g** is a set of equality constraints (e.g., the power balance equations) and **h** is a set of inequality constraints (such as line flows).

Two Example OPF Solution Methods

- Non-linear approach using Newton's method
  - handles marginal losses well, but is relatively slow and has problems determining binding constraints
  - Generation costs (and other costs) represented by quadratic or cubic functions
- Linear Programming
  - fast and efficient in determining binding constraints, but can have difficulty with marginal losses.
  - used in PowerWorld Simulator
  - generation costs (and other costs) represented by piecewise linear functions
- Both can be implemented using an ac or dc power flow.

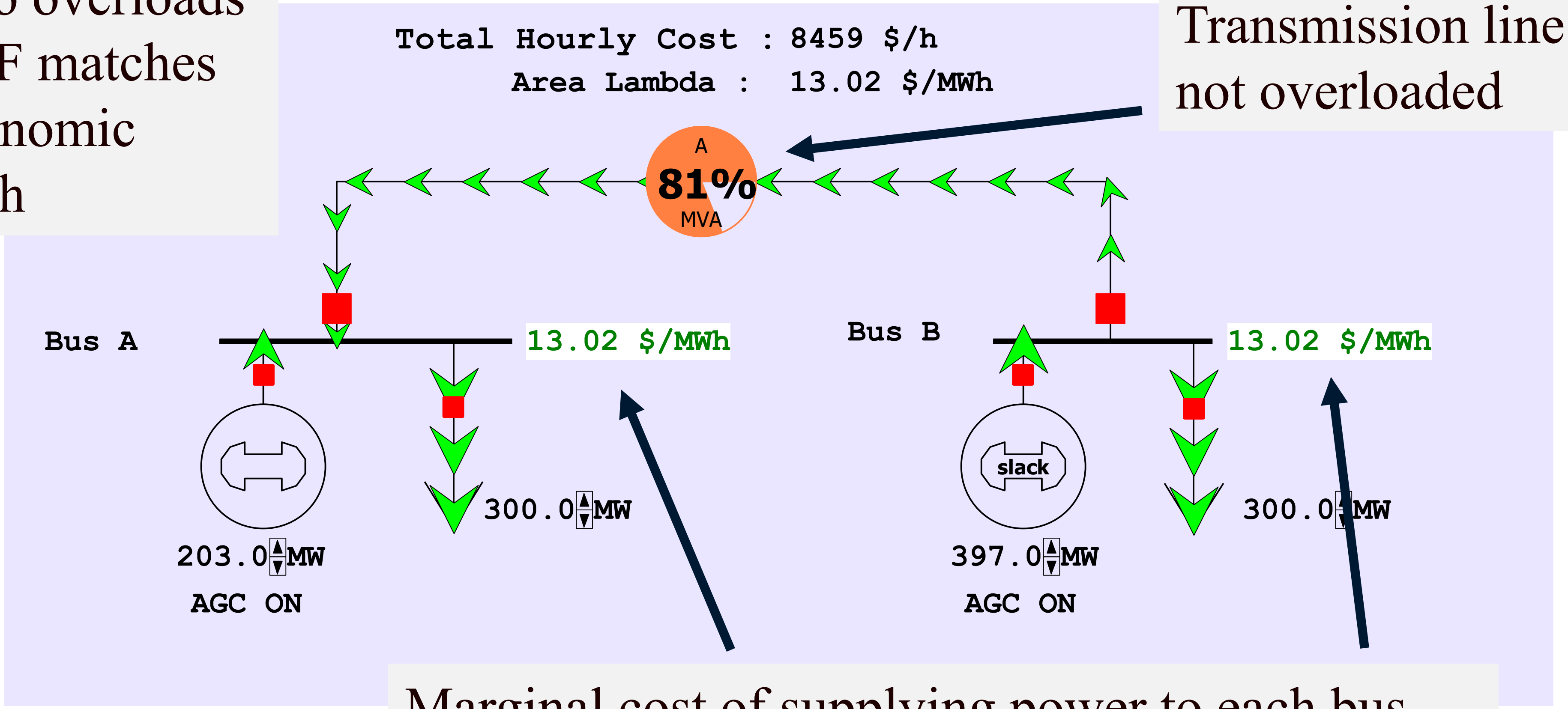
LP OPF Solution Method

- Solution iterates between
  - solving a full ac or dc power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account non-linearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost

Two Bus with Unconstrained Line

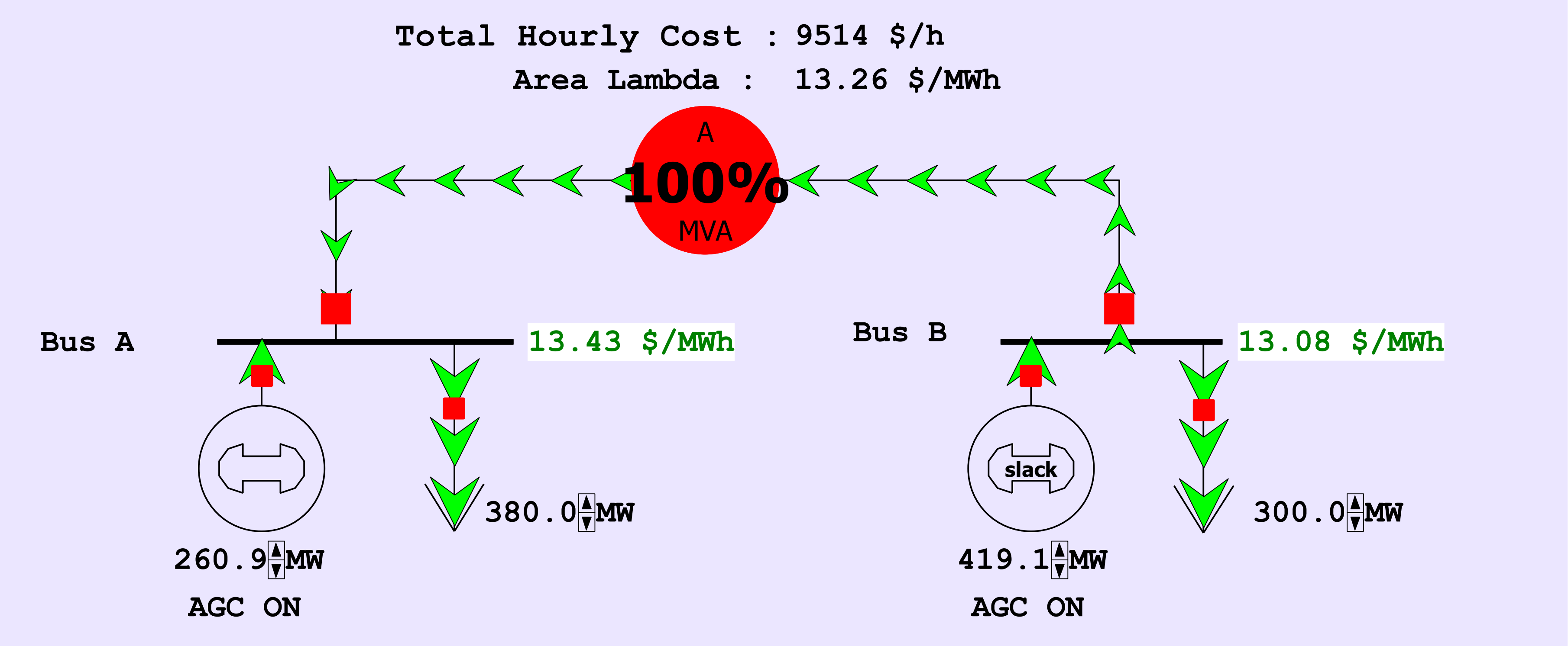
With no overloads the OPF matches the economic dispatch

Transmission line is not overloaded



Marginal cost of supplying power to each bus (locational marginal costs)

Two Bus with Constrained Line

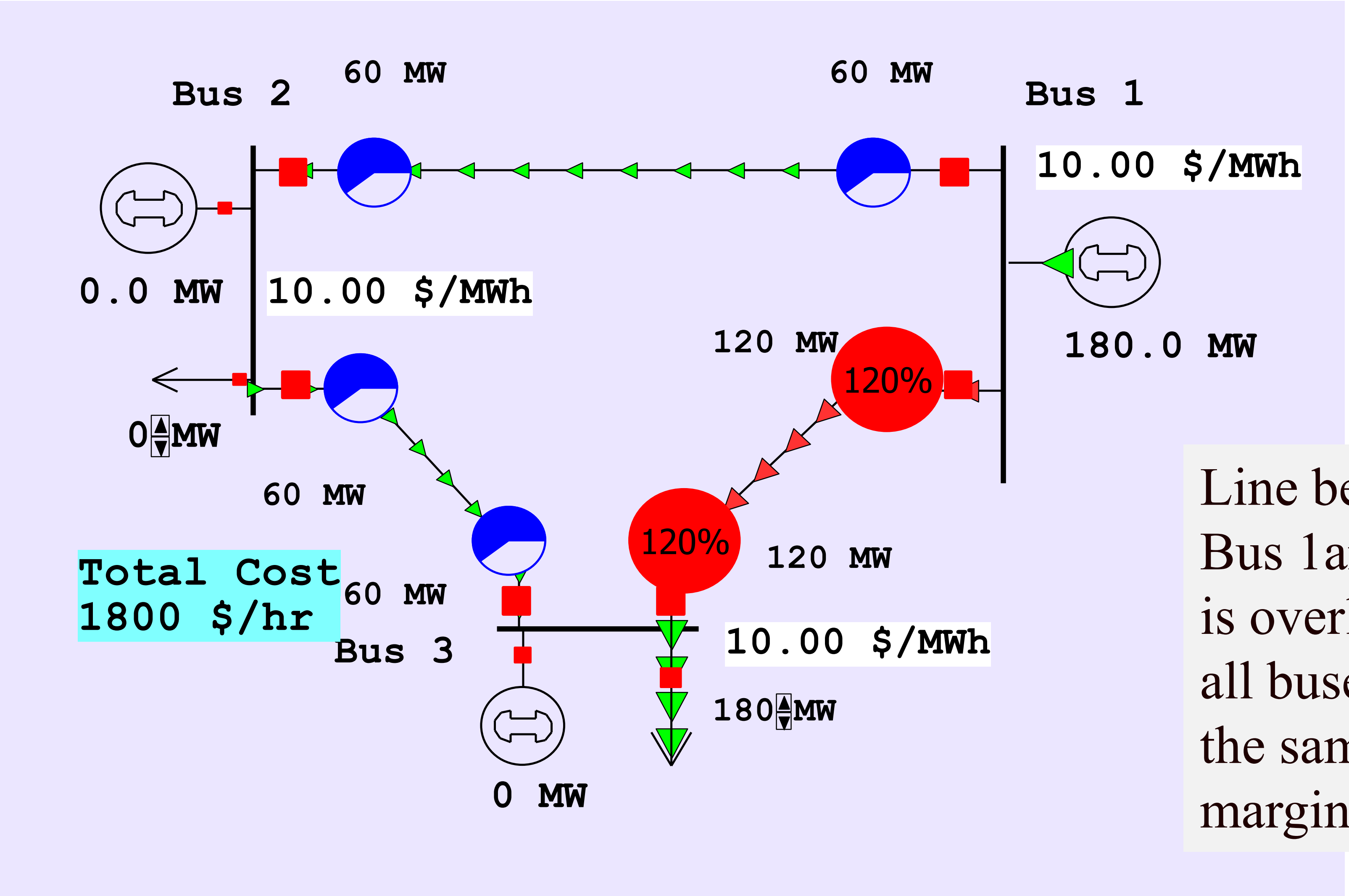


With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

Three Bus (B3) Example

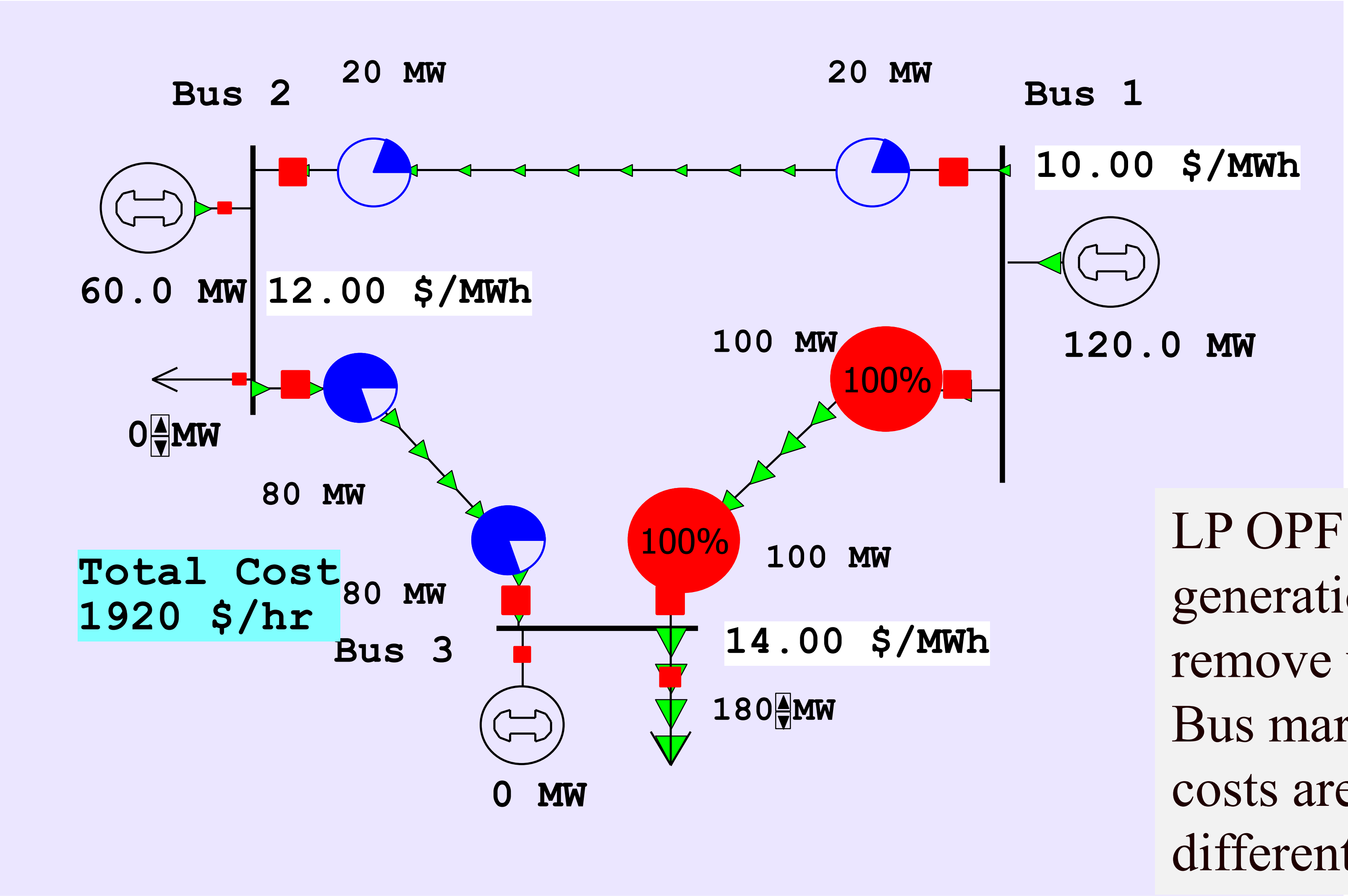
- Consider a three bus case (Bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit.
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 3.

B3 with Line Limits NOT Enforced



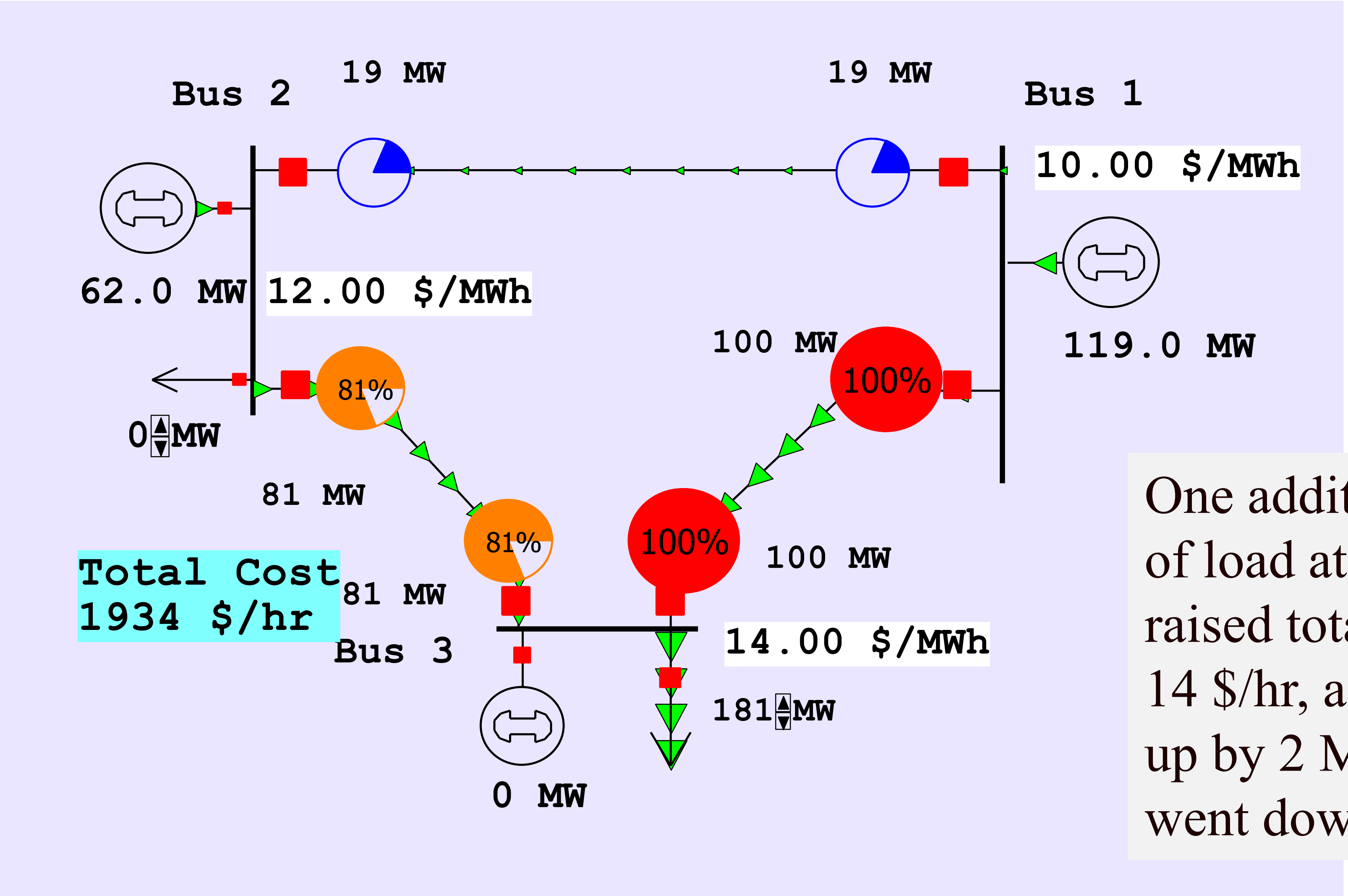
Line between Bus 1and Bus 3 is overloaded; all buses have the same marginal cost

B3 with Line Limits Enforced



LP OPF changes generation to remove violation. Bus marginal costs are now different.

Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

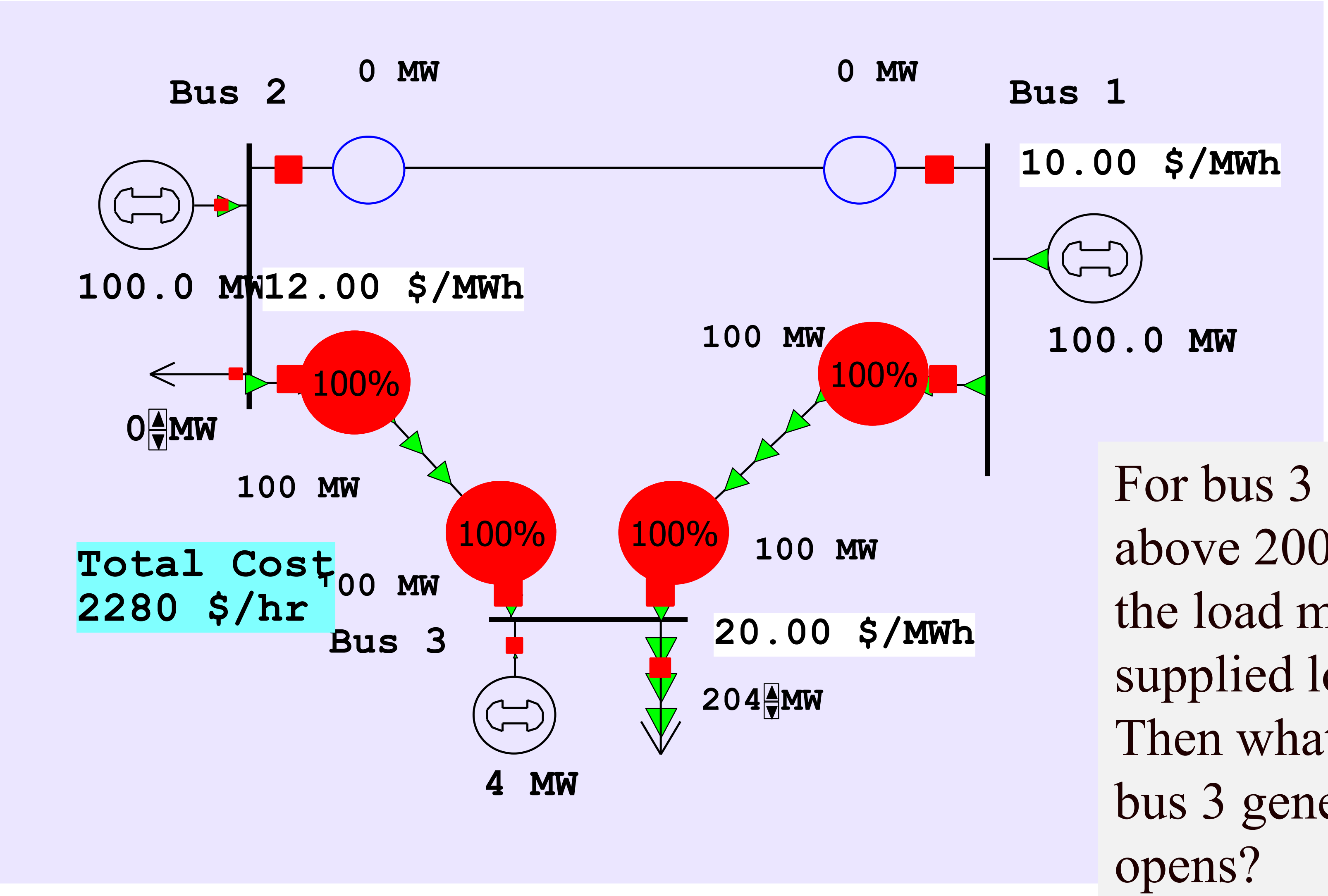
Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would “loop around” from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1to 3.

Why is bus 3 LMP \$ 14 / MWh, cont'd

- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - $\Delta P_{G1} + \Delta P_{G2} = 1$ MW
  - $2/3\ \Delta P_{G1} + 1/3\ \Delta P_{G2} = 0$; (no more flow on 1-3)
- Solving requires we up P_{G2} by 2 MW and drop P_{G1} by 1 MW -- a net increase of \$24 – \$10 = \$14.

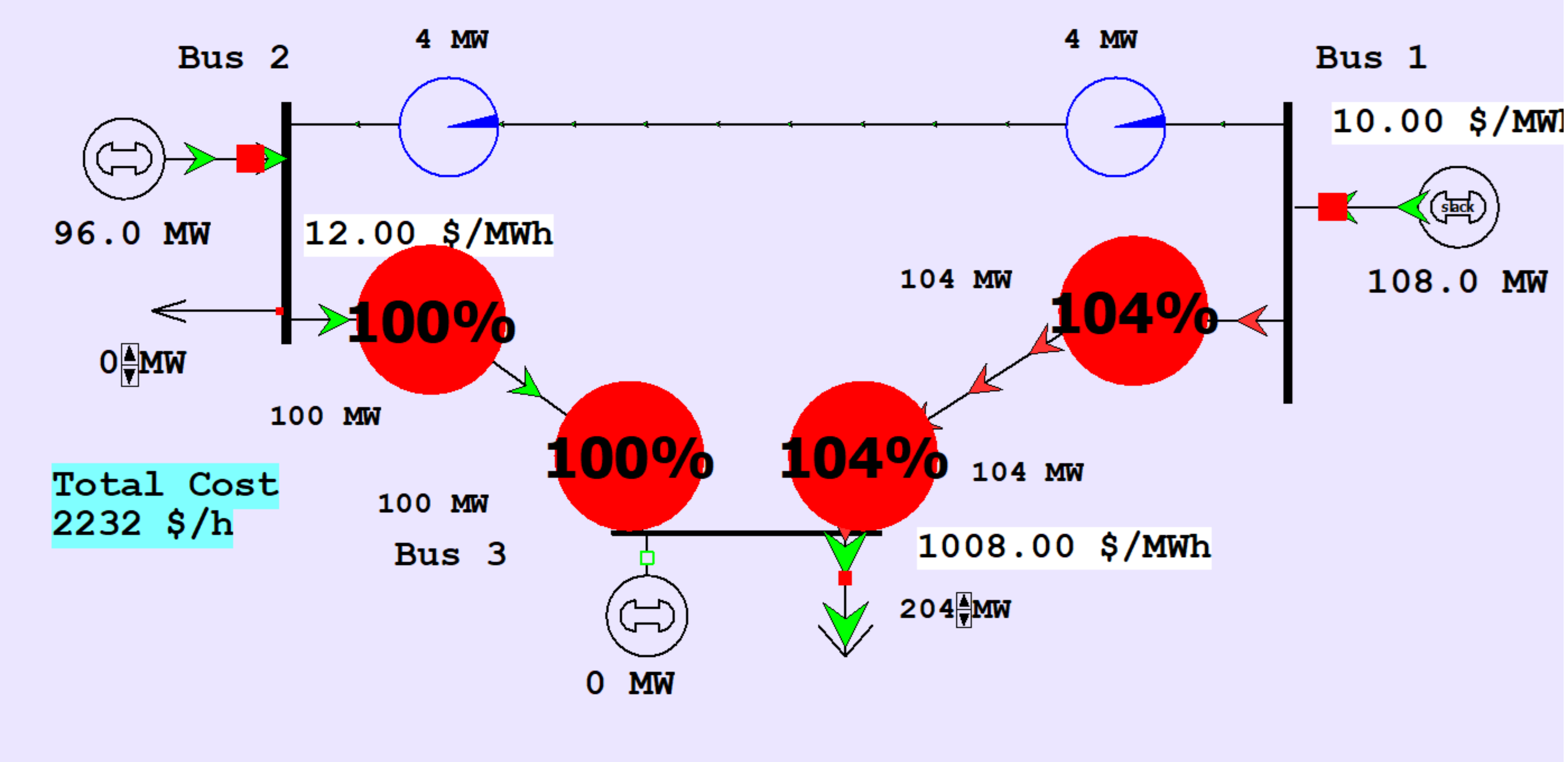
Both lines into Bus 3 Congested



For bus 3 loads above 200 MW, the load must be supplied locally. Then what if the bus 3 generator opens?

Both lines into Bus 3 Congested

An infeasible example can be created by opening the generator at Bus 3 with the Bus 3 load above 200 MW. There is no way to serve the load without overloading a transmission line.



Quick Coverage of Linear Programming (LP)

- LP is probably the most widely used mathematical programming technique.
- It is used to solve linear, constrained minimization (or maximization) problems in which the objective function and the constraints can be written as linear functions.
- Linear programming got its start during WWII, but it was secret throughout the war.
- George Dantzig published the simplex method in 1947, and John von Neuman developed the theory of duality around the same time; it became widely used .

Example Problem 1

- Assume that you operate a lumber mill which makes both construction-grade and finish-grade boards from the logs it receives. Suppose it takes 2 hours to rough-saw and 3 hours to plane each 1000 board feet of construction-grade boards. Finish-grade boards take 2 hours to rough-saw and 5 hours to plane for each 1000 board feet. Assume that the saw is available 8 hours per day, while the plane is available 15 hours per day. If the profit per 1000 board feet is $100 for construction-grade and $120 for finish-grade, how many board feet of each should you make per day to maximize your profit?

Problem 1 Setup

Let x_1=amount of cg, x_2= amount of fg

Maximize $100x_1 + 120x_2$

s.t. $2x_1 + 2x_2 \le 8$

$3x_1 + 5x_2 \le 15$

$x_1, x_2 \ge 0$

Notice that all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are seeking to determine the values of x_1 and x_2

Example Problem 2 (Nutritionist Problem)

- A nutritionist is planning a meal with 2 foods: A and B. Each ounce of A costs $ 0.20, and has 2 units of fat, 1 of carbohydrate, and 4 of protein. Each ounce of B costs $0.25, and has 3 units of fat, 3 of carbohydrate, and 3 of protein. Provide the least cost meal which has no more than 20 units of fat, but with at least 12 units of carbohydrates and 24 units of protein.

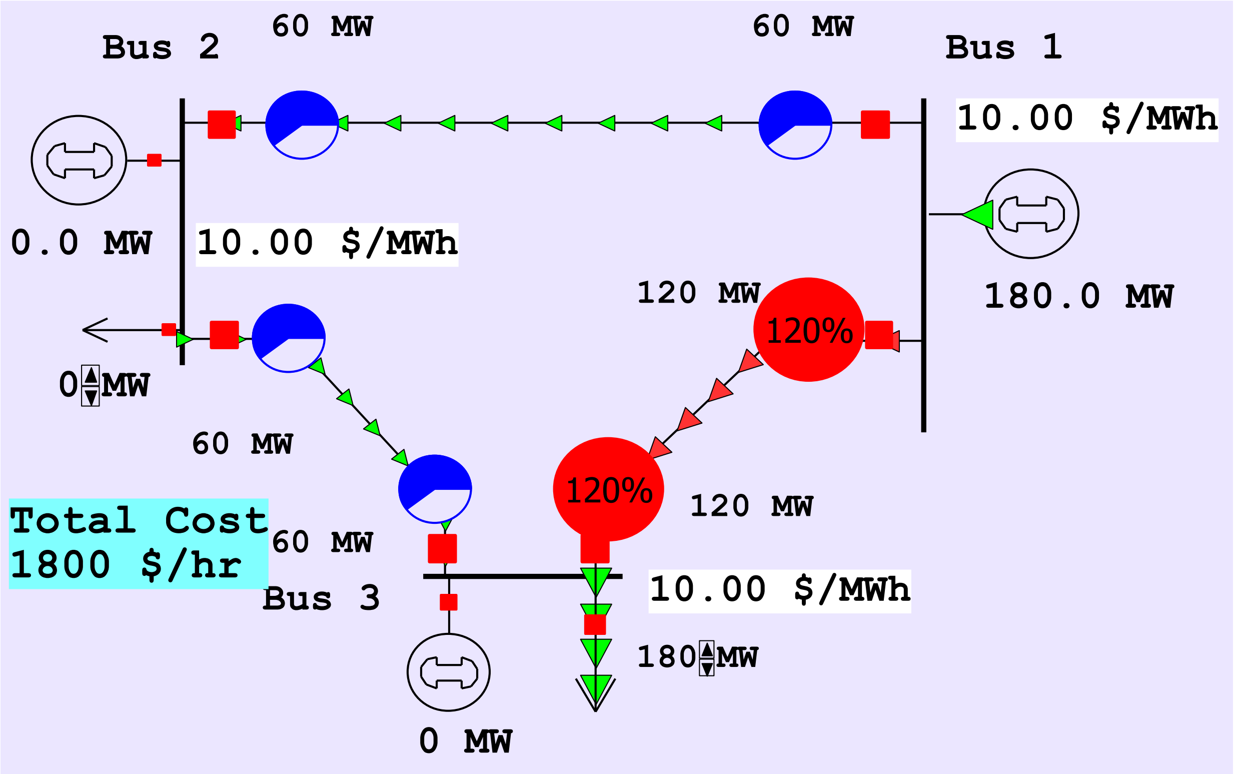
Let x_1=ounces of A, x_2= ounces of B

$$\begin{aligned} \text{Minimize} \quad & 0.20x_1 + 0.25x_2 \\ \text{s.t.} \quad & 2x_1 + 3x_2 \le 20 \\ & x_1 + 3x_2 \ge 12 \\ & 4x_1 + 3x_2 \ge 24 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Again all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are again seeking to determine the values of x_1 and x_2; notice there are also more constraints than solution variables

Three Bus Case Formulation

- For the earlier three bus system given the initial condition of an overloaded transmission line, minimize the cost of generation such that the change in generation is zero, and the flow on the line between buses 1 and 3 is not violating its limit.
- Can be setup considering the change in generation, $(\Delta P_{G1}, \Delta P_{G2}, \Delta P_{G3})$



Three Bus Case Problem Setup

Let $x_1 = \Delta P_{G1}$, $x_2 = \Delta P_{G2}$, $x_3 = \Delta P_{G3}$

Minimize $\quad 10x_1 + 12x_2 + 20x_3$

s.t. $\quad \frac{2}{3}x_1 + \frac{1}{3}x_2 \le -20$ — Line flow constraint

$x_1 + x_2 + x_3 = 0$ — Power balance constraint

enforcing limits on $x_1,\ x_2,\ x_3$

LP Standard Form

The standard form of the LP problem is

Minimize $\mathbf{c}\,\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} = \mathbf{b}$

$\mathbf{x} \geq \mathbf{0}$

Maximum problems can be treated as minimizing the negative

where $\mathbf{x}$ = n-dimensional column vector

$\mathbf{c}$ = n-dimensional row vector

$\mathbf{b}$ = m-dimensional column vector

$\mathbf{A}$ = m×n matrix

For the LP problem usually n>> m

The previous examples were not in this form!