Reliable Calculation of Power System Low-voltage Solutions

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Abstract

This paper develops a method for reliably determining the set of low-voltage solutions which are closest to the operable power flow solution. These solutions are often used in energy methods along with other techniques for power system voltage stability assessment. An algorithm is presented which provides good initial guesses for these solutions. Results are demonstrated on small systems and the IEEE 118 bus system.

I. Introduction

Due to increased competitive pressure, the use of high redundancy to maintain adequate power system reliability is becoming a luxury that we can no longer afford. Future power systems must do more with less, and in particular will need to be pushed much closer to their operational limits. Such operation will require quicker and more accurate determination of these limits, along with effective techniques for control intervention following unexpected system disturbances. Of particular concern is the determination of the system voltage stability limits.

One class of methods for determination of the voltage stability limits utilizes the distance between the operable power flow solution and an alternative (or low-voltage) solution. This includes the use of state space measures [1], [2] and energy methods [3] for quasi-static voltage stability assessment. Additionally, low-voltage solutions are used in the determination of the exit point for dynamic voltage stability assessment [4], [5]. The common element of these methods is the need to determine at least one of these low-voltage solutions. It has long been recognized that the nonlinear power flow equations can have multiple solutions [6]. While the actual number of solutions depends upon system loading, the number of potential solutions can be quite large; the maximum number for an n+1 bus system is estimated to be 2^n in [7] and $\binom{2n}{n}$ in [8]. For these methods to be practical, there is a need for an efficient method to determine the appropriate low-voltage solutions reliably.

A number of methods have been proposed for determining some (or all) of these solutions. In [7] an algorithm is presented that attempts to determine all the low-voltage solutions. The algorithm can be summarized as follows. Assume the operable solution $\mathbf{V}^s$ has been calculated (where $\mathbf{V}$ is a vector of the complex voltages at all buses in the system). Then for each bus in the system a low initial voltage guess V_i^u is calculated using a closed form equation [7], with the assumption that the voltages at all other buses are fixed. Then select either V_i^u or V_i^s as elements of the initial voltage guess for the rectangular Newton-Raphson (N-R) power flow solution. Form all the 2^n possible combinations of the initial voltage guess vectors, and attempt to solve the power flow. The number of solutions actually found with this method depends upon the convergence characteristics of the N-R power flow. In [8] a homotopy-based method is developed which offers a method for determining all of the system solutions, but requires computation proportional to $\binom{2n}{n}$. An improved homotopy method to determine all solutions is presented in [9], which requires computation proportional to the product of the system size and the number of solutions the system actually has. Genetic algorithms for solving this problem are discussed in [10]. While these methods promise the determination of all the system solutions, they all require computation well beyond that considered reasonable for on-line usage.

Luckily, results from [11] indicate that the search for low-voltage solutions can be restricted to those solutions corresponding to equilibrium points with a single positive eigenvalue associated with the Jacobian of the linearized system dynamic equations (i.e., the type-one solutions). For certain realistic classes of load models, the stability of an equilibrium point can be determined from the eigenvalues of the polar form of the power flow Jacobian [12], [13], [14]. Furthermore, only those solutions associated with relatively low energy margins need to be determined. Nevertheless, determining these solutions quickly enough for on-line usage has proven difficult. In [7] a method which requires approximately n power flow solutions was suggested. The computation was further reduced in [11] and [15] by employing screening techniques using equivalent systems. In [16] a method is presented which exploits the convergence characteristics of the rectangular N-R power flow to determine the closest low-voltage solution. This method works well, provided the two solutions are close enough; otherwise no information is provided about existing low-voltage solutions.

The goal of this paper is to develop a robust method for determining this set of low-voltage solutions "closest" to the operable solution. The method presented here utilizes the structure of the power flow equations to determine good initial guesses for these solutions. It is general enough to determine more than just a single low-voltage

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solution. Results are presented for tests on small systems and the IEEE 118 bus system.

II. Power Flow Low-Voltage Solutions

The problem is introduced by briefly considering the characteristics of low-voltage solutions using the symmetric three bus system shown in Figure 1 (line impedances are per unit using a 100 MVA base). Buses 1 and 2 are modeled as constant PQ load buses, while bus 3 is the system slack with a constant per unit voltage of 1.0 + j0.0. For the base case loading shown in the figure, the system has four real power flow solutions given in Table 1. Solution A corresponds to the operable solution, solutions B and C are type-one, while solution D is type-two.

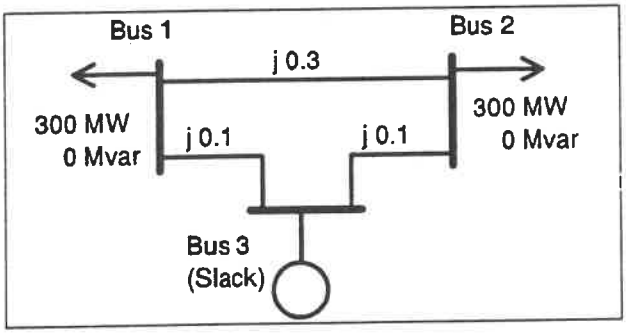

Figure 1: Three Bus System

Voltage	Solution A	Solution B	Solution C	Solution D
V_1	0.9487	0.2597	0.6930	0.3162
θ_1	-18.43°	-78.78°	-29.88°	-71.56°
V_2	0.9487	0.6930	0.2597	0.3162
θ_2	-18.43°	-29.88°	-78.78°	-71.56°

Table 1: Three Bus Base Case Solutions

As the system loading is varied, the locations of these solutions will also vary. In particular, as the loading is increased towards a point of maximum loadability, the number of solutions will tend to decrease until at some point only a single type-one solution and the operable solution remain. Maximum loadability occurs when these two solutions coalesce at a point on the maximum loadability boundary (denoted by Σ) in a saddle-node bifurcation. System loadability can be monitored by tracking the distance between the operable solution and this type-one solution. However a key point is that unless one knows a priori how system loading will change, it is impossible to know with absolute (and sometimes even reasonable) certainty the identity of this type-one solution. Therefore a number of solutions which are close to the operable solution should be determined.

The advantage of the approach presented here is that it provides a method of voltage stability assessment which is independent of any assumed path to the loadability boundary. This quality can be extremely important in trying to accurately assess system security under the unusual operating conditions often associated with proximity to voltage instability. While it is true that many utilities have a good idea of load participation under normal operating conditions, this information is much more difficult to ascertain under such stressed conditions for which little historical data is available. Furthermore, as access to transmission systems becomes more open, there will be an increase in potentially harmful third party transactions of which the operator may have little knowledge and control.

As an example of how system loading affects the solutions, consider the following three figures which show solution variation for the three bus system in the V_1-V_2 plane as load is increased with different load participation factors. In Figure 2 the load increase at bus 1 is twice the load increase at bus 2, while in Figure 3 the opposite occurs, with the load increase at bus 2 twice the value at bus 1. In the first case the heavy load increase at bus 1 ultimately causes solutions A and B to coalesce as maximum loadability is reached (at a total load of 973 MW). Note that solutions C and D also coalesce, but at a substantially lower load of only 756 MW. The opposite occurs in the second case with solutions A and C now coalescing at maximum loadability (B and D coalesce at a much lower load).

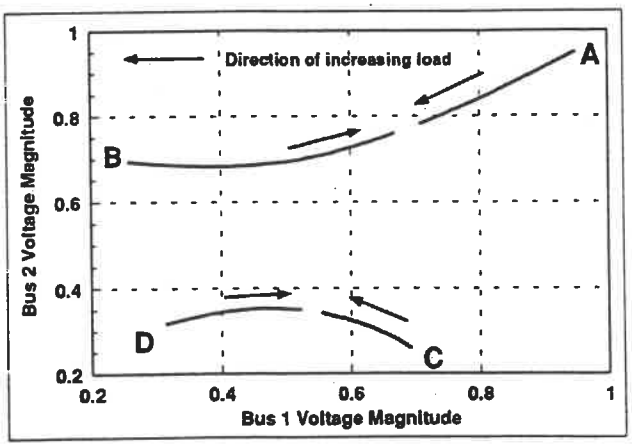

Figure 2: Maximum Load Participation at Bus 1

In the Figure 4 case the load participation is initially higher at bus 1, causing solutions A and B to move toward each other. However, when the bus 1 load is at 1.3 times the base value, the participation is switched so that most of

the load increase occurs at bus 2. The maximum loadability boundary is then ultimately reached through solutions A and C coalescing. This example illustrates that even though a low-voltage solution may at some point be closest to the operable solution (as B is early on), changing system conditions can result in a final bifurcation with another solution. Hence the potential need to calculate more than one low-voltage solution.

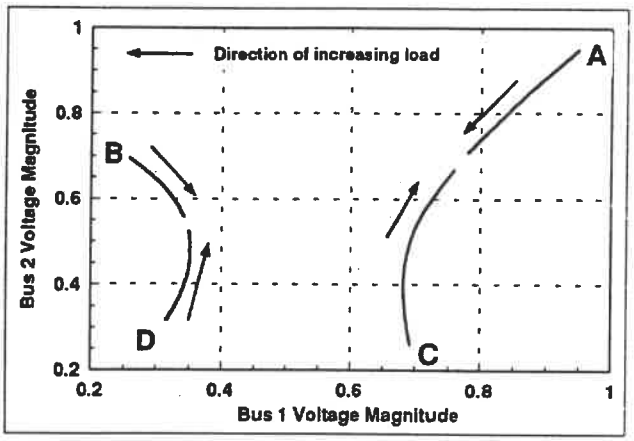

Figure 3: Maximum Load Participation at Bus 2

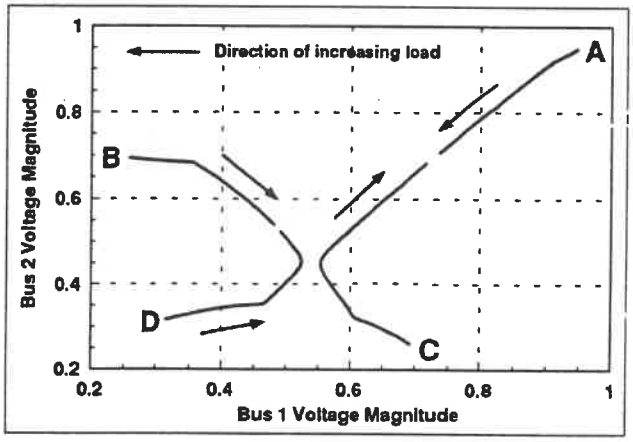

Figure 4: Varying Load Participation

The type-one low-voltage solutions are often identified by the bus number with the lowest voltage magnitude. Alternatively, and almost always equivalently, the solutions can be classified based upon information derived from an eigenvector of the polar form of the solution Jacobian. This is based upon the fact that at the point of bifurcation the Jacobian is singular, with the initial direction of voltage collapse along the eigenvector corresponding to the zero eigenvalue of the polar form of the power flow Jacobian [17], [18]. This eigenvector usually has relatively large components at only a few buses. Additionally, for loading significantly less than this critical value, the eigenvector associated with the positive eigenvalue of $J(x^u)$ still tends to approximate this direction [11]. The solution can be identified from the bus number corresponding to the largest magnitude in this eigenvector. Thus, for the three-bus example, solution B is denoted as the bus 1 low-voltage solution, while solution C is the bus 2 solution (solution A is the operable solution, while solution D is type-two).

For loading very close to the bifurcation point, the eigenvector associated with the least negative eigenvalue of the operable solution also tends to approximate this direction. However, numerical testing indicates that this approximation is valid over a much smaller range of parameter variation back from the bifurcation surface than for the low-voltage solutions. Therefore it does not appear that this eigenvector can be reliably used to identify vulnerable areas in large systems.

III. Relationships between Solutions

To develop the necessary relationships between the operable solution and the low-voltage solutions, consider the power flow equations for an n+1 bus system (with bus n+1 as the system slack):

$$S = f(x^*) \tag{1}$$

where S is a vector of the constant real and reactive power load minus generation at all the non-generator (PQ) buses, and the real power plus voltage magnitude equality constraints at all generator (PV) buses (except the slack). For convenience assume that the first m buses are PV, and the remainder are PQ:

$$S = [P_1, ..., P_n, V_1^2, ..., V_m^2, Q_{m+1}, ..., Q_n]^T \tag{2}$$

Additionally, x^* is a power flow solution in rectangular coordinates, $V_i = e_i + jf_i$,

$$x = [e_1, e_2, ..., e_n, f_1, f_2, ..., f_n]^T, \tag{3}$$

f is the function of the bus power/voltage constraints

$$f = [f_{p1}(x), ..., f_{pn}(x), f_{q1}(x), ..., f_{qn}(x)]^T \tag{4}$$

with

$$f_{pi} = - \sum_{j=1}^{n} [\, e_i(e_jG_{ij} - f_jB_{ij}) + f_i(f_jG_{ij} + e_jB_{ij}) \,] \tag{5a}$$

and

$$f_{qi} = e_i^2 + f_i^2 \tag{5b}$$

at the PV buses and

$$f_{qi} = -\sum_{j=1}^{n} [\, f_i(e_j G_{ij} - f_j B_{ij}) - e_i(f_j G_{ij} + e_j B_{ij}) \,] \tag{5c}$$

at the PQ buses. The bus admittance matrix is G + jB.

Because the power balance equations are a set of quadratic equations with no first order terms, the Taylor series expansion of (1) can be written exactly as [19]:

$$S = f(x^*) = f(x + \Delta x) = f(x) + J(x)\,\Delta x + f(\Delta x) \tag{6}$$

where Δx is the difference between an arbitrary voltage x and a solution x^*, while $J(x)$ is the power flow Jacobian. An additional consequence of (1) being quadratic is that for an arbitrary scalar μ,

$$f(\mu \Delta x) = \mu^2 f(\Delta x) \tag{7}$$

In order to develop an improved algorithm to determine the low-voltage solutions, we make four observations. First, consider two unique power flow solutions where x^s is the operable power flow solution, while x^u is a type-one low-voltage solution. Furthermore let x^m be the midpoint between the two solutions. Then define

$$\Delta x^m = x^s - x^m = x^m - x^u \tag{8}$$

From (6) and (7) it is clear

$$f(x^s) = f(x^m) + J(x^m)\,\Delta x^m + f(\Delta x^m) = S \tag{9}$$

$$f(x^u) = f(x^m) - J(x^m)\,\Delta x^m + f(\Delta x^m) = S \tag{10}$$

Subtracting (10) from (9) gives

$$0 = 2\,J(x^m)\,\Delta x^m \tag{11}$$

Since by definition $\Delta x^m \neq 0$, $J(x^m)$ is singular indicating that x^m is a point on the maximum loadability boundary Σ, and that Δx^m is the eigenvector associated with the zero eigenvalue of $J(x^m)$. Since x^m is on the line segment joining x^s and x^u, it is the boundary point closest (in state space) to both solutions. The corresponding load level for x^m is $f(x^m)$. Thus, starting from the operable solution x^s, by knowing x^m we could directly determine x^u.

The second observation provides a method for approximating Δx^m. Again using (6), we can write

$$f(x^m) = f(x^s) - J(x^s)\,\Delta x^m + f(\Delta x^m) \tag{12}$$

Then by subtracting (9) from (12), and recognizing that $J(x^m)\,\Delta x^m = 0$, we can directly solve for Δx^m

$$f(x^m) - f(x^s) = -f(x^m) + f(x^s) - J(x^s)\,\Delta x^m \tag{13}$$

$$2\,[J(x^s)]^{-1}\,[f(x^m) - f(x^s)] = \Delta x^m \tag{14}$$

Thus, if we know the "midpoint mismatch", $[f(x^m) - f(x^s)]$, or at least have a reasonable approximation, we could determine Δx^m.

The third and fourth observations are used to get this approximation. First, we use the optimal multiplier result of [19] which states that once a direction Δx has been determined, we can directly (and efficiently) solve for a scalar optimal multiplier μ which minimizes the cost function in that direction.

$$F(x) = \frac{1}{2}\,[S - f(x^s + \mu \Delta x)]^T\,[S - f(x^s + \mu \Delta x)] \tag{15}$$

The optimal multiplier result is derived by first using the exact Taylor series expansion from (6) to expand

$$f(x^s + \mu \Delta x) = f(x^s) + \mu\,J(x^s)\,\Delta x + \mu^2 f(\Delta x)$$

Then define

$$a = S - f(x^s) \tag{16}$$

$$b = -J(x^s)\,\Delta x \tag{17}$$

$$c = -f(\Delta x) \tag{18}$$

and rewrite (15)

$$F(x) = \frac{1}{2}\,[a + \mu\, b + \mu^2 c]^T\,[a + \mu\, b + \mu^2 c] \tag{19}$$

The values of μ which minimize the cost function are determined by differentiating (19) with respect to μ, which gives

$$g_0 + g_1 \mu + g_2 \mu^2 + g_3 \mu^3 = 0 \tag{20}$$

where for the general problem

$$g_0 = a^T b \tag{21}$$

$$g_1 = b^T b + 2\,a^T c \tag{22}$$

$$g_2 = 3\,b^T c \tag{23}$$

$$g_3 = 2\,c^T c \tag{24}$$

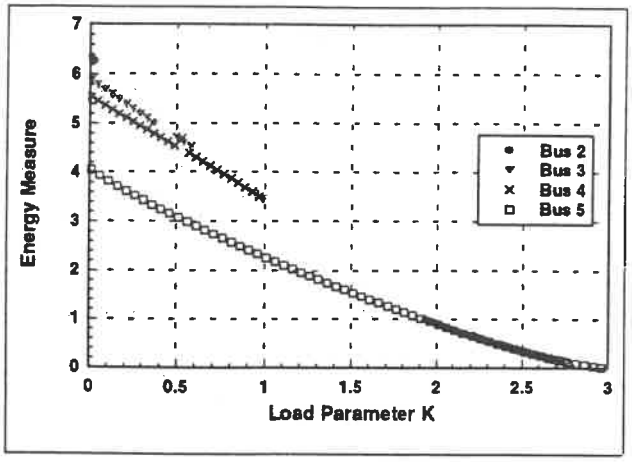

Figure 6: Results of Algorithm for 5-Bus System

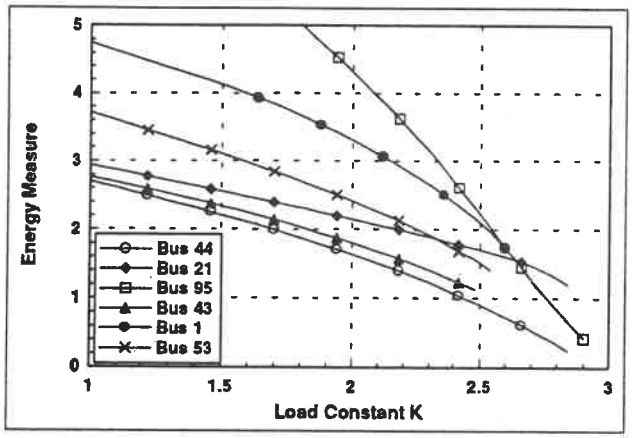

Figure 7: Results of Algorithm for 118-Bus System

and Figure 7 were obtained with prior knowledge of some of the most stressed buses in the system. Since computational expense is a primary concern, efficiently determining the set K of buses at which a low-voltage solution is likely to occur is a high priority. As discussed in the previous section, for very large systems, an initial screening method such as the FBBV method could be employed. However, because the intial guess x^0 is quickly determined, screening may often be accomplished most conveniently by monitoring the norm of $f(x^0)$-S. Figure 8 illustrates how the cost function at the initial guess x^0 varies with the energy measure corresponding to the type-one solution found from the initial guess for k between 2.0 and 3.0. If more than one initial guess leads to a particular solution, then only the one with the least initial cost is included in this plot. This approach was deemed appropriate because guesses which converge to the same solution tend to correspond to buses which neighbor each other. Thus, while the identity of the most stressed bus may be somewhat clouded, its location in the system is retained. With few exceptions, it seems that the initial guesses which incur the least relative costs will ultimately converge to the lowest-energy, and therefore most critical, low-voltage solutions. Thus, once x^0 has been determined efficiently for each bus in the system, the search for low-voltage solutions may be originated from only the least-cost initial guesses. The merits of this screening method were demonstrated on the IEEE 118-bus system as shown in Figure 9. The buses examined in Figure 9 are among the twenty buses for which the initial value of $f(x^0)$-S, averaged over 2.0<=k<=3.0, is smallest (the remaining buses of this set either do not converge or converge over only a very small range in k). Included in this set of buses are all of those identified in Figure 5 and several additional curves which suggest the presence of additional low-energy solutions. Indeed, when the Simplified Method was employed again for these newfound solutions, nearly all of them were recovered, as shown in Figure 10. Consequently, a screening method based upon the norm of $f(x^0)$-S provides an efficient means for tracking several low-voltage solutions by narrowing the list of candidates for low-voltage solutions to a select few. More work is required to evaluate further the reliability of this screening method.

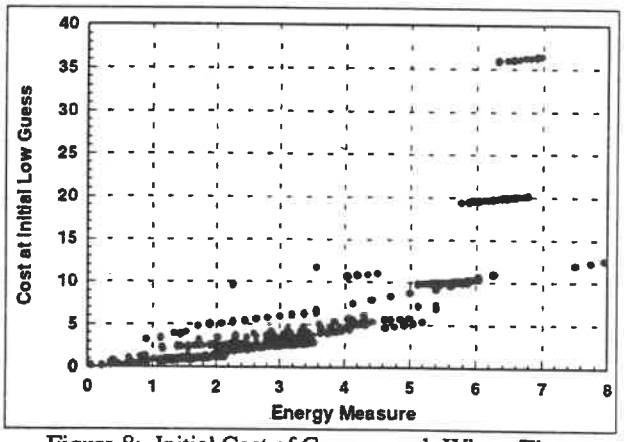

Figure 8: Initial Cost of Guesses and Where They Convergence

A final issue to address is the strength of the approximation of the components of ΔS. Recall that it has been observed that when collapse occurs, it tends to be centered about a particular bus, and that the initial direction of collapse usually involves large changes only in reactive power at that bus. Thus, it was suggested that all elements of ΔS except the reactive power mismatch at the center bus be set to zero. Figure 11 evaluates this approximation for the 5-bus system by showing how the real and reactive mismatches at x^m at all buses, normalized with respect to the bus 5 reactive mismatch,

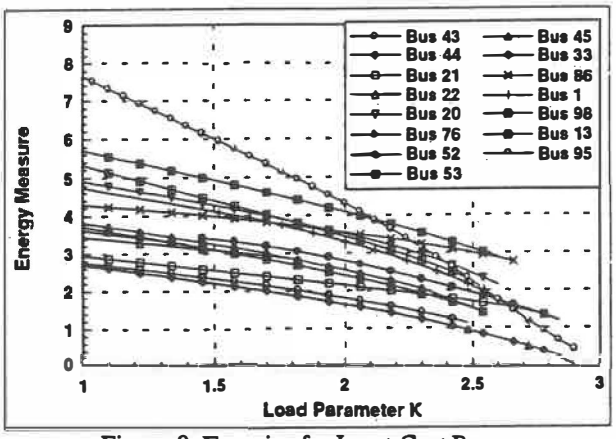

Figure 9: Energies for Least-Cost Buses

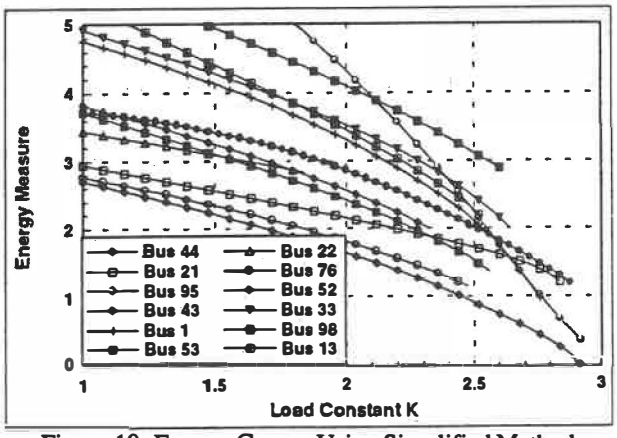

Figure 10: Energy Curves Using Simplified Method

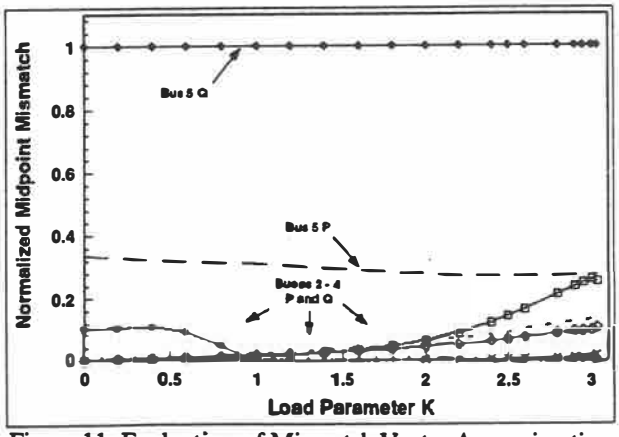

Figure 11: Evaluation of Mismatch Vector Approximation

vary with k as the bus 5 low solution is traced. These curves suggest that the center bus's reactive power mismatch at x^{m} generally tends to be at least three times as large as any of the other components of ΔS. This observation lends some credence to the approximation. Of course, the actual components of $f(x^{m})$-S will be system-dependent and will hinge heavily on load participation. Since some preliminary experimentation with the Stagg system revealed that modifying the elements of ΔS according to these normalized mismatches improved convergence of this algorithm as collapse approached, further study of this approximation may be worthwhile.

VI. Conclusion

A computationally efficient and reliable method for calculating the lowest-energy type-one solutions has been introduced and demonstrated on both the Stagg-El Abiad 5-bus and IEEE 118-bus systems. The algorithm poses little additional computation and converges to the low-voltage solution within a few iterations. Moreover, it is general enough to track several low-voltage solutions, and it provides a convenient screening facility for discerning which low-voltage solutions to investigate. Future work will investigate how the algorithm performs for other load participations and how the components of ΔS may be adjusted to improve convergence.

VII. Acknowledgments

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