Quickly Detecting and Correcting Alternative Power Flow Solutions with Large Grid Results

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Abstract—**This paper presents a process for quickly detecting and correcting alternative power flow solutions to the desired (operable) solution. As is well known, the positive sequence power flow has a potentially large number of solutions, with typically only one operable solution. Usually, the power flow algorithm correctly converges to this solution, but sometimes it converges to an undesired alternative solution that might be difficult to differentiate from the operable solution. To address this issue the paper presents a method for quickly and reliably detecting these situations and for attempting to correct the undesired solution to the operable solution. Results are demonstrated on power systems ranging in size from three to 23,600 buses. The results show alternative solutions can be quickly detected relative to the power flow solution time and corrected more than 80% of the time.**

Index Terms—*power flow convergence, alternative power flow solutions*

I. Introduction

The power flow (PF) is widely used in power system analysis to solve the set of nonlinear power balance equations to determine the bus voltage magnitudes and angles, with the Newton-Raphson (NR) algorithm widely used [1]. Usually the PF converges to the solution at which the electric grid would actually operate, denoted here as the "operable solution" (OprS). However, as has been known for more than fifty years [2] [3], sometimes the PF converges to an alternative solution.

In the vast majority of cases the PF solution with the highest per unit (pu) voltages corresponds to the desired OprS since it represents how the grid would actually operate. In the literature this solution is sometimes called the stable equilibrium point (SEP) since with the use of common dynamic models it would be stable [4], [5]. The other solutions usually, but not always, have lower voltages and are known as either low-voltage solutions or alternative solutions [6], [7]; here the abbreviation AltS is used for these solutions. Since these solutions would be unstable with common dynamics they are also sometimes called unstable equilibrium points (UEPs), with the type of the solution denoted by the number of positive eigenvalues in the power flow Jacobian [5].

This paper's contribution is to present a computationally efficient algorithm to determine whether the NR has converged to an AltS and if it has to attempt to correct the AltS so it can be resolved to converge to the OprS. The paper's organization is as follows. Section II provides useful background material on the AltSs, and how the PF convergence is impacted by the initial voltage guess. Section III then presents the algorithms and the test power flow cases. Section IV then provides results for a large-system, while Section V concludes with recommendations and directions for future work. All the computational results presented here have been done using PowerWorld Simulator version 24 [8].

II. Background

The PF solves the bus power balance equations to determine the phasor voltages at each bus in the electric grid model. Since the impedances and bus voltages can be represented in either rectangular or polar form, there are a few common power flow formulations. Two common representations are the rectangular form from [9] (NRR) with the voltages given as $V_k = e_k + jf_k$, and the polar form [10] (NRP) with the voltages given as $V_k = V_k \angle \theta_k$. For a grid with N buses the real and reactive NRR power balance equations are given in (1) and (2),

$$P_{Gk} - P_{Lk}(V_k) = \sum_{n=1}^{N} \left[e_k \left(e_n G_{kn} + f_n B_{kn} \right) + f_k \left(f_n G_{kn} - e_n B_{kn} \right) \right] \quad (1)$$

$$Q_{Gk} - Q_{Lk}(V_k) = \sum_{n=1}^{N} \left[f_k \left(e_n G_{kn} + f_n B_{kn} \right) - e_k \left(f_n G_{kn} - e_n B_{kn} \right) \right] \quad (2)$$

where P_{Gk} and Q_{Gk} are the real and reactive generation at bus k, $P_{Lk}(V_k)$ and $Q_{Lk}(V_k)$ are the real and reactive loads, and G_{kn} and B_{kn} are the real and imaginary bus admittance matrix values. Note, this formulation allows the load values to be a function of their bus voltage magnitudes. While a constant power model is common (with a potential modification noted below), the well-known ZIP model [11] could also be used. Also, in the PF not all the voltages are independent variables. For example, at the slack bus the voltage magnitude and angle are fixed, and at generator (PV) buses the voltage magnitude is fixed, within their reactive power limits.

Since the power balance equations are nonlinear, if they have a solution they almost always have multiple solutions. An intuitive insight into the nature of these solution can be seen by noting that if the net power injection at bus k is zero (e.g., in an

unloaded grid) the power balance equations are satisfied by either an open circuit or a short circuit condition (i.e., $V_k = 0$) at the bus. Since in an unloaded network this could be true simultaneously at each bus, the N+1 bus grid could have potentially at least 2^N solutions [12], with other authors noting the possibility of even more solutions [13]. However, the number of actual solutions is usually much less, decreasing as the loading on the grid increases [7], [14] until right before the point of maximum loadability just the OprS and a single AltS exist. Using the notation of [5] this is a type-1 AltS. A consequence of this insight is while each AltS applies to the entire grid, each AltS can be roughly associated with inoperable conditions at one or more buses. These buses, and nearby neighboring buses, will be denoted as the AltS regions. For a type-1 AltS there is a single region.

A simple example of this is illustrated using the lossless three-bus grid from [15]. This example's oneline is shown in **Figure 1** with the figure also showing the OprS for the indicated load using the flow visualization approach of [16] in which the green arrows show the real power flow and the blue arrows the reactive power. At this loading level the case also has three AltSs with all four solutions given in Table 1. These four solutions are labeled as the OprS, AltS1 (the AltS with a low voltage at Bus 1), AltS2 (the AltS with a low voltage at Bus 2), and AltS3 (the low voltage at Buses 1 and 2). AltS1 and AltS2 are type-1 solutions, and AltS3 is a type-2 solution. Figure 2 shows the power flows associated with AltS1, with the blue arrows highlighting the unrealistically large amount of reactive power being supplied by the slack bus.
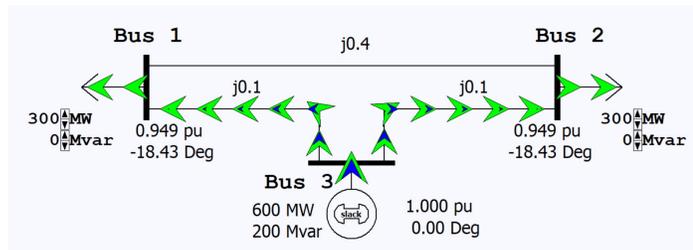

Figure 1: Lossless Three-Bus Oneline Showing the OprS

Table 1: Three-Bus Case Solutions

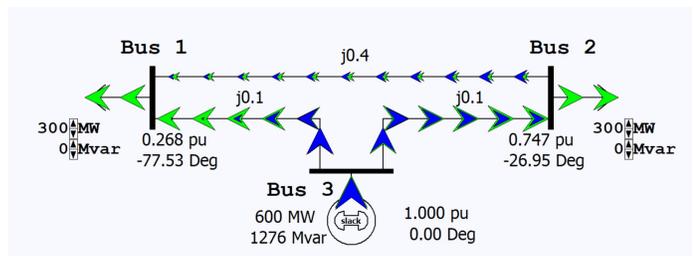| Voltage	OprS	AltS1	AltS2	AltS3
V_1 (deg)	0.949	0.268	0.747	0.316
θ_1 (deg)	-18.43	-77.53	-26.95	-71.57
V_2 (pu)	0.949	0.747	0.268	0.316
θ_2 (deg)	-18.43	-26.95	77.53	-71.57

Figure 2: Lossless Three-Bus Oneline Showing an AltS1

While the AltSs are occasionally desired, such as in some voltage stability applications ([17], [18], [19]), otherwise almost always the PF objective is to only converge to the OprS. There are a number of different techniques for helping to improve PF convergence to the AltS, with a few examples given in [20] [21], [22], [23]. One way to quantify the robustness of the NR PF's convergence to the OprS is to consider its region of attraction (ROA), defined as the set of voltage guesses that converge to a particular solution. The dimension of the ROA depends on the number of independent voltages values, which for an N bus system will be between N and 2N. Papers showing a 2-dimensional projection of the ROA include [7], [24] and [25].

One common PF solution technique, which has been used in commercial software for many years and is particularly germane here, is to automatically modify the constant power or constant current load during the NR iterations to add voltage dependence when the load's terminal voltage goes below a specified threshold, denoted here as minSVolt. The equation for this transformation is given in (3),

$$P_{L,k}(V_k) = \frac{1}{2} P_{Lbase,k}\left(1 - \cos(\pi \frac{V_k}{\text{minSVolt}})\right) \quad (3)$$

where $P_{Lbase,k}$ is the specified constant power load at Bus k, V_k is the bus's pu voltage magnitude, and $P_{L,k}$ is the modified constant power component of the load. The rationale for this function is that realistic loads cannot continue to be constant power for low voltages, with a cosine function used to ensure that the function has a smooth derivative. The shape of this function for a minSVolt of 0.7 is shown in **Figure 3**. A similar function is used to modify the constant power portion of the reactive load. In the ZIP load model the real and reactive constant current portion of the load can also be modified in a similar manner except replacing in (3) minSVolt with minIVolt; usually minIVolt has a lower value than minSVolt.
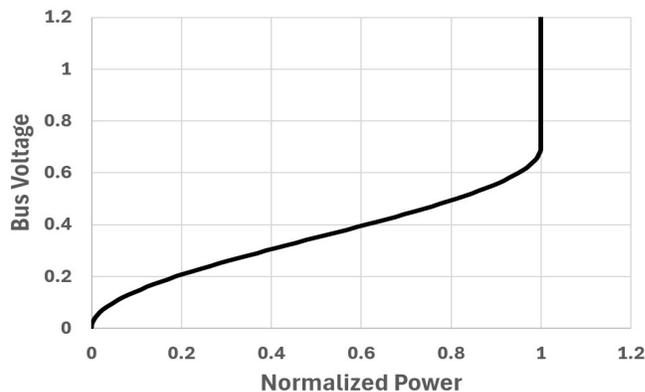

Figure 3: Voltage Variation in the Specified Constant Power Loads

As noted in [25] the use of this constant power load modification does not necessarily increase the size of the ROA. Rather, what is does do is to change the nature of the AltSs since now at each load bus k the short circuit condition (i.e., V_k of zero) corresponds to a valid PF AltS. Table 2 shows the modified three-bus case solutions using a minSVolt of 0.7 pu.

Now the short circuit condition (V=0) is a valid solution for the power balance equations, though it certainly does not correspond to an operable condition (unless the grid has a balanced three-phase fault).

Table 2: Modified Three-Bus Case Solutions with MinSVolt of 0.7 pu

Voltage	OprS (A)	AltS 1 (B)	AltS 2 (C)	AltS 3 (D)
V_1 (deg)	0.949	0	0.729	0
θ_1 (deg)	-18.43	0	-24.30	0
V_2 (pu)	0.949	0.729	0	0
θ_2 (deg)	-18.43	-24.30	0	0

The focus of this paper is to detect whether the PF has converged to an AltS, and if so, to attempt to correct it. For the three-bus case at this loading level the AltSs are readily apparent by the one or more low voltages shown in the table. Also, for such a small system there would not be a need to attempt to correct the solution since convergence to the OprS can be obtained just by resetting all the voltage magnitudes and angles to what is known as a flat start voltage guess. That is, setting all the voltage magnitudes to 1.0 pu and all the angles to zero. However, for larger systems the situation is more complex, and the flat start guess often does not work.

This is illustrated with the paper's second example case, the a 23,600 bus synthetic electric grid from [26], which has a geographic footprint covering the central portion of the US. Its oneline, along with a color contour of the per unit bus voltage magnitudes [27] at its OprS, is shown in **Figure 4**. In the figure the nominal voltage level for the transmission lines is indicated by the line's color with orange for 500 kV, red for 345 kV, blue for 230 kV, and black for lower levels; alpha-blending is used to highlight the highest nominal voltage lines [28]. The number of nominal voltages are shown in **Figure 5**. For the OprS the lowest voltage magnitude is 0.984 pu and the highest is 1.079 pu. This case is publicly available at the repository described in [29]. As is common in practice, the case has a minSVolt of 0.7 pu.
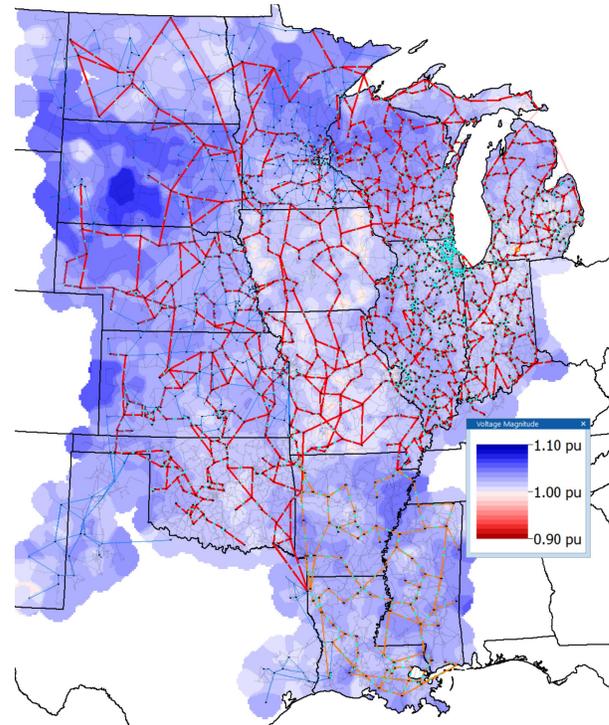

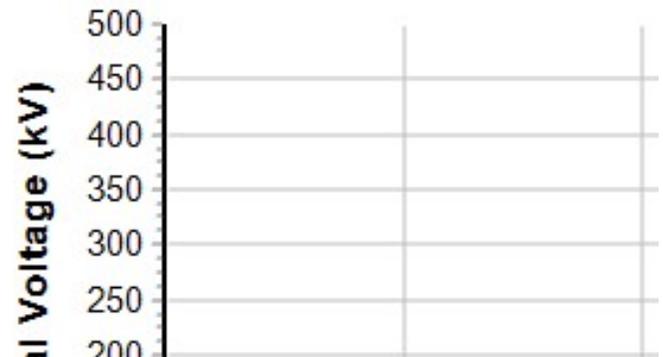Figure 4: 23,600-Bus Grid Voltage Magnitude Contour at the OprS

Figure 5: Nominal Voltage Levels used in the 23,600-Bus Example

At this loading level the grid is not close to maximum loadability so it has many AltSs. While usually the PF goal is usually not to converge to an AltS, to illustrate the paper's algorithm of detecting and correcting AltSs in the next section a large number of these AltS are calculated by sequentially solving the PF, initializing from the OprS except setting the voltage guess to $0.25\angle 0°$ at a single bus. As an example, such a voltage guess is used at bus 270047, which is in the eastern part of the US state of South Dakota; this bus has a nominal voltage level of 115 kV. At the AltS the lowest voltage magnitude is now 0.0 pu, with 16 buses having voltage magnitudes below 0.7 pu. **Figure 6** shows a zoomed view of the voltage contour in the region. Here the AltS region is roughly the sixteen buses with the low voltage magnitudes. Given that the AltS is created with a low initial voltage guess at a single bus, again using the notation of [5] this is a type-1 AltS.
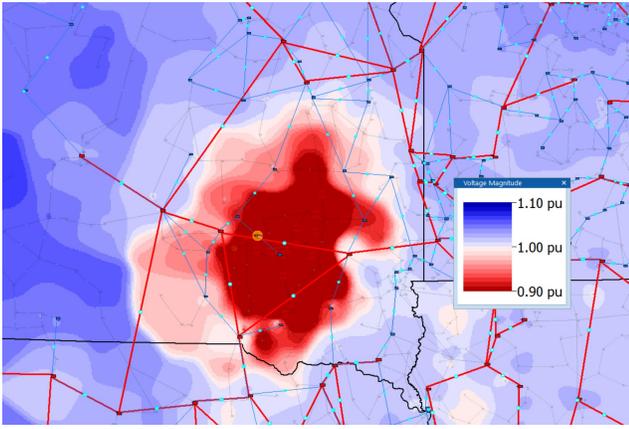
Figure 6: AltS Voltage Contour for a Low Voltage Guess at Bus 270047

However, there is nothing to preclude having low voltage guesses at multiple buses. This can result in multiple AltS regions and AltSs with types greater than 1. The voltage magnitude contour for such a situation is shown in **Figure 7**, in which a low voltage guess is again made at Bus 270047, and now also at Bus 291358 (161 kV nominal voltage), which is in the western part of the US state of Wisconsin. This results in a type-2 AltS, now with two AltS regions and voltage magnitudes below 0.7 pu at 26 buses and below 0.9 pu at 80 buses.
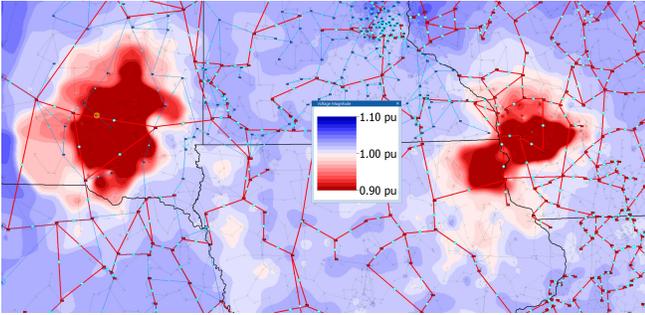

Figure 7: AltS Voltage Contour for Low Guesses at Buses 270047 and 291358

While in both these figures the voltages magnitudes are clearly low around the buses with the initial low voltage guesses, an important observation, leveraged in the next section, is in most of the remainder of the grid the voltages are often almost unchanged from their AltS values. Hence the focus of this paper is to present an algorithm that can detect and often correct AltSs of any type to the OprS.

Before presenting the algorithm, it is useful to briefly consider why the PF would converge to an AltS when the user is presumably not making such initial low voltage guesses. The short answer is because regardless of user intent, the voltage guess is not within the ROA for the new OprS. Unpacking this some, as noted in [25] because of the normal bus voltage angle variation across a large grid can be large, it is not unusual for a flat start voltage guess at a bus (e.g. $1\angle 0°$) to not within this ROA. So a potential situation for such a scenario is when a user is entering new buses and explicitly setting their voltages to flat start values. This could mean that even though the previous solution is at an OprS, the voltage guess for the modified system is no longer within the ROA for the new OprS. Thus when the

PF is solved the solution either diverges or converges to an AltS. Other scenarios could be when changes due to a contingency result in the current AltS (i.e., the voltage guess for the new PF) out of the ROA for the post-contingent OprS.

The final issue for this section is the consequences of converging to an AltS. Unless the PF algorithm has a mechanism to automatically detect AltSs, it is certainly possible for them to go unnoticed particularly for large grids. Whether this matters depends upon the location of the AltS region(s) relative to the footprint of interest. If they are remote then it might not matter, however it could matter a lot if they are close or within the footprint of interest. Then, if an AltS is detected but can't be corrected, there is a need to indicate an erroneous PF. The contribution of this paper is to present an algorithm for both quickly detecting and often correcting the AltS to the OprS.

III. QUICKLY DETECTING AND CORRECTING ALTS

Building on the background from the prior section, this section presents an algorithm that can be embedded in any NR PF to quickly detect an AltS, and if found, often to correct it to the OprS. The input to the algorithm is a solved PF, a correction iteration counter (k, initialized to zero), and a maximum iteration value, $k^{max} > 0$. The algorithm then consists of the five steps shown in Algorithm 1.

Algorithm 1 AltS Detection and Correction Procedure

1: **Step 1:** Set $k \leftarrow 0$
2: **Step 2:** Check whether the PF has converged to an AltS and identify the buses at which it occurs
3: **if** no AltS detected **or** $k = k^{max}$ **then**
4:     **if** $k = k^{max}$ **then**
5:         Indicate AltS convergence
6:     **end if**
7:     **stop**
8: **end if**
9: **Step 3:** Determine the AltS region(s) and the boundary buses adjacent to each region
10: **Step 4:** Modify the voltage magnitude and angle guesses at buses inside the AltS region(s); optionally reset PF controls, such as generators that solved at reactive power limits, back to voltage regulation
11: **Step 5:** Resolve the PF, increment k and return to Step 2

Given that in most applications the PF usually solves to the AltS, it is crucial that Step 2 (AltS Detection) be extremely computationally efficient. That is, the time needed to screen whether the PF has converged to an AltS must be small relative to the PF solution time. There are several methods for detecting likely AltSs. The first is to just check for abnormally large or small voltage magnitudes. This has the advantage of being fast and can certainly detect many AltSs, particularly when the common load modification approach of (3) is used resulting in solutions with voltage magnitudes of zero. Its key disadvantage is it can miss some AltSs that have voltages similar to normal values [30] and the ones that are missed are the ones that can most easily fool a user into thinking they are at the OprS.

The second approach is to calculate the largest eigenvalues of the power flow Jacobian. This approach has historically been

used, with power methods used to calculate the Jacobian's dominant eigenvalues. However, as noted in [31], this approach does not work well if the grid has negative reactive branches. While such branches were initially uncommon in early power grid models, they are now extremely common due to three-winding transformer equivalent circuits and sometimes series compensation. Therefore, this approach is not used here.

The third approach is to consider the sign of the dV_i/dQ_i sensitivities with reactive power assumed to be positive for generation. For an n-bus grid this values can be calculated by using sparse vector methods with computation on the order of $n^{0.45}$ [32], [33]. While this approach can also have issues with negative reactance branches, these issues can be managed using the techniques of [30]. This technique is used here, with the modification that only a small subset of the system buses are checked, with the subset consisting of those with the lowest and highest voltage magnitudes. The center of each AltS region is then indicated by the bus with a negative sensitivity value. The computational statistics given in the next section for the 23,600-bus grid indicate that this screening technique has sufficient computational efficiency so it could be routinely applied at the end of PF solutions.

Steps 3 and 4 are then associated with determining the AltS regions, and with modifying the voltage guesses for buses within those regions. The approach used here is relatively straightforward to describe, yet simultaneously provides impressive correction results. The approach to check the voltage magnitude for each bus indicated by the Step 2 screening as the center of an AltS region to determine whether it is below or above 1.0 pu. If below 1.0 pu, do a search out from this bus identifying all the buses that have voltages magnitudes below a threshold V^{low}; if it is above 1.0 pu find the buses with voltage magnitudes above a threshold V^{high}. These buses are then defined as an AltS correction region. Then identify all the boundary buses connected to this set, and calculate the average of their bus voltage angles. Then, set the voltage guesses for all buses in this AltS correction region to have a magnitude of 1.0 pu, and an angle equal to the boundary bus angle average. Finally, for all the generators within this AltS correction region, if they are at their reactive power limits reset them to be PV buses. For iterations beyond the first one (i.e., when $k > 0$) the solution would likely be restored to the original AltS. The next section demonstrates the effectiveness of this approach on the 23,600-bus system.

IV. LARGE GRID RESULTS AND DISCUSSION

This section presents the results of applying this algorithm to the previously mentioned 23,600-bus grid. For testing the grid is first solved to determine its AltS with its voltage contour shown in **Figure 4**. Next, sequentially for all the buses in the system, except for the slack bus and any three winding transformer star equivalent buses, an AltS solution is attempted by setting the voltage guess at the bus to $0.25\angle0°$. The NR PF is then solved; this is done for 23,222 buses here. If the PF converged, using the previous mentioned screening approach of calculating the dV_i/dQ_i for all buses with voltage magnitudes below 0.8 pu or above 1.15 pu, a total of 14,535 AltSs are found.

Using the Step 4 correction approach with V^{low} of 0.75 pu, and V^{high} of 1.25, 12,183 (83.8%) of these AltS are successfully restored to the OprS. For this example k^{max} is set to 1, with additional correction iterations considered at the end of this section.

Digging further into these results, as noted earlier key for being able to use this approach for routine PF solutions is that the screening to determine if the solution is an AltS is that the screening be fast relative to the time needed to solve the power flow. With the 23,600-bus case the average screening time is 0.042 seconds, compared to a typical PF solution time of about two seconds (a value that is, of course, contingency dependent). The average number of buses checked with the algorithm is 21, with a high of more than 1100!

As has been noted, the correction percentage for all the AltS is 83.8%. However, as might be expected this value depends upon the severity of the AltS, quantified based on the number of buses included in the Step 2 screening. For the simplest AltSs, defined as having five or fewer buses in the screening (occurring with 4467 AltSs [30.7% of the total]), the correction percentage is 96.2%, requiring an average of 9.7 iterations. In contrast, for the more severe AltSs, defined as having twenty or more buses in the screening (occurring with 4865 AltSs [33.5%]), the correction percentage drops to 66.7%, requiring an average of 20.9 iterations. When the number screening has more than 100 buses (at 449 AltSs [3.1%], the correction percentage is only 33.1%, requiring an average of 25.9 iterations. Still, some quite severe AltSs can be corrected as indicated by the bus 190209 AltS (in the US state of Minnesota) shown in **Figure 8**; here there are voltage magnitudes below 0.7 pu at 563 buses and below 0.9 pu at 1363 buses. The correction took 23 iterations.
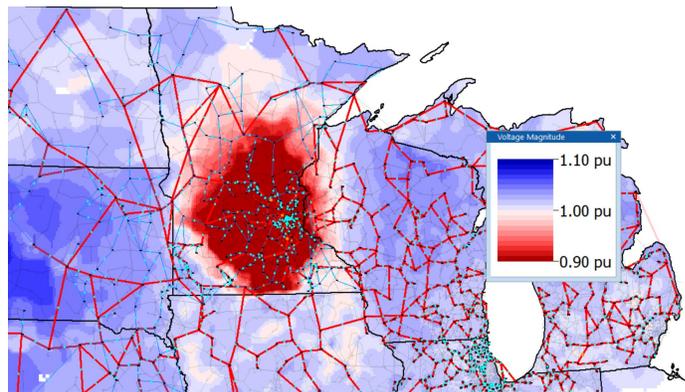

Figure 8: AltS Voltage Contour for Low Guesses at Buses 190209

Given that the previous results are done for k^{max} set to 1, the correction percentage could likely be increased by additional iterations using different correction approaches. The advantage of the paper's algorithm is that multiple approaches can be tried on the AltS, recognizing that only one needs to work and that provided the original AltS is stored its values can be reset when k is incremented. Insight into how the correction procedure could be modified can be gained by considering the above AltSs for which the previous approach failed. While full consideration is left for future work, one simple modification

would be to set k^{max} to 2 and then change V^{low} from 0.75 pu for the first iteration to 0.9 pu for the second. With this change the correction percentage for the 4467 AltSs having five or fewer screening buses increases from 96.2% to 99.4%.

V. CONCLUSIONS AND FUTURE WORK

The key contribution of this paper is providing an algorithm to quickly determine if a PF has solved to an AltS, and if it has, to provide an approach for attempting to correct it back to the OprS. Results using a 23,600-bus with about 14,500 AltS indicate that correction can be achieved about 83.8% of the time, with the best results for AltSs that have relatively small AltS region. Needed future work is to further test the algorithm on addition grids, and to develop additional approaches for correcting the alternative solution, with a goal of increasing the correction rate to as close to 100% percent as possible.

VI. ACKNOWLEDGEMENTS

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