Phase Shift Groups in Power Flow Analysis for Improved Power Systems Modeling

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***Abstract*—This letter presents an algorithm that efficiently and consistently represents, in the positive-sequence power flow, the phase shifts introduced by wye–delta-connected transformers. As is well known, wye–delta transformers impose fixed voltage and current phase shifts in multiples of 30 degrees. Because these shifts do not affect bus voltage magnitudes or branch power flows, they are frequently omitted or modeled inconsistently. However, non-uniform representation of transformer phase shifts can introduce artificial voltage-angle discontinuities that negatively impact Newton-based power flow convergence. To address this issue, the letter introduces the concept of Phase Shift Groups (PSGs), which enforce consistent voltage-angle representation across the power system model. The proposed PSG-aware initialization yields a physically consistent starting point that improves Newton-Raphson convergence robustness compared to conventional flat-start methods. Beyond initialization, PSGs provide a systematic mechanism for data-quality validation, flagging incomplete or inconsistent transformer connection data. Scalability is demonstrated on systems with up to 110,000 buses with negligible computational overhead.**

***Index Terms*—Power system analysis, transformer modeling, phase shift groups, power flow initialization.**

I. Introduction and Background

POSITIVE-SEQUENCE power flow analysis is fundamental to large-scale electric grid modeling, determining bus voltage magnitudes and angles that drive real and reactive power flow calculations. As grid models increasingly include wye-delta (Y–Δ) transformers, common at transmission-distribution interfaces and in generator step-up (GSU) applications, phase-shift modeling has become a critical concern. Y–Δ connections introduce fixed phase shifts in 30° multiples, which, while not affecting voltage magnitudes or power flows in steady-state analysis, create voltage angle inconsistencies when modeled non-uniformly across the network.

Current practice follows three approaches. First, explicit modeling uses the transformer phase-shift parameter [1], setting fixed angles (typically $\pm 30^\circ$) for some, but not all, Y–Δ transformers. Second, and historically the most common, is just to ignore them. For most applications, this is acceptable because these phase shifts do not change the solved voltage magnitudes or the solved power flows [2]. Third, vector group specification provides definitive phase relationship information [3], [4], increasingly required for geomagnetic disturbance (GMD) studies [5]–[7]. However, these approaches often coexist inconsistently within the same model, creating angle discontinuities across transformer boundaries.

This inconsistency causes practical problems. Non-uniform phase shift representation complicates power flow initialization, as a flat-start approach (initializing all angles to zero) may place the initial state outside the Newton-Raphson solver's region of attraction when Y–Δ transformers impose fixed angle offsets [8], [9]. Initializing a 30° phase difference to 0° creates massive artificial active power flows across low-impedance transformer branches, resulting in severe initial power mismatches. This can cause convergence issues, leading the solution to an infeasible one or even failure [10]. Furthermore, flat-start initialization provides no consistent phase reference, leaving phase-shift discrepancies to be reconciled manually, which is a time-consuming and error-prone process.

This letter introduces Phase Shift Groups (PSGs), a computationally efficient framework for identifying and managing sub-networks whose voltage angles are collectively displaced by transformer-induced phase shifts. Unlike existing approaches that treat phase shifts as isolated transformer parameters, PSGs recognize these displacements as a topological property of the network. The contribution is threefold: (i) a linear-time algorithm for automatic PSG identification, (ii) demonstration of improved power flow convergence through PSG-aware initialization, and (iii) validation on industry-scale systems up to 110,000 buses with negligible computational overhead. This framework provides a systematic foundation for consistent handling of phase shifts across power flow initialization and network data validation.

II. Algorithm

A PSG is defined as a set of buses whose voltage angles are uniformly offset due to one or more modeled Y–Δ transformer phase shifts, with the corresponding transformers forming the boundary of the group. Although many PSGs consist of a single bus bounded by a single transformer, larger and nested PSGs may arise in practice, particularly in systems with multiple voltage levels such as distribution networks. Proposed

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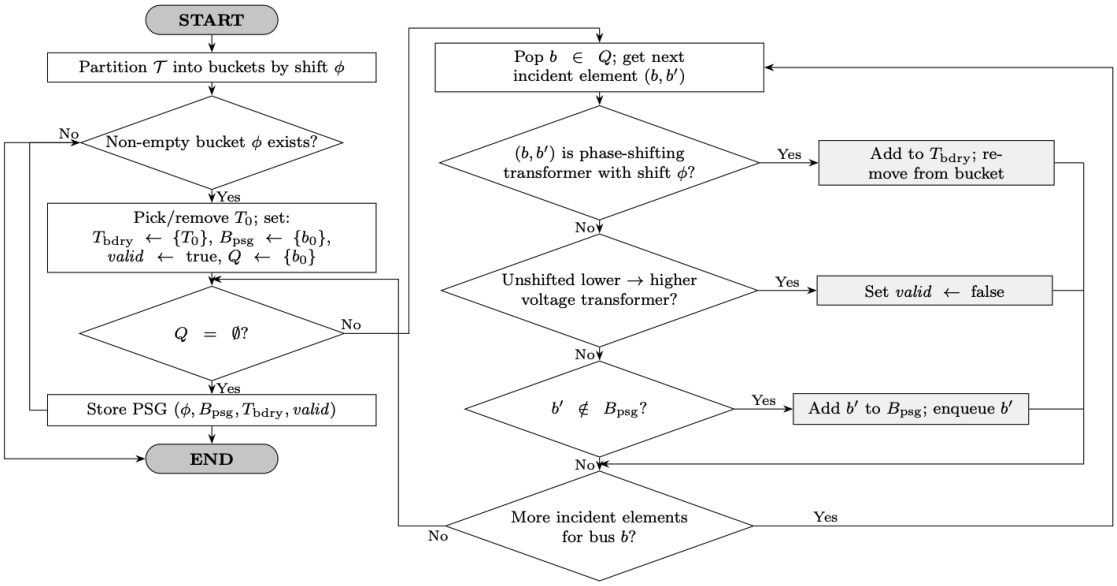

Fig. 1. PSG algorithm block-diagram

Algorithm 1, illustrated in Fig. 1, identifies PSGs by expanding from the low-side (star) bus of each phase-shifting transformer and grouping all buses connected through transformers with the same fixed nonzero phase displacement. Transformers that introduce phase shifts define PSG boundaries, while all buses reached during the expansion constitute the internal PSG. Inconsistent voltage-level transitions encountered during the process are flagged, indicating potentially incomplete or inconsistent transformer modeling. The resulting PSGs provide a structured representation of phase-shifted sub-networks that is later used to demonstrate the algorithm's behavior and scalability through representative case studies.

Algorithm 1 Phase Shift Group (PSG) Construction

Require: Network $(\mathcal{B}, \mathcal{E})$; transformers $\mathcal{T}$ with fixed shifts $\phi \in 30^\circ\mathbb{Z},\ \phi \neq 0$

Ensure: PSGs, each defined by internal buses B_{psg}, boundary transformers T_{bdry}, and flag *valid*

1: **Step 1 – Init.** Partition $\mathcal{T}$ into buckets by shift value ϕ.

2: **Step 2 – Seed.** For each non-empty bucket ϕ, pick and remove a transformer T_0, set $T_{\text{bdry}} \leftarrow \{T_0\}$, $B_{\text{psg}} \leftarrow \{b_0\}$, $valid \leftarrow$ true, and identify its low-side bus (or star bus for 3-winding transformers) as b_0.

3: **Step 3 – Flood-fill.** While $Q \neq \emptyset$, pop bus b and inspect each incident element (b, b'):

- *Phase-shifting transformer with shift ϕ:* add to T_{bdry} and remove from bucket.
- *Unshifted lower → higher voltage transformer:* set $valid \leftarrow$ false.
- *Otherwise:* if $b' \notin B_{\text{psg}}$, add b' to B_{psg} and enqueue.

4: **Step 4 – Store.** Save $(\phi, B_{\text{psg}}, T_{\text{bdry}}, valid)$ and repeat Steps 2-3 until all buckets are empty.

Because each transformer is removed from its bucket exactly once and each bus is added to the queue at most once, the total number of operations is proportional to the size of the network graph. Thus, the complexity of the procedure is $\mathcal{O}(|\mathcal{B}| + |\mathcal{E}|)$, where $|\mathcal{B}|$ is the number of buses and $|\mathcal{E}|$ is the number of branches. This linear-time complexity ensures scalability to large-scale transmission networks with thousands of buses and transformers, as the runtime grows approximately linearly with the system size [11].

Once PSGs are identified, the uniform phase displacement of each group can be applied directly to its bus voltage angles.

In the positive-sequence model, each Y-Δ transformer is represented by an off-nominal complex turns ratio whose angle equals its fixed shift $\phi \in 30^\circ\mathbb{Z}$, so the winding connection rotates the low-side voltage phasor by ϕ. Unlike the conventional flat start, the proposed PSG-aware initialization encodes these phase shifts directly in the initial state vector: each bus i in a PSG is assigned

$$\theta_i^{(0)} = \theta_{\text{slack}} + \phi_{\text{PSG}}, \tag{1}$$

where ϕ_{PSG} is the phase displacement of the group's boundary-transformer winding connections. This yields a physically consistent starting point. Because the branch active power transfer $P_{ij} \approx (V_i V_j / X)\sin(\theta_i - \theta_j)$ scales inversely with the small transformer reactance, a 30° offset left at zero would otherwise drive a massive artificial flow across the transformer branch [9]. Assignment (1) removes this contribution, reducing the initial mismatch and improving Newton-Raphson convergence, as demonstrated in Section III.

The same group displacement can also be removed to rotate a group back to a common reference, or applied to initialize newly added buses and branches.

III. Case Studies and Results

This section demonstrates how PSGs can be applied in practice, focusing on their role in power flow initialization. The initialization quality is assessed through the initial power

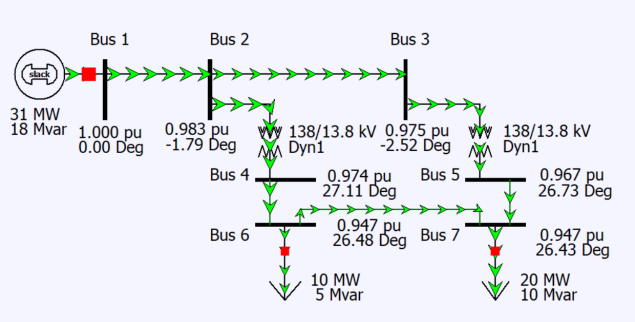

Fig. 2. 7-bus example with transformer-induced phase shifts.

mismatch $\|\Delta P^{(0)}\|_\infty$, which measures how far the starting point lies from the solution before any iteration begins. Four test cases of increasing scale are presented: a 7-bus illustrative example, the 2000-bus Texas synthetic grid, the 28,755-bus WECC and the 110,000-bus interconnection-scale model.

A. 7-bus case

An illustrative 7-bus example is shown in Fig. 2. The system contains two 138/13.8 kV D-Y-connected transformers (Dyn1), each introducing a fixed 30° phase displacement between the high- and low-voltage sides. Consequently, all buses electrically downstream of these transformers experience a uniform voltage-angle shift relative to the upstream transmission network. In this case, buses 4 through 7 form a single PSG, with their voltage angles collectively offset by approximately 30° from buses 1 through 3. The two D–Y transformers define the PSG boundary, separating the phase-shifted sub-network from the remainder of the system.

Because the flat start ignores these phase shifts, it can degrade convergence in systems with multiple wye–delta transformers [9], PSG-based adjustment of the initial group angles instead provides a physically consistent starting point.

In the 7-bus example, a flat start, where all bus angles are initialized to the slack-bus angle, is inconsistent with the transformer-induced phase shifts. This inconsistency can cause the Newton-Raphson solver to diverge, as illustrated in Fig. 3.

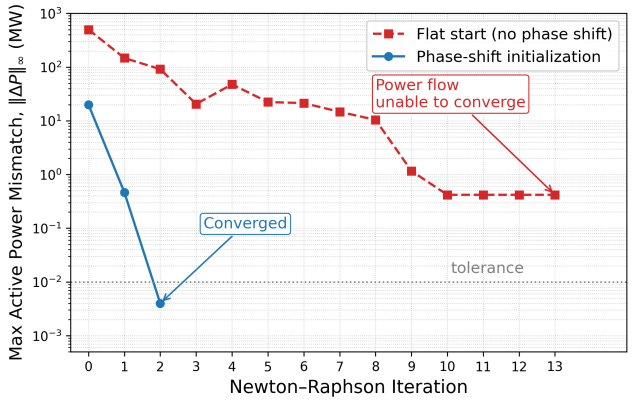

Fig. 3. Newton–Raphson power flow convergence comparison between flat start and phase-shift initialization for 7-bus case.

The quality of an initialization can be quantified by the initial power mismatch $\|\Delta P^{(0)}\|_\infty$, the largest active power residual across all buses at iteration 0. A smaller value indicates a starting point geometrically closer to the solution, which directly improves convergence robustness.

The initial active power mismatch under flat start ($\|\Delta P^{(0)}\|_\infty = 500$ MW) is 25 times larger than under phase-shift initialization ($\|\Delta P^{(0)}\|_\infty = 20$ MW), quantifying the distance between the starting point and the solution manifold and directly explaining the convergence failure observed.

B. Texas 2k - bus synthetic case

As a further validation, the 2000-bus Texas synthetic grid [12] is considered. In this network, six phase-shifting transformers were added, making initialization non-trivial. Fig. 4 compares the Newton-Raphson convergence behavior under flat start and phase-shift initialization.

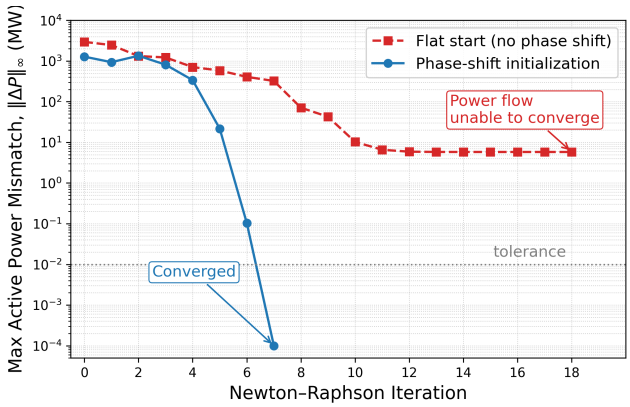

Fig. 4. Newton–Raphson power flow convergence comparison between flat start and phase-shift initialization for 2k-bus Texas synthetic case.

At iteration 0, the flat start yields $\|\Delta P^{(0)}\|_\infty = 2953$ MW compared to 1280 MW under phase-shift initialization, a 2.3 times reduction, resulting in convergence within 7 iterations, while flat start fails after 18.

C. Algorithm Scalability

To evaluate computational performance, the PSG identification was applied to two large-scale industry models: the Western Electricity Coordinating Council (WECC) system with 28,755 buses, and the combined North American Eastern and Western Interconnects with 110,000 buses [13]. All computations were performed on a desktop computer with an Intel Core i9-12900K processor (16 cores, 3.2 GHz) and 80 GB of RAM. The key metrics are summarized in Table I.

TABLE I
PSG IDENTIFICATION SCALABILITY RESULTS

Buses	WECC 28,755	Eastern + Western 110,000
Phase-shift transformers	696	1,950
PSGs identified	681	3,095
Invalid PSGs	1	51
Runtime	6 ms	30 ms

For the WECC model, all 681 PSGs were identified in 6 ms, with only 1 flagged as invalid. The 110,000-bus interconnection completed in 30 ms, which is a 3.8 increase in network size, yields only a 5 increase in runtime, confirming near-linear scaling. In both cases, invalid PSGs corresponded to incomplete transformer connection data in the original datasets, highlighting the algorithm's secondary utility for data quality validation [14].

Without the PSG framework, initialization of the 110,000-bus model required manual adjustment of phase angles across 1,950 transformer connections, taking hours of engineering time and prone to errors [15]. By contrast, the PSG-based approach eliminates the need for manual phase-angle adjustments while adding negligible runtime to the overall power flow computation.

IV. Conclusion

This letter introduced the Phase Shift Group (PSG) framework as a systematic and scalable approach for managing transformer-induced phase displacements in large-scale positive-sequence power flow models. Unlike common practices that ignore transformer phase shifts, embed them inconsistently through phase-angle parameters, or rely on partially available vector group data, the PSG framework explicitly identifies and organizes phase-shifted sub-networks as a topological property of the grid.

PSG-aware initialization places the Newton-Raphson solver in a physically consistent starting point, improving convergence robustness compared to conventional flat-start methods, which fail to converge in the presence of unmodeled transformer phase shifts. PSG identification executes in under 30 ms on systems with up to 110,000 buses, confirming near-linear scaling with negligible preprocessing overhead. Beyond initialization, PSGs provide a systematic mechanism for data-quality validation, flagging incomplete or inconsistent transformer connection data.

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